

# Testing Non-nested Demand Relations: Linear Expenditure System versus Indirect Addilog\*

Paul de Boer<sup>†</sup>  
*Econometric Institute*  
*Erasmus University Rotterdam*

Richard Paap  
*Econometric Institute*  
*Erasmus University Rotterdam*

ECONOMETRIC INSTITUTE REPORT EI 2009-07

## Abstract

In applied economic research computable general equilibrium [CGE] models in which the behavior of economic agents are modeled, are widely used. In many CGE models, the Linear Expenditure System [LES] is used to model behavior of the household sector. The disadvantage of LES is that the Engel curves, describing the relationship between expenditure on a certain commodity and total expenditure, are straight lines. Moreover, the LES does not allow for the existence of inferior commodities, elastic demand and gross substitution. An alternative model for the household block is the Indirect Addilog System [IAS] which is as simple to implement as LES, but which does not suffer from these theoretical deficiencies. Consequently, IAS provides a theoretically richer description of household behavior than LES, while it is as easy to implement.

In this paper we test the LES specification against the IAS specification in case one disposes of a budget survey. It is not possible to use a standard likelihood ratio test as both models are not nested. We propose to use the likelihood ratio test for non-nested hypotheses due to VUONG (1989), or, alternatively, the distribution-free test due to CLARKE (2007). We apply both tests to the Palestinian Expenditure and Consumption Survey (PECS, 2005) and find that there is overwhelming evidence that the IAS specification is to be preferred to the LES specification.

**Keywords:** models of household behavior, non-nested hypothesis testing.

---

\*Calculations in this paper are done in Gauss 6.0. Gauss programs are available upon request. This paper is dedicated to Professor W.H. Somermeyer, Director of the Econometric Institute from 1966 until his untimely death on 31 May 1982. Wim Somermeyer was friend and guide to author Paul de Boer, as well as supervisor of his Ph.D. Thesis.

<sup>†</sup>Corresponding author: Paul de Boer, Econometric Institute, Erasmus University Rotterdam, P.O. Box 1738, 3000 DR Rotterdam, The Netherlands, e-mail: [pmdeboer@ese.eur.nl](mailto:pmdeboer@ese.eur.nl), phone: +31-10-4081271, fax: +31-10-4089162.

# 1 Introduction

In applied economic research computable general equilibrium [CGE] models are widely used. In these models, the behavior of several economic actors (for example, firms, households, government, rest of the world) is modeled in blocks, the links between these blocks are modeled, as well.

The CPB Netherlands Bureau for Economic Policy Analysis uses a recursively dynamic CGE model for the world economy, named WorldScan. For a technical description of this model we refer to LEJOUR *et al.* (2006)<sup>1</sup>. The model is used both as a tool to construct long-term scenarios and as an instrument for policy assessments. Recently, it has been used in the fields of economic integration (DE BRUIJN *et al.*, 2008 and LEJOUR *et al.*, 2008) and climate change (VEENENDAAL and MANDERS, 2008). For a description of the various models used by the CPB Netherlands Bureau of Economic Policy Research we refer to DON and VERBRUGGEN (2006).

In this paper we focus on one of these blocks, the household model. We quote LEJOUR *et al.* (2006, page 65):

“On the basis of the preferences consumers decide how to spend their budget on consumer goods and services. The Linear Expenditure System [LES] is suitable to model this consumption decision, because it combines simplicity with some flexibility. (. . . . .). The modeling of consumer choice is also important as it enables explicit welfare analyses”.

It follows that in WorldScan LES is adopted for modeling the household block. The explicit welfare indexes that are commonly adopted to measure the impact of changes in economic policy are the so-called “equivalent variation” and/or “compensating variation” (VARIAN, 1992). The outcome of these indexes depends on the choice of the household model.

Besides WorldScan, other well-known CGE models use LES specifications, as well. MIRAGE, the CGE model of CEPII (Centre d'Études Prospectives et d'Informations Internationales) uses a combination of the Constant Elasticities of Substitution [CES] functional form (ARROW *et al.*, 1961) combined with LES (BCHIR, 2002, p. 47). Linkage, a CGE model of the World Bank, uses as default the LES augmented with savings (VAN DER MENSBRUGGHE, 2005, p. 21). GTAP (the Global Trade Analysis Project of the Purdue University), besides LES, allows for a generalization of LES, the so-called AIDADS (CRANFIELD *et al.*, 2000; REIMER and HERTEL, 2004). This model, an implicit additive demand system, is due to RIMMER and POWELL (1996). VAN DER MENSBRUGGHE (2005) discusses this model in his Appendix G. AIDADS allows for a richer description of Engel curves (the relationship between expenditure on a certain commodity, good or service, and total expenditure) than LES, but “comes at the expense of an additional  $(J - 1)$  parameters” (RIMMER and POWELL, 1996, p. 1615). In many practical applications, this prevents AIDADS from being used<sup>2</sup>.

The disadvantage of LES is that the Engel curves are straight lines. Moreover, the LES does not allow for the existence of inferior commodities (income elasticities smaller than zero), elastic demand (absolute value of the own price elasticity larger than one) and gross substitution (negative cross price elasticity), see CHUNG (1994, Chapter 2). An alternative model for the household block is the Indirect Addilog System [IAS] which is as simple to implement as LES, but which exhibits non-linear Engel curves and allows for the existence of inferior commodities, elastic demand and gross substitution. Consequently, IAS provides for a theoretically richer description of household behavior than LES, while it is as easy to implement.

The purpose of this paper is to test the LES specification against the IAS specification when both models are estimated using budget survey data. As both models are not nested, it is not possible to use a standard likelihood ratio test. To compare both specifications we first express the log-likelihood functions of both models in terms of a density for the

demands for the commodities. Next, we propose to use the likelihood ratio test for non-nested hypotheses of VUONG (1989), or, alternatively, the distribution-free test due to CLARKE (2007) for a formal statistical comparison.

The outline of this paper is as follows. In Section 2 we present the economic theoretical background of these two systems of demand relations, Section 3 deals with parameter estimation and the derivation of the test statistics using the approach of Vuong and of Clarke. As this dataset is readily available, we apply in Section 4 both tests to the Palestinian Expenditure and Consumption Survey (PECS, 2005). It turns out that there is overwhelming evidence for IAS specification over the LES specification. Section 5, finally, contains some concluding remarks.

## 2 The Two Systems of Demand Relations

In an economy there are  $J$  commodities with prices  $p_j$  ( $j = 1, \dots, J$ ) which are given for a household. Let  $p$  denote the (row) vector with typical element  $p_j$ . Given its budget  $m$  (income minus savings) the household has to decide which quantity  $y_j$  to purchase from each commodity  $j$ . Let  $y$  denote the (row) vector with typical element  $y_j$  ( $j = 1, \dots, J$ ). The decision is based on the maximization of its *direct utility function*  $U(y) = f(y)$ . Let the solution of this constrained optimization problem be denoted by  $y_j = g_j(p, m)$ . Substitution of this solution into the utility function yields the so-called indirect utility function  $V(p, m) = f(g_1(p, m), \dots, g_J(p, m))$ . Applying the implicit function theorem to the *indirect utility function* (in the framework of the theory of household demand called *Roy's identity*) yields the demand for commodity  $j$ :

$$x_j(p, m) = -\frac{\partial V(p, m)/\partial p_j}{\partial V(p, m)/\partial m}. \quad (1)$$

## 2.1 The linear expenditure system [LES]

TINBERGEN (1942) proposed to generalize the Cobb-Douglas production function (COBB and DOUGLAS, 1928) by introducing positive minimum amounts of capital and labor. Shortly after the 2nd World War this idea was introduced in the theory of household behavior in a series of articles: KLEIN and RUBIN (1948-1949), SAMUELSON (1948), GEARY (1949-1950) and STONE (1954). This function is known as the Stone-Geary utility function and the ensuing demand model as the Linear Expenditure System [LES]. It is based on the maximization of the direct utility function:

$$U(y) = \begin{cases} \prod_{j=1}^J (y_j - \mu_j)^{\alpha_j} & \text{if } y_j > \mu_j \\ 0 & \text{if } y_j \leq \mu_j \end{cases} \quad (2)$$

with parameters:  $\alpha_j > 0$  (called *marginal budget shares*), and  $\mu_j \geq 0$  (*subsistence quantities*).

Since utility is ordinal, any monotonous transformation of the utility function is a utility function as well. Therefore, without any loss of generality we impose the restriction:

$$\sum_{j=1}^J \alpha_j = 1. \quad (3)$$

The household maximizes its utility function (2) subject to its budget constraint:

$$\sum_{j=1}^J p_j y_j = m. \quad (4)$$

The demand relations easily follow and read:

$$y_j = \mu_j + \alpha_j p_j^{-1} \left( m - \sum_{k=1}^J p_k \mu_k \right) \quad (5)$$

so that *Engel curve*, the relationship between expenditure on commodity  $j$  ( $p_j y_j$ ) and its budget  $m$ , is a linear function:

$$p_j y_j = p_j \mu_j + \alpha_j \left( m - \sum_{k=1}^J p_k \mu_k \right). \quad (6)$$

Since  $\mu_k$  is interpreted to be the minimum quantity demanded of commodity  $k$ ,  $\sum_{k=1}^J p_k \mu_k$  represents the subsistence expenditure of the household and  $(m - \sum_{k=1}^J p_k \mu_k)$  is called its *supernumerary* or *discretionary* expenditure. According to this model, the household purchases first the minimum quantities of each commodity, and secondly, allocates its discretionary expenditure in fixed fractions over the commodities.

Defining the budget shares:

$$w_j = \frac{p_j y_j}{m} \text{ for } j = 1, \dots, J \quad (7)$$

the *income elasticity* (which measures the percentage change in the demand for a commodity  $y_j$  given a one percent change in the household's budget  $m$ ) is easily derived from (5):

$$E(y_j, m) = \frac{\partial \ln y_j}{\partial \ln m} = \frac{\alpha_j}{w_j} \text{ for } j = 1, \dots, J. \quad (8)$$

We assume that each household in the budget survey faces the same utility function. Moreover, prices are not recorded, so that it is assumed that all households face the same price. Without loss of generality all prices are put equal to one. Introducing the index  $i$  to denote the respondent ( $i = 1, \dots, N$ , where  $N$  denotes the number of respondents), and an additive disturbance  $\varepsilon_{ij}$ , the LES (5) boils down to:

$$y_{ij} = \mu_j + \alpha_j \left( m_i - \sum_{k=1}^J \mu_k \right) + \varepsilon_{ij} = \gamma_j + \alpha_j m_i + \varepsilon_{ij}, \quad (9)$$

where

$$\gamma_j = \mu_j - \alpha_j \sum_{k=1}^J \mu_k. \quad (10)$$

Because of the adding up restriction (3), it follows from (10) that  $\sum_{j=1}^J \gamma_j = 0$ . Consequently, the parameters  $\mu_j$  are not identified (in Appendix A we shortly describe how they are usually identified in CGE-modeling). Moreover, summation of (9) over  $j$  implies, taking account of (3) and (4) (with all prices put equal to one), that  $\sum_{j=1}^J \varepsilon_{ij} = 0$  for

all  $i$ . Consequently, the covariance matrix of the disturbances is singular. This problem is solved by deleting an arbitrary demand relation from the system, without loss of generality the first one.

## 2.2 The indirect addilog system [IAS]

The IAS has been introduced by LESER (1941) and, independently, by SOMERMEYER and WIT (1956) by directly specifying the functional form. HOUTHAKKER (1960) derived the system by applying Roy's identity (1) to the indirect addilog utility function<sup>3</sup>:

$$v(p, m) = \sum_{j=1}^J c_j \frac{(m/p_j)^{\beta_j} - 1}{\beta_j}. \quad (11)$$

The budget share equations<sup>4</sup> of IAS are:

$$w_j = \frac{c_j (m/p_j)^{\beta_j}}{\sum_{k=1}^J c_k (m/p_k)^{\beta_k}}. \quad (12)$$

In the literature there is confusion about the restrictions to be imposed on the parameters<sup>5</sup>. MURTY (1982), without proof, gives the correct restrictions:

$$c_j \geq 0 \text{ and } \beta_j \geq -1 \quad (13)$$

for all  $j$ , the equality holding for at most  $J - 1$  commodities in the first case and at most for one commodity in the second case. The proof has been supplied by DE BOER *et al.* (2006).

The parameters  $c_j$ , called “*preference coefficients*”, are indeterminate, that is to say: if we multiply each of them by the same factor, the equations (12) do not change. Therefore, we impose the identifying restriction that the preference coefficients sum up to one:

$$\sum_{j=1}^J c_j = 1. \quad (14)$$

From (12) we derive the Engel curve:

$$p_j y_j = \left( \frac{c_j (m/p_j)^{\beta_j}}{\sum_{k=1}^J c_k (m/p_k)^{\beta_k}} \right) \times m. \quad (15)$$

Since the first term in (15) is a non-linear function in  $m$ , the Engel curve is *non-linear*. It can easily be shown (SOMERMEYER and LANGHOUT, 1972) that the income elasticities are:

$$E(y_j, m) = \frac{\partial \ln y_j}{\partial \ln m} = 1 + \beta_j - \sum_{k=1}^J w_k \beta_k. \quad (16)$$

Like we did for LES, we put all prices equal to one and introduce the index  $i$  to denote the respondent, ( $i = 1, \dots, N$ ). Then, IAS (12) boils down to:

$$w_{ij} = \frac{y_{ij}}{\sum_{k=1}^J y_{ik}} = \frac{c_j m_i^{\beta_j}}{\sum_{k=1}^J c_k m_i^{\beta_k}}. \quad (17)$$

Selecting a reference commodity, without loss of generality commodity 1, it easily follows, after introducing an additive disturbance, that:

$$\tilde{y}_{ij} = \ln \left( \frac{y_{ij}}{y_{i1}} \right) = \delta_j + (\beta_j - \beta_1) \ln m_i + \varepsilon_{ij} \text{ for } j = 2, \dots, J \quad (18)$$

with

$$\delta_j = \log c_j - \log c_1. \quad (19)$$

By taking the logarithm of the ratio of the demand for a commodity  $j$  ( $= 2, \dots, J$ ) and the demand for the reference commodity 1 we circumvent the singularity problem of the covariance matrix of the disturbances. It follows from (18) and (19) that the reaction coefficients  $\beta_j$  and the preference coefficients  $c_j$  are not identified (in Appendix A we propose a method of identification of the parameters) .

Having estimated the *differences* of the parameters of interest, we obtain the estimates of the income elasticities by rewriting (16) to:

$$E(y_j, m) = 1 + \beta_j - \sum_{k=1}^J w_k \beta_k = 1 + (\beta_j - \beta_1) - \sum_{k=1}^J w_k (\beta_k - \beta_1). \quad (20)$$

### 3 Parameter Estimation and Testing

In Section 3.1 we discuss estimation of the parameters of the IAS and LES specification. The tests to compare both specification are discussed in Section 3.2

#### 3.1 Parameter Estimation

To estimate the parameters of the LES we consider the multivariate regression model (9) written in vector notation

$$y_i = \Pi x_i + \varepsilon_i \quad (21)$$

for  $i = 1, \dots, N$ , where  $y_i$  is a  $(J - 1)$ -dimensional vector containing the demands for the last  $J - 1$  commodities  $(y_{i2}, \dots, y_{iJ})$  and where  $x_i$  is 2-dimensional vector containing an intercept and the budget restriction  $m_i$ . The  $(J - 1) \times 2$  matrix  $\Pi$  contains the  $J - 1$  intercept parameters and the  $J - 1$  marginal budget shares. We assume that the  $J - 1$  dimensional error term  $\varepsilon_i = (\varepsilon_{i2}, \dots, \varepsilon_{iJ})'$  is normal distributed with mean zero and  $(J - 1) \times (J - 1)$  dimensional covariance matrix  $\Sigma$ .

The log-likelihood function belonging to this multivariate regression model is given by

$$\begin{aligned} \ell_{\text{LES}}(y|\Pi, \Sigma) &= \sum_{i=1}^N \ln f_{\text{LES}}(y_i|\Pi, \Sigma) \\ &= \sum_{i=1}^N \left( -\frac{J-1}{2} \ln 2\pi - \frac{1}{2} \ln |\Sigma| - \frac{1}{2} (y_i - \Pi x_i)' \Sigma^{-1} (y_i - \Pi x_i) \right). \end{aligned} \quad (22)$$

As the multivariate regression model contains the same regressors in all equations, the ML estimates of the parameters in  $\Pi$  are equal to the ordinary least squares [OLS] estimates in (9). The ML estimate of  $\Sigma$  is equal to  $\frac{1}{N} \sum_{i=1}^N \hat{\varepsilon}_i \hat{\varepsilon}_i'$ , where  $\hat{\varepsilon}_i$  denotes the vector of OLS residuals.

To estimate the parameters of the IAS we consider the multivariate regression model (18) in vector notation

$$\tilde{y}_i = \tilde{\Pi} \tilde{x}_i + \tilde{\varepsilon}_i \quad (23)$$

for  $i = 1, \dots, N$  with  $\tilde{\varepsilon}_i \sim N(0, \tilde{\Sigma})$ , where  $\tilde{y}_i$  is a  $(J-1)$ -dimensional vector containing the log demands of the last  $J-1$  commodities in deviation from the log demand of commodity 1 ( $\ln(y_{i2}) - \ln(y_{i1}), \dots, \ln(y_{iJ}) - \ln(y_{i1})$ ). The vector  $\tilde{x}_i$  contains an intercept and the log budget restriction  $\ln m_i$ . The  $(J-1) \times 2$  matrix  $\tilde{\Pi}$  contains the  $J-1$  intercept parameters and the  $J-1$  differences of reaction coefficients  $(\beta_j - \beta_1)$ .  $\tilde{\Sigma}$  is the  $(J-1) \times (J-1)$  dimensional covariance matrix of the  $J-1$  dimensional error term  $\tilde{\varepsilon}_i = (\tilde{\varepsilon}_{i2}, \dots, \tilde{\varepsilon}_{iJ})'$ .

The log-likelihood function belonging to this multivariate regression model is given by

$$\begin{aligned} \ell_{\text{IAS}}(\tilde{y}|\tilde{\Pi}, \tilde{\Sigma}) &= \sum_{i=1}^N \ln f_{\text{IAS}}(\tilde{y}_i|\tilde{\Pi}, \tilde{\Sigma}) \\ &= \sum_{i=1}^N \left( -\frac{J-1}{2} \ln 2\pi - \frac{1}{2} \ln |\tilde{\Sigma}| - \frac{1}{2} (\tilde{y}_i - \tilde{\Pi}\tilde{x}_i)' \tilde{\Sigma}^{-1} (\tilde{y}_i - \tilde{\Pi}\tilde{x}_i) \right). \end{aligned} \quad (24)$$

The ML estimate of  $\tilde{\Pi}$  contains the OLS estimates in (18) and  $\tilde{\Sigma}$  can be estimated using the OLS residuals in a similar way as for the LES specification.

It is not possible to compare the values of the log-likelihood functions in a direct way as for the LES specification the likelihood function is expressed in terms of the density of  $y$ , while for the IAS specification we consider the density of  $\tilde{y}$ . To make the likelihood function of the LES and IAS comparable we consider the log-likelihood function of the IAS as a density of  $y$ , that is,  $\ell_{\text{IAS}}(y|\tilde{\Pi}, \tilde{\Sigma}) = \sum_{i=1}^N \ln f_{\text{IAS}}(y_i|\tilde{\Pi}, \tilde{\Sigma})$ . The log-likelihood contributions of this likelihood are given by

$$\ln f_{\text{IAS}}(y_i|\tilde{\Pi}, \tilde{\Sigma}) = \ln f_{\text{IAS}}(\tilde{y}_i|\tilde{\Pi}, \tilde{\Sigma}) + \ln \left| \frac{\partial \tilde{y}_i}{\partial y_i} \right|, \quad (25)$$

where the last term is the log of the Jacobian of the transformation from  $\tilde{y}_i$  to  $y_i$ . This Jacobian equals

$$\left| \frac{\partial \tilde{y}_i}{\partial y_i} \right| = \left| \frac{\sum_{j=1}^J y_i}{\prod_{j=1}^J y_i} \right|, \quad (26)$$

see Appendix B for the derivation.

## 3.2 Testing

Although we now can compare the likelihood values of both specifications, it is not possible to test the LES specification against the IAS specification using a standard likelihood ratio test as both models are not nested. To test both specifications against each other we use the approach of VUONG (1989) and CLARKE (2007).

VUONG (1989) considers the null hypothesis

$$H_0 : E_0 \left[ \ln \left( \frac{f_{\text{IAS}}(y_i | \tilde{\Pi}, \tilde{\Sigma})}{f_{\text{LES}}(y_i | \Pi, \Sigma)} \right) \right] = 0, \quad (27)$$

which corresponds to the hypothesis that two models are equally close to the true specification. The test statistic boils down to

$$\frac{\ell_{\text{IAS}}(y | \hat{\Pi}, \hat{\Sigma}) - \ell_{\text{LES}}(y | \hat{\Pi}, \hat{\Sigma})}{\sqrt{N} \hat{\omega}_N}, \quad (28)$$

where  $\hat{\Pi}, \hat{\Sigma}$  and  $\tilde{\Pi}, \tilde{\Sigma}$  are the maximum likelihood estimates of the parameters of the LES and IAS, respectively, and where

$$\hat{\omega}_N^2 = \frac{1}{N} \sum_{i=1}^N \ln \left( \frac{f_{\text{IAS}}(y_i | \hat{\Pi}, \hat{\Sigma})}{f_{\text{LES}}(y_i | \hat{\Pi}, \hat{\Sigma})} \right)^2 - \left( \frac{1}{N} \sum_{i=1}^N \ln \left( \frac{f_{\text{IAS}}(y_i | \hat{\Pi}, \hat{\Sigma})}{f_{\text{LES}}(y_i | \hat{\Pi}, \hat{\Sigma})} \right) \right)^2. \quad (29)$$

The test statistic is asymptotically standard normal distributed under the null hypothesis. Note that both models have the same number of parameters and hence no degrees of freedom correction is necessary.

CLARKE (2007) proposes a distribution-free test and considers the null hypothesis

$$H_0 : \Pr_0 \left[ \ln \left( \frac{f_{\text{IAS}}(y_i | \tilde{\Pi}, \tilde{\Sigma})}{f_{\text{LES}}(y_i | \Pi, \Sigma)} \right) > 0 \right] = 0.5. \quad (30)$$

The test statistics is simply the number of times that  $\ln f_{\text{IAS}}(y_i | \tilde{\Pi}, \tilde{\Sigma})$  is larger than  $\ln f_{\text{LES}}(y_i | \hat{\Pi}, \hat{\Sigma})$ . The test statistics is under the null hypothesis  $\text{Bin}(N, 0.5)$  distributed.

Note that to compare the log-likelihood functions of both models we have expressed the density of the IAS in levels  $y$  using the Jacobian transformation. Another possibility is to express the log-likelihood function of the LES in terms of  $\tilde{y}$ . This does however not lead to a different value of the test statistics as  $\ln f_{\text{LES}}(\tilde{y}_i|\Pi, \Sigma) = \ln f_{\text{LES}}(y_i|\Pi, \Sigma) + \ln |\partial y_i/\partial \tilde{y}_i|$  and  $\ln |\partial y_i/\partial \tilde{y}_i| = -\ln |\partial \tilde{y}_i/\partial y_i|$ . Furthermore, given the fact that value of the maximum likelihood and the Jacobian of the transformation is independent of the chosen reference commodity the values of the test statistics also do not depend on the choice of the reference commodity.

In the next section we will illustrate the use of the Vuong and Clarke tests on the Palestinian Expenditure and Consumption Survey (PECS, 2005).

## 4 Application

To illustrate the use of the VUONG (1989) and CLARKE (2007) test we estimate demand relations for the Palestinian Expenditure and Consumption Survey (PECS, 2005). We consider 10 sectors, that is, expenditures on food, beverages, clothing, housing, furniture, recreation, education, transport, medical expenditures, and a miscellaneous category. We have observations on total expenditures and expenditures in the 10 sectors for  $N = 2152$  individuals.

We estimate the parameters of a linear expenditure system and an indirect addilog system following the approach in Section 3. The average log-likelihood contribution of the LES is  $-65.364$ , while for the IAS we obtain  $-52.567$  (in levels). The values of the log-likelihood functions of both models suggest that the IAS is better than the LES. To analyze whether this difference is statistically significant, we consider the Vuong and Clarke tests discussed in Section 3.2. The value of the Vuong test statistic equals 31.033 and hence we reject the null that both specifications are equally close to the true specification versus the alternative that IAS is closer ( $p$ -value is 0.000). If we apply the Clarke test we find that in

Table 1: Parameter estimates and elasticities with estimated standard error for the indirect addilog system

Commodity	Intercept		$\beta_j - \beta_1$		Elasticity <sup>a</sup>	
	Estimate	s.e.	Estimate	s.e.	Estimate	s.e.
Food	–	–	0	–	0.493	0.018
Beverages	–9.072	0.514	0.819	0.063	1.312	0.064
Clothing	–18.907	0.926	1.920	0.114	2.413	0.113
Housing	1.506	0.177	–0.219	0.022	0.274	0.025
Furniture	–9.113	0.447	0.834	0.055	1.327	0.054
Medical	–11.359	1.081	0.808	0.133	1.301	0.133
Transport	–11.233	0.461	1.199	0.057	1.692	0.055
Recreation	–20.892	0.973	1.941	0.120	2.434	0.117
Education	–18.736	1.004	1.667	0.124	2.160	0.124
Miscellaneous	–8.914	0.508	0.800	0.063	1.293	0.061

<sup>a</sup>Elasticities are given in (20), where  $w_j$  is set equal to the total budget share in the sample.

91% of the cases the log-likelihood contributions of the IAS specification are larger than the log-likelihood contributions of the LES specification ( $p$ -value based on a Binomial distribution with  $N = 2152$  and  $p = 0.5$  is 0.000). Hence, also the Clarke test indicates that the IAS is significantly closer to the true specification.

Table 1 provides the parameter estimates and the estimated elasticities of the indirect addilog system. The elasticities are computed using (20), where  $w_j$  is set equal to the total budget share of the commodities in the sample. It follows that the expenditure elasticities of food and of housing are lower than one, which means that these are necessary commodities. This confirms the famous law of ENGEL (1857) that food is necessary and the less known law of SCHWABE (1868) that housing is a necessary commodity, as well. The other major groups turn out to be luxury commodities, since their expenditure elasticity is larger than one.

## 5 Conclusion

In this paper we have considered the LES specification and the IAS specification to model the behavior of the household sector. The parameters of both specification are estimated using budget survey information. As both specifications are non-nested we have proposed to use the likelihood-ratio based tests of VUONG (1989) and CLARKE (2007) for non-nested hypotheses. To illustrate our approach we have applied both tests to the Palestinian Expenditure and Consumption Survey (PECS, 2005) and find that there is overwhelming evidence for the IAS specification over the LES specification.

Our test statistic is based on the normality assumption of the disturbances in the multivariate regression equations which are needed to estimate the LES and IAS specification. One can easily relax this assumption by assuming a finite mixture of multivariate normals for the distribution of the disturbances. It is well known that this mixture specification can approximate many types of multivariate distributions. The mixture of multivariate normals specification can also be used to account for potential heteroskedasticity in the disturbances across individuals. We consider these extensions a topic for future research.

## A Calibration of Parameters

### A.1 Linear expenditure system

The minimum subsistence levels  $\mu_j$  are not identified. In CGE-modeling one usually assigns a value to the expenditure elasticity of the marginal utility of income ( $\lambda$ ), the so-called Frisch parameter  $\varphi_m$ . In the framework of LES, it is equal to minus the inverse of the fraction of supernumerary expenditure in total expenditure (BLONIGEN *et al.*, 1997):

$$\varphi_m = \frac{\partial \log \lambda}{\partial \log m} = -\frac{m}{\left(m - \sum_{k=1}^J p_k \mu_k\right)}. \quad (31)$$

Substituting this expression in (5), putting the price equal to 1, we arrive, after rewriting, at:

$$\mu_j = y_j + \alpha_j m^{-1}, \quad (32)$$

where  $y_j$  is the average expenditure on commodity  $j$  in the budget survey, and  $m$  denotes the average total expenditure in the budget survey.

To clarify the procedure, suppose that a consumer commits 20% of its expenditure to the purchase of its subsistence expenditure. Then, its Frisch parameter is equal to  $-1/0.80 = -1.25$ .

### A.2 Indirect addilog system

The reaction coefficients  $\beta_j$  are not identified. One might, like in the case of the LES assign a value to the Frisch parameter<sup>6</sup>. However, in the framework of IAS it does not have a clear link to an economic concept as in case of LES. Alternatively, one may fix a value of one of the price elasticities. Suppose for instance that it is reasonable to fix the own price elasticity of the reference commodity food at  $-0.8$ . This means that if the price of food increases by one percent the demand for food decreases by 0.8 percent. It can be derived from (12) that the own price elasticity of food is equal to:

$$E(x_1, p_1) = -(1 - w_1)\beta_1 - 1, \quad (33)$$

see SOMERMEYER and LANGHOUT (1972).

Since  $w_1$ , the average budget share of food, is known from the survey, fixing the own price elasticity at  $-0.8$  yields the calibrated value of  $\beta_1$ . The calibrated value of the other reaction coefficients follow from Table 1. Putting prices equal to one, taking the identifying restriction (14) into account, we can use (12) to calibrate the preference coefficients  $c_j$ :

$$c_j = \frac{w_j m^{-\beta_j}}{\sum_{k=1}^J w_k m^{-\beta_k}}. \quad (34)$$

## B Derivation of the Jacobian

To derive the Jacobian we use that  $\ln(\tilde{y}_{ij}) = \ln(y_{ij}) - \ln(m_i - \sum_{j=1}^J y_{ij})$ . Hence,

$$\left| \frac{\partial \tilde{y}_i}{\partial y_i} \right| = \left| \begin{pmatrix} 1/y_{i2} + 1/y_{i1} & 1/y_{i1} & \cdots & 1/y_{i1} & 1/y_{i1} \\ 1/y_{i1} & 1/y_{i3} + 1/y_{i1} & 1/y_{i1} & \cdots & 1/y_{i1} \\ \vdots & 1/y_{i1} & \ddots & 1/y_{i1} & \vdots \\ 1/y_{i1} & \vdots & 1/y_{i1} & 1/y_{iJ-1} + 1/y_{i1} & 1/y_{i1} \\ 1/y_{i1} & 1/y_{i1} & \cdots & 1/y_{i1} & 1/y_{iJ} + 1/y_{i1} \end{pmatrix} \right| \quad (35)$$

$$= \left| \text{diag}(1/y_{i2} + 1/y_{i1}, \dots, 1/y_{iJ} + 1/y_{i1}) + 1/y_{i1} \iota_{J-1} \iota'_{J-1} \right|,$$

where  $\iota_{J-1}$  is a  $(J-1)$ -dimensional vector of ones. If we use that for a nonsingular diagonal  $(J-1) \times (J-1)$ -dimensional matrix  $D$ , a scalar  $\alpha$ , and a  $J-1$ -dimensional vector  $d$  it holds that  $|D + \alpha d d'| = |D| |1 + \alpha d' D^{-1} d|$  (DHRYMES, 1978, Proposition 1) it is easy to show that

$$\left| \frac{\partial \tilde{y}_i}{\partial y_i} \right| = \left| \prod_{j=2}^J \frac{1}{y_{ij}} \left( 1 + \frac{\sum_{j=2}^J y_{ij}}{y_{i1}} \right) \right| = \left| \frac{\sum_{j=1}^J y_{ij}}{\prod_{j=1}^J y_{ij}} \right|. \quad (36)$$

## Notes

<sup>1</sup>Documents and discussion papers can be obtained at: [www.cpb.nl/eng/research/](http://www.cpb.nl/eng/research/)

<sup>2</sup>Calibration needs  $J$  equations (VAN DER MENSBRUGGHE, 2005, p. 91) instead of 1 in case of LES or IAS. As shown in Appendix A of this paper, knowledge of one own price elasticity leads to identification of all parameters. AIDADS requires the knowledge of *all*  $J$  own price elasticities.

<sup>3</sup>The specification of HOUTHAKKER (1960) reads:  $\sum_{j=1}^J c_j^* (m/p_j)^{\beta_j}$ . Using the reparametrization  $c_j = c_j^* \beta_j$  and subtracting the constant  $\sum_{j=1}^J c_j / \beta_j$  we arrive at (11). Both specifications represent the same preferences, but the advantage of (11) is that the parameter restrictions can readily be derived, see MURTY (1982) and DE BOER *et al.* (2006), and that the special case  $\beta_j = 0$  is defined to be equal to  $\ln(m/p_j)$ .

<sup>4</sup>In LESER (1941), SOMERMEYER and WIT (1956) and SOMERMEYER and LANGHOUT (1972) the reaction coefficients are denoted by  $\alpha_j$  ( $= -\beta_j$ ). If all  $\beta_j = \beta$  IAS reduces to the constant elasticity of substitution [CES] function (ARROW *et al.*, 1961). If, moreover,  $\beta = 0$ , we obtain the Cobb-Douglas function.

<sup>5</sup>HANOCH (1975), DEATON and MUELBAUER (1980) and CHUNG (1991) give as restriction  $\beta_j > 0$ , excluding the region  $-1 \leq \beta_j < 0$ . As a consequence, the existence of inelastic demand and of gross complementarity is, erroneously, excluded.

<sup>6</sup>It can be shown that  $\phi_m = \sum_{k=1}^J w_k \beta_k - 1 > -2$  in view of  $\beta_j \geq -1$ .

## References

- ARROW, K.J., H.B. CHENERY, B. MINHAS and R.M. SOLOW (1961), Capital-labor substitution and economic efficiency, *Review of Economics and Statistics* **43**, 225-250.
- BCHIR, M.H., Y. DECREUX, J.-L. GUÉRIN AND S. JEAN (2002), MIRAGE, a computable general equilibrium model for trade policy analysis, Working Paper 2002-17, CEPII, Paris, France.
- BLONIGEN, B.A., J.E. FLYNN and K.A. REINERT (1997), Sector-focused general equilibrium modeling, in: J.F. FRANCOIS and K.A. REINERT (eds.), *Applied methods of trade policy analysis: a handbook*, Cambridge University Press, Cambridge, UK, 189-230.
- CHUNG, J.W. (1994), *Utility and production functions: theory and applications*, Blackwell, Oxford, UK.
- CLARKE, K.A. (2007), A simple distribution-free test for nonnested model selection, *Political Analysis* **15**, 347-363.
- COBB, C.W. and P.H. DOUGLAS (1928), A theory of production, *American Economic Review*, Papers and Proceedings **XVIII**, 139-165.
- CRANFIELD, J.A.L., P.V. PRECKEL, J.S. EALES and T.W. HERTEL (2000), On the estimation of 'an implicitly additive demand system', *Applied Economics* **32**, 1907-1915.
- DEATON, A.S. and J. MUELLBAUER (1980), *Economics and consumer behavior*, Cambridge University Press, Cambridge, UK.
- DE BOER, P.M.C., J. BRÖCKER, B.S. JENSEN and J. VAN DAAL (2006), The expenditure system of the CDES indirect utility function, Econometric Institute Report EI 2006-11, Erasmus University Rotterdam (<http://hdl.handle.net/1765/7757>)
- DE BRUIJN, R. H. KOX and A. LEJOUR (2008), Economic benefits of an integrated European market for services, *Journal of Policy Modelling* **30**, 301-319.
- DHRYMES, P.J. (1978), *Mathematics for econometrics*, Springer-Verlag, New York.
- DON, F.J.H. and J.P. VERBRUGGEN (2006), Models and methods for economic policy: 60 years of evolution at CPB, *Statistica Neerlandica* **60**, 145-170.
- ENGEL, E. (1857), Die Productions- und Consumptionsverhältnisse des Königreichs Sachsen, Zeitschrift des Statistisches Bureaus des Königlich Sächsischen Ministerium des Innern, reprinted in: *Bulletin of the International Statistical Institute* (1895) **9**, 1-54.
- GEARY, R.C. (1949-1950), A note on 'A constant utility index of the cost of living', *Review*

- of *Economic Studies* **18**, 65-66.
- HANOCH G. (1975), Production and demand models with direct or indirect implicit additivity, *Econometrica* **43**, 395-419.
- HOUTHAKKER, H.S. (1960), Additive preferences, *Econometrica* **28**, 244-257.
- KLEIN, L.R. and H. RUBIN (1948-1949), A constant –utility index of the cost of living, *Review of Economic Studies* **15**, 84-87.
- LEJOUR, A., H. ROJAS-ROMAGOSA and G. VERWEIJ (2008), Opening services markets within Europe; modelling foreign establishments in a CGE framework, *Economic Modelling* **25**, 1022-1039.
- LEJOUR, A., P. VEENENDAAL, G. VERWEY and N. VAN LEEUWEN (2006), WorldScan: a model for international economic policy analysis, CPB Document 111, CPB Netherlands Bureau for Economic Policy Analysis, The Hague, the Netherlands.
- LESER, C.E.V. (1941), Family budget data and price-elasticities of demand, *Review of Economic Studies* **9**, 40-57.
- MURTY, K.N. (1982), Theoretical restrictions on the parameters of indirect addilog demand equations - a comment, *Econometrica* **50**, 225-227.
- PECS (2005), Palestinian Central Bureau of Statistics, Ramallah, Palestine.
- REIMER, J.J. and T.W. HERTEL (2004), Estimation of international demand behaviour for use with input-output based data, *Economic Systems Research* **16**, 347-366.
- RIMMER, M.T. and A.A. POWELL (1996), An implicitly additive demand system, *Applied Economics* **28**, 1613-1622.
- SAMUELSON, P.A. (1948), Some implications of linearity, *Review of Economic Studies* **30**, 88-90.
- SCHWABE, H. (1868), Das Verhältnis von Miethe und Einkommen in Berlin, in: *Berlin und seine Entwicklung- Gemeinde-Kalender und städtisches Jahrbuch für 1868*.
- SOMERMEYER, W.H. and J.W.W.A. WIT (1956), Een verdeelmodel (an allocation model), Report M14, Statistics Netherlands, Voorburg.
- SOMERMEYER, W.H. and A. LANGHOUT (1972), Shapes of Engel curves and demand curves: implications of the expenditure model, applied to Dutch data, *European Economic Review* **3**, 351-386.

- STONE, R. (1954), Linear expenditure system and demand analysis: an application to the British pattern of demand, *Economic Journal* **64**, 511-532.
- TINBERGEN, J. (1942), Professor Douglas' production function, *Revue de l'Institut International de Statistique* **10**, 37-48.
- VAN DER MENSBRUGGHE, D. (2005), Linkage technical reference document, version, 6.0, The World Bank, Washington State DC.
- VARIAN, H.R. (1992), *Microeconomic analysis*, third edition, Norton, New York.
- VEENENDAAL, P. and T. MANDERS (2008), Border tax adjustment and the EU-ETS, a quantitative assessment, CPB Document 171, CPB Netherlands Bureau for Economic Policy Analysis, The Hague, the Netherlands.
- VUONG, Q.H. (1989) Likelihood ratio tests for model selection and non-nested hypotheses, *Econometrica* **57**, 307-333.