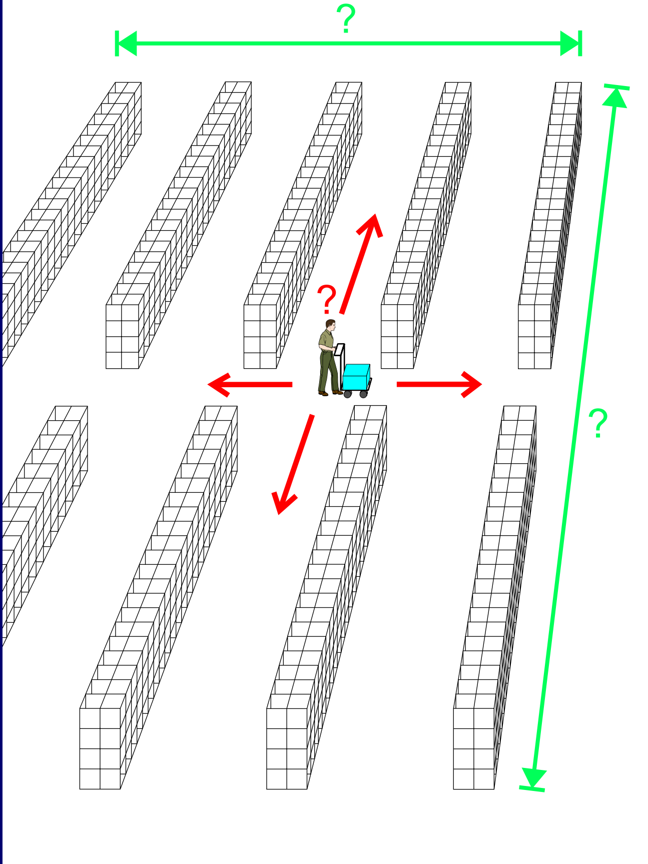
KEES JAN ROODBERGEN

Layout and Routing Methods for Warehouses



Layout and Routing Methods for Warehouses

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Lay-out en routeringsmethoden voor magazijnen

Proefschrift

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In memory of my mother

Voorwoord

Voor u ligt het resultaat van vier jaar onderzoek naar logistieke processen in magazijnen. In 1996 ben ik begonnen als Assistent In Opleiding (AIO) aan de Faculteit Bedrijfskunde van de Erasmus Universiteit Rotterdam. Ik bedank Edo van der Poort, die samen met Gerard Sierksma mij uitstekend begeleidde bij het schrijven van mijn scriptie aan de Rijksuniversiteit Groningen, nog hartelijk voor het feit dat hij mij attendeerde op de beschikbare promotieplaatsen in Rotterdam.

In Rotterdam leerde ik al snel de AIO's kennen die tegelijk met mij waren begonnen. Enkelen van hen zijn tot mijn vriendenkring gaan behoren. Met name Robert van der Meer wil ik hier noemen. Robert en ik deelden het eerste jaar op de universiteit een werkkamer. Dit leverde veel gezelligheid en relevante en irrelevante discussies op. Ook het feit, dat je je buik moest inhouden om je bureaustoel zover tegen je eigen bureau te kunnen aanschuiven, dat de ander zijn bureaulade volledig kon openen, vergeet je nooit meer. In deze eerste weken in Rotterdam had ik nog geen eigen woonruimte kunnen vinden en woonde ik bij de familie Rodenburg. Hun gastvrijheid was enorm, waardoor ik mij meteen thuisvoelde in het westen van Nederland. In de loop der tijd leerde ik mijn collega's van de vakgroep Management van Technologie en Innovatie kennen. Ik bedank hen voor de goede en rustige sfeer en voor het inzicht dat zij mij hebben geboden in allerlei vakgebieden, waar ik nog geen kennis van had. Verder mag AnneMarie Stolkwijk niet ongenoemd blijven. Zeker in de begintijd, maar ook daarna, biedt zij aan AIO's veel hulp bij allerlei aangelegenheden. Eigenlijk hoort haar naam in haast elk voorwoord van een Rotterdams bedrijfskundeproefschrift voor te komen.

Een belangrijke factor in mijn bestaan als AIO was René de Koster. René nam zijn taak als promotor zeer serieus en ruimde, zeker de eerste jaren, veel tijd in voor gesprekken. Zijn praktische kennis van zaken was zeer nuttig voor het opzetten van mijn experimenten. Ook voor de ruimte die hij mij gaf om zaken te onderzoeken, die niet direct in mijn onderzoeksvoorstel stonden, ben ik hem dankbaar. Ook mijn tweede promotor Jo van Nunen mag niet onvermeld blijven. Jo zit boordevol ideeën, die te pas en te onpas naar boven komen borrelen. Dit leverde nuttige aanvullingen op mijn proefschrift op en ook nog stof tot nadenken voor in de toekomst.

Wetenschap bedrijven wordt wel gezien als een tamelijk solistische arbeid. Vaak wanneer je alleen in je kamer op de universiteit zit, lijkt dat ook waar te zijn. Toch leent de wetenschap zich ook goed tot samenwerking. Met een aantal mensen heb ik samen aan artikelen gewerkt. *Among others, I would like to thank Charles Petersen for working with me on a paper and presentation for the educational forum of Promat '99. It was great to meet him for the first time, some time after we had written the paper together. My visit at the Georgia Institute of Technology in Atlanta has greatly benefitted my research. For about three months, I had the opportunity to work with Gunter Sharp on layout issues in order picking. Chapter 5 contains the results of this period. I thank Gunter Sharp for his hospitality, his help in my research and the discussions we had on cultural differences between Europe and the United States.*

In de afgelopen jaren ben ik geregeld in het buitenland geweest om presentaties te geven over de methoden, de resultaten en de praktische bruikbaarheid van mijn onderzoek. Vele instanties ben ik dankbaar voor hun financiële bijdragen en het vertrouwen dat zij daarmee in mijn onderzoek stelden. In dit kader wil ik de Faculteit Bedrijfskunde van de Erasmus Universiteit en de Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO) noemen. *Furthermore, I am grateful for the opportunities offered to me by the Material Handling Industry of America (MHIA). The Material Handling Institute of MHIA, by means of its president Dick Ward, gave me the possibility to present my research at two Material Handling Research Colloquia and to present the practical aspects of my work at the educational forum of Promat in 1999 and 2001. I probably learned at least as much from the questions as the audience learned from my answers.*

Naast het doen van onderzoek heb ik vele cursussen gevolgd, lesgegeven en studenten begeleid bij het schrijven van hun scriptie. Ronald van Voorden was de eerste student die bij mij kwam voor begeleiding bij het schrijven van zijn scriptie. Zijn onderzoek betrof onder meer het routeren van orderverzamelaars. Ter ondersteuning van zijn onderzoek ontwierp ik een aantal routeringsmethoden. Hiermee was mijn interesse gewekt voor problemen in orderverzamelgebieden bestaande uit meerdere blokken. Een aanzienlijk deel van dit proefschrift gaat over soortgelijke situaties.

Mijn eerste grote ervaring met lesgeven was in 1999 toen ik Bert De Reyck assisteerde bij het geven van het vak Simulatie van Logistieke Systemen. De studenten bedrijfskunde die dit vak volgen, zijn vierdejaars en volgen de specialisatiefase. Van Bert leerde ik hoe je een goede voorbereiding, een inzichtelijke presentatie, enthousiasme en inzet kunt combineren tot een vak dat veel kennis bijbrengt en hoog wordt gewaardeerd door studenten.

Om mijn onderzoek naar de buitenwereld te promoten heb ik een website gemaakt. Op deze website kan de bezoeker een groot aantal routeringsmethoden toepassen in een grafische weergave van een magazijn. Ook kan de bezoeker zelf de muis ter hand nemen om het eigen inzicht in routeringsproblemen te testen. De website is te vinden op het adres http://www.mediaport.org/~kjr/warehouse/. Verder heb ik een bijdrage geleverd aan de Erasmus Logistica Warehouse Website. Deze website is te vinden op http://www.fbk.eur.nl/OZ/LOGISTICA/. Hier kan het door mij geprogrammeerde simulatieprogramma voor routering van orderverzamelaars gebruikt worden. De websites leverden ook publicaties op onder meer in vaktijdschriften, zie de referenties [1], [61], [62], [68] en [266].

Bij het schrijven van een boek is het altijd prettig als iemand het manuscript goed voor je naleest. Voor het nalezen en corrigeren van de Nederlandstalige gedeelten bedank ik Eri Vis-Theunessen hartelijk. Iris Vis bedank ik voor het lezen, corrigeren en bediscussiëren van de Engelstalige en wiskundige gedeeltes. Ook bedank ik Iris voor het feit dat zij altijd voor mij klaar staat en voor al het begrip dat zij de afgelopen tijd heeft getoond. Ik zal proberen komend jaar hetzelfde voor haar te doen.

Rotterdam, 2001

Kees Jan Roodbergen

Contents

List of Figures

1

Introduction

Nowadays, customers can order products electronically with their computers or mobile phones via the Internet. Because the process of ordering is fast, the customers expect comparable speed in delivery. Existing organizations that supply products to customers are trying to adapt to this new situation and new organizations emerge. Traditionally, the aim of logistics was to assure that the right product is at the right place at the right time in the right quantity. The two main activities carried out in logistics were considered to be transportation and keeping inventories. Transportation is used to get products at the right place by moving them from one location to another location. Inventories can be useful to supply products at the right time by storing them for a certain period of time until they are needed. Such a storage location is generally called a *warehouse*. However, the situation and the opinions have changed. According to the *Council of Logistics Management* logistics can now be defined as "that part of the supply chain process that plans, implements, and controls the efficient, effective flow and storage of goods, services, and related information from the point of origin to the point of consumption in order to meet customers' requirements." This means that logistics is more than just moving and storing goods. We sketch some of these developments in logistics below.

The Council of Logistics Management sees logistics as a part of the supply chain process. The *supply chain* encompasses all activities associated with the flow and transformation of goods from the raw material stage, through to the end user, as well as the associated information flows, see Handfield and Nichols [119]. The supply chain concept has gained in importance, since individual companies are realizing that they can no longer manage to satisfy customer demand on their own. It is necessary to work together with several companies within the supply chain to be able to meet delivery schedules, product specifications, service requirements and so on. The efforts to harmonize the processes in the supply chain are called *supply chain management*. Many trends impact the supply chain; we will discuss a few important ones below. We will discuss increasing customer power, shorter product life cycles, value added logistics, reduction of response times, return flows and shortening of throughput times.

As we just briefly noted, a very important trend affecting supply chains is increasing customer power. In earlier times, producers made their products and the customers bought what was available. Nowadays, customers shift easily from one brand to another. Producers try to keep up with changes in customer demand. And if necessary, they try to produce to order. Efficient consumer response is one of the goals. One effect is that the variety of products is ever-increasing while at the same time, life cycles of the products are shortening. There is an enormous impact of all this on the part of warehousing, such as a need to make the storage time of products shorter. This is necessary to reduce costs, increase flexibility and to prevent companies from having large amounts of stock left when there is no more demand for the products. Additionally, the production process can be altered to reduce risk of storage. For example, producers can try to give their products a modular structure. This enables them to decide on the final appearance of the product at a later stage in production. This may result in situations where the final composition of products is performed in the warehouse, instead of at the production plant. For example, in the computer industry a number of components is delivered to a warehouse. Only when a customer orders a computer, is the requested composition of computer, keyboard, power cable and manual created. The fact that activities like this are performed in warehouses is part of what is often called *value added logistics*. Other value added activities are for example quality control, custom packaging and labeling.

Having some form of assembly in the warehouse makes it possible to customize products based on the requests of the customers. And because the warehouse is generally closer to the customer than the factory, the response time of the company to a customer order is lower. Furthermore, technological developments, such as automated warehouses and transportation systems, computer integrated manufacturing and advances in telecommunications, such as Electronic Data Interchange (EDI) and multimedia communication, enable faster and more reliable delivery of orders to the warehouse and goods to the customer. From a study by Rogers *et al.* [230] it appeared that firms using more warehousing information technologies had a significantly better performance in the areas of quality improvements, cycle time reductions and productivity improvements. The speed and accuracy of logistics can actually serve as a competitive weapon for companies. Fast and accurate deliveries can be charged for. Or if that is not possible, fast deliveries against the lowest possible costs can be attempted.

Finally, we note a trend that is concerned with the increase in return flows. Return flows consist, for example, of products that customers ordered via the Internet but decided not to buy, products that are defective and need to be repaired or products that have reached the end of their life-cycle and need to be dismantled, remanufactured or recycled. New products that are returned need to be checked before they can be sold again. The volume of returned products may be such that a separate space in the warehouse is required. If it is decided that a returned product can be sold again, it needs to be decided whether it is stored together with the regular stock or separately. Re-use of products, components and materials – initiated by the desire for environmental improvements – is enforced by legislation. It may induce cost savings, but can also constitute a competitive advantage, due to the 'green image' of the company. Return flows are thus becoming increasingly important in logistics.

Several adaptations in the logistics branch are needed to respond to these trends. Shapiro *et al.* [258] discussed how to improve the *order management cycle* from planning to post-sales service, which can result in improved customer satisfaction, reduced interdepartmental problems and improved financial performance. One specific response is to try and shorten the throughput time. This increases the flexibility for reactions to changes in customer demand and it decreases the risk of unsalable stock and reduces inventories. A way to shorten the throughput time is to design a supply network without warehouses. Products are then shipped either directly or via cross-docking centers from the production facility to the customer. Throughput time is mainly decreased because products are no longer stored in the warehouse. However, the feasibility of this option may depend on several factors, such as flexibility in the production, communication facilities between companies and reliability of production and distribution. Of importance in this respect is also the Just-In-Time concept; it does not matter how long it takes for a product to arrive as long as it arrives exactly at the time it is needed.

At first sight it may seem that due to all the developments in logistics, warehouses are gradually becoming superfluous. In some cases this may actually be so, but in many other situations warehouses will continue to play an important role. For example, as is the case for a company which makes products in the United States and sells them in Europe. Products can be transported either by airplane or by ship. Often air transport is too expensive, but transport by ship too slow. Customers may not be prepared to wait for weeks for the products they ordered. A warehouse can solve the problem. The producer temporarily stores products in a European warehouse and ships products to the customers from the warehouse on demand.

We distinguish four main reasons why warehouses may be useful: to facilitate the coordination between production and customer demand by buffering products for a certain period of time, to accumulate and consolidate products from various producers for combined shipment to common customers, to provide same-day delivery to important customers, and to support product customization activities, such as packaging, labeling, marking, pricing or even final assembly, see [92] and [113]. Thus, warehouses will continue to play an important role in logistics.

Section 1.1 gives an overview of logistic functions in warehouses. In Section 1.2 an overview is given of systems used in warehouses to store and handle the products. Warehouse design issues are taken up in Section 1.3. Methods to control warehouse processes are given in Section 1.4. An outline of this thesis is given in Section 1.5. Special attention is given to one aspect of warehousing: the order picking process. Order picking is the process by which products are retrieved from storage on the basis of customer orders. The order picking process will be the main object of study in this thesis.

1.1 Warehousing

As we noted in the previous section, warehouses are of considerable importance in logistics networks. Basically, we can distinguish two types of warehouses: distribution centers and production warehouses. A *distribution center* is a warehouse in which products from one or more suppliers are collected for delivery to a number of customers. A *production warehouse* is used for the storage and distribution of raw materials, semi-finished products and finished products in a production environment.

There are many activities that occur as part of the process of getting materials into and out of the warehouse. Numerous product transformations take place. For example, incoming products on pallets are stored in a pallet rack. Individual cases, i.e. boxes filled with products, are removed from a pallet and stored in another area of the warehouse. Individual products (*items*) are removed from the cases in response to customer orders.

A warehouse generally consists of several areas. We describe some of the most common areas and some product flows from area to area. This description is given to relate an impression of warehousing activities and by no means exhaustive of all situations possible. At the *receiving area* incoming shipments are unloaded from a truck and inspected to see whether quantity and quality of the products are as ordered. After receiving, products are either transported directly to the shipping area (this is called *cross docking* or *transshipment*) or transported to and placed in a storage area. Roughly, we can distinguish three types of picking areas: *pallet area*, *case pick area* and *item pick area.* A warehouse may have more than one area of the same type, or may not have a certain type of area at all. Retrieved quantities are transported either to another area, to the accumulation and sorting area or to the shipping dock. Movement of items from one storage area to another storage area is most often carried out for replenishment. For example, a pallet is moved from the pallet area to the case pick area. Individual cases can now be removed from the pallet for the case picking operation. Products which are retrieved based on customer orders are moved from the storage area to the accumulation and sorting area or directly to the shipping area. The *accumulation and sorting area* (if present) is used to regroup products such that they match with the customer orders. For example, a customer orders a list of products. Some products have to be retrieved from a case pick area and other products from an item pick area. These products have to be merged before shipment to the customer. It may occur that products for several customers are retrieved simultaneously from a storage area. These products have to be split before they can be sent to the right customer. After accumulation and sorting, products can be grouped. Small products are put in cases; cases can be stacked on pallets. Finally, the outgoing orders are loaded onto vehicles in the *shipping area.* Storage systems and material handling equipment will be discussed in more detail in Section 1.2.

Management decisions concerning warehousing can be subdivided into strategic decisions, tactical decisions and operational decisions. Strategic management decisions are long-term decisions and concern the determination of broad policies and plans for using the resources of a company to best support its long-term competitive strategy. For example: how many distribution centers do we need? To which markets do we supply our products? In which locations do we place warehouses? Which performance criteria does the warehouse have to meet? Decisions made at the strategic level define a framework or constraints set under which the company must operate in both the intermediate and short term.

Tactical management decisions primarily address how to schedule material and labor efficiently within the constraints of previously made strategic decisions. For example: what is the layout of the warehouse? Which storage systems do we use? Which products do we keep in storage? Do we use order picking per customer order or batch picking? These tactical decisions become the constraints under which operational decisions are made.

Decisions with respect to the operational control are narrow and short-term by comparison. For example: how do we divide the order picking area into zones? What type of batching do we use? Which routing algorithm is preferred?

Furthermore, we have on-line decisions, which are made in modern warehouses by the computer system using the control systems chosen at the tactical and operational level. These on-line decisions include: when do we pick a certain order? Where do we store incoming loads? In which sequence do we pick the products?

The on-line decisions in a warehouse are often made by applying operating policies. Operating policies are decision rules that determine how various warehouse operations are performed. For example, a storage policy determines where to store each product and a routing policy determines how an order picker has to travel through the warehouse to retrieve a specific order from storage.

1.2 Warehousing systems

A wide variety of warehousing systems is used to store, retrieve and sort products. Pallet systems are used for large products or for handling large quantities of products. For handling smaller quantities of products we may use cases or we may even handle individual pieces. Often a warehouse will contain some combination of pallet, case and individual product handling. For example, incoming products may be stored initially on pallets. After a while a pallet is moved to the case picking area. Orders containing large quantities of the product can be satisfied by retrieving a full case from this area. Orders with one or a few products are satisfied from the item picking area, which is replenished with cases from the case picking area. See also Yoon [299] and Yoon and Sharp [301].

We discuss storage systems and material handling equipment for pallets, cases and individual items. Furthermore, we describe part-to-picker systems and systems for sorting. For more information on warehousing systems see e.g. Frazelle [91, 92], Kulwiec [158] and Tompkins *et al.* [269].

1.2.1 Storage systems

The simplest form of storage is *block stacking*. In this situation, the warehouse is one open space in which pallets, filled with products, are stored on the floor or on top of each other. Multiple pallets with the same product are grouped. Only the pallets on the edges of the group can be reached. A more advanced way to store pallets is with the use of *pallet racks*. A pallet rack is a metal construction that makes it possible to stack pallets higher than with block stacking while keeping the possibility to manually access the pallets on the lower levels directly. Generally, several racks are placed in rows with *aisles* in between where vehicles or people can move for handling the pallets. Pallet racks can be *single-deep,* but also *multi-deep*. That is, each pallet is accessible directly from the pick face or the pallet positions are two or more pallets deep, which means that one pallet is positioned in front of the other pallet. Another way to store several pallets behind each other are so-called *drive-in* or *drive-through racks*. In this case, the vehicle can drive into the rack to retrieve or store pallets.

In situations where the throughput is very high, it may be a good choice to use *pallet flow racks*. In a pallet flow rack each position can contain multiple pallets, which are positioned behind each other. Products are retrieved from one side of the rack and refills are done from the other side. Pallets roll on a conveyor from one side of the rack to the other. A variation on this is the *push-back rack*, where refills and retrievals are done from the same side. If a pallet is stored in a slot, the other pallets in the slot are pushed backwards and the new pallet takes the position at the front.

There are also systems in which products are not stored on pallets, often in quantities that are (much) less than a full pallet. These systems include shelves, storage drawers and gravity case flow racks. A system with shelves is inexpensive, easy to use and easy to reconfigure. A system with drawers offers higher storage density as compared to shelves, because there is for example no extra space needed above each pick location to enable the order picker to put his hand in. Furthermore, drawers offer more protection against theft and influences from the environment, since the drawers are not open to the environment and can be locked. Gravity flow racks may be used for products that experience high demand rates. Cases with products are replenished from the back of the rack and roll towards the front if an (empty) case is removed at the front. Order pickers take products from the cases in response to customer orders and remove empty cases from the rack to allow the next box to come forward.

1.2.2 Handling equipment

Various types of equipment exist for putting pallets in storage or retrieving pallets from storage. Basic equipment are *pedestrian stackers* and *counterbalance lift trucks*. A pedestrian stacker is a simple piece of equipment that can lift and stack a pallet. When traveling, the operator walks behind this vehicle. A counterbalance lift truck generally has a fork that can go up and down a mast to lift pallets. The operator sits or stands on the vehicle when traveling. This vehicle is fairly cheap and very flexible. A disadvantage of this vehicle is its wide turning radius which requires wide aisles, i.e. the racks have to be far apart. Several variations on the counterbalance lift truck exist to enable operations in narrow aisles. For example, *wide-wheelbase trucks*, that have support arms at the front for stability instead of a counterbalance. Or *reach trucks*, which have a fork or mast that can 'reach' to the load while the vehicle stays at a fixed position. There is a version of this truck where forks can reach into the racks to store pallets multi-deep. Furthermore, we have *swing-mast* or *swing-fork* trucks, which are able to retrieve pallets from the side without having to turn the entire vehicle, thus saving valuable floor space. The most advanced pallet handling system is the *automated storage / retrieval system*. This system consists of racks served by cranes running on rails. It is capable of handling pallets without the interference of an operator; the system is fully automated. Horizontal and vertical movement of the cranes can occur at the same time. Aisles can be very narrow, thus saving floor space. The investment costs for such a system are, however, very high.

For case picking other equipment can be used. The base-line in case picking is the usage of pallet trucks. A *pallet truck* is a vehicle that is either hand or motor operated and can carry one or two pallets. It has no lifting capability, except to raise the load a few centimeters off the floor to allow movement. All cases that have to be picked by the operator are positioned at floor level. Replenishment is generally done from locations above the pick location with a vehicle of the type mentioned for pallet handling. These kinds of systems are very common in the grocery industry. To avoid being restricted to picking from the floor level, one may use an *order picking truck*. This truck is able to both drive and lift. The operator goes up and down with the pick pallet, which enables the operator to pick a case from a storage location and to put it on the pick pallet.

Several other systems for case picking exist. For example, a *pick-to-belt* system. In this system the order pickers take cases from the rack and put them directly onto a conveyor (belt) that runs along the aisle. Storage of products is often in pallet flow racks, see Section 1.2.1. Advanced systems use lights on the rack to indicate which case has to be picked. Operators can attach a label to each box as identification for the process of sorting cases into customer orders in a later stage.

Item picking requires yet another type of vehicle, although some of the previously mentioned vehicles can be used for item picking as well. A wide variety of picking carts exists. A *picking cart* may just be used to transport the picked items, but may also have a printer, a barcode scanner or indicator lights. The cart may have multiple levels to facilitate picking of several orders at the same time. Another possibility is to use a *conveyor with totes*. Picked products are put in a tote, which is thereafter transported to other pickers or another part of the warehouse. In all storage systems for item picking it has to be possible for the order picker to reach the highest shelf or drawer. Therefore, these systems are limited to a height of at most 1.8 meters. To increase space utilization the systems can be placed on mezzanines. In this way, the warehouse can have several floors of shelves, drawers or racks. This is advantageous since most warehouse buildings are much higher than one picker-to-stock system anyway, due to the height of other (pallet) systems. It is also possible to use a *man-up system* for item picking. This system overcomes the limitation of maximum height. The order picker can use, for example, an order picking truck. Or a man-aboard automated storage / retrieval system may be used. This is an automated storage / retrieval system on which the order picker rides aboard the machine to pick the products.

1.2.3 Part-to-picker systems

The classical picking systems described above are called *picker-to-part* systems. In these systems, the order picker walks or drives through the warehouse to one or more locations to retrieve products. An alternative to these systems are the *part-to-picker* systems. In part-to-picker systems the order picker stays at a fixed location and the products move towards him. Several systems exist. A *carousel* consists of a number of bins or shelves that rotate either horizontally or vertically. Control of the carousel can be either manual by the order picker or automatically. The speed of picking depends partly on the rotation speed. It can be useful to install carousels such that one order picker can handle more than one carousel.

Another advanced picking system uses an automated storage / retrieval system. Pallets with cases are brought automatically to fixed pick positions where an operator takes the requested cases from the pallet. After that, the pallet is automatically returned to the rack. The same method is used by a miniload system; the main difference is that storage containers for small items are used instead of pallets.

One step further in automation is fully automated systems that collect items without the need of an order picker. For example, a*utomated item dispensing systems* drop items on a conveyor. This high-speed system is especially suitable for small items of uniform size.

1.2.4 Transport, accumulation and sorting systems

In warehouses products frequently have to be transported from one area to another. This can be done with some of the handling equipment mentioned in Section 1.2.2. For example, counterbalance lift trucks are often used for this transport. Another method is to use conveyors. *Conveyors* can be used to move products between specific locations over a fixed path. A large variety of conveyors exist, where products move on a belt, on rollers, on chains and so on. There are conveyors which move, for example, pallets, bins or cases. Besides for transport, conveyors can also be very useful in applications such as accumulation and sorting.

Accumulating and sorting of products can be performed to regroup products such that they match the customer orders, see Section 1.1. For example, product flows on conveyors from different areas are first merged to one flow. After merging, the resulting flow has to be split into customer orders. Products arrive on a recirculating conveyor that has a number of lanes. Each lane corresponds to a customer or a truck with a group of destinations. If a product on the recirculating conveyor passes the right lane a mechanism diverts the product to this lane. All products with the same destination accumulate in a single lane. There is a huge variety of systems to divert products from a conveyor to for example a lane, another conveyor or into totes.

1.3 Warehouse design

Facility design planning concerns the decision making needed to establish a facility for efficient handling of incoming and outgoing products. The first decision to take is whether a warehouse is needed at all. In some cases, it may be possible to ship products directly from the producer to the customer. However, as noted in the beginning of this chapter, many reasons exist why a warehouse may be useful in the logistics network. If a warehouse is needed, then a choice has to be made between contract warehousing (i.e. a third party takes care of the warehousing) or warehousing by the company itself. Next, the number and location(s) of the warehouses have to be determined. The decisions noted here could be made sequentially, but can also be treated as joint decisions.

After deciding that a warehouse is to be built at a certain location, its size, shape and contents need to be established. A methodology for comparing alternatives is given in Brynzer *et al.* [35]; benchmarking of warehousing practice is considered in Hollingworth [127]. Practical methods for comparing alternatives with respect to equipment and layout is described in Moerkerken [198] and Perlmann and Bailey [213]. Other literature on design of warehouses includes [69], [56], [105], [194], [211], [219], [248], [300] and [301].

Regarding practical implementations it is often the case that various aspects of the design are executed in a fixed sequence. Typically, one may first decide on the storage requirements and layout, then determine the storage systems and handling equipment and finally give thought to the operating policies and information flows, see De Koster [59]. However, for a good design it is necessary to consider all aspects of the design in relation to each other. In Yoon and Sharp [301] a procedure is given for the design of order picking systems and it is noted that the selection of equipment, operating policies, physical transformation of products and the information transformation must be performed together. This reasoning is important because, for example, the equipment choice influences the floor space needed. When determining a layout without considering the operating policies, one may at some point find out that it is impossible to achieve the proposed performance with any existing operating policy, because the layout is inadequate. It may not be necessary to specify operating policies in full detail at an early point in time, but they have to be accounted for. We will study the interaction between an operating policy and warehouse layout in Chapters 4 and 5.

Without passing over the interaction between the various components of design, we discuss the aspects of sizing, system selection and layout separately in the next few sections. The aspect of the operating policies is discussed in Section 1.4.

1.3.1 Sizing of warehouses

One important part of designing a warehouse is the determination of its size. The size can usually not be chosen freely. Height and total floor space may be restricted due to government regulations or nearby constructions. When determining warehouse size it is not only important to consider current needs, but also to take into account seasonal influences and estimates of future trends. Additionally, the type of operation that will be performed inside the building is very important. A picking operation where order pickers walk through the facility needs more floor space than an automated storage / retrieval system that can store pallets at heights of 20 meters.

The calculation of the required number of storage locations to hold a given number of products, when demand is known, is discussed in Hall [114]. The amount of storage space needed may also depend on production lot sizes, see e.g. [126], [187], [189] and [295] . Cormier and Gunn [50] concentrate on the establishment of a warehouse size and underlying inventory policy that are jointly optimal. A warehouse sizing model for a company that produces one product and plans on having one warehouse is considered in Rao and Rao [226]. Furthermore, the product demand is seasonal and the company is assumed to be able to rent space from a public warehouse. Warehouse sizing is also discussed in [129], [244] and [294]. Leasing of warehouse space is taken into account in [51], [144] and [181]. The problem of capacity expansion of an existing facility is considered in Cormier and Gunn [52].

1.3.2 System selection

The warehousing systems have to be selected as well. According to Ashayeri and Gelders [7] the main issues for the warehouse designer are to select the best storage system, choose the appropriate handling equipment and to determine the warehouse layout. The first two issues will be discussed in this section; the layout will be discussed in the next section.

Several factors play a role in the system selection. For example, the physical properties of the products (e.g. size, weight, required temperature and flammability), are important to know before deciding on storage and handling equipment. Also transaction data such as order data are important since a warehouse that receives only a few orders per day may need a different structure from a warehouse that gets thousands of orders a day. Economic constraints, such as budget and project life, and environmental constraints, such as safety requirements, each have their effect on design. Furthermore, an analysis of system requirements (throughput, inventory) and system alternatives (hardware, operators) gives insight into capabilities and requirements for the system. Finally, the performance of the systems as influenced by the operating policies must be analyzed, see [299] and [301]. Selection of warehousing systems is discussed in [47], [86], [153], [193] and [204].

1.3.3 Layout of warehouses

We can distinguish two types of layout decisions. Firstly, the decision on where to place the various departments (receiving, order picking, sorting, etc.) and secondly to determine the layout within the various departments. The Systematic Layout Procedure described in Muther [203] is a layout procedure that starts with an activity relationship diagram, determines the amount of space needed for each activity and finally results in a block layout. Each department is given a certain part (block) of the total floor space available. The objective of layout optimizations is generally to find a layout that results in the lowest material handling costs. These material handling costs are often interpreted as transportation costs, which in turn are often treated as a function of travel distance. These costs are often approximations. For example, it is common to assume that all flows go from the center of one block along a straight line to the center of another block. This is in contrast to real flows of products with inbound and outbound locations per block and routes that have to go around other blocks that are in between the two locations. See Meller and Gau [196] for a literature review on this topic.

The second layout decision is concerned with the contents of the blocks. For example, we have to determine the number of aisles in the order picking area. If we wanted to design an order picking area with a certain fixed storage capacity, then we could choose to have a few long aisles or to have many short aisles. The goal is to determine a layout that is good, for example with respect to travel times. Chapters 4 and 5 of this thesis extensively deal with the problem of determining a layout for the order picking area extensively. Literature is discussed and new results are given.

1.4 Operating policies

Operating policies are decision rules that can be used to control various processes in a warehouse. For example, the assignment of orders to order pickers needs to be determined frequently. Another decision that has to be made is the determination of storage locations for incoming loads. The effectiveness of operating policies depends, apart from the efficiency of the policy itself, also on factors such as layout, truck schedules that have to be met and order data. For example, if the warehouse receives many small orders, then it may be advantageous to let the order pickers pick several orders at the same time. However, if orders are large they may have to be split to make the order fit in the picking vehicle. An overview of operating policies is given in Van den Berg [279].

For any existing warehouse, it may be advantageous to re-evaluate the operating policies occasionally. For example, an important trend in warehousing is that the number of orders increases, with a simultaneous decrease in the size of each individual order. If this is the case, the company may need to shift to a different means of retrieving orders from storage.

We divide the operating policies into four types: storage policies, order picking policies, sorting policies and other policies. These policies and their functions will be discussed in the remainder of this section.

1.4.1 Storage policies

Before products can be retrieved from storage, they first have to be stored. There are numerous ways to assign products to storage locations. We describe five frequently used types of storage assignment: random storage, closest open location storage, dedicated storage, full turnover storage and class based storage. For *random storage* every incoming pallet (or an amount of similar products) is assigned a location in the warehouse that is selected randomly from all eligible empty locations with equal probability. This storage policy will only work in a computer-controlled environment. If the order pickers can choose the location for storage themselves we would probably get a system known as *closest open location storage.* The first empty location that is encountered will be used to store the products. This typically leads to a warehouse where racks are full around the depot and gradually more empty towards the back (if there is excess capacity). The *depot* is the location where order pickers start and finish their picking routes. An empty pick cart and a pick list can be picked up at the depot. A pick list is a printed sheet of paper, on which the requested products are sorted indicating the order picking sequence.

Another possibility is to store each product at a fixed location, which is called *dedicated storage.* A disadvantage of dedicated storage is that a location is reserved even for products that are out of stock. Moreover, for every product sufficient space has to be reserved such that the maximum inventory level can be stored. Thus the space utilization is low. An advantage is that order pickers become familiar with product locations. It is also possible to match the layout of the warehouse with the layout of the stores. This can save work in the stores because the products are logically grouped. Finally, dedicated storage can be helpful if products have different weights. Heavy products have to be on the bottom of the pallet and light products on top. By storing products in order of weight and routing the order pickers accordingly, a good stacking sequence is obtained without additional effort. However, order picking routes will be longer compared to routing without weight restrictions, see Boerrigter [24]. Dedicated storage can be applied in pick areas, with a bulk area for replenishment that may have, for example, random storage. In this way, the advantages of dedicated storage still hold, but the disadvantages are only minor because dedicated storage is applied only to a small area.

A fourth storage policy is *full-turnover storage*. This policy distributes products over the storage area according to their turnover. The products with the highest sales rates are located at the easiest accessible locations, usually near the depot. Slow moving products are located somewhere towards the back of the warehouse. An early storage policy of this type is the cube-per-order index (COI) rule, see Heskett [122] and [123]. The COI of an item is defined as the ratio of the item's total required space to the number of trips required to satisfy its demand per period. The algorithm consists of locating the items with the lowest COI closest to the depot. A practical implementation of full turnover policies would be easiest if combined with dedicated storage. The problem is that demand rates vary constantly and the product assortment changes frequently. Each change would require a new ordering of products in the warehouse resulting in a large amount of reshuffling of stock. A solution might be to carry out the restocking once per period. The loss of flexibility and consequently the loss of efficiency might be substantial when using full-turnover storage. The concept of *class-based storage* combines some of the methods mentioned so far. This method divides the products into a number of classes. Each class is then assigned to a dedicated area of the warehouse. Storage within an area is random. Classes are determined by some measure of demand frequency of the products. Fast moving items are generally called *A-items*. The next fastest moving category of products are called *B-items*, and so on. Often the number of classes is restricted to three, although in some cases more classes can give additional gains with respect to travel times. The advantage of this way of storing is that fast moving products can be stored close to the depot and simultaneously the flexibility and low storage space requirements of random storage are applicable.

All five storage assignment policies discussed so far have not entailed possible relations between products. For example, customers may tend to order a certain product together with another product. In this case, it may be interesting to located these two products close to each other. An example of this is called *family-grouping*, where similar products are located in the same region of the storage area. Clearly, grouping of products can be combined with some of the previously mentioned storage policies. For example, it is possible to use class-based storage and simultaneously group related items. However, the decision in which class to locate the products has to depend on a combination of the properties of all products in the group.

Literature on storage assignment includes [2], [34], [38], [39], [43], [85], [99], [103], [104], [109], [110], [120], [121], [132], [140], [145], [148], [149], [151], [152], [157], [161], [162], [167], [168], [180], [184], [186], [189], [191], [190], [192], [199], [200], [210], [217],

[218], [231], [232], [239], [241], [242], [243], [246], [250], [251], [252], [256], [265], [268], [276], [277], [283], [286], [298].

1.4.2 Order picking policies

Order picking concerns the retrieval of products from storage to meet the demand of customers. This order picking process is often the most laborious activity in a warehouse. On average the costs of order picking may amount to about 55% of the operational costs in a warehouse, see Tompkins *et al.* [269]. The efficiency of the order picking process depends on factors such as the methods for storage and transport (flow racks, shelves, order picking trucks, picking carts, etcetera), on the layout of the area and on the control mechanisms. In this section we will mainly focus on order picking policies for picker-to-part systems, since these systems are the subject of this thesis. Some attention to policies for part-to-picker systems is given in Section 1.2.3.

In a typical conventional picker-to-part warehouse, the order picking process may consist of the following steps:

1. A customer places an order at the warehouse, for example by phone, e-mail or EDI message.
2. The order is processed by the Warehouse Management System (a computer system),
3. A pick list (a list with products ordered by customers) is printed,
4. An order picker (person that is responsible for retrieving products from storage) gets the pick list and a pick device and walks or drives through the warehouse to get the products, that were ordered.
5. After retrieving all products, the order picker drops off the products at a designated location (depot).

The time needed to pick an order consists of several components. We distinguish: *walking or driving between items*, *picking of items* and *remaining activities*. Driving to the pick location may include lifting, since often order picking trucks are used that are capable of simultaneously driving and lifting. A model for measuring efficiency of order picking tasks is given in Riaz Khan [229]. The time needed for driving to pick locations obviously depends on the layout of the picking area, see also Chapters 4 and 5.

Picking the items consists of a series of actions. For example, an order picker driving a truck or crane has to perform the following pick activities:

- position the truck or crane at the pick location,
- pick the proper quantity from the location,
- put the picked items on a product carrier,
- confirm the pick on a pick list or mobile terminal,
- read information about the next pick.

The remaining activities of the order picking process include picking up an empty pick carrier, the acquisition of information on which items have to be picked at which locations, and dropping off the full pick carrier at some point after the pick.

A typical order picking situation is depicted in Figure 1.1. The warehouse is rectangular with no unused space and consists of a number of parallel *pick aisles.* The warehouse is divided into a number of *blocks,* each of which contains a number of subaisles. A *subaisle* is that part of a pick aisle that is within one block. At the front and back of the warehouse and between each pair of blocks, there is a *cross aisle.* Cross aisles do not contain storage locations, but can be used to change aisles. Every block has a *front cross aisle* and a *back cross aisle*; the front cross aisle of one block is the back cross aisle of another block, except for the first block. Usually, there are no storage locations accessible from the cross aisles. The order pickers can walk or drive in any of the pick aisles or cross aisles. In the figure each square corresponds to a storage location. Each solid black square indicates a location that has to be visited by the order picker on his route to pick products. Figure 1.1 also depicts the depot, where order pickers start and end their routes.

In this thesis, we will focus on methods to achieve a decrease in the time needed to pick an order. First of all, there is the choice of equipment. This includes storage and transport systems, but also control equipment such as bar code scanners. Most equipment choices are influenced by factors such as the type (shape, size) of the products and by storage and demand quantities and frequencies. For example, products that are ordered frequently may be placed in flow racks or small items may be placed in storage drawers, see also Section 1.2.1.

Secondly, we can decide where to store products. Often a warehouse has several areas, such as a pallet area, case pick area and item pick area, see Section 1.1. Products can be stored in one or in several of these areas. For order picking it is attractive to have an area that is as small as possible. However, a smaller picking area has to be replenished more often. Therefore, sizes of areas have to be balanced and given the area sizes it has to be decided where to store which product. See also the forward-reserve problem described in Section 1.4.4.

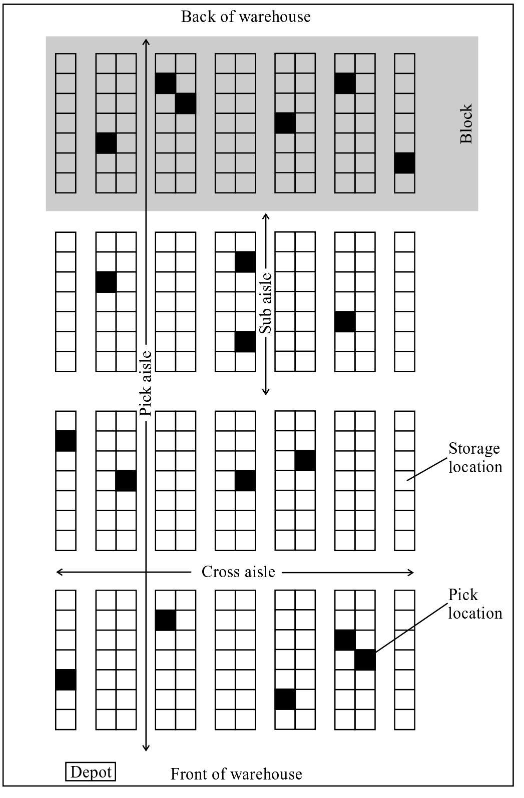

FIGURE 1.1. A typical order picking situation.

As a third method to achieve a decrease in the time needed to pick an order, we consider the operating policies. An interesting aspect of a change in operating policies is that such a change is often possible in an existing warehouse without additional capital investments. Sometimes policies can be introduced or changed solely by making changes in the control system (for example in the software of the warehouse management system). For example, for a given order, we could try to minimize the distance that has to be traveled by the order picker to retrieve all products from storage. This would change the routes taken by the order pickers, but equipment like the racks and order picking vehicles can remain unchanged.

A fourth option to achieve a decrease in the time needed to pick an order is to determine a good layout for the order picking storage area. Clearly, physical changes to the warehouse can be very involving. Therefore, this option is especially suitable for warehouses, that still need to be built. When determining a layout we have to take into account factors such as aisle length and the number of aisles and cross aisles.

This issue was discussed in Section 1.3.3 and we will give new results on layout issues in Chapters 4 and 5.

We can distinguish a number of operating policies for order picking. Three order picking policies are concerned with zoning, batching and routing, see also Cormier and Gunn [49]. *Zoning* occurs if each order picker only picks products from an order if those products are located in his assigned zone, i.e. part of the warehouse. *Routing* policies determine the sequence in which products have to be retrieved. That is, for a given list of products we try to reduce the distance that order pickers have to walk. For *batching,* suppose we have a number of orders that have to be retrieved from storage. Then we could have the order picker retrieving the orders one at a time or we could try to combine several orders and retrieve them simultaneously. Retrieving several orders in one route generally reduces the distance that has to be traveled by the order picker. However, more effort is needed to keep track of which product belongs to which order or to sort products later on. We will first discuss routing policies and thereafter zoning and batching policies will be explained.

Routing policies offer an important possibility to achieve savings on order pickers and equipment by optimizing order picking routes. Depending on the situation, the problem of finding a good sequence for retrieving products is more or less complicated. Broadly, the problem is in which sequence the order picker or automated vehicle needs to visit a number of locations to minimize total travel time. The simplest case occurs in a situation where single command cycles are performed, for example by an automated storage / retrieval system. *Single command cycles* means that either a load (e.g. a pallet or bin) is moved from the input point into a rack (storage) or a load is moved from a rack to the output point (retrieval). More complicated situations arise when *dual command cycles* are performed, i.e. a load is picked up at the input point and put in the rack, then another load is retrieved from the rack and deposited at the output point. This saves some travel distance, since the system does not have to go forth and back to the input–output point twice. The objective is to combine storage and retrieval commands such that the travel is minimized. Routes for automated storage / retrieval cranes are generally restricted to one storage and one retrieval command within one aisle. In case picking and item picking operations it often occurs that the order pickers have to pick several products in one route. Literature on routing with several picks in one aisle includes [14], [27], [138], [255] and [285]. Furthermore, a route can go through several aisles in the warehouse. In the remainder of this section we will focus on order picking situations where order pickers can visit multiple aisles to pick one or more products for an order.

The objective of operating policies for routing is to sequence the items on the pick list to ensure a good route through the warehouse. The problem of routing order pickers in a warehouse is actually a special case of the Traveling Salesman Problem, see also Lawler *et al.* [164]. The traveling salesman problem owes its name to the problem described by the following situation. A salesman, starting in his home city, has to visit a number of cities exactly once and return home. He knows the distance between each pair of cities and wants to determine the order in which he has to visit the cities such that the total traveled distance is as small as possible. Clearly, the situation of the traveling salesman has many similarities with that of an order picker in a warehouse. The order picker starts at the depot (home city), where he receives a pick list, has to visit all pick locations (cities) and finally has to return to the depot. An example layout of a warehouse with pick locations is given in Figure 1.1. A graph representation of the situation of this figure is given in Figure 1.2. A graph is a pair (V, E) where V is a finite, non-empty set of elements called vertices or nodes and E is a finite set of unordered pairs of elements of V called edges.

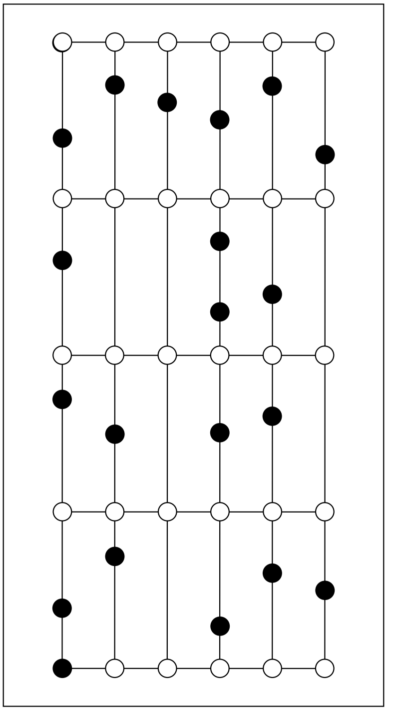

FIGURE 1.2. Graph representation of the situation depicted in Figure 1.1.

Some differences exist between the classical Traveling Salesman Problem and the situation of order picking in warehouses. First of all, if we look at the graph in Figure 1.2, there are a number of nodes that do not have to be visited (indicated with white circles). These nodes are the cross points between aisles and cross aisles. The order picker is allowed to visit them, but does not have to. The black circles represent the pick locations and the depot; these nodes must be visited. It is permissible to visit the pick locations and depot more than once. The problem of order picking classifies as a *Steiner* Traveling Salesman Problem because of the two facts that some of the nodes do not have to be visited and that the other nodes can be visited more than once, see e.g. Cornuéjols *et al.* [53]. The difficulty with the (Steiner) Traveling Salesman Problem is that it is in general 'hard' to solve. That is, there is no algorithm, i.e. solution method, known that can solve instances of the Traveling Salesman in polynomial time. We define a polynomial-time algorithm as an algorithm for which the maximum number of elementary operations used on a problem instance of size n is bounded from above by a polynomial in the input size.

However, for one type of warehouses it was shown by Ratliff and Rosenthal [227] that there does exist an algorithm that can solve the problem in running time linear in the number of aisles and the number of pick locations. The situation considered in this paper is a warehouse consisting of one block. In Cornuéjols *et al.* [53] it is shown that the algorithm of Ratliff and Rosenthal [227] can be extended to solve the Steiner Traveling Salesman Problem in all series-parallel graphs. A graph G is a series-parallel graph if starting from a graph consisting of two vertices u, v joined by an edge (u, v), G can be constructed by a number of 'series' or 'parallel' operations. A 'series' operation is defined as replacing an edge (u, v) by two edges (u, w) and (w, v) with w a new vertex and a 'parallel' operation as duplicating an edge between two vertices.

In Van Dal [275], De Koster and Van der Poort [64] and in Chapter 3 the algorithm by Ratliff and Rosenthal [227] is extended to different warehouse situations that cannot be represented as series-parallel graphs. The algorithm from De Koster and Van der Poort [64] can determine shortest order picking routes in a warehouse of one block with decentralized depositing. *Decentralized depositing* means that order picker can deposit picked items at the head of every aisle, for example on a conveyor. Instructions for the next route are given via a computer terminal. In Chapter 3 we develop an algorithm for a warehouse with three cross aisles, one in the front, one in the back, and one in the middle. The graph for this layout is not series-parallel. The problem of routing is solved in Gelders and Heeremans [95] by applying a branch-and-bound algorithm (see Little *et al.* [179]) to a warehouse layout with multiple blocks.

In practice, the problem of routing order pickers in a warehouse is mainly solved by using heuristics. A heuristic is an algorithm that generates a feasible solution, which cannot be guaranteed to be optimal. In Hall [115] a number of common routing heuristics are given for warehouses consisting of one block and also approximate estimates for average travel time for these heuristics and the optimal algorithm are given. Performance comparisons between optimal routing and heuristics for warehouses of one block are given in e.g. De Koster and Van der Poort [64], De Koster *et al.* [65] and Petersen [216]. The effect of routing in combination with storage assignment rules (see Section 1.4.1) is analyzed in Petersen [217] and Petersen and Schmenner [218]. A preliminary study on heuristic routing in warehouses with multiple blocks was done in Roodbergen and De Koster [236]. They compare three heuristics for a number of situations, including a narrow-aisle high-bay warehouse where order picking trucks are used. A routing heuristic, using dynamic programming, for warehouses with multiple blocks is presented in Vaughan and Petersen [288]. Heuristics from these articles as well as a new heuristic are given in Chapter 2 of this thesis.

Other issues may arise when trying to find good routes for order pickers. All articles discussed so far assume that the aisles of the warehouse are narrow enough to allow the order picker to retrieve products from both sides of the aisle without changing position. In Goetschalckx and Ratliff [102] an polynomial-time optimal algorithm is developed that solves the problem of routing order pickers in wide aisles. Another problem with routing may arise if products are stored at multiple locations in a warehouse. In this case a choice has to be made from which location the products have to be retrieved. A model for the problem of simultaneous assignment of products to locations and routing of order pickers is given in Daniels *et al.* [57]. Furthermore, heuristics are given to solve the problem. A further routing problem is that of allowing the order picker to do multiple picks per stop. That is, the order picker travels through a warehouse with a vehicle. He stops the vehicle and walks forth and back to a number of pick locations to retrieve products. Then he continues to the next stop location. And so on. The trade-off is between the time to start and stop the vehicle and the distance walked by the order picker. This problem was analyzed and solved optimally in Goetschalckx and Ratliff [101].

Part of the research on routing consists of travel time estimation. Using techniques from statistics and operations research an attempt is made to give an estimate of how much time (or distance) it takes to collect an order. Many results are known for systems where the vehicle is confined to a single aisle, see for example [29], [44], [83] and [118]. For travel time estimates of single and dual command cycles in multiple aisle systems see e.g. [21], [89], [162] and [207].

Few researchers have looked for travel time estimates for picking in systems with multiple aisles and multiple picks per route. Kunder and Gudehus [159] give travel time estimations for three routing heuristics in a warehouse consisting of one block. This work is extended in Hall [115] with more advanced routing heuristics for one block warehouses. Furthermore, a lower bound on travel time for the optimal algorithm from Ratliff and Rosenthal [227] is given. Formulations for average travel time in a situation with decentralized depositing are given in De Koster *et al.* [65]. Both Hall [115] and Kunder and Gudehus [159] assume that pick locations are distributed randomly over the order picking area according to a uniform distribution. In Jarvis and McDowell [140] travel time estimates are determined and used to determine which products (fast moving, slow moving) should be located in which aisles. A travel time analysis for a general product to location assignment is given in Chew and Tang [43] and Tang and Chew [265]. That is, demand rates for products can vary throughout the warehouse. They use the travel time estimates to evaluate batching strategies. Expected travel distances for two routing methods in a warehouse consisting of two blocks is given in Caron *et al.* [38]. The depot is located between the two blocks, which is different from what is depicted in Figure 1.1. Furthermore, items are assumed to be distributed according to the cube-per-order index, see Section 1.4.1. Some travel time estimates which are used in an analysis of batching strategies including time-window batching are given in Choe [46].

None of the articles mentioned here has an explicit modeling of the depot location. Furthermore, some parts of the travel time estimates can be improved. We will discuss the depot location and an improvement of the estimates in Chapter 4. Travel time estimation in warehouses with multiple blocks and any number of picks has also not been analyzed before. Chapter 5 deals with this issue.

Zoning and batching policies can be used to distribute orders over a number of order pickers. *Zoning* is a method to divide the total order picking area into smaller units. Order pickers only retrieve products that are located in their zone. *Batching* is a method to determine which order picker retrieves which (partial) orders. The simplest form of distributing orders over order pickers is *single order picking*, i.e. each order picker retrieves one complete customer order within the picking area. For this he may have to traverse the whole order picking area.

As an alternative to single order picking, the order picking area can be divided into zones. Each order picker is assigned to pick the part of the order that is in his assigned zone. One disadvantage of zoning is that orders are split and must be consolidated again before shipment to the customer. Two approaches can be used to cope with this. The first approach is that of *progressive assembly* of an order. Using this approach one order picker starts on the order. When he finishes his part, the tote and pick list (or any other means that are used) are handed to the next picker, who continues the assembly of the order. This system is also called *pick-and-pass*. The second approach for zoning is *parallel picking*, where a number of order pickers start on the same order, each order picker in his own zone. The partial orders are merged after picking. In practice, zoning is partially based on product properties, like size, weight, required temperature and safety requirements. Developing a method to determine zone sizes within a specified warehouse area would however be an interesting research topic. Important for zone sizing is that the workload is equally distributed over the order pickers. Little literature on zoning is available. A generic discussion on zoning is given in Speaker [262]. In Choe [46] the problem of simultaneous zoning, batching and sorting is considered. In Malmborg [183] the problem of assigning products to locations is studied with zoning constraints. A case study with zoning and batching is given in Brynzer and Johansson [33]. Zoning also occurs in *pick-to-bin systems.* In these systems, order pickers generally work on one rack; each order picker picks products from only a part of the rack. Picked products are put in a bin and if a picker is done with his part then the bin is handed to the next picker, see De Koster [58] and [60].

An alternative for the type of zoning described above would be a more dynamic way of assigning order pickers to zones. An example of this is the use of bucket-brigades, see Bartholdi [15], Bartholdi and Eisenstein [17] and Bartholdi *et al.* [16]. Bucket brigades can be especially suitable in situations such as pick-to-bin, where all order pickers are working on a single straight line. The idea is roughly as follows. There is one rack from which the products are to be retrieved. One order picker starts an order at the far left of the rack. He picks a number of products and at some point gives the partially fulfilled order to the next order picker, who continues picking the products along the line. The order is handed from picker to picker until it reaches the far right of the line, where it is put on a conveyor for further transport. The special feature of the bucket brigades is the way in which it is determined when an order is handed from one order picker to the next. Suppose, at some point in time all order pickers are working on separate orders, if the order picker closest to the end of the line deposits his finished order, he walks back along the line towards the starting point. If he meets another order picker, he then takes over the order from the other person and continues picking this order. The order picker from which the order was taken moves back along the line until he meets another order picker, and so on. One order picker starts all orders. The order pickers have to be in sequence of their respective speed of working for the system to function adequately. The main advantage of bucket brigades is that they are self-balancing with respect to workload.

As discussed, zoning is a way of distributing an order over a number of order pickers. Thus several order pickers are working on the same order. However, we could also let one order picker retrieve several (small) orders at the same time. This is called *batching*. Each order picker then collects a batch of orders, while sorting the items per order (called *sort-while-pick*). Most heuristic batching methods basically follow three steps: a method of initiating batches, a method of allocating orders to batches and a stopping rule to determine when a batch has been completed.

Another way of batching is to add up all available orders per item, retrieve all items individually and sort them afterwards into customer orders. Various intermediate forms exist. Combinations of zoning and batching are possible as well. An advantage of batching is that the length of the route necessary to fill a batch of orders is shorter than the sum of the route lengths if the orders are retrieved individually. Batching may require a different type of vehicle, the travel speed may be different, and the order picker or someone else may have to perform extra functions such as separating the batch into the individual orders. A limit on the size of a batch is most often determined by the capacity of the vehicle or an upper limit on response time. Batching becomes more difficult when orders arrive on-line. In many warehouses there is a batch-arrival component (for example, orders left from the previous day that did not make the departure time) and an on-line component. For on-line arrivals, there is a trade-off between waiting until more orders arrive so that more efficient pick routes can be formed (shorter pick times per item) and, on the other hand, minimizing the workload and realizing due times, see also Bhaskaran and Malmborg [23].

Batching is discussed in for example [4], [46], [78], [79], [80], [81], [82], [83], [90], [98], [130], [135] and [173]. An overview and comparison of existing order batching heuristics is given in Pan and Liu [205] for the case of order picking in one aisle. In De Koster *et al.* [66] and Rosenwein [247] batching overviews are given for the case of order pickers that can visit multiple aisles. A branch-and-bound algorithm to solve the problem of minimizing the maximum pick time of any of the batches to optimality is given in Gademann *et al.* [94].

Part-to-picker policies are used to control part-to-picker systems. For example, an important issue in the control of carousels is the sequencing of picks, such that the system has to make as few movements as possible. An operator can extract items from one or from multiple carousels. Many aspects discussed for the picker-to-part systems also play a role in part-to-picker systems, such as storage assignment.

Carousel literature includes [19], [97], [110], [117], [131], [132], [133], [249], [270], [278] and [293]. Furthermore, automated storage / retrieval systems and miniload systems can be used for picking systems where products move to the picker. These systems are frequently studied under the name *end-of-aisle* order picking systems, see e.g. [30], [31], [88], [182] and [209].

1.4.3 Accumulation and sorting policies

If batching or zoning is applied, then some extra effort is needed to ensure that in the end all products of each order can be shipped to the customers together. One way to do this is to incorporate the order consolidation into the order picking process. For example, by using a sort-while-pick or a pick-and-pass system. Alternatively, accumulation and sorting is performed after all products are picked. For this process a sorting system can be used.

In general, a sorting system can perform the following actions:

- loading – products are fed to the sorter with sufficient space between them;
- accumulation – it may be necessary to temporarily buffer products before merging to compensate for imbalance between flows;
- merging flows – when two or more flows come together (for example from different areas), they have to be merged into one flow;
- transport and induct – before sorting, usually the distances between the products have to be optimized;
- sorting – each product is diverted to the sorting lane corresponding to the order to which the product belongs.

Equipment properties that are of influence on the throughput capacity of the sorter include the speed of the sorter conveyor and the space needed between products. Operating strategies like the assignment of orders to lanes also influence throughput.

Before sorting, different product flows often have to be merged. Possible merging policies are those of alternating, priority and train merge. With train merge policy, boxes are accumulated before the merge. After accumulation, the boxes continue their route as a group (a train) of equidistantly spaced boxes that can be sorted easily. After merging, the products have to be sorted. An important factor in sorting is to determine the assignment of orders to lanes.

Merging of flows is analyzed in Arantes and Deng [3] and De Koster en Wijnen [67]. Simulation is used in Bozer and Sharp [28] to examine the advantages of using a recirculation loop to avoid lane blocking when a shipping lane is full. In Bozer *et al.* [26] simulation is used to examine the system when there are more orders than lanes, and orders that are not assigned to a lane are forced to recirculate around the loop. A case study on the implementation of a sorting system is given in Johnson and Lofgren [143] and in Meller [195] the optimal order-to-lane assignment is determined. Downstream sorting is analyzed in Choe [46]. Other literature on merging and sorting includes [141], [201], [291] [296] and [302]. More on conveyors can be found in [76], [77], [106], [202], [220], [221], [254], [261] and [297].

1.4.4 Other operating policies

Many other processes have to be managed in a warehouse. Each process can be optimized individually, even though an optimization of the parts will not guarantee a good overall solution. To sketch the wide range of optimization problems that arise in warehouses, we give in this section some further areas of interest and some of the literature involved.

Policies for automated storage / retrieval systems

Generally, automated storage / retrieval systems and miniload systems operate with single or dual command cycles, see Section 1.4.2. For dual command cycles it is interesting to schedule loads such that combinations of the load to store and the load to retrieve is such that travel times are minimized. More picks can also occur. If there are more picks in one aisle, then the routing problem is in a two-dimensional plane. However, the plane is far from euclidean because horizontal travel speed may depend on the height at which the vehicle is traveling and accelerating and deceleration play an important role. For the situation of multiple picks in multiple aisles, see Section 1.4.2. For travel time minimization it is also important to position frequently requested loads close to the input-output point, where pallets have to be deposited after retrieval. For class-based storage the shapes of the classes have to be determined and the assignment of products to locations has to be determined. Batching has also been evaluated within the context of automated systems. Another point to consider is where to park the vehicle if it is idle; *dwell-point policies* have been developed for this positioning.

See [5], [6], [8], [9], [10], [11], [12], [13], [27], [29], [41], [42], [44], [45], [71], [72], [73], [74], [75], [80], [81], [84], [85], [88], [100], [104], [112], [118], [121], [128], [130], [131], [132], [133], [134], [135], [136], [137], [139], [142], [146], [147], [149], [150], [151], [152], [154], [155], [156], [157], [160], [165], [166], [167], [169], [170], [172], [174], [175], [176],

[177], [178], [182], [192], [197], [200], [206], [208], [214], [223], [224], [233], [242], [245], [253], [256], [257], [259], [264], [268], [277], [280], [281], [289] and [292].

Policies for forward-reserve allocation

Many warehouses have a system for order picking with a forward area and a reserve area. The forward area is used for order picking and the reserve area is used to replenish the forward area. The size of the forward area is restricted; the smaller the area the lower the average travel times of the order pickers will be. It is important to decide how much of each product is placed in the forward area and where in the area it has to be located. It may even be the case that it is advantageous to store some of the products only in the reserve area. This may occur if demand quantities are very high or if demand frequencies are very low. Furthermore, replenishments are often restricted to times at which there is no order picking activity, which gives additional constraints. The decisions concerning the problems noted here are summarized under the name *forward-reserve problem*. Literature includes Frazelle *et al.* [93], Hackman and Platzman [111] and Van den Berg *et al.* [282].

Policies for internal transport

One of the problems in a warehouse is the transportation of goods from one area of the warehouse to another area. For example, pallets with products that arrive at the warehouse have to be transported to the labeling area to be labeled with barcode for identification purposes. After that the pallet has to be transported to the storage area. This transport can be done using, for example, counterbalance lift trucks. Lift trucks are in wide use since they are inexpensive and very flexible. The assignment of jobs to lift trucks is a complex problem that is closely related to the Vehicle Routing Problem, see e.g. Fisher [87]. Instead of lift trucks, conveyors or automated guided vehicles can be used to transport goods from one location to another location. Automated guided vehicles are capable of driving from location to location without the need of an operator. Conveyors are especially fit for transportation of large amounts of cases, but can also be used for full pallets. Policies for accumulating and sorting with conveyors are dealt with in Section 1.4.3. A study on control policies for lift trucks and automated guided vehicles is given in Van der Meer [284]. Another issue that appears when designing the system is the number of vehicles that are needed to transport all products in time. Although the context of the article is within container terminals, the algorithm given in Vis *et al.* [290] could be used to determine the minimum number of vehicles needed in a warehouse for internal transport.

Policies for palletizing

In the process of putting products on pallets it might be worthwhile trying to fit all products on as few pallets as possible. Restriction may include weight limitations, having to keep customer orders together, environmental restrictions or different temperature requirements. The problem of palletizing is closely related to the bin packing problem, see e.g. Coffman *et al.* [48]. Pallet loading schemes are treated in [36], [37], [40], [70], [124], [125], [212], [222], [228], [260], [263], [267], [271] and [272].

Related to this is the problem of packing a number of unit loads, e.g. pallets, as efficiently as possible into a vehicle or container. Literature on this subject includes [96] and [116].

Policies for cross docking

In a typical cross-docking operation, a number of trucks arrive and loads are unloaded from these trucks. Other trucks arrive and take the loads that correspond to their destination. Thus, incoming loads may be grouped by supplier; outgoing loads by customer. The problems to solve are finding a good layout and a good assignment of trucks to dock doors in order to minimize workload (travel time of workers) and to avoid congestion. The literature on this subject includes [18], [107], [108], [273] and [274].

1.5 Outline of the thesis

In the remainder of this thesis we focus on the order picking area. We identified order picking as one of the major cost components in warehousing. Therefore, any improvement in this area can have a significant impact on profitability of the warehouse. In Chapter 2 we focus on routing of order pickers in a picker-to-part environment. Routing of order pickers has been extensively researched in layouts consisting of one block. So far only one publication has dealt with routing in layouts with any number of blocks. We start Chapter 2 with a short overview of literature on routing in one block warehouses. Thereafter, we describe a number of routing heuristics for multiple block warehouses. A comparison of routing methods is given as well. The chapter ends with a case study of a company that actually implemented a heuristic we developed. This chapter is based in part on the publications [63], [65], [236], [238] and [239].

Since heuristics only give a feasible solution which cannot be guaranteed to be optimal, it would also be interesting to have an algorithm that is capable of generating shortest order picking routes in warehouses with multiple blocks. In Chapter 3, we give such an algorithm which finds shortest routes in layouts of two blocks. Clearly this algorithm is useful only in practice if a layout with two blocks has on average lower travel times than a layout with one block. Therefore, we analyze the difference in average travel time between a layout with one block and a layout with two blocks in Chapter 3 as well. This chapter is based on our research in [237].

Routing methods, like those in Chapters 2 and 3, try to find good routes for a given layout. But in one layout average travel time may very well be much higher than in another layout. We investigate the influence of layout on average travel distance of order pickers in Chapter 4 and 5. In Chapter 4 we restrict the analysis to layouts of one block. Chapter 5 analyzes layouts of multiple blocks. For the analysis in these chapters we develop statistical estimates for average travel distance based on routes as generated by a routing heuristic. Layout factors that are taken into account are, for example, the number of aisles and the number of blocks. Moreover, we prove in Chapter 4 that in a one block layout the best location for the depot is the middle of the front cross aisle. The proof holds for all routing heuristics that have so far been discussed in literature. Chapters 4 and 5 are in part based on our research in [234] and [240].

2

Heuristic routing of order pickers

In Chapter 1 we have identified routing of order pickers as one of the important aspects to consider when improving the efficiency of order picking. Routing of order pickers concerns the determination of a sequence in which a certain number of products have to be retrieved from storage. The common objective in routing problems is to develop a method that is capable of generating routes that are as short as possible. Travel time reduction can lead to a significant decrease in warehousing costs, since typically 55% of warehouse operating costs comes from order picking and 50% of the order pickers' time is spent on traveling, see Tompkins *et al.* [269]. The potential for gains in practice is illustrated in Section 2.5 by means of a case study.

Besides the objective of short routes, there are other considerations. For example, it may be advisable to keep in mind that an order picker has to actually execute the route. One important reason in favor of heuristics – when compared to an algorithm that calculates shortest routes – is that they can create routes with an easy-to-understand structure. Order pickers have to follow the instructions on their pick list to form the route calculated by the computer system. If the route is not logically formed according to the order pickers then they may accidentally walk the wrong way or they may decide to override the system and walk the way they think is best. Clearly, this will not give the highest performance. Therefore, a route that is easy to understand can enhance picker productivity. Furthermore, determining shortest routes for order pickers is far from trivial. Although an algorithm exists for one-block warehouses, there is no algorithm available that can calculate shortest routes for any number of blocks efficiently. We do give an algorithm for two-block warehouses in Chapter 3. And even if an algorithm exists for a certain situation, it will be difficult to change it if the shape of the picking zones is changed. For example, the algorithm may work well in a zone consisting of one block, but if three zones are combined to form one new zone consisting of three blocks, the algorithm can no longer be used. If one of the heuristics from Section 2.2 is used, then no adaptation would be necessary.

Section 2.1 contains an overview of routing issues for warehouses consisting of one block. Several routing policies are described. Furthermore, differences between routing heuristics are discussed based on a literature study. Interactions with other operating policies such as storage assignment and batching are also taken into account. In Section 2.2 routing policies for warehouses consisting of multiple blocks are given. One method comes from existing literature; two other heuristics are extensions of heuristics for the one-block layout. An entirely new heuristic for a layout with multiple blocks, the combined$^+$ heuristic, is given in Section 2.3. To analyze the performance of the heuristics, an non-polynomial-time optimal algorithm is used that generates shortest order picking routes. Performance comparisons between heuristics and the optimal algorithm are given in Section 2.4 for various warehouse layouts and order sizes. For the majority of the instances with two or more blocks, the combined$^+$ heuristic appears to perform better than the other heuristics. In Section 2.4 some consequences for layout are also discussed. From the results it appears that the addition of cross aisles, i.e. replacing one block by multiple smaller blocks, can decrease handling time of the orders by lowering average travel times. However, adding a large number of cross aisles may increase average travel times because the space occupied by the cross aisles has to be traversed as well.

2.1 Routing in one block

One-block layouts are those which are analyzed most frequently in literature. Numerous operating policies have been tested in a one-block environment. Therefore, we will start with a discussion of this layout before going to the more general layout with any number of blocks. Routing policies can be split into two categories. Firstly, there are those methods that determine shortest routes, which we will call *optimal algorithms*. Secondly, we have routing *heuristics*, methods that determine a feasible route, that is not necessarily the shortest route. For the one-block layout there exists an efficient optimal algorithm, see Ratliff and Rosenthal [227]. Therefore it does not seem to be necessary to look into heuristics for this layout. However in Hall [115] and

Petersen [216] it was noted that in practical situations there may be reasons to prefer heuristic routing. Important reasons in favor of heuristics are their adaptability and the fact that they can create routes with an easy-to-understand structure.

We will first describe some routing policies for one-block layouts and then we will give an overview of results from literature. Note that these routing policies may be used for any pick list size. However, there is not much use for these policies with single or dual command cycles. Single command cycles simply go from the depot via the shortest path to the location and back to the depot. Dual command cycles require the visit of two locations. Route length will be the same regardless of the sequence of the two visits. Besides, dual command cycles often occur in full pallet load situations, where the vehicle first has to store a pallet (to empty the vehicle) before retrieving another pallet. Thus, differences in route length between policies will only occur for pick lists of three or more items.

2.1.1 S-shape heuristic

A simple way to route order pickers is by using the S-shape (also called transversal) policy. Any aisle containing at least one item is traversed through the entire length. Aisles with no picks are not entered. After picking the last item, the order picker returns to the front aisle. This heuristic is likely to be the most frequently used routing policy in practice. For an example route see Figure 2.1.

Numerous papers use the S-shape heuristic in their analyses in one or another way, see [64], [65], [66], [115], [159], [216], [217], [218], [250].

2.1.2 Return heuristic

With the return heuristic the aisles are always entered from the front and left on the same side after picking the items in this aisle. For an example route see Figure 2.1. The return heuristic is as easy to implement and use as the S-shape heuristic. The choice between these two heuristics is basically the same as the choice between single-sided and double-sided picking. *Single-sided picking* means that the order picker picks items from just one side of the aisle at a time. Therefore if both sides of an aisle have picks, the aisle has to be traversed twice. *Double-sided picking* means that the order picker retrieves items from both sides of the aisle at the same time. This means that the order picker constantly has to move from one side of the aisle to the other side. The S-shape heuristic is suitable for double-sided picking since every aisle is visited just once. The return heuristic is especially suitable for single-sided picking. As a consequence, one of the important criteria to choose between return and S-shape is the width of the aisles. From Goetschalckx and Ratliff [102] it appeared that the aisles have to be very wide or pick density has to be very high for double-sided picking to be useful. This leaves us with one main application for the return heuristic, which is in situations where there is only the possibility for changing aisles in the front cross aisle.

2.1.3 Midpoint heuristic

For this heuristic the warehouse is essentially divided into two halves. Picks in the front half are accessed from the front cross aisle and picks in the back half are accessed from the back cross aisle. Only the first and the last aisle with items are traversed entirely, see also Figure 2.1. This routing policy could be a good alternative to S-shape as long as there is on average only one pick per aisle. However, the largest gap heuristic, described below, always has a performance that is better than or at least equal to that of midpoint, see Hall [115]. The midpoint does offer a simple approach that is easy to implement.

2.1.4 Largest Gap heuristic

The picker enters the first aisle and traverses this aisle to the back of the warehouse. Each subsequent aisle is entered up to the 'largest gap' and left from the same side as it was entered. A gap represents the distance between any two adjacent items, or between a cross aisle and the nearest item. The last aisle is traversed entirely and the picker returns to the depot along the front entering again each aisle up to the largest gap. Thus, the largest gap is the part of the aisle that is not traversed. An example route is given in Figure 2.1.

2.1.5 Composite heuristic

This routing policy combines features of the S-shape and return heuristics. This heuristic decides for each aisle individually whether it is shorter to traverse it entirely or to make a return route. See Petersen [215]. An example route is given in Figure 2.1.

2.1.6 Combined heuristic

A routing policy that gives routes that look somewhat similar to the composite heuristic is the combined heuristic. This routing policy will be explained in detail in

Section 2.3. It has a dynamic programming component, which – in a way – makes it possible to look one aisle ahead. For example, consider an aisle for which it would be shortest to make a return. The combined heuristic could choose to traverse this aisle anyway, because this gives a better starting point for the next aisle. This will in turn lead to a better overall result. The combined heuristic has appeared to perform well in warehouses with multiple cross aisles, see Section 2.4. An example route is given in Figure 2.1.

2.1.7 Optimal algorithm

All routing policies mentioned before in this section restrict the possibilities of creating a route. For example, the S-shape heuristic forces order pickers to traverse each aisle entirely. To obtain the shortest route possible, we need a routing policy that is capable of considering all possibilities for travelling in and between aisles. In Ratliff and Rosenthal [227] such an optimal procedure is presented for routing workers in a rectangular warehouse. An example route is given in Figure 2.1. This algorithm can be run on a personal computer and finds the optimal route within seconds. See Section 1.4.2 for more information on determining shortest order picking routes.

2.1.8 Comparison of routing policies for one block

First of all we have to note that the shortest routes will be given by the algorithm described in Ratliff and Rosenthal [227]. For decentralized depositing the method described in De Koster and Van der Poort [64] can be used. If one prefers to use a routing heuristic, several decision problems arise. A specific heuristic may be very good in one situation but perform poorly in another situation. For example, pick density plays an important role. If the order picker has to visit very many locations in each aisle then it is highly likely that routes generated by the S-shape heuristic are close to optimal. But if there are only few picks per aisle then another heuristic might be better, see e.g. Hall [115].

The equipment is also an important aspect to consider when choosing between routing heuristics. For example, a vehicle that has a lower speed in the cross aisles than in the pick aisles will favor a heuristic that minimizes cross aisle travel. A vehicle that requires a considerable amount of time to enter aisles will favor a routing policy that enters aisles only once, which is essentially the same as minimizing cross aisle travel. See e.g. De Koster and Van der Poort [64] and De Koster *et al.* [65] for an analysis of the differences between optimal and heuristic routing in such an environment.

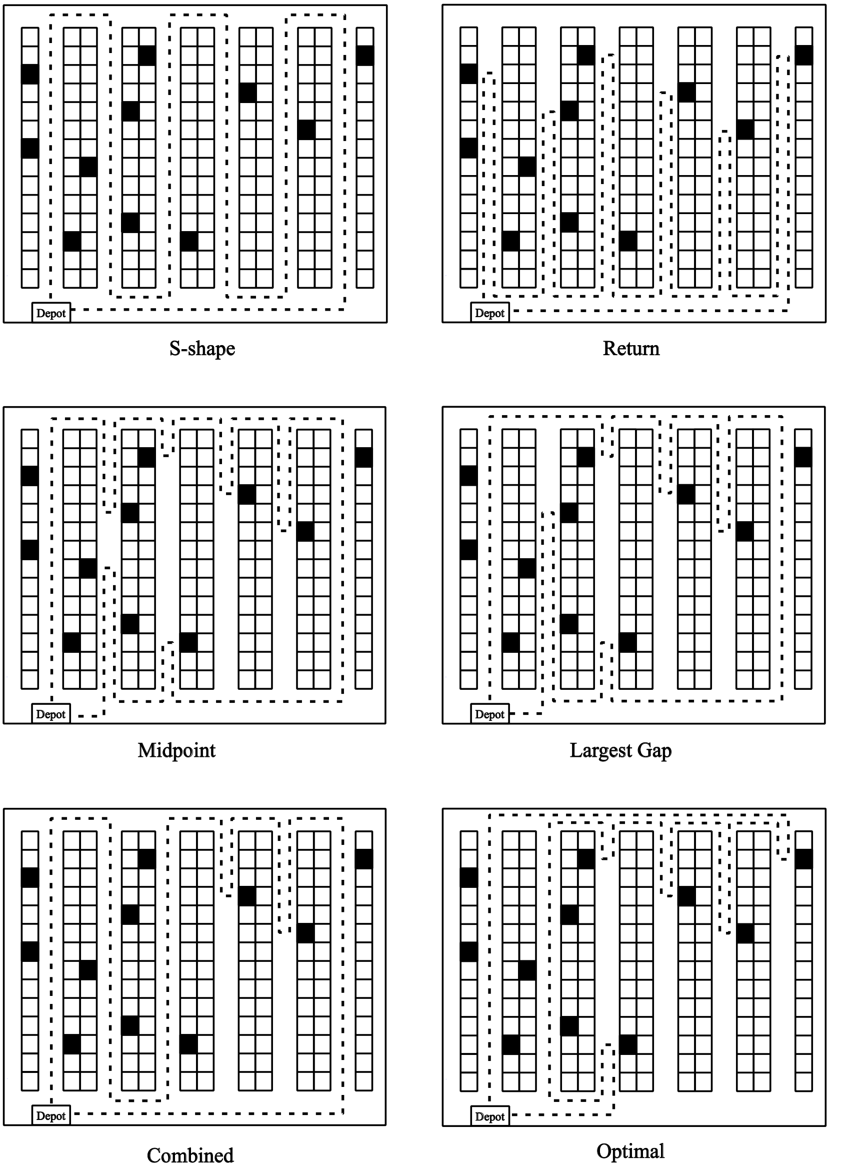

FIGURE 2.1. Example of a number of routing policies for a layout of one block.

Product properties play a role, too. If there are weight restrictions, i.e. heavy products cannot be stacked on light products, then this restricts the possibilities for routing. Heavy products have to be retrieved first, otherwise pallets have to be restacked while picking. In practice, this problem is often solved by positioning products in the racks in increasing product weight. A routing policy should then adhere to this ordering.

Thirdly, there is a strong interaction between routing and storage assignment policies. For example, S-shape always entirely traverses aisles. Therefore, it would make sense to assign storage locations such that the most frequently demanded items are in one aisle, the somewhat less frequently demanded items in the next aisle and so on, see Jarvis and McDowell [140]. A comparison between routing policies can therefore only be performed with a predefined storage assignment rule or with simultaneous consideration of storage assignment. Under random storage assignment, Hall [115] gives, for example, a rule of thumb for choosing between S-shape and largest gap. S-shape is preferred if the number of picks is on average less than 3.8 per aisle; otherwise largest gap is preferred. Again we note that the choice between routing policies also depends on the number of items per route. In Petersen [216] several routing policies are compared under random storage. The interaction between class-based storage and routing is studied in Petersen [217] and Petersen and Schmenner [218]. An open problem is to determine the optimal allocation of items when using an optimal algorithm for routing.

Furthermore, there is the issue of choosing a batching policy. The effectiveness of a batching policy depends on the underlying routing policy. For example, we can consider the following rule for adding an order to a batch: add the order that gives the least number of extra aisles to visit. Then, if the routing policy is S-shape this batching method is likely to perform fairly well. However, if the routing policy was largest gap then there may have been a much better batching candidate, see e.g. De Koster *et al.* [66]. We note that batching also has an additional effect on the decision concerning the routing policy because batching influences the number of picks per route.

Finally, there is a relation between the layout of the order picking area and the choice of a routing heuristic. If aisles are very wide a return policy may be preferred to a S-shape policy, see Goetschalckx and Ratliff [102]. And if the layout is such that there is only one cross aisle, then the return policy is the only one feasible of the heuristics mentioned here. The number of cross aisles is an important factor as well. In a layout with two cross aisles the largest gap heuristic might be better than S-shape, but by adding one cross aisle, the balance may turn. See Section 2.4 for the evaluation of various routing heuristics in a number of different layouts. In short, the routing heuristic can only be decided upon when keeping the situation in mind.

2.2 Routing in multiple blocks

A graphical sketch of the warehouse layout considered in this section is given in Figure 1.1. The warehouse is rectangular with no unused space and consists of a number of parallel *pick aisles*. The warehouse is divided into a number of *blocks*, each of which contains a number of subaisles. A *subaisle* is that part of a pick aisle that is within one block. The term *aisle* is used when a statement holds for both pick aisles and subaisles. At the front and back of the warehouse and between each pair of blocks, there is a *cross aisle*. Cross aisles do not contain storage locations, but can be used to change aisles. Every block has a *front cross aisle* and a *back cross aisle*; the front cross aisle of one block is the back cross aisle of another block, except for the first block. The number of cross aisles equals the number of blocks plus one. This holds because there is one cross aisle in the front, one in the back and one between each two adjacent blocks.

Order pickers are assumed to be able to traverse the aisles in both directions and to be able to change direction within the aisles. The aisles are narrow enough to allow picking from both sides of the aisle without changing position. Each order consists of a number of items that are usually spread out over a number of subaisles. We assume that the items of an order can and will be picked in a single route. Aisle changes are possible in any of the cross aisles. Picked orders have to be deposited at the depot, where the picker also receives the instructions for the next route. The depot is located at the head of the first pick aisle in the front cross aisle. Note that the location of the depot can potentially influence the average travel time. The effect of depot location for one-block layouts is evaluated in Chapter 4.

In the remainder of this section we describe four different types of routing. Two heuristics are based on well known heuristics for a layout with two cross aisles: S-shape and largest gap. Furthermore, the routing policy of Vaughan and Petersen [288] is described briefly. The fourth routing method, consists of finding a shortest route.

2.2.1 S-shape heuristic

Basically, any subaisle containing at least one pick location is traversed through the entire length. Subaisles where nothing has to be picked are not entered. In the following more elaborate description of the heuristic, letters between brackets correspond to the letters in the example route depicted in Figure 2.2a.

1. Determine the left-most pick aisle that contains at least one pick location (called *left pick aisle*) and determine the block farthest from the depot that contains at least one pick location (called *farthest block*).

2. The route starts by going from the depot to the front of the left pick aisle **(a)**.

3. Traverse the left pick aisle up to the front cross aisle of the farthest block **(b)**.

4. Go to the right through the front cross aisle of the farthest block until a subaisle with a pick is reached **(c)**. If this is the only subaisle in this block with pick locations then pick all items and return to the front cross aisle of this block. If there are two or more subaisles with picks in this block, then entirely traverse the subaisle **(d)**.

5. At this point, the order picker is in the back cross aisle of a block, call this block the *current block*. There are two possibilities.

   (1) There are picks remaining in the current block (not picked in any previous step). Determine the distance from the current position to the left-most subaisle and the right-most subaisle of this block with picks. Go to the closer of these two **(e)**. Entirely traverse this subaisle **(f)** and continue with step 6.

   (2) There are no items left in the current block that have to be picked. In this case, continue in the same pick aisle (i.e. the last pick aisle that was visited in either step 7 or in this step) to get to the next cross aisle and continue with step 8.

6. If there are items left in the current block that have to be picked, then traverse the cross aisle towards the next subaisle with a pick location **(g)** and entirely traverse that subaisle **(h)**. Repeat this step until there is exactly one subaisle left with pick locations in the current block.

7. Go to the last subaisle with pick locations of the current block **(i)**. Retrieve the items from the last subaisle and go to the front cross aisle of the current block **(j)**. This step can actually result in two different ways of traveling through the subaisle (1) entirely traversing the subaisle or (2) enter and leave the subaisle from the same side.

8. If the block closest to the depot has not yet been examined, then return to step 5.

9. Finally, return to the depot **(k)**.

2.2.2 Largest gap heuristic

The largest gap heuristic basically follows the perimeter of each block entering subaisles when needed. The heuristic first goes to the farthest block and then proceeds block by block to the front of the warehouse. A route resulting from this heuristic is depicted in Figure 2.2b. Letters between brackets correspond to the letters in Figure 2.2b. In this description we say that each subaisle is entered as far as the 'largest gap'. By a *gap* we mean the distance between any two adjacent pick locations within a subaisle, or between a cross aisle and the nearest pick location. The *largest gap* is the largest of all gaps in a subaisle. The largest gap divides the pick locations in a subaisle into two sets. One set of pick locations is accessed from the back cross aisle; the other set from the front cross aisle. We note that one or both of the sets may be empty, making it unnecessary to enter the subaisle from that side.

1. Determine the left-most pick aisle that contains at least one pick location (called *left pick aisle*) and determine the block farthest from the depot that contains at least one pick location (called *farthest block*).

2. The route starts by going from the depot to the front of the left pick aisle **(a)**.

3. Traverse the left pick aisle up to the front cross aisle of the farthest block **(b)**.

4. Go to the right through the front cross aisle of the farthest block until a subaisle with a pick is reached **(c)**. If this is the only subaisle in this block with pick locations then pick all items and return to the front cross aisle of this block. If there are two or more subaisles with picks in this block, then entirely traverse the subaisle **(d)**.

5. At this point, the order picker is in the back cross aisle of a block, call this block the *current block*. There are two possibilities.

   (1) There are picks remaining in the current block (not picked in any previous step). Determine the subaisle of the current block with pick locations that is farthest from the current position. Call this subaisle the *last subaisle of the current block*. Continue with step 6.

(2) There are no items left in the current block that have to be picked. Continue in the same pick aisle (i.e. the last pick aisle that was visited in either step 7, step 8 or in this step) to get to the next cross aisle and continue with step 9.

6. Follow the shortest path through the back cross aisle starting at the current position, visiting all subaisles that have to be entered from the back **(e)** and ending at the last subaisle of the current block **(f)**. Each subaisle that is passed has to be entered up to the largest gap. Note that this step may require the order picker to walk part of the cross aisle both from left to right and from right to left (see the example of Figure 2.2b)
7. Entirely traverse the last subaisle of the current block to get to the front cross aisle **(g)**.
8. Start at the last subaisle of the current block and move past all subaisles of the current block that have picks left. Enter these subaisles up to the largest gap to pick the items **(h)**.
9. If the block closest to the depot has not yet been examined, then return to step 5.
10. Finally, return to the depot **(k)**.

2.2.3 Aisle-by-aisle heuristic

This heuristic for warehouses with multiple cross aisles was presented in Vaughan and Petersen [288]. Order picking routes resulting from this heuristic visit every pick aisle exactly once. That is, first all items in pick aisle 1 are picked, then all items in pick aisle 2, and so on. Dynamic programming is used to determine the best cross aisles to go from pick aisle to pick aisle.

The order picking route starts at the depot. For every cross aisle i the distance is calculated that is needed to start at the depot, pick all items in pick aisle 1 and exit the pick aisle via cross aisle i. If there are m cross aisles, then this results in m distances each with a corresponding partial order picking route. Now for each cross aisle j, we determine cross aisle i such that the distance to start at the depot, pick all items in pick aisle 1, pick all items in pick aisle 2 and exit pick aisle 2 at cross aisle j, is shortest if we go from pick aisle 1 to pick aisle 2 via cross aisle i. This gives us again m distances and partial order picking routes. Continuing in a similar fashion, we determine for each cross aisle j exiting pick aisle 3 the best cross aisle to go from pick aisle 2 to pick aisle 3. This process is repeated until all pick aisles have been considered. Then the order picker returns to the depot. An example route is given in Figure 2.2c.

Note that the algorithm originally assumed that the order picker starts at the head of the left-most pick aisle of the warehouse and ends at the right-most pick aisle of the warehouse. For reasons of compatibility with the other routing methods, a minor change in the heuristic was made such that routes start and end at the depot.

2.2.4 Optimal algorithm

The routing of order pickers in a warehouse is a special case of the Traveling Salesman Problem. A number of locations have to be visited with the objective of traveling as little as possible. For the Traveling Salesman Problem there is no polynomial-time algorithm known that can find shortest routes. For warehouses with two cross aisles however, an efficient routing algorithm is given in Ratliff and Rosenthal [227]. Their method uses dynamic programming to solve the problem. Extensions of the algorithm to more cross aisles are non-trivial and the number of equivalence classes and possible transitions increase rapidly (see Chapter 3 for an algorithm for a layout with three cross aisles). The shortest order picking routes calculated for this chapter can be obtained with any enumeration method, such as a branch-and-bound procedure for the Traveling Salesman Problem (see e.g. Little *et al.* [179]). In Figure 2.2d an example route is depicted. The results from optimal routing will be used as a benchmark in the performance analysis of the heuristics described in this chapter.

2.3 Combined heuristic

For practice, it is generally preferred to have a routing method that generates routes that have a clear and easy to understand structure. Routes having a clear pattern reduce the time spent by order pickers on searching for locations and reduce the risk of pick errors. The combined routing method generates such routes. Every subaisle that contains items, is visited exactly once. The route starts and ends at the depot. The order picker goes through the left-most pick aisle that contains items towards the block farthest from the depot that contains items. The subaisles of the farthest block are visited sequentially from left to right. Then the order picker goes to the next block, which is one block closer to the depot. The items in this block are picked. This process is repeated until all blocks with items have been visited. See Figure 2.6a for an example route. The subaisles are either entirely traversed or the order picker enters and leaves the subaisle from the same side. These choices are made

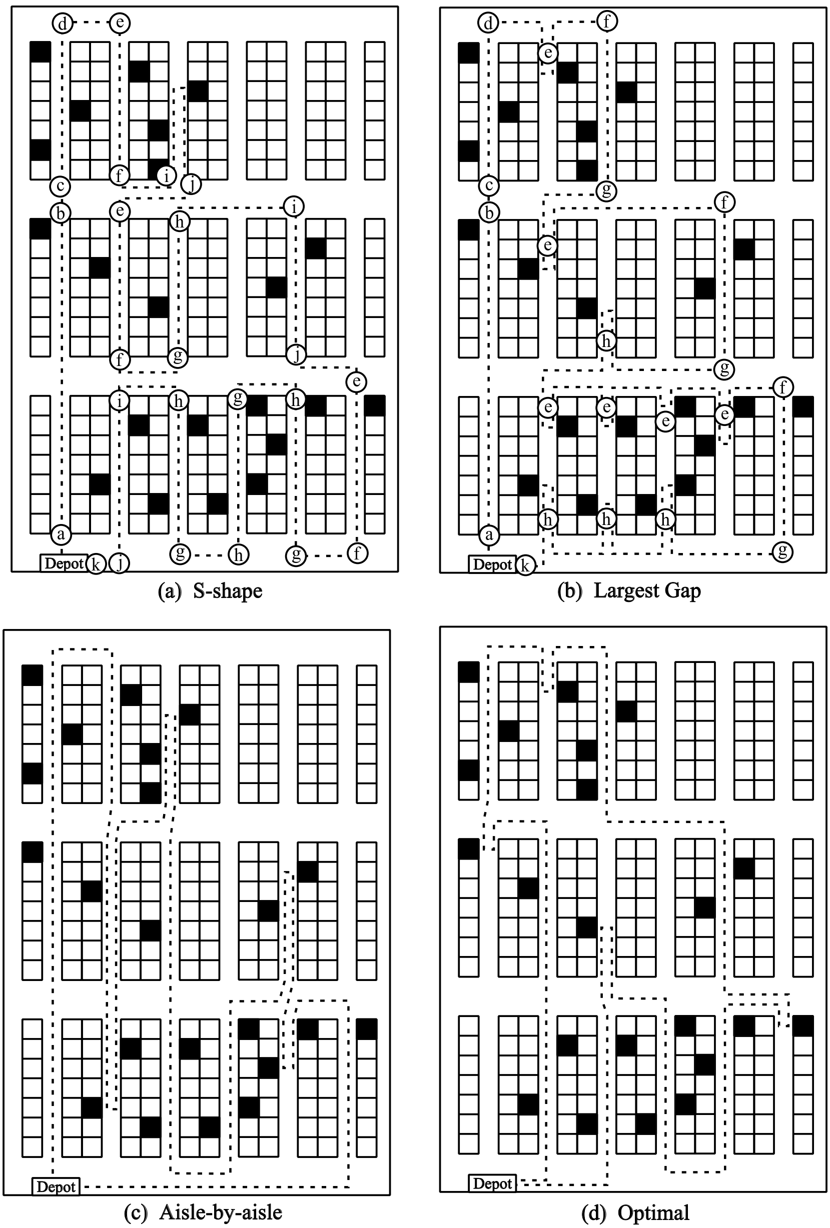

FIGURE 2.2. Example routes for four routing methods in a multiple block layout.

with a dynamic programming method which will be explained in Section 2.3.2. The construction of a complete route is discussed in Section 2.3.4.

2.3.1 Definitions of variables and parameters

We define the following variables:

- k the number of blocks
- n the number of pick aisles

We denote some physical locations in the warehouse as follows:

- a_{ij} the back end of subaisle j in block i, for each block $i, i = 1, ..., k$ and for each subaisle $j, j = 1, ..., n$
- b_{ij} the front end of subaisle j in block i, for each block $i, i = 1, ..., k$ and for each subaisle $j, j = 1, ..., n$
- d the depot

Note that for $i = 1, 2, ..., k - 1$ it holds that $b_{ij} = a_{i+1,j}$. This holds because we assume that the order pickers walk through the middle of the cross aisles. The distance from the end of a subaisle to the center of a cross aisle is administrated as if it belongs to the subaisle.

For each block we will use a dynamic programming method. We will first describe this dynamic programming method for a single block in Section 2.3.2. In Section 2.3.4 we describe how the heuristic creates routes, using the dynamic programming method for each single block.

2.3.2 Dynamic programming method for one block

This section describes a dynamic programming method to route an order picker through a single block i ($i = 1, ..., k$). The route starts at the left-most subaisle that contains items (ℓ) and ends at the right-most subaisle that contains items (r). We define L_j to be a partial route visiting all pick locations in subaisles ℓ through j and we distinguish two equivalence classes of partial routes:

L_j^a which is a partial route that ends at the back of subaisle j
L_j^b which is a partial route that ends at the front of subaisle j

We distinguish two ways to go from subaisle $j-1$ to subaisle j, see Figure 2.3:

t_a which goes along the back of the block
t_b which goes along the front of the block

Furthermore, we distinguish four ways to pick all items in subaisle j. These four transitions are (see Figure 2.3):

t_1 entirely traverse the subaisle
t_2 do not enter this subaisle at all
t_3 enter and leave the subaisle from the front of the block
t_4 enter and leave the subaisle from the back of the block

Clearly, transition t_2 is only allowed if the subaisle does not contain any items.

With $L_j + t_w$ we denote that partial route L_j is extended with transition t_w ($w = 1, 2, 3, 4, a, b$). The function $c(.)$ gives the travel time associated with its argument, e.g. $c(L_j^b + t_1)$ gives the time needed to walk the partial route L_j^b plus the time needed to walk transition t_1. Note that the transitions only contain information on how to enter and leave the subaisles. The exact path within the subaisle – and therefore the travel time associated with a transition – is dependent on the item locations within the subaisle under consideration.

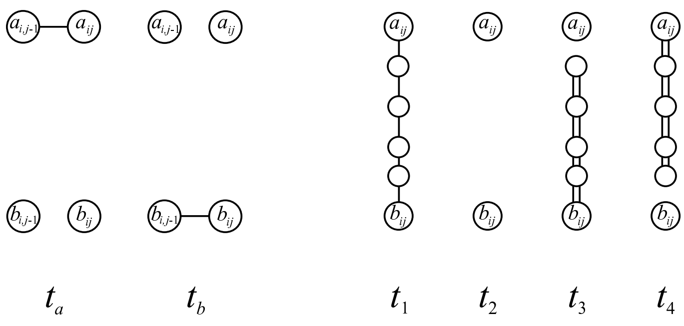

FIGURE 2.3. Transitions used by the combined routing heuristic.

Using the potential states, the possible transitions between the states and the costs (travel time) involved in such transition, we now give the dynamic programming method. This method will determine a partial route, going through one block. The construction of the full order picking path, connecting the partial routes for the individual blocks, will be described in Section 2.3.4.

Step 1

The block under consideration is block i.
If block i is the block farthest from the depot, that contains items, then start with the two partial routes:

L^a_ℓ which starts at node $b_{i\ell}$, ends at node $a_{i\ell}$ and consists of transition t_1
L^b_ℓ which starts and ends at node $b_{i\ell}$ and consists of transition t_3

Otherwise, start with two partial routes:

L^a_ℓ which starts and ends at node $a_{i\ell}$ and consists of transition t_4
L^b_ℓ which starts at node $a_{i\ell}$, ends at node $b_{i\ell}$ and consists of transition t_1

Step 2

For each consecutive subaisle j $(\ell < j < r)$ we determine L^a_j and L^b_j as follows.
If subaisle j contains items then:

$$L^a_j = \begin{cases} L^a_{j-1} + t_a + t_4 & \text{if } c(L^a_{j-1} + t_a + t_4) < c(L^b_{j-1} + t_b + t_1) \\ L^b_{j-1} + t_b + t_1 & \text{otherwise} \end{cases}$$

$$L^b_j = \begin{cases} L^b_{j-1} + t_b + t_3 & \text{if } c(L^b_{j-1} + t_b + t_3) < c(L^a_{j-1} + t_a + t_1) \\ L^a_{j-1} + t_a + t_1 & \text{otherwise} \end{cases}$$

If subaisle j does not contain items then:

$$\begin{aligned} L^a_j &= L^a_{j-1} + t_a \\ L^b_j &= L^b_{j-1} + t_b \end{aligned}$$

Step 3

For the last subaisle of the block (subaisle r), we determine

$$L^b_r = \begin{cases} L^b_{r-1} + t_b + t_3 & \text{if } c(L^b_{r-1} + t_b + t_3) < c(L^a_{r-1} + t_a + t_1) \\ L^a_{r-1} + t_a + t_1 & \text{otherwise} \end{cases}$$

The resulting partial route L^b_r will be used to form the complete order picking route. In this step we do not need L^a_r anymore. This is because once all items have been picked in a block, we need to go to the front of the block to be able to continue to the next block, see Section 2.3.4.

2.3.3 Example of the dynamic programming method

We consider one block with 3 subaisles for which we will apply the dynamic programming algorithm. Figure 2.4a gives the block with pick locations for this example. This block is assumed to be the block farthest from the depot that contains items. In total 7 items have to be picked from this block. The length of a subaisle is 8 meters (1 meter for each section plus 0.5 meter on both sides to go to the center of the cross aisle). The distance between two neighboring subaisles is 4 meters. Travel speed is 1 meter per second. Figure 2.4b depicts the situation of Figure 2.4a with nodes for the pick locations and the heads of the subaisles and with edges for the possible travel paths. Figure 2.5 visualizes the steps of the dynamic programming algorithm. Note that in this example $\ell = 1$ and $r = 3$. All travel times in this example are expressed in seconds.

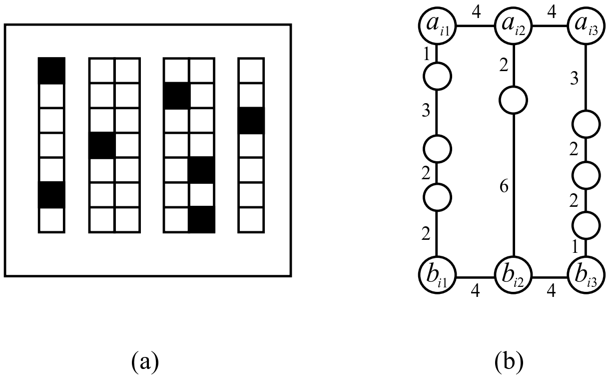

FIGURE 2.4. Example situation for the dynamic programming method.

Step 1 Since the block under consideration (block i) is assumed to be the block farthest from the depot, we start with two partial routes L_1^a and L_1^b. L_1^a starts at node b_{i1}, ends at node a_{i1} and consists of transition t_1, with associated travel time $c(t_1) = 8$. L_1^b starts and ends at node b_{i1} and consists of transition t_3, with associated travel time $c(t_3) = 14$.

Step 2 We have two possibilities for creating L_2^a, namely as $L_1^a + t_a + t_4$ (travel time $8 + 4 + 4 = 16$) or as $L_1^b + t_b + t_1$ (travel time $14 + 4 + 8 = 26$). We choose the shortest of the two. Thus $L_2^a = L_1^a + t_a + t_4$.

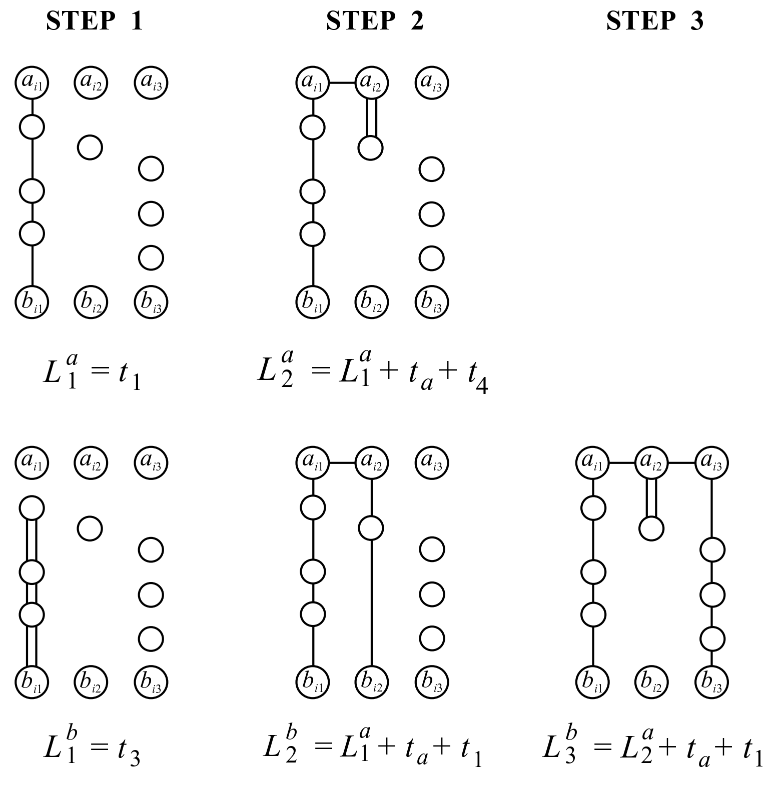

FIGURE 2.5. Visualization of the steps taken by the dynamic programming method.

Similarly, we have two possibilities for creating L_2^b, namely as $L_1^b+t_b+t_3$ (travel time $14+4+12=30$) or as $L_1^a+t_a+t_1$ (travel time $8+4+8=20$). We choose the shortest of the two. Thus $L_2^b = L_1^a + t_a + t_1$.

Step 3 We have two possibilities to create L_3^b. Clearly, $L_2^a + t_a + t_1$ (travel time $16+4+8=28$) is faster than $L_2^b+t_b+t_3$ (travel time $20+4+10=34$). Therefore, $L_3^b = L_2^a + t_a + t_1$.

This completes the partial route created by the dynamic programming algorithm. The creation of a full order picking route, going through multiple blocks, will be discussed in the next section.

2.3.4 Route construction for multiple blocks

So far, we have described a dynamic programming method for routing in a single block. In this section we give a description of the combined routing heuristic, which uses this dynamic programming method for the individual blocks. The route starts and ends at the depot. First, the order picker goes to the block farthest from the depot that contains items via the left-most pick aisle that contains items. The items in the left-most pick aisle are picked and next the items in the farthest block are picked. Then the order picker moves one block towards the depot and picks all items from that block. This process is repeated until all blocks with items have been visited.

An example route is depicted in Figure 2.6a. Each number in this figure corresponds to one of the numbers of the steps in the description below. Note that step 6 requires applying the dynamic programming algorithm from Section 2.3.2. It can easily be seen that we can use the results from the example in Section 2.3.3 for the farthest block in the example of Figure 2.6a. The stepwise procedure for the combined heuristic is given below.

1. Determine the left-most pick aisle that contains at least one pick location (called *left pick aisle*) and determine the block farthest from the depot that contains at least one pick location (called *farthest block*).

2. The route starts by going from the depot to the front of the left pick aisle.

3. Traverse the left pick aisle up to the front cross aisle of the farthest block (block $i_{\min}$).

4. Set $i = i_{\min}$.

5. Determine whether or not block i contains items that have not been picked in step 3.

   If no items have to be picked in block i: traverse the nearest subaisle of block i to reach the next block. Continue with step 7.

   If items have to be picked in block i: determine the left-most subaisle and the right-most subaisle that contains items (respectively subaisle ℓ and subaisle r), excluding any subaisle that was already visited in step 3. Go from the current position to the nearest of these two ($j_{\min}$).

6. Apply the dynamic programming method of Section 2.3.2 to block i. If in step 5 $j_{\min} = \ell$, then add the partial route resulting from the dynamic programming algorithm to the order picking path. If $j_{\min} = r$, then reverse the partial route resulting from the dynamic programming algorithm (the route then starts in subaisle r and ends in subaisle ℓ). Add this reversed partial route to the order picking path. Reversing the path means that the order picker will visit the subaisles from right to left. The calculations were performed from left to right.

7. When block k (the block closest to the depot) has been evaluated, the order picker returns to the depot. Otherwise increase i by 1 and return to step 5.

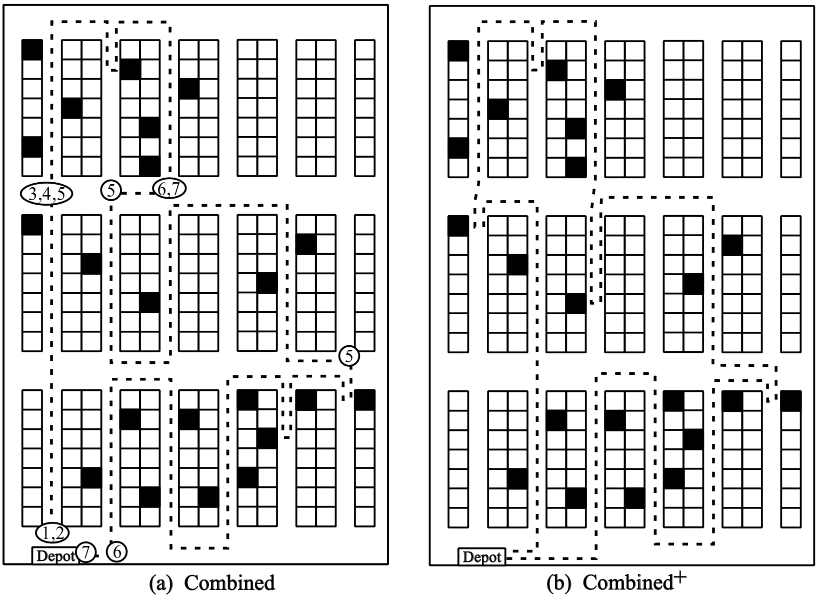

FIGURE 2.6. Routes resulting from applying the combined and combined$^+$ routing methods.

2.3.5 Improvements of the combined heuristic

First of all, we consider the routing in the block closest to the depot. The starting point for routing in this block is determined by the position where the order picker ends his route through the previous block. This could lead to a route in which aisles are visited from the left to the right. This implies that the order picker ends his route somewhere at the right of the front cross aisle. After this, a considerable part of the front cross aisle has to be traversed before reaching the depot. This can be prevented by forcing the route to visit aisles from the right to the left in the block closest to the depot. It can easily be seen that this change in the heuristic will either decrease travel time or leave the travel time unaltered.

Secondly, we consider the path of the order picker from the depot to the farthest block. This path goes through the left-most pick aisle with pick locations. However, one can think of situations where it can be advantageous to deviate from this path.

In the example of Figure 2.6a, travel time can be decreased by going through the second pick aisle towards the back of the warehouse instead of through the first pick aisle. In general terms, we could create routes such that the order picker picks items from the left x pick aisles on his way to the farthest block and picks items from the right $n - x$ pick aisles when returning to the front. The dynamic programming method is applied to the left x subaisles of each block on the way to the back and to the right $n - x$ subaisles of each block on the way to the front. By optimizing over x we obtain a route that is guaranteed to be shorter or at most as long as the route generated by the original combined heuristic.

We adapt the combined heuristic to incorporate both improvements suggested in this section and call the result the *combined*$^+$ heuristic. An example route is given in Figure 2.6b. Note that the two improvements could also be added to the largest gap and S-shape heuristic. However, the main advantage of these two heuristics is meant to be their ease of use, which would diminish by such substantial alterations. Therefore, we do not alter the other heuristics.

2.4 A comparison of heuristics for multiple blocks

This section compares the optimal and heuristic solutions in a practical order picking system, namely a shelf area. We consider a shelf area where order pickers walk through the warehouse to pick small items, using a small pick cart. The following assumptions are made. The average walking speed in both cross aisles and pick aisles is 0.6 meters per second. The center-to-center distance between two neighboring pick aisles is 2.5 meters and no additional time is needed for aisle changing. Cross aisle width is 2.5 meters. Picking of items can be performed simultaneously from both sides of a pick aisle since the aisles are fairly narrow. Order pickers are assumed to walk through the middle of the pick aisles and cross aisles.

For this type of warehouse we assume the following measures to be representative. Pick aisle length varies between 10 and 30 meters. Each order picker works in a zone consisting of 7 to 15 pick aisles. Each picking route has to visit between 10 and 30 locations. These values are based on observations of numerous actual manual shelf warehouse operations. We use the extremes of these values for our simulation experiments, which gives eight different configurations. For each configuration, we generate a number of random orders. The locations of the items in an order are uniformly and independently distributed over the order picking area. That is we assume that products are stored randomly in the storage area. No positioning according to demand frequencies is used. See for example Caron *et al.* [38] for issues involving non-random storage.

For each random order, the route length in a warehouse with two cross aisles is calculated for the S-shape, largest gap, aisle-by-aisle, combined and combined$^+$ heuristics and for the optimal algorithm. Then, an additional cross aisle is added and route length for each of the routing methods is calculated. Another cross aisle is added, route length is calculated, and so on. Averages are taken over the instances with the same number of cross aisles.

We can distinguish two ways to increase the number of cross aisles. Firstly, we can fix the number of storage locations. In this approach the length of the warehouse will increase if cross aisles are added. Secondly, we could fix the size of the warehouse and add cross aisles by losing storage locations. Since in design the amount of storage space is usually decided in advance, we choose the first approach. This is consistent with Vaughan and Petersen [288]. The minimum number of cross aisles is two. Such a warehouse with two cross aisles has one cross aisle in the front and one cross aisle in the back. This restriction is needed since all routing methods assume that the order picker is able to enter and leave the subaisles both from the front and from the back. Additional cross aisles are inserted such that the center distance between any two adjacent cross aisles is equal. Other ways of cross aisle distribution may be interesting especially in situations where a storage method other than random storage is used. The maximum number of cross aisles is 11 for the experiments. Since the shortest pick aisles in the experiments are 10 meters, having 11 cross aisles means that the shortest subaisle encountered in the experiments is 1 meter. One meter can be considered to be the minimum rack length that is practically feasible in a shelf warehouse.

For each simulation experiment, the necessary number of replications needs to be determined such that the estimate for the mean travel time has a relative error smaller than some γ, for $0 < \gamma < 1$. An approximation for the necessary number of replications, such that the relative error is smaller than γ with a probability of $1 - \alpha$, is given in Law and Kelton [163]. For all situations considered in this section, a replication size of 2000 orders has appeared to be sufficient to guarantee a relative error of at most 1% with a probability of 95%.

Table 2.1 gives the average travel time for each combination of the 8 instances, 10 cross aisle configurations and 6 routing methods. From the table we can see that the S-shape heuristic never had the best performance of the five heuristics. Largest gap had the best performance in 5 situations each of which has a layout with two cross aisles. Aisle-by-aisle had the best performance in 4 situations, of which 3 equal the travel time of the combined and combined$^+$ heuristics. The combined$^+$ heuristic gave the best results in 74 of the 80 instances, of which 3 equal the travel time of the aisle-by-aisle and combined heuristics. For each of the heuristics the average calculation time for a single route was less than 0.1 seconds on a 350Mhz computer.

Optimal			number of cross aisles									
aisles	length	items	2	3	4	5	6	7	8	9	10	11
7	10	10	138.7	129.7	131.5	135.7	141.7	148.0	155.5	162.0	169.6	177.4
7	10	30	186.6	191.4	198.6	207.1	216.3	224.9	235.0	243.2	252.8	261.6
15	10	10	219.6	202.0	201.4	205.2	211.4	218.2	226.7	233.8	242.2	251.0
15	10	30	337.5	314.3	311.6	315.7	324.5	333.4	346.1	355.6	368.7	381.4
7	30	10	269.6	222.9	211.1	209.0	211.4	215.8	221.3	227.4	233.9	240.2
7	30	30	398.3	361.1	342.9	336.5	333.8	334.2	337.0	340.8	345.0	349.7
15	30	10	377.3	308.0	290.9	287.7	289.3	293.5	299.2	305.4	312.0	318.5
15	30	30	665.5	540.6	495.9	479.8	473.3	472.8	475.9	480.7	486.0	491.7

Largest gap			number of cross aisles									
aisles	length	items	2	3	4	5	6	7	8	9	10	11
7	10	10	146.6	156.9	164.6	169.3	176.3	181.9	187.4	194.7	201.9	209.3
7	10	30	208.6	240.9	273.9	303.5	330.3	348.6	363.4	379.6	394.4	408.4
15	10	10	227.3	265.2	287.2	296.2	305.7	312.2	317.1	324.6	331.4	338.4
15	10	30	357.5	413.5	484.5	552.1	614.6	660.1	692.3	724.7	750.1	776.3
7	30	10	295.1	259.9	250.7	246.3	247.4	250.3	254.5	259.7	264.9	270.6
7	30	30	451.7	424.7	425.7	435.9	446.9	456.1	464.1	472.2	478.4	488.7
15	30	10	401.0	377.6	379.1	377.2	379.5	382.4	386.6	390.6	395.2	400.6
15	30	30	715.6	646.0	665.4	705.2	746.3	779.9	805.0	826.5	842.8	863.7

S-shape			number of cross aisles									
aisles	length	items	2	3	4	5	6	7	8	9	10	11
7	10	10	165.1	145.7	152.6	155.7	161.4	167.7	174.6	181.8	188.6	196.4
7	10	30	203.5	210.3	250.1	253.4	278.3	287.8	301.0	311.9	322.3	332.4
15	10	10	266.2	224.6	245.0	252.6	261.2	270.2	278.9	288.3	295.5	304.6
15	10	30	391.3	359.5	431.1	422.8	478.3	491.1	518.4	539.0	557.8	576.0
7	30	10	353.1	276.2	256.5	245.0	242.5	243.4	247.1	251.7	257.1	262.5
7	30	30	452.0	426.8	438.2	420.1	427.7	423.4	427.0	429.7	432.9	437.1
15	30	10	517.6	376.9	361.0	349.4	347.6	350.1	354.7	360.4	366.4	372.7
15	30	30	833.3	686.0	688.2	636.4	663.4	653.6	666.5	675.2	684.4	695.1

Aisle-by-aisle			number of cross aisles									
aisles	length	items	2	3	4	5	6	7	8	9	10	11
7	10	10	148.5	144.3	153.7	164.6	177.5	189.9	205.4	216.4	230.3	245.6
7	10	30	192.1	207.2	227.0	246.7	268.5	289.5	315.7	333.6	357.6	383.8
15	10	10	235.2	220.7	229.4	241.0	255.1	268.9	286.3	298.4	313.6	330.9
15	10	30	356.7	349.3	369.1	392.5	421.3	449.8	486.3	509.8	542.1	578.6
7	30	10	304.7	268.0	268.4	276.5	287.5	299.6	313.0	325.7	339.0	351.8
7	30	30	418.8	413.9	422.8	438.5	457.2	477.7	500.4	522.5	544.1	565.8
15	30	10	427.2	362.0	358.6	366.8	378.3	391.7	406.6	420.6	435.3	449.7
15	30	30	732.7	648.8	642.8	658.6	682.0	709.1	739.7	769.4	798.9	828.9

Combined			number of cross aisles									
aisles	length	items	2	3	4	5	6	7	8	9	10	11
7	10	10	148.5	134.6	145.4	151.2	158.2	165.5	172.9	180.4	187.5	195.6
7	10	30	192.1	196.5	236.1	240.4	267.0	277.7	292.2	304.2	315.4	326.3
15	10	10	235.2	208.6	235.3	246.7	257.1	267.2	276.6	286.6	294.1	303.6
15	10	30	356.7	324.6	402.5	399.5	459.0	474.8	504.2	526.8	547.1	566.7
7	30	10	304.7	243.4	235.1	231.2	232.5	236.1	241.5	247.4	253.5	259.7
7	30	30	418.8	386.2	397.4	382.2	394.9	394.7	401.7	407.5	413.8	420.7
15	30	10	427.2	330.1	332.5	331.8	334.8	340.8	347.6	355.0	362.0	369.2
15	30	30	732.7	584.6	605.6	569.7	609.2	608.6	628.2	642.5	656.5	671.3

Combined$^+$			number of cross aisles									
aisles	length	items	2	3	4	5	6	7	8	9	10	11
7	10	10	148.5	133.2	136.2	140.2	146.0	152.1	159.4	165.9	173.3	180.8
7	10	30	192.1	196.0	224.7	232.0	244.2	253.2	260.7	268.9	276.6	283.9
15	10	10	235.2	207.0	214.5	220.8	227.6	234.3	242.0	249.3	257.0	264.7
15	10	30	356.7	323.6	373.3	379.6	401.4	415.3	425.5	437.2	447.6	457.2
7	30	10	304.7	235.4	219.3	214.9	216.2	219.8	225.2	230.8	237.2	243.4
7	30	30	418.8	381.7	379.3	365.8	362.7	360.6	361.6	363.6	366.6	370.6
15	30	10	427.2	323.1	305.3	300.5	301.0	304.3	310.1	315.7	322.0	328.5
15	30	30	732.7	577.6	567.6	539.8	538.5	536.7	537.8	541.6	545.2	550.7

TABLE 2.1. Average travel time for six routing methods.

If we compare the combined and the S-shape heuristic on theoretical grounds, we can state that for each individual order combined gives a route that is equal to or shorter than the S-shape route. This is because S-shape entirely traverses every subaisle containing items, whereas the combined heuristic chooses between entirely traversing the subaisle or returning to the same side of the subaisle, depending on which gives the shortest travel time. Thus, the combined heuristic is capable of generating routes that are exactly the same as those of S-shape, however – if possible – it will give shorter order picking routes by returning within subaisles. The difference in performance between S-shape and combined depends on the situation under consideration. For the situations analyzed in this section it holds that average travel time for S-shape is at least 7% higher than for combined in warehouses with three cross aisles. The difference between S-shape and combined tends to zero when increasing the number of cross aisles. The percentage difference between combined and S-shape is smaller if the pick density is high, i.e. 30 items instead of 10 items. This is due to the fact that optimal routes generally tend to entirely traverse more aisles if pick density is higher. Therefore, S-shape routes are closer to optimal and the room for improvement is smaller for the combined heuristic.

A second interesting property of the heuristics is the fact that routes of the aisle-by-aisle, combined and combined$^+$ heuristics are identical for warehouses with two cross aisles. This is due to the fact that the heuristics use the same system of dynamic programming. Aisle-by-aisle creates routes through all blocks and uses one equivalence class for each cross aisle. Combined and combined$^+$ apply dynamic programming to each block individually and use two equivalence classes, for each cross aisle one. If the warehouse has two cross aisles, i.e. consists of one block, then both heuristics use the same two equivalence classes and consequently give the same routes.

Now let us look at the effect of the improvements for the combined heuristic suggested in Section 2.3.5. Combined and combined$^+$ are the same for warehouses with two cross aisles, because the changes only apply to blocks between the depot and the farthest block. For situations with three cross aisles, the difference between combined and combined$^+$ is minor. The improvements are however substantial for situations with more than three cross aisles. Average travel time of combined was in these cases up to 24.6% over that of combined$^+$.

In the situations with three or more cross aisles, combined$^+$ had the best performance of the heuristics for all situations except one. For pick lists of 10 items and with three or more cross aisles, the difference between combined$^+$ and optimal is less than 7.5%. For the situations considered in this section, the size of the gap between combined$^+$ and the optimal algorithm varies between 1% and 25%, see Table 2.2. The largest differences occurred for pick lists of 30 items. The situations with 7 pick aisles gave a smaller difference between optimal and combined$^+$ than the situations with

15 pick aisles. Generally, we can say that the gap between optimal and combined$^+$ tends to be larger if the situation is more complex, i.e. more aisles and/or more items.

Combined$^+$			number of cross aisles									
aisles	length	items	2	3	4	5	6	7	8	9	10	11
7	10	10	7.1	2.7	3.6	3.3	3.0	2.8	2.5	2.4	2.2	2.0
7	10	30	2.9	2.4	13.2	12.0	12.9	12.6	10.9	10.6	9.4	8.5
15	10	10	7.1	2.5	6.5	7.6	7.7	7.4	6.8	6.6	6.1	5.5
15	10	30	5.7	2.9	19.8	20.2	23.7	24.6	22.9	22.9	21.4	19.9
7	30	10	13.0	5.6	3.9	2.8	2.2	1.9	1.8	1.5	1.4	1.4
7	30	30	5.2	5.7	10.6	8.7	8.6	7.9	7.3	6.7	6.3	6.0
15	30	10	13.2	4.9	4.9	4.5	4.1	3.7	3.6	3.4	3.2	3.2
15	30	30	10.1	6.8	14.5	12.5	13.8	13.5	13.0	12.7	12.2	12.0

TABLE 2.2. Percentage difference in average travel time between the combined$^+$ and the optimal routing method.

The observed gap between the best heuristic and the optimal algorithm gives rise to two different approaches for further research: develop better heuristics or use an optimal algorithm for routing order pickers. Both approaches have their advantages and disadvantages. The heuristics are fairly simple in structure and therefore easy to implement in a practical situation. For situations with a large number of cross aisles, it may be worthwhile analyzing the performance of standard routing heuristics for the traveling salesman problem. On the other hand, optimal routing gives significantly shorter routes. However, the logic may be unclear to the order picker which may cause him to accidentally walk the wrong way or to override the system and walk the way he thinks is best. Furthermore, in this chapter we used a non-polynomial-time algorithm to calculate the shortest routes. Such a method has unpredictable computation times, which is an undesirable property for practical implementations.

From Table 2.1, we can also see the effect of adding cross aisles to the layout. In general we can say that travel time decreases if we change the layout from two to three cross aisles. Two exceptions occur: firstly, with largest gap for warehouses with short pick aisles; and, secondly, for a small warehouse with many picks. Both exceptions seem intuitively clear. For the first exception we consider that if a warehouse has short pick aisles then the distance travelled in the cross aisles accounts for a relatively large amount of the travel time. This is even more the case when using largest gap in warehouses with three or more cross aisles, since routes resulting from this heuristic often traverse cross aisles (except those at the back and front) twice. Consequently, a large increase in travel time occurs if a third cross aisle is added to the layout. For the second exception we consider a warehouse with a few short pick aisles and many pick locations. The main advantage of adding cross aisles is that more possibilities arise to route the order picker. However, if the order picker has to visit many locations, then entirely traversing pick aisles is close to optimal. Any extra cross aisle only increases the warehouse size and therefore travel times.

More cross aisles may or may not improve the average time needed to pick an order. For most situations it holds that adding cross aisles decreases average travel time up to a certain point after which average travel time starts increasing again. The optimal number of cross aisles with respect to travel time seems to depend on the number of pick aisles, the aisle length and the number of items. Aisle length especially seems to be important in the sense that longer aisles most often require more cross aisles. It is important to be aware of the fact that for warehouse design more cross aisles implies higher space requirements. Therefore, the cost reductions from adding cross aisles have to be weighed against increased costs for the building.

2.5 Case study: De Bijenkorf

In many distribution centers there is a constant pressure to reduce the order throughput times. One such distribution center is the DC of De Bijenkorf, a retail organization in The Netherlands with 7 subsidiaries and a product assortment of about 300,000 SKUs (stock keeping units). The orders for the subsidiaries are picked manually in this warehouse, which is very labor intensive. Furthermore, many shipments have to be finished at about the same time, which leads to peak loads in the picking process. The picking process is therefore a costly operation.

In a study we have investigated the possibilities of picking the orders more efficiently, without altering the storage or material handling equipment used or the storage policies, see De Koster *et al.* [63]. It appeared to be possible to obtain a reduction between 17 and 34% in walking time, by simply routing the pickers more efficiently. The amount of walking time reduction depends on the routing algorithm used. The largest saving is obtained by using the optimal routing algorithm described in Chapter 3. The main reason for this substantial reduction in walking time is the change from one-sided picking to two-sided picking in the narrow aisles.

2.5.1 Situation description

The area of the warehouse we have studied is the bin storage area. In this area, the products are stored in plastic bins. They are also picked in the same type of bins. In this area the picking is the most labor intensive. The bins are stored on two different floors in shelf racks. Both floors consist of 12 blocks each having 11 aisles of 18.57 meters long with a center-to-center distance between aisles of 2.115 meters. The aisle width available to the order pickers is 0.9 meter. Cross aisles are 2.6 meters wide. Every rack has 42 sections, each offering space for 8 plastic bins. Two adjacent blocks form a so-called *preferred zone*. Every article stored in the bin storage area is assigned to one preferred zone, depending on the cash register from which it is sold in the department stores. Therefore, products sold at a single cash register are grouped together in one preferred storage zone, a form of family grouping.

Within the area, storage is arranged as follows. If a bin has to be stored, the information system indicates the preferred storage zone where this should occur. Within this zone, the warehouse employee may store the bin at any free location. This results in a closest open location storage rule, see Section 1.4.1.

A pick order is picked by one order picker in one preferred zone, although exceptions occur. Each order picker usually picks one order at a time. The picked items are put in plastic bins on a small pick cart. The sequence in which the order lines (SKUs) have to be picked is indicated on the pick list. This sequence is created by simply sorting the pick locations in increasing order.

An important change at the De Bijenkorf was that the sales area in the stores was enlarged. In this way a larger number of SKUs could be displayed in the stores, rather than stored in a stock room in the store. This was done in order to increase sales and resulted in a decrease of the stock per SKU at the stores. This has put more pressure on the warehouse. All stores are now supplied daily, instead of once every two weeks. In the bin storage area, this has lead to many small orders instead of few and very large orders. The total walking distance needed to collect all orders has increased significantly. The study in Van Voorden [287] found that an order picker in the bin storage area walks on average 7 km on a daily basis to collect the items.

The research project at De Bijenkorf focused on a reduction of these walking distances. We have focused on routing and batching policies within a preferred zone. The approach that was chosen consists of an order analysis of orders in two representative preferred zones, a simulation of the current situation and of alternatives, a comparison of results, recommendations and implementation.

Over two consecutive weeks all orders in the two preferred zones were recorded. Some data of these orders is listed in Table 2.3. Full frequency distributions were obtained for the order sizes. From the collected data it was also possible to find a pick frequency distribution for each storage location. This was used later in a simulation program to generate random pick locations. The storage location distribution is quite skewed, due to closest open location storage rule. More information on the data analysis can be found in De Koster *et al.* [63].

	Preferred zone 1	Preferred zone 2
Average orders per day	47	29
Average lines per order	16.9	31.2

TABLE 2.3. Some data collected for two preferred storage zones.

2.5.2 Current routing method

From the analysis of the pick routes it appeared that the large distances that are traveled in the picking process can be attributed mainly to the single-sided order picking. It is a known fact that double-sided order picking greatly outperforms single-sided order picking unless the number of pick locations per aisle is very large or the aisles are very wide, see Goetschalckx and Ratliff [102]. The single-sided picking is a result from the fact that pick lists are generated by printing location numbers in increasing order. The location numbering is indicated in an aisle in Figure 2.7. The figure is a simplification of the real situation since less aisles and only 56 (instead of the actual 168) locations per aisle have been drawn.

The pickers, in fact, use two different traveling strategies. Some pickers work from the middle cross aisle: they leave their pick cart in the middle aisle when entering an aisle. Other pickers always take their pick cart with them and then consequently travel the full aisle when they have to switch to a neighboring aisle. However, all pickers pick strictly in the sequence indicated on the pick list. After simulation, it appeared that the second routing method is slightly better on average than the first one. For the comparison, only this best one was used.

In Figure 2.7 an example is given of this last type of pick routes, in which 16 locations have to be visited. In the upper storage block, the picker enters the aisle from the middle aisle and returns to the middle aisle after the furthest pick in the aisle. In the lower storage block, the picker traverses the full aisle, starting from the middle aisle, if the next pick location is also in the lower storage block and the traveling direction matches with the location numbering sequence.

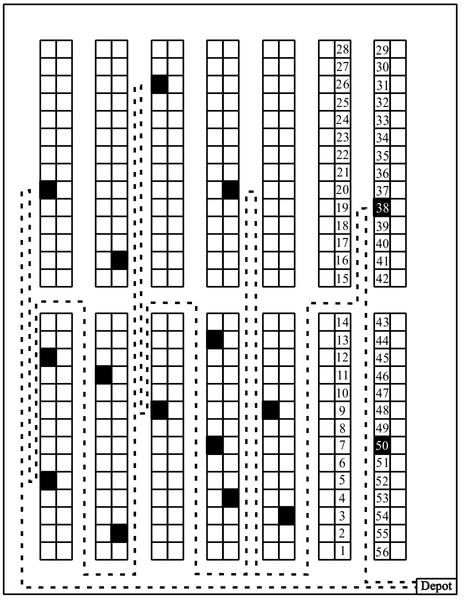

FIGURE 2.7. Orderpicking route in the current method with full aisle traversal (lower block) and return (upper block).

2.5.3 Simulation results

In our search to decrease the travel times, we could try to improve the location numbering, see e.g. Bartholdi and Platzman [20]. We will, however, look into the possibilities of improving the routing method. For each of the two preferred storage zones, 10,000 orders have been randomly generated. The travel times needed to pick these orders were calculated with different routing policies, including the current one. The number of order lines of an order and the pick location of each of the articles are drawn from the corresponding probability distributions based on the order analysis. In total, 3 different heuristic routing methods and an optimal algorithm have been compared with the current routing method. The heuristic routing methods are S-shape, largest gap (see Section 2.2) and the combined heuristic (see Section 2.3). The optimal algorithm (see Chapter 3) is based on dynamic programming. The results can be found in Table 2.4.

	Average daily travel distance (in meters)	Total average daily pick time (in hours)
current	151,352	153.5
S-shape	125,738	145.6
largest gap	116,979	142.9
combined	106,193	139.6
optimal	99,349	137.5

TABLE 2.4. Comparison of three heuristics and the optimal routing method with the current routing method.

The results in Table 2.4 have been obtained by extrapolation of the simulation results of 2 preferred storage zones to all 12 preferred storage zones in the bin storage area. It appears that even a simple heuristic such as S-shape or largest gap yields a significant reduction in travel distance, namely 16.9% and 22.7%, respectively. This magnitude of the reduction is mainly due to the change to double-sided picking. If the smarter combined heuristic or an optimal algorithm is used, improvements of 29.8% or 34.4% are obtainable.

Even though the reduction in walking distance is significant, the improvement of the total pick time (which also includes picking of items and remaining activities, see Section 1.4.2) is far less. This is due to the fact that a large part of the non-travel time is spent on removing bins from the racks, waiting for a non-occupied computer terminal to confirm the picks and other administrative tasks. It is clear that further improvements are possible here. The result is that the savings in total pick time vary from 5.2% for S-shape upto 10.5% for the optimal algorithm. Assuming that a productive man-day is on average 7 hours, there would be a reduction of 1.1 to 2.3 pickers.

From the simulation results it appeared that the combined heuristic has a very good performance, but that the routing is much less complicated than that of the optimal algorithm. Therefore, the management of De Bijenkorf decided to implement the combined heuristic for routing the order pickers. Besides the routing policies some batching policies have also been evaluated for De Bijenkorf. If orders are batched, with a time-savings method and the combined routing heuristic, savings of about 68% in travel distance can be reached. This is equivalent to a saving of 3 to 4 pickers. The interested reader is referred to Van Voorden [287] or De Koster *et al.* [63].

2.6 Concluding remarks

Performances of heuristics in warehouses with two cross aisles have been studied extensively in literature. In this chapter, we have introduced several methods for routing order pickers in a warehouse with multiple cross aisles. Two methods, the S-shape and largest gap heuristics, are straightforward extensions of existing methods for warehouses with two cross aisles. The aisle-by-aisle heuristic was introduced in Vaughan and Petersen [288]. The combined and combined$^+$ heuristics are introduced in this chapter.

In a comparison of the heuristics, the combined$^+$ heuristic had the best performance of the heuristics for the majority of the situations, 74 of 80, we evaluated in Section 2.4. Largest gap was found to be useful in situations with two cross aisles and low pick densities, which is consistent with Hall [115].

The performance of the heuristics was also compared to the results of a non-polynomial-time algorithm that generates shortest order picking routes. It has to be noted that the gap between this optimal routing method and the best heuristic varies substantially. Implementation of the optimal procedure in practical situations may however give rise to problems such as unpredictable computation times. It could therefore be desirable to improve heuristic performance or find more efficient methods to calculate shortest routes.

From the case study in Section 2.5 we found that the heuristics can be very useful in practical situations. Estimated travel distance could be reduced by 30.6% when changing from their current routing method to the combined heuristic. The management of the distribution center has therefore decided to implement the combined heuristic.

In Section 2.4 we considered only situations where products are stored randomly. Other storage assignment rules may cause a different ranking among the heuristics. Furthermore, the positioning of the cross aisles may be an interesting aspect to consider when using non-random storage, since there will be much activity in the area with the frequently requested products. Batching policies have not yet been studied in a layout with multiple cross aisles.

Generally, average travel times decrease when changing the layout from two to three cross aisles. Two exceptions are in the situation of a small warehouse with many picks for all routing methods and in the situations with short pick aisles for largest gap. More cross aisles may or may not decrease travel times, depending on the routing method and the situation under consideration. Further layout issues will be discussed in Chapters 4 and 5.

3

Optimal routing of order pickers: Two blocks

In Chapter 2 we have calculated shortest order picking routes using enumeration. However, calculation times for such a procedure are unpredictable. We also evaluated a number of routing heuristics in Chapter 2. The combined$^+$ heuristic was found to give fairly good routes in situations with two or more blocks. However, we have also seen that the difference in average travel time between the combined$^+$ heuristic and optimal routing can be as large as 25%. It might therefore be desirable to have either better heuristics or an algorithm that can efficiently calculate shortest order picking routes.

Ratliff and Rosenthal [227] created an algorithm based on dynamic programming that is capable of determining shortest routes in warehouses consisting of one block and with a central depot. The algorithm has a running time linear in the number of aisles and the number of pick locations. De Koster and Van der Poort [64] created an algorithm for routing in a warehouse of one block with decentralized depositing. Both algorithms are not capable of determining routes in warehouses with more than one block. In this chapter we give an algorithm that is capable of determining shortest order picking routes in warehouses consisting of two blocks with a central depot. If desired, the algorithm can also be used for warehouses of one block. Future extensions to more blocks would be interesting, since we have shown in Chapter 2 that travel time savings are sometimes possible by increasing the number of blocks up to six.

The question that rises when developing an algorithm for a layout with two instead of one block, is whether such an algorithm can be useful in practice. From Chapter 2 we can already derive some information. It was found that if the layout was changed from one to two blocks then average travel time decreased for nearly all routing heuristics and situations evaluated. In this chapter we give a more comprehensive testing of the implications for average travel time when changing the number of blocks from one to two. We optimize the layout of a warehouse under the condition that it consists of one block and we optimize the layout of this warehouse under the condition that it consists of two blocks. Then we compare the average travel times of these two situations. We do this for pick lists varying in size from 1 to 50 items. It appears that average travel time is often lower in warehouses with two blocks. However, this is dependent on the size of the pick list. Large pick lists in small warehouses are best picked in a one-block layout. Large warehouses profit from a second block, regardless of pick list size.

In Section 3.1 we model the warehouse and order picking locations using graph theory. A dynamic programming formulation is given in Section 3.2 in order to solve the problem of finding a shortest order picking route in warehouses of two blocks. In Section 3.3 we compare average route length in one-block warehouses to the average route length in two-block warehouses. Section 3.4 contains concluding remarks.

3.1 The warehouse

A warehouse consists of a number of parallel aisles. The items are stored on both sides of the aisles. Order pickers are assumed to be able to traverse the aisles in both directions and to change direction within the aisles. Each order consists of a number of items that are usually spread out over a number of aisles. We assume that the items of an order can be picked in a single tour. Aisle changes are possible at the front end, the rear end and in the middle of the aisles. That is, the warehouse consists of two blocks. Picked orders have to be deposited at the depot, where the picker also receives the instructions for the next tour (i.e. route).

In order to determine an order picking tour of minimum length, the travel time between each pair of adjacent item locations in the warehouse needs to be specified. In the specification of the travel time we can take into account the time for entering an aisle and the time for accelerating and decelerating while driving from one location to another. We will focus only on minimizing the travel time. Other order picking activities, like positioning the truck or crane at the pick location, picking items from the pick location and putting them onto a product carrier, have to be performed anyway. Therefore, they do not impact the choice of an order picking tour.

We consider a warehouse with n aisles where we can change aisles at the front, the rear and the middle of each aisle. Stated differently, we have a parallel aisle warehouse consisting of two blocks of each n aisles. See Figure 3.1(a) for an example layout. In this warehouse we have to pick an order of m items. Of these m items there are m_x items located in block X and m_y items in block Y $(m_x + m_y = m)$.

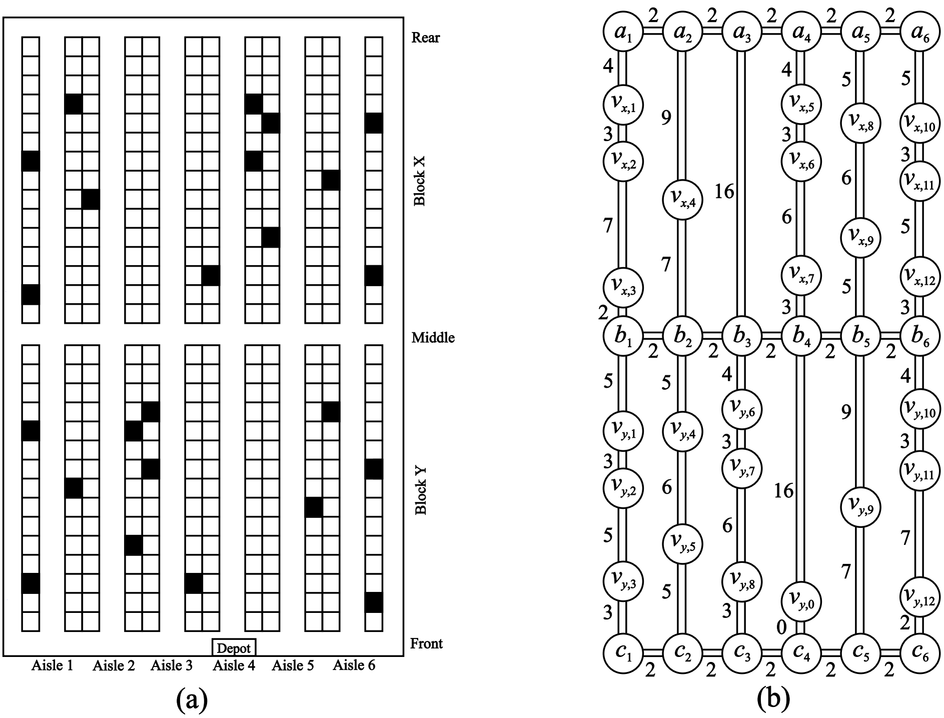

FIGURE 3.1. Part (a) gives an example layout of a warehouse consisting of two blocks. Each solid square corresponds to a location, where items have to be picked. Part (b) gives a graph representation of this warehouse with the pick locations.

The warehouse with order picking locations can be modeled as a graph G with vertices:

$v_{x,i}$ $(i = 1, ..., m_x)$ corresponding to the pick locations in block X,

$v_{y,i}$ $(i = 1, ..., m_y)$ corresponding to the pick locations in block Y,

a_i $(i = 1, ..., N)$ corresponding to the rear end of aisle i,

b_i $(i = 1, ..., N)$ corresponding to the middle of aisle i,

c_i $(i = 1, ..., N)$ corresponding to the front end of aisle i,

$v_{y,0}$ corresponding to the depot.

Any two vertices that correspond to adjacent locations in the warehouse are connected by two parallel edges. No more than two parallel edges are needed, since a shortest tour contains no more than two edges between any pair of vertices (see Appendix A, Corollary 1.1). The length of the edges indicates the travel times in the warehouse. See Figure 3.1(b) for a graph representation of Figure 3.1(a).

Any order picking tour will be considered as being a special kind of subgraph of the warehouse graph, and is called a *tour subgraph*. That is, any subgraph T of the warehouse graph G is called a tour subgraph if its edges form a cycle that includes the depot and each of the pick locations at least once (see Appendix A, Theorem 1 for a more exact description). The length of a subgraph is defined as the sum of the length of the edges in this subgraph. In Ratliff and Rosenthal [227] an algorithm is given that constructs an order picking tour from a given tour subgraph. The problem of finding a shortest order picking tour can therefore be solved by finding a tour subgraph of minimum length.

3.2 Finding a shortest tour subgraph

Let L_j^- be the subgraph of the warehouse graph, consisting of vertices a_j, b_j and c_j together with all edges and vertices to the left of a_j, b_j and c_j. Let Y_j be the subgraph of the warehouse graph consisting of vertices b_j and c_j together with all edges and vertices between b_j and c_j and define $L_j^{+y} = L_j^- \cup Y_j$. Similarly, let X_j be the subgraph of the warehouse graph consisting of vertices a_j and b_j together with all edges and vertices between a_j and b_j and define $L_j^{+x} = L_j^{+y} \cup X_j$. We use L_j to indicate that a result holds if we let $L_j = L_j^-$, $L_j = L_j^{+y}$ or $L_j = L_j^{+x}$.

For any subgraph $L_j \subset G$, a subgraph $T_j \subset L_j$ is called a L_j *partial tour subgraph* if another subgraph of G (called *completion*) exists consisting of edges and vertices not contained in L_j, such that the union of these two subgraphs forms a tour subgraph (see also Appendix A, Theorem 2). Two L_j partial tour subgraphs are *equivalent* if any completion of one partial tour subgraph is a completion for the other (see also Appendix A, Theorem 3).

The algorithm uses the concept of dynamic programming to construct a minimum tour subgraph. We start with all L_1^{+y} partial tour subgraphs consisting only of vertices and edges between b_1 and c_1. In the next step, we extend the L_1^{+y} partial tour subgraphs by adding vertices and edges between a_1 and b_1 to obtain L_1^{+x} partial tour subgraphs. Next, L_2^- partial tour subgraphs are determined by extending L_1^{+x} partial tour subgraphs with edges between aisle 1 and aisle 2. From L_2^- partial tour subgraphs we can obtain L_2^{+y} partial tour subgraphs. Continuing in this way, we finally get the L_n^{+x} partial tour subgraphs, which are precisely the tour subgraphs.

To describe the algorithm within the concept of dynamic programming, we define the potential *states*, the possible *transitions* between states, and the *costs* (tour lengths) involved in such a transition.

3.2.1 States for the dynamic programming model

The classes of equivalent L_j partial tour subgraphs can be characterized by the five features (see Appendix A, Theorem 2): degree parity of a_j, degree parity of b_j, degree parity of c_j, connectivity, and distribution of a_j, b_j and c_j over the various components.

Degree parity describes whether the number of edges incident with the vertex is odd, even or zero. The term *connectivity* gives the number of connected components of the partial tour subgraph. *The distribution of a_j, b_j and c_j over the various components* indicates which of a_j, b_j and c_j are contained in the same component.

We denote the five features in a quintuplet. The degree parity is given by u (odd, u for 'uneven'), e (even) or 0 (zero). The connectivity, giving the number of components, is an integer between 0 and 3. For ease of notation we suppress the distribution of a_j, b_j and c_j over the various components if there is only one possibility, given the other four features. In fact, we only need to give the distribution of a_j, b_j and c_j over the various components for the case of 2 components each having even degree parity (see Appendix A, Theorem 3). Therefore, the fifth feature only needs the following possibilities: *a-bc*, *b-ac*, *c-ab*. For example, with *a-bc* we denote that a_j is in one component and b_j and c_j are in the other.

As an example we consider the equivalence class $(u, e, u, 2)$. This class has odd degree parity in a_j and c_j, even degree parity in b_j and consists of two components (the fifth feature is not given, since the only valid possibility is a_j and c_j in one component and b_j in the other. Any other possibility would violate condition (b) of Theorem 2 in Appendix A, because if a_j has odd degree parity, then at least one other vertex in the same component must have odd degree parity as well. Theorem 2(b) in Appendix A implies that this other vertex must be either b_j or c_j. Only c_j has odd degree parity and must therefore be in the same component as a_j).

Using theorems and corollaries from Appendix A, it can be proven that the only 25 equivalence classes are:

$(0,0,0,0)$, $(0,0,0,1)$, $(e,e,e,1)$, $(e,e,e,3)$,
$(e,0,0,1)$, $(0,e,0,1)$, $(0,0,e,1)$, $(e,e,0,1)$, $(e,0,e,1)$, $(0,e,e,1)$,
$(u,u,0,1)$, $(u,0,u,1)$, $(0,u,u,1)$, $(e,u,u,1)$, $(u,e,u,1)$, $(u,u,e,1)$,
$(e,e,0,2)$, $(e,0,e,2)$, $(0,e,e,2)$, $(e,u,u,2)$, $(u,e,u,2)$, $(u,u,e,2)$,
$(e,e,e,2,a\text{-}bc)$, $(e,e,e,2,b\text{-}ac)$, $(e,e,e,2,c\text{-}ab)$.

We note that $(0,0,0,0)$ is only possible if none of the aisles in L_j contains an item to be picked and $(0,0,0,1)$ is only possible if none of the aisles in $G - L_j$ contain an item to be picked ($G - L_j$ denotes the subgraph that remains after all edges and vertices in L_j have been deleted from G).

Using Theorem 1 from Appendix A, it can be derived that after calculating the L_n^{+x} partial tour subgraphs, the minimum length tour subgraph is the shortest of the following partial tour subgraphs:

$(0,0,0,1)$, $(e,0,0,1)$, $(0,e,0,1)$, $(0,0,e,1)$,
$(e,e,0,1)$, $(e,0,e,1)$, $(0,e,e,1)$, $(e,e,e,1)$.

3.2.2 Transitions for the dynamic programming model

The transitions between states consist of adding vertices and edges. We distinguish three different transitions. In the first type of transition, from L_j^- to L_j^{+y}, vertices and edges between b_j and c_j are added. In the second type, from L_j^{+y} to L_j^{+x} vertices and edges between a_j and b_j are added. In the last type, from L_j^{+x} to L_{j+1}^- the connection between aisle j and aisle $j+1$ is made.

Transition from L_j^- to L_j^{+y}

We consider any aisle j. We know that we never need more than two edges between any pair of vertices (see Appendix A, Corollary 1.1). Therefore, the edges between b_j and c_j can be configured as one of the possibilities in Figure 3.2. The equivalence classes that we obtain by this transition are given in Table 1 of Appendix B.

Transition from L_j^{+y} to L_j^{+x}

This transition is very similar to the transition from L_j^- to L_j^{+y}. Again we can distinguish six ways to traverse the edges in aisle j between a_j and b_j. The equivalence classes that we obtain by this transition are given in Table 2 of Appendix B.

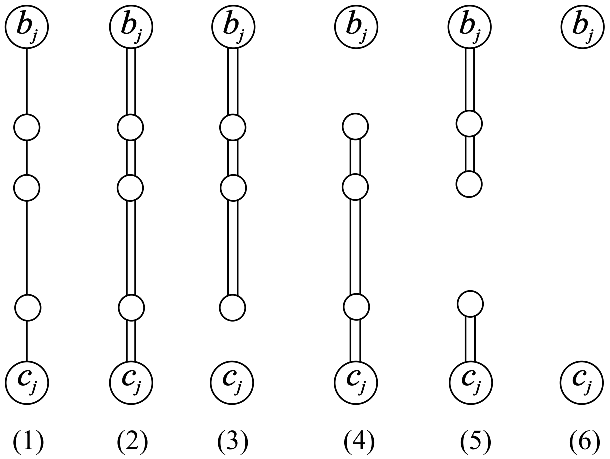

FIGURE 3.2. Six ways to traverse the edges in aisle j between b_j and c_j. In transition (5) only the longest double edge is not traversed. Transitions (3) and (4) are only possible if there is at least one item in this part of the aisle. Transition (5) is only possible if there are two or more items in this part of the aisle and transition (6) is only allowed if this part of the aisle is empty.

Transition from L_j^{+x} to L_{j+1}^{-}

This transition makes the connection between aisle j and aisle $j+1$ by adding configurations given in Figure 3.3. The equivalence classes that we obtain by this transition are given in Table 3 of Appendix B.

3.2.3 Costs for the dynamic programming model

The third and last piece of information we need to be able to apply dynamic programming to the problem of routing order pickers in a warehouse, is the cost involved in each transition. The cost of each transition is simply equal to the sum of the lengths of the edges added in the transition.

3.2.4 Applicability of the algorithm

The algorithm is applicable to a wide variety of warehouses. Clearly, the middle aisle does not have to be exactly in the middle, but can be placed anywhere between the front and the rear of the warehouse. Furthermore, the algorithm can be used in warehouses with only one or two possibilities for aisle changing, by setting the appropriate distances between the aisles to infinity. In this way, the algorithm can

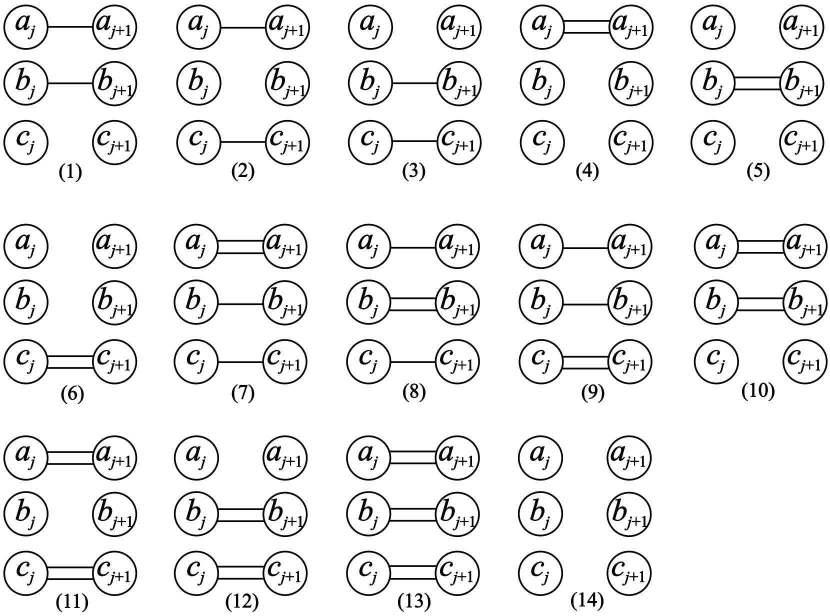

FIGURE 3.3. 14 possible arc configurations to change from aisle j to aisle $j+1$.

also be used to find order picking tours in warehouses with a one-block layout. In spite of its apparent complexity, the algorithm solves any practical sized problem within fractions of a second.

The algorithm considers all aisles and items, and for each aisle and item a constant number of operations has to be done. Hence, the time-complexity function of the algorithm is linear in the number of aisles and the number of items.

3.3 Performance comparison

In this section a performance comparison is made between warehouses with a middle aisle and warehouses without a middle aisle. In order to compare the two types of warehouse layouts, we use simulation to determine the average travel time needed to pick an order, when using the optimal routing algorithm we developed in Section 3.2. However, average travel time is not only influenced by the presence or absence of a middle aisle, but also by factors like warehouse type, warehouse size, number of aisles, location of the depot, order picking equipment, pick list size, storage assignment rules, and the location of the middle aisle (if present).

For our comparisons, we consider a very common type of warehouse, namely a shelf area. Order pickers walk through this warehouse to pick items, using a small pick cart. Before starting a tour, order pickers collect a pick list at the depot, which we assume to be located at the head of the left-most aisle. Other positions of the depot are possible, see Chapter 4 for an analysis of the depot location when using the S-shape heuristic. The average walking speed in this type of warehouses is usually around 0.6 meters per second. The distance between two neighboring aisles is 2.5 meters. Products are assumed to be assigned randomly to storage locations according to a uniform distribution. Average travel time is determined by taking the average of the travel times of 10,000 simulated orders.

If we consider this warehouse type, then three important factors influencing travel time remain, namely (1) warehouse size, (2) warehouse layout and (3) pick list size. Warehouse layout covers (a) the presence or absence of a middle aisle, (b) the number of aisles and (c) the location of the middle aisle. Aisle length is determined as the ratio of warehouse size and the number of aisles. For this analysis we assume that a middle aisle will always be located in the exact middle of the picking aisles. One may consider a middle aisle that is closer to the rear of the warehouse than to the front. This might actually be better than a location in the exact middle for example in a situation with two aisles and many items to pick. In this case a 'good' route visits all pick locations in the first aisle, goes to the second aisle through the nearest cross aisle and then visits all pick locations in the second aisle. A good location for the middle aisle would then be around the expected location of the pick that is closest to the rear of the warehouse. However, in most situations with a moderate number of items and aisles, the exact middle will be close to optimal due to the uniform distribution of the items. Besides, for practical reasons of flexibility it may be undesirable to locate the middle aisle anywhere else than in the exact middle. If at some point the depot location is changed from front to back then the initial advantage of an eccentric middle aisle may turn into a disadvantage.

In this section, we will evaluate two warehouse sizes, namely a warehouse with a total aisle length of 70 meters and one with a total aisle length of 450 meters. The size of a shelf area served by one order picker is in practice generally between these two extremes. Now, we can determine the optimal layout (i.e. the number of aisles) for the warehouse with a middle aisle and for the warehouse without a middle aisle for each pick list size. Then we compare average travel time in a warehouse of optimal layout with a middle aisle to average travel time in a warehouse of optimal layout without a middle aisle.

Firstly, we determine average travel time for pick lists of 30 items. The number of aisles is varied from 1 to 50. Total aisle length is kept constant at 450 meters. Thus, we consider warehouses with the following layouts: 1 aisle of 450 meters, 2 aisles of 225 meters, ..., 50 aisles of 9 meters. In each of the 10,000 replications per warehouse layout, a random order is generated for the warehouse without middle aisle and the travel time is calculated. Thereafter, a middle aisle is inserted in the layout and the travel time is again calculated for the same order.

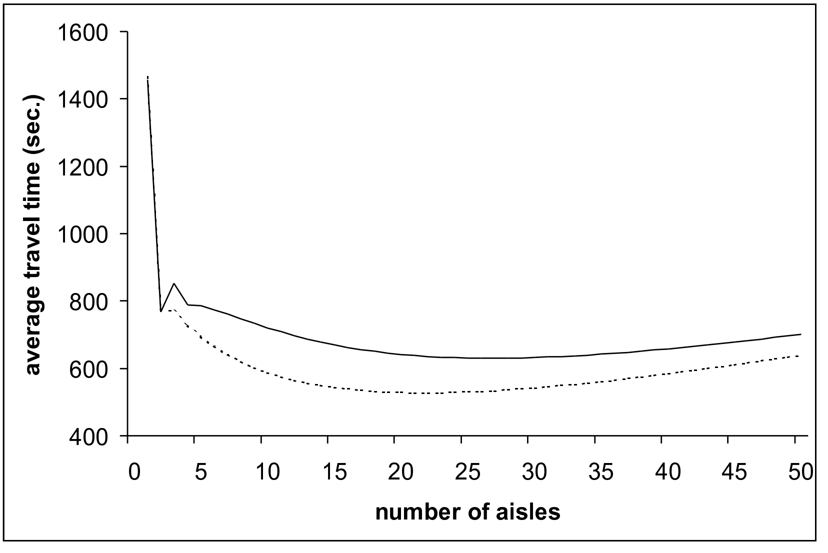

FIGURE 3.4. Average travel time for an order of 30 items as a function of the number of aisles for a warehouse with 450 meters of aisles, with a middle aisle (dashed line) and without a middle aisle (solid line).

Figure 3.4 depicts the average travel time as a function of the number of aisles. One curve gives the results for a warehouse without a middle aisle, the other curve for a warehouse with a middle aisle. The curve for the warehouse without middle aisle is above the other curve for all configurations, except for warehouses with one or two aisles. It follows that average travel time is lower for the warehouse with a middle aisle for any number of aisles, with the exception of warehouses with 1 or 2 aisles. Also, the best possible layout without a middle aisle (27 aisles of 16.7 meters with an average travel time of 630 seconds) results in a considerably higher travel time than the best possible layout with a middle aisle (22 aisles of 20.5 meters with an average travel time of 527 seconds). The small peak for 3 aisles in the curve for the warehouse without cross aisles, is due to the fact that at least one of the aisles has to be entered and left from the same side to ensure that the order picker ends his tour at the front of the warehouse.

We have analyzed average travel time for warehouses with and without a middle aisle for a fixed pick list size of 30 items. Also, we determined the optimal number of aisles in each of the two types of warehouses by locating the minimum of the curves in Figure 3.4. In a similar fashion, we determine the optimal number of aisles for all pick list sizes ranging from 1 to 50 items. Total aisle length is kept constant at 450 meters. Thus, for each pick list size we determine the optimal number of aisles separately. The corresponding average travel time is depicted in Figure 3.5 as a function of the pick list size. There are two curves in Figure 3.5, one curve for a warehouse layout with a middle aisle and one for a layout without a middle aisle. It appears from this figure that for all pick list sizes (except size 1), the layout with a middle aisle gives lower average travel times. Savings on travel time of more than 15% are possible.

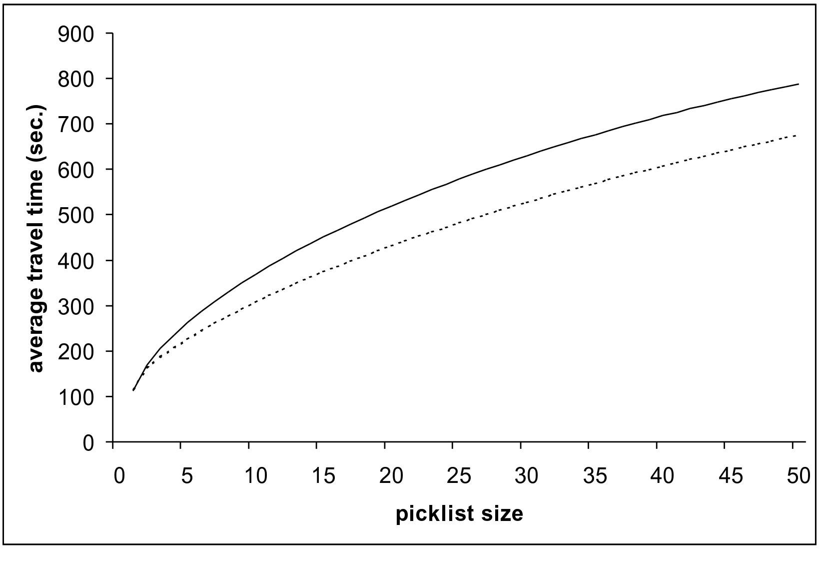

FIGURE 3.5. Average travel time as a function of the pick list size for a warehouse with 450 meters of aisles, with a middle aisle (dashed line) and without a middle aisle (solid line).

Repeating the previous experiment for warehouses with a total aisle length of 70 meters gives the results as depicted in Figure 3.6. As before we determine for each pick list size separately the optimal number of aisles for a warehouse without a middle aisle and for a warehouse with a middle aisle. Average travel time for the optimal number of aisles is depicted as a function of the pick list size. In this warehouse, a middle aisle is beneficial only for pick list sizes from 3 to 22 items. For pick lists containing 1, 2 or more than 22 items, a layout without a middle aisle would give lower average travel times. In fact, if we continued to increase the pick list size beyond 50 items, then at some point the two curves for the warehouse with 450 meters of aisles (Figure 3.5) would also intersect. This can be explained as follows. For small pick lists, the introduction of a middle aisle offers more possibilities for creating tours. This will result in shorter tours. If the number of picks increases, then at some point the optimal tour will traverse nearly every aisle entirely. In this situation the middle aisle is not in use anymore to reduce the number of aisles visited, but order pickers still have to cross the middle aisle. This results in higher travel times compared to a situation without middle aisle. However, for most practically sized orders a middle aisle will give savings on travel time in such a large warehouse.

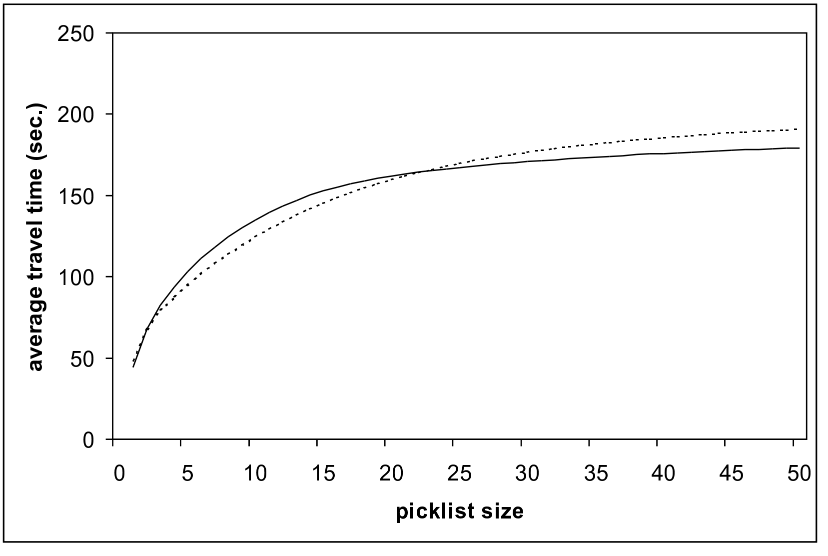

FIGURE 3.6. Average travel time as a function of the pick list size for a warehouse with 70 meters of aisles, with a middle aisle (dashed line) and without a middle aisle (solid line).

3.4 Concluding remarks

Average travel time in warehouses depends on many factors such as warehouse type, warehouse size, number of aisles, location of the depot, order picking equipment, pick list size and storage assignment rules. Each of these factors may have a significant influence on travel time. In this chapter we have evaluated the impact on average travel time of warehouse layout. Specifically, we evaluated whether or not a middle aisle could improve efficiency. To this end, we have constructed a dynamic programming algorithm for calculating order picking tours of minimal length in warehouses with up to three cross aisles. The algorithm is more complex than the algorithm for the one-block layout of Ratliff and Rosenthal [227]. For the one-block layout only 7 equivalence classes are needed, whereas the dynamic programming algorithm of this chapter needs 25 equivalence classes. This is caused by the increased number of valid combinations that arise due to an extra cross aisle. Calculations for any practically sized problem only take fractions of a second. Further extensions to more blocks are clearly possible, but may not be interesting for practice.

In a simulation experiment, we used the algorithm to determine average travel time for two warehouse sizes and varying pick list sizes. We determined the number of aisles that minimize average travel time for each combination of warehouse size and pick list size. The number of aisles was determined separately for the situation where there is a middle aisle and for the situation where there is no middle aisle. In the majority of the situations evaluated it appeared that the layout of two blocks, i.e. with a middle aisle, resulted in lower average travel time than the one-block layout. Possible efficiency gains by introducing a middle aisle are especially present for large warehouses.

4

A layout for the order picking area: One block

In Chapters 2 and 3 we developed a number of routing methods and discussed some results for the layout of the order picking area. In this chapter and in Chapter 5 we will focus entirely on the layout of the order picking area. This chapter describes an approach to determine a layout for the order picking area with the restriction that the layout consists of one block, i.e. there are exactly two cross aisles, one in the front and one in the back. In Chapter 5 a similar methodology is used to find layouts that are not restricted to one block. The objective is to minimize the average travel distance needed to pick an order or batch of orders.

We start by developing an analytical formula with which the average length of an order picking route in a picking area can be calculated. The calculated values are compared with results from simulations. Using the analytical formula for average travel distance as an objective function in a non-linear programming model, we can determine the optimal layout for a given situation. The model assumes that the required storage space, the number of picks per route and the properties of the equipment are known. Equipment properties include, for example, center-to-center distance between aisles and the speed of the picking vehicle. Using these inputs we can determine the number of aisles, the aisle length and the depot location that minimize average travel distance.

Furthermore, we will prove in this chapter that a depot located in the middle of the front cross aisle gives better results with respect to average travel distances than any other location in the front cross aisle. From the experiments it appears that the optimal number of aisles in an order picking area depends strongly on both required storage space and pick list size.

In Section 4.1 we describe a model to find the best layout in a picking area consisting of one block, if a specific routing method is employed. In Section 4.2 an estimate for the average travel distance is developed. In Section 4.3 the location of the depot is discussed and Section 4.4 describes some other layout experiments. Conclusions are given in Section 4.5.

4.1 Layout optimization

In this section we will describe a non-linear programming model to optimize the layout of the order picking area with respect to average travel distance. We will model the objective function with an estimate of average travel distance that is based on statistical properties of a routing method. Clearly, average travel distance is only one of many possible objectives, see for example Larson *et al.* [162]. Another objective would be to minimize total cost, which includes both material handling and building costs. Building costs can, for example, be modeled by a cost per unit distance of the warehouse perimeter, see Francis [89]. However, response time is nowadays one of the major concerns of warehouses, therefore we will seek for a layout that minimizes the travel distance needed for picking an order. The time needed to pick an order consists of several components. We distinguish walking or driving between items, picking of items and remaining activities; see Section 1.4.2 for an explanation. Since picking of items and remaining activities are to a large extent independent of the layout, we can focus on walking and driving time. In a manual environment this travel time is just a linear function of travel distance, therefore we will focus on minimizing travel distance in this chapter. Some considerations concerning travel time instead of travel distance are given in Section 4.2.3. This approach is similar to that of Hall [115], Kunder and Gudehus [159] and Pandit and Palekar [207]. We will give the model and explain the assumptions in this section. In Section 4.2 the travel distance estimate is developed.

We consider a manual order picking operation, where order pickers walk through a picking area to retrieve products from storage. Picked items are placed on a vehicle, such as a cart or pallet jack, which the order picker takes with him on his route. With some minor changes, we can also optimize other picking environments with this model, see Section 4.2.3. The picking area is rectangular with no unused space and consists of a number of parallel aisles. At the front and rear of the picking area there is a cross aisle. Cross aisles do not contain storage locations, but can be used to change aisles. An example of a layout of such a picking area is given in Figure 4.1. Solid black squares in the figure indicate sections in the rack where items have to be picked. The dotted lines indicate where the order picker may walk or drive.

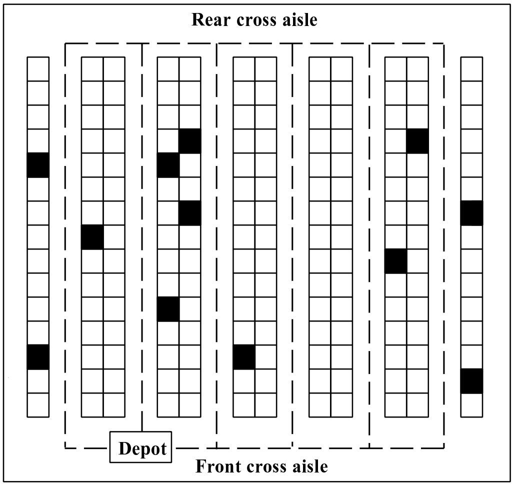

FIGURE 4.1. Example layout of an order picking area.

We will develop a model specifically for warehouses where the order pickers retrieve items from multiple aisles. For travel distance estimation in systems where order picking is restricted to a single aisle (such as an automated storage / retrieval system) see e.g. Bozer and White [29], Chiang *et al.* [44], Elsayed and Unal [83] and Han *et al.* [118]. For a block stacking environment see Berry [22].

Order pickers are assumed to be able to traverse an aisle in either direction and to change direction within an aisle. Items are stored on both sides of the aisles. Item locations are determined randomly according to a uniform distribution. Clearly, activity-based item location could possibly require a different layout. Here, we will consider only random storage assignment since this strategy can be considered as a base-line against which layouts with activity-based storage assignment can be compared. Previous work on travel distance estimation for random storage is described in Hall [115]. Estimations in an activity based storage environment are described in for example Caron *et al.* [38, 39], Chew and Tang [43], and Tang and Chew [265].

The aisles are narrow enough to allow picking from both sides of the aisle without changing position. See Goetschalckx and Ratliff [102] for routing in warehouses with wide aisles. Every item can be picked from the rack by the order picker without climbing or using a lifting device. Each order consists of a number of items that are usually spread out over a number of aisles. In this chapter, we assume that the items of an order can and will be picked in a single route. Since we assume that the order consists of any number of items, single and dual command operations (for a definition see Section 1.4.2) can also be described accurately with this model. Literature on layout and travel distance estimation in single and dual command environments includes Bassan *et al.* [21], Francis [89], Larson *et al.* [162] and Pandit and Palekar [207]. Picked orders have to be deposited at the depot, where the picker also receives the instructions for the next route. The depot is located in the front cross aisle.

We use a routing strategy called the traversal or S-shape strategy. This strategy is widely used in practice. Evaluations of the S-shape routing method can for example be found in Hall [115] and Petersen [216] for picking areas consisting of one block and in Chapter 2 for picking areas with multiple blocks. With the S-shape strategy, any aisle containing at least one item is traversed through the entire length. Aisles where nothing has to be picked are not entered. After picking the last item, the order picker returns to the front end of the aisle. In Figure 4.2 a route is given, that is found by applying the S-shape heuristic to the layout of Figure 4.1. Travel distance approximations of other routing methods can be found in Caron *et al.* [38], Hall [115] and Kunder and Gudehus [159].

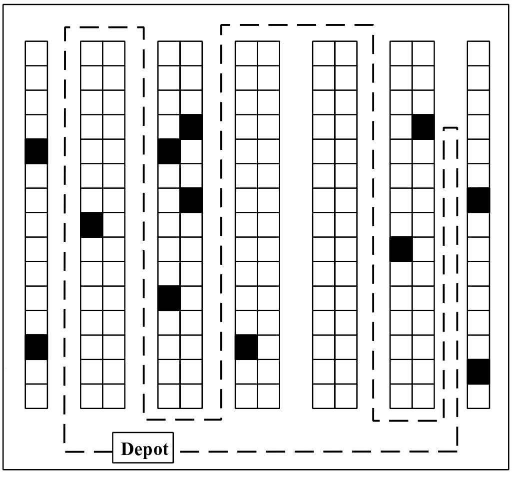

FIGURE 4.2. Route found by the S-shape heuristic in the example situation of Figure 4.1.

To find the layout that results in minimal average travel distance, we first have to find an analytical expression that expresses the average travel distance as a function of a number of layout arguments. We distinguish the following major factors that influence average travel distance:

- n the number of aisles (integer)
- y the length of the aisles (real)
- m the number of picks per route (integer)
- d the depot location, $1 \leq d \leq n$ (real)

The depot can be located anywhere in the front cross aisle between the left-most aisle (aisle 1) and the right-most aisle (aisle n). The position is indicated with a number. For example, $d = 1$ indicates that the depot is located at the head of aisle 1; $d = 3.5$ indicates that the depot is located between aisles 3 and 4.

Furthermore, we define the following parameters:

- w_a center-to-center distance between two adjacent aisles (i.e. width of an aisle including the storage racks),
- w_c width of a cross aisle,
- S total aisle length, measured along the pick face.

Once we have an expression for the average travel distance, then we can try to minimize this expression. If $T_m(n, y, d)$ gives the average travel distance for a picking area with n aisles of length y and the depot located at d, given that m products have to be picked per route, then our problem can be formulated as:

$$
\begin{aligned}
&\min T_m(n, y, d) \\
&n \cdot y = S \\
&n \geq 1 \quad \text{(integer)} \\
&y \geq 1.0 \\
&1 \leq d \leq n
\end{aligned}
$$

Thus, we try to find the minimum value of $T_m(n, y, d)$ under the conditions that total aisle length equals S, the number of aisles is 1 or more, the depot is located in the front cross aisle between aisle 1 and n, and the minimum length of an aisle is

1.0 meter (which is the minimum that would be physically possible to build). In the next section we will derive an expression for $T_m(n, y, d)$. In Sections 4.3 and 4.4 we will investigate properties of good layouts for the order picking area.

4.2 Average travel distance estimation

In this section we will derive a formulation for the average travel distance in a picking area consisting of one block. For the rectilinear or euclidean space, travel distance estimates are given in Daganzo [54, 55]. Travel distance estimation in warehouses is however different because the order pickers are confined to the aisles and cross aisles. Previous work on travel distance estimation in similar situations as we evaluate, includes Hall [115] and Kunder and Gudehus [159]. This research differs in two aspects. First, we give a better estimate for the distance traveled if the order picker has to make a turn. For a graphical example, see the right-most aisle in Figure 4.2. Secondly, we develop an entirely new estimate for the distance traveled in the cross aisles.

The travel distance consists of two components: (1) distance traveled within the aisles and (2) distance traveled in the cross aisles. We will derive estimates for both components separately.

4.2.1 Estimate for travel within the aisles

Under the assumption that products are distributed uniformly over the aisles and locations, we can easily derive that the number of aisles containing at least one pick location, has an expected value of:

$$E[A] = n \cdot \left(1 - \left(\frac{n-1}{n}\right)^m\right)$$

which is n times the probability that an aisle contains at least one pick. This is similar to the formulations in Caron *et al.* [38, 39], Chew and Tang [43], Hall [115] and Tang and Chew [265]. Actually this is an approximation of the expected number of aisles, see Kunder and Gudehus [159].

The expected value of the distance traveled inside the aisles, D_y, can then be stated as:

$$E[D_y] = y' \cdot E[A] + C$$

where $y' = y + w_c$. That is, y' is the length of an aisle plus two times the distance to go from the end of an aisle to the center of the cross aisle (two times $\frac{1}{2}w_c$). This distance is added since we assume that the order pickers walk through the middle of the cross aisles. C is a correction term that accounts for extra travel in the last aisle that is visited. This extra travel distance occurs if the number of aisles that has to be visited is an odd number. In this case the last aisle is both entered and exited from the front (see Figure 4.2 for an example). In Hall [115] and Kunder and Gudehus [159] it is assumed that if the order picker has to turn in the last aisle, then the distance traveled in this aisle is $2 \cdot y'$. That is, the last aisle first has to be traversed entirely to the back of the warehouse before the order picker can return to the front. Hall [115] gives as an estimate for the resulting extra travel of $0.5 \cdot y'$. That is, an extra aisle traversal with probability 0.5. However, it can easily be seen that if the number of items to pick is high, then this approximation is either about $0.5 \cdot y'$ too high (if there is an even number of aisles) or $0.5 \cdot y'$ too low (odd number of aisles). See also De Koster *et al.* [65]. In Kunder and Gudehus [159] a better approximation is given, but this formulation still suffers from the problem that in practice the order picker will usually return to the front directly after picking the last item, instead of first going to the back of the warehouse as is assumed in the article. Caron *et al.* [38, 39] use a slightly different routing method. They assume that the order picker makes the turn in the aisle where the turn will be the shortest, instead of always turning in the last aisle. Furthermore, they assume that the warehouse always has an even number of aisles.

To determine the correction term C consider the following. Suppose that a turn has to be made and the number of picks in this last aisle is b_1, then the distance traveled in this aisle would equal:

$$2 \cdot \frac{b_1}{b_1 + 1} \cdot y'$$

which is based on the well know property that the maximum of b_1 continuous uniformly distributed [0,1] variables equals $b_1/(b_1+1)$. Since we already accounted for a distance of y' in the estimate $y' \cdot E[A]$, we find that the additional travel for turns is given by:

$$2 \cdot \frac{b_1}{b_1 + 1} \cdot y' - y'$$

Let us now determine the probability that such a turn occurs. First, we need to determine the probability that all picks fall into exactly g aisles, where g is an odd number. This probability is given by:

$$\begin{pmatrix} n \\ g \end{pmatrix} \left(\frac{g}{n}\right)^m \cdot X \tag{4.1}$$

where X is 1 minus the probability that all m picks fall into less than g aisles, conditional on the fact that all m items fall in at most g specific aisles. We will not write out X explicitly, because this term will cancel out in the final formulation. See De Koster *et al.* [65] or Kunder and Gudehus [159] for the full formulation of X.

Given that at most g aisles contain a pick location, we find that the probability that there are b_i picks in aisle i $(1 \leq i \leq n)$, is given by a multinomial probability distribution function:

$$\frac{m!}{b_1! \cdot b_2! \cdot \ldots \cdot b_g!}\left(\frac{1}{g}\right)^{b_1} \cdot \left(\frac{1}{g}\right)^{b_2} \cdot \ldots \cdot \left(\frac{1}{g}\right)^{b_g} = \frac{m!}{\prod_{i=1}^{g} b_i!}\left(\frac{1}{g}\right)^{m}$$

for

$$\sum_{i=1}^{g} b_i = m$$

However, we also have the restriction that $b_i \geq 1$, because all g aisles are assumed to contain at least one pick. Therefore, we have to divide the previous probabilities by X, which is 1 minus the probability that at least one of the aisles does not contain picks. By taking the sum over all possible values of $b = (b_1, b_2, ..., b_g)$ and multiplying each probability with the corresponding extra travel distance, we obtain:

$$\frac{1}{X}\left(\frac{1}{g}\right)^{m} \sum_{b \in B} \frac{m!}{\prod_{i=1}^{g} b_i!} \cdot \left(2 \cdot y' \cdot \frac{b_1}{b_1+1} - y'\right) \tag{4.2}$$

where

$$B = \left\{ b \quad | \quad b_i \geq 1, \sum_{i=1}^{g} b_i = m \right\}$$

The right-most term between brackets gives the expected extra travel distance if there are exactly g aisles and there are exactly b_i picks in the aisle. By multiplying equations 4.1 and 4.2 and taking the sum over all possible values of g, we obtain the desired estimate for extra travel distance due to turns:

$$C = \sum_{g \in G} \binom{n}{g} \left(\frac{g}{n}\right)^{m} \left(\frac{1}{g}\right)^{m} \sum_{b \in B} \frac{m!}{\prod_{i=1}^{g} b_i!} \cdot \left(2 \cdot y' \cdot \frac{b_1}{b_1+1} - y'\right)$$

where

$$G = \{ g \quad | \quad 1 \leq g \leq n,\ g \leq m \text{ and } g \text{ is odd} \}$$

An easier to use approximation (i.e. an approximation that uses less computer time for calculations) results if we use $\frac{m}{g}$ as an estimate for the number of items in the last aisle. Then the correction term for the extra travel distance in the last aisle can be formulated as (see also De Koster *et al.* [65]):

$$C_2 = \sum_{g \in G} \left[\begin{pmatrix} n \\ g \end{pmatrix} \left(\frac{g}{n}\right)^m \cdot X \cdot \left(2 \cdot y' \cdot \frac{\frac{m}{g}}{\frac{m}{g}+1} - y'\right)\right]$$

where

$$X = 1 - \sum_{i=1}^{g-1} (-1)^{i+1} \binom{g}{g-i} \left(\frac{g-i}{g}\right)^m$$

4.2.2 Estimate for travel within the cross aisles

The estimate for travel in the cross aisles consists of three components (1) travel from the depot to the left-most aisle with picks, (2) travel from aisle to aisle while picking items, and (3) travel from the right-most aisle with picks to the depot. See also Figure 4.3 for a graphical illustration of the components.

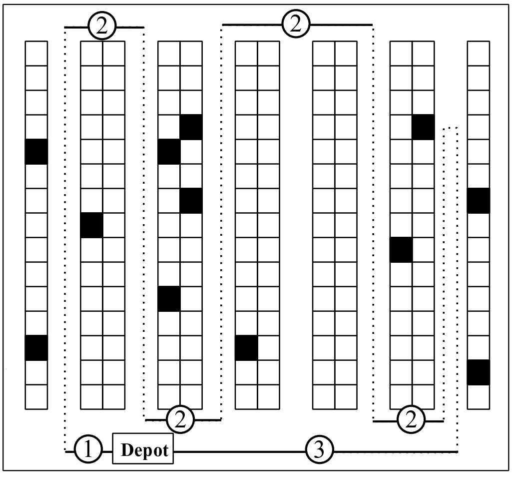

FIGURE 4.3. Illustration of the three components for travel in the cross aisles.

Both Hall [115] and Kunder and Gudehus [159] assume that the depot is located in the middle of the front cross aisle. Kunder and Gudehus [159] determined the probability that all items are distributed over a certain number, say f, aisles and estimates the distance traveled in the cross aisles by $w_a \cdot (n-1) \cdot \frac{f-1}{f+1}$. That is, the distance in the cross aisles is estimated using a property of f points distributed over a line according to a continuous uniform distribution. Hall [115] uses the same property in a different way, estimating the distance in the cross aisle by $w_a \cdot (n-1) \cdot \frac{m-1}{m+1}$. Both approximations suffer from the same two problems. First of all, a discrete process is approximated using expected values for the continuous case. Secondly, it is assumed that for each route the depot is located between the left-most and right-most aisle with picks. Situations in which all aisles that have picks are on one side of the depot are neglected. This holds because the estimates are equal to two times the distance from the left-most aisle to the right-most aisle. Any distance to reach the left (right) most aisle from a depot located on the left (right) side of the warehouse is not accounted for. Since a good evaluation of the depot location is not possible with the existing estimates, we will give a new formulation.

We start with the estimate of the distance traveled in the cross aisle while picking items. We have n aisles numbered $1, ..., n$ from left to right. The estimated distance between the left-most aisle containing picks and the right-most aisle containing picks is given by (for a proof see Appendix C):

$$w_a \cdot \left((n-1) - 2 \cdot \sum_{i=1}^{n-1} \left(\frac{i}{n}\right)^m \right)$$

Next, we estimate the average distance from the depot to the right-most aisle. The probability that i is the right-most aisle to be visited is:

$$\left(\frac{i}{n}\right)^m - \left(\frac{i-1}{n}\right)^m$$

which is the probability that all picks fall in aisles $1, ..., i$ minus the probability that all picks fall in aisles $1, ..., i-1$. The distance to be traveled is then the distance from depot location to the right-most aisle. If aisle i would be the right-most aisle with items, then the distance to travel would be:

$$w_a \cdot |i - d|$$

Taking the sum over all values of i and multiplying by their probability of occurrence gives the expected distance to be traveled from the depot to the right-most aisle:

$$w_a \cdot \sum_{i=1}^{n} \left(|i - d| \cdot \left[\left(\frac{i}{n}\right)^m - \left(\frac{i-1}{n}\right)^m \right] \right) \tag{4.3}$$

Similarly, we can find the distance from the depot to the left-most aisle to be:

$$w_a \cdot \sum_{i=1}^{n} \left(|i - (n - d + 1)| \cdot \left[\left(\frac{i}{n}\right)^m - \left(\frac{i-1}{n}\right)^m \right] \right) \tag{4.4}$$

The expected value of the distance traveled in the cross aisles, D_x, can now be obtained by adding all three components for cross aisle travel:

$$\begin{aligned} E[D_x] &= w_a \cdot \left(n - 1 - 2 \cdot \sum_{i=1}^{n-1} \left(\frac{i}{n}\right)^m \right) \\ &\quad + w_a \cdot \sum_{i=1}^{n} \left((|i-d| + |i-n+d-1|) \cdot \left[\left(\frac{i}{n}\right)^m - \left(\frac{i-1}{n}\right)^m \right] \right) \end{aligned}$$

4.2.3 Estimate for total average travel distance

Adding the two components (distance traveled in aisles and in cross aisles) gives the total expected travel distance in the picking area:

$$T_m(n, y, d) = E[D_y] + E[D_x]$$

This formulation has been developed for a manual picking operation. Therefore, we were able to use the average travel distance as a performance criterium. If we analyzed, for example, an automated storage / retrieval system, then a better performance criterium would be average travel time. This is the case because travel speed in aisles and cross aisles is unequal in such an environment. If we define t_y as the travel speed in the aisles and t_x as the travel speed in the cross aisles then the estimate for average travel time would be $T'_m(n, y, d) = t_y \cdot E[D_y] + t_x \cdot E[D_x]$. If orders are retrieved by an order picker walking through the warehouse with a small picking cart, there will be no additional time for changing aisles. However, if a relatively large vehicle is used in narrow aisles, then additional time is required for each change of aisles to position the vehicle correctly. Since this extra time has to be added each time the vehicle enters an aisle, we can just add $t_c \cdot E[A]$ to the travel time estimate, where t_c is the time needed to enter an aisle.

In practical situations it will often be the case that the pick list size is variable. The formula we developed is valid only for a fixed pick list size. However, it can easily be adapted for a variable pick list size. For example, assume that we know for every pick list size m that it will occur with probability p_m, then the estimate for average travel distance is given by $\sum_{m=1}^{\infty} p_m \cdot T_m(n, y, d)$.

Clearly, due to the nature of the approach we used (e.g. we assume that expected travel distance in aisles and expected travel distance in cross aisles are independent), this formulation gives an approximation of actual average travel distance. We will test the performance of the formula in Section 4.4.

4.3 Optimization of the depot location

Order pickers are assumed to start and end their routes at the depot. There may be several reasons why such a central location is in use. For example, if pick lists are generated by a printer, then order pickers have to go to the printer before starting a route. The printer would then function as depot. Or the depot may be the location where order pickers pick up an empty pick cart before starting a new route.

The depot can be located anywhere around the picking area. A depot can also serve more than one picking zone. Since aisles are accessed from the cross aisles, the depot is most often located in one of the cross aisles. If the depot is located elsewhere, it can still be modeled as if it is located at the left or right of one of the cross aisles. This is possible since order pickers always have to go through a cross aisle before entering the aisles. A virtual depot can be located at the corner of the picking area that is closest to the actual depot. In this way, order pickers will always pass the virtual depot before starting their picking route. The difference in travel distance between a route starting at the actual depot and a route starting at the virtual depot is just a constant, i.e. twice the distance between the actual and the virtual depot.

Intuitively, a depot in the middle of the front cross aisle seems to be the best with respect to average travel time. In this way, the probability seems to be highest that the depot is located between the left-most and right-most aisle of a picking route, thus preventing extra travel time to go forth and back to this region from a depot that is out of the middle. However, in the literature depot locations vary. A depot located in the middle is used in Goetschalckx and Ratliff [102], Hall [115], Kunder and Gudehus [159] and Petersen [217]. A depot located in a corner is used in Chew and Tang [43], De Koster *et al.* [66], Gibson and Sharp [98], Rosenwein [247] and Tang and Chew [265]. Both middle and corner options are considered in Jarvis and McDowell [140], Petersen [216] and Petersen and Schmenner [218]. In Ruben and Jacobs [250] travel to and from the depot is not taken into account, they only use travel between picks for their calculations. De Koster and Van der Poort [64] evaluated decentralized depositing (no depot at all). And finally Caron *et al.* [38, 39] assume the depot to be located between two blocks.

In this section, we will show that under the condition of random storage, the depot should be located in the middle of the front cross aisle, if it is possible to choose the depot location freely. Actually, a depot located in the middle of the back cross aisle would be equally good. This holds because 'front' and 'back' depend on the direction from which you look at the picking aisles. It is shown that the position in the middle of the front cross aisle minimizes the average travel distance for any choice of the number of aisles. A proof for this was given in Bassan *et al.* [21] in the context of a single command environment. We will show that it holds for any pick list size.

Theorem
The depot location that minimizes average travel distance is the exact middle of the front cross aisle.

Proof
Clearly, the depot location only influences average travel distance through the term (see equations 4.3 and 4.4):

$$w_a \cdot \sum_{i=1}^{n} \left((|i-d| + |i-n+d-1|) \cdot \left[\left(\frac{i}{n}\right)^m - \left(\frac{i-1}{n}\right)^m \right] \right)$$

We distinguish 4 cases.

Case 1: $i \geq d$ and $i + d \geq n + 1$

$$\begin{aligned} |i-d| + |i-n+d-1| &= i-d+i-n+d-1 \\ &= 2i-n-1 \end{aligned}$$

Since $2i - n - 1$ is independent of d there is no influence of the depot location on average travel distance.

Case 2: $i \leq d$ and $i + d \geq n + 1$

$$\begin{aligned} |i-d| + |i-n+d-1| &= d-i+i-n+d-1 \\ &= 2d-n-1 \end{aligned}$$

Travel distance is minimized by choosing d as small as possible under the condition that $i \leq d$ and $i + d \geq n + 1$. Substituting i by d gives $2d \geq n + 1$ or $d \geq \frac{n+1}{2}$. This implies that travel distance is minimized if $d = \frac{n+1}{2}$, i.e. the depot is located in the middle of the front cross aisle.

Case 3: $i \geq d$ and $i + d \leq n + 1$

$$\begin{aligned} |i-d| + |i-n+d-1| &= i-d-i+n-d+1 \\ &= -2d+n+1 \end{aligned}$$

Travel distance is minimized by choosing d as large as possible under the condition that $i \geq d$ and $i + d \leq n + 1$. Substituting i by d gives $2d \leq n + 1$ or $d \leq \frac{n+1}{2}$. This implies that travel distance is minimized if $d = \frac{n+1}{2}$, i.e. the depot is located in the middle of the front cross aisle.

Case 4: $i \leq d$ and $i + d \leq n + 1$

$$\begin{aligned} |i-d| + |i-(n-d+1)| &= d-i-i+n-d+1 \\ &= -2i+n+1 \end{aligned}$$

This is independent of the depot location.

Average travel distance is either not influenced by the depot location or it is minimized by taking $d = \frac{n+1}{2}$. Therefore, it holds that average travel distance is minimized with respect to the depot location by taking $d = \frac{n+1}{2}$, i.e. by locating the depot in the middle of the front cross aisle. ∎

We note that we have proven that the depot is to be located in the middle of the front cross aisle under the conditions that (1) S-shape is used for routing, (2) random storage is used and (3) the depot location is restricted to the front cross aisle. Furthermore,we note that the depot has to be located in the exact middle of the front cross aisle. Therefore, if the number of aisles is even, the depot will not be located exactly at the head of an aisle but between two aisles.

Looking at other routing heuristics (see Chapter 2 or Petersen [216]), it is straightforward to check that the S-shape, largest gap, combined, composite, midpoint and return heuristics all have the same amount of cross aisle travel per route in one-block layouts. This holds because each of the heuristics travels the distance between the left-most and right-most aisle with items twice, plus the distance from the depot to the left-most (right-most) aisle is traveled twice if the depot is positioned on the far left (right). The above proof is therefore valid for any of the other routing heuristics as well.

An evaluation of the percentage difference in average travel distance between a picking area with a depot located at the left and a picking area with a depot located in middle is given in the next section. The next section also contains experiments concerning the number of aisles required to minimize average travel distance.

4.4 Layout experiments

First of all, we will compare the values calculated with the formula from Section 4.2 with the results from simulation. To this end, we consider a manual picking operation in a shelf area, where several order pickers may be assigned to the same zone. The center-to-center distance between two neighboring aisles is 2.5 meters. Order pickers are assumed to travel through the exact middle of the aisles and cross aisles. Cross aisles are 2.5 meters wide. For this type of picking areas we assume the following measures to be representative. Aisle length varies between 10 and 30 meters. Each order picker works in a zone consisting of 7 to 15 aisles. Each picking route has to visit between 10 and 30 locations. We use the extremes of these values for our comparison, which gives eight different situations. Furthermore, for each situation we consider two depot locations: at the left (at the head of aisle 1) and in the middle of the front cross aisle. For the simulation we generated 2000 orders for each situation. The number of replications is sufficient to obtain a 95% confidence interval with a half-width of less than 2% of the sample mean. Pick locations are distributed over the aisles and locations according to a uniform distribution. No picks occur in the cross aisles. Table 4.1 gives the results from the simulation and from the formula described in Section 4.2.

The percentage difference in Table 4.1 between simulated and calculated values has been calculated before rounding of the route length. As is apparent, the results from the formula follow the behavior of the simulation closely. Therefore, we can determine average route length by straightforward calculations with the formula instead of developing a simulation model for the order picking area.

aisle length	# aisles	# items	depot	Sim.	Formula	Difference
10	7	10	left	99.0	99.0	0.03%
10	7	10	middle	97.5	97.7	−0.21%
10	7	30	left	121.8	123.2	−1.11%
10	7	30	middle	121.7	123.1	−1.21%
10	15	10	left	159.7	159.5	0.10%
10	15	10	middle	154.9	155.0	−0.03%
10	15	30	left	234.7	235.1	−0.17%
10	15	30	middle	234.0	234.4	−0.18%
30	7	10	left	211.8	211.4	0.21%
30	7	10	middle	210.0	210.1	−0.07%
30	7	30	left	271.0	272.3	−0.51%
30	7	30	middle	270.7	272.3	−0.58%
30	15	10	left	310.5	310.2	0.11%
30	15	10	middle	305.3	305.6	−0.08%
30	15	30	left	499.9	500.4	−0.09%
30	15	30	middle	499.2	499.7	−0.09%

TABLE 4.1. Average route length in meters for picking an order.

We have already proven that the depot is best located in the middle of the front cross aisle. However, it would be interesting to know the magnitude of the difference between various depot locations. From Table 4.1 we can see that the difference between a depot on the left and a depot in the middle for situations with a large pick list (30 items) is very small, less than 0.5%. For small pick lists (10 items) however, the difference is larger. Figure 4.4 depicts average travel distance (calculated with the formula of Section 4.2) as a function of the depot location for the situation with 15 aisles of 10 meters and a pick list of 10 items. From this figure we can see that the best location for the depot is at the head of aisle 8. Obviously, this is the middle of the front cross aisle. We can also see that the route length is fairly insensitive to small changes in the depot location around the optimum.

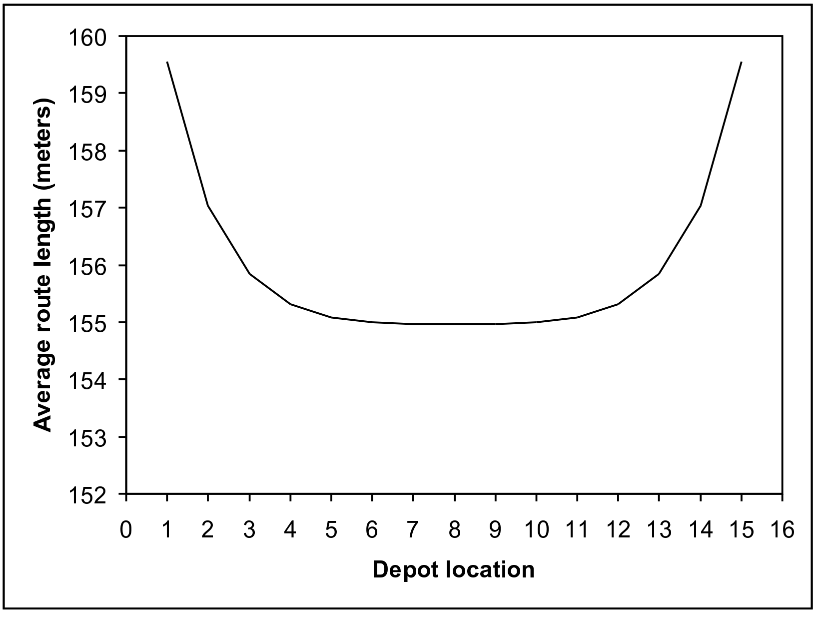

FIGURE 4.4. Average route length in meters as a function of the depot location for a warehouse with 15 aisles of 10 meters and a picklist size of 10.

We can use the model of Section 4.1 to determine the best layout for various values of the total aisle length. Consider the 16 situations from Table 4.1. These situations consist of 4 different values of total aisle length (70, 150, 210 and 450 meters) and two values for pick list size (10 and 30 picks). Using the formula we determine average route length for 1, 2, 3, ... aisles with decreasing aisle length such that total aisle length is constant at either 70, 150, 210 or 450 meters. The depot is located in the middle of the front cross aisle, because we already proved in the previous section that the middle is the best location. Aisle length is determined by dividing total aisle length by the number of aisles and adding twice the distance needed to go from the end of the aisle to the middle of the cross aisle. By taking the number of aisles that results in minimum average route length, we obtain the best layout with respect to travel distance.

Figure 4.5 depicts average route length as a function of the number of aisles. The minimum average route length for each curve is indicated with an asterisk. For example, in a picking area with 450 meters of aisle and a pick list size of 10 we find that the optimal layout is one with 28 aisles. From this figure we can see that larger picking areas require more aisles. A layout with exactly two aisles seems to give relatively good results. Only minor improvements can be obtained by having more aisles (for total aisle length 150 or 210 meters). A layout with three aisles is always worse than one with two aisles. It can also be seen that average route length changes only slightly if the layout deviates from the optimal layout (i.e. the upper three curves are fairly flat around the optimum).

If we increase the pick list size to 30, we obtain a totally different picture. First of all, the erratic behavior of the curves in Figure 4.6 is striking. This can be explained as follows. If the picking area has an odd number of aisles and if all aisles have to be visited, then the order picker has to make a turn in the last aisle. Thus, part of the last aisle is traveled twice. If the number of picks is high then the probability that all aisles have to be visited is high and the distance traveled twice in the last aisle is large. Therefore, the distance in picking areas with an odd number of aisles tends to be higher than similar picking areas with an even number of aisles. Another point that follows from Figure 4.6 is that if large orders have to be picked, then it is very likely that it is best to have a picking area consisting of exactly two picking aisles. This does not necessarily imply that there should be only one picking area consisting of two aisles, but is does suggest that the best situation would be one where every order picker is responsible for picking in just two aisles.

It is interesting to note from a practical point of view that most of the curves are quite flat. That is, around the optimum there are a large number of other layouts that have average travel distances which are only a few percent higher. A designer can use this flexibility to meet other requirements. Such requirements may be prevention of congestion, flexibilty for redesign or stability with respect to changes in pick list size.

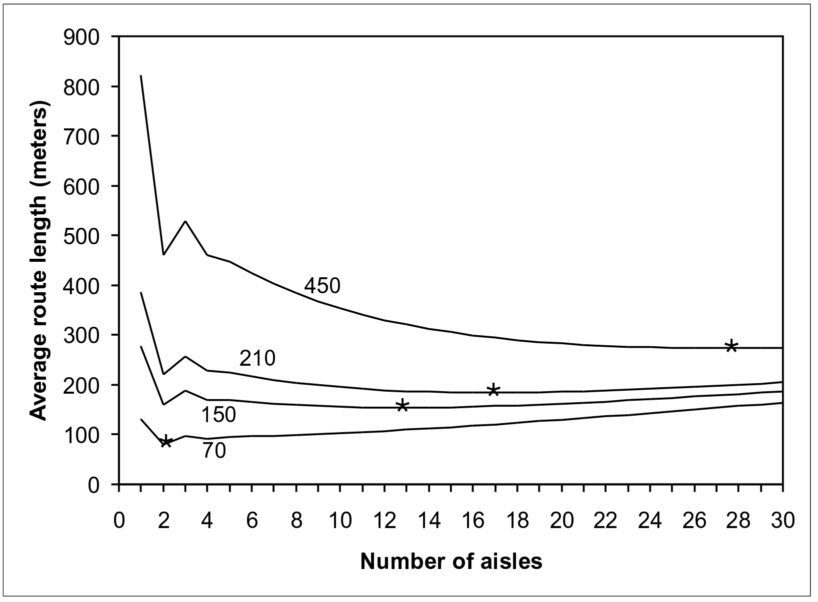

FIGURE 4.5. Average route length in meters as a function of the number of aisles for four values of the total aisle length (indicated next to the curves in meters). Pick list size is 10. An asterisk indicates the optimal layout.

4.5 Concluding remarks

In this chapter we have evaluated the relation between the layout of the order picking area and the average length of a picking route for areas consisting of one block. First an analytical formula is presented with which the average route length can be calculated based on the number of aisles, aisle length, depot location and number of picks per route. The results from this formula are compared with simulation results. It appeared that the difference between simulated and calculated values was between 0% and 1.2% in our experiments. Therefore, we consider the analytical formula to be accurate enough to be used instead of simulation when determining average route length.

We proved that the best depot location is in the middle of the front cross aisle. The proof is valid for a wide variety of routing heuristics under the conditions that random storage is used and that the depot has to be located at some point in the front cross aisle. Furthermore, we investigated the behavior of the average route length if we moved the depot to various positions along the front cross aisle. It appeared that

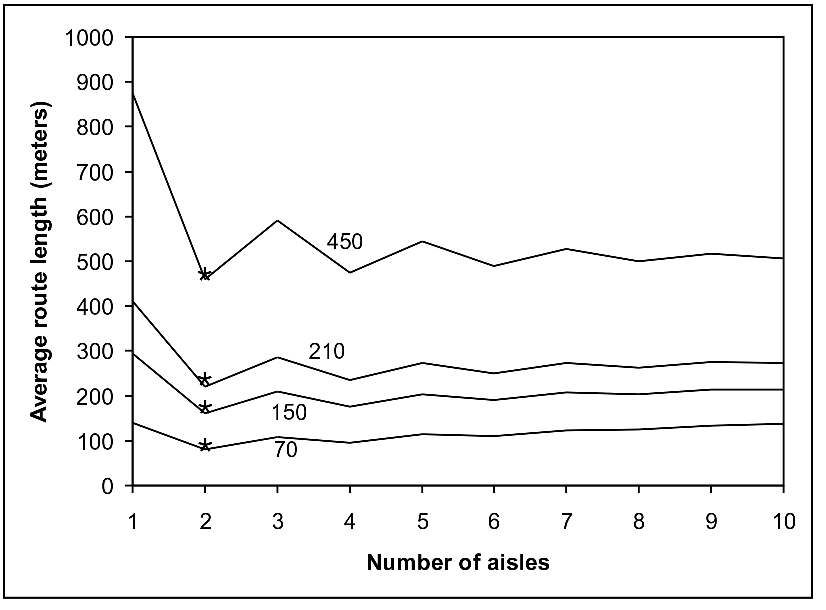

FIGURE 4.6. Average route length in meters as a function of the number of aisles for four values of the total aisle length (indicated next to the curves in meters). Pick list size is 30. An asterisk indicates the optimal layout.

positions somewhat out of the middle give an average route length that differs only slightly from the optimum.

Finally, we have used the analytical formula to determine the optimal layout for eight different situations. From these analyses we found the following behavior. Larger picking areas require more aisles. From the viewpoint of strict travel distance minimization, a very high pick density is best dealt with in a picking area where each picking zone consists of exactly two aisles. For practical implementations, other considerations than travel distance can be taken into account easily too, since there are generally many layouts that have an average travel distance that is very close to the optimum. Picking areas with an even number of aisles give a lower average route length than picking areas with an odd number of aisles, when there are many picks and the S-shape heuristic is used for routing.

5

A layout for the order picking area: Multiple blocks

In the previous chapter we developed statistical estimates for the average travel distance required to pick an order in one-block warehouses. Based on these estimates we were able to determine certain characteristics of good layouts. The limitation of the formulas described so far is that they are valid only in warehouses consisting of one block. It would, however, be interesting to have statistical estimates for multiple-block warehouses as well. After all, in Chapters 2 and 3 we have shown that in many cases warehouses consisting of more than one block have lower average travel times than warehouses consisting of only one block, given a certain storage capacity. Therefore, we will extend the work of Chapter 4 to situations with multiple blocks.

In this chapter, we describe a method to determine a layout for the order picking area without any restriction on the number of aisles or blocks. An estimate is derived for the average travel distance. This estimate is based on statistical properties of the S-shape routing heuristic; see Section 2.2.1 for a description of this heuristic. It is assumed that the random storage policy, see Section 1.4.1, is applied to determine storage location for products. The resulting estimate for average travel distance is a function of the order size, the number of aisles, the number of blocks, aisle width and aisle length. By minimizing this function for a given order size and a given storage capacity, we can obtain the desired layout for the order picking area.

In Section 5.1 we describe the situation for which the average travel distance estimate will be developed. Section 5.2 describes the objective of layout optimization for the order picking area in more detail. The optimization model is presented in Section 5.2 as well. The objective function of this model is derived in Section 5.3. The accuracy of the travel distance estimate is checked by comparing the results with simulation in Section 5.4. An example of a layout optimization is given in Section 5.5. Section 5.6 gives some concluding remarks.

5.1 Layout and routing

Although most of the assumptions and definitions in this chapter are the same as those in previous chapters we will summarize the important ones for the sake of completeness. We consider a manual order picking operation, where order pickers walk through a warehouse to retrieve products from storage. Picked items are placed on a pick device, which the order picker takes with him on his route. An example of such a warehouse layout is given in Figure 5.1. The warehouse is rectangular with no unused space and consists of a number of parallel *pick aisles*. The warehouse is divided into a number of *blocks,* each of which contains a number of subaisles. A *subaisle* is that part of a pick aisle that is within one block. The term *aisle* is used when a statement holds for both pick aisles and subaisles. At the front and back of the warehouse and between each pair of blocks, there is a *cross aisle*. Cross aisles do not contain storage locations, but can be used to change aisles. Every block has a *front cross aisle* and a *back cross aisle*; the front cross aisle of one block is the back cross aisle of another block, except for the first block.

The main advantage of having extra cross aisles in a warehouse consists of the increased routing options, which may result in lower travel distances. On the other hand, warehouse size must increase if more cross aisles are added, because total storage space must be kept constant to meet predefined requirements. Research on the subject of travel time estimation and/or layout determination has mainly been restricted to warehouses consisting of a single block, see e.g. Chew and Tang [43], Choe [46], De Koster [59], Hall [115], Jarvis and McDowell [140], Kunder and Gudehus [159], Tang and Chew [265] and Chapter 4 of this thesis. Caron *et al.* [38] study a warehouse consisting of two blocks, with the depot located between the two blocks.

Order pickers are assumed to be able to traverse an aisle in either direction and to change direction within an aisle. Items are stored at both sides of the aisles. Item locations are determined randomly according to a uniform distribution. Clearly, activity-based item location could possibly ask for a different layout. In this chapter we will consider only random storage assignment since this strategy can be considered as a base-line against which layouts with activity-based storage assignment can be

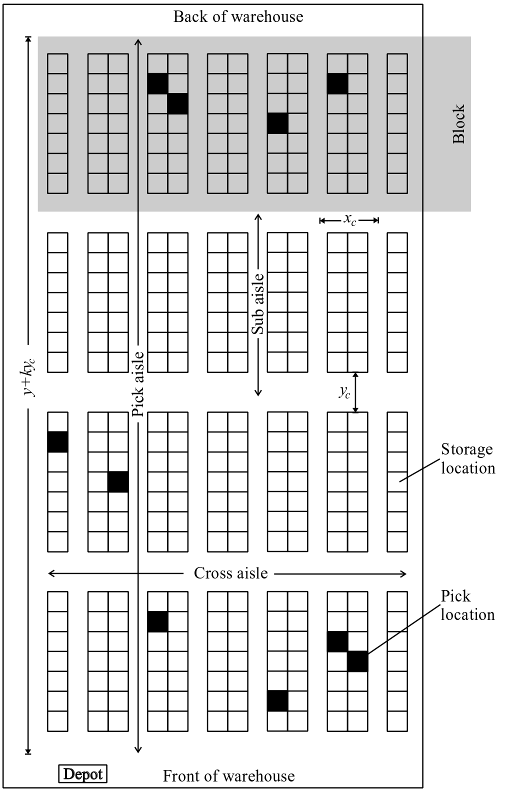

FIGURE 5.1. Example layout of an order picking area.

compared. The aisles are narrow enough to allow picking from both sides of the aisle without changing position. Every item can be picked from the rack by the order picker without climbing or using a lifting device. Each order consists of a number of items that are usually spread out over a number of aisles. We assume that the items of an order can and will be picked in a single route. Aisle changes are possible in any of the cross aisles. Picked orders have to be deposited at the depot, where the picker also receives the instructions for the next route. The depot is located in the front most cross aisle.

In this chapter we will use a routing strategy called the traversal or *S-shape strategy*. This strategy is an extension for situations with multiple cross aisles of a routing strategy that is widely used in practice. The version for multiple blocks is described in Section 2.2.1. Evaluations of the performance of the S-shape routing method for warehouses consisting of multiple blocks can be found in Section 2.4. With the S-shape strategy, any subaisle containing at least one pick is generally traversed through the entire length, with few exceptions. Subaisles where nothing has to be picked are not entered. An example route for the S-shape heuristic is given in Figure 2.2.

5.2 Layout optimization

In this chapter we describe a method to determine a layout for an order picking area without a restriction on the number of blocks. The objective is to find a layout that results in minimal average travel distance.

For order picking with a cart or pallet jack, average travel distance is influenced by five major factors:

1. Length of the pick aisles
2. Number of pick aisles
3. Number of blocks
4. Location of the depot.
5. Number of picks per route

Only the first four factors can be considered as layout factors. However, the number of picks per route also has a large influence on average travel distance. Therefore, we will optimize the layout for a fixed number of picks. This constraint can easily be relaxed if a probability distribution is known for the number of picks per route. The approach for this is identical to that described in Section 4.2.3.

Generally, when designing a warehouse, the storage space that is required to store all products is determined in advance. Only after this decision is made, does a layout have to be determined. Therefore, we take the total storage space as an input for our model.

We define the following variables:

n number of pick aisles (integer)

k number of blocks (integer)

y length of a pick aisle along the pick face (i.e. length of a pick aisle minus the width of the cross aisles)

d the location of the depot, $1 \leq d \leq n$ (real).

The depot can be located anywhere in the front most cross aisle between the left-most pick aisle (pick aisle 1) and the right-most pick aisle (pick aisle n). The position is indicated with a number. For example, $d = 1$ indicates that the depot is located at the head of pick aisle 1; $d = 3.5$ indicates that the depot is located between pick aisles 3 and 4.

We define the following parameters:

m the number of picks (integer)

y_c width of a cross aisle

x_c center-to-center distance between two adjacent pick aisles

S total aisle length, measured along the pick face

See Figure 5.1 for a graphical illustration of y, y_c and x_c.
Furthermore, we will use the following conventions:

i, g are used to index the pick aisles

j, h are used to index the blocks

u, t are used to index the picks

We can model the problem as follows:

$$\begin{aligned}
&\min T_m(n, k, y, d)\\
&\text{s.t.}\\
&n \cdot y = S\\
&n \geq 1\\
&k \geq 1\\
&1 \leq d \leq n\\
&n, k \text{ integer}
\end{aligned}$$

The objective is to minimize average travel distance, T_m, which is a function of the number of pick aisles n, the number of blocks k and the depot location. Travel distance T_m also depends on the number of picks m, which is a fixed value for this model. The minimization is to be performed under the condition that total storage space is kept constant. This is modeled as $n \cdot y = S$, i.e. total aisle length along the pick face is constant. This is possible since the storage space per meter aisle length is constant.

The major difficulty in this model is to determine $T_m(n, k, y, d)$. In the next section we will derive an expression for average travel distance $T_m(n, k, y, d)$ for the case where the order pickers travel through the warehouse according to the S-shape heuristic.

5.3 Average travel distance estimation

In this section we will give a statistically based estimate for the average travel in a warehouse with multiple cross aisles when using the S-shape strategy for routing. Before giving the complete estimate, we will first decompose the order picking routes into seven parts that will be estimated separately. We distinguish:

1. Traveling through subaisles to retrieve products
2. Traveling through subaisles without picks, from front to back
3. Traveling through subaisles without picks, from back to front
4. Correction of travel distance for turns within subaisles
5. Traveling in cross aisles
6. Traveling to and from the depot in the front cross aisle.
7. Extra time needed to change aisles by the picking vehicle, such as slowing down for turns or intersections.

Each of these parts (except part 7) is indicated in Figure 5.2 with a number corresponding to the above list. The estimates and explanations of these parts is given in Sections 5.3.1 - 5.3.7. We denote the first estimate, for traveling through subaisles to retrieve products, by E_1; the second estimate will be denoted by E_2 and so on. The total estimate for average travel distance will then be given by $T_m(n,k,y,d) = \sum_{i=1}^{6} E_i$.

To facilitate reading, we will first give the probability that items are distributed over the order picking area in a certain way. This formulation will be used several times in subsequent sections. We divide the warehouse into seven mutually exclusive areas, which are shown in Figure 5.3. Borders between areas are partially defined by the left-most pick aisle that contains pick locations ('left pick aisle') and the block farthest from the depot that contains pick locations ('farthest block'). The first area, labeled with the letter A, consists of the subaisle that is in the left pick aisle and in the farthest block. The second area, labeled B, consists of all subaisles in the farthest block to the right of area A. Areas C and D together (denoted as $C \cup D$) consist of all subaisles that are in the left pick aisle and below area A. Area D consists of one

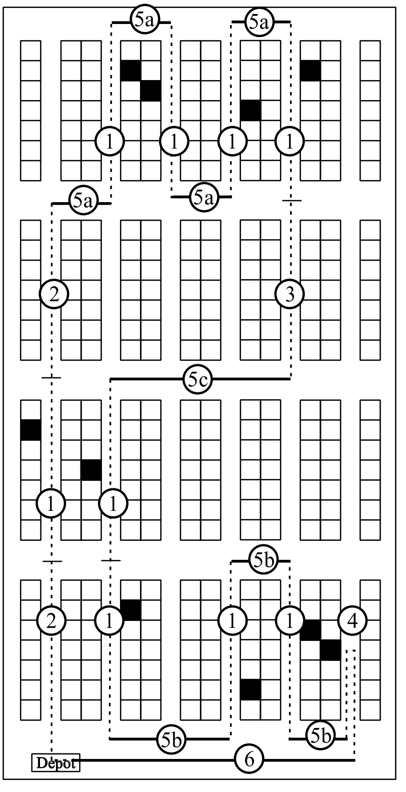

FIGURE 5.2. Graphical illustration of the various components of the travel distance estimate.

(random) subaisle in $C \cup D$ and area C consists of the remaining subaisles in $C \cup D$. Areas E and F together (denoted as $E \cup F$) include all subaisles that are both to the right and below area A (not including any subaisle from either area A, B, C, or D). Area F consists of a group of subaisles that are in the intersection of $E \cup F$ and a single block. The block used for this intersection is randomly chosen, such that the intersection is non-empty (provided that $E \cup F$ is non-empty). The seventh area consists of all remaining subaisles and is not given a name, since it is not used in any of the calculations. Note that since $A, ..., F$ are sets, we will say that an area is empty if it does not exist. If an area is said to be non-empty, i.e. it exists, it may still be empty with regards to the number of pick locations, i.e. there may be no picks in the area.

Suppose we know that all pick locations fall into or to the right of pick aisle i $(1 \leq i \leq n)$, where the right-most pick aisle of the warehouse is indexed as 1 and

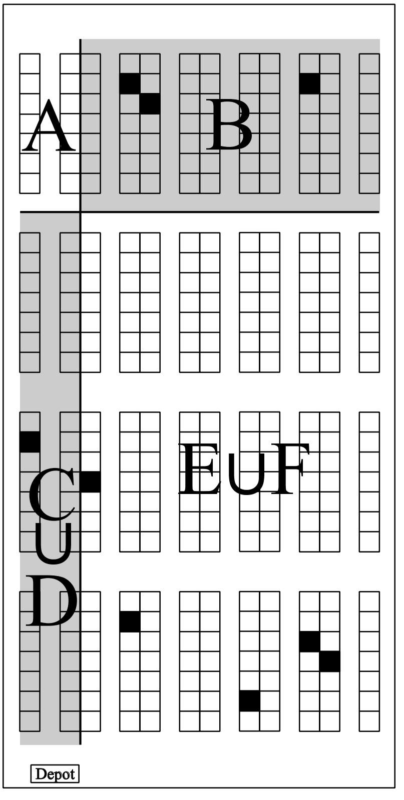

FIGURE 5.3. Illustration of the six different areas used in creating the travel distance estimates.

the left-most pick aisle of the warehouse as n. Furthermore, suppose that we know that all pick locations fall into or below block j $(1 \leq j \leq k)$, where the block closest to the depot is indexed as 1 and the block farthest from the depot (regardless of pick locations) as k.

Suppose we also know that there is at least one pick location in pick aisle i and at least one pick location in the block j. This means that one of the following two conditions holds (1) area A contains at least one pick location, (2) both area B and area $C \cup D$ contain each at least one pick location.

Define $U_a, U_b, U_c, U_d, U_e, U_f$ to be random variables that denote the number of pick locations that fall in areas A, B, C, D, E, F respectively. Define the vector $U = (U_a, U_b, U_c, U_d, U_e, U_f)$ with observations denoted as $u = (u_a, u_b, u_c, u_d, u_e, u_f)$. Then the probability that $U = u$ given that all items are in pick aisles $1...i$ and blocks $1...j$

and given that either area A is non-empty or both areas B and $C \cup D$ are non-empty, is given by:

$$P_{ij}(U = u \mid u \in R) = \frac{m!}{u_a! u_b! u_c! u_d! u_e! u_f!} \cdot \frac{P_{ij}^a \cdot P_{ij}^b \cdot P_{ij}^c \cdot P_{ij}^d \cdot P_{ij}^e \cdot P_{ij}^f}{P_{ij}\,(u \in R)}$$

where

$$R = \left\{ u \quad \middle| \quad \begin{array}{l} u_a, u_b, u_c, u_d, u_e, u_f \geq 0 \\ u_a > 0 \text{ or } (u_b > 0 \text{ and } u_c + u_d > 0) \\ u_a + u_b + u_c + u_d + u_e + u_f = m \\ \text{if } j = 1 \text{ then } u_c = 0, u_d = 0, u_e = 0 \text{ and } u_f = 0 \\ \text{if } j = 2 \text{ then } u_c = 0 \text{ and } u_e = 0 \end{array} \right\}$$

$$P_{ij}\,(u \in R) = 1 - \left(\frac{i(j-1)}{ij}\right)^m - \left(\frac{j(i-1)}{ij}\right)^m + \left(\frac{(i-1)(j-1)}{ij}\right)^m$$

$$\begin{aligned}
P_{ij}^a &= \left(\frac{1}{ij}\right)^{u_a} \\
P_{ij}^b &= \left(\frac{i-1}{ij}\right)^{u_b} \\
P_{ij}^c &= \left(\frac{j-2}{ij}\right)^{u_c} \cdot I_{\{j>2\}} + I_{\{j\leq 2\}} \\
P_{ij}^d &= \left(\frac{1}{ij}\right)^{u_d} \cdot I_{\{j>1\}} + I_{\{j=1\}} \\
P_{ij}^e &= \left(\frac{ij-2i-j+2}{ij}\right)^{u_e} \cdot I_{\{j>2\}} + I_{\{j\leq 2\}} \\
P_{ij}^f &= \left(\frac{i-1}{ij}\right)^{u_f} \cdot I_{\{j>1\}} + I_{\{j=1\}}
\end{aligned}$$

$$I_{\{\text{condition}\}} = \begin{cases} 1 & \text{if condition is true} \\ 0 & \text{otherwise} \end{cases}$$

The conditions as imposed on $(u_a, u_b, u_c, u_d, u_e, u_f)$ in the definition of the set R state that (1) the number of pick locations in each area is non-negative, (2) area A contains at least one pick location or area B and $C \cup D$ each contain at one least pick location, (3) the total number of pick locations equals the sum of the pick locations in each of the individual areas, (4) if all pick locations fall into just two blocks then there are no picks in areas C and E. (5) if all picks fall into one block, then there are no picks in areas C, D, E and F. Conditions (4) and (5) hold because from the definition of the areas it is clear that the areas do not exist under the conditions stated. Consequently, no picks are then possible in these areas.

To explain $P_{ij}\,(u \in R)$ observe that the set R defines all feasible values of u. All values are feasible except if there are no picks in areas A and B and/or if there are no picks in areas A and $C \cup D$. The probability that there are no picks in areas A and B is $\left(\frac{i(j-1)}{ij}\right)^m$ and the probability that there are no picks in areas A and $C \cup D$ is $\left(\frac{j(i-1)}{ij}\right)^m$. However, if we subtract these two probabilities from 1, then we would subtract the probability that all items fall in area $E \cup F$ twice. Therefore, we again add the probability that all items fall in area $E \cup F$, which is $\left(\frac{(i-1)(j-1)}{ij}\right)^m$.

The index function I is necessary because in the cases where $I = 0$ it holds that the corresponding area is empty. That is, if $j = 1$ then the probability that a pick falls in either area C, D, E or F is zero, because these areas are empty by definition. If $j = 2$, then areas C and E are empty, but areas D and F are non-empty.

For simplicity we will use the following notation:

$$P_{ij}(U_f = 0) \equiv \sum_{\substack{u \in R \\ u_f = 0}} P_{ij}\,(U = u \mid u \in R)$$

Similar statements will be used for conditions other than $U_f = 0$.

5.3.1 Traveling through subaisles to retrieve products

In this section we give the expected travel distance if every subaisle that contains a pick location is traversed its entire length. This estimate is formed as the product of two terms: (1) the expected number of subaisles that contain a pick location and (2) the length of a subaisle. The expected number of subaisles that have to be visited is:

$$nk\left[1 - \left(\frac{nk-1}{nk}\right)^m\right]$$

where nk is the total number of subaisles in the warehouse. The term between square brackets is 1 minus the probability that a certain subaisle does not contain pick locations. A more precise estimate for the number of subaisles to be visited is possible, however, in Kunder and Gudehus [159] it is shown that an estimate like this is, in general, adequate.

The length of a subaisle is given by:

$$\frac{y + ky_c}{k}$$

This is the length of a subaisle along the pick face (y/k) plus two times half the width of a cross aisle. This is because we assume that the order picker travels exactly through the middle of the aisles and cross aisles. Thus, if the order picker exits a subaisle, he has to travel $\frac{1}{2}y_c$ before he is at the middle of the cross aisle and can continue his travel there. This also holds for entering the subaisle from the cross aisle, thus adding another $\frac{1}{2}y_c$ to the length of a subaisle.

This gives the estimate E_1 for traveling through subaisles to retrieve products:

$$E_1 = nk\left[1 - \left(\frac{nk-1}{nk}\right)^m\right] \cdot \frac{y + ky_c}{k}$$

5.3.2 *Traveling through subaisles without picks, from front to back*

We recall from the description of the S-shape heuristic that the left-most pick aisle containing picks is traversed from the front of the warehouse to the front cross aisle of the farthest block. The estimate E_1, described in the previous section, only accounts for subaisles with pick locations. However, in the left-most pick aisle there is a chance that one or more subaisles are to be traversed that have no pick locations.

In this section, we give an estimate for the number of subaisles in the left-most pick aisle that will be traversed but do not contain pick locations. By multiplying that estimate with the length of a subaisle, we obtain the travel distance that is the result of traveling through empty subaisles from the front to the back of the warehouse.

We first need to determine the position of the farthest block. If we index the blocks from front to back (i.e. the block closest to the depot is block 1, the next block 2 etc.), then the probability that the farthest block to visit is block j, is given by:

$$P(j \text{ is farthest block}) = \left[\left(\frac{j}{k}\right)^m - \left(\frac{j-1}{k}\right)^m\right]$$

This is the probability that all pick locations fall into blocks 1 through j minus the probability that all pick locations fall into blocks 1 through $j-1$. This equals the probability that all picks fall into blocks 1 through j and that there is at least one pick location in block j.

We index the pick aisles from right to left. We determine the probability that the left-most pick aisle with picks is pick aisle i. Analogous to the previous formulation for the number of blocks, we find that:

$$P(i \text{ is left-most pick aisle}) = \left[\left(\frac{i}{n}\right)^m - \left(\frac{i-1}{n}\right)^m\right]$$

By multiplying $P(j$ is farthest block) with $P(i$ is left-most pick aisle), we obtain the probability that all pick locations fall in pick aisles $1...i$ and blocks $1...j$ and there is at least one pick location in pick aisle i and at least one pick location in block j.

$$A_{ij} = \left[\left(\frac{i}{n}\right)^m - \left(\frac{i-1}{n}\right)^m\right] \cdot \left[\left(\frac{j}{k}\right)^m - \left(\frac{j-1}{k}\right)^m\right]$$

Now we can formulate the estimate E_2 for traveling through subaisles without picks, from front to back, as:

$$E_2 = \frac{y + ky_c}{k} \cdot \sum_{i=1}^{n}\sum_{j=1}^{k}\left[(j-1) \cdot A_{ij} \cdot P_{ij}(U_d = 0)\right]$$

The first part is the length of a subaisle. The second part consists of a summation over all i and j. The probability that an empty subaisle has to be traversed from front to back can be determined by taking the sum of $P_{ij}(U = u \mid u \in R)$ over all u for which $u_d = 0$. This is multiplied by A_{ij} and by $j-1$, which is the number of blocks where the situation of traversing an empty subaisle from front to back can occur. Note that by definition:

$$P_{ij}(U_d = 0) \equiv \sum_{\substack{u \in R \\ u_d = 0}} P_{ij}\left(U = u \mid u \in R\right)$$

5.3.3 Traveling through subaisles without picks, from back to front

The estimate E_3 for traveling through subaisles without picks, from back to front can be obtained in a similar fashion as the estimate in the previous section and is given by:

$$E_3 = \frac{y + ky_c}{k} \cdot \sum_{i=1}^{n}\sum_{j=1}^{k}\left[(j-1) \cdot A_{ij} \cdot P_{ij}(U_f = 0)\right]$$

5.3.4 Correction of travel distance for turns within subaisles

After the order picker picks the last item in a block, he has to return to the front of the block to be able to continue with the next block or to return to the depot. If the last subaisle of a block was entered from the back, then this subaisle can be traversed through its entire length. However, if the subaisle was entered from the front, then the order picker has to turn and go back to the front of the block after picking the last item. Since we already accounted for traveling the subaisle in its entire length in estimate E_1, we have to make a correction for the situations in which a turn may occur.

Suppose we know that there are t picks in a subaisle and that the order picker has to make a turn in this subaisle. Then the extra distance needed for the turn as compared to a full subaisle traversal is given by:

$$2 \cdot \frac{y + ky_c}{k} \cdot \frac{t}{t+1} - \frac{y + ky_c}{k} \tag{5.1}$$

In this formulation $\frac{t}{t+1}$ is the expected value of the maximum of t continuous uniformly distributed [0,1] variables. The average distance from the middle of the cross aisle to the farthest pick in the subaisle is $\frac{y+ky_c}{k}\frac{t}{t+1}$. Since the order picker also has to return the same way, the travel distance is twice this amount. In estimate E_1 we already accounted for $\frac{y+ky_c}{k}$, so we have to subtract this amount, which results in equation 5.1.

There are three areas in which a turn can occur, namely areas A, B and $E \cup F$. We first consider the extra travel distance for turns in area A.

Area A

A turn in area A can only occur if area B is empty ($U_b = 0$). Given that the left-most pick aisle is pick aisle i and that the farthest block is block j, the probability that a turn in area A occurs with t pick locations in the only subaisle of area A is given by:

$$P_{ij}\left(U_a = t \wedge U_b = 0\right) = \sum_{\substack{u \in R \\ u_a = t \\ u_b = 0}} P_{ij}\left(U = u \mid u \in R\right)$$

By multiplying the probability that a turn occurs by the corresponding expected extra travel distance and taking the sum over all possible values of t, we obtain the expected extra travel distance in area A (given that the left-most pick aisle is pick aisle i and that the farthest block is block j):

$$\sum_{t=1}^{m} \left[P_{ij}\left(U_a = t \wedge U_b = 0\right) \cdot \left(2 \cdot \frac{y + ky_c}{k} \frac{t}{t+1} - \frac{y + ky_c}{k} \right) \right] \tag{5.2}$$

Area B

For area B we distinguish two situations. The first situation occurs when there are no picks in area A and the second situation when there are picks in area A. We have to distinguish between these two situations because if area A contains no picks, then a turn will occur in area B if the number of subaisles to be visited in area B is odd.

On the other hand, if area A contains picks then an even number of subaisles with picks in area B will result in a turn.

Suppose we know that the left-most pick aisle is i (area B consists of $i-1$ subaisles) and that t picks fall into area B, then the probability that these picks fall into exactly g of the $i-1$ subaisles is given by:

$$\binom{i-1}{g}\left(\frac{g}{i-1}\right)^t \cdot X \tag{5.3}$$

where X is 1 minus the probability that all t picks fall into $g-1$ or less subaisles out of a given set of g subaisles, see also De Koster *et al.* [65]. To obtain this, use is made of the inclusion-exclusion rule. Thus,

$$X = 1 - \sum_{\gamma=1}^{g-1}(-1)^{\gamma+1}\binom{g}{g-\gamma}\left(\frac{g-\gamma}{g}\right)^t$$

Since all subaisles have equal probabilities of containing picks, the subaisle where the order picker has to make a turn will on average contain t/g picks. The additional travel for a turn in the last subaisle can be approximated by:

$$2 \cdot \frac{y+ky_c}{k} \cdot \frac{\frac{t}{g}}{\frac{t}{g}+1} - \frac{y+ky_c}{k} \tag{5.4}$$

using the property that the expected value of the maximum of β continuous uniformly distributed [0,1] variables is $\beta/(\beta+1)$.

By multiplying equations 5.3 and 5.4 we obtain

$$C_i(g,t) = \binom{i-1}{g}\left(\frac{g}{i-1}\right)^t \cdot X \cdot \left(2 \cdot \frac{y+ky_c}{k}\frac{\frac{t}{g}}{\frac{t}{g}+1} - \frac{y+ky_c}{k}\right)$$

which is the average extra travel distance for a subaisle where a turn has to occur, under the conditions that the left-most pick aisle is pick aisle i, there are exactly g subaisles that have at least one pick and there are in total exactly t picks in these g subaisles.

The expected extra travel distance in area B (given that there is at least one pick in area A, the left-most pick aisle is pick aisle i and the farthest block is block j) can now be stated as:

$$\sum_{t=1}^{m}\left[P_{ij}(U_a > 0 \wedge U_b = t) \cdot \sum_{g \in G_{even}} C_i(g,t)\right]$$

where

$$G_{even} = \{g \mid 1 \leq g \leq i-1, g \leq t \text{ and } g \text{ is even}\}$$

This result follows by aggregating $C_i(g,t)$ for all situations in which a turn may occur (i.e. the number of subaisles with a pick in area B is even) for a given value of t and multiplying it by the probability that there are actually t picks in area B and at least one pick in area A.

The estimate for the situation in which there is a turn in area B when there are no picks in area A, as well as the situation in which a turn occurs in area $E \cup F$ can be derived in a similar fashion. For this, we also need to define a set G_{odd} which use and derivation is basically similar to that of G_{even}. By taking the sum over all possible values of i and j and multiplying by the probability that i is the left-most pick aisle and j is the farthest block, we obtain the estimate E_4 for the extra travel distance resulting from turns in subaisles:

$$\begin{aligned} E_4 \quad = \quad \sum_{i=1}^{n}\sum_{j=1}^{k}\Bigg[A_{ij}\cdot\sum_{t=1}^{m}\Bigg(&P_{ij}\left(U_a=t\wedge U_b=0\right)\cdot\left(2\cdot\frac{y+ky_c}{k}\frac{t}{t+1}-\frac{y+ky_c}{k}\right)\\ &+P_{ij}(U_a>0\wedge U_b=t)\cdot\sum_{g\in G_{even}}C_i(g,t)\\ &+P_{ij}(U_a=0\wedge U_b=t)\cdot\sum_{g\in G_{odd}}C_i(g,t)\\ &+(j-1)\cdot P_{ij}\left(U_f=t\right)\cdot\sum_{g\in G_{even}}C_i(g,t)\Bigg)\Bigg] \end{aligned}$$

where

$$\begin{aligned} C_i(g,t) &= \binom{i-1}{g}\left(\frac{g}{i-1}\right)^t\cdot X\cdot\left(2\cdot\frac{y+ky_c}{k}\frac{\frac{t}{g}}{\frac{t}{g}+1}-\frac{y+ky_c}{k}\right)\\ X &= 1-\sum_{\gamma=1}^{g-1}(-1)^{\gamma+1}\binom{g}{g-\gamma}\left(\frac{g-\gamma}{g}\right)^t\\ G_{even} &= \{g \mid 1\leq g\leq i-1, g\leq t \text{ and } g \text{ is even}\}\\ G_{odd} &= \{g \mid 1\leq g\leq i-1, g\leq t \text{ and } g \text{ is odd}\} \end{aligned}$$

5.3.5 *Travel distance within cross aisles*

The distance traveled by the order picker in the cross aisles can be divided into the following components:

a. The distance traveled in cross aisles of the block farthest from the depot that contains picks

b. The distance traveled in cross aisles of blocks other than the farthest block while picking items. That is, all of the distances traveled between two subaisles in the same block, with at least one pick in each of the two subaisles.

c. The distance traveled in the cross aisles while connecting one block with the next. This travel results from step 5 of procedure for S-shape as described in Section 2.2.1.

The remaining distances traveled in cross aisles concerns the distance from the depot to the left-most pick aisle and the distance traveled in the front most cross aisle from the last pick back to the depot. These will be given in Section 5.3.6.

Estimates for the first three components will be given in this section. For a graphical illustration see Figure 5.2, where the three components are marked 5a, 5b and 5c.

First we will give an estimate for the distance traveled in the cross aisles of the block farthest from the depot that contains picks. Suppose that i is the left-most pick aisle with picks and that j is the block farthest from the depot with picks. We introduce g, where g indexes the subaisles from right to left. Note that i indexes the pick aisles, right to left as well. If g is the right-most subaisle in block j with picks, then the distance traveled in the cross aisles of block j is $(i-g)x_c$. If $L_{ij}(g)$ gives the probability that g is the right-most subaisle in block j with picks, then the estimate for cross aisle travel in the farthest block is given by:

$$E_5' = x_c \cdot \sum_{g=1}^{i-1} [(i-g) \cdot L_{ij}(g)]$$

In this formulation $L_{ij}(g)$ is given by:

$$\begin{aligned} L_{ij}(g) &= P(\text{exit block } j \text{ at subaisle } g) \\ &= \sum_{u_b=1}^{m-1} P_{ij}(U_b = u_b) \cdot \left[\left(\frac{i-g}{i-1}\right)^{u_b} - \left(\frac{i-g-1}{i-1}\right)^{u_b} \right] \end{aligned} \tag{5.5}$$

This can be explained as follows. The area under consideration is area B. This area consists of $i-1$ subaisles. Suppose there are u_b items to be picked in this area.

Then the probability that all u_b items fall in the subaisles $g, ..., i-1$ and subaisle g contains at least one pick, is given by:

$$\left(\frac{i-g}{i-1}\right)^{u_b} - \left(\frac{i-g-1}{i-1}\right)^{u_b}$$

Taking the sum over all possible values of u_b and multiplying by the corresponding probabilities yields equation 5.5.

Next we consider the estimate for the distance traveled in cross aisles of blocks other than block j while picking items. Consider area F where we have $i-1$ subaisles. The probability that all picks fall in $\ell+1$ consecutive subaisles is given by:

$$Q(\ell) = \left(\frac{\ell+1}{i-1}\right)^{u_f} - 2\left(\frac{\ell}{i-1}\right)^{u_f} + \left(\frac{\ell-1}{i-1}\right)^{u_f} \tag{5.6}$$

$Q(\ell)$ gives the probability that the left-most and right-most subaisles with picks in area F are exactly a distance of $\ell \cdot x_c$ apart. That is, equation 5.6 gives the probability that all items of this area fall in subaisles $\ell_1, ..., \ell_2$, at least one pick falls in ℓ_1, at least one pick falls in ℓ_2 and $\ell_2 - \ell_1 = \ell$. For an explanation refer to Appendix C where the probability $Q(\ell, r)$ is introduced, which is largely similar to equation 5.6.

There are $(i-1-\ell)$ different combinations of $\ell+1$ consecutive subaisles possible from a set of $i-1$ subaisles. The distance traveled is $x_c \cdot \ell$. Furthermore, we have to sum over all possible values of the number of picks in this area and multiply by the corresponding probability. Finally, since there are $j-1$ areas where cross aisle travel can occur during picking we have to multiply by $j-1$. Summarizing, we get as an estimate:

$$E_5'' = x_c \cdot (j-1) \cdot \sum_{u_f=1}^{m-1} \sum_{\ell=1}^{i-2} [\ell \cdot (i-1-\ell) \cdot Q(\ell) \cdot P_{ij}(U_f = u_f)]$$

The third component consists of cross aisle travel while connecting the blocks between picking. We need a factor to account for the expected travel from the end subaisle of the previous block to the start subaisle of the next block. We have E_5'' as an estimate for the distance between the left and right-most subaisle in a block containing items. The distance between the subaisles 1 and $i-1$ is equal to $i-2$. Therefore, the end point of the previous block can vary over a distance of $x_c(i-2) - E_5''$. The start point of the next block and end point of the previous block are uniformly distributed over this distance. Treating aisle locations as continuous random variables, an approximation for the expected distance between the location of the two points is:

$$\frac{1}{3} \cdot ((i-2) \cdot x_c - E_5'')$$

This holds for $j-1$ blocks. Furthermore, the distance only needs to be traveled if there is at least one pick in the next block. Otherwise, the order picker just traverses the closest subaisle to go straight to the next block. Therefore, we can estimate the distance traveled in the cross aisles while connecting one block with the next, by:

$$E_5''' = (j-1) \cdot \frac{1}{3} \cdot ((i-2) \cdot x_c - E_5'') \cdot (1 - P_{ij}(U_f = 0))$$

The total estimate E_5 for cross aisle travel thus becomes:

$$E_5 = \sum_{i=1}^{n} \sum_{j=1}^{k} A_{ij} \left(E_5' + E_5'' + E_5''' \right)$$

5.3.6 Travel distance to and from the depot

In this section we will give an estimate for the distance that has to be traveled through the front-most cross aisle from the depot to the left-most pick aisle with items and for the distance traveled through the front most cross aisle after all items have been picked.

First of all the order picker goes from the depot to the left-most pick aisle that contains items to be picked. The estimate for this can be obtained analogous to that given in equation 4.4 of Chapter 4, this gives:

$$E_6' = x_c \cdot \sum_{g=1}^{n} \left(|g - (n-d+1)| \cdot \left[\left(\frac{g}{n}\right)^m - \left(\frac{g-1}{n}\right)^m \right] \right) \tag{5.7}$$

After picking all items the order picker returns to the depot through the front cross aisle. It is important to know whether the path through the last block with picks went from left to right or from right to left through the block. If the subaisles in the last block were visited left to right then the distance to a depot located at the head of the left-most pick aisle is much larger, then if the subaisles were visited right to left. We will first determine the probability that the number of blocks that have to be visited is odd. If the number of blocks to be visited is odd then we will assume that the order picker visits subaisles in the last block from right to left. We assume this because subaisles in the first block, the 'farthest block', will always be visited from left to right. Subaisles in the block, that is visited next, are likely to be visited from right to left and so on. Likewise, if the number of blocks to be visited is even, we assume that the subaisles in the last block are visited from right to left.

The probability that there are exactly h blocks to be visited out of a total of k can be derived analogous to equation 5.3:

$$\binom{k}{h}\left(\frac{h}{k}\right)^m \cdot X' \tag{5.8}$$

where

$$X' = 1 - \sum_{\gamma=1}^{h-1} (-1)^{\gamma+1} \binom{h}{h-\gamma}\left(\frac{h-\gamma}{h}\right)^m$$

Then the probability that the number of blocks to be visited is even, is given by:

$$P_{even} = \sum_{h \in H_{even}} \binom{k}{h}\left(\frac{h}{k}\right)^m \cdot X'$$

$$H_{even} = \{h \mid 1 \leq h \leq k,\ h \leq m \text{ and } h \text{ is even}\}$$

The probability that the number of blocks to be visited is odd, is given by:

$$P_{odd} = 1 - P_{even}$$

Given that the path through the last block goes from right to left, i.e. the number of blocks visited is even, we can determine the expected travel distance in the front cross aisle. Suppose that the left-most pick aisle with items is pick aisle i, the farthest block with picks is block j and the number of picks in this last block is u_f, then, similar to equation 5.7, the expected distance to the depot is:

$$x_c \cdot \sum_{g=1}^{i-1} \left(|g-(n-d+1)| \cdot \left[\left(\frac{g}{i-1}\right)^{u_f} - \left(\frac{g-1}{i-1}\right)^{u_f} \right] \right)$$

Taking the sum over all possible values of u_f and multiplying by the corresponding probability gives:

$$E_6^{even} = x_c \cdot \sum_{g=1}^{i-1} \sum_{u_f=1}^{m-1} \left(P_{ij}(U_f = u_f) \cdot |g-(n-d+1)| \cdot \left[\left(\frac{g}{i-1}\right)^{u_f} - \left(\frac{g-1}{i-1}\right)^{u_f} \right] \right)$$

Similarly, we can determine the expected distance from the right-most subaisle of the last block to the depot, in case of an odd number of blocks to visit, to be:

$$E_6^{odd} = x_c \cdot \sum_{g=1}^{i-1} \sum_{u_f=1}^{m-1} \left(P_{ij}(U_f = u_f) \cdot |g-(n-d+1)| \cdot \left[\left(\frac{i-g}{i-1}\right)^{u_f} - \left(\frac{i-g-1}{i-1}\right)^{u_f} \right] \right)$$

These formulations, however, do not include the distance that has to be traveled through the front cross aisle if $U_f = 0$. That is if the block closest to the depot has no picks. We will assume that the order picker in this case has to travel half the distance. That is, we assume that the order picker – on average – ends in the middle of the last area between subaisles 1 and i. The middle of this is $x_c \cdot i/2$ from pick aisle i and $x_c \cdot (n - i + 1) + x_c \cdot i/2$ from the left side of the warehouse. The distance from this location to the depot is therefore given by:

$$x_c \cdot \left(n - \frac{i}{2} + 1 - d\right)$$

with probability $P_{ij}(U_f = 0)$. Thus,

$$E_6'' = x_c \cdot P_{ij}(U_f = 0) \cdot \left(n - \frac{i}{2} + 1 - d\right)$$

Finally, we need to determine the distance the order picker travels if all picks are in the block closest to the depot. That is , if $j = 1$. Similar to the previous estimates, this estimate can be derived to be:

$$E_6''' = \sum_{i=1}^{n} \sum_{g=1}^{n} \left(A_{i1} \cdot x_c \cdot |g - d| \cdot \left[\left(\frac{g}{n}\right)^m - \left(\frac{g-1}{n}\right)^m \right] \right)$$

In total, we find as estimate E_6 for the distance traveled to and from the depot:

$$E_6 = E_6' + \sum_{i=1}^{n} \sum_{j=2}^{k} \left[A_{ij} \left(P_{even} \cdot E_6^{even} + P_{odd} \cdot E_6^{odd} + E_6'' \right) \right] + E_6'''$$

5.3.7 Extra time needed to change aisles by the picking vehicle

The extra time needed for changing aisles depends on the situation and type of vehicle used. Where orders are retrieved by an order picker walking through the warehouse with a small picking cart, there will be no additional time for changing aisles. However, if a relatively large vehicle is used in narrow aisles, then additional time is required for each change of pick aisles to position the vehicle correctly.

The number of subaisles to visit can easily be identified to equal:

$$E[A] = (E_1 + E_2 + E_3) \cdot \frac{k}{y + k y_c}$$

If it takes t_c seconds to maneuver the vehicle per subaisle, then an estimate for travel time needs to be adjusted by $t_c \cdot E[A]$. We note that this adjustment is only possible if the entire estimate is given in seconds, instead of meters. For methods to do this, as well as to incorporate different travel speeds within and outside the pick aisles, see Section 4.2.3.

5.3.8 Estimate for total average travel distance

The total estimate for average travel distance in a warehouse with n pick aisles, k blocks, d the location of the depot and m items to retrieve, can now be formulated as:

$$T_m(n,k,y,d) = \sum_{i=1}^{6} E_i \tag{5.9}$$

5.4 Comparison between simulation and the estimate

In this section, we will compare the values of the estimate developed in this chapter with the results from simulation. We consider the same manual picking operation in a shelf area as in Chapter 2. The center-to-center distance between two neighboring pick aisles is 2.5 meters. Order pickers are assumed to travel through the middle of the aisles and cross aisles. Cross aisles are 2.5 meters wide. For this type of picking areas we assume the following measures to be representative. Pick aisle length varies between 10 and 30 meters. Each order picker works in a zone consisting of 7 to 15 pick aisles. Each picking route has to visit between 10 and 30 locations. We use the extremes of these values for our comparison, which gives eight different situations. Furthermore, for each situation we consider five different values for the number of blocks. That is, we make a comparison for one block situations. Then we insert cross aisles to increase the number of blocks to 2, 3, 4 and 5. The cross aisles are equally spaced over the distance from the front of the warehouse to the back. For the simulation we generated 10,000 orders for each situation. Pick locations are distributed over the aisles and locations according to a uniform distribution. No picks occur in the cross aisles. Table 5.1 gives the results from the simulation. Table 5.2 gives the results from equation 5.9. The percentage difference between the simulation and the formula is given in Table 5.3.

The percentage differences in Table 5.3 between simulated and calculated values have been calculated before rounding off the route length. The results from the formula follow the behavior of the simulation fairly closely. The absolute difference varies from 0.01% upto 3.39%.

simulation			number of blocks				
aisles	length	items	1	2	3	4	5
7	10	10	98.7	87.1	91.3	93.5	96.6
7	10	30	121.8	126.2	150.5	152.6	168.2
15	10	10	159.5	134.6	147.6	152.2	157.3
15	10	30	234.8	215.8	259.7	254.9	288.9
7	30	10	211.1	164.9	153.4	147.1	145.3
7	30	30	270.9	255.9	263.4	252.3	257.6
15	30	10	310.3	225.7	216.7	210.1	209.0
15	30	30	500.2	411.8	413.8	382.8	398.9

Table 5.1 Average travel distance as determined by simulation.

formula			number of blocks				
aisles	length	items	1	2	3	4	5
7	10	10	99.3	85.8	92.4	90.9	95.3
7	10	30	123.8	127.4	150.8	150.2	168.2
15	10	10	159.8	130.9	150.6	147.0	155.9
15	10	30	235.6	215.4	257.7	247.6	287.0
7	30	10	212.3	163.8	154.5	144.6	144.0
7	30	30	274.0	257.8	264.3	250.3	257.9
15	30	10	310.8	222.1	219.6	205.1	207.7
15	30	30	501.6	411.8	412.1	375.7	397.2

Table 5.2 Average travel distance as determined by equation 5.9.

percentage difference			number of blocks				
aisles	length	items	1	2	3	4	5
7	10	10	-0.66	1.53	-1.21	2.72	1.38
7	10	30	-1.67	-0.92	-0.19	1.61	-0.01
15	10	10	-0.18	2.71	-2.02	3.39	0.87
15	10	30	-0.35	0.19	0.80	2.84	0.67
7	30	10	-0.55	0.69	-0.72	1.68	0.85
7	30	30	-1.17	-0.75	-0.34	0.81	-0.14
15	30	10	-0.15	1.60	-1.38	2.41	0.62
15	30	30	-0.29	0.01	0.41	1.85	0.43

Table 5.3 Percentage difference between the average travel distance determined by simulation and the average distance determined with equation 5.9.

5.5 An example of layout optimization

In Section 5.2 we have given a model to determine the layout for the order picking area. The objective function for this model is derived in Section 5.3. Here we will illustrate the model with an example. We return to the case study of Section 2.5. A picking zone in this warehouse consists of 11 pick aisles and two blocks. Each side of a subaisle offers space for 42 sections with 8 bins each. Thus a picking zone offers space to 1848 sections of 8 bins, ignoring any space required for roof support and staircases. A subaisle is 18.57 meters long, implying that a bin section requires 0.442 meters. That is, 0.40 meters for the bins and 4.2 centimeters for the racks and some clearance. The center-to-center distance between aisles is 2.115 meters and the cross aisle width is 2.6 meters. We will search a layout for 'preferred zone 1' and assume that products are uniformly distributed over the area and that the pick lists contain 17 items.

Since we intend to design a picking area for bins, aisle length must be a multiple of 0.442 meters. Otherwise, we would have some dead space at the end of the aisle, where no bins fit in. Furthermore, we require that the design must be such that the total number of bin sections in the area is at least 1848. The actual number of sections depends on the number of pick aisles.

This gives us the following model:

$$
\begin{aligned}
&\min T_m(n,k,y)\\
&\text{s.t.}\\
&y = 0.442 \cdot \left\lceil \tfrac{924}{n} \right\rceil\\
&n \geq 1\\
&k \geq 1\\
&n, k \text{ integer}
\end{aligned}
$$

with parameters:

$$
\begin{aligned}
m &= 17\\
d &= 1\\
x_c &= 2.115\\
y_c &= 2.6
\end{aligned}
$$

In this model we have replaced the equation $y \cdot n = S$ of the original model by $y = 0.442 \cdot \left\lceil \frac{924}{n} \right\rceil$. This is done because there must be at least 1848 bin sections. Bins can be on both sides of the aisles, so if we created a warehouse of one aisle, then this aisle would be $0.442 \cdot 924$ meters long. The bin sections are divided equally over the n pick aisles. Dividing 924 by n may result in a fractional number, therefore the result is rounded upto the nearest integer larger than $924/n$. To obtain the length of an aisle we multiply the number of sections per pick aisle by 0.442 meters, which gives $0.442 \cdot \lceil \frac{924}{n} \rceil$. The number of picks per route is 17, the depot is assumed to be located at the head of the left-most pick aisle, center-to-center distance between aisles is 2.115 meters and cross aisle width is 2.6 meters.

For the optimization, we have evaluated average travel distance for all situations with $1, 2, 3, ..., 10$ blocks and $1, 2, 3, ..., 30$ pick aisles. From these 300 different layouts, the situation with 8 blocks and 10 pick aisles had the lowest average travel distance at 247.4 meters. The original situation of 11 pick aisles and 2 blocks gave an average travel distance of 307.6. Thus, another layout may reduce average travel distance by almost 20%.

Furthermore, there is a huge variety of layouts that give average travel distances that are within 5% of the optimal value. In total, we found that 58 layouts of the 300 layouts we evaluated were within 5% of the optimal value. All of these alternative layouts are listed in Appendix D. As we already noted in Chapter 4, such a situation can be very useful for designing a picking area in practice. In this way, other objectives can be included without having to sacrifice too much in average travel distance. For instance, in this example a layout with fewer blocks may be desirable. In Figure 5.4, we have depicted all layouts that give an average travel distance that is within 5% of the optimal value. As can be seen, the area has a clear structure. In many cases adding or deleting one or two pick aisles or cross aisles will not alter average travel distance too much. Note that there are no layouts present in the figure with one or three blocks. This is due to the fact that these layouts are inefficient with respect to the amount of cross aisles travel.

5.6 Concluding remarks

In this chapter we have presented a model that can be used to determine a layout for the order picking area such that the average travel distance is minimized. In Chapter 4 a similar model was given, but that model restricted the number of blocks to one. The model of this chapter has no restrictions on the layout variables. The objective function of the model is based on statistical properties of the routes generated by the S-shape heuristic. The accuracy of the statistical estimates was shown by comparing calculated average travel distances with travel distances determined by simulation. For this the simulation programs described in Chapter 2 were used. It appeared that

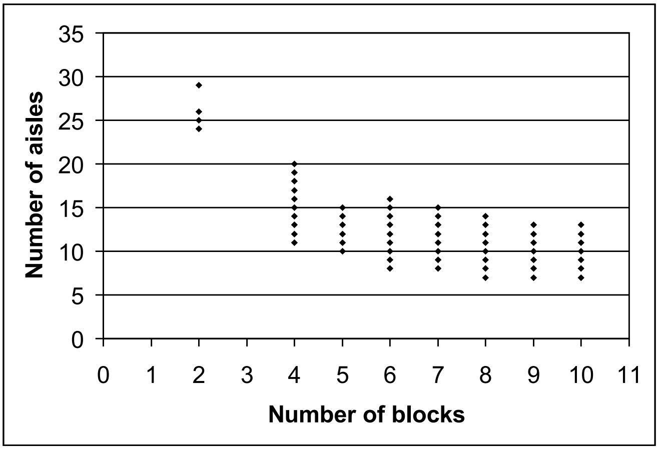

FIGURE 5.4. All layouts with an average travel distance within 5% of the optimal value.

the estimate for average travel distance was at most 3.4% off the simulated values. In more than half the instances the difference was less than 1%. To illustrate the application of the model we applied it to a case study. The original situation was a warehouse with 2 blocks and 11 pick aisles, giving an estimated average travel distance of 307.6 meters. The best layout we found had 8 blocks and 10 pick aisles resulting in an average travel distance almost 20% lower than in the original situation.

6
Conclusions and further research

Warehouses form an important part of the supply chain. Warehouses are needed for a number of reasons such as to facilitate the coordination between production and customer demand, by buffering products for a certain period of time, to accumulate and consolidate products from various producers for combined shipment to common customers, to provide same-day delivery to important customers, and to perform product customization activities, such as packaging, labeling, marking and pricing. A more elaborate description of the functions of warehouses and several trends in warehousing and logistics are discussed in Chapter 1.

This thesis describes several new concepts to improve the efficiency of order picking in warehouses. *Order picking* concerns the retrieval of products from storage to meet the demand of customers. This order picking process is often one of the most laborious and time-consuming activities in a warehouse. The efficiency of the order picking process depends on factors such as the methods for storage and transport, on the layout of the storage area and on the control mechanisms. In Chapter 1 of this thesis we give an overview of various types of warehouse equipment, storage systems and methods for warehouse sizing, system selection and warehouse layout. Furthermore, numerous control policies, i.e. rules that determine many warehouse operations, are described in detail in Chapter 1.

In *picker-to-part systems*, the order picker walks or drives through the warehouse to one or more locations to retrieve products. Picker-to-part systems are the most frequently used systems for retrieving items from storage in warehouses. Usually the most time-consuming task for the order pickers in these systems is traveling from location to location. This thesis gives new methods to reduce the travel time of order pickers in picker-to-part systems. These methods include routing methods and methods to determine a layout for the order picking area.

6.1 Routing methods for order pickers

The changes required in logistics due to the emergence of e-commerce force warehouses, among other things, to improve the speed of their processes. Routing methods can play a role in the efforts to improve warehousing speed. Routing of order pickers, as treated in Chapters 2 and 3, concerns the determination of a sequence in which a certain number of products have to be retrieved from storage. The common objective in routing problems is to develop a method that is capable of generating routes that are as short as possible. Routes of order pickers are restricted to the aisles and cross aisles of the picking area. Products are stored on both sides of the aisles. The order pickers can change aisles in one of the cross aisles, which are perpendicular to the aisles. A *block* is that part of the picking area that is between two cross aisles. A graphical representation is given in Figure 1.1.

In Chapter 2 we discuss some existing routing policies for warehouses consisting of one block, i.e. a layout with two cross aisles, one in the front and one in the back. Differences between routing heuristics for one-block layouts are discussed based on a literature study. Furthermore, we present several heuristics for layouts with multiple blocks. One method comes from existing literature; two other heuristics are newly developed extensions of heuristics for the one-block layout. New heuristics for a layout with multiple blocks, the combined and combined$^+$ heuristics, are presented as well. Performance comparisons between heuristics and an algorithm, that can generate shortest order picking routes, are given for various warehouse layouts and order sizes. For the majority of the instances with two or more blocks, the our combined$^+$ heuristic appeared to perform better than any of the other heuristics.

It might also be desirable to have an algorithm that can efficiently calculate shortest order picking routes. Such an algorithm exists for warehouses consisting of one block and with a central depot. This algorithm is, however, not capable of determining routes in warehouses with more than one block. In Chapter 3 we give a new algorithm that is capable of determining shortest order picking routes in warehouses consisting of two blocks with a central depot. If desired, the algorithm can also be used for warehouses of one block. To show the relevance of such an algorithm for practice, we have given a comprehensive test of the implications for average travel time when changing the number of blocks from one to two. The average travel times for situations with one and two blocks were compared for pick lists varying in size from 1 to 50 items. It appeared that average travel time is often lower in warehouses with two blocks. However, this is dependent on the size of the pick list. Large pick lists in small warehouses are best picked in a one-block layout. Large warehouses profit of a second block regardless of pick list size.

All experiments in Chapters 2 and 3 and the case study in Section 2.5 show the importance of good methods for routing order pickers. Decreases in average travel time of up to 35% appeared to be possible by introducing another routing method. The combined$^+$ heuristic we developed in Chapter 2 of this thesis appeared to have superior performance for warehouses consisting of multiple blocks. The new optimal routing method for warehouses with two blocks we developed in Chapter 3, appeared to have good potential, since order picking routes are often shorter in two-block warehouses than in one-block situations. These improvements in travel time are especially important since order picking is one of the most time-consuming aspects of warehousing.

6.2 Layout of the order picking area

The time needed for driving to pick locations obviously depends on the layout of the picking area. In Chapters 2 and 3 we have discussed some consequences of layout of the order picking area for average travel time. From the results it appeared, among other things, that the addition of cross aisles (i.e. replacing one block by multiple smaller blocks) can decrease handling time of the orders by lowering average travel time. However, adding a large number of cross aisles may increase average travel times because the space occupied by the cross aisles has to be traversed as well.

In Chapters 4 and 5 we have developed a model to determine the layout that minimizes average travel distances. The model in Chapter 4 has the restriction that the layout consists of one block. This means that the variables that are to be determined, are the number of aisles, the length of the aisles and the depot location. The inputs for the model are properties of the system and equipment, the distribution of the number of picks per route and the total storage capacity. In Chapter 5 a similar methodology is used to find layouts where the number of blocks is not restricted.

The models of Chapters 4 and 5 are very simple, except for the objective function. The objective function is a statistical estimate of average travel distance based on the properties of a commonly used routing method, the S-shape heuristic. The objective functions for the models are derived in detail. To check the accuracy of the estimates for average travel distance, we compared the calculated values with results from simulation. The difference between calculated and simulated values was for all instances at most a few percent.

Existing literature does include travel distance estimates for one-block situations. However, none of the existing estimates included the depot location explicitly. Therefore, optimization of the depot location was not possible. Furthermore, the estimates of travel distance within the pick aisles, was not entirely accurate in existing literature. If the layout is restricted to one block, we have proven in Chapter 4 that a depot located in the middle of the front cross aisle gives better results with respect to average travel distances than any other location in the front cross aisle, regardless of any of the other layout variables. From the experiments it appeared that the optimal number of aisles in an order picking area depends strongly on both required storage space and pick list size.

To overcome the limitations of the model of Chapter 4 we have developed the model of Chapter 5, which has no limitations with regards to the number of blocks. This is useful since we have shown in Chapters 2 and 3 that in many cases warehouses consisting of more than one block have lower average travel distances than warehouses consisting of only one block, given a certain storage capacity. Existing literature only gives a travel distance estimate for a two-block layout with rather restrictive assumptions.

The applicability of the model of Chapter 5 was shown by means of a case study. It turned out that 58 of 300 layouts evaluated had average travel distances less than 5% over the optimum. This implies that the designer of a warehouse has ample opportunity to include other objectives, like prevention of congestion or flexibility for future redesign, without having to sacrifice too much in travel distance.

6.3 Further research

Answering one question often raises several new questions. This has been the case with the research presented in this thesis as well. Unsolved questions with regards to routing of order pickers include: can we find an even better heuristic to route order pickers in warehouses with multiple blocks? Can we develop a heuristic that can be guaranteed to give solutions that are at most x% over the optimal solution? Can we develop an optimal algorithm for more than two blocks? Other interesting questions with regards to routing are those that involve the interaction with other operating polices such as storage assignment and batching. Some research has been performed with regards to good combinations of routing and storage assignment. The performance of batching heuristics also depends strongly on the routing method that is used. Therefore, some interesting questions to answer are: what are good combinations of routing heuristics and storage assignment rules? For a given multi-block layout and routing method, how do we have to store products according to their demand rate to minimize average travel distance? How do batching methods perform in a multi-block environment?

In the area of warehouse layout a number of questions also remain unanswered. We have shown that adding cross aisles can be very beneficial. However, we have only analyzed layouts with the restriction that products are assigned randomly to storage locations. What is a good layout if class-based storage is used? What are the interactions between layout and storage assignment? In our analysis we have also limited the positions of the cross aisles such that the distance between any two neighboring cross aisles is the same. However, in some cases it might be better to place the cross aisles at other positions. This was argued in Chapter 3 for the position of the middle aisle. It is likely that the positioning of cross aisles becomes an even bigger issue if a class-based storage assignment rule is used. For example, it might be a good idea to have a cross aisle directly behind the A-items, the most frequently requested products. Another question that arises: is it optimal with regards to travel distance to have a rectangular layout? Or is there a better shape? For example, with class-based storage it might be an idea to store all A-items in two long aisles and to store the other items in many short aisles.

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Appendix A

Theorems of Chapter 3

Theorem 1 *Ratliff and Rosenthal [227]*

A subgraph $T \subset G$ is a tour subgraph if and only if

(a) all vertices v_i for $i = 0, 1, 2, ..., m$ have positive degree in T,

(b) excluding vertices with zero degree, T is connected,

(c) every vertex in T has even or zero degree.

Corollary 1.1 *Ratliff and Rosenthal [227]*

A minimum length tour subgraph contains no more than two edges between any pair of vertices.

Corollary 1.2 *Ratliff and Rosenthal [227]*

If (P_1, P_2) is any node partition of a tour subgraph, there is an even number of edges with one end in P_1 and the other end in P_2.

Theorem 2

Necessary and sufficient conditions for $T_j \subset L_j$ to be an L_j partial tour subgraph are

(a) for all $v_i \in L_j$, the degree of v_i is positive in T_j,

(b) every vertex, except possibly a_j, b_j and c_j, has even degree or zero degree,

(c) excluding vertices with zero degree, T_j has either

- no connected component,
- a single connected component containing at least one of a_j, b_j and c_j,
- two connected components with in each component at least one of a_j, b_j and c_j, and each of a_j, b_j and c_j contained in at most one component,
- three connected components with a_j, b_j and c_j each in a different component.

Proof

The proof is similar to that of Theorem 2 of Ratliff and Rosenthal [227].

Theorem 3

Two L_j partial tour subgraphs are equivalent if

(a) a_j, b_j and c_j each have the same degree parity (i.e. even, odd or zero) in both partial tour subgraphs.

(b) excluding vertices with zero degree, both partial tour subgraphs have either

- no connected component,
- a single connected component containing at least one of a_j, b_j and c_j,
- two connected components with in each component at least one of a_j, b_j and c_j, and each of a_j, b_j and c_j contained in at most one component,
- three connected components with a_j, b_j and c_j each in a different component.

(c) the distribution of a_j, b_j and c_j over the various components is the same for both partial tour subgraphs.

Proof

The theorem and therefore the proof is largely similar to Theorem 2 of Ratliff and Rosenthal [227], only condition (c) is added.

Note: For the layout considered in Ratliff and Rosenthal [227], condition (c) is always satisfied. However, with a middle aisle this is not the case for 2 components with each of a_j, b_j and c_j having even degree parity. In all other cases, it can be proven that (c) is satisfied if (a) and (b) are satisfied.

Appendix B

Tables of Chapter 3

Table 1 gives the possible transitions from L_j^- equivalence classes to L_j^{+y} equivalence classes. Table 2 for L_j^{+y} to L_j^{+x} and Table 3 for L_j^{+x} to L_{j+1}^-. Denote an entry in row i, column j by (i, j). An entry (i, j) gives the equivalence class resulting from adding the edge configuration of column j (as depicted in either Figure 3.2 or 3.3) to the equivalence class of row i. Edge configurations from Figure 3.2 are used for Tables 1 and 2; edge configurations from Figure 3.3 are used for Table 3.

Some of the transitions are not possible or will never lead to the optimal solution. We can distinguish 2 reasons why a transition between two equivalence classes is not possible. (1) The transition would give a configuration violating condition (b) of Theorem 2 in Appendix A. (2) The transition would give a configuration violating condition (c) of Theorem 2 in Appendix A.

For example, consider Table 3. Adding edge configuration (2) from Figure 3.3 to equivalence class $(u, u, 0, 1)$ may seem to result in equivalence class $(u, 0, u, 1)$. However, this would violate condition (b) of Theorem 2 in Appendix A. The theorem implies that after the transition a_j, b_j and c_j should have even degree or zero degree and that a_{j+1}, b_{j+1} and c_{j+1} may have odd degree. Adding configuration (2) to class $(u, u, 0, 1)$ would leave b_j and c_j with odd degree and is therefore not permitted. Now in Table 3, consider adding arc configuration (1) from Figure 3.3 to class $(u, u, e, 2)$. Theorem 2(c) in Appendix A requires that after the transition each of the two components contains at least one of a_{j+1}, b_{j+1} and c_{j+1}. Since a_j and b_j

are in the same component, a_{j+1} and b_{j+1} will be in the same component as well. So the connection with the second component (containing c_j) will be lost with this transition. Therefore, it is not allowed.

Table 1

L_j^- to L_j^{+y}	(1)	(2)	(3)	(4)	(5)	(6) a)
(0,0,0,0) b)	(0,u,u,1)	(0,e,e,1)	(0,e,0,1)	(0,0,e,1)	(0,e,e,2)	(0,0,0,0)
(0,0,0,1) c)	e)	e)	e)	e)	e)	(0,0,0,1)
(e,0,0,1)	(e,u,u,2)	(e,e,e,2,a-bc)	(e,e,0,2)	(e,0,e,2)	(e,e,e,3)	(e,0,0,1)
(0,e,0,1)	(0,u,u,1)	(0,e,e,1)	(0,e,0,1)	(0,e,e,2)	(0,e,e,2)	(0,e,0,1)
(0,0,e,1)	(0,u,u,1)	(0,e,e,1)	(0,e,e,2)	(0,0,e,1)	(0,e,e,2)	(0,0,e,1)
(e,e,0,1)	(e,u,u,1)	(e,e,e,1)	(e,e,0,1)	(e,e,e,2,c-ab)	(e,e,e,2,c-ab)	(e,e,0,1)
(e,0,e,1)	(e,u,u,1)	(e,e,e,1)	(e,e,e,2,b-ac)	(e,0,e,1)	(e,e,e,2,b-ac)	(e,0,e,1)
(0,e,e,1)	(0,u,u,1)	d)	(0,e,e,1)	(0,e,e,1)	(0,e,e,1)	(0,e,e,1)
(e,e,e,1)	(e,u,u,1)	d)	(e,e,e,1)	(e,e,e,1)	(e,e,e,1)	(e,e,e,1)
(u,u,0,1)	(u,e,u,1)	(u,u,e,1)	(u,u,0,1)	(u,u,e,2)	(u,u,e,2)	(u,u,0,1)
(u,0,u,1)	(u,u,e,1)	(u,e,u,1)	(u,e,u,2)	(u,0,u,1)	(u,e,u,2)	(u,0,u,1)
(0,u,u,1)	(0,e,e,1)	d)	(0,u,u,1)	(0,u,u,1)	(0,u,u,1)	(0,u,u,1)
(e,u,u,1)	(e,e,e,1)	d)	(e,u,u,1)	(e,u,u,1)	(e,u,u,1)	(e,u,u,1)
(u,e,u,1)	(u,u,e,1)	d)	(u,e,u,1)	(u,e,u,1)	(u,e,u,1)	(u,e,u,1)
(u,u,e,1)	(u,e,u,1)	d)	(u,u,e,1)	(u,u,e,1)	(u,u,e,1)	(u,u,e,1)
(e,e,0,2)	(e,u,u,2)	(e,e,e,2,a-bc)	(e,e,0,2)	(e,e,e,3)	(e,e,e,3)	(e,e,0,2)
(e,0,e,2)	(e,u,u,2)	(e,e,e,2,a-bc)	(e,e,e,3)	(e,0,e,2)	(e,e,e,3)	(e,0,e,2)
(0,e,e,2)	(0,u,u,1)	(0,e,e,1)	(0,e,e,2)	(0,e,e,2)	(0,e,e,2)	(0,e,e,2)
(e,e,e,2,a-bc)	(e,u,u,2)	d)	(e,e,e,2,a-bc)	(e,e,e,2,a-bc)	(e,e,e,2,a-bc)	(e,e,e,2,a-bc)
(e,e,e,2,b-ac)	(e,u,u,1)	(e,e,e,1)	(e,e,e,2,b-ac)	(e,e,e,2,b-ac)	(e,e,e,2,b-ac)	(e,e,e,2,b-ac)
(e,e,e,2,c-ab)	(e,u,u,1)	(e,e,e,1)	(e,e,e,2,c-ab)	(e,e,e,2,c-ab)	(e,e,e,2,c-ab)	(e,e,e,2,c-ab)
(e,u,u,2)	(e,e,e,2,a-bc)	d)	(e,u,u,2)	(e,u,u,2)	(e,u,u,2)	(e,u,u,2)
(u,e,u,2)	(u,u,e,1)	(u,e,u,1)	(u,e,u,2)	(u,e,u,2)	(u,e,u,2)	(u,e,u,2)
(u,u,e,2)	(u,e,u,1)	(u,u,e,1)	(u,u,e,2)	(u,u,e,2)	(u,u,e,2)	(u,u,e,2)
(e,e,e,3)	(e,u,u,2)	(e,e,e,2,a-bc)	(e,e,e,3)	(e,e,e,3)	(e,e,e,3)	(e,e,e,3)

a) This transition is only allowed if there are no items in this part of the aisle

b) This class can occur only if there are no items to be picked in L_j^-

c) This class can only occur if there are no items to be picked in $G - L_j^-$

d) This transition will never lead to an optimal solution

e) This transition would violate condition (c) of Theorem 2 in Appendix A

Table 2

L_j^{+y} to L_j^{+x}	(1)	(2)	(3)	(4)	(5)	(6) [a]
(0,0,0,0) [b]	(u,u,0,1)	(e,e,0,1)	(e,0,0,1)	(0,e,0,1)	(e,e,0,2)	(0,0,0,0)
(0,0,0,1) [c]	e)	e)	e)	e)	e)	(0,0,0,1)
(e,0,0,1)	(u,u,0,1)	(e,e,0,1)	(e,0,0,1)	(e,e,0,2)	(e,e,0,2)	(e,0,0,1)
(0,e,0,1)	(u,u,0,1)	(e,e,0,1)	(e,e,0,2)	(0,e,0,1)	(e,e,0,2)	(0,e,0,1)
(0,0,e,1)	(u,u,e,2)	(e,e,e,2,c-ab)	(e,0,e,2)	(0,e,e,2)	(e,e,e,3)	(0,0,e,1)
(e,e,0,1)	(u,u,0,1)	d)	(e,e,0,1)	(e,e,0,1)	(e,e,0,1)	(e,e,0,1)
(e,0,e,1)	(u,u,e,1)	(e,e,e,1)	(e,0,e,1)	(e,e,e,2,b-ac)	(e,e,e,2,b-ac)	(e,0,e,1)
(0,e,e,1)	(u,u,e,1)	(e,e,e,1)	(e,e,e,2,a-bc)	(0,e,e,1)	(e,e,e,2,a-bc)	(0,e,e,1)
(e,e,e,1)	(u,u,e,1)	d)	(e,e,e,1)	(e,e,e,1)	(e,e,e,1)	(e,e,e,1)
(u,u,0,1)	(e,e,0,1)	d)	(u,u,0,1)	(u,u,0,1)	(u,u,0,1)	(u,u,0,1)
(u,0,u,1)	(e,u,u,1)	(u,e,u,1)	(u,0,u,1)	(u,e,u,2)	(u,e,u,2)	(u,0,u,1)
(0,u,u,1)	(u,e,u,1)	(e,u,u,1)	(e,u,u,2)	(0,u,u,1)	(e,u,u,2)	(0,u,u,1)
(e,u,u,1)	(u,e,u,1)	d)	(e,u,u,1)	(e,u,u,1)	(e,u,u,1)	(e,u,u,1)
(u,e,u,1)	(e,u,u,1)	d)	(u,e,u,1)	(u,e,u,1)	(u,e,u,1)	(u,e,u,1)
(u,u,e,1)	(e,e,e,1)	d)	(u,u,e,1)	(u,u,e,1)	(u,u,e,1)	(u,u,e,1)
(e,e,0,2)	(u,u,0,1)	(e,e,0,1)	(e,e,0,2)	(e,e,0,2)	(e,e,0,2)	(e,e,0,2)
(e,0,e,2)	(u,u,e,2)	(e,e,e,2,c-ab)	(e,0,e,2)	(e,e,e,3)	(e,e,e,3)	(e,0,e,2)
(0,e,e,2)	(u,u,e,2)	(e,e,e,2,c-ab)	(e,e,e,3)	(0,e,e,2)	(e,e,e,3)	(0,e,e,2)
(e,e,e,2,a-bc)	(u,u,e,1)	(e,e,e,1)	(e,e,e,2,a-bc)	(e,e,e,2,a-bc)	(e,e,e,2,a-bc)	(e,e,e,2,a-bc)
(e,e,e,2,b-ac)	(u,u,e,1)	(e,e,e,1)	(e,e,e,2,b-ac)	(e,e,e,2,b-ac)	(e,e,e,2,b-ac)	(e,e,e,2,b-ac)
(e,e,e,2,c-ab)	(u,u,e,2)	d)	(e,e,e,2,c-ab)	(e,e,e,2,c-ab)	(e,e,e,2,c-ab)	(e,e,e,2,c-ab)
(e,u,u,2)	(u,e,u,1)	(e,u,u,1)	(e,u,u,2)	(e,u,u,2)	(e,u,u,2)	(e,u,u,2)
(u,e,u,2)	(e,u,u,1)	(u,e,u,1)	(u,e,u,2)	(u,e,u,2)	(u,e,u,2)	(u,e,u,2)
(u,u,e,2)	(e,e,e,2,c-ab)	d)	(u,u,e,2)	(u,u,e,2)	(u,u,e,2)	(u,u,e,2)
(e,e,e,3)	(u,u,e,2)	(e,e,e,2,c-ab)	(e,e,e,3)	(e,e,e,3)	(e,e,e,3)	(e,e,e,3)

a) This transition is only allowed if there are no items in this part of the aisle

b) This class can occur only if there are no items to be picked in L_j^{+y}

c) This class can occur only if there are no items to be picked in $G - L_j^{+y}$

d) This transition will never lead to an optimal solution

e) This transition would violate condition (c) of Theorem 2 in Appendix A

Table 3

L_j^{+x} to L_{j+1}^{-}	(1)	(2)	(3)	(7)	(8)	(9)
(u,u,0,1)	(u,u,0,1)	a)	a)	a)	a)	c)
(u,0,u,1)	a)	(u,0,u,1)	a)	a)	c)	a)
(0,u,u,1)	a)	a)	(0,u,u,1)	c)	a)	a)
(e,u,u,1)	a)	a)	(0,u,u,1)	(e,u,u,1)	a)	a)
(u,e,u,1)	a)	(u,0,u,1)	a)	a)	(u,e,u,1)	a)
(u,u,e,1)	(u,u,0,1)	a)	a)	a)	a)	(u,u,e,1)
(e,u,u,2)	a)	a)	b)	(e,u,u,2)	a)	a)
(u,e,u,2)	a)	b)	a)	a)	(u,e,u,2)	a)
(u,u,e,2)	b)	a)	a)	a)	a)	(u,u,e,2)

L_j^{+x} to L_{j+1}^{-}	(10)	(11)	(12)	(13)	(14)
(0,0,0,0) $d)$	c)	c)	c)	c)	(0,0,0,0)
(0,0,0,1) $e)$	b)	b)	b)	b)	(0,0,0,1) $f)$
(e,0,0,1)	c)	c)	b)	c)	(0,0,0,1) $f)$
(0,e,0,1)	c)	b)	c)	c)	(0,0,0,1) $f)$
(0,0,e,1)	b)	c)	c)	c)	(0,0,0,1) $f)$
(e,e,0,1)	(e,e,0,1)	c)	c)	c)	(0,0,0,1) $f)$
(e,0,e,1)	c)	(e,0,e,1)	c)	c)	(0,0,0,1) $f)$
(0,e,e,1)	c)	c)	(0,e,e,1)	c)	(0,0,0,1) $f)$
(e,e,e,1)	(e,e,0,1)	(e,0,e,1)	(0,e,e,1)	(e,e,e,1)	(0,0,0,1) $f)$
(e,e,0,2)	(e,e,0,2)	b)	b)	c)	b)
(e,0,e,2)	b)	(e,0,e,2)	b)	c)	b)
(0,e,e,2)	b)	b)	(0,e,e,2)	c)	b)
(e,e,e,2,a-bc)	(e,e,0,2)	(e,0,e,2)	b)	(e,e,e,2,a-bc)	b)
(e,e,e,2,b-ac)	(e,e,0,2)	b)	(0,e,e,2)	(e,e,e,2,b-ac)	b)
(e,e,e,2,c-ab)	b)	(e,0,e,2)	(0,e,e,2)	(e,e,e,2,c-ab)	b)
(e,e,e,3)	b)	b)	b)	(e,e,e,3)	b)

L_j^{+x} to L_{j+1}^-	(4)	(5)	(6)
(0,0,0,0) $^{d)}$	c)	c)	c)
(0,0,0,1) $^{e)}$	b)	b)	b)
(e,0,0,1)	(e,0,0,1)	b)	b)
(0,e,0,1)	b)	(0,e,0,1)	b)
(0,0,e,1)	b)	b)	(0,0,e,1)
(e,e,0,1)	(e,0,0,1)	(0,e,0,1)	b)
(e,0,e,1)	(e,0,0,1)	b)	(0,0,e,1)
(0,e,e,1)	b)	(0,e,0,1)	(0,0,e,1)
(e,e,e,1)	(e,0,0,1)	(0,e,0,1)	(0,0,e,1)

a) This transition would violate condition (b) of Theorem 2 in Appendix A

b) This transition would violate condition (c) of Theorem 2 in Appendix A

c) This transition will never lead to an optimal solution

d) This class can occur only if there are no items to be picked in L_j^{+x}

e) This class can occur only if there are no items to be picked in $G - L_j^{+x}$

f) This transition is allowed only if there are no items to be picked in $G - L_j^{+x}$

This table has been split into three parts for layout reasons. All combinations that are not in any part would violate condition (b) of Theorem 2 in Appendix A.

Appendix C

Proof of Chapter 4

In this appendix we will prove that the estimated distance between the left most aisle containing picks and the right most aisle containing picks is given by:

$$E[D_x^1] = w_a \cdot \left((n-1) - 2 \cdot \sum_{i=1}^{n-1} \left(\frac{i}{n} \right)^m \right)$$

The probability that all m items fall in subaisles $\ell, ..., r$, at least one pick falls in ℓ and at least one pick falls in r, is given by:

$$Q(\ell, r) = \begin{cases} \left(\frac{r-\ell+1}{n}\right)^m - 2\left(\frac{r-\ell}{n}\right)^m + \left(\frac{r-\ell-1}{n}\right)^m & \ell < r \\ \\ \left(\frac{1}{n}\right)^m & \ell = r \end{cases}$$

This can be explained as follows. Clearly the probability that all items fall within the range of aisles ℓ and r is given by $\left(\frac{r-\ell+1}{n}\right)^m$. However, this includes the probability that aisle ℓ and/or aisle r is empty. Therefore, we have to substract the probability that all items are in aisles $\ell+1$ through r and the probability that all items are in aisles ℓ through $r-1$. That is we have to substract 2 times $\left(\frac{r-\ell}{n}\right)^m$. However, now we have substracted the probability that all items are in aisles $\ell+1$ through $r-1$ twice; we should only have substracted it once. Therefore, we have to add $\left(\frac{r-\ell-1}{n}\right)^m$. This gives the desired probability.

Now, we can write the expected travel distance as:

$$E[D_x^1]/w_a = \sum_{\substack{\ell=1,r=1\\ \ell\leq r}}^{n} (r-\ell)\cdot Q(\ell,r)$$

Because $(r-\ell)\cdot Q(\ell,r)=0$ for $\ell = r$ we can rewrite as:

$$E[D_x^1]/w_a = \sum_{\substack{\ell=1,r=2\\ \ell< r}}^{n} (r-\ell)\cdot Q(\ell,r)$$

By definition of $Q(\ell,r)$:

$$E[D_x^1]/w_a = \sum_{\substack{\ell=1,r=2\\ \ell< r}}^{n} (r-\ell)\cdot \left[\left(\frac{r-\ell+1}{n}\right)^m - 2\left(\frac{r-\ell}{n}\right)^m + \left(\frac{r-\ell-1}{n}\right)^m\right]$$

Substitute $r-\ell$ by i and note that there are $n-(r-\ell)$ possibilities to have all items in $\ell - r$ aisles. This gives:

$$\begin{aligned}
&E[D_x^1]/w_a = \\
= \quad & \sum_{\substack{\ell=1,r=2\\ \ell< r}}^{n} (r-\ell)\cdot \left[\left(\frac{r-\ell+1}{n}\right)^m - 2\left(\frac{r-\ell}{n}\right)^m + \left(\frac{r-\ell-1}{n}\right)^m\right] \\
= \quad & \sum_{r-\ell=1}^{n-1} (n-r+\ell)\cdot(r-\ell)\cdot \left[\left(\frac{r-\ell+1}{n}\right)^m - 2\left(\frac{r-\ell}{n}\right)^m + \left(\frac{r-\ell-1}{n}\right)^m\right] \\
= \quad & \sum_{i=1}^{n-1} i\cdot(n-i)\cdot\left[\left(\frac{i+1}{n}\right)^m - 2\left(\frac{i}{n}\right)^m + \left(\frac{i-1}{n}\right)^m\right]
\end{aligned}$$

We split the summation and delete $i=1$ from the last sum. The latter operation is possible since the term equals zero for $i=1$. This gives:

$$\begin{aligned}
E[D_x^1]/w_a \quad = \quad & \sum_{i=1}^{n-1} i(n-i)\left(\frac{i+1}{n}\right)^m - 2\sum_{i=1}^{n-1} i(n-i)\left(\frac{i}{n}\right)^m \\
& + \sum_{i=2}^{n-1} i(n-i)\left(\frac{i-1}{n}\right)^m
\end{aligned}$$

Rewriting summations gives:

$$\begin{aligned} E[D_x^1]/w_a &= \sum_{i=2}^{n}(i-1)(n-i+1)\left(\frac{i}{n}\right)^m - 2\sum_{i=1}^{n-1} i(n-i)\left(\frac{i}{n}\right)^m \\ &\quad + \sum_{i=1}^{n-2}(i+1)(n-i-1)\left(\frac{i}{n}\right)^m \end{aligned}$$

Taking out one term of the first summation and expanding the first summation with $i = 1$ and the last summation with $i = n - 1$ (that is we add zero twice) gives:

$$\begin{aligned} E[D_x^1]/w_a &= (n-1) + \sum_{i=1}^{n-1}(i-1)(n-i+1)\left(\frac{i}{n}\right)^m - 2\sum_{i=1}^{n-1} i(n-i)\left(\frac{i}{n}\right)^m \\ &\quad + \sum_{i=1}^{n-1}(i+1)(n-i-1)\left(\frac{i}{n}\right)^m \end{aligned}$$

Which equals:

$$\begin{aligned} E[D_x^1]/w_a &= (n-1) + \\ &\quad \sum_{i=1}^{n-1}\left[(i-1)(n-i+1) - 2i(n-i) + (i+1)(n-i-1)\right]\left(\frac{i}{n}\right)^m \\ &= (n-1) + \sum_{i=1}^{n-1}(-2)\left(\frac{i}{n}\right)^m \end{aligned}$$

■

Appendix D

Table of Chapter 5

This table gives all layouts with an average travel distance that is within 5% of the optimal value. The best layout is one with 8 blocks and 10 aisles.

blocks	aisles	distance (m.)	blocks	aisles	distance (m.)
8	10	247.3868	6	15	252.6510
6	12	247.8374	8	13	253.2190
6	11	247.9171	4	18	253.3962
8	9	248.1740	4	12	254.4820
9	10	248.2411	7	8	254.6183
9	9	248.3366	4	19	254.8636
8	11	248.4643	10	12	255.2884
7	10	248.6737	7	14	255.6708
6	13	248.7688	6	16	255.7390
6	10	248.8478	9	13	255.9767
7	11	249.0420	5	12	256.6335
10	9	249.5166	5	13	256.8715
4	15	249.7890	2	25	256.9965
4	16	249.9526	8	14	257.0850
9	11	249.9665	10	7	257.2897
10	10	249.9774	9	7	257.3441
7	9	250.2260	5	11	257.4548
7	12	250.2497	6	8	257.6392
8	12	250.3718	4	11	257.9129
6	14	250.8278	4	20	258.1298
9	8	251.1386	5	14	258.3178
4	14	251.1826	2	26	258.5068
10	8	251.7167	2	24	258.6780
8	8	251.7262	7	15	258.7202
4	17	251.7686	8	7	258.7217
6	9	251.7737	2	29	259.0924
4	13	252.1719	5	10	259.1316
10	11	252.2252	10	13	259.2484
7	13	252.4277	5	15	259.4628
9	12	252.5139			

Samenvatting

Tegenwoordig kunnen klanten vele producten bestellen met behulp van een computer of mobiele telefoon via het internet. Een snel verloop van het bestelproces schept echter ook verwachtingen ten aanzien van de aflevering. Bestaande organisaties proberen zich aan deze nieuwe situatie aan te passen en nieuwe organisaties ontstaan. De distributieketen omvat alle activiteiten die te maken hebben met de stroom en transformatie van goederen van het stadium van grondstoffen tot aan de klant, evenals de bijkomende informatiestromen. Magazijnen vormen een belangrijke schakel in de distributieketens. Door producten enige tijd in magazijnen op te slaan, kan de coördinatie tussen productie en vraag worden vereenvoudigd. Daarnaast kunnen in magazijnen producten van verschillende producenten worden gecombineerd voor verzending naar gezamenlijke klanten, kan worden gezorgd voor levering aan belangrijke klanten op dezelfde dag als waarop wordt besteld en kunnen producten klantspecifiek worden gemaakt door ze bijvoorbeeld te voorzien van verpakkingen, etiketten en prijzen.

In veel magazijnen vervult het orderverzamelen een centrale rol. *Orderverzamelen* is het proces waarbij producten uit de opslag worden gehaald op basis van orders van klanten. Meestal is deze taak niet geautomatiseerd wegens het feit, dat producten van allerlei afmetingen uit het rek moeten worden gepakt en op een ladingdrager, zoals een krat of pallet, moeten worden gestapeld. Het orderverzamelen is door de vele handelingen een zeer arbeidsintensief en tijdrovend proces.

In dit proefschrift is een aantal methoden beschreven om de efficiëntie van het orderverzamelen te verhogen, teneinde tegemoet te komen aan de toenemende druk om alle schakels van de distributieketen te versnellen. De efficiëntie van het orderverzamelen hangt onder meer af van factoren zoals de gebruikte opslagsystemen en transportmiddelen, de lay-out en de besturingsregels. Hoofdstuk 1 beschrijft verschillende systemen voor het opslaan van goederen en verschillende soorten transportmaterieel om de goederen te verplaatsen. Verder wordt beschreven welke methoden bestaan om de grootte van het magazijn te bepalen, om de juiste systemen te selecteren en om de lay-out te bepalen. Ook wordt in detail ingegaan op verschillende besturingsregels. Deze besturingsregels bepalen voor een aanzienlijk deel de gang van zaken, bijvoorbeeld door beslissingen te nemen over het volgende: wanneer wordt welke order verzameld, welke orderverzamelaar voert welke order uit, wat is een goede looproute voor de orderverzamelaar en wanneer moet een voorraadlocatie worden bijgevuld.

Voor het orderverzamelen zijn twee systemen te onderscheiden, namelijk systemen waarbij de orderverzamelaar naar de goederen toe gaat en systemen waarbij de goederen naar de orderzamelaar komen. Dit proefschrift gaat over de systemen waarbij de orderverzamelaar naar de goederen toe gaat. Veelal liggen de producten dan in rekken. De rekken zijn opgesteld in rijen, waartussen de orderverzamelaars zich kunnen voortbewegen. Het centrale doel van de hier beschreven methoden is het reduceren van de tijd die de orderverzamelaars besteden aan het zich verplaatsen van locatie naar locatie, dus van de gemiddelde reistijd per order. We bewandelen hiertoe twee wegen, namelijk het bepalen van goede routeringsmethoden en het bepalen van goede lay-outs.

Routeringsmethoden bepalen de route, die een orderverzamelaar door het magazijn moet volgen, zodat elke locatie waar een product uit de order opgeslagen ligt, wordt bezocht. Van alle handelingen die een orderverzamelaar uitvoert, is dit in het algemeen de tijdrovendste. Het ligt daarom voor de hand om te proberen hier tijdsbesparingen te realiseren. Bovendien zijn veel van de andere handelingen niet eenvoudig te veranderen of te versnellen. Bepaalde handelingen, zoals het pakken en neerzetten van producten, moeten altijd worden uitgevoerd en de snelheid hiervan hangt vooral af van de handigheid van de orderverzamelaar. Doelstelling voor de routeringsmethoden is primair om zo kort mogelijke routes te vinden. Verder kan het verstandig zijn om rekening te houden met andere aspecten, zoals begrijpelijkheid van de route voor de orderverzamelaar of congestie indien veel orderverzamelaars zich in hetzelfde gebied bevinden.

Het *bepalen van een lay-out* voor een orderverzamelgebied betreft onder andere het bepalen van het aantal gangen, het aantal blokken en de lengte van de gangen.

De *gangen* worden gevormd door de ruimte tussen twee rekken. *Dwarsgangen* staan loodrecht op de gangen en dienen alleen voor het wisselen van gang. Orderverzamelaars bewegen zich door de gangen en dwarsgangen naar de locaties, waar de gezochte producten liggen opgeslagen. Dat deel van het verzamelgebied dat tussen twee dwarsgangen in ligt, wordt aangeduid met de term *blok*. Een voorbeeld van een lay-out van een orderverzamelgebied is weergegeven in Figuur 1.1.

Het vinden van routes voor orderverzamelaars is feitelijk een speciaal geval van het bekende handelsreizigersprobleem. Het bijzondere is echter dat het – in tegenstelling tot het algemene handelsreizigersprobleem – voor magazijnen bestaande uit een blok mogelijk is om op efficiënte wijze kortste routes te bepalen. Voor magazijnen bestaande uit twee blokken is in Hoofdstuk 3 van dit proefschrift een nieuw efficiënt algoritme beschreven. Voor situaties met meer dan twee blokken worden in Hoofdstuk 2 enkele nieuwe heuristieken gegeven.

Allereerst geeft Hoofdstuk 2 een overzicht van een aantal routeringsmethoden voor magazijnen bestaande uit een blok. Verder wordt kort ingegaan op de verschillen tussen deze methoden. Vervolgens wordt een aantal heuristieken beschreven om routes te bepalen in magazijnen bestaande uit meerdere blokken. Een van deze heuristieken komt uit de bestaande literatuur en twee heuristieken zijn aanpassingen van heuristieken, die zijn ontwikkeld voor situaties met een blok. Verder wordt een nieuwe heuristiek gegeven die voor de overgrote meerderheid van de testinstanties beter presteert dan de andere heuristieken. Het praktische nut van een lay-out met meerdere blokken – in tegenstelling tot een lay-out met maar een blok – wordt zowel in Hoofdstuk 2 als Hoofdstuk 3 besproken. In zeer veel gevallen leidt het toevoegen van een extra dwarsgang tot lagere gemiddelde reistijden.

De andere manier, die in dit proefschrift aan bod komt, om de gemiddelde reistijd per order te reduceren, is het bepalen van een goede lay-out. Zoals gezegd bestaat het orderverzamelgebied uit een aantal gangen en dwarsgangen. De orderverzamelaars beginnen en eindigen hun routes vaak bij een depot. Bij het depot kan bijvoorbeeld een lege ladingdrager of een lijst met te verzamelen producten worden opgehaald. Volle ladingdragers kunnen hier weer worden afgezet. Het doel van Hoofdstukken 4 en 5 is om het aantal gangen, het aantal blokken, de lengte van de gangen en de positie van het depot zodanig te bepalen, dat de gemiddeld afgelegde afstand om een order te verzamelen wordt geminimaliseerd. Hoofdstuk 4 geeft een model dat kan worden gebruikt om de lay-out te bepalen onder de voorwaarde dat het aantal blokken beperkt is tot een. Het model van Hoofdstuk 5 kent deze beperking niet en kan dus worden gebruikt om lay-outs met meerdere blokken te vinden. Als parameters voor de modellen gelden de eigenschappen van de opslagsystemen en voertuigen, de kansverdeling voor het aantal verschillende producten per route en de totale opslagcapaciteit van het magazijn.

De modellen van Hoofdstukken 4 en 5 zijn simpel, met uitzondering van de doelstellingsfuncties. De doelstelingsfuncties zijn statistische schattingen van de gemiddeld afgelegde afstand, gebaseerd op een veel gebruikte routeringsmethode, de S-shape heuristiek, en op de aanname dat opslaglocaties voor producten willekeurig worden bepaald. Een groot gedeelte van de beide hoofdstukken is gewijd aan de afleidingen van deze schattingen. De precisie van de schattingen wordt aangetoond door een vergelijking met de resultaten van simulatie. Het verschil is ten hoogste een paar procent, maar veelal minder dan een procent.

In het geval dat het aantal blokken is gerestricteerd tot een, is in Hoofdstuk 4 bewezen dat het midden van de voorste dwarsgang een betere locatie is voor het depot dan enige andere locatie in de voorste dwarsgang, wanneer het minimaliseren van de gemiddeld afgelegde afstand de doelstelling is. Deze optimale locatie van het depot geldt voor alle in Hoofdstuk 2 genoemde heuristieken en is onafhankelijk van de andere lay-out variabelen. Het optimale aantal gangen blijkt sterk af te hangen van zowel de vereiste totale opslagruimte als van het aantal te verzamelen producten. De toepasbaarheid van het model uit Hoofdstuk 5 is gedemonstreerd aan de hand van een voorbeeld. Hierbij bleek dat de gemiddeld afgelegde afstand in 58 van de 300 geteste lay-outs minder dan 5% afweek van de kortste gemiddeld afgelegde afstand. Dit betekent dat een ontwerper nog ruim de mogelijkheid heeft om aan andere eisen, zoals voorkoming van congestie of behoud van flexibiliteit voor toekomstige veranderingen, te voldoen zonder dat de gemiddeld afgelegde afstand sterk stijgt.

Curriculum Vitae

Kees Jan Roodbergen was born in 1971 in Harfsen (district of Gorssel), The Netherlands. He attended the secondary school 'Rijksscholengemeenschap' in Lochem, The Netherlands, from 1983 to 1989. Next, he studied econometrics at the University of Groningen, where he graduated in 1996 on his Master's thesis 'The symmetric traveling salesman problem: removing the influence of constant matrices on heuristic performances'. In 1996 he started as a Ph.D. candidate at the Rotterdam School of Management / Faculteit Bedrijfskunde of the Erasmus University Rotterdam. For four years, he performed research on order picking in warehouses. As a result of a case study, one of his routing methods was implemented in real life. Kees Jan Roodbergen has given presentations, seminars and workshops on order picking in both Europe and North America. He has been a speaker at major warehousing trade meetings, such as Logistica and Promat. Several of his articles have appeared or have been accepted for publication in scientific and professional journals. In 1999 he has been a visiting scholar for three months at the Georgia Institute of Technology in Atlanta, United States of America. Since September 2000, Kees Jan Roodbergen is assistant professor of logistics and operations management at the Rotterdam School of Management / Faculteit Bedrijfskunde. He now teaches courses on simulation and supply chain management and continues his research on warehousing.

In this thesis, we have consistently used the words "he" and "his" when referring to a person working in a warehouse. The reader can freely substitute these words with "she" and "her". Statistically, this would probably be more correct anyway, since in many warehouses most employees are female.

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Layout and Routing Methods for Warehouses

discusses aspects of order picking in warehouses. Order picking is the process by which products are retrieved from storage to meet customer demand. Various new routing methods are introduced to determine efficient sequences in which products have to be retrieved from storage. Furthermore, a new method is given to determine a layout for the order picking area. The objective is to minimize the average distance traveled per route by the order pickers.

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