A Column Generation Approach for Locating Roadside Clinics in Africa based on Effectiveness and Equity

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Abstract

Long distance truck drivers in Sub-Saharan Africa are extremely vulnerable to HIV and other infectious diseases. The NGO North Star Alliance aims to alleviate this situation by placing so-called Roadside Wellness Centers (RWCs) at busy truck stops along major truck routes. Currently, locations for new RWCs are chosen so as to maximize the expected patient volume and to ensure continuity of access along the routes. As North Star's network grows larger, the objective to provide equal access to healthcare along the different truck routes gains importance. This paper considers the problem to locate a fixed number of RWCs based on these effectiveness and equity objectives. We come up with a novel, set-partitioning type of formulation for the problem and propose a column generation algorithm to solve it. Additionally, we propose and analyze several state-of-the-art acceleration techniques, including dual stabilization, column pool management, and accelerated pricing, which solves the pricing problem as a sequence of shortest path problems. Though the facility location problem is strongly $\mathcal{NP}$-hard, our algorithm yields near-optimal solutions to large randomly generated problem instances within an acceptable amount of time. Our analysis of the trade-off between the equity criterion and North Star's current criteria shows that solutions that are close to optimal with respect to each of the effectiveness and equity objectives are likely to be attainable.

Keywords: humanitarian logistics, facility location, equity, column generation

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1 Introduction

Long distance truck drivers in Sub-Saharan Africa are extremely vulnerable to HIV and other infectious diseases (Apostolopoulos and Sönmez 2007). The underlying determinants seem to be loneliness, long separation from home, monotony, and stress, which make them engage in high-risk sexual behaviors (Morris and Ferguson 2007). This context thereby brings about huge health (and thereby social and economic) risks to this population, and seems to fuel the spread of HIV and other infectious diseases (Gatignon and Wassenhove 2008).

Providing long distance truck drivers with continuous access to basic (HIV) prevention, treatment, and care services is therefore believed to be an effective way to combat HIV and its consequences. Traditional healthcare facilities, however, are generally incapable of delivering these services. The main reasons are that truck drivers do not have time or permission to deviate from their routes, that these facilities cannot be accessed by truck, and that the opening hours are inconvenient to the drivers (Gatignon and Wassenhove 2008, Ferguson and Morris 2007).

Non-governmental organization North Star Alliance (North Star) aims to fill this gap by placing so-called Roadside Wellness Centers (RWCs) at busy truck stops along major truck routes in Sub-Saharan Africa. These RWCs provide basic healthcare services, like clinical services, HIV testing and counseling, and behavior change communication to truck drivers and surrounding populations. North Star's current network consists of 35 RWCs, and will expand considerably in the next couple of years. This is expected to bring about large health benefits to truck drivers and surrounding populations. In the first place, by placing new RWCs at busy truck stops, North Star makes sure that many truck drivers are provided with (at least a basic level of) access to the most needed health services when they spend the night there. In addition, establishing a dense network of RWCs ensures that many truck drivers also have continuous access to the needed health services. That is, they are sufficiently close to an RWC at every moment during their trip, which is a requirement for several health services to be effective (De Vries et al. 2014a).

Choosing the locations of a given number of new RWCs presents novel and complex optimization problems. Decision makers face a huge number of possible location decisions, and have to balance multiple objectives. De Vries et al. (2014b) propose a mixed-integer programming (MIP) formulation for the problem to locate a given number of new RWCs and to decide on the set of health services to be offered by these RWCs. The objectives are to maximize the impact of the new RWCs in terms of the patient volume served and in terms of the extent to which the truck drivers have continuous access to healthcare. Numerical and analytical results suggest that the model potentially yields significant improvement for the location decisions taken by North Star.

In spite of that, further improvement of the model is needed in two directions. First, as many other non-profit organizations, North Star operates in a complex environment that is characterized by many different objectives and stakeholders. As North Star's network grows larger, a third objective is gaining importance among them: to provide equitable access to healthcare. More specifically, inequalities in access among truck drivers using different truck routes are to be minimized, both for ethical reasons, as inspired by the right to health (United Nations 1946), as well as for medical and financial reasons.

The second direction for improvement deals with the complexity of the location problem. De Vries et al. (2014b) show that the location problem is strongly $\mathcal{NP}$-complete, and numerical experiments show that solving large problem instances becomes extremely difficult. Moreover, the complexity of the problem will be increased considerably by including an equity criterion. Alternative model formulations and solution methods are required to deal with this.

This paper considers these two directions in the context of the problem of *locating a given number of RWCs* (i.e., we do not consider the decisions on the health services to offer at the RWCs). Our contributions are fourfold. First, we introduce and motivate the equity criterion, and propose several measures for (in)equality in access to healthcare among mobile patient populations. Second, we propose and analyze a novel set partitioning type of formulation for this type of facility location problem. The strengths of this formulation are that it allows for a variety of objective functions (e.g., maximizing patient volume, ensuring continuous access, and providing equity), and that the integrality gap is very small. Third, to deal with the exponential number of variables our formulation brings about, we propose and analyze a column generation approach to solve it. Moreover, we investigate several strategies to speed up our algorithm, including dual stabilization, column pool management, and accelerated pricing. The latter solves the pricing problem to near-optimality as a sequence of shortest path problems. Last, we numerically assess the trade-off between the equity criterion and North Star's current criteria (patient volume and continuous access) based on randomly generated instances. Our results suggest that solutions that are close to optimal with respect to each of the optimization criteria are attainable.

Though we specifically focus on the problem to select locations for a given number of RWCs, these contributions also apply to the extension in which decisions on the health services to offer are considered. Additionally, our contributions are also applicable to a variety of related facility location problems that deal with moving demand units. Examples include the positioning of refueling stations, billboards, detection or inspection stations, convenience stores, and ambulances.

The remainder of this paper is organized as follows. Section 2 describes the problem and the optimization criteria in detail. Next, we present the set partitioning type of formulation in Section 3. Section 4 describes our basic column generation approach. After this, we describe our acceleration techniques in Section 5. Section 6 presents our results and is followed by Section 7, which investigates the trade-off between equity and North Star's current objectives. Finally, in Section 8 we draw some conclusions and discuss possible directions for future research.

2 Problem Description

Before we describe our problem in detail, let us introduce some notations. Let KP denote the set of potential RWC locations – potential locations for new RWCs – and let KC denote the set of current RWC locations – locations corresponding to the current RWCs. The union of these sets $KC \cup KP$ is the set of RWC locations K. Furthermore, let binary variable x_k indicate whether or not an RWC is available at a given RWC location $k \in KP$. We consider the problem to select locations for p new RWCs from the set KP, and measure the quality of a solution with respect to the patient volume criterion, the continuous access criterion, and the equity criterion by variables Z_{PV}, Z_{CA}, and Z_{EQ}, respectively. Finally, w_{PV}, w_{CA}, and w_{EQ} represent the relative importance of the three criteria. Then our objective function is defined as:

$$\max \quad w_{PV} Z_{PV} + w_{CA} Z_{CA} + w_{EQ} Z_{EQ} \tag{1}$$

The definitions of Z_{PV}, Z_{CA}, and Z_{EQ} use the following notations. Parameter d_k denotes the expected daily number of patients entering an RWC at location k. The set of long distance truck routes using the road network is denoted by Q and indexed by q. We measure the level of access along truck route q by means of the variable $c_q \in [0, 1]$, which we refer to as the *coverage score* of route q. Additionally, we relate to route q an ordered set of RWC locations $K_q \subseteq K$, where elements $K_q(1), K_q(2), ...$ represent the $1^{st}, 2^{nd}, ...$ RWC location passed when traveling the route. Similarly, KP_q and KC_q denote the ordered sets of potential and current RWC locations, respectively. Given location decisions $\mathbf{x} = \{x_k\}$, $K_q^{\mathbf{x}}$ represents the ordered set of RWC locations along route q for which $x_k = 1$. Finally, the number of truck drivers traveling route q is represented by f_q.

The facility location problem described above is unique in the sense that it deals with servicing *mobile* patients. Traditional healthcare facility location models, in contrast, assume patients to be static (see Daskin and Dean (2004) and Rahman and Smith (2000) for literature reviews). De Vries et al. (2014b) propose definitions of Z_{PV} and Z_{CA} based on interviews with North Star's CEO and staff. This section briefly describes these definitions and the rationale behind them, and introduces and motivates the equity criterion and corresponding measures.

2.1 Patient Volume

The patient volume criterion refers to the objective of serving as many truck drivers as possible. The rationale behind this objective is that the more truck drivers can be provided with at least a *basic level of access* to the needed health services, the better it is. Namely, such basic level of access suffices to provide these truck drivers with many important "single shot" services like HIV testing and counseling, behavioral change communication, and condom distribution.

Given location decisions $\mathbf{x}$, we measure patient volume Z_{PV} as the total expected daily number of patients entering an RWC:

$$Z_{PV} = \sum_{k \in KP} d_k x_k \tag{2}$$

The patient volume objective provides an incentive to place RWCs at *busy truck stops*, as many truck drivers can be reached there.

2.2 Continuous Access

Whereas providing truck drivers with a *basic level of access* suffices to effectively provide some health services, providing *continuous access* is beneficial or even required for other health services. Continuous access provides truck drivers with adequate access at any point in time, and enables truck drivers to receive continuous and coordinated care over time (as the clinics share information, truck drivers can obtain or continue a service at every clinic) (De Vries et al. 2014a). These characteristics have been shown to lessen treatment delay and to stimulate treatment adherence, and thereby to decrease disease progression, disease transmission, and drug resistance (see, e.g., Starfield et al. 2005). Continuous access seems to be particularly crucial for the provision of HIV treatment and TB treatment (Munro et al. 2007, Mills et al. 2006). These treatments rely on continuous adherence, and require frequent patient-provider contacts. In addition, adequate access at any point in time is necessary in case of complications (e.g., opportunistic infections) and in case that medicines need to be refilled.

Variable c_q, the coverage score of truck drivers traveling route q, indicates to what extent the level of access to healthcare provided along this route is "sufficient", and ranges from 0 (insufficiently covered) to 1 (sufficiently covered). Given location decisions $\mathbf{x}$, we measure the *total* level of continuous access provided as the sum of the coverage scores of all truck drivers in the network, and refer to this measure as the continuous access score:

$$Z_{CA} = \sum_{q \in Q} f_q c_q \tag{3}$$

Before we formally define c_q, we make the following assumption:

Assumption 1. *RWC equivalents are accessible at the origin and destination of each truck route.*

An RWC equivalent refers to other healthcare facilities that also contribute to continuity of access along a truck route. As many truck route origins and destinations correspond to large cities, this assumption holds in most cases. We regard these facilities as RWCs in our model, implying that $K_q^{\mathbf{x}}(1)$ and $K_q^{\mathbf{x}}(m)$ correspond to current RWC locations at the origin and destination of route q, respectively (here, $m = |K_q^{\mathbf{x}}|$). Based on this assumption, we can split up the truck route in $m-1$ sub-routes between two adjacent RWCs along the route: $\left(K_q^{\mathbf{x}}(j), K_q^{\mathbf{x}}(j+1)\right), j \in \{1, 2, ..., m-1\}$, corresponding to a travel time of $t_{\left(K_q^{\mathbf{x}}(j), K_q^{\mathbf{x}}(j+1)\right)}$ time units.

Now consider a given moment during the trip of a truck driver along route q. We define him to be "safe" if the travel time to the next RWC along his route (i.e., his access time) is at most a given critical time-limit τ, and measure c_q as a piece-wise linear function $g(\cdot)$ of the fraction of time he is "safe" during his trip:

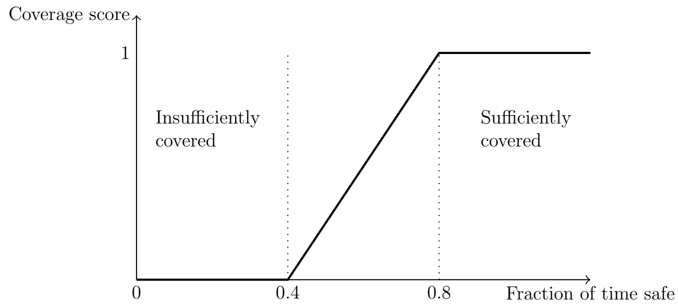


Figure 1: Coverage score as a piece-wise linear function of the fraction of time safe, with $\mu_1 = 0.4, \mu_2 = 0.8$.

$$c_q = g\left(\frac{\sum_{j=1}^{m-1} \min\{t_{\left(K_q^*(j),K_q^*(j+1)\right)}, \tau\}}{T_q}\right) \tag{4}$$

Here, T_q denotes the total travel time for route q and the numerator represents the total time a truck driver traveling route q is safe. Figure 1 shows an example of the function $g(\cdot)$ with three different sections. In this example, if the truck driver is safe less than 40% of the time, he is regarded as being insufficiently covered. If he is safe 80% or more of the time, then we regard him as being sufficiently covered. For later use, we denote the two breakpoints of the function by μ_1 and μ_2, respectively, and represent the three segments of by $\sigma_1 = [0, \mu_1], \sigma_2 = [\mu_1, \mu_2]$, and $\sigma_3 = [\mu_2, 1]$.

2.3 Equity

Equity involves the comparison of two or more populations (or individuals) along some dimension. In the context of the health sector, this comparison is made based on the service or utility the health system provides to the different populations. Examples of specific concepts that can be used to make this comparison include health status, distribution of resources, expenditures, utilization, and access (Goddard and Smith 2001, Culyer and Wagstaff 1993, Musgrove 1986).

The equity issue arises in a natural way in our healthcare facility location problem. Truck drivers generally drive along the same truck route for years (personal communication, North Star, 2011), so that, to a large extent, each truck route corresponds to a unique population of truck drivers. Different location allocation decisions have a different impact in terms of access to healthcare along these truck routes, so that these decisions implicitly determine the (in)equality in access to healthcare among the different populations.

As North Star's network is growing larger, this issue is gaining more and more importance. Whereas it can easily be defended why North Star should assign its first RWCs to the largest populations, one faces a dilemma as soon as the number of RWCs in the network grows. Namely, one has to decide whether to assign new RWCs to large and relatively well-served truck populations, where the total health impact may be largest, or to underserved populations, where the total health impact may be smaller. Hence, the principle of providing equal access to healthcare for truck drivers who (supposedly) have equal needs starts playing a role. Next to this ethical reason, there also is a pragmatic reason for taking the equity criterion into account: donors seem to be more willing to finance North Star's operations in case that a large part of the truck driver population benefits from them.

To account for the equity criterion in facility location decisions, one needs to clarify the exact meaning of equity. Though literature on equity stresses the importance of the subject, particularly in a resource allocation context, there is no consensus on its definition (Waters 2000). Young (1995) classifies resource allocation rules based on three equity concepts: parity (claimants should be treated equally), proportionality (goods should be divided in proportion to differences among claimants), and priority (the person with the greatest claim to the good should get it). Depending on the weights assigned to these concepts, many views on equity are possible, ranging from a totally egalitarian perspective (inequalities are unacceptable) and a Marxist perspective (inequalities should represent differences in need) to a Rawlsian perspective (inequality is only allowed if it benefits those least advantaged) (Williams and Cookson 2000).

Equity measures compare *effects* of actions on different groups, and possibly weigh such effect based on the characteristics of a group (e.g., needs or size). In contrast to the other two objectives, an abundance of literature is available on how to measure equity. Marsh and Schilling (1994) provide a list of 20 equity measures that have been developed in the facility location context. Though each of these measures prefer a completely equitable distribution over any other distribution, they differ in their valuations of inequitable distributions. More specifically, they assign different weights to the concepts of parity, proportionality, and priority. Though the measures might have been developed in a different context, they are easily transferrable to our situation. Let the *effect* for population q be represented by c_q. Some of the most common equity measures are the *sum of absolute deviations* (SAD), the *mean absolute deviation* (MAD), the *minimum effect* (ME), and the *Gini coefficient* (GC):

$$
\begin{aligned}
Z_{EQ}^{SAD} &= \sum_{q_1,q_2 \in Q | q_2 > q_1} |c_{q_1} - c_{q_2}| f_{q_1} f_{q_2} && (5)\\
Z_{EQ}^{MAD} &= \sum_{q \in Q} |c_q - \bar{c}| f_q && (6)\\
Z_{EQ}^{ME} &= \min_{q \in Q} c_q && (7)\\
Z_{EQ}^{GC} &= \frac{\sum_{q_1,q_2 \in Q | q_2 > q_1} |c_{q_1} - c_{q_2}| f_{q_1} f_{q_2}}{2 \sum_{q \in Q} c_q f_q} && (8)
\end{aligned}
$$

Here, $\bar{c} = \frac{\sum_{q \in Q} c_q f_q}{\sum_{q \in Q} f_q}$. Measure (7) only considers the population that is least well-off, and thereby neglects inequalities among the other populations. Note that increasing the effect for the least well-off is often not possible, as it might be impossible to provide each population of truck drivers with access to healthcare. Hence, this measure would probably have hardly any effect on the facility location decisions. The Gini coefficient (8), which is defined as the mean absolute deviation over the mean effect, in contrast, does consider the inequalities among all populations, but has the disadvantage that it is highly non-linear. We therefore believe that measures (5) and (6) (or comparable alternatives) are most useful in the context of our facility location problem. For the remainder of the paper, we measure equity as Z_{EQ}^{SAD}, and henceforth refer to this measure as the *equity score* Z_{EQ}. Note though that with some minor adaptations, the models and solution methods presented next can also be applied when equity is measured by Z_{EQ}^{MAD} or Z_{EQ}^{ME}.

There is a clear trade-off between this equity objective and the other two objectives. Locating RWCs so as to obtain more equitable access to healthcare will generally imply that RWCs are allocated to small and underserved populations of truck drivers. One thereby neglects the possibility to capture a much larger patient volume, and/or to ensure continuous access to a relatively well-served, but much larger population.

3 Set Partitioning Type Formulation

De Vries et al. (2014b) propose a MIP formulation for the location problem defined in Section 2. This formulation uses binary variables x_k to indicate whether or not an RWC is placed at potential location k and i_{klq} to identify pairs of RWCs (k, l) that are adjacent along route q. The latter variables are used to calculate the travel time gaps between facilities along the route (which yield the coverage scores c_q). We describe this formulation in Appendix A and refer to it as the *direct formulation*.

This section proposes an alternative, set partitioning type of problem formulation, which we refer to as the *set partitioning formulation*. In contrast with the direct formulation, this formulation uses binary variables to indicate whether or not to establish a *configuration of RWCs* along a given route. A configuration of RWCs defines for each RWC location along a path whether or not an RWC is located there, as illustrated in Figure 2. The main advantage of introducing these decision variables is that this allows us to pre-calculate the coverage scores c_q, in contrast with the direct formulation, which defines c_q as a variable.

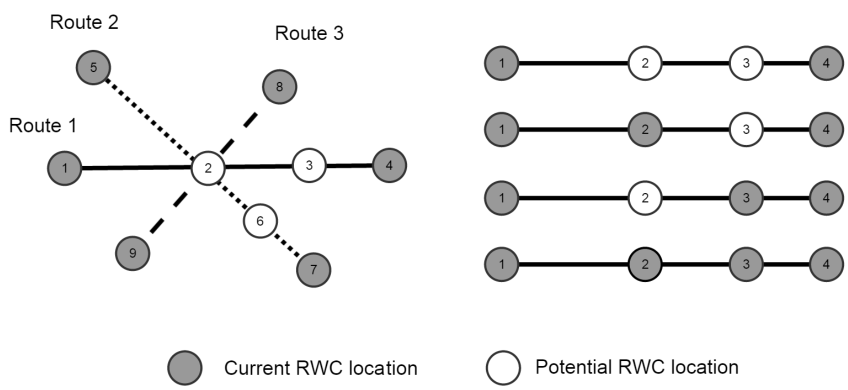


Figure 2: Four possible configurations of RWCs along truck route 1.

Let N_q denote the set of possible configurations for route q, indexed by n, and let N_q^k be the subset of configurations for route q with an RWC located at potential RWC location k. We use binary decision variables y_q^n to indicate whether or not configuration $n \in N_q$ is chosen for route q. Furthermore, parameter c_q^n denotes the coverage score of route q given that configuration $n \in N_q$ is realized, as calculated by (4). Using these notations, the location problem defined in Section 2 can be formulated as the following MIP model:

$$\max \quad w_{PV} \sum_{k \in KP} d_k x_k + w_{CA} \sum_{q \in Q} \sum_{n \in N_q} f_q c_q^n y_q^n - w_{EQ} \sum_{q_1 \in Q} \sum_{\substack{q_2 \in Q \\ q_2 > q_1}} \left(\Delta^+_{q_1 q_2} + \Delta^-_{q_1 q_2} \right) f_{q_1} f_{q_2} \tag{9}$$

$$\text{s.t.} \quad \sum_{n \in N_q} y_q^n = 1 \qquad q \in Q \tag{10}$$

$$\sum_{k \in KP} x_k = p \tag{11}$$

$$x_k - \sum_{n \in N_q^k} y_q^n = 0 \qquad k \in KP, q \in Q \tag{12}$$

$$\sum_{n \in N_{q_1}} c_{q_1}^n y_{q_1}^n - \sum_{n \in N_{q_2}} c_{q_2}^n y_{q_2}^n = \Delta^+_{q_1 q_2} - \Delta^-_{q_1 q_2} \qquad q_1, q_2 \in Q, q_2 > q_1 \tag{13}$$

$$x_k, y_q^n \in \{0, 1\} \qquad k \in KP, q \in Q, n \in N_q \tag{14}$$

$$\Delta^+_{q_1 q_2}, \Delta^-_{q_1 q_2} \geq 0 \qquad q_1, q_2 \in Q, q_2 > q_1 \tag{15}$$

Here, $\Delta^+_{q_1 q_2} = \max\{c_{q_1} - c_{q_2}, 0\}$ and $\Delta^+_{q_1 q_2} = \max\{-(c_{q_1} - c_{q_2}), 0\}$, so that $|c_{q_1} - c_{q_2}| = \Delta^+_{q_1 q_2} + \Delta^-_{q_1 q_2}$. The objective function (9) maximizes a weighted sum of the patient volume, the continuous access score and the equity score. Constraints (10) ensure that for each route $q \in Q$ only one configuration $n \in N_q$ is chosen. Next, constraint (11) ensures that the total number of newly placed RWCs is equal to p. The variables x_k and y_q^n are linked by constraints (12), enforcing the choice of a configuration $n \in N_q^k$ if and only if x_k equals 1. Constraints (13) determine the value of $\Delta^+_{q_1 q_2}$ and $\Delta^-_{q_1 q_2}$. Finally, constraints (14) define y_q^n and x_k as binary variables, and constraints (15) define $\Delta^+_{q_1 q_2}, \Delta^-_{q_1 q_2}$ as non-negative variables. Note that this model is closely connected to the set partitioning problem, as constraints (10) represent set partitioning constraints, and as we could express the model in terms of the "partitioning variables" y_q^n only (by substituting the variables x_k).

4 Column Generation Approach

Column generation has been proven to be a suitable technique for solving integer linear programming problems (ILPs) that involve a large number of variables. Instead of directly solving the linear programming relaxation of an ILP, called the Master Problem (MP), the method starts with a small subset of variables and solves the LP relaxation using this restricted variable set. This reduced problem is called the Restricted Master Problem (RMP).

Using the resulting dual variables as an input, the method identifies excluded variables that may improve the solution value, and adds them to the set of included variables. The problem to find these variables is called the Pricing Problem. This process repeats till optimality is proven. Detailed information about column generation techniques can be found in Desaulniers et al. (2005) and Lübbecke (2010).

As our set partitioning formulation induces an exponential number of variables, we set out to develop a column generation approach to solve its LP relaxation. As we will show in Section 6, the set partitioning formulation has a very tight LP relaxation. Therefore, instead of embedding our column generation approach in a computationally expensive Branch-and-Bound scheme, we propose a heuristic that directly constructs a feasible integer solution from the LP relaxation solution. This section presents our basic solution approach, which solves the pricing problem exactly using a MIP formulation. Section 5 proposes several acceleration techniques, including an alternative method for solving the pricing problem.

4.1 Restricted Master Problem

The RMP is obtained by relaxing the variables x_k and y_q^n from the set partitioning formulation (i.e., by defining them as continuous variables instead of discrete variables) and by including only a subset of all variables y_q^n. We denote the subset of configurations n for route q for which the variable y_q^n is included in the RMP by N_q^*. Furthermore, we represent the dual variables associated with constraints (10), (11), (12), and (13) by α_q, β, γ_{kq}, and $\delta_{q_1 q_2}$, respectively.

4.2 Pricing Problem

Generating a column for the RMP corresponds to finding a configuration $n \in N_q \setminus N_q^*$ for some route q for which the so-called reduced costs of variable y_q^n is positive. Let parameter a_{kq}^n indicate whether or not configuration n for route q has an RWC located at potential RWC location k. Then the reduced costs of the corresponding variable y_q^n are calculated as:

$$r_n^q = -\alpha_q + \sum_{k \in KP_q} a_{kq}^n \gamma_{kq} - c_q^n \rho_q \tag{16}$$

Here, $\rho_q = \sum_{u<q} \delta_{uq} - \sum_{u>q} \delta_{qu} - w_{CA} f_q$. Note that the reduced costs for y_q^n are independent of the characteristics of other routes, and hence that the pricing problem can be separated per route q:

$$\max_{n \in N_q} -\alpha_q + \sum_{k \in KP_q} a^n_{kq}\gamma_{kq} - c^n_q \rho_q \tag{17}$$

This problem can be regarded as an instance of the Roadside Healthcare Facility Location Problem with $|Q| = 1$, and hence can be solved using the MIP formulation presented in De Vries et al. (2014b). We describe the MIP formulation of (17) in Appendix B.

4.3 Initialization and Termination Criterion

We start the column generation approach with a small subset of variables y^n_q for route q: the ones corresponding to configurations *without* new RWCs, configurations with *one* new RWC and configurations with an RWC at *each* potential RWC location along the route.

Solving the pricing problem for route q to optimality, we obtain one configuration n^*_q having reduced costs $r^*_q = r^q_{n^*_q}$. Because $y^n_q \leq 1$, LP duality implies:

$$z_{RMP} \leq z^*_{MP} \leq z_{RMP} + \sum_{q \in Q} r^*_q = UB$$

Here, z^*_{MP}, z_{RMP}, and UB denote the optimal solution value of the Master Problem, the optimal solution value of the RMP, and an upper bound on the optimal solution value of our location problem, respectively. The last equality follows from the observation that the MP is a relaxation of the location problem. Our column generation algorithm stops when $\frac{UB - z_{RMP}}{z_{RMP}} < \epsilon$, where ϵ is a fixed threshold.

4.4 Obtaining Integer Solutions

In each iteration (i.e., each time we solve the RMP), we use refined search to build a feasible solution from the fractional RMP solution. We first select the $p+s$ potential RWC locations with the largest value for x_k, where s denotes some strictly positive integer. Afterwards, we perform a refined search, identifying p out of the $p+s$ locations that yield a high solution value. Specifically, we first choose the $p-s$ locations with the largest value for x_k. Next, we complement these by s out of the $2s$ remaining locations, evaluating each of the $\binom{2s}{s}$ possibilities. If the resulting solution has a higher solution value then the best known integer solution, we replace the best solution by the current solution.

Our numerical experiments show that such heuristic solution often has a higher solution value than the corresponding solution of the RMP. We therefore add the variables corresponding to the configurations that produce a heuristic solution to the variable set of the RMP.

5 Acceleration Strategies

Column generation solution approaches often converge slowly towards the optimum. Causes include a large number of iterations needed to reach convergence, and large solution times for the pricing problem and/or the restricted master problem. This section proposes five acceleration strategies tackling these causes: accelerated pricing, adding multiple columns, column pool management, dual stabilization, and a 2-stage approach.

5.1 Accelerated Pricing

Solving the pricing problem using the MIP model is computationally expensive. Instead of directly solving the pricing problem to optimality, the method presented next tries to quickly identify an attractive (note, not necessarily optimal) variable using a sequence of shortest path problems, and only uses the MIP model if no attractive column is identified. Next, we introduce some theoretical results inspiring this approach.

Let e_q^n represent the fraction of time a truck driver traveling route q is "safe" when choosing configuration n (see Section 2.2). Furthermore, let $g_1(e_q^n) = 0$, $g_2(e_q^n) = \frac{e_q^n - \mu_1}{\mu_2 - \mu_1}$, and let $g_3(e_q^n) = 1$, i.e. these functions are the extrapolations of the three function segments of $g(\cdot)$. Next, consider the set of configurations n for which e_q^n lies in a given segment σ_i: N_{qi}. We relate to this set the acyclic graph G_{qi} depicted in Figure 3. This graph has one node for each RWC location along route q, including origin node $O := KC_q(1)$ and destination node $D := KC_q(m)$. Let ϕ^n denote the $O - D$ path visiting the nodes corresponding to the RWCs in configuration n. In Appendix C, we prove the following result:

Proposition 5.1. *There exist arc weights for graph G_{qi} such that for each $n \in N_{qi}$ holds that the length of the corresponding $O - D$ path $l(\phi^n)$ equals $-r_q^n$*

Proof. See Appendix C $\square$

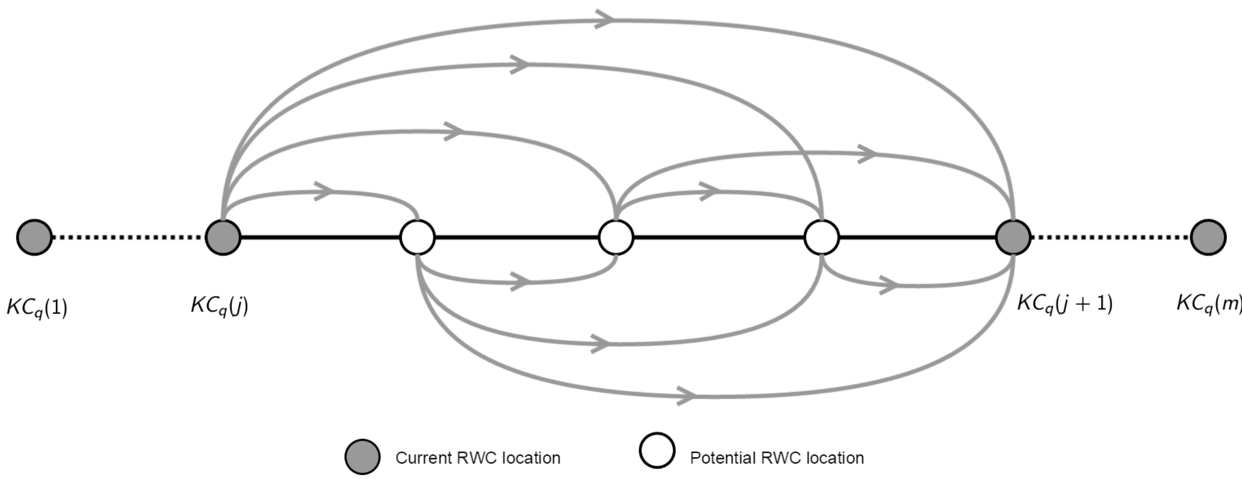


Figure 3: Directed graph G_{qi} for route q. Each node corresponds to an RWC location. The nodes visited in a path ϕ^n from $KC_q(1)$ to $KC_q(m)$ correspond to the RWCs located in configuration n.

A direct implication of Proposition 5.1 is that configurations $n \in N_{qi}$ for which r_q^n is larger correspond to shorter paths in the acyclic graph. This inspires the following solution approach for the pricing problem corresponding to route q (see Algorithm 1). We construct the acyclic graph corresponding to segment σ_1 and solve the shortest path problem for this graph using the Bellman-Ford algorithm. Next, we calculate $r_q^{\hat{n}}$ for the configuration $\hat{n}$ corresponding to the shortest path. If $r_q^{\hat{n}} > 0$, we mark variable $y_q^{\hat{n}}$ as an attractive variable. We repeat this procedure for segment σ_2, and add the attractive variable having the largest reduced costs to the set of variables included in the RMP. (The proof of Proposition 5.1 implies that the configurations obtained for segments σ_1 and σ_3 are the same, so that we do not need to repeat the procedure for σ_3).

Note that this first part of our solution approach is not exact. Since the graph corresponding to segment σ_i misrepresents the reduced costs corresponding to configurations $n \notin N_{qi}$, the configuration found may not be the one with the largest reduced costs. In some cases, however, we can use the configurations obtained to prove that no configuration with positive reduced costs exists. For example, let the configurations found for the graphs G_{q1} and G_{q2} be represented by n_1 and n_2, respectively. Suppose that $e_q^{n_1} \in \sigma_3$ and $e_q^{n_2} \in \sigma_2$, that $0 \geq r_q^{n_2} \geq r_q^{n_1}$, and that $\rho_q > 0$. By optimality of n_2 know that for each of the configurations n for which $e_q^n \in \sigma_2$ holds that $r_q^n \leq r_q^{n_2}$. Furthermore, because $\rho_q > 0$, we know that for each of the configurations n for which $e_q^n \in \sigma_1 \cup \sigma_3$ holds that $r_q^n \leq r_q^{n_1} \leq r_q^{n_2}$ (see equation (16)), showing that for each n holds that $r_q^n \leq r_q^{n_2} \leq 0$.

In case that we cannot prove that a column with positive reduced costs does not exist, we set out to find one using a refined search. Specifically, we propose to solve the pricing problem $|KP_q|$ times. In each time, we adapt the graphs G_{qi} such that they enforce locating an RWC at a specific potential RWC location $k \in KP_q$ and optimize the other location decisions using the Bellman-Ford algorithm. Next, we add the attractive variable having the largest reduced costs to the set of variables included in the RMP. If also the refined search fails to identify an attractive variable, we solve the pricing problem using the MIP model presented in Appendix B.

Algorithm 1 *Pricing algorithm for route q*

Require:
route q and dual variables α_q, β, γ_{kq}, and $\delta_{q_1q_2}$
Ensure:
variable with positive reduced cost or certificate that no such variable exists

construct graphs G_{q1} and G_{q2}
obtain configuration(s) using the Bellman-Ford algorithm
if reduced costs of at least one of the corresponding variables are positive **then**
    return the variable with the largest reduced costs
else if certificate that no such variable exists **then**
    stop
else apply refined search
    if variable with positive reduced cost found **then**
        return the variable with the largest reduced costs
    else
        solve pricing problem as a MIP
        return a variable with positive reduced costs or a certificate that no such variable exists
    end if
end if

Note that we apply this solution method to each route q, yielding up to $|Q|$ variables to be included in the RMP per iteration.

5.2 Adding Multiple Columns

The algorithm presented in the previous subsection adds in each iteration at most one variable per route to the set of variables included in the RMP. Adding *multiple* attractive variables per iteration is a common acceleration technique. The ideas underlying it are that any variable with positive reduced costs is attractive for entering the RMP, that it is often easy to find multiple attractive variables, and that adding multiple variables tends to significantly decrease the number of iterations required to solve the RMP. We propose to adapt steps 3 - 7 of Algorithm 1 as follows: we start with obtaining the optimal configuration(s) corresponding to the two generated graphs *and* perform the refined search algorithm. All variables with positive reduced costs found are then added to the variable set of the RMP.

5.3 Column Pool Management

We save each column generated in our the column generation approach in a column pool. Before each iteration, we determine for each column in the pool whether or not to include it in the RMP. The aim is to balance the purpose to maximize the objective value of the RMP (by including as many attractive columns as possible) and on the other hand the aim to minimize the solution time for the RMP (by minimizing the number of columns included). Specifically, we include a column if at least one of the following conditions holds: (1) it was generated in the initialization, (2) it resulted from the previous call to the pricing problem, (3) it has been part of the basis at least once in the last κ iterations, where κ denotes some strictly positive integer (4) it has reduced costs greater than a given percentage of the best reduced cost among all columns available for that path, (5) it is part of an integer solution found that has an objective value greater than the corresponding RMP relaxation.

5.4 Dual Stabilization

Column generation is often characterized by large oscillations in the dual variables from iteration to iteration. This leads to slow convergence and often to degeneracy. Several techniques have appeared that stabilize the dual variables and thereby accelerate the convergence (see Desaulniers et al. 2005). We propose to call the pricing problem twice per iteration. The first call provides the dual solution obtained in the current iteration, π^*, as an input. The second call provides $\pi_{ST} = \lambda\hat{\pi} + (1-\lambda)\pi^*$ as an input, as suggested by Pessoa et al. (2010). Here, $\hat{\pi}$ represents the best known dual solution (i.e., the dual that produces the smallest upper bound) and λ denotes some constant between 0 and 1.

5.5 2-Stage Approach

The time required to solve the RMP is relatively large. This seems to be caused by the large number of variables and constraints needed to calculate the equity score. In response to that, we implement a 2-Stage column generation approach. In the 1^{st} stage we solve the RMP without constraints (13). This corresponds to leaving the equity score out of consideration. In the 2^{nd} stage, we include constraints (13), and solve the full RMP with the columns generated so far.

6 Computational Experiments

This section numerically analyzes the column generation approach presented in Sections 4 and 5. Section 6.1 starts with describing the randomly generated networks used for this analysis. Next, we analyze the integrality gap for the direct formulation and the set partitioning formulation in Section 6.2. For our baseline case, Section 6.3 describes the convergence of the column generation approach and the impact of each of the acceleration strategies. Finally, Section 6.4 analyzes the computational performance of the column generation approach in detail for the entire set of instances. All mathematical models are implemented in Java, using ILOG CPLEX v12.61 as a solver, and analyzed using a computer running Windows 8.1 with a 2.3 GHz i7-4712HQ processor and 16GB of RAM.

6.1 Randomly Generated Instances

We generate our problem instances as follows. First, we generate the locations of n_{OD} O-D nodes in $[0, 1000]^2$ according to a continuous uniform distribution. We calculate Euclidian distances between nodes, and take the minimum spanning tree of the full graph. Next, the full road network is obtained by adding for each node n_a additional arcs, connecting it to the n_a closest nodes it has not yet been connected to in the minimum spanning tree. We assume that each O-D node represents a current RWC location (in accordance with Assumption 1), and that currently no RWC is located elsewhere. The locations of n_p potential RWC locations are generated by drawing completely random positions at the full road network (hence, each location along the full road network has equal probability of being selected). Next, we generate for each O-D node n_r truck routes that have this node as an origin, and randomly select one of the other O-D nodes as its destination (if a generated flow already exists, we draw a different destination). The specific routes corresponding to the truck flows are obtained by applying a shortest-path algorithm. The truck driver volume parameters f_q are calculated as $f_q = \eta_{O_q}\eta_{D_q}$, and normalized such that $\sum_{q \in Q} f_q = 100 n_{OD} n_r$ (hence, the average truck driver volume equals 100). Here, η_{O_q} and η_{D_q} denote the population at the origin and destination of flow q, respectively, which are drawn from a uniform distribution on $[0, 1]$. Finally, we generate the patient volume parameters as $d_k = d_k^f + d_k^v$, where $d_k^f = 10$

represents the fixed patient volume and d_k^v the patient volume that depends on the flow volume passing this location, f_k. The first reflects the fact that North Star only selects potential locations at which the expected patient volume exceeds a given threshold. We draw d_k^v from a Gamma distribution with shape parameter f_k and scale parameter 1, and normalize them such that $\sum_{k \in K} d_k^v = 20|K|$ afterwards (hence, the average patient volume equals 10 + 20).

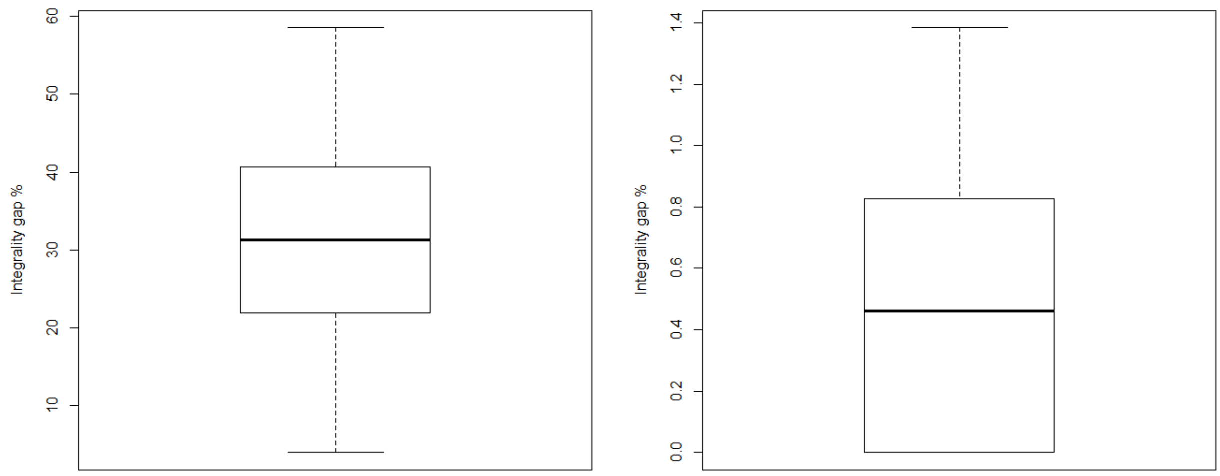
	Instance Names	Quantity	n_{OD}	n_r	n_p
Medium	r75p150n1, r75p150n2,...	10	15	5	150
	r125p150n1, r75p150n2,...	10	25	5	150
	r200p100n1, r200p100n2,...	10	40	5	100
	r200p200n1, r200p200n2,...	10	40	5	200
Large	r320p600n1, r320p600n2,...	3	80	4	600
	r500p1000n1, r500p1000n2,...	3	100	5	1000
	r2400p5000n1, r2400p5000n2,...	3	300	8	5000

Table 1: Characteristics of the randomly generated instances

Table 1 describes the characteristics of 49 randomly generated instances used for our numerical experiments. As for notation, an instance named $r75p150n1$ is the first instance in the set of instances having 75 routes and 150 potential locations. The instances can be accessed at `http://people.few.eur.nl/hdevries`. Finally, we use the following parameter values in our experiments: $p = 20, \mu_1 = 0.4, \mu_2 = 0.8, \tau = 100, w_{PV} = 10, w_{CA} = 1.5$ and $w_{EQ} = 10^{-4}$. The values of the latter three parameters were chosen so as to reflect the trade-offs decision makers face in practice.

6.2 Integrality Gap

Figure 4 shows boxplots of the integrality gap (i.e., the gap between the optimal solution value and the optimal solution value of the LP relaxation) for the direct formulation and the set partitioning formulations, based on the 40 medium instances (large instances were excluded, as many could not be solved to optimality). We observe that the integrality gap for the direct formulation is much larger: it generally ranges between 20% and 40% whereas the gap for the set partitioning formulation is generally smaller than 1%. Furthermore, solving the LP relaxation of the direct formulation results in an integer (i.e., an optimal) solution for 20% of the instances, highlighting the strength of the formulation.



(a) Direct formulation

(b) Set partitioning formulation

Figure 4: Integrality Gap

6.3 Effect of Acceleration Strategies

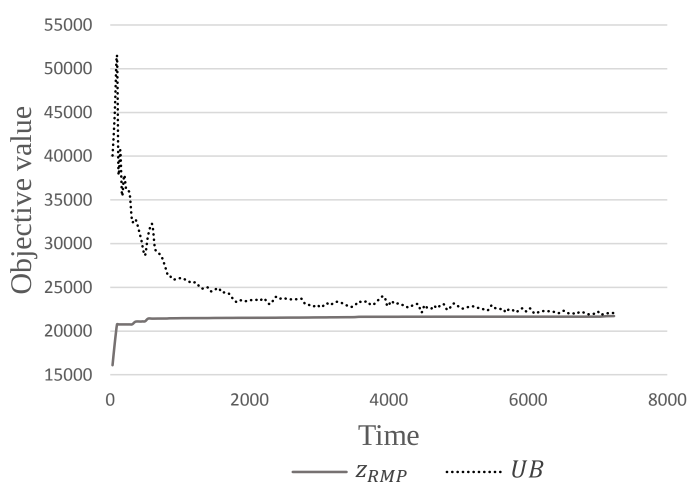


Figure 5: Convergence of the basic column generation approach for instance r200p200n1

Figure 5 shows the convergence of the basic column generation approach for our baseline case $r200p200n1$. It confirms the slow convergence behavior that has motivated several of our acceleration techniques. The effects of these techniques in terms of the optimality gap reached after 500 seconds (note, we know the optimal solution by solving the location problem exactly), the time/number of iterations needed to reach convergence, the time/number of iterations needed to find the final solution (i.e., the best solution found), and the total number of columns generated/included in the last iteration are described for the baseline case in Table 2. This table provides these statistics for the following solution approaches: (1) the basic approach (BA), as presented in Section 4, (2) approach 1 + accelerated pricing (AP), (3) approach 2 + adding multiple columns (MC), (4) approach 3 + column pool management (CM), (5) approach 4 + dual stabilization (DS), and (6) approach 5 + 2-Stage approach (2S).

Solution approach	**1**	**2**	**3**	**4**	**5**	**6**
Opt. gap after 500 sec. (%)	2.314	0.759	1.130	0.000	0.684	0.000
Time final solution (sec.)	3026	6368	1410	495	946	375
Time convergence (sec.)	>7200	>7200	1410	1387	946	393
Iterations to final solution	57	120	28	10	22	23
Iterations to convergence	>130	>133	28	35	22	24
Columns last iteration	7573	5764	14432	13232	12616	9073
Total columns generated	7573	5764	14432	16706	18458	10031

Table 2: Impact of acceleration strategies for instance $r200p200n1$. Solution approaches: 1 = BA, 2 = 1 + AP, 3 = 2 + MC, 4 = 3 + CM, 5 = 4 + DS, and 6 = 5 + 2S

Though each approach identified the optimal solution, the table shows that the acceleration techniques jointly result in a vast improvement in the computational performance of the column generation algorithm. Particularly adding multiple columns, column pool management, and the 2-stage approach have a large impact. This seems to be explained by the fact that MC greatly reduces the number of iterations and that CM and 2S significantly reduce the time required to solve the RMP (which accounts for approximately 99% of the solution time).

Figure 6 provides a more detailed view on the performance of the acceleration techniques, showing the development of the solution value for solution approaches 1, 4, and 6 (for sake of clarity we excluded the others from the figure). It shows that the approach including all acceleration techniques obtains near-optimal solutions within a couple of seconds.

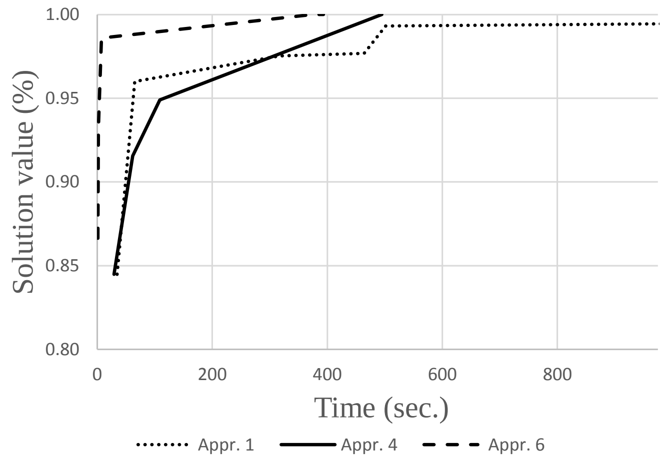


Figure 6: Development of the solution value over time for solution approaches 1 (BA), 4 (BA + AP + MC + CM), and 6 (BA + AP + MC + CM + DS + 2S), measured as a fraction of the optimal solution value

6.4 Overall Results

Next, we analyze the performance of the column generation approach for the full set of instances and compare it with the performance of CPLEX 12.61 on the direct formulation. The specific column generation approach we consider here includes each of the acceleration strategies, and will be referred to as the 2-stage approach from now on. Table 3 describes the performance of the two approaches for the 40 medium instances. The 2-stage approach obtains the optimal solution in 34 of the 40 instances, and yields a negligible optimality gap for the others. Columns t_{CONV} and t_{SOL} contain the average time to convergence and the average time to finding the final solution (i.e., the best solution found) for both approaches. They show that average solution times for both approaches are comparable and that the approach using CPLEX on the direct formulation seems to outperform the 2-stage approach in terms of time to convergence. Columns t_{RMP} and t_{PP}, providing the average time the 2-stage approach spends on the RMP and the pricing problem, highlight the efficiency of our approach for solving the pricing problem and stress the importance to decrease computation times for the RMP.

Table 4 describes some more details about the 2-stage solution approach, including the average number of iterations till convergence, the average number of columns included in the last iteration, the average total number of columns generated during the execution of the algorithm, and the average total number of columns in the instances (i.e., included in the

	Avg. optimality gap (%)		Avg. computation time					
	DF	2S	DF		2S			
			t_{SOL}	t_{CONV}	t_{SOL}	t_{CONV}	t_{RMP}	t_{PP}
r75p150	0.00%	0.01%	3.28	4.75	5.61	12.05	3.32	0.30
r125p150	0.00%	0.03%	44.38	88.84	19.75	143.52	73.70	5.09
r200p100	0.00%	0.03%	30.11	57.49	40.45	60.28	32.29	0.53
r200p200	0.00%	0.00%	133.84	230.05	169.2	1056.68	890.1	7.05

Table 3: Performance of CPLEX 12.61 on the direct formulation (DF) and the column generation approach including each of the acceleration strategies (2S) on the 40 medium problem instances

master problem). The huge number of columns corresponding to the larger instances show that a column generation solution approach is crucial for the set partitioning formulation.

	#Iterations	#Columns		
		Last Iteration	Generated	Total
r75p150	32	4,224	4,950	345,623
r125p150	60	9,704	16,871	976,515
r200p100	23	3,035	3,158	19,237
r200p200	58	14,677	32,164	2,476,941

Table 4: Average numbers of iterations performed and columns generated by the 2-stage approach on the 40 medium problem instances

The computational benefits of the 2-stage approach become particularly visible when large problem instances are considered. Though both formulations did not converge after the time limit of 1 hour, Table 5 shows that the value of the best solution obtained by the 2-stage approach is on average much higher. For the set of largest instances, CPLEX even fails to produce an integer solution within the time limit.

Instance	Obj. value		Difference
	DF	2S	
r320p600n1	16888.75	17001.31	112.57
r320p600n2	24863.54	24704.83	-158.70
r320p600n3	23539.43	22933.96	-605.48
r500p1000n1	13397.37	14759.82	1362.45
r500p1000n2	7977.70	8594.52	616.81
r500p1000n3	-6661.87	15562.55	22224.42
r2400p5000n1	-	227090.11	-
r2400p5000n2	-	222021.25	-
r2400p5000n3	-	222671.04	-

Table 5: Objective value obtained after 1 hour for the 9 large problem instances using approaches DF and 2S.

7 Sensitivity Analysis on Equity

As motivated, it is becoming more and more relevant to have insight in the trade-off between the equity criterion and North Star's current optimization criteria. This section investigates this trade-off for a small problem instance: $r200p50n1$. We solve this instance to optimality for 300 settings of the weight parameters w_{PV}, w_{CA}, and w_{EQ}, yielding a large number of Pareto efficient solutions. Figures 7 and 8 show their relative optimality with respect to each of the three criteria, measured as a fraction of the highest attainable value for these criteria. The filled dots in such figure represent solutions that are not only Pareto efficient with respect to the three criteria, but also with respect to the two criteria considered in the figure.

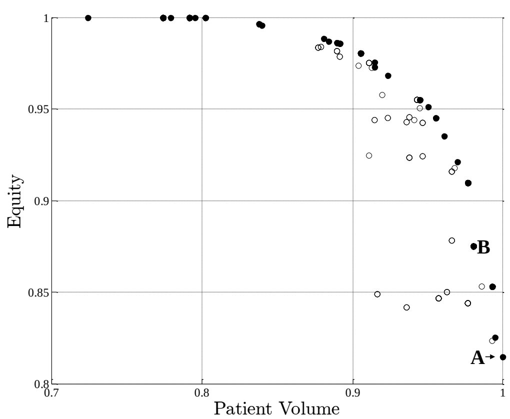


Figure 7: Relative optimality with respect to patient volume and equity.

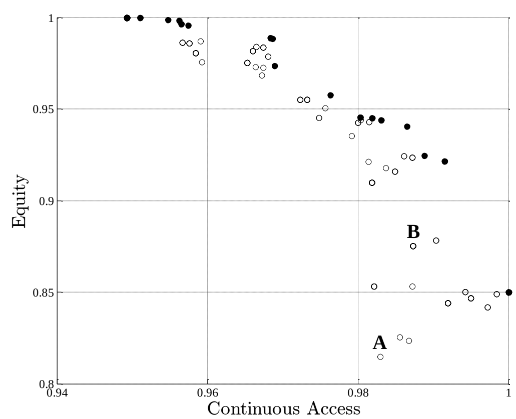


Figure 8: Relative optimality with respect to continuous access and equity.

Note: A point in these figures shows the relative optimality of the corresponding solution with respect to two optimization criteria. Here, relative optimality with respect to a criterion is measured as the fraction of the highest attainable value for that criterion (based on p new facilities).

We see that considerable gains in terms of equity can be made at a marginal cost in terms of continuous access and/or patient volume. For example, choosing the solution corresponding to point B (equity is assigned a significant weight) in these figures instead of the solution corresponding to point A (equity is hardly considered) increases the equity score by 7.2% and decreases the patient volume by only 2.0% (and increases the continuous access score by 0.4%). More generally, our results suggest that it is possible to obtain solutions that are close to optimal with respect to each optimization criterion, as illustrated by the fact that for 6 of the Pareto efficient solutions the relative optimality with respect to *each* of the three criteria equals at least 94%.

Figure 9 illustrates the impact of including the equity criterion on the actual levels of access provided in solutions A and B, showing the distribution of the coverage scores for the entire truck driver population we consider. As expected, increasing the importance of equity significantly decreases the number of truck drivers having a coverage score between 0 and 0.25.

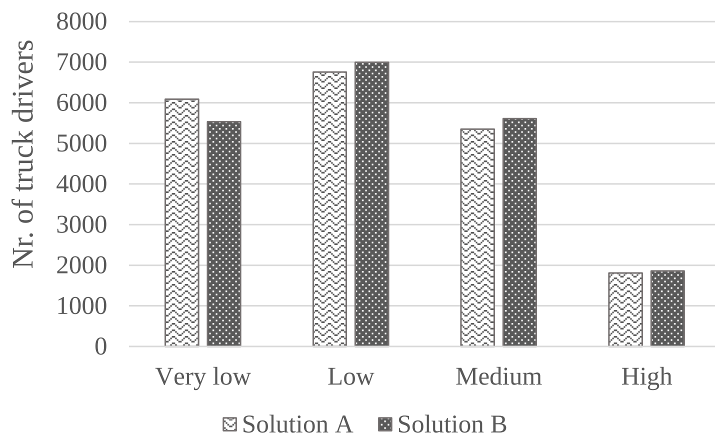


Figure 9: Coverage scores for truck driver population in Pareto efficient solutions A (equity is hardly considered) and B (equity is assigned a significant weight). Very low: $c_q \in [0, 0.25)$, low:$c_q \in [0.25, 0.5)$, medium: $c_q \in [0.5, 0.75)$, high: $c_q \in [0.75, 1]$.

8 Conclusions and Discussion

This paper considers the problem to locate a given number of new RWCs based on three optimization criteria: maximize patient volume, ensure continuity of access, and establish an equitable health system. We first propose several measures of equity of access to healthcare in the context of a mobile patient population, based on a review of the literature available. Measures for the patient volume criterion and the continuous access criterion are taken from previous work on this problem. The equity criterion significantly increases the complexity of the location problem, and existing models and solution methods will be unsuitable for solving large instances. We therefore come up with a novel, set-partitioning type of formulation for the problem. Numerical experiments show that, in contrast with the formulation previously introduced, the integrality gap for this formulation is very small. As this formulation requires an exponential number of variables, we propose a column generation algorithm to solve it. Additionally, we propose and analyze several state-of-the-art acceleration techniques, including accelerated pricing, dual stabilization and a 2-stage approach. Though the facility location problem is strongly $\mathcal{NP}$-hard, our algorithm yields near-optimal solutions to large randomly generated problem instances within an acceptable amount of time.

Our numerical analysis of the trade-off between equity and North Star's current optimization criteria provides some interesting insights. Considerable gains in terms of equity can be made at a marginal loss in terms of patient volume and/or continuous access. Furthermore, our results suggest that solutions that are close to optimal in terms of each of the three criteria are attainable by appropriately balancing the weights attached to them, providing a strong argument for considering the equity criterion in future network design decisions.

Although our solution approach solves large problem instances to near-optimality, we see several opportunities for improvement. First, our approach can be transformed into an exact approach by embedding it in a Branch-and-Price scheme. Furthermore, we notice that our formulation weakens when the relative importance of the equity criterion increases. Cutting plane techniques may be beneficial or even necessary in this context. Finally, we use a relatively simple heuristic for generating feasible integer solutions. More advanced (meta)heuristics, taking better advantage of the benefits of placing multiple facilities along a route, seem to be promising. We believe that our solution method and subsequently the solutions proposed can greatly benefit from advancing further research in these areas.

Acknowledgments

We are grateful to Kees Kooiman, Danielle Nativ and Jolien Rip for their contributions at an early stage of this work.

A Direct Formulation

The direct formulation uses the following notations in addition to those introduced in the paper. K_{kq} represents the set of locations that are passed *after* location k during a trip from along route q. Variable $i_{klq}, k \in K_q, l \in K_{kq}$ equals 1 if locations k and l are adjacent RWCs along route q (i.e., RWCs are located at both locations, and there is no RWC between them), and equals 0 otherwise. Finally, λ_{iq} and z_{iq} are auxiliary variables to model the piecewise linear function $g(\cdot)$

$$\max \quad w_{PV}\sum_{k\in K} d_k x_k + w_{CA}\sum_{q\in Q} f_q c_q - w_{EQ}\sum_{q_1\in Q}\sum_{\substack{q_2\in Q\\ q_2>q_1}}\left(\Delta^+_{q_1q_2} - \Delta^-_{q_1q_2}\right) f_{q_1} f_{q_2} \tag{A.1}$$

$$\text{s.t. } c_q = \lambda_{3q} + \lambda_{4q} \qquad q \in Q \tag{A.2}$$

$$\lambda_{1q}0 + \lambda_{2q}\mu_1 + \lambda_{3q}\mu_2 + \lambda_{4q} = \frac{1}{T_q}\left(\sum_{k\in K_q}\sum_{l\in K_{kq}} i_{klq}\min\{t_{kl},\tau\}\right) \qquad q \in Q \tag{A.3}$$

$$\lambda_{1q} + \lambda_{2q} + \lambda_{3q} + \lambda_{4q} = 1 \qquad q \in Q \tag{A.4}$$

$$\lambda_{1q} \leq z_{1q} \qquad q \in Q \tag{A.5}$$

$$\lambda_{2q} \leq z_{1q} + z_{2q} \qquad q \in Q \tag{A.6}$$

$$\lambda_{3q} \leq z_{2q} + z_{3q} \qquad q \in Q \tag{A.7}$$

$$\lambda_{4q} \leq z_{3q} \qquad q \in Q \tag{A.8}$$

$$z_{1q} + z_{2q} + z_{3q} = 1 \qquad q \in Q \tag{A.9}$$

$$c_{q_1} - c_{q_2} = \Delta^+_{q_1q_2} - \Delta^-_{q_1q_2} \qquad q_1, q_2 \in Q, q_2 > q_1 \tag{A.10}$$

$$x_k = 1 \qquad k \in KC \tag{A.11}$$

$$\sum_{k\in KP} x_k = p \tag{A.12}$$

$$\sum_{l\in K_q} i_{klq} = x_k \qquad k \in K_q, q \in Q \tag{A.13}$$

$$\sum_{l\in K_q} i_{klq} = 1 \qquad k \in KC_q(1), q \in Q \tag{A.14}$$

$$\sum_{k\in K_q} i_{klq} = x_l \qquad l \in K_q, q \in Q \tag{A.15}$$

$$\sum_{k\in K_q} i_{klq} = 1 \qquad l \in KC_q(m), q \in Q \tag{A.16}$$

$$x_k \in \{0,1\} \qquad k \in K_q, q \in Q \tag{A.17}$$

$$\Delta^+_{q_1q_2}, \Delta^-_{q_1q_2} \geq 0 \qquad q_1, q_2 \in Q, q_2 > q_1 \tag{A.18}$$

$$i_{klq} \in [0,1] \qquad k \in K_q, l \in K_{kq}, q \in Q \tag{A.19}$$

$$\lambda_{iq} \geq 0, z_{iq} \in \{0,1\} \qquad i \in \{1,2,3,4\}, q \in Q \tag{A.20}$$

The objective function (A.1) represents a weighted sum of the patient volume, the continuous access score, and the equity score. Constraints (A.3) - (A.9) define c_q as a piece-wise linear function of the fraction of time a truck driver travelling route q is "safe" during his trip. This fraction is calculated by the right-hand side of (A.3). Next, constraints (A.10) enable the calculation of $|c_{q_1} - c_{q_2}|$. The current network of RWCs is described in constraint (A.11). Constraints (A.13)-(A.16) ensure that the variables i_{klq} attain the correct value. Finally, the decision variables are defined in (A.17) - (A.20).

B MIP Formulation Pricing Problem

$$\max \quad -\alpha_q + \sum_{k \in KP_q} \gamma_{kq} x_k - \rho_q c_q \tag{B.1}$$

$$\text{s.t. } c_q = \lambda_{3q} + \lambda_{4q} \tag{B.2}$$

$$\lambda_{1q}0 + \lambda_{2q}\mu_1 + \lambda_{3q}\mu_2 + \lambda_{4q} = \frac{1}{T_q}\left(\sum_{k \in K_q}\sum_{l \in K_{kq}} i_{klq} \min\{t_{kl}, \tau\}\right) \tag{B.3}$$

$$\lambda_{1q} + \lambda_{2q} + \lambda_{3q} + \lambda_{4q} = 1 \tag{B.4}$$

$$\lambda_{1q} \leq z_{1q} \tag{B.5}$$

$$\lambda_{2q} \leq z_{1q} + z_{2q} \tag{B.6}$$

$$\lambda_{3q} \leq z_{2q} + z_{3q} \tag{B.7}$$

$$\lambda_{4q} \leq z_{3q} \tag{B.8}$$

$$z_{1q} + z_{2q} + z_{3q} = 1 \tag{B.9}$$

$$x_k = 1 \qquad k \in KC_q \tag{B.10}$$

$$\sum_{l \in K_q} i_{klq} = x_k \qquad k \in K_q \tag{B.11}$$

$$\sum_{l \in K_q} i_{klq} = 1 \qquad k \in KC_q(1) \tag{B.12}$$

$$\sum_{k \in K_q} i_{klq} = x_l \qquad l \in K_q \tag{B.13}$$

$$\sum_{k \in K_q} i_{klq} = 1 \qquad l \in KC_q(m) \tag{B.14}$$

$$x_k \in \{0,1\} \qquad k \in K_q \tag{B.15}$$

$$i_{klq} \in [0,1] \qquad k \in K_q, l \in K_{kq} \tag{B.16}$$

$$\lambda_{iq} \geq 0, z_{iq} \in \{0,1\} \qquad i \in \{1,2,3,4\} \tag{B.17}$$

See Appendix A for the interpretation of this model. Solving the problem formulated in this model yields location decisions x_k, which define a configuration of RWCs n for route q.

C Proof of Proposition 5.1

Let K_q^n denote the ordered set of RWC locations along route q for which $x_k = 1$ in configuration n. Our proof of Proposition 5.1 makes use of the following lemma:

Lemma C.1. *Suppose that e_q^n lies in segment σ_i of the piecewise linear function $g(\cdot)$. Then*

$$c_q^n = \sum_{j=1}^{m-1} g_i \left(\frac{\min\{t_{\left(K_q^n(j), K_q^n(j+1)\right)}, \tau\}}{T_q} \right) \tag{C.1}$$

Proof. This immediately follows from definition (4), and from the fact that the linearity of $g_i(\cdot)$ implies that the summation can be taken out of this function. □

Proposition 5.1. *There exist arc weights for graph G_{qi} such that for each $n \in N_{qi}$ holds that the length of the corresponding $O - D$ path $l(\phi^n)$ equals $-r_q^n$.*

Proof. We define the weights corresponding to the arcs in this graph as follows. First, we add α_q to all arcs departing from the $K_q(1)$ (the current RWC location at the origin of the route) and add $-\gamma_{kq}$ to all incoming arcs for potential RWC location $k \in KP_q$. Finally, we add $g_i\left(\frac{\min\{t_{kl}, \tau\}}{T_q}\right)\rho_q$ to each arc (k,l).

To prove our claim, let us define $l(\phi^n)$. Note that path ϕ^n must use an arc departing from the node corresponding to $K_q(1)$, so that it's length includes the term α_q. Furthermore, it includes the term $-\gamma_{kq}$ for each visited node corresponding to a potential RWC location $k \in KP_q$. Finally, the length includes the term $g_i\left(\frac{\min\left\{t_{\left(K_q^n(j), K_q^n(j+1)\right)}, \tau\right\}}{T_q}\right)\rho_q$ for each of the arcs corresponding to the pair of RWC locations $\left(K_q^n(j), K_q^n(j+1)\right)$. Hence, using Lemma C.1 we obtain that:

$$
\begin{aligned}
l\left(\phi^{n}\right) &= \alpha_{q} - \sum_{k \in KP_{q}} a_{kq}^{n} \gamma_{kq} + \sum_{j=1}^{m-1} g_{i} \left(\frac{\min\{t_{\left(K_{q}^{n}(j), K_{q}^{n}(j+1)\right)}, \tau\}}{T_{q}} \right) \rho_{q} \\
&= \alpha_{q} - \sum_{k \in KP_{q}} a_{kq}^{n} \gamma_{kq} + c_{q}^{n} \rho_{q} = -r_{q}^{n}
\end{aligned}
$$

This concluses the proof. $\square$

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