A Test for Mean-Variance Efficiency of a Given Portfolio under Restrictions

Thierry Post

ERIM REPORT SERIES *RESEARCH IN MANAGEMENT*	
ERIM Report Series reference number	ERS-2005-032-F&A
Publication	May 2005
Number of pages	20
Persistent paper URL	
Email address corresponding author	gtpost@few.eur.nl
Address	Erasmus Research Institute of Management (ERIM) RSM Erasmus University / Erasmus School of Economics Erasmus Universiteit Rotterdam P.O.Box 1738 3000 DR Rotterdam, The Netherlands Phone: + 31 10 408 1182 Fax: + 31 10 408 9640 Email: info@erim.eur.nl Internet: www.erim.eur.nl

Bibliographic data and classifications of all the ERIM reports are also available on the ERIM website:
www.erim.eur.nl

ERASMUS RESEARCH INSTITUTE OF MANAGEMENT

REPORT SERIES
RESEARCH IN MANAGEMENT

ABSTRACT AND KEYWORDS	
Abstract	This study proposes a test for mean-variance efficiency of a given portfolio under general linear investment restrictions. We introduce a new definition of pricing error or "alpha" and as an efficiency measure we propose to use the largest positive alpha for any vertex of the portfolio possibilities set. To allow for statistical inference, we derive the asymptotic least favorable sampling distribution of this test statistic. Using the new test, we cannot reject market portfolio efficiency relative to beta decile stock portfolios if short-selling is not allowed.
Free Keywords	Mean-variance Efficiency, Portfolio Constraints, Asset Pricing, Portfolio Analysis
Availability	The ERIM Report Series is distributed through the following platforms: Academic Repository at Erasmus University (DEAR), DEAR ERIM Series Portal Social Science Research Network (SSRN), SSRN ERIM Series Webpage Research Papers in Economics (REPEC), REPEC ERIM Series Webpage
Classifications	The electronic versions of the papers in the ERIM report Series contain bibliographic metadata by the following classification systems: Library of Congress Classification, (LCC) LCC Webpage Journal of Economic Literature, (JEL), JEL Webpage ACM Computing Classification System CCS Webpage Inspec Classification scheme (ICS), ICS Webpage

A test for mean-variance efficiency of a given portfolio under restrictions

Thierry Post*

*Post is corresponding author: Erasmus University Rotterdam, P.O. Box 1738, 3000 DR, Rotterdam, The Netherlands, email: gtpost@few.eur.nl, tel: +31-104081428. Financial support by Tinbergen Institute, Erasmus Research Institute of Management and Erasmus Center of Financial Research is gratefully acknowledged. Any remaining errors are the authors' responsibility.

A test for mean-variance efficiency of a given portfolio under restrictions

This study proposes a test for mean-variance efficiency of a given portfolio under general linear investment restrictions. We introduce a new definition of pricing error or "alpha" and as an efficiency measure we propose to use the largest positive alpha for any vertex of the portfolio possibilities set. To allow for statistical inference, we derive the asymptotic least favorable sampling distribution of this test statistic. Using the new test, we cannot reject market portfolio efficiency relative to beta decile stock portfolios if short-selling is not allowed.

TESTS FOR MEAN-VARIANCE EFFICIENCY of a given portfolio are useful tools for portfolio management applications and empirical asset pricing research. Most efficiency tests including the classic Gibbons, Ross and Shanken (GRS; 1989) test focus on the case where the portfolio weights are unrestricted. However, in practice, investors typically face investment restrictions such as short-selling constraints and position limits. Restrictions also play an important role in some asset pricing theories such as Black's zero-beta model where riskless borrowing is not allowed.

Mean-variance efficiency is typically gauged by the empirical pricing errors or "alphas" of the individual assets relative to the evaluated portfolio. The alphas measure the effect of marginal changes of the portfolio weights. A positive alpha means that an asset should be overweighed relative to its current weight in the portfolio and a negative alpha means that underweighting is required. For example, the GRS test essentially uses a weighted sum of squared alphas to test if the alphas are jointly equal to zero. The same approach generally does not apply in case of restrictions, because a positive (negative) alpha does not imply inefficiency if the weight of the asset cannot be raised (lowered).

This study develops a mean-variance efficiency test that can account for general linear investment restrictions. In the spirit of the traditional mean-variance efficiency tests, our test focuses on alphas. To account for the above complications when using alphas under restrictions, we develop an alternative definition of alpha. In contrast to the traditional alphas, our alphas (1) are measured for vertices of the portfolio possibilities set, or 'extreme portfolios', rather than individual assets, (2) are measured using returns in excess of the return to the evaluated portfolio rather than in excess of the riskless rate, (3) may take negative values (but not positive values) for the 'inactive' vertices. To test mean-variance efficiency under constraints, we propose to use the maximum positive sample alpha as a test statistic. To allow for statistical inference, we derive the asymptotic least favorable sampling distribution of this test statistic.

An alternative to the traditional mean-variance efficiency tests is the Bayesian approach by Kandel *et al.* (1995). This approach does not use alphas but rather measures efficiency directly by the maximum improvement in mean return given the variance of the evaluated portfolio. Interestingly, as shown by Wang (1998), this approach can be extended in a straightforward manner to account for investment restrictions. We do not wish to enter the on-going debate about the relative merits of classical and Bayesian statistical inference here. Rather, our purpose is to enrich the classical approach to mean-variance analysis with an efficiency test that applies under general restrictions.

Our test is in the spirit of Post's (2003) test for second-order stochastic dominance (SSD) efficiency of a given portfolio, which can also account for general portfolio constraints. The advantage of the stochastic dominance approach is that it avoids the possible specification error associated the mean-variance framework if for example measures of downside risk and upside potential are relevant in addition to the mean and the variance. On the other hand, the stochastic dominance approach is very general and may lack statistical power (ability to detect inefficient portfolios) in small and medium-sized samples. For this reason, the empirical researcher is probably well-advised to use stochastic dominance tests and mean-variance tests in combination.

The remainder of this study is structured as follows. Section I introduces some preliminary notation and definitions. Section II introduces our new definition of alpha. Section III illustrates this new definition by means of a numerical example. In Section IV, we discuss how the largest positive sample alpha can be used as test statistic and proposes a test procedure. Next, Section V analyzes the statistical properties of the test procedure by means of a simulation study with a simulation design based on representative stock return data. Section VI investigates if the value-weighted stock market portfolio is mean-variance efficient relative to benchmark portfolios formed on market beta if we impose short-selling restrictions. Finally, section VII presents conclusions and suggestions for further research. The appendix includes a formal proof of the asymptotic least favorable sampling distribution presented in Section IV.

I. Preliminaries

The investment universe consists of K normally distributed assets, one of which may be a riskless asset. Let the excess returns to these assets be represented by $\boldsymbol{x} \in \Re^K$. Investors may diversify between the assets and the portfolio possibilities take the form of a polytope of general form

$$\Lambda \equiv \left\{ \boldsymbol{\lambda} \in \Re^K : \mathbf{A}\boldsymbol{\lambda} \leq \mathbf{b} \right\} \qquad \textbf{(1)}$$

with $\mathbf{A} \in \Re^{N \times M}$ and $\mathbf{b} \in \Re^M$ for the coefficients of M linear restrictions imposed on the portfolio weights. Many practical investment constraints have a linear form, including short-selling constraints, position limits and restrictions on risk factor loadings (betas). Furthermore, non-linear restrictions often can be approximated with high precision by a set of linear restrictions.

All feasible portfolios represent convex combinations of the N vertices of the portfolio possibilities set, or 'extreme portfolios'. Collect the vertices in the matrix $\mathbf{V} \equiv (\boldsymbol{v}_1 \cdots \boldsymbol{v}_N)$, with $\boldsymbol{v}_i \in \Lambda$, $i = 1, \cdots, N$.[1] Using the vertices, the portfolio possibilities set can equivalently be represented as

$$\Lambda = \left\{ \boldsymbol{\lambda} \in \Re^K : \mathbf{V}\boldsymbol{\sigma} = \boldsymbol{\lambda}; \boldsymbol{\sigma} \in \Sigma \right\} \qquad \textbf{(2)}$$

with $\Sigma = \left\{ \boldsymbol{\sigma} \in \Re_+^N : \boldsymbol{\sigma}^{\mathrm{T}} \mathbf{1}_N = 1 \right\}$ for a basic simplex. Note that in the special case with only short selling restrictions, the extreme portfolios reduce to simply the individual assets and $\Lambda = \left\{ \boldsymbol{\lambda} \in \Re_+^K : \boldsymbol{\lambda}^{\mathrm{T}} \mathbf{1}_K = 1 \right\}$.

---

[1] To enumerate the vertices of a polytope, we may use e.g. the Double Description Method, first introduced by Motzkin *et al.* (1953), and revisited by Fukuda and Prodon (1996).

II. Redefining alphas

As discussed in the introduction, the traditional approach of testing if the alphas of all assets are jointly equal to zero generally does not apply in case of investment restrictions. A positive (negative) alpha does not imply inefficiency if the weight of the asset cannot be raised (lowered). Still, we may use a variation to the traditional theme to test mean-variance efficiency with constraints.

Our analysis starts with the optimality conditions for mean-variance optimization under restrictions. For simplicity, we formulate the problem in terms of expected utility maximization, following e.g. Hanoch and Levy (1969). A portfolio $\boldsymbol{\tau} \in \Lambda$ is mean-variance efficient if and only if it represents the optimal portfolio for some investor with (standardized) quadratic utility function $u(x) = (1 - bE[\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau}])x + 0.5bx^2$, with b for a risk aversion coefficient.[2,3] The optimization problem can be represented as

$$\max_{\boldsymbol{\lambda} \in \Lambda} E[u(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\lambda})] = \max_{\boldsymbol{\sigma} \in \Sigma} E[u(\boldsymbol{x}^{\mathrm{T}}\mathbf{V}\boldsymbol{\sigma})] \tag{3}$$

Our alphas are defined as follows:

$$\boldsymbol{\alpha} \equiv E[u'(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau})(\mathbf{V}^{\mathrm{T}}\boldsymbol{x} - \boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau})] = E[(1 + b(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau} - E[\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau}])(\mathbf{V}^{\mathrm{T}}\boldsymbol{x} - \boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau})] \tag{4}$$

Each alpha measures the marginal effect (in terms of additional expected utility for a mean-variance optimizer) of deviating from the evaluated portfolio by moving in the direction of one of the extreme portfolios. Note that the alphas are defined for the *N* extreme portfolios rather than the *K* individual assets. Further, our alphas are measured using returns in excess of the return on the evaluated portfolio $(\mathbf{V}^{\mathrm{T}}\boldsymbol{x} - \boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau})$. This reflects the fact that the traditional alphas may be non-zero if riskless lending and/or borrowing is restricted.

PROPOSITION 1 *The evaluated portfolio is mean-variance efficient if and only if*

$$\begin{cases} \alpha_i = 0 & i \in \Theta \\ \alpha_i \leq 0 & i \notin \Theta \end{cases} \tag{5}$$

with

$$\Theta \equiv \left\{ i = 1, \cdots, K : \gamma_i > 0 \quad \gamma \in \Lambda : \gamma^{\mathrm{T}}\mathbf{V} = \boldsymbol{\tau} \right\} \tag{6}$$

---

[2] A quadratic utility function implies preferences over mean and variance only. This generally is meaningful only for elliptical distributions (including the normal), which are completely characterized by their mean and variance. A well-known drawback of the quadratic function is that it allows for decreasing utility, while not allowing for decreasing absolute risk aversion. For this reason, the quadratic is hardly used outside the context of elliptical distributions. In fact, the quadratic is best seen as a second-order Taylor series approximation to the true utility function.

[3] This function is standardized such that $E[u'(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau})] = 1$, a conventional standardization when testing market portfolio efficiency in the asset pricing literature.

Here, Θ represents the 'active set' or the set of extreme portfolios that can be used to form the evaluated portfolio. For these extreme portfolios, the alphas must equal zero. For the remaining extreme portfolios, negative alphas are allowed. Such negative alphas represent hypothetical improvement possibilities that cannot be exploited because a set of investment restrictions is binding.

A remaining problem is the selection of the risk aversion parameter b. The GRS test effectively selects this parameter by setting the alpha of the evaluated portfolio equal to zero. Since we use returns in excess of the return on the evaluated portfolio, this condition is already satisfied in our approach. We therefore use another standardization that uses a zero alpha for $\boldsymbol{\kappa}$, the equal-weighted average of the active extreme portfolios:[4]

$$\begin{cases} \kappa_i = 1/\sum_{j=1}^{N} 1(\gamma_j > 0) & i \in \Theta \\ \kappa_i = 0 & i \notin \Theta \end{cases} \tag{7}$$

It follows directly from Proposition 1 that this portfolio must have a zero alpha, or $\boldsymbol{\alpha}^{\mathrm{T}}\boldsymbol{\kappa} = E[(1+b(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau} - E[\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau}])(\boldsymbol{x}^{\mathrm{T}}\mathbf{V}\boldsymbol{\kappa} - \boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau})] = 0$, and hence

$$b = -\frac{E[\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau} - \boldsymbol{x}^{\mathrm{T}}\mathbf{V}\boldsymbol{\kappa}]}{E[(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau} - E[\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau}])(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau} - \boldsymbol{x}^{\mathrm{T}}\mathbf{V}\boldsymbol{\kappa})]} \tag{8}$$

To summarize, our alphas differ in three respects from the traditional alphas: (1) they refer to extreme portfolios rather than individual assets, (2) they use returns in excess of the return to the evaluated portfolio rather than the riskless rate, and (3) the alphas of the inactive extreme portfolios may be negative.

For some purposes, it is convenient to express the alphas in terms of means and "betas", or regression coefficients with the evaluated portfolio:

$$\boldsymbol{\beta} \equiv \frac{Cov[\boldsymbol{x}, \boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau}]}{Var[\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau}]} \tag{9}$$

Specifically, using some algebra, we may reformulate (4) in the following manner:

$$\boldsymbol{\alpha} = E[\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau}] - \theta_1 - \theta_2\boldsymbol{\beta} \tag{10}$$

where

$$\theta_1 \equiv E[\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau}] - bVar[\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau}] \tag{11}$$

and

$$\theta_2 \equiv -bVar[\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau}] \tag{12}$$

[4] We use this standardization because it applies under general restrictions. However, alternative standardizations may be employed here, provided they are consistent with the condition (5). An alternative approach is to select the value for b that maximizes the empirical fit (minimizes the alphas of the active set).

represent a zero-beta rate and a beta premium respectively.

This formulation is reminiscent of Black's zero-beta model where riskless borrowing is not allowed and hence the zero-beta rate may be greater than zero and the beta risk premium may be smaller than the equity premium. However, our test in general does not test the zero-beta model, because (1) the evaluated portfolio need not be the market portfolio, (2) alphas are measured for extreme portfolios rather than the individual assets, and (3) the alphas for inactive extreme portfolios may be negative. Still, if we evaluate market portfolio efficiency under short sales constraints, then the model essentially reduces to a test for the zero-beta model, as will be discussed in Section IV.

III. Numerical Example

We now turn to a numerical example to illustrate our alphas. Consider K=3 assets that obey a trivariate normal distribution with mean vector and covariance matrix as follows:

$$E[\boldsymbol{x}] = [0.12 \quad 0.08 \quad 0.02]^{\mathrm{T}}$$

$$E[(\boldsymbol{x}-E[\boldsymbol{x}])(\boldsymbol{x}-E[\boldsymbol{x}])^{\mathrm{T}}] = \begin{bmatrix} 0.08 & 0.02 & 0.05 \\ 0.02 & 0.07 & 0.04 \\ 0.05 & 0.04 & 0.06 \end{bmatrix}$$

Short sales are not allowed and a position limit of 80% applies for every asset. Thus, the portfolio possibilities are given by

$$\Lambda = \left\{ \boldsymbol{\lambda} \in \Re^3_+ : \boldsymbol{\lambda}^{\mathrm{T}} \mathbf{1}_3 = 1; \boldsymbol{\lambda} \leq 0.8 \mathbf{1}_3 \right\}$$

The N=6 vertices of this portfolio possibilities set or extreme portfolios are given by

$$\mathbf{V} = \begin{bmatrix} 0.8 & 0.2 & 0.2 & 0.8 & 0 & 0 \\ 0.2 & 0.8 & 0 & 0 & 0.8 & 0.2 \\ 0 & 0 & 0.8 & 0.2 & 0.2 & 0.8 \end{bmatrix}$$

Figure 1 displays the portfolio possibilities set and the associated extreme portfolios.

[Insert Figure 1 about here]

Figure 2 shows the mean-standard deviation diagram for this example. The black curve represents the efficient frontier of the portfolio possibilities set Λ.

[Insert Figure 2 about here]

Consider the portfolio

$$\boldsymbol{\tau} = [0.62 \quad 0.38 \quad 0]^{\mathrm{T}}$$

Clearly, this portfolio is mean-variance efficient. Hence, Proposition 1 must apply. In this case, the 'active set' of extreme portfolios that can be used to form the evaluated portfolio is simply $\Theta = \{1, 2\}$; the evaluated portfolio can only be formed as $\boldsymbol{\tau} = 0.7\boldsymbol{v}_1 + 0.3\boldsymbol{v}_2$. Hence, according to Proposition 1, $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ must have a zero alpha, while the other four extreme portfolios ($\boldsymbol{v}_3$ through $\boldsymbol{v}_6$) may have a negative alpha. Recall that our alphas are standardized by requiring a zero alpha for the equal-weighted average of all 'active' extreme portfolios. Thus, in this case, we have

$$\boldsymbol{\kappa} = [0.5 \quad 0.5 \quad 0 \quad 0 \quad 0 \quad 0]^{\mathrm{T}}$$

Figure 3 shows a mean-beta diagram. Alphas are measured as the distance to the black straight line through the evaluated portfolio and the equal-weighted portfolio. Clearly, the 'active' extreme portfolios $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ have a zero alpha. By contrast, the 'inactive' extreme portfolios $\boldsymbol{v}_3$ through $\boldsymbol{v}_6$ have a negative alpha. These alphas reflect hypothetical improvement possibilities that cannot be exploited because investment restrictions are binding. Most notably, the evaluated portfolio assigns a zero weight to the third asset, i.e., τ_3=0, and hence the short sales constraint for this asset is binding for this portfolio. This explains why all extreme portfolios with a strictly positive weight for the third asset have a negative pricing error. The other five constraints (no short sales for the first two assets and position limits for all three assets) are not binding.

[Insert Figure 3 about here]

IV. Empirical testing

Our null hypothesis is that the evaluated portfolio is mean-variance efficient, or (5). In order to test this null, we must provide a scalar valued population functional that separates the null from the alternative. As discussed above, negative alphas do not necessarily imply inefficiency and hence the traditional (weighted) sum of squared alphas does not apply here. Rather, to test if a given portfolio $\boldsymbol{\tau} \in \Lambda$ is mean-variance efficient, we propose to analyze the largest positive alpha:

$$\theta \equiv \max_{i=1,\cdots,N} \{\alpha_i\} \qquad \textbf{(13)}$$

The maximum positive alpha is an attractive efficiency measure because any inefficient portfolio—regardless of the restrictions imposed on the portfolio—must involve at least one positive alpha, and any efficient portfolio may involve only negative alphas.

In practice, the return distribution and the true alphas are not known. Typically, information is limited to a discrete set of time-series observations, which are here assumed to be serially independently and identically distributed (IID) random draws from the return distribution.[5] Throughout the text, we will represent the observations by $\boldsymbol{x}_t \equiv (x_{1t} \cdots x_{Nt})^{\mathrm{T}}$, $t = 1, \cdots, T$. Using the return observations and their sample means $\bar{\boldsymbol{x}}$, we can construct the following empirical alphas:

$$\hat{\boldsymbol{\alpha}} \equiv T^{-1} \sum_{t=1}^{T} [(1 - \hat{b}\boldsymbol{x}_t^{\mathrm{T}}\boldsymbol{\tau})(\mathbf{V}^{\mathrm{T}}\boldsymbol{x}_t - \boldsymbol{x}_t^{\mathrm{T}}\boldsymbol{\tau})] \quad \textbf{(14)}$$

where

$$\hat{b} \equiv -\frac{\sum_{t=1}^{T} (\boldsymbol{x}_t^{\mathrm{T}}\boldsymbol{\tau} - \boldsymbol{x}_t^{\mathrm{T}}\mathbf{V}\boldsymbol{\kappa})}{\sum_{t=1}^{T} (\boldsymbol{x}_t^{\mathrm{T}}\boldsymbol{\tau} - \bar{\boldsymbol{x}}^{\mathrm{T}}\boldsymbol{\tau})(\boldsymbol{x}_t^{\mathrm{T}}\boldsymbol{\tau} - \boldsymbol{x}_t^{\mathrm{T}}\mathbf{V}\boldsymbol{\kappa})} \quad \textbf{(15)}$$

As an empirical test statistic, we may use the largest positive empirical alpha:

$$\hat{\theta} \equiv \max_{i=1,\cdots,N} \{\hat{\alpha}_i\} \quad \textbf{(16)}$$

Using $\boldsymbol{\Omega} \equiv E[(1 - b(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau} - E[\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau}])^2(\mathbf{V}^{\mathrm{T}}\boldsymbol{x} - \boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau})(\mathbf{V}^{\mathrm{T}}\boldsymbol{x} - \boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau})^{\mathrm{T}}]$ and $\Phi(\cdot \mid \mathbf{0}_N, T^{-1}\boldsymbol{\Omega})$ for a cumulative multivariate normal distribution function with means $\mathbf{0}_N$ and covariance matrix $T^{-1}\boldsymbol{\Omega}$, we may characterize the sampling distribution of the test statistic in the following way

PROPOSITION 2 *Under the asymptotic least favourable distribution,*

$$\Pr[\hat{\theta} > y] = \Gamma(y \mid \mathbf{0}_N, T^{-1}\boldsymbol{\Omega}) \equiv (1 - \int_{z \leq y\mathbf{1}_N} d\Phi(z \mid \mathbf{0}_N, T^{-1}\boldsymbol{\Omega})) \quad \textbf{(17)}$$

Thus, to test efficiency, we may compare $\Gamma(\hat{\theta} \mid \mathbf{0}_N, T^{-1}\boldsymbol{\Omega})$, with a predefined level of significance $\alpha \in [0,1]$, and reject efficiency if $\Gamma(\hat{\theta} \mid \mathbf{0}_N, T^{-1}\boldsymbol{\Omega}) \leq \alpha$. Equivalently, we may reject efficiency if the observed value of θ is greater than or equal to the critical value $\Gamma^{-1}(\alpha \mid \mathbf{0}_N, T^{-1}\boldsymbol{\Omega}) \equiv \inf_{y \geq 0}\{y : \Gamma(y \mid \mathbf{0}_N, T^{-1}\boldsymbol{\Omega}) \leq a\}$.

Two results are useful for implementing Theorem 1 in practice. First, computing *p*-values and critical values requires the variance-covariance matrix $\boldsymbol{\Omega}$. We may estimate $\boldsymbol{\Omega}$ in a distribution-free and consistent manner using the sample equivalent

---

[5] The assumption of a serial IID return distribution may be relaxed to a stationarity and ergodic distribution, provided we correct for serial correlation and heteroskedasticity in addition to contemporaneous correlation and heteroskedasticity (see for example MacKinlay and Richardson (1991)). This seems especially useful for applications to high-frequency stock returns, where strong and predictable patterns of serial correlation and heteroskedasticity have been documented. However, for low-frequency stock returns, such patterns seem less important. Therefore, we use all available data to recover the patterns of contemporaneous correlation and heteroskedasticity, which generally are very strong for stock returns. We leave the issue of accounting for serial correlation and heteroskedasticity for high-frequency data for further research.

$$\hat{\boldsymbol{\Omega}} \equiv T^{-1}\sum_{t=1}^{T}[(1-\hat{b}(\boldsymbol{x}_t^{\mathrm{T}}\boldsymbol{\tau}-\bar{\boldsymbol{x}}^{\mathrm{T}}\boldsymbol{\tau})^2(\mathbf{V}^{\mathrm{T}}\boldsymbol{x}_t-\boldsymbol{x}_t^{\mathrm{T}}\boldsymbol{\tau})(\mathbf{V}^{\mathrm{T}}\boldsymbol{x}_t-\boldsymbol{x}_t^{\mathrm{T}}\boldsymbol{\tau})^{\mathrm{T}}] \quad \textbf{(18)}$$

Second, we may approximate $\Gamma(y|\mathbf{0}_N, T^{-1}\hat{\boldsymbol{\Omega}})$, using Monte-Carlo simulation. In this paper, we will use the following approach. We first draw S=100,000 random vectors $\boldsymbol{w}_s \in \Re^N$, $s \in \{1,\cdots,S\}$, drawn from a normal distribution with mean $\mathbf{0}_N$ and variance-covariance matrix $T^{-1}\hat{\boldsymbol{\Omega}}$. Next, $\Gamma(y|\mathbf{0}_N, T^{-1}\hat{\boldsymbol{\Omega}})$ is approximated by the relative frequency of the vectors $\boldsymbol{z}_s$, $s \in \{1,\cdots,S\}$, that fall outside the integration region $\{\boldsymbol{z} \in \Re^N : \boldsymbol{z} \leq y\mathbf{1}_N\}$.

Note that the p-values are derived from the asymptotic distribution and it is not clear how good the results will be in small samples. Also, the p-values are derived under the least favorable distribution, meaning that the true p-values may be smaller than $\Gamma(\hat{\theta}|\mathbf{0}_N, T^{-1}\boldsymbol{\Omega})$. This approach stems from the desire to be protected from Type I error (wrongly classifying an efficient portfolio as inefficient). The other side of the coin is a possible loss of statistical power for distributions that depart from the least favorable distribution.

V. Simulation

To shed some light on the statistical properties of the proposed test procedure under realistic conditions, this section reports the results of a simulation experiment. Our simulation design is based on representative stock return data. Specifically, we consider the case where an investor can invest ten risky assets and a riskless asset. Short sales are not allowed and the portfolio possibilities are given by $\Lambda = \{\boldsymbol{\lambda} \in \Re_+^{11} : \boldsymbol{\lambda}^{\mathrm{T}}\mathbf{1}_{11} = 1\}$. The assets obey a multivariate normal population distribution with mean and covariance parameters equal to the sample parameters of the monthly excess returns (month-end to month-end) to the ten CRSP beta decile portfolio and the one-month US Treasury bill in the sample period from January 1933 to December 2002.[6]

From the normal population distribution, we draw 10,000 random samples through Monte-Carlo simulation. For every random sample, we apply our test procedure to two test portfolios. The equal weighted portfolio (EP) is known to be mean-variance inefficient relative to the normal population distribution. We analyze the statistical power of the test procedures by its ability to correctly classify EP as inefficient. By contrast, the ex ante tangency portfolio (TP) is efficient, and we analyze the statistical size by the relative frequency of random samples in which this portfolio is wrongly classified as inefficient. This experiment is performed for a sample size (T) of 50 to 2,000 observations and for a significance level (α) of 2.5, 5, and 10 percent. Figure 4 further illustrates our simulation conditions by means of a mean-standard deviation diagram.

[6] The beta decile portfolios are formed with the following procedure. Every year, all individual stocks listed on NYSE, AMEX, and Nasdaq, and covered by CRSP, are sorted based on their market beta in the past five years, ten deciles are formed, and value-weighted portfolios are formed from the stocks in each decile.

[Insert Figure 4 about here]

Figure 5 shows the simulation results. The top graph shows the statistical size as a function of sample size and significance level and the bottom graph shows the statistical power. For the sake of comparison, both graphs also include the results for Post's (2003) test procedure for second-order stochastic dominance (SSD) efficiency. Since this test is more general than the mean-variance efficiency test, the rejection rates are lower.

Ignoring very small samples, the statistical size is generally substantially smaller than the nominal level of significance α, and it converges to zero. In fact, using a level of significance of ten percent, the size is smaller than five percent for samples as small as 150 observations. This reflects our focus on the least favorable distribution, which minimizes Type I error. By contrast, the statistical power goes to unity as we increase the sample size. In small and medium-size samples, the mean-variance procedure is substantially more powerful than the SSD procedure. For example, using a ten percent significance level, the MV procedure achieves a rejection rate above 50 percent already for samples of about 250 observations. By contrast, The SSD procedure achieves this rejection rate only for samples of about 500 observations.

In conclusion, the simulations indicate that the suggested test procedure has favorable statistical properties in a realistic setup and is useful for real-life applications. The results also indicate the potential loss of power by using stochastic dominance tests. Of course, the other side of the coin is that the statistical size of the mean-variance test will be higher than reported (more erroneous rejections) if the returns deviate significantly from a normal distribution. Therefore, in practice, the empirical researcher is probably well-advised to use stochastic dominance tests and mean-variance tests in combination.

[Insert Figure 5 about here]

VI. Empirical Application

In this section, we will illustrate our approach to testing mean-variance efficiency under constraints by means of an empirical application. We will examine US stock market data to test if the market portfolio is mean-variance efficient. For various reasons, market portfolio efficiency is an interesting hypothesis. First, the Sharpe-Lintner-Mossin Capital Asset Pricing Model (CAPM) predicts that the market portfolio is efficient. Second, market portfolio efficiency seems consistent with the popularity of passive mutual funds and exchange traded funds that track broad value-weighted equity indexes.

To test market portfolio efficiency, we need a proxy for the market portfolio and proxies for the individual assets in the investment universe. For the market portfolio, we will use the CRSP all-share index, which is the value-weighted average of all common stocks listed on NYSE, AMEX, and Nasdaq, and covered by CRSP. for the individual assets, we use the ten beta decile portfolios and the one-month US Treasury bill. We use data on monthly returns (month-end to month-end) from January 1933 to December 2002 (840 months). Recall that the same data material also served as input for our simulations. However, we now deviate from simulations by (1) analyzing the CRSP all-share index rather than the tangency portfolio and the equal-weighted portfolio and (2) analyzing the original sample rather than random samples drawn from a normal distribution with population parameters equal to the original sample parameters.

Beta-sorted portfolios have been used extensively to test the CAPM; see Black, Jensen and Scholes (1972), Friend and Blume (1973), Fama and MacBeth (1973), Reinganum (1981) and Fama and French (1992), among others. The empirical results suggest that the CAPM is violated, because the spread in the means is too small relative to the spread in the betas. In other words, by buying low-beta stocks and selling high-beta stocks, we can "beat the market" (achieve a higher Sharpe-ratio than the market portfolio). Still, the results may be consistent with Black's zero-beta CAPM, which allows for a higher zero-beta rate and a lower beta risk premium.

We first test if the market portfolio is mean-variance efficient using the Gibbons, Ross and Shanken (GRS; 1989) test. Given the existing evidence for beta portfolios, we expect to find a high degree of mean-variance efficiency. Following Wang (1998), we may ask if this inefficiency can be explained by short-selling restrictions. Short selling typically is difficult to implement in practice due to margin requirements and explicit or implicit restrictions on short selling for institutional investors (see, for example, Sharpe (1991)). To answer this question, we will apply our test to the case where the portfolio set is reduced to only convex combinations of T-bills and the ten stock portfolios.

In this case, our test reduces to a test for the zero-beta model. The extreme portfolios are simply the 11 individual benchmark portfolios. Further, the market portfolio includes all risky assets with a strictly positive weight and hence the short sales constraints for the ten stock portfolios are not binding and do not affect the alphas. After all, an investor holding the market portfolio can improve his or her portfolio without short selling by simply selling his or her current position in the assets with a negative alpha.[7] By contrast, the market portfolio does not include the riskless asset and hence the short sales constraint for the T-bills is binding. Thus, the only deviation from the Sharpe-Lintner-Mossin CAPM is that we may observe a negative alpha for the T-bills.

Table I reports our test results. In line with the existing evidence for beta-sorted portfolios, we must reject unconstrained mean-variance efficiency, with a p-value of 0.002. As expected, the low-beta stocks are substantially underpriced and the high-beta stocks are substantially overpriced. The pricing errors range from 0.236 (or 2.83% per annum) for the lowest-beta stocks to -0.380 (or -4.56% per annum) for the highest-beta stocks. Hence, investors can beat the market by short selling high-beta stocks and using the proceeds to buy low-beta stocks. Note that no alpha is reported for the T-bills. The GRS test uses returns in excess of the riskless rate and hence the alpha of the riskless asset by construction equals zero.

Interestingly, the evidence against efficiency of the market portfolio is substantially weaker if short-selling is not allowed. This is reflected in our alphas, which are generally lower than the traditional alphas. Recall that we use an alternative definition of alpha: alphas are measured using returns in excess of the return to evaluated portfolio rather than the riskless rate. Our test focuses on the largest positive

[7] By contrast, in Wang's (1998) test, short-selling constraints for the risky assets do affect the test results because he employs a global measure for the degree of mean-variance efficiency—the maximum improvement in mean return given the variance. By contrast, in line with the traditional approach, we use a marginal measure based on the effect of small changes to the portfolio weights—the alphas. If constraints are not binding, then they do not affect the alphas.

alpha, which in this case amounts to 0.177 (or 2.12% per annum) and is achieved for the medium-beta portfolio #5. Associated with this maximum alpha is a p-value of 41.7%, far above the range of conventional significance levels. Note that the large negative pricing error for the T-bills of -0.326 (or -3.91% per annum) does not constitute evidence against efficiency, because riskless borrowing is not allowed.

[Insert Table I about here]

Figure 6 further illustrates our results by means of a mean-beta diagram. Our approach basically uses the black straight line through the market portfolio and the equal-weighted portfolio as a benchmark for measuring alphas and it considers only deviations above this line. By contrast, the GRS test for unconstrained mean-variance efficiency uses the grey straight line through the market portfolio and the origin and considers deviations both above and below the line.

[Insert Figure 6 about here]

VII. Conclusions

We propose a test for mean-variance efficiency of a given portfolio under general investment restrictions. Our test relies on a new definition of pricing error or "alpha". Our alphas differ in three respects from the traditional alphas: (1) they refer to extreme portfolios, or vertices of the portfolio possibilities set, rather than individual assets, (2) they use returns in excess of the return to the evaluated portfolio rather than the riskless rate, and (3) the alphas of the inactive extreme portfolios may be negative. As an efficiency measure we propose to use the largest positive alpha for any extreme portfolio, or vertex of the portfolio possibilities set. To allow for statistical inference, we derive the asymptotic least favorable sampling distribution of this test statistic. We apply our new test to test mean-variance efficiency of the market portfolio. In this application, our test effectively gives a test for Black's zero-beta model. Consistent with existing evidence, we cannot reject market portfolio efficiency relative to beta decile stock portfolios.

Appendix

Proof to Proposition 1 The Karush-Kuhn-Tucker conditions for optimization problem (3) can be formulated in terms of the gradient vector $\partial E[u(\boldsymbol{x}^{\mathrm{T}}\mathbf{V}\boldsymbol{\sigma})]/\partial\boldsymbol{\sigma} = E[u'(\boldsymbol{x}^{\mathrm{T}}\mathbf{V}\boldsymbol{\sigma})(\mathbf{V}^{\mathrm{T}}\boldsymbol{x})]$. Specifically, optimality of $\boldsymbol{\tau} \in \Lambda$ requires

$$\begin{cases} E[u'(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau})(\boldsymbol{v}_i^{\mathrm{T}}\boldsymbol{x})] = c & i \in \Theta \\ E[u'(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau})(\boldsymbol{v}_i^{\mathrm{T}}\boldsymbol{x})] \le c & i \notin \Theta \end{cases} \tag{i}$$

with c for some constant, measuring the shadow price of the budget constraint $\boldsymbol{\sigma} \in \Sigma$. Using the definition of the active set Θ, (i) implies

$$E[u'(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau})(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau})] = c \tag{ii}$$

Subtracting $E[u'(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau})(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau})]$ from the left-hand sides and c from the right-hand sides of (i), we find (5). *Q.E.D.*

Proof to Proposition 2 The terms $(1-b(\boldsymbol{x}_t^{\mathrm{T}}\boldsymbol{\tau}-E[\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau}]))(\mathbf{V}^{\mathrm{T}}\boldsymbol{x}-\boldsymbol{x}_t^{\mathrm{T}}\boldsymbol{\tau})$, $t=1,\cdots,T$, are serially IID with mean $\boldsymbol{\alpha}$ and covariance matrix $\boldsymbol{\Omega}$. Hence it follows from the Lindeberg-Levy central limit theorem that $T^{-1}\sum_{t=1}^{T}(1-b(\boldsymbol{x}_t^{\mathrm{T}}\boldsymbol{\tau}-E[\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau}]))(\mathbf{V}^{\mathrm{T}}\boldsymbol{x}-\boldsymbol{x}_t^{\mathrm{T}}\boldsymbol{\tau})$ obeys an asymptotic normal distribution with mean $\boldsymbol{\alpha}$ and covariance matrix $T^{-1}\boldsymbol{\Omega}$. Since $\hat{b}$ and $\bar{\boldsymbol{x}}^{\mathrm{T}}\boldsymbol{\tau}$ are consistent estimators for b and $E[\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\tau}]$, respectively, the alphas $\hat{\boldsymbol{\alpha}}\equiv T^{-1}\sum_{t=1}^{T}[(1-\hat{b}(\boldsymbol{x}_t^{\mathrm{T}}\boldsymbol{\tau}-\bar{\boldsymbol{x}}^{\mathrm{T}}\boldsymbol{\tau}))(\mathbf{V}^{\mathrm{T}}\boldsymbol{x}_t-\boldsymbol{x}_t^{\mathrm{T}}\boldsymbol{\tau})]$ obey the same distribution, i.e., $\hat{\boldsymbol{\alpha}}\overset{Asy}{\sim}N(\boldsymbol{\alpha},T^{-1}\boldsymbol{\Omega})$. We adhere to the statistical convention of using the least favorable distribution that maximizes the *p*-value under the null hypothesis. Under the null of efficiency, or $\boldsymbol{\alpha}\le\mathbf{0}_N$, the p-value $\Pr[\hat{\theta}>y]$ is maximal if $\boldsymbol{\alpha}=\mathbf{0}_N$. Thus, the asymptotic least favorable distribution of $\hat{\boldsymbol{\alpha}}$ is $N(\boldsymbol{\alpha},T^{-1}\boldsymbol{\Omega})$. Hence, under the asymptotic least favorable distribution, $\hat{\theta}$ is the largest order statistic of *N* random variables with a joint normal distribution, and we find $\Pr[\hat{\theta}>y]=(1-\int_{z\le y}d\Phi(z\,|\,\mathbf{0}_N,T^{-1}\boldsymbol{\Omega}))$. *Q.E.D.*

References

Black, F., M. C. Jensen, and M. Scholes, 1972, The capital asset pricing model: Some empirical tests, In: Jensen, M. C. (ed.), *Studies in the Theory of Capital Markets* (Praeger, NY), 79-121.

Fama, Eugene F., and Kenneth R. French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427-465.

Fama, E. F. and J. Macbeth, 1973, Risk, return and equilibrium: Empirical tests, *Journal of Political Economy* 81, 607-636.

Friend, I. and M. Blume, 1973, A New Look at the Capital Asset Pricing Model, *Journal of Finance* 28, 283-299.

Fukuda and Prodon, 1996, *'Double Description Method Revisited'*, in: Deza, M., R. Euler and I. Manoussakis, eds., 1996, *'Combinatorics and Computer Science'*, Lecture Notes in Computer Science, volume 1120, Springer-Verlag.

Gibbons, M. R., S. A. Ross, and J. Shanken, 1989, A test of the efficiency of a given portfolio, *Econometrica* 57, 1121-1152.

Hanoch, G., and H. Levy, 1969, The Efficiency Analysis of Choices Involving Risk, *Review of Economic Studies* 36, 335-346.

Kandel, S., R. McCulloch, and R. Stambaugh, 1995, Bayesian inference and portfolio efficiency. *Review of Financial Studies* 8, 1-53.

MacKinlay, A. C., and M. P. Richardson, 1991, Using generalized method of moments to test mean-variance efficiency, *Journal of Finance* 46, 511-527.

Motzkin, T.S, H. Raiffa, G.L. Thompson and R.M. Thrall, 1953, *'The Double Description Method'*, in H.W. Kuhn and A.W. Tucker, eds., 1953, *'Contributions to the Theory of Games'*, volume 2, Princeton University Press, Princeton, N.J.

Post, T., 2003, 'Empirical tests for stochastic dominance efficiency', *Journal of Finance* 58, 1905—1932.

Reinganum, Marc R., 1981, A new empirical perspective on the CAPM, *Journal of Financial and Quantitative Analysis* 16, 439-462.

Sharpe, William F., 1991, Capital asset prices with and without negative holdings, *Journal of Finance* 64,489-509.

Wang, Zhenyu, 1998, Efficiency loss and constraints on portfolio holdings, *Journal of Financial Economics* 48, 359-375.

Table I

Test results

The table shows the pricing errors, overall p-value and risk aversion parameter *b* for the GRS test and our mean-variance test for efficiency of the CRSP all-share index relative to the ten beta portfolios in the sample period from January 1933 to December 2002 (840 months).

		GRS	Our test
Alphas	Low beta	0.236	0.105
	2	0.162	0.076
	3	0.077	0.042
	4	-0.013	-0.027
	5	0.139	0.177
	6	-0.071	-0.016
	7	-0.109	-0.019
	8	-0.258	-0.116
	9	-0.256	-0.078
	High beta	-0.380	-0.144
	T-bill		-0.326
	p-value	0.002	0.410
	b	-0.031	-0.017

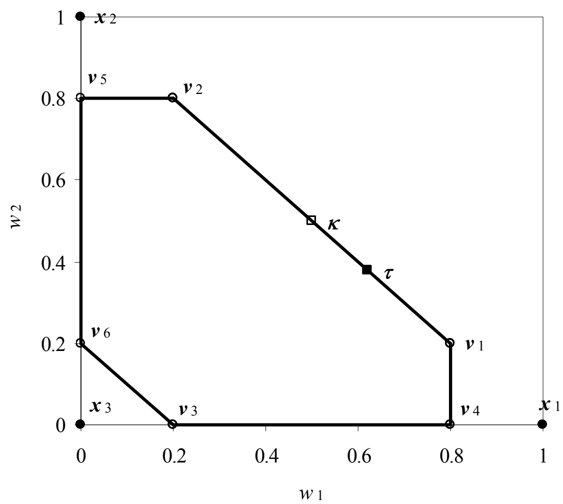

Figure 1: The portfolio possibilities set. The thick black lines represent the contours of the portfolio possibilities set $\Lambda = \{\boldsymbol{\lambda} \in \Re_+^3 : \boldsymbol{\lambda}^{\mathrm{T}}\mathbf{1}_3 = 1; \boldsymbol{\lambda} \leq 0.8\mathbf{1}_3\}$. The portfolio possibilities set involves six vertices ($\boldsymbol{v}_1$, $\boldsymbol{v}_2$, $\boldsymbol{v}_3$, $\boldsymbol{v}_4$, $\boldsymbol{v}_5$, and $\boldsymbol{v}_6$). Also shown are the three individual assets ($\boldsymbol{x}_1$, $\boldsymbol{x}_2$, and $\boldsymbol{x}_3$), the evaluated portfolio ($\boldsymbol{\tau}$) and the equal-weighted average of the active vertices ($\boldsymbol{\kappa}$).

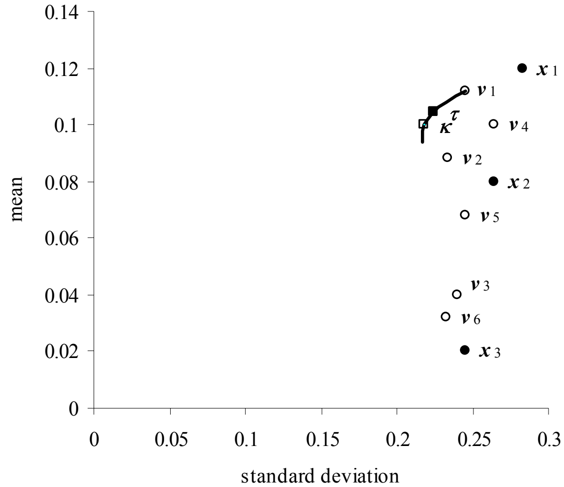

Figure 2: Mean-standard deviation diagram. The figure shows the mean-standard deviation combinations of the six vertices (v_1, v_2, v_3, v_4, v_5, and v_6), the three individual assets (x_1, x_2, and x_3), the evaluated portfolio (τ) and the equal-weighted average of the active vertices (κ). The black curve represents the mean-variance efficient frontier of the portfolio possibilities set $\Lambda = \{\lambda \in \Re^3_+ : \lambda^{\mathrm{T}}\mathbf{1}_3 = 1; \lambda \le 0.8\mathbf{1}_3\}$.

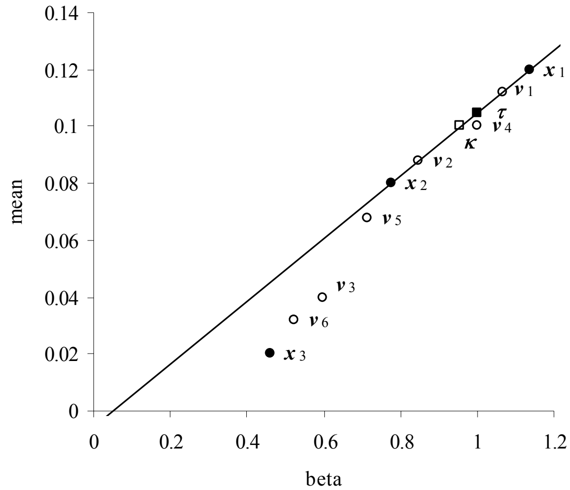

Figure 3: Mean-beta diagram. The figure shows the mean-beta combinations of the six vertices ($\boldsymbol{v}_1$, $\boldsymbol{v}_2$, $\boldsymbol{v}_3$, $\boldsymbol{v}_4$, $\boldsymbol{v}_5$, and $\boldsymbol{v}_6$), the three individual assets ($\boldsymbol{x}_1$, $\boldsymbol{x}_2$, and $\boldsymbol{x}_3$), the evaluated portfolio ($\boldsymbol{\tau}$) and the equal-weighted average of the active vertices ($\boldsymbol{\kappa}$). Alphas are measured relative to the black straight line through the evaluated portfolio and the equal-weighted portfolio.

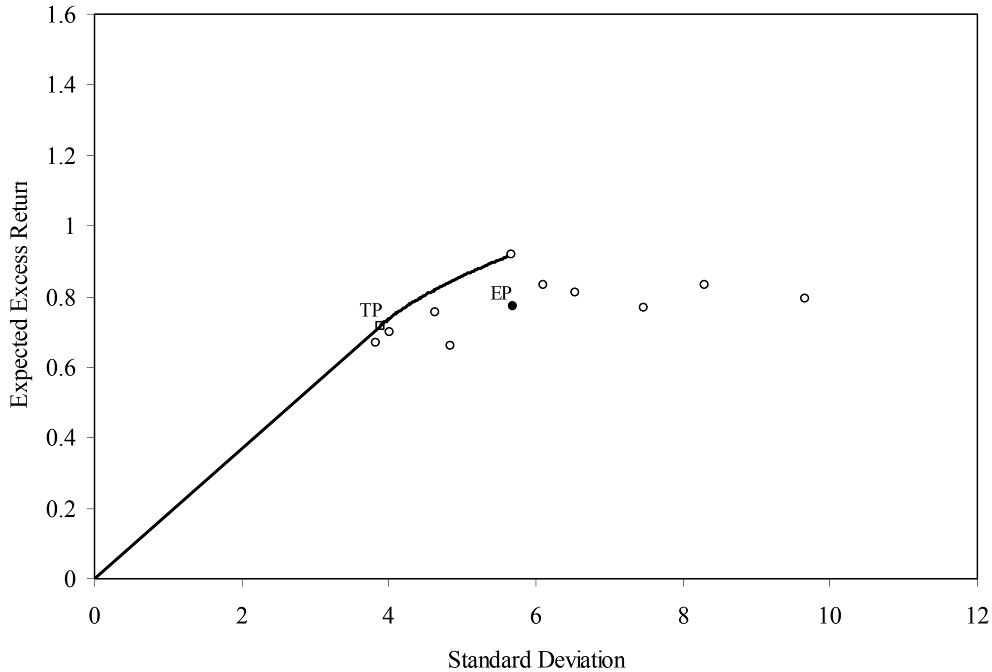

Figure 4: Mean-Standard Deviation Diagram. The figure shows the mean-standard deviation diagram based on the monthly excess returns of the ten beta portfolios in the sample from January 1933 to December 2002 (840 months). The diagram includes the individual benchmark portfolios (the clear dots), the equal weighted test portfolio (EP, the filled dot), the tangency portfolio (TP, the clear square), and the mean-variance efficient frontier of the portfolio possibilities set without short sales $\Lambda = \left\{\lambda \in \Re_{+}^{11} : \boldsymbol{\lambda}^{\mathrm{T}}\mathbf{1}_{11} = 1\right\}$.

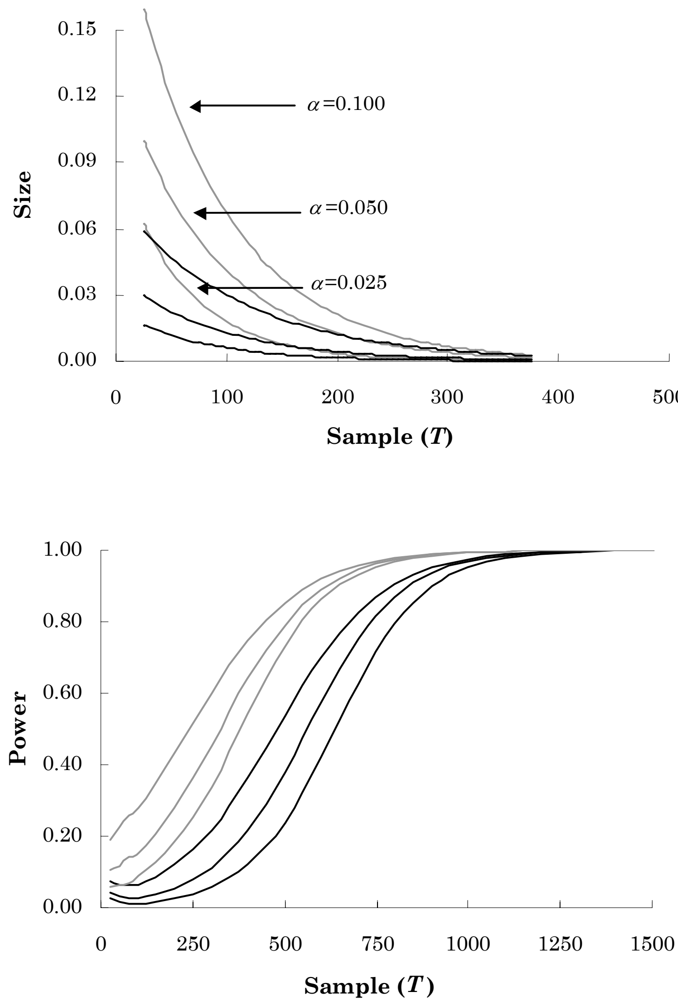

Figure 5: Statistical properties of the mean-variance and stochastic dominance test procedures. The figure displays the size and power of the test procedures for various numbers of time-series observations (T) and for a significance level (α) of 2.5, five and ten percent. The results are based on 10,000 random samples from a multivariate normal distribution with joint moments equal to the sample moments of the monthly excess returns of the ten beta portfolios and the U.S. Treasury bill for the period from January 1933 to December 2002. The portfolio possibilities set equals the basic simplex, i.e. sort-selling is not allowed. The dark lines show the results for the stochastic dominance test, and the gray lines show the results for the mean-variance test. Size is measured as the relative frequency of random samples in which the efficient tangency portfolio is wrongly classified as inefficient. Power is measured as the relative frequency of random samples in which the inefficient equally weighted portfolio is correctly classified as inefficient.

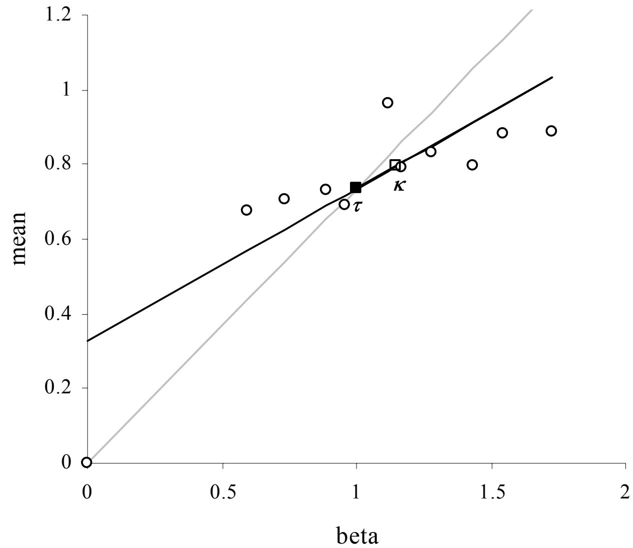

Figure 6: Market portfolio efficiency with and without short sales. The figure shows the mean-beta combinations for the ten beta portfolios (clear dots), the CRSP all-share index (filled square) and the equal-weighted portfolio (clear square) in the sample period from January 1933 to December 2002 (840 months). The GRS alphas are measured relative to the grey straight line through the origin and the market portfolio; our alphas are measured relative to the black straight line through the market portfolio and the equal-weighted portfolio.

Publications in the Report Series Research* in Management

ERIM Research Program: "Finance and Accounting"

2005

Royal Ahold: A Failure Of Corporate Governance
Abe De Jong, Douglas V. Dejong, Gerard Mertens en Peter Roosenboom
ERS-2005-002-F&A
http://hdl.handle.net/1765/1863

Capital Structure Policies in Europe: Survey Evidence
Dirk Brounen, Abe de Jong and Kees Koedijk
ERS-2005-005-F&A
http://hdl.handle.net/1765/1923

A Comparison of Single Factor Markov-Functional and Multi Factor Market Models
Raoul Pietersz, Antoon A. J. Pelsser
ERS-2005-008-F&A
http://hdl.handle.net/1765/1930

Efficient Rank Reduction of Correlation Matrices
Igor Grubišić and Raoul Pietersz
ERS-2005-009-F&A
http://hdl.handle.net/1765/1933

Generic Market Models
Raoul Pietersz and Marcel van Regenmortel
ERS-2005-010-F&A
http://hdl.handle.net/1765/1907

The price of power: valuing the controlling position of owner-managers in french ipo firms
Peter Roosenboom and Willem Schramade
ERS-2005-011-F&A
http://hdl.handle.net/1765/1921

The Success of Stock Selection Strategies in Emerging Markets: Is it Risk or Behavioral Bias?
Jaap van der Hart, Gerben de Zwart and Dick van Dijk
ERS-2005-012-F&A
http://hdl.handle.net/1765/1922

Sustainable Rangeland Management Using a Multi-Fuzzy Model: How to Deal with Heterogeneous Experts' Knowledge
Hossein Azadi, Mansour Shahvali, Jan van den Berg and Nezamodin Faghih
ERS-2005-016-F&A
http://hdl.handle.net/1765/1934

A Test for Mean-Variance Efficiency of a given Portfolio under Restrictions
Thierry Post
ERS-2005-016-F&A

Testing for Stochastic Dominance Efficiency
Oliver Linton, Thierry Post and Yoon-Jae Whang
ERS-2005-033-F&A

Wanted: A Test for FSD Optimality of a Given Portfolio
Thierry Post
ERS-2005-034-F&A

How Domestic is the Fama and French Three-Factor Model? An Application to the Euro Area
Gerard A. Moerman
ERS-2005-035-F&A
http://hdl.handle.net/1765/6626

[*] A complete overview of the ERIM Report Series Research in Management:
https://ep.eur.nl/handle/1765/1

ERIM Research Programs:
LIS Business Processes, Logistics and Information Systems
ORG Organizing for Performance
MKT Marketing
F&A Finance and Accounting
STR Strategy and Entrepreneurship