TI 2006-075/1
Tinbergen Institute Discussion Paper

Going where the Ad leads you: On High Advertised Prices and Search where to buy

Maarten C.W. Janssen
Marielle C. Non

Faculty of Economics, Erasmus Universiteit Rotterdam, and Tinbergen Institute.

Tinbergen Institute
The Tinbergen Institute is the institute for economic research of the Erasmus Universiteit Rotterdam, Universiteit van Amsterdam, and Vrije Universiteit Amsterdam.

Tinbergen Institute Amsterdam
Roetersstraat 31
1018 WB Amsterdam
The Netherlands
Tel.: +31(0)20 551 3500
Fax: +31(0)20 551 3555

Tinbergen Institute Rotterdam
Burg. Oudlaan 50
3062 PA Rotterdam
The Netherlands
Tel.: +31(0)10 408 8900
Fax: +31(0)10 408 9031

Going Where the Ad Leads You: On High Advertised Prices and Searching Where to Buy*

Maarten C.W. Janssen† Marielle C. Non‡

August 1, 2006

Abstract

The search literature assumes that consumers know which firms sell products they are looking for, but are unaware of the particular variety and the prices at which each firm sells. In this paper, we consider the situation where consumers are uncertain whether a firm carries the product at all by proposing a model where in the first stage firms decide on whether or not to carry the product. Firms may advertise, informing consumers not only of the price they charge, but also of the basic fact that they sell the product. In this way, advertising lowers the expected search cost. We show that this role of advertising can lead to a situation where advertised prices are higher than non-advertised prices in equilibrium.

Keywords: consumer search, informative advertising

JEL codes: D83, L11, L13, M37

*We thank David Soberman for a discussion at the early stages of this paper. We thank Chaim Fersthman, Matthijs Wildenbeest and seminar participants at Tinbergen Institute Rotterdam for their helpful comments and suggestions.

†Dept. of Economics, Erasmus University Rotterdam and Tinbergen Institute, P.O. Box 1738, 3000 DR Rotterdam, The Netherlands. E-mail: janssen@few.eur.nl.

‡Corresponding author, Tinbergen Institute and Dept. of Economics, Erasmus University Rotterdam, P.O. Box 1738, 3000 DR Rotterdam, The Netherlands. E-mail: non@few.eur.nl

1 Introduction

Imagine yourself attending a conference in a foreign country. You sit with your laptop in your hotel room and you realize that when your battery expires you cannot charge it because of a different electricity outlet system. You are going out in town to search for an electricity converter. You enter a first, electronics, shop where the charming sales representative tells you that unfortunately they do not carry such an item in their store. The same story repeats in a number of stores, after which you disappointingly go back to the hotel. In a last desperate attempt you ask at the hotel lobby whether they by any chance would know a shop where they carry the item you are looking for. Triumphantly, the clerk at the desk tells you a firm has left an advertisement behind informing people that they carry all different types of electronic converters one may ever wish to use (possibly with the prices at which they sell). You are very happy for this piece of information, immediately go to the shop and are prepared to buy at any (somewhat reasonable) price.

This story contains an element that we believe is important in many markets, not just when hanging around in far away destinations: namely that a large part of the search activities of people is not about "searching for firms with the lowest price", but rather about "searching for firms that sell the product". This distinction has not been made in the economics literature on consumer search. The typical search model only considers situations where all firms in the market carry the product and the only reason for consumers to search (further) is to look for a price-quality combination that better fits the individual's preferences. That is, the literature on consumer search is not about the "real" search activity of consumers when they are uncertain about which firms carry the product.

Another important aspect of the above story is that a potential important role of advertising is simply to inform consumers about the fact that the advertising firm carries the product, thereby helping the consumer to save on his expected search cost. If the uncertainty about which firm carries the product one is looking for is very large, then the reduction in "real" search cost may be quite significant. This in turn may help advertising firms to charge (substantially) higher prices and in this way recover the advertising expenditures. In this way, so the present paper argues, advertised prices may actually be higher than non-advertised prices. This is contrary to conventional wisdom expressed in the literature on informative advertising, according to which informative price advertising leads to better informed consumers, and therefore, to more competition and lower prices (see, e.g., Farris and Albion (1980) and Tirole (1998, section 7.3)). Thus, the paper contributes to the strategic literature on advertising by arguing that in the presence of uncertainty about which firms sell the product, informative price advertising may lead to higher prices compared to the non-advertised prices. This insight is important in empirical work on advertising. It shows that one should be careful to conclude from an observed positive correlation between advertising and prices that advertising is persuasive, see e.g. Boulding et al. (1994) and Clark (2005).[1]

This paper studies "searching for the product" and "high prices through informative advertising" in relation. To this end, we develop a three-stage model. In the first stage, firms decide whether or not they want to allocate shelve-space to a particular type of product. Doing so has an opportunity cost of not using that space for having some other commodity on display. We call all firms who decide to carry the product "active firms". In the second stage, active firms decide on their price and on whether they advertise the fact that they carry the product (and the price at which they sell it) by sending an advertisement to consumers. For simplicity, advertising is modeled as an all-or-nothing decision.[2] In the third stage, after potentially having received some ads, consumers decide whether or not to search for a firm that carries the product, with a potentially lower price than the firms that advertised. However, if the firm has not advertised, the consumer does not know whether or not the firm carries the product in the first place.

The simplest search model we can imagine that makes the point that advertised prices can be higher than non-advertised prices, even under informative price advertisement, has two types of consumers: low demand consumers have search cost c_L, whereas high demand consumers have higher search cost c_H. One may think of the high demand consumers as having high income from demanding jobs to justify the correlation between the size of consumers' search cost and their willingness to pay. In such a model, there may be many different types of equilibria, depending on the parameter configurations. We focus on equilibria where advertised prices are higher than non-advertised prices. The simplest such equilibrium has the following structure. As soon as consumers receive advertisements, they buy at the advertising firm. When the low demand consumers get no advertisement, they search for an active firm. High demand consumers who get no advertisements however do not search for an active firm as they find the probability that these firms do not carry the product too high compared to

[1] An interesting method to empirically distinguish between informative and persuasive advertising is proposed by Ackerberg (2001). He argues that informative advertising mainly influences inexperienced buyers, since informative advertising provides these buyers with the information they need. Persuasive advertising, on the other hand, will influence both experienced and inexperienced buyers.

[2] Otherwise, complications arise of the form that a firm from which no advertisement has been received, may be either a firm that did not advertise or a firm that advertises but whose advertisement is simply not received.

the search costs they have to make. Non-advertising firms therefore completely concentrate on the low demand consumers, whereas advertising firms concentrate on both types. The probability that firms are active and the intensity with which active firms advertise are determined in such a way that firms are indifferent between being inactive, advertising high prices and not-advertising and setting low prices. To be able to make firms indifferent we impose a condition on the proportion of high and low valuation consumers in the population.

The paper is, of course, related to the large literatures on consumer search and advertising (see, e.g., the seminal papers by Stigler (1961), Stahl (1989, 1994), Butters, (1977)). The main difference with the consumer search literature is, as we mentioned, the fact that consumers really have to search for the good if firms do not advertise, as firms may not carry the product. Contributing to the strategic literature on advertising, Meurer and Stahl (1994) are the first to note that informative advertising may lead to higher prices. Soberman (2004) also outlines a model where informative advertising and prices can be positively correlated. In both Meurer and Stahl (1994) and Soberman (2004) products are assumed to be heterogenous. Informative advertising plays two roles in this context. First, it creates awareness of products so that consumers know what best fits their tastes. This strengthens the product differentiation aspect, giving firms an incentive to raise prices. On the other hand, it also leads to more consumers with full information and this gives firms an incentive to reduce prices. When product differentiation is important enough, the first aspect is more important than the second so that advertising can lead to price increases. To contrast our results with these two papers, we present a model where products are homogeneous. The channel through which advertising leads to higher prices in the present paper, as described above, is therefore completely different in our model.

The literature that combines consumer search and advertising is much more limited.[3] Robert and Stahl (1993) is the first paper where consumers' ignorance about prices can be resolved by consumers searching for prices or by firms informing consumers about the prices they charge through advertising. Following on their work, Stahl (2000) and Janssen and Non (2005) check the robustness of the model by investigating the properties of different modeling assumptions. Janssen and Non assume that a fraction of consumers is fully

[3]Apart from the literature mentioned here, there is also a recent paper by Stivers and Tremblay (2005). Their model is however very different from the standard search models as they model search costs as the wedge between producer prices and consumer prices, very much like the analysis in traditional tax studies. Moreover, they assume that advertising lowers the search costs of consumers. In such a world, they show that it is possible that advertising raises the price the firms ask, while at the same time decreasing the price (including search costs) that consumers have to pay.

informed about prices and that less-informed consumers can both decide to search and to buy at firms that advertise. Under these conditions they characterize all equilibria of the model and show that in some equilibria advertised prices may be higher than non-advertised prices. They, however, make the unrealistic assumption that less-informed consumers can buy at firms that advertise without incurring search cost, giving advertising firms an advantage above non-advertising firms that have to be searched for. In the present paper we formally explain how this difference in search cost to buy from an advertising and a non-advertising firm can emerge out of the uncertainty consumers face when they visit a shop that did not advertise.

The rest of this note is organized as follows. Section 2 presents the model. Section 3 formally characterizes one equilibrium where advertised prices are higher than non-advertised prices. In section 4 we will elaborate on the existence of other equilibria. Section 5 concludes.

2 The model

In this section we will specify the three-stage model we use. We assume a homogenous good oligopoly, with $n \geq 2$ firms in the market. For simplicity, the firms have no production costs, but there is an opportunity cost C for shelving the product. In the first stage of the model the firms decide on whether or not to carry the product. We will denote the probability of a firm being active by β. Note that an inactive firm has profit 0.

In the "second stage" of the model, firms decide simultaneously on their advertising strategy and price. Firms do not know the outcome of the first stage (the decision to be active) and therefore, it is just as if firms play these two stages simultaneously. An active firm can decide to advertise that it sells the product and at which price. Note here that advertising is purely informative: an advertisement informs about existence and price. Advertising is an 'all-or-nothing' decision, that is, a firm either advertises to the complete market or does not advertise at all. The cost of advertising is A. We will denote the probability with which a firm advertises by α. The pricing strategy depends on whether a shop advertises or not. We will therefore specify a price strategy conditional on advertising and a price strategy conditional on not advertising. Denote by $\overline{p}_1$ the highest advertised price, and by $\underline{p}_1$ the lowest advertised price. Prices of non-advertising firms are denoted by p_0.

There is a unit mass of consumers, who can be divided into two types. A fraction γ has a low demand $L-p$ and low search costs c^L. A fraction $1-\gamma$ has a high demand $H-p$ and high search costs c^H. Note that $H > L$ and $c^H > c^L$ and that demand is linear in prices p. In the third stage of the game, consumers receive the advertisements that are sent and decide on their search strategy. Consumers search sequentially; conditional on the advertisements that are sent. Sequential search means that the consumers first look at the advertisements they have received. They then decide whether or not to visit one additional firm. After visiting this firm they decide on whether or not to visit a second firm, and so on. Note that consumers only search non-advertising firms, since they already know that advertising firms are active and the ad also tells at which price the active firm sells. We assume that every visit to a firm costs c^L (c^H). Visiting an advertising firm, a consumer has to make the search costs, but is sure he will find the product. Visiting a non-advertising firm, a consumer has to make the same search costs, but in this case is not sure he will find the product. This implies that the 'real' search costs for searching a non-advertising firm is higher than the search costs for searching an advertising firm.

We look for a perfect Bayesian equilibrium of this game. In such an equilibrium, the consumer strategy in principle depends on the full set of advertised prices that the consumer has seen. However, the only part of this set of advertised prices that is important in making the decision whether or not to search is the lowest observed price, since consumers would never buy from an advertising firm if they have seen an ad from another firm with a lower price. We can therefore specify the consumer strategy in terms of the lowest observed price. Out-of-equilibrium beliefs are not really important in this game as firms do not come in different types, i.e., they all sell the same commodity. Consumers, however, do update their beliefs (using Bayes' rule) about the probability the firm carries the product after not having received an advertisement. As we will see, this occurs with positive probability along the equilibrium path.

3 An equilibrium

The model that has been specified above has many possible equilibria. In this section we will characterize one of the more interesting ones in which advertising firms set higher prices than non-advertising firms. For such an equilibrium to exist we must have that $0 < \beta < 1$ and $0 < \alpha < 1$. An equilibrium with $\beta = 0$ would mean there is no market at all, while $\beta = 1$ would bring us back to a standard search model where consumers are sure that every firm is active. If $\alpha = 0$ or $\alpha = 1$, we can not compare advertised and non-advertised prices.

For different parameter values there are also potentially different equilibria with advertising firms unambiguously setting higher prices than non-advertising firms. To have this it is clear that non-advertising firms should focus their pricing decision on a group of consumers that has on average more low type consumers than the group of consumers the advertising firms focus on. In this section, we concentrate on such a case, namely where consumers choose the following search strategy.

Assumption 3.1 *(Consumer Search Strategy). All consumers who get an advertisement with a price at or below $\frac{1}{2}(\gamma L + (1-\gamma)H)$ do not search for non-advertising firms and buy immediately from the lowest priced advertising firm $L-p$ or $H-p$ units, respectively.*[4] *Low demand consumers who get no advertisements search for a non-advertising firm carrying the product. They search on until they find an active firm, and as soon as they found one with price at or below $\frac{1}{2}L$, they will buy there.*[5] *High demand consumers who get no advertisements do not search, and as a consequence do not buy.*

Given this strategy of the consumers, we claim that the optimal pricing, advertising and activity strategy of the firms is given in the following lemma.

Lemma 3.2 *If consumers follow the search strategy specified in Assumption 3.1 and $H < (1+\frac{1}{\sqrt{1-\gamma}})L$, then the following forms the optimal response for firms. Non-advertising firms set a price $p_0 = \frac{1}{2}L$. Advertising firms choose a price according to a price distribution with cdf*

$$F_1(p) = \frac{1}{\alpha\beta} - \frac{1-\alpha\beta}{\alpha\beta}\left(\frac{(\gamma L + (1-\gamma)H)^2}{4p(\gamma L + (1-\gamma)H - p)}\right)^{\frac{1}{n-1}},$$

with $\overline{p}_1 = \frac{1}{2}(\gamma L + (1-\gamma)H)$. Lastly, the probability with which firms advertise α and their activity probability β are implicitly defined by

$$(1-\alpha\beta)^{n-1} = \frac{4(C+A)}{(\gamma L + (1-\gamma)H)^2}$$

and

$$\gamma L^2 \frac{C+A}{(\gamma L + (1-\gamma)H)^2} = C + C(n-1)\frac{\beta(1-\alpha)}{1-\alpha\beta}.$$

Proof

See appendix. □

[4] It is more difficult to specify fully rational behavior in case the lowest observed advertised price is above $\frac{1}{2}(\gamma L + (1-\gamma)H)$. For the derivation of the equilibrium it is not necessary to specify the consumer behavior in this case, since even if the consumers would buy immediately after seeing such a high price, firms would not deviate. Therefore, we do not fully specify consumer behavior for this case. We will provide more details in the proof of lemma 3.2.

[5] Here the same applies as in footnote 4.

The consumer search behavior that is assumed, gives advertising firms the possibility to ask a higher price than non-advertising firms. As non-advertising firms only sell to the low demand consumers and since they do not compete with one another, it is optimal for them to ask the monopoly price given low demand. Setting an advertising price that is higher than the non-advertised price, both consumer types still would buy from the advertising firm. Since advertising firms sell to both consumer types, one of the optimal prices, the maximum price, is a weighted average of $\frac{1}{2}L$, the optimal price when only selling to the low demand consumers, and $\frac{1}{2}H$, the optimal price when only selling to the high demand consumers. However, as advertising firms also compete with one another (in case there are two or more firms advertising), the mixed strategy price distribution optimally balances the monopoly power they may have with the possibility they compete with each other. The condition $H < (1 + \frac{1}{\sqrt{1-\gamma}})L$ is used to guarantee that an advertising firm wants to serve both types of demand and not concentrate its sales activities on the high demand consumers only. The condition requires H/L not to be too large and γ, the proportion of low demand consumers, to be relatively large.

We now turn to check the optimality of the assumed consumer strategy.

Lemma 3.3 *Suppose that* $\frac{1-\alpha\beta}{(1-\alpha)\beta} > \frac{(L-\frac{1}{2}L)^2}{(L-\frac{\gamma L+(1-\gamma)H}{2})^2}$ *and* $H < \frac{2-\gamma}{1-\gamma}L$. *Given the strategy of firms specified in the previous lemma, there are values of* c^L *and* c^H, *with* $c^H > c^L$, *such that the optimal search and buying strategy of consumers is as given in assumption 3.1.*

Proof
See appendix. □

There are several parameter restrictions for the consumer search strategy to be optimal. In the proof of the previous lemma, we have specified several restrictions on the search cost parameters c^L and c^H. Both search cost parameters have a lower bound and an upper bound. The upper bound on the search costs follows from the fact that consumers are assumed to buy immediately after receiving an advertisement. If the cost of buying from an advertising firm is too high, a consumer would prefer not to buy at all. On the other hand, when getting an ad, consumers do not search on. This leads to a lower bound on the search costs, since if search costs were negligibly small, consumers would continue to search for another active firm, even after receiving ads.

Next to the upper and lower bound on search costs that originate from the behavior of consumers who get an ad, there is an extra upper bound on the search costs for the low demand consumers and an extra lower bound on the high demand consumers' search costs. These bounds originate from the behavior of the consumers who do not receive an ad. Low demand consumers who do not receive advertisements decide to search. This gives a second upper bound on c^L, since if the search costs are too high, it is not profitable to search. In the proof it is shown that this second upper bound is below the upper bound that originates from the fact that consumers who get an ad buy immediately at the advertising firm: the expected surplus from searching after not having received an ad apparently is lower than the surplus from buying at an advertising firm, even though the advertised price can be high. The reason is that searching consumers have a strictly positive probability of not finding an active firm.

High demand consumers who do not receive advertisements do not search. This gives a second lower bound on c^H, since for really low search costs these consumers would prefer to search. This second lower bound is above the lower bound that originates from consumers who received ads. To see this, note that the second bound says that c^H should be higher than the expected surplus from searching *without having received any ads.* The other lower bound says that c^H should be higher than the expected surplus from searching *after having received some advertisements.* This surplus is below the surplus from searching when no advertisements have been received.

The condition $\frac{1-\alpha\beta}{(1-\alpha)\beta} > \frac{(L-\frac{1}{2}L)^2}{(L-\frac{\gamma L+(1-\gamma)H}{2})^2}$ mentioned in the lemma takes care of the fact that one is able to find search cost parameters c^L and c^H such that the search conditions discussed above can be satisfied. Next to the restrictions on the search costs, there is a second restriction, $H < \frac{2-\gamma}{1-\gamma}L$, which simply guarantees that low demand consumers are willing to buy even at the maximum price advertised as specified in their search strategy.

After this discussion on the conditions pertaining to the optimality of the consumer search strategy, we are now ready to state the full equilibrium in which advertising firms set higher prices than non-advertising firms. In addition to the conditions specified in the previous two lemmas, the equilibrium requires that $0 < \alpha < 1$ and $0 < \beta < 1$.

Proposition 3.4 *If* $\gamma > \frac{4C}{L^2}$ *and* $H < (1 + \frac{1}{\sqrt{1-\gamma}})L$*, then there are values of* c_L *and* c_H*, with* $c_H > c_L$*, and values of* $n > 2$ *and* $A > 0$ *such that the strategies of firms and consumers specified in the previous two lemmas form a perfect Bayes-Nash equilibrium, with* $0 < \alpha, \beta < 1$.

Proof
See appendix. □

The equilibrium we specified has indeed the feature that advertised prices are above the non-advertised price. Even so, consumers who receive an advertisement do not search for the lower priced non-advertising firms. The reason is that they do not know which firms are active and which firms are not. Possibly, a consumer has to search many times, making a lot of search costs, in order to find an active firm. The alternative is to pay search costs once, to be able to buy at a high price for sure.

The equilibrium we characterized holds under some restrictions. In the proof we formally characterize upper and lower bounds for both advertising costs A and the shelving costs C. This is quite intuitive, since in equilibrium firms are indifferent between advertising and not advertising and between being active and not active. If the advertising costs are too high, no firm would advertise, while if the advertising costs are too small, every active firm would advertise destroying the incentives to search as in this case all non-advertising firms are inactive. In the same way, if the shelving costs are too high, no firm would be active, while if the shelving costs are too small, every firm would be active.

Another restriction, namely $\gamma > \frac{4C}{L^2}$, can be understood as follows. A non-advertising firm makes its maximum profit when it is the only active firm, and in that case profits equal $\frac{1}{4}\gamma L^2$. Since we impose that in equilibrium at least some firms are active, these profits should be higher than the shelving costs C, leading to a lower bound on γ. If this condition holds, the proof shows that we can always find values of A and C such that firms are indifferent between being active and not active in this market and between advertising and non-advertising. On the other hand, the proof also shows that the restrictions for this equilibrium to hold become easier to be satisfied, the larger n. It can be shown that if n increases, β decreases in such a way that for n going to infinity the number of active firms βn is a constant. Consequently, the number of inactive firms grows infinitely large. Therefore, the larger n, the larger the possible difference in expected search cost between following up on an advertisement and searching for an active firm among all non-advertising firms.

4 Other equilibria

In the previous section we characterized one of the possible equilibria of our model. There are however many more equilibria possible even if we restrict ourselves to cases where both α and β are between 0 and 1. In this section we will discuss some of the possible forms other equilibria can take and discuss the reasons why a full characterization is quite difficult indeed.

In section 3 we derived an equilibrium by following several steps. The first step is to assume a consumer search strategy. Based on this consumer strategy one can derive an optimal firm strategy. This firm strategy then is used to check whether the assumed consumer strategy is indeed optimal. This procedure can also be followed to derive other equilibria. However, consumer behavior for each type of consumer consists of three elements. After not having received any advertisements, a consumer has to decide whether to search or not. After having received one or more advertisements, consumers have to decide whether to search further or not, and whether it would be profitable to buy from the lowest priced advertising firm or not. This gives 2^6 possible search behaviors. It would be quite a large task to investigate all these possibilities. We note that some of these alternative search strategies can be part of an equilibrium where advertised prices are also higher than non-advertised prices. Take for example the following situation. High demand consumers who get an advertisement buy immediately from the lowest priced advertising firm. If a high demand consumers does not receive an advertisement, he will search for an active firm. Low demand consumers always search for an active non-advertising firm no matter whether or not they received an advertisement. In such a situation advertising firms mainly sell to the high demand consumers, and therefore may choose prices close to $H/2$, while non-advertising firms mainly sell to the low demand consumers. Under some conditions, this search behaviour could indeed give rise to an equilibrium where advertised prices are higher than non-advertised prices. The condition $H < (1 + \frac{1}{\sqrt{1-\gamma}})L$ does not need to hold in this equilibrium as advertising firms will never want to set prices above $H/2$. It is, of course, also clear that equilibria with advertised prices below non-advertised prices exist.

The last remark we would like to make on other equilibrium possibilities is that price distributions with a 'gap' are also possible. This could happen if consumers observing prices close to $\underline{p}_1$ buy immediately from an advertising firm, while consumers observing prices close to $\overline{p}_1$ as lowest advertised price first search for another active firm. In this case there is a price p^* between $\underline{p}_1$ and $\overline{p}_1$ such that consumers are indifferent between searching and not searching. Asking a price slightly above p^* would give less expected sales and consequently less expected profits than when asking a price slightly below p^*. However, asking a price that is far enough above p^* could give expected profits that equal the expected profits from asking a price slightly below p^*. As we cannot rule out equilibria of this type a full characterization of all equilibria is beyond what one possibly could hope to obtain in this model.

5 Conclusion

The core of this paper centers around the uncertainty consumers face concerning the shops that carry the product they are looking for: some firms do have the product, others don't. This uncertainty is important in explaining consumer search behaviour, but so far this type of uncertainty has not been considered in the large literature on consumer search. An important role of advertising in such a situation is to inform consumers that the advertising firm indeed sells the product. Advertising therefore can lower consumers' expected search costs. Since visiting an advertising firm comes with lower expected search costs than finding the product in a non-advertising firm, advertising firms have an advantage above non-advertising firms. In this paper we address the question whether advertising firms can use this advantage to set higher prices. We show that this is indeed the case. This result is important for empirical work on advertising as it shows that a positive relation between advertising and prices is not necessarily the result of persuasive advertising. Our model makes clear that such a positive relation can also arise from informative advertising.

We have characterized one equilibrium where advertising firms set higher prices than non-advertising firms. We have also indicated that there are other equilibria that have this property and that there are also equilibria where despite the comparative advantage advertising firms set lower prices than non-advertising firms.

Appendix

Proof of lemma 3.2

We will first turn to advertising firms. We first note that $F_1(p)$ specified in the lemma is strictly increasing in p, and $F_1(\overline{p}_1) = 1$. The minimum price $\underline{p}_1$ is defined as the price where $F_1(p) = 0$. These observations make clear that $F_1(p)$ is indeed a cumulative distribution function.

Consumers buy immediately from advertising firms that ask a price at or below $\frac{1}{2}(\gamma L + (1-\gamma)H)$. The profit function for these prices therefore is

$$\pi_1(p) = p(\gamma(L-p) + (1-\gamma)(H-p))(1-\alpha\beta+\alpha\beta(1-F_1(p)))^{n-1} - A.$$

The profits are given by the price times the expected sales minus the advertising costs A. Expected sales are given by $(\gamma(L-p) + (1-\gamma)(H-p))(1-\alpha\beta+\alpha\beta(1-F_1(p)))^{n-1}$, that is, an advertising firm only sells if there is no other firm advertising a lower price, which happens with probability $(1-\alpha\beta+\alpha\beta(1-F_1(p)))^{n-1}$, and if an advertising firm advertises the lowest price it sells $L-p$ items to the low demand consumers and $H-p$ items to the high demand consumers.

For prices between $\underline{p}_1$ and $\overline{p}_1$, inserting the equilibrium cdf $F_1(p) = \frac{1}{\alpha\beta} - \frac{1-\alpha\beta}{\alpha\beta}(\frac{(\gamma L+(1-\gamma)H)^2}{4p(\gamma L+(1-\gamma)H-p)})^{\frac{1}{n-1}}$ gives $\pi_1(p) = (1-\alpha\beta)^{n-1}\frac{(\gamma L+(1-\gamma)H)^2}{4} - A = \pi_1$.

For prices below $\underline{p}_1$, inserting $F_1(p) = 0$ in the profit function gives $\pi_1(p) = p(\gamma(L-p)+(1-\gamma)(H-p)) - A$; a function that is increasing in p. Deviating to a price below $\underline{p}_1$ therefore is not profitable.

For $F_1(p)$ to be the equilibrium pricing strategy, it also should not be profitable to deviate to higher prices. What could happen when a firm advertises a price above $\overline{p}_1$? If there are other advertising firms, consumers will never buy from the deviating firm. However, with probability $(1-\alpha\beta)^{n-1}$ the deviating firm is the only advertising firm, and has a possibility to sell to some or all consumers. In assumption 3.1 we did not specify the consumer behavior if consumers' lowest observed advertised price is above $\frac{1}{2}(\gamma L+(1-\gamma)H)$. This also is not necessary. If the deviation price is below L, the best that can happen to a firm is that all consumers still buy immediately from the advertising firm. In that case the profits are as specified above, with $F_1(p) = 1$. This is a parabola with top at $\overline{p}_1$, so even if consumers would all immediately buy, deviating is not profitable.

If the deviation price is above L but below H,[6] only the high demand consumers possibly buy from the deviating firm. In the best case, all the high demand consumers buy immediately from the deviating firm. This would give profits $p(1-\gamma)(H-p)(1-\alpha\beta)^{n-1} - A$. Note that for $p > L > \overline{p}_1$, $p(1-\gamma)(H-p)(1-\alpha\beta)^{n-1} - A > p(\gamma(L-p)+(1-\gamma)(H-p))(1-\alpha\beta)^{n-1} - A$,

[6] It is clear that deviating to a price above H is never optimal.

so in this case it is possible that deviating is profitable. Maximum profits are obtained when the price is set to $\frac{1}{2}H$, and the profits from deviating then are $\frac{1}{4}H^2(1-\gamma)(1-\alpha\beta)^{n-1} - A$. For $F_1(p)$ to be an equilibrium strategy, it has to be the case that

$$\frac{1}{4}H^2(1-\gamma)(1-\alpha\beta)^{n-1} - A < (1-\alpha\beta)^{n-1}\frac{(\gamma L + (1-\gamma)H)^2}{4} - A,$$

or

$$H < (1 + \frac{1}{\sqrt{1-\gamma}})L. \tag{1}$$

We now turn to the behavior of non-advertising firms. Given the search strategy specified, non-advertising firms only sell when no other firm is active and advertises. Moreover, they will never sell to high demand consumers as these consumers never search. Low demand consumers who get no advertisements, on the other hand, search for an active firm. So, the expected profits for a non-advertising firm asking a price at or below $\frac{1}{2}L$ are

$$\pi_0(p) = \gamma p(L-p)(1-\alpha\beta)^{n-1}\frac{1}{1+(n-1)\frac{\beta(1-\alpha)}{1-\alpha\beta}}.$$

The low demand consumers give a profit $\gamma p(L-p)$, but only if no firm advertises, what happens with probability $(1-\alpha\beta)^{n-1}$, and profits have to be shared between all active non-advertising firms. The expected number of active non-advertising firms is $(n-1)$ times the probability a firm is active and non-advertising, conditional on the fact that no firm advertises. This conditional probability is given by $\frac{\beta(1-\alpha)}{1-\alpha\beta}$. From the profit function it is easy to see that it is optimal for non-advertising firms to set $p_0 = \frac{1}{2}L$. A firm that deviates to a higher price will in the best case still sell to all consumers who search for it. This would give expected profits equal to the expression given above, which is a parabola with top at $\frac{1}{2}L$. Deviating therefore is not profitable.

The last thing we have to show is that in equilibrium α and β are implicitly defined by $(1-\alpha\beta)^{n-1} = \frac{4(C+A)}{(\gamma L+(1-\gamma)H)^2}$ and $\gamma L^2\frac{C+A}{(\gamma L+(1-\gamma)H)^2} = C + C(n-1)\frac{\beta(1-\alpha)}{1-\alpha\beta}$. These two equalities originate from the requirement that $\pi_0 = C$ and $\pi_1 = C$, such that firms are indifferent between advertising and not advertising, and indifferent between being active and being inactive.

$\square$

Proof of lemma 3.3

The first condition we have to impose is that it is optimal for low demand consumers who get an advertisement with price at or below $\overline{p}_1$ to not search, but buy immediately from the lowest priced advertising firm even if this is the highest price advertised. Consumers are willing to buy if the advertised price is below their valuation for the product, that is,

$$\overline{p}_1 < L, \tag{2}$$

which reduces to $H < \frac{2-\gamma}{1-\gamma}L$, and if their surplus from buying is above the search costs they have to make to visit the advertising firm, that is,

$$\frac{1}{2}(L - \overline{p}_1)^2 > c^L. \tag{3}$$

Note that the surplus from buying is the area above the price $\overline{p}_1$ and below the demand function $L - p$.

The low demand consumers who get an advertisement are not willing to search if the expected surplus from searching is below the surplus from buying immediately. The expected surplus from searching depends on the number of firms that have not yet been searched. Denote by S_k the expected surplus from searching if there are k firms yet to be searched. The probability a consumer finds an active firm conditional on the firm not having sent an advertisement is given by $\frac{(1-\alpha)\beta}{1-\alpha\beta}$. With the remaining probability, the consumer does not find an active firm. In this case a consumer can decide to search on, giving expected surplus S_{k-1}, or to buy from the lowest priced advertising firm, asking price p_1^*. Combining this, and taking into account that all non-advertising firms ask a single price p_0, we get

$$S_k = -c^L + \frac{(1-\alpha)\beta}{1-\alpha\beta}\frac{1}{2}(L-p_0)^2 + (1 - \frac{(1-\alpha)\beta}{1-\alpha\beta})\max(S_{k-1}, \frac{1}{2}(L-p_1^*)^2 - c^L)$$

and in particular

$$S_1 = -c^L + \frac{(1-\alpha)\beta}{1-\alpha\beta}\frac{1}{2}(L-p_0)^2 + (1 - \frac{(1-\alpha)\beta}{1-\alpha\beta})(\frac{1}{2}(L-p_1^*)^2 - c^L).$$

We note that if $S_1 > \frac{1}{2}(L-p_1^*)^2 - c^L$ (that is, if there is only one firm to be searched, it is profitable to search instead of buying immediately at the advertising firm) then $S_2 > S_1 > \frac{1}{2}(L-p_1^*)^2 - c^L$, $S_3 > S_2 > \frac{1}{2}(L-p_1^*)^2 - c^L$, $S_4 > S_3 > \frac{1}{2}(L-p_1^*)^2 - c^L$, and so on. So whenever $S_1 > \frac{1}{2}(L-p_1^*)^2 - c^L$, it is always profitable to search until one active non-advertising firm is found. Furthermore, if $S_1 < \frac{1}{2}(L-p_1^*)^2 - c^L$, then $S_2 = S_1 < \frac{1}{2}(L-p_1^*)^2 - c^L$, $S_3 = S_2 < \frac{1}{2}(L-p_1^*)^2 - c^L$ and so on. So whenever $S_1 < \frac{1}{2}(L-p_1^*)^2 - c^L$, it is never profitable to search.

Low demand consumers who get an advertisement should not be willing to search even if they only observe the highest price that possibly is advertised, leading to the restriction

$$-c^L+\frac{(1-\alpha)\beta}{1-\alpha\beta}\frac{1}{2}(L-p_0)^2+(1-\frac{(1-\alpha)\beta}{1-\alpha\beta})(\frac{1}{2}(L-\overline{p}_1)^2-c^L) < \frac{1}{2}(L-\overline{p}_1)^2-c^L. \tag{4}$$

For high demand consumers we have the same kind of restrictions. High demand consumers who get an advertisement also do not search on but buy immediately. This gives

$$\frac{1}{2}(H-\overline{p}_1)^2 > c^H \tag{5}$$

and

$$-c^H+\frac{(1-\alpha)\beta}{1-\alpha\beta}\frac{1}{2}(H-p_0)^2+(1-\frac{(1-\alpha)\beta}{1-\alpha\beta})(\frac{1}{2}(H-\overline{p}_1)^2-c^H) < \frac{1}{2}(H-\overline{p}_1)^2-c^H. \tag{6}$$

There are two more restrictions on consumer search. Low demand consumers who get no advertisements search for an active firm. Expected surplus from searching is the same as the expected surplus from searching after having received one or more ads, except that $\max(S_{k-1}, \frac{1}{2}(L-p_1^*)^2-c^L)$ is replaced by $\max(S_{k-1}, 0)$. The analysis is the same; in particular, if $S_1 = -c^L + \frac{(1-\alpha)\beta}{1-\alpha\beta}\frac{1}{2}(L-p_0)^2 > 0$ then it is profitable to search until finding an active firm. If $S_1 < 0$, if is not profitable to search at all. This gives

$$\frac{(1-\alpha)\beta}{1-\alpha\beta}\frac{1}{2}(L-p_0)^2 - c^L > 0. \tag{7}$$

In contrast, high demand consumers who get no advertisements do not search, giving restriction

$$\frac{(1-\alpha)\beta}{1-\alpha\beta}\frac{1}{2}(H-p_0)^2 - c^H < 0. \tag{8}$$

The three restrictions on the low type of consumers can be written as restrictions on c^L. This gives

$$c^L < \frac{1}{2}(L-\frac{\gamma L+(1-\gamma)H}{2})^2,$$

$$c^L < \frac{1}{2}\frac{(1-\alpha)\beta}{1-\alpha\beta}(L-\frac{1}{2}L)^2$$

and

$$c^L > \frac{(1-\alpha)\beta}{1-\beta}\frac{1}{2}(L-\frac{1}{2}L)^2 - \frac{(1-\alpha)\beta}{1-\beta}\frac{1}{2}(L-\frac{\gamma L+(1-\gamma)H}{2})^2.$$

This is only possible if $\frac{(1-\alpha)\beta}{1-\beta}\frac{1}{2}(L-\frac{1}{2}L)^2 - \frac{(1-\alpha)\beta}{1-\beta}\frac{1}{2}(L-\frac{\gamma L+(1-\gamma)H}{2})^2 < \frac{1}{2}(L-\frac{\gamma L+(1-\gamma)H}{2})^2$, and in that case $\frac{1}{2}(L-\frac{\gamma L+(1-\gamma)H}{2})^2 > \frac{1}{2}\frac{(1-\alpha)\beta}{1-\alpha\beta}(L-\frac{1}{2}L)^2$. So restrictions (3), (4) and (7) are equivalent to

$$\frac{(1-\alpha)\beta}{1-\beta}\frac{1}{2}(L-\frac{1}{2}L)^2-\frac{(1-\alpha)\beta}{1-\beta}\frac{1}{2}(L-\frac{\gamma L+(1-\gamma)H}{2})^2 < c^L < \frac{1}{2}\frac{(1-\alpha)\beta}{1-\alpha\beta}(L-\frac{1}{2}L)^2. \tag{9}$$

Thus, we can find a value of c_L that satisfies these inequalities if

$$\frac{1-\alpha\beta}{(1-\alpha)\beta} > \frac{(L-\frac{1}{2}L)^2}{(L-\frac{\gamma L+(1-\gamma)H}{2})^2}. \tag{10}$$

Similarly, for the high demand consumer. The three relevant restrictions only hold if $\frac{1}{2}(H-\frac{\gamma L+(1-\gamma)H}{2})^2 > \frac{(1-\alpha)\beta}{1-\alpha\beta}\frac{1}{2}(H-\frac{1}{2}L)^2$, which can be rewritten as $\frac{(1-\alpha)\beta}{1-\alpha\beta}\frac{1}{2}(H-\frac{1}{2}L)^2 > \frac{(1-\alpha)\beta}{1-\beta}\frac{1}{2}(H-\frac{1}{2}L)^2 - \frac{(1-\alpha)\beta}{1-\beta}\frac{1}{2}(H-\frac{\gamma L+(1-\gamma)H}{2})^2$. The three restrictions are equivalent then to require

$$\frac{(1-\alpha)\beta}{1-\alpha\beta}\frac{1}{2}(H-\frac{1}{2}L)^2 < c^H < \frac{1}{2}(H-\frac{\gamma L+(1-\gamma)H}{2})^2, \tag{11}$$

which can hold only if

$$\frac{1-\alpha\beta}{(1-\alpha)\beta} > \frac{(H-\frac{1}{2}L)^2}{(H-\frac{\gamma L+(1-\gamma)H}{2})^2}. \tag{12}$$

As the RHS of this inequality is smaller than the RHS of (10), this inequality always holds when (10) holds. Moreover, note that the lower bound on c^H is larger than the upper bound on c^L, so the condition $c^L < c^H$ is automatically satisfied.

□

Proof of proposition 3.4

To begin with, we note that $1+\frac{1}{\sqrt{1-\gamma}} < \frac{2-\gamma}{1-\gamma}$. This implies that the condition $H < \frac{2-\gamma}{1-\gamma}L$ that is necessary for the consumer search strategy to be optimal, is automatically satisfied whenever the condition $H < (1+\frac{1}{\sqrt{1-\gamma}})L$, needed for the firm strategy to be optimal, is satisfied.

We restate the respective equilibrium conditions α and β have to satisfy: $(1-\alpha\beta)^{n-1} = \frac{4(C+A)}{(\gamma L+(1-\gamma)H)^2}$ and $\gamma L^2\frac{C+A}{(\gamma L+(1-\gamma)H)^2} = C + C(n-1)\frac{\beta(1-\alpha)}{1-\alpha\beta}$. Using this last equilibrium condition we can reformulate the condition $\frac{1-\alpha\beta}{(1-\alpha)\beta} > \frac{(L-\frac{1}{2}L)^2}{(L-\frac{\gamma L+(1-\gamma)H}{2})^2}$ needed to guarantee the optimality of the consumer search strategy in lemma 3.3 fully in terms of exogenous parameter values:

$$\frac{(n-1)C\left[\gamma L+(1-\gamma)H\right]^2}{\gamma(A+C)L^2 - C\left[\gamma L+(1-\gamma)H\right]^2} > \frac{(L-\frac{1}{2}L)^2}{(L-\frac{\gamma L+(1-\gamma)H}{2})^2},$$

or

$$\frac{(C+A)}{(\gamma L+(1-\gamma)H)^2} < \frac{C}{\gamma L^2}\left(1+(n-1)\frac{(L-\frac{\gamma L+(1-\gamma)H}{2})^2}{(L-\frac{1}{2}L)^2}\right). \tag{13}$$

We define x as $x=\frac{4(C+A)}{(\gamma L+(1-\gamma)H)^2}$ and note that $x>0$. We can now write

$$(1-\alpha\beta)^{n-1}=x$$

and

$$\beta(1-\alpha)=\frac{\gamma L^2}{4C(n-1)}x^{\frac{n}{n-1}}-\frac{1}{n-1}x^{\frac{1}{n-1}}.$$

From the equations above we can then isolate α and β as

$$\beta=1+\frac{\gamma L^2}{4C(n-1)}x^{\frac{n}{n-1}}-\frac{n}{n-1}x^{\frac{1}{n-1}}$$

and

$$\alpha=\frac{1-x^{\frac{1}{n-1}}}{\beta}=\frac{1-x^{\frac{1}{n-1}}}{1+\frac{\gamma L^2}{4C(n-1)}x^{\frac{n}{n-1}}-\frac{n}{n-1}x^{\frac{1}{n-1}}}.$$

Using these expressions, restrictions $0<\alpha<1$ and $0<\beta<1$ can be rewritten as follows:

$$\alpha>0 \Longleftrightarrow x<1;$$

$$\begin{aligned}\alpha &< 1 \Longleftrightarrow 1-x^{\frac{1}{n-1}}<1+\frac{\gamma L^2}{4C(n-1)}x^{\frac{n}{n-1}}-\frac{n}{n-1}x^{\frac{1}{n-1}} \Longleftrightarrow\\ -1 &< \frac{\gamma L^2}{4C(n-1)}x-\frac{n}{n-1} \Longleftrightarrow \gamma L^2 x>4C;\end{aligned}$$

$$\beta<1 \Longleftrightarrow \frac{\gamma L^2}{4C(n-1)}x^{\frac{n}{n-1}}-\frac{n}{n-1}x^{\frac{1}{n-1}}<0 \Longleftrightarrow \gamma L^2 x<4nC$$

$$\beta>0 \Longleftrightarrow x^{\frac{1}{n-1}}(\frac{\gamma L^2}{(n-1)4C}x-\frac{n}{n-1})>-1$$

It is not difficult to see that when the first three restrictions are satisfied, the last restriction also holds. To see this, note that the first restriction implies that $x^{\frac{1}{n-1}}<1$. The second and third restriction together yield $-1<\frac{\gamma L^2}{(n-1)4C}x-\frac{n}{n-1}<0$, and so $x^{\frac{1}{n-1}}(\frac{\gamma L^2}{(n-1)4C}x-\frac{n}{n-1})>-1$. Using the definition of x, we can thus rewrite $0<\alpha,\beta<1$ as

$$\frac{C}{\gamma L^2} < \frac{(C+A)}{(\gamma L+(1-\gamma)H)^2} < \min(\frac{1}{4}, \frac{nC}{\gamma L^2}). \tag{14}$$

As $n > \left(1+(n-1)\frac{(L-\frac{\gamma L+(1-\gamma)H}{2})^2}{(L-\frac{1}{2}L)^2}\right)$, conditions (13) and (14) can be combined to

$$\frac{C}{\gamma L^2} < \frac{(C+A)}{(\gamma L+(1-\gamma)H)^2} < \min(\frac{1}{4}, \frac{C}{\gamma L^2}\left(1+(n-1)\frac{(L-\frac{\gamma L+(1-\gamma)H}{2})^2}{(L-\frac{1}{2}L)^2}\right)). \tag{15}$$

These inequalities can only be satisfied if $\frac{C}{\gamma L^2} < \frac{1}{4}$. On the other hand, it is easy to see that the term in the middle is increasing in A and that for $A = 0$, the middle terms is smaller than the LHS. Therefore, if $\frac{C}{\gamma L^2} < \frac{1}{4}$, one can always find a value of $A > 0$ such that both inequalities in (15) are satisfied.

□

References

[1] Daniel A. Ackerberg. Empirically distinguishing informative and prestige effects of advertising. *RAND Journal of Economics*, 32(2):316–333, 2001.

[2] William Boulding, Eunkyu Lee, and Richard Staelin. Mastering the mix: do advertising, promotion, and sales force activities lead to differentiation? *Journal of Marketing Research*, 31(2):159–172, 1994.

[3] Gerard R. Butters. Equilibrium distribution of sales and advertising prices. *Review of Economic Studies*, 44(3):465–491, 1977.

[4] C. Robert Clark. Advertising restrictions and competition in the children's breakfast cereal industry. mimeograph, January 2005.

[5] Paul W. Farris and Mark S. Albion. The impact of advertising on the price of consumer products. *Journal of Marketing*, 44:17–35, 1980.

[6] Maarten C.W. Janssen and Mariëlle C. Non. Advertising and consumer search in a duopoly model. Tinbergen Institute Discussion Paper TI 2005-022/1, 2005.

[7] Michael Meurer and Dale O. Stahl. Informative advertising and product match. *International Journal of Industrial Organization*, 12:1–19, 1994.

[8] Jacques Robert and Dale O. Stahl, II. Informative price advertising in a sequential search model. *Econometrica*, 61(3):657–686, 1993.

[9] David A. Soberman. Research note: Additional learning and implictions on the role of informative advertising. *Management Science*, 50(12):1744–1750, 2004.

[10] Dale O. Stahl, II. Oligopolistic pricing with sequential consumer search. *American Economic Review*, 79:700–712, 1989.

[11] Dale O. Stahl, II. Oligopolistic pricing and advertising. *Journal of Economic Theory*, 64:162–177, 1994.

[12] Dale O. Stahl, II. Strategic advertising and pricing in e-commerce. In Michael R. Baye, editor, *Industrial Organization, Advances in Applied Microeconomics*, volume 9 of *Industrial Organization*, pages 69–100. JAI Press, Amsterdam, 2000.

[13] George J. Stigler. The economics of information. *Journal of Political Economy*, 69(3):213–225, 1961.

[14] Andrew Stivers and Victor J. Tremblay. Advertising, search costs, and social welfare. *Information Economics and Policy*, 17:317–333, 2005.

[15] Jean Tirole. *The Theory of Industrial Organization*. MIT Press, 1998.