PORTFOLIO CONCENTRATION
PORTFOLIO CONCENTRATION
HET CONCENTRATIE NIVEAU VAN PORTEFEUILLES

Thesis

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by

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To Carmen and Maria Nour
for sharing with me the values of unity and perseverance.
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Ghassan A. Chammas
Beirut, 15 September 2016
Chapter 1

Introduction

I hold that anything that does not start with the basis that *techné* (know how) is superior to *epistémé* (know what), especially in complex systems, is highly suspicious.

*Nassim Nicholas Taleb*, in his foreword to *Lecturing Birds on Flying*, 2009

1.1 Investment Portfolio Selection Process

In the process of managing investment portfolios we can distinguish three stages:

A. Choosing the specialization of the portfolio. For instance, European corporate bonds, Asian chemical stocks, etc.

B. Choosing the allocation within the chosen sectors: Concentration

C. Monitoring the portfolio
Chapter 1. Introduction

The portfolio manager uses a great variety of information in every stage of the process, both as input for decision-making and for monitoring purposes.\(^1\)

Theoretically, the manager starts with all the available stocks in all the possible and available investment universes. This is to say that the manager starts with a very large number of stocks belonging to all possible jurisdictions and markets. Eventually, the manager will start narrowing down his choices to those markets and sectors where he believes he wants to put his “bet” or, where he wants to allocate his wealth. This process of narrowing down to focus on the desired stocks to be included in the portfolio ideally relies on the information available to the manager which will assist him in making his decision. Defined as “portfolio specialization”, this process is related to the stage A depicted in Figure 1.2.

Once the focus of the portfolio is defined, the manager will allocate the wealth available to each and every stock of the \(n\) stocks he chooses from the available \(N\) stocks of the universe. He will end up with a weights matrix \([w_i], i = 0 \ldots n,\) and where \(0 \leq [w_i] \leq 1\)

\(^1\)The past ten years have seen a new approach to investment and a more heuristic point of view towards the process that we are about to describe as a whole, as seen in Chapter 4 of this present thesis. Far from supporting the portfolio decisions on risk-return considerations, the new approach varies from risk parity asset allocation (Qian, 2005) to a totally heuristic and subjective approach, as suggested by Taleb in his book the Black Swan (Taleb, 2007) and later when he introduces the concept of Antifragility (Taleb, 2012). Around the same time, a rather deeper criticism of algorithms and mathematical and statistical approaches was presented by Triana (Triana, 2009).

\(^2\)In theory, this is what is normally expected. In practice, we see investors making a lot of unconsciously naive and illogical choices. We believe they should not: the process of information analysis remains essentially rational.
subject to $\sum_{i=1}^{n} w_i = 1$. This process is defined as “portfolio concentration” and is related to the stage B depicted above. (Please refer to Figure 1.2 for a pictorial representation of the specialization and concentration phases.)

The type of information needed depends on the goals the manager wants to achieve and constraints that need to be observed (risk, return, SRI, geographic preference, sector avoidance, etc...).

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**Figure 1.2:** Asset Allocation process. Please note the Specialization and the Concentration phases.

The final portfolio can be described ex-ante and ex-post. Ex-ante, the manager has historical data at his disposal showing the performance of each element of his portfolio like the observed returns and their volatility around their average. He also has non-estimative

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*SRI: Socially Responsible Investment*
information like the industrial sector and the geographic region of the elements of the portfolio, among many other “static” characteristics\(^4\).

Ex-post, the manager has the observed returns and hence their volatility or variance at his disposal. He can also have an estimation error descriptor comparing the historical forecasts with the ones realized and observed at a certain time during the horizon of the investment. He can also observe the changes in concentration levels because of weight changes due to price variation at the time of observation.

## Available Information

Different types of information are available:

a. Historical prices, company reports, company financial statements, specialized market reports, etc.

b. Current status of the investment object (geography, industry, asset class, ownership structure, etc.)

c. Trading volumes, market data, expert analysis, news, trends and indexes.

Obviously, the manager is interested in the future of the portfolio. At the time of inception of the portfolio, the manager will basically rely upon an expected return preference and the standard deviation of this return, based on historical observation of the performance of the stocks within his portfolio.

The standard deviation is used to measure the amount of risk involved in investing in such a stock or portfolio, along with the intrinsic risk inherent to the stock itself. Harry Markowitz (Markowitz, 1952), in a seminal paper that marks the beginning of modern portfolio theory, showed how to create a frontier of investment portfolios where each of them achieves the highest possible return given its level of risk. This complex computational method for the times was complemented by William Sharpe (Sharpe, 1963) with a simplified technique which is now referred to as the single-index model.

The performance expected and the performance realized depend on the horizon and on some evaluation points until the horizon of the investment. Therefore, a lot of attention has been given in literature to the estimation of future returns, and the deviation involved from a calculated average, to proxy the risk involved. There is a lot of literature that casts doubt on the quality of these estimates (Stein, 1955) and also (Michaud and Davis, 1982). The predictability of future value of return depends on statistical estimation which is prone

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\(^4\)Static characteristics are those characteristics which do not normally change with time. In fact, those characteristics might change if, for example, two companies merge and hence the industrial sector changes, or if a company is de-listed in a jurisdiction and listed in another or if a company passes from family ownership to non-family ownership during the life of the investment. But for all practical purposes, we will assume that those characteristics are static and remain the same during the investment life.
1.2 Research Questions and Objectives

In this thesis we will be using terms and concepts related to the specialization and concentration of investment portfolios. What follows is a list of definitions related to the terms used hereafter.

**Definition 1. Investment Universe:**
We define the Investment Universe as all the possible investment assets that an investor can invest his wealth in. This is an exhaustive list of all possible assets available. In this thesis, we will restrict our field of research to stocks listed on a stock exchange. Hence, the Investment Universe will include all available listed stocks in the world, at a certain moment in time.

**Definition 2. Investment Constraints:**
Sometimes referred to as Screening or Filters, an investment constraint is a filter or screen imposed on a set of assets that reduces the size of the set, obeying preferences and criteria of the investor.

**Definition 3. Investment Opportunity Set (IOS):**
The universe of choices as to investments available to an investor.
Chapter 1. Introduction

Definition 4. Portfolio Opportunity Set (POS):
We define a POS as the set of resulting investment opportunities resulting from subjecting an IOS to a set of constraints. When aiming at specific investment objectives and satisfying specific investment constraints, a universe of feasible portfolios can be identified. Such a universe is the portfolio opportunity set. A POS is the set of all possible portfolios made of a given number of individual stocks which were chosen from among all stocks of an IOS after subjecting it to constraints (or screening criteria).

When we look at a portfolio selection process, we have a number of comments. Investors often rely more on econometrics and statistical techniques and neglect some available information that can provide them with a better insight on their investments. In fact, referring to Figure 1.2 and considering the Stage A of the process we introduced earlier (namely the definition of the specialization of the portfolio), we realize that not much literature and theory\(^5\) are dedicated to this initial process of narrowing the available set of opportunities: The constraints imposed by the investors on the initial available investment universe reduces the available remaining stocks that will constitute the portfolio. This reduction in size of the available opportunities is the specialization process. Investors often make preliminary choices at this stage of the process which are not well documented and often are ad-hoc. We will introduce the concept of portfolio specialization and relate them to the investment portfolio process.

Although concentration measures, and similarly inequality measures, have been widespread in assessing economic welfare and income distribution in societies and countries, their use in investment portfolios is not very noticeable in the investment and banking realm. The investor usually imposes several criteria and screens to subject the available investment universe to his investment preferences. This screening criteria will reduce the available assets, making the portfolio specialized in some asset attributes. After the process of asset choice, the wealth allocation per asset will further introduce an additional concentration within the portfolio.

We propose to research the concentration and the specialization measurements in a portfolio of investment, as well as showing that the new measurements we introduce will improve the description of an investment portfolio, filling in a gap in the investment information process. Our dissertation evolves around introducing an additional family of\(^5\) (Fama and French, 1993) sorting the US stock market into 6 different portfolios based on the ratio of their equity book value to market value (BE/ME) and hence picking the stocks with higher ratio, is one example of stock picking techniques based on some information related to the stocks involved. Another stock picking theory involves high idiosyncratic volatility, discussed in the paper of Duan (Duan et al., 2009). This study finds that mutual fund managers have stock-picking ability for stocks with high idiosyncratic volatility, which is an information not related to the inherent attributes of the stocks themselves but rather an error prone measurement since it involves volatility(calculated from data series) as a criteria for choice of stock.

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1.2. Research Questions and Objectives

descriptors, namely the concentration and the specialization measures of an investment portfolio, answering the following research questions and leading to the research’s main objective.

**Research question 1:** The investment process includes various steps that the investors undergo before they take the final decision of wealth allocation. In all the steps of the investment process, the investors require certain information that is necessary for the final decision of wealth allocation. What are the various stages of an investment process? What is the type and the characteristics of the information that investors normally seek in each stage of the investment process? What valuable information is distilled from including the concentration and the specialization of the portfolio in the information required in the investment process?

**Research question 2:** The concepts of concentration and specialization are widely used in the economic studies of welfare and income inequality as well as international trade and industrial specialization and monopolies, among many other related topics. What is the relevant existing literature available for the researcher on concentration and specialization measures? What are the desired principles and characteristics that a concentration and a specialization measure, applied to an investment portfolio, will result in relevant and useful information for the investor? What measures of concentration and specialization do we choose that we believe they are adequate to be applied to an investment portfolio, from the standpoint of an investor?

**Research question 3:** How to measure the concentration and specialization levels of a portfolio using the indexes chosen in Research Question 2? What is the information that these chosen measures provide to the investors and, consequently, what is the toolbox that our proposed measures provide to the investors in the investment process?

**Research question 4:** How do we apply our methodology to an actual portfolio? What are the observed variations in concentration and specialization in an actual portfolio throughout the horizon of the investment? What conclusions and observations would an investor draw from using the concentration and specialization measures as descriptors to his investment?

**Research objective:** The main objective of this research is to introduce a toolbox using the specialization and concentration measures as an additional descriptor of the investment portfolio. The outputs of this toolbox (specialization and concentration of an investment portfolio) are an additional information provided to the investor that can be used in the investment process as well as to assess and dynamically adjust the portfolio in the monitoring process.
1.3 Contribution

The first contribution of this thesis is the use of a new measure, concentration, to describe a portfolio. We show that it makes sense to measure and monitor the concentration and the specialization levels of a portfolio.

The second contribution is essentially that the concentration measure (how much invested in each stock) and the specialization measure (how much invested in each sector or in each attribute) are additional descriptors that the investor will use to know exactly where his wealth is placed and hence be able to compare among his various investments and funds. The investor will learn, for example, that in portfolio No.1 he is more concentrated on pharmaceutics in Asia then portfolio No.2 which is more specialized in real estate business in Europe. More importantly, this description is estimation error free since it does not depend on time series regression or projection. It is rather an independent measure to describe (and monitor) the portfolio of investment. The investor can monitor and observe quickly the changes from one time to another. This additional information that this tool extends to the investor will eventually contribute to his portfolio management, but this particular issue is not within the scope of this thesis.

Thirdly, this thesis provides rules and procedures to measure the concentration and specialization levels hence describing the influence of the constraints on a portfolio.

1.4 The Road map

In Chapter 2 we will explore the process of describing investment opportunities and the variety of information required and used by portfolio managers in each and every stage of the investment process. We will focus on the choice process from specifying the goals and horizon of the investment until the asset choice and the wealth allocation. Chapter 2 concludes by introducing the concept of specialization vector\(^6\) that a manager specifies in order to filter-in those stocks that respond to his investment aspirations and criteria (specialization). This will obviously pave the road to defining the concept of portfolio concentration which will measure the weight allocation of wealth per stock and hence per specialized sector.

In Chapter 3 we will introduce the concentration measures used in economic studies as witnessed in the literature. We will also collect, define and discuss the general properties of those measures. This will lead us to defining concentration and specialization in investment portfolios. We will select two particular measures of concentration namely the Hirschman-
1.4. The Road map

Herfindahl Index (HHI)\(^7\) and the Gini index\(^8\) and we will present their characteristics and various properties.

In Chapter 4 we will relate our approach in this research to the main stream financial markets. We will comment thoroughly on the development of the investment optimization process from the mean-variance method reaching the latest development in risk parity approach. The present time tendencies is to opt for a more heuristic approach rather then a “quant” rigorous algorithm, where more market and sectoral information is required and where estimation error jeopardizes the reliance on past quotes estimates and moments. This chapter bridges the theoretical study of the concentration and specialization measures with the rationale of the requirement of an error free descriptor in the wealth allocation as well as in the investment monitoring and management, and introduces the application of our measures and the contribution of our approach to the investment decision making process.

In Chapter 5 we will show how to use the above mentioned concentration and specialization measures in describing a portfolio of investment. We will show how the values of the concentration measures will change after constraints are applied to the investment universe (the specialization vector). We will compare the values of the indexes before and after the application of the constraints on an investment universe. We will also show the changes in those values when combining portfolios.

Chapter 6 is a direct application to the methodology and measures introduced in this research. We will apply our toolbox formulas to two series of quarterly portfolios composed of stocks listed in the United States. We will choose the top 500 stocks per capitalization in each period and we will form two portfolios. The first one consists of a market-capitalization weighted portfolio \(P_{mcap}\) and the second one an equally weighted portfolio \(P_{eq}\).

Throughout the time interval of our consideration, Our analysis is two-fold. First, we shall describe the trends and the variation over time of the concentration and the specialization of the quarterly portfolios. Then at a second stage, we shall describe the concentration and the specialization of equally weighted and market capitalizations portfolios of three particular quarters that we believe are outstanding examples of the methodology we wish to convey in this research.

Chapter 7 concludes the research with a conjecture on neutral and biased portfolios using the criteria of concentration and specialization that were derived earlier.

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7The Hirschman-Herfindahl Index or HHI (also known as Herfindahl Index) is essentially used to measure the size of firms in relation to the industry and as an indicator of the amount of competition among them. Named after economists Orris C. Herfindahl (Herfindahl, 1955) and Albert O. Hirschman (Hirschman, 1964).

8The Gini Index (also known as the Gini coefficient or Gini ratio) is a measure of concentration developed by the Italian statistician and sociologist Corrado Gini and published in his 1912 paper “Variability and Mutability”.
1.5 Declaration of contribution

In this section, I declare my contribution to the different chapters of this dissertation and also acknowledge the contribution of other parties where relevant. In general, and where it is otherwise specified, the author formulated the research questions, performed the literature review, conducted the data analysis, interpreted the findings, and wrote the manuscript with the feedback and directives of the promoter.

Chapter 1: The majority of the work in this chapter has been done independently by the author of this dissertation, and the feedback from the promoter has also been implemented.

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Chapter 5: The majority of the work in this chapter has been done independently by the author of this dissertation. The author formulated the research question, performed the literature review, collected the data, conducted the data analysis, interpreted the findings, and wrote the manuscript. Obviously, at several points during the process, each part of this chapter was improved by implementing the detailed feedback provided by the promoter.

Chapter 6: The majority of the work in this chapter has been done independently by the author of this dissertation, and the feedback from the promoter has also been implemented. The data used to conduct the analysis was downloaded from CRSP/Compustat. At several points during the process of data treatment and analysis, each part of this chapter was improved by implementing the detailed feedback provided by the promoter.

Chapter 7: The majority of the work in this chapter has been done independently by the author of this dissertation, and the feedback from the promoter has also been implemented.
1.6 Concluding Remarks

**Appendix A:** The majority of the work in this appendix has been done and or compiled independently by the author of this dissertation from various sources. The nature of this appendix being descriptive rather then analytical, various sources, duly referenced, where employed to compile the formulas and decomposition methods mentioned.

**Appendix B:** This appendix was inspired by the work of Professors Benedetto Matarazzo, Salvatore Greco and Jaap Spronk, the promoter of this thesis. This is an unpublished and partial study that the author was able to explore thanks to the generosity of its owners.

### 1.6 Concluding Remarks

This PhD dissertation advances investment allocation and management literature by contributing to the knowledge about the concentration and specialization of investment portfolio. The description of an investment portfolio with respect to its concentration and specialization has high practical relevance and adds to the information that an investor seeks, but theory and empirical evidence about it are lacking. Due to this scarcity, I quite often referred to related research areas such as income inequality, social welfare and industrial geographical distribution.

I believe that the portfolio descriptors introduced in this dissertation not only fill certain research gaps, but they also suggest future avenues of research and present valuable recommendations for investors and investment managers alike.
Chapter 2

Describing Investment Opportunities

In this day and age, conventionalist thinking would dictate, stochastic calculus and econometrics become the most essential tools for aspiring finance stars, with staid accounting and fundamental analysis consigned to the dustbin of unacceptable simplicity.

_Pablo Triana, Lecturing Birds on Flying, preface, 2009_

This chapter addresses the first research question related to the investment process and introduces the concepts of specialization and concentration in a portfolio\(^1\).

What is the type and the characteristics of the information that investors normally seek in each stage of the investment process? What valuable information is distilled from including the concentration and the specialization of the portfolio in the information required in the investment process?

Investors and investment managers\(^2\) usually seek to meet a variety of conditions when

\(^1\)In all our subsequent analysis and throughout this thesis, we will assume that the asset class of choice of the investor are the listed stocks. For that matter, and referring to Definition 1 in Chapter 1, we will assume that the Investment universe is holding all the possible listed stocks in the world.

\(^2\)In this thesis we refer to investors or investment management or even managers to denote a party, an institution or a person or group of persons investing their wealth in the financial market without consideration to their gender. While we refer to “An Investor or The Investor” as a person or entity involved in financial investment and financial markets irrespective of his/her gender, we will refer to this investor as using “he or his” as pronouns.
allocating their wealth in a portfolio. Preservation of capital is an important determinant of the choice of investment opportunities along with liquidity of the investment at hand. However, maximizing future return remains a key determinant of the investment process. Maximizing return heavily relies on historical performance data and hence bears estimation error. In many cases, investors impose return-based goal constraints on the investment managers like a minimum return or a maximum acceptable variance of the return. This is due to the risk profile of the investor and to his expectations of the future performance.

Investors may add additional characteristics and impose side constraints on their choice. Besides the directly-return related constraints, indirectly-return related constraints might be defined like size or market capitalization footage of the company, its PE ratio, its ownership structure or the asset class and industry sector.

Other financial objectives or constraints that managers seek are liquidity considerations, leverage level of the stock, dividend and other balance sheet related ratios. Non-financial objectives are also sought after. Characteristics like SRI oriented or environment friendly investment along with certain preferences on some particular geographic locations (USA stock market, EU or Asia and emerging markets).

In fact, the investor imposes various additional “conditions” on his portfolio’s wealth allocation that represent his preference for placing his money on the table: from an initial available investment universe consisting of \( N \) possible stocks the investor will eventually choose \( n \) stocks representing his preferences.

This initial process will build up what we define as the investment universe for this particular investor. From all the possible universes of available investment, his preferences will reduce the available stocks to a smaller universe. This process we defined as specialization in Chapter 1.

Eventually, the investor will end up with one specialized portfolio consisting of his “chosen” stocks from all combinations of stocks in the possible investment universe. As defined in Chapter 1, Definition 4, the Portfolio Opportunity Set or POS is the set of all possible compositions of a portfolio given the set of assets one could invest in, the investment opportunity set, and the constraints that a portfolio manager must obey (Hallerbach et al., 2004) and (Hallerbach and Spronk, 1997) and (Pouchkarev et al., 2006). The final choice of the investor (and or manager) will be a portfolio belonging to the POS set of portfolios with the weight matrix of his choice \( [w_i] \), \( i = 0 \ldots n \), and where \( 0 \leq [w_i] \leq 1 \) subject to

\[
\sum_{i=1}^{n} w_i = 1.
\]

The investor thus faces two different decisions when determining where and how much of his wealth to allocate:

1. The selection of the Investment Opportunity Set, IOS (as defined in Definition 3, Page 5) from the Investment Universe in line with the specialization desired by the investor. (Specialized POS)

2. The choice of position or wealth allocation within the specialized POS (Concentrated
What will help the investor decide upon what stocks to include and how much to place on each stock depends largely on the information he has on the stocks and the environment of investment as a whole besides his own subjective preferences.

2.1 Portfolio Information

A considerable number of investors and investment professionals rely on the risk and return to describe, select and manage a portfolio. These measures are based on estimates of historical returns over a selected period of time. Both measures, which are estimates, are used to predict the possible future performance of the portfolio (or more precisely the stocks that constitute it). As these estimates are based on various assumptions, they are not error free. In fact, portfolio managers talk about estimation error which is the difference (or deviation) between the return estimated for time $t_1$ and the true return realized and observed at time $t_1$. This deviation can be very costly if a heavy weight (a high amount of the total wealth) was invested in this particular stock. Thus, the weights also play a very important role in investment and allocation decisions.

Different types of information is available:

1. Historical prices, company reports and calculated variables like variance and other moments.
2. Intrinsic characteristics of the investment object like its geographical location, industry sector, asset class, ownership structure among others.
3. Market data and analysts reports, like trading volumes, trends, indexes and market outlooks among other data.

In practice, investors often focus on estimates of risk and return that are based on historical returns and carry considerable estimation error. Objective data like the characteristics of the stocks such as geographic location, PE ratio, industrial sector, market capitalization and accounting fundamentals that are usually published in the listing of the stock exchange are also available.

The investor might seek additional information like the ownership structure, the involvement of the company on specific activities or attitude towards some social issues (equal opportunity for genders, employment of minors, pollution levels, environmental-friendly policies, etc.). All of the information that the investor has represents an input in the “investment decision process”. The question that the investor will always ask is how exact is this data, how precise is the estimation he is relying upon? This limit to the quality of the information at hand will influence the final allocation decision and is a modulator of the final weight vector $w$. In other words, the preference of the investor along with the information he
gathers on the stocks will make him/her deviate from the $1/n$ portfolio, representing an equally weighted portfolio and hence with no preference of one stock in particular over the other. This preference, derived essentially from the information he gathered from the different sources will therefore affect the specialization and the concentration of the portfolio. Figure 2.1 illustrates the various information types that an investor seeks when composing his portfolio.

Figure 2.1: Different types of information needed for the investment process.

As illustrated earlier in Figure 1.1, each stage of the investment process has its own requirement of information. While in stage A (choice of assets and filtering process: Specialization stage) we would primarily require non-financial characteristics and indirectly return related information, in stage B (Weight allocation within specialized portfolio: Concentration stage) the investor would primarily require directly return-related as well as financial characteristics. In stage C (monitoring the performance of the final portfolio), the investor seeks to see the big image as well the detailed characteristics of the portfolio. We believe the measures of specialization and of concentration that we will derive in this thesis will lead to additional insights for monitoring as well as managing the portfolio of investment.

Our focus in this thesis is essentially stages A and B where we will produce a descriptor of the portfolio elements and composition in terms of specialization (Stage A) and concentration (Stage B). As for the Monitoring process (Stage C) we implicitly refer to our suggested measures to be included in the toolbox for investors and portfolio managers alike to enhance their decision making process and monitoring techniques.
2.1.1 Portfolio Information and Estimation Error

Usually, an investor trades off one characteristic for another or constrains his portfolio on general preferences. Trading off between characteristics (like preferring more pharmaceutics stocks over oil and gas companies for example), or including a more volatile stock in the portfolio against a less volatile stock with very low return is a common allocation practice.

He might as well constrain his portfolio to exclude companies dealing with alcoholic beverages or armaments or preferring non-family owned companies or those located outside the USA, as an example. It is a matter of choice, judgment and taste, and subjectivity plays a major role in the composition of the portfolio.

Among the modulators that the manager refers to when making his choices are some measures that are inherently error free like some characteristics belonging mainly to financial and non-financial objectives and also to indirectly return related information, as shown in Figure 2.1. For example the size of the firm and its ownership structure are estimation error free. To the contrary, return and variance, beta and correlation are estimates based on historical data and hence carry an element of estimation error.

The seminal work of Markowitz (1952 and 1959) suggested using the rate of return \( r_i \) and the standard deviation \( \sigma_i \) of a stock \( i \), over a period of time, as decision tools for the stock selection and wealth allocation. The choice of the investor is initially guided by the concept of maximizing the expected return with the least possible risk, i.e. with the least possible standard deviation, according to the investor’s risk appetite or aversion. This premise is valid and seems to be intuitively logical: Inversely, the investor wants to remunerate the risk he takes with the maximum possible available return. However, the estimation error of the indicators, namely the expected rate of return \( r_i \) and its standard deviation \( \sigma_i \), can be substantial and may distort the initial forecast. The quest here is twofold: 1) it is to use a descriptor that does not inherently carry any estimation error i.e. to use a descriptor that is not based on estimated values and 2) whether using such and estimation error free descriptor would add to the quality of the investment process.

2.1.2 Portfolio Characteristics and objectives

As discussed earlier in this chapter, the investor seeks reliable information on the stocks included in his universe before he makes his final choice and wealth allocation. Some characteristics of the stocks an investor might consider to analyze are:

(a) Asset class like equity (stocks, fixed income bonds and money market instruments are some available instruments for investment). It should be noted that in addition to the three main asset classes, some investors add real estate and commodities, and possibly other types of investments to their asset class mix. Whatever the asset class line-up, each one is expected to reflect different risk and return investment characteristics, and will perform differently in any given market environment.
(b) Geographic location. The manager might specify a certain geographic scope to his investment focusing on some particular countries or excluding some others.

(c) Industrial sector. Investors tend to screen out some industrial sectors according to their economic conjecture and outlook. The past sub-prime crisis ruled out many real estate developers and even financial institutions from active portfolios due to the sector’s crisis. Many investors tend to filter out those industries whose activities are considered harmful to the society or to the environment according to some personal criteria. A wider screening process is applied by Socially Responsible Investors (SRI) where some industries or countries are ruled out of the investment choice.

(d) PE (Price earning ratio). The manager normally specifies a range of PE ratio that is acceptable. For example, the investor might impose a bracket on the PE ratio of the stocks to be included in his portfolio like “PE ratio between 13 and 17” \((13 \leq PE \leq 17)\) or he can specify a minimum acceptable PE ratio of 15 \((PE > 15)\), reflecting the investor’s preference for stocks with growth expectations.

(e) Specific financial ratios. Some investors look at certain financial ratios to assess the “health” of the company in question. Islamic investors, for example, often use ratios like the gearing or indebtedness ratio, liquidity ratio, income from interest to total income ratio among other available ratios to screen out some stocks.

(f) Ownership structure. Investors tend to categorize companies by their ownership structure. Family owned companies tend to behave and react differently to economic surprises and events as compared to non-family owned companies. When management and ownership are concentrated in one family structure, agency problems could arise and serious corporate governance flags can be raised. Some investors seek to invest in family owned businesses while others avoid them to allocate a very small percentage of their wealth on such stocks.

(g) Market capitalization footing. Investors tend to categorize the company by the amount of their market capitalization, commonly referred to as market cap. While some investors prefer big-cap companies, others would rule them out in favor of medium cap or small-cap stocks.

The list above is not exhaustive and the choice criteria can be numerous and diverse. The investor will decide upon which characteristic or attribute to use for his screening process. His choice of those criteria will influence and shape the specialization of his resulting portfolio. This first screening process is applied to the \(N\) available stocks in the investment universe. The resulting specialized universe consists of \(n\) stocks where \(n \leq N\).
2.1.3 The Specialized Investment Universe

Let $U$ denote the universe of all existing $N$ stocks. The elements of this universe are the stocks $u_i$ such that $1 \leq i \leq N$. $U$ is the Investment Universe as per Definition 1, Chapter 1. Let $S$ denote the resulting screened investment universe with $n$ stocks. The elements of $S$ are the stocks $s_j$, where $1 \leq j \leq n$.

Furthermore, let $\vec{p}$ denote the vector of $m$ desired attributes that the investor would like each investment to achieve. We now define a function, $\pi(u_i)$, that operates on the elements $u_i$ of the universe of stocks $U$ and returns a screened or selected universe $S$ whose elements $s_j$ obey the vector of the stock’s attributes $\vec{p}$. Thus, by applying the function $\pi$ to the universe of available stocks, we obtain:

$$S = \{u_i | \pi(u_i) = \vec{p}\}.$$

As an illustration, consider the following vector of screening attributes:

$$\vec{p} = \{\text{USA stocks, non-fincl Co., } P/E \geq 15, \text{ Debt/Assets} \leq 33.3\%\} \quad (2.1)$$

This first specialization process $\pi(u_i)$, according to $\vec{p}$, will result in a specialized investment universe $S$. Within this specialized universe $S$, the investor will choose the weight vector of wealth allocation $[w]$ reflecting the level of concentration of the portfolio, given the specialized universe. We will discuss the concept of specialization and concentration in investment portfolio in more details in Chapter 3.

In the resulting portfolio, $S$ subjected to $\vec{p}$, there is a specialization in the US market only, excluding all financial companies and subjecting the remaining US and non-financial companies to a financial screening of PE ratio threshold level of 15 and a Debt/Asset ratio limit of maximum 33.33%.
Chapter 2. Describing Investment Opportunities

All possible markets
USA stocks
Non-Financial companies
P/E > 15
D/A ≤ 33.3%

Desired Investment universe

Figure 2.2: Graphical representation of the specialization vector \( \vec{p} = \{ \text{USA stocks, non-financial Co., } P/E \geq 15, \text{ Debt/Assets } \leq 33.3\% \} \).

Accordingly, the final desired universe of investment \( S \), after applying the specialization filters \( \pi_{ui} \) can be expressed as:

\[
S = S_{(\text{OnlyUSA})} \cap S_{(\text{financial})} \cap S_{(P/E<15)} \cap S_{(D/A>33.3\%)}
\]

The reduction in number of included stocks can be very drastic according to the number of filters and the level of cut-offs desired.

2.2 Weight-attribute Matrix and Impact Matrix

Let us assume, for the sake of illustration that an investor, or a manager, wishes to invest in a portfolio whose characteristics are the following:

1. include stocks with their expected return \( E_r > 7\% \)
2. include stocks whose Standard Deviation \( \sigma < 15\% \)
3. include stocks with \( PE > 15 \)
4. Only USA and Japan stocks to be included
5. include only stocks belonging to the following sectors: a) pharmaceutics b) oil and gas c) retail service d) metallurgy and e) mining$^3$.

$^3$We will use common names of industrial sectors in this example to illustrate our point. However, we are aware of the usage of the GICS (Global Industrial Classification Standard) utilized in Compustat listing, as well the SIC (Standard Industrial Classification) or the NAICs (North American Industrial Classification Standard) all using digits and numbers.
2.2. Weight-attribute Matrix and Impact Matrix

This can be represented by a specialization vector $\vec{p}_1$, according to the annotation used above as follows:

$$\vec{p}_1 = \{E_r > 7\%, \sigma < 15\%, PE > 12, USA + Jap, Pharma + Oil + Retail + Metal + Mining\}$$

Assume for the sake of this example that the investor intends to choose to place his investment on 10 stocks answering the above mentioned criteria or, in other words, he will choose 10 stocks ($S_1, \ldots, S_{10}$) that answer the $\vec{p}_1$ specialization vector. The following table 2.1 illustrates this final choice:

to describe and categorize an industrial sector We will be using the GICS symbolization in subsequent chapters of this thesis, but we will only use common sectors names in the present chapter for illustration purposes.
Table 2.1: Hypothetical 10-stocks portfolio resulting from the screening vector: \( \vec{p}_1 = \{E_r > 7\%, \sigma < 15\%, PE > 12, USA + Jap, Pharma + Oil + Retail + Metal + Mining\} \). (This portfolio will be used to illustrate the usage of the toolbox later in Chapter 5).

To be able to analyze the specialization and the concentration level of this hypothetical portfolio resulting from applying the vector \( \vec{p}_1 \) to the investment universe, we need to establish a matrix like table where all those criteria can be expressed numerically. This will be called the impact matrix and is established according to the following rules:

A- Create a detailed weight-attribute matrix by expanding the specialization conditions (filtering criteria), as seen in Table 2.2:

1. Stock names and weights are aligned vertically in a column
2. Filtering criteria are aligned horizontally in a row.
3. Transform the numerical criteria into several numerical intervals. For example, the criteria \( E_r > 7\% \) is to be translated into several intervals like shown in Table 2.3.
4. For non-numerical criteria include a column for each category: USA and Japan for...
the geographical criteria have two separate columns, and each sector of the industry has its independent column.

5. Insert a 1 in the squares where the attribute is valid for each stock.
### Table 2.2: Detailed weight-attribute matrix of a hypothetical specialization vector \( \vec{p}_1 = \{E_r > 7\%, \sigma < 15\%, PE > 12, USA + Jap, Pharma + Oil + Retail + Metal + Mining\} \)

<table>
<thead>
<tr>
<th>( S_i )</th>
<th>( w_i )</th>
<th>( E_r &gt; 7% )</th>
<th>( \sigma &lt; 15% )</th>
<th>( PE &gt; 12 )</th>
<th>Location</th>
<th>Industrial Sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0.07</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>S2</td>
<td>0.06</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>S3</td>
<td>0.07</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>S4</td>
<td>0.08</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>S5</td>
<td>0.09</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>S6</td>
<td>0.11</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>S7</td>
<td>0.14</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>S8</td>
<td>0.18</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>S9</td>
<td>0.09</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>S10</td>
<td>0.11</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Tot</td>
<td>1.00</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>
### Table 2.3: Detailed Impact Matrix of the hypothetical specialization vector $\vec{p}_1 = \{E_r > 7\%, \sigma < 15\%, PE > 12, USA + Jap, Pharma + Oil + Retail + Metal + Mining\}$. 

<table>
<thead>
<tr>
<th>$S_i$</th>
<th>$w_i$</th>
<th>$E_r &gt; 7%, E_r % \in: [7, 10]$</th>
<th>$[10, 13]$</th>
<th>$[13, +]$</th>
<th>$\sigma &lt; 15%, \sigma % \in: [0, 10]$</th>
<th>$[10, 15]$</th>
<th>$[12, 14]$</th>
<th>$[14, 16]$</th>
<th>$[16, +]$</th>
<th>Location</th>
<th>Industrial Sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0.07</td>
<td>0</td>
<td>0</td>
<td>0.07</td>
<td>0</td>
<td>0.07</td>
<td>0</td>
<td>0</td>
<td>0.07</td>
<td>0</td>
<td>0.07</td>
</tr>
<tr>
<td>S2</td>
<td>0.06</td>
<td>0</td>
<td>0</td>
<td>0.06</td>
<td>0</td>
<td>0.06</td>
<td>0</td>
<td>0</td>
<td>0.06</td>
<td>0</td>
<td>0.06</td>
</tr>
<tr>
<td>S3</td>
<td>0.07</td>
<td>0</td>
<td>0</td>
<td>0.07</td>
<td>0</td>
<td>0.07</td>
<td>0</td>
<td>0</td>
<td>0.07</td>
<td>0</td>
<td>0.07</td>
</tr>
<tr>
<td>S4</td>
<td>0.08</td>
<td>0</td>
<td>0</td>
<td>0.08</td>
<td>0</td>
<td>0.08</td>
<td>0</td>
<td>0</td>
<td>0.08</td>
<td>0</td>
<td>0.08</td>
</tr>
<tr>
<td>S5</td>
<td>0.09</td>
<td>0.09</td>
<td>0</td>
<td>0.09</td>
<td>0</td>
<td>0.09</td>
<td>0</td>
<td>0</td>
<td>0.09</td>
<td>0</td>
<td>0.09</td>
</tr>
<tr>
<td>S6</td>
<td>0.11</td>
<td>0.11</td>
<td>0</td>
<td>0.11</td>
<td>0</td>
<td>0.11</td>
<td>0</td>
<td>0</td>
<td>0.11</td>
<td>0</td>
<td>0.11</td>
</tr>
<tr>
<td>S7</td>
<td>0.14</td>
<td>0.14</td>
<td>0</td>
<td>0.14</td>
<td>0</td>
<td>0.14</td>
<td>0</td>
<td>0</td>
<td>0.14</td>
<td>0</td>
<td>0.14</td>
</tr>
<tr>
<td>S8</td>
<td>0.18</td>
<td>0.18</td>
<td>0</td>
<td>0.18</td>
<td>0</td>
<td>0.18</td>
<td>0</td>
<td>0</td>
<td>0.18</td>
<td>0</td>
<td>0.18</td>
</tr>
<tr>
<td>S9</td>
<td>0.09</td>
<td>0.09</td>
<td>0</td>
<td>0.09</td>
<td>0</td>
<td>0.09</td>
<td>0</td>
<td>0</td>
<td>0.09</td>
<td>0</td>
<td>0.09</td>
</tr>
<tr>
<td>S10</td>
<td>0.11</td>
<td>0.11</td>
<td>0</td>
<td>0.11</td>
<td>0</td>
<td>0.11</td>
<td>0</td>
<td>0</td>
<td>0.11</td>
<td>0</td>
<td>0.11</td>
</tr>
<tr>
<td>tot</td>
<td>1.00</td>
<td>0.31</td>
<td>0.31</td>
<td>0.38</td>
<td>0.39</td>
<td>0.61</td>
<td>0.28</td>
<td>0.43</td>
<td>0.29</td>
<td>0.58</td>
<td>0.42</td>
</tr>
</tbody>
</table>

Table 25: Weight-attribute Matrix and Impact Matrix
B- Create a detailed Impact matrix by multiplying the weight matrix \([w_i]\) by each category (column) of the Criteria matrix, as seen in Table 2.1 and in Table 2.3. The resulting is an Impact matrix showing the weights of each stock \(S_i\) per each category. Please note that the sum of weights per category should be equal to 1, which is the budget constraint.
2.2. Weight-attribute Matrix and Impact Matrix

As an illustration, the Expected return impact matrix \( IMP_{exp.ret} \) was obtained by multiplying the transpose of the weight matrix \( w_i \) by the Expected return weight criteria matrix \( C_{exp.Ret} \):

\[
IMP_{exp.ret} = [w_i]^{-1} \times C_{exp.Ret}
\]

and where:

\[
[w_i]^{-1} = \begin{bmatrix} .07 & .06 & .07 & .08 & .09 & .11 & .14 & .18 & .09 & .11 \end{bmatrix}
\]

\[
C_{exp.Ret} = \begin{bmatrix}
0 & 0 & 1 \\
0 & 0 & 1 \\
0 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 0 \\
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 0 \\
\end{bmatrix}
\]

Which is shown in Table 2.3 in the first left quadrant. The same methodology applies to all the criteria included in the specialization vector until the final impact matrix is reached.

It is worth noting that the attributes included in the specialization vector \( \vec{p}_1 \) of this example are arbitrary as well as the weights used which are for illustration purposes.

In theory, the manager or the investor would specify his requirement for a portfolio with a maximum of \( n \) stocks with \( m \) attributes. Those attributes can be numerous.

In theory \( m \) can be greater than \( n \), in the sense that the manager might decide to include 10 stocks in his portfolio with 15 different attributes or criteria. Each stock chosen in the portfolio must have each and every criteria attribute or else it would have been filtered out.

On the other hand, using the individual impact matrix per stock, we have a glimpse description of the contribution of this stock to the portfolio. Consider for example stock \( s_3 \) as an illustration. Its impact matrix is the following:

\[
IMP_{s_3} = \begin{bmatrix} 0 & 0.08 & 0.08 & 0.08 & 0.08 & 0.08 & 0.08 & 0.08 \end{bmatrix}
\]

Assuming linearity of weight aggregation, we can sum up all impact matrices of the individual stocks, \( IMP_{s_i} \) to reach the portfolio impact matrix, \( IMP_{portf} \):

\[
IMP_{portf} = \sum_{i=1}^{n} IMP_{s_i} = \begin{bmatrix} .31 & .31 & .38 & .61 & .39 & .28 & .43 & .29 & .58 & .42 & .37 & .13 & .09 & .32 & .09 \end{bmatrix}
\]
Chapter 2. Describing Investment Opportunities

So when we see this matrix we can have an idea or a description of what the portfolio is and we can compare two different portfolios assuming the specialization vectors have the same filtering criteria. This impact matrix reflects the impact of the aggregate weights of the portfolio on each and every attribute.

2.3 Introduction to Portfolio Description Using Concentration

At the level of portfolio information, many helpful insights can be drawn from an impact matrix. Not only it shows, at a glance, the proper geographical distribution within the portfolio but also the investor can see where is his wealth allocated and how it is distributed. Looking at the tables 2.2 and 2.3 we can have an idea of the contents of the portfolio. At a glance we can describe it and draw some first hand conclusions.

- The portfolio has slightly more weight on the high return bracket of $E_r > 13\%$ because it has 4 included in this group compared to 3 stocks in each of the other remaining two Expected return groups.
- Half of the portfolio stocks have a standard deviation between $15\% > \sigma > 10\%$. However, this is reflected in 0.61 of cumulative weights of those 5 stocks.
- 6 stocks are Japanese and 4 stocks are from USA. This is reflected also in the weights where 0.58 in weight represent Japanese stocks.
- 4 out of the 10 available stocks belong to the pharmaceutical industry sector with 0.37 in weight against 0.13 to its nearest sector, oil and gas.
- On the weight side, we observe that $S_8$ is the heaviest with 18% weight contribution in the portfolio followed by $S_7$ with 14%. The lightest stock is $S_2$ with 6% in weight contribution.
- $S_8 + S_7 + S_6 = 43\%$ representing slightly less then half of the wealth invested in the portfolio.

A lot of conclusions can be drawn from the simple observation of Tables 2.2 and 2.3 above. It is a multidimensional impact matrix showing where the influences of the weights are and where the most of the stocks are, or in other words, this matrix gives an idea on the concentration of some attributes around some stocks.

From the matrices depicted we can see the specialization of the portfolio and its concentration.

We can observe that the portfolio above is concentrated around 3 stocks $S_8, S_7$ and $S_6$ and that it is specialized in Japanese stocks rather then US stocks to a certain degree. It is also a pharmaceuticals and metallurgy specialized portfolio with most of its stocks exhibiting a standard deviation between 10% and 15%.
This qualitative description based on the observations of the tables of the weight-attribute and impact matrices give a rapid idea on how the portfolio is constituted and can be a handy tool to compare two different portfolios, at least from the stocks and their attributes. We can tell exactly that one portfolio is more concentrated then another in a certain given attribute. We can also say with confidence that the portfolio is concentrated in weights around some identified stock. What we need is to quantify this “degree of specialization” and this “degree of specialization”. We will derive concentration and specialization measures that quantify our description, allowing the investor to compare with a fair degree of confidence across several portfolios and investment platform.

2.4 Concluding Remarks

The investor interest is in the future performance of his investment, and the future performance depends on the horizon of the investment and the time to the horizon from the day of inception of the portfolio. For that reason, a lot of attention has been given in the literature to the estimation of the future returns. A lot of attention is also given in the literature putting the quality of these estimates into doubt, in fact, when the estimates are not reliable, using optimization processes will result in unreliable output that will tend to concentrate the portfolio in very few assets.

There is an obvious need to construct an additional descriptor, aside the usual tool-kit of investment professionals, i.e. the return and variance, to include an estimation error-free measurement to describe an investment portfolio. A usual strategy followed by investors to lower the estimation error, at least intuitively, is to allocate $1/n$ of the wealth in each chosen asset. In fact, (DeMiguel et al., 2009) in their paper on the $1/n$ allocation strategy show that, a $1/n$ allocation strategy will reduce the impact of estimation error.

However, even if an investor decides to allocate $1/n$ of his wealth in each stock, choosing to minimize concentration level, the portfolio remains biased in his initial choice of the attributes of the assets to be included in his portfolio. The screening or specialization vector introduced in this Chapter, shows clearly that if an investor wants to allocate with optimal diversification strategy, which is intuitively a $1/n$ allocation, the choice of the assets to be included in his portfolio should engulf all the possible existing assets in the investment Universe.

The impact matrix generated by the choices of the investor describes the portfolio with measures, concentration and specialization, that are estimation error free since the characteristics of the attributes measured are not time dependent but rather intrinsic to the assets and very rarely change.

---

4we assume that those attributes do not change with time, unless a merger and acquisition happens and the company changes its industry sector or geographic location among all other possible attributes.
Chapter 3

Concentration and Specialization Measures

Someone told me that each equation I included in the book would halve the sales.

Stephen Hawking

The aim of this chapter is to introduce the concentration and specialization measures used in the study of welfare and income inequality. This chapter answers Research Question 2. It will explore the related literature review on those measures. Additionally, this chapter will define the principles and characteristics that a concentration and a specialization measure, applied to an investment portfolio, should have that will result in relevant and useful information for the investor. The chapter will conclude by specifying those measures of concentration and specialization that we believe they are adequate to be applied to an investment portfolio, from the standpoint of an investor.

3.1 Concentration and Specialization Measures in the Literature

As we saw earlier in Chapter 2, the concepts of concentration and specialization are complementary. In fact, specialization is a concentration in attributes or properties. It is the result of a filtering process where an investment universe is subjected to a set of desired
attributes resulting in a specialized sub-universe\(^1\). The concept of concentration is widely applied in economics and income theories. It is often used as a tool to define and detect a group of dominant industries in a market or to study the income distribution between nations or within societies as well as defining the poverty lines and inequality of a related social situation. Inequality as a measure is widely used in welfare studies and is a term indicating uneven distribution of wealth between citizens. It is considered that when inequality exists among members of a society, some form of wealth concentration must have lead to it. Hence, in some texts Inequality and Concentration are used interchangeably. In fact, when equality of income exists among members of a society, no wealth concentration is observed. In our thesis we will use the term concentration. However, in this section, the term “inequality” will be used when it is referred to by the author of the literature under review. The concepts of inequality and concentration are further discussed in Section 3.2.

Various theories concerning international trade, economic geography and socio-economic policies use the concept of concentration as a quantitative tool. As mentioned earlier, industrial specialization of regions uses inequality or concentration measures to describe the industrial specialization of a region compared to other regions in a geographic area. In the coming subsections we will explore the concentration and the specialization measures in the literature and we will pave the way to derive from the available economic research, the general properties that a concentration and specialization measures, applied to investment portfolio, must have.

### 3.1.1 Measures of Concentration.

Early works on inequality are based on the work of Max Otto Lorenz (Lorenz, 1905) and what is known as the Lorenz curve. Essentially, the Lorenz curve represents the relationship between the cumulative portion of the population and the cumulative amount of a given resource or a scalar of interest held by the population. The scalars, like individual income, units of production or market share per company etc. are ranked in increasing order and the cumulative participation of each rank is then calculated. For example, consider a population of \( n \) individuals and consider a scalar \( x \) that is a positive and natural number\(^2\) associated

\(^1\)Our discussion hinges on concentration and specialization in investment portfolios. The concept of specialization, or geographical and spatial specialization, widely used in industrial specialization of regions, relates to the attribute of industrial specialization of a given region, comparing its industrial output of some specific industrial sector with other regions.'

\(^2\)In our thesis, we will be dealing essentially with positive and natural numbers associated with a measurable attribute (in our case the weights of the individual stocks in a portfolio). The concentration measures encountered in the literature mainly deal with positive numbers. When negative numbers are involved (like negative income for example), it is obvious that the coefficient or index of concentration takes value greater then one, as demonstrated theoretically by Hagerbaumer (1977) and empirically by Pyatt et al. (1980). Many research dealt with reformulating and normalizing some concentration measures to include negative elements, we mention mainly the reformulation of the Gini index to include negative income
3.1. Concentration and Specialization Measures in the Literature

with a measurable attribute like income level, units of production or market share. For each individual \( i, i = 1, \ldots, n \) we associate a value \( x_i \) where the vector \( x_1, \ldots, x_n \) is sorted in ascending order of magnitude. The Lorenz curve is then obtained by plotting the points \((\frac{k}{n}, \frac{S_k}{S_n})\), with \( k = 0, \ldots, n \) and with \( S_0 = 0 \) and \( S_k = \sum_{i=1}^{k} x_i \) representing the attribute cumulative sum of the first \( k \) individuals of the population. Joining the points yields the curve \( M \) connecting the origin at \((0, 0)\) with the point \((1, 1)\), as shown in Fig 3.1

![The Lorenz Curve M](image)

Figure 3.1: The Lorenz curve \( M \), showing the line of perfect equality.

The Lorenz curve \( M \) is always below the line of total equality since \( S_k = \sum_{i=1}^{k} x_i \) becomes \( S_k = \sum_{i=1}^{k} \frac{1}{n} = \frac{k}{n} \) only when total equality is achieved.

Greater inequality among members of the population is registered by the deviation of the Lorenz curve from the total equality line. Maximum concentration or total inequality occurs when all of the attribute (wealth, income, production, etc.) is concentrated in one single member of the population and the plot \((\frac{k}{n}, \frac{S_k}{S_n})\) is reduced to a single point \((1, 1)\) representing the maximum concentration or the maximum inequality\(^3\).

During the same period of time, around 1905 a relevant scientific event also took place in the University of Bologna, Italy. Corrado Gini defended his doctoral thesis on the statistical analysis of birth by gender. The Lorenz (1905) paper mentioned earlier greatly influenced further development in stochastic dominance, probability and distribution theories, while by Chen et al. (1982).

\(^3\)Note that it is intuitive to find graphically the point on \( M \) representing the average value of the attribute measured. Since the average of the scalar is \( 1/n \) and since the total equality line is a straight line with slope \( 1/n \), we conclude that the tangent to the curve \( M \), parallel to the total equality line, is tangent at a point representing the cumulative count of the population whose cumulative value of the attribute is the average of the distribution.
Chapter 3. Concentration and Specialization

Gini focused on income inequality that was primed by his criticism of the Pareto inequality parameter. This led Gini later on to formulate his proposition of the famous inequality ratio, or what we call today the Gini index or, simply, the Gini of a distribution and which he published in 1914 (Gini, 1914).

Those two analytical elements, namely the Lorentz curve and the Gini index are widely used in welfare, income and generally in most economic studies concerned with distribution, dominance or deviation from a standard: poverty line (Dalton, 1920), poverty and deprivation (Sen, 1976), Internet bandwidth usage (Ogryczak, 2007), industrial specialization versus geographic concentration (Ceapraz, 2008) and (Aiginger and Davies, 2004), the effect of merger and acquisitions between firms and oligopoly (Watt and de Quinto, 2003) or political science (Taagapera, 1979) to cite some examples.

Various developments and reinterpretation of the Lorenz-Gini initial approaches resulted in new indexes and measurements of inequality or concentration using a welfare economic approach pioneered by Dalton (Dalton, 1920), Kolm (Kolm, 1969) and Atkinson (Atkinson, 1970).

In this line of development, Camilo Dagum has published three seminal papers on personal income distribution, inequality measures between income distributions and the relationship between income inequality measures and social welfare functions. In fact his model of income distribution (Dagum, 1977) is a continuous probability function defined over all positive numbers. This marks the start of creating special distribution functions to cater for and explain inequality as such. Dagum (1980) (and later 1990) suggested a measure of the direct economic distance between income distributions to assess relative affluence between the subgroups of the population under study. In this paper, Dagum uses the Gini index as a measure of inequality (Gini, 1914). The Dagum distance measure evaluates the economic distance, in units of monetary measurement, between the subgroups under study. This distance is directional and is calculated from observed data and hence, it is not an estimation. It describes the wealth distribution (and consequently the wealth distribution function) among the members of the society under study, divided into independent non-overlapping groups. For example, the wealth distribution distance can be captured between males and females, or between the northern–southern–eastern and western areas of a country. Traditionally, theses measures were assessed using statistical measurements like the median or average of the distribution considered. The simplicity in using the Gini ratio or index in such a measurement is that the calculation does not make any assumption on the distribution itself and hence it is not an estimated value but rather a precise, estimation error free measurement. In a further development, Dagum suggests a decomposition of the Gini index (Dagum, 1997b) among three different ratios, namely: the Gini inequality ratio within a subgroup of the population, the contribution of this subgroup to the total inequality ratio of the whole population and the distance between the inequality between subgroups of the population. A similar approach to decomposing inequality measures was presented by
3.1. Concentration and Specialization Measures in the Literature

Bourguignon (Bourguignon, 1979) (and later 1988) whose ratio is based on the log of the proportion of inequality within a subgroup to the average inequality within the full group.

In his book “Measuring Inequality”, Cowell (2009) describes various methods of measuring income inequality in a population. He derives a general entropy measurement which is also found in various literatures of information theory. Cowell derives a measure of the “degree of disorder” within a system according to the probability distribution of the events in question within the system, a measurement defined as entropy. Theil (1967) argues that the entropy concept is a useful measurement of inequality if the concept of \( n \) possible events is re-interpreted as \( n \) members of the population.

Although the available literature on income distribution is extensive, very limited research was found related to portfolio concentration levels or measurement. For instance, building on the entropy concept, Kapur and Kesavan (1992) proposed an entropy maximization model and a cross-entropy minimization model as a variation of Markowitz model. Later, Raghunathan (1995), uses entropy to describe the diversification\(^4\) of companies and to qualify the amount of concentration of a particular firm in a market or in an industrial sector. He also used the entropy measure to compare a firm’s diversification (the different output it produces against focusing on one unit of produced output) against its level of specialization and concentration. Meucci (2009) describes a diversification distribution and later derives a diversification index of the portfolio based on the entropy of the distribution.

As for concentration levels in investment portfolios, the literature available predominantly relates to measures of diversification in portfolios. Heterogeneity of markets was explored by Pouchkarev (Pouchkarev et al., 2006), applying the methodology of Portfolio Opportunity Sets\(^5\). A related paper on concentration of portfolio based on the bias of local versus foreign investors (due to asymmetry of information available to locals versus the non-locals, biasing the portfolio selection and concentrating it in few local stocks) was discussed by Ivkovic et al. (2008) using the \( HHI \) index as a concentration measure. Portfolio selection based on fuzzy cross-entropy was explored by Qin et al. (2009).

Still, some articles relevant to our study of concentration in investment portfolios must be mentioned, namely, an unpublished paper by Matarazzo B. et al.\(^6\), on pretension level

\(^4\)Diversification is essential in portfolio management and in portfolio description. The term diversification is used in economic and welfare study to describe a non-concentrated set of elements but not necessarily an equally distributed attribute among the elements of the set. By intuition it is assumed that a diversified set is a set whose total value is divided equally among all members of the set (the \( 1/n \) allocation scheme, see (DeMiguel et al., 2009)). We will further explore diversification measures in a portfolio in coming sections of this thesis.

\(^5\)POS: Portfolio Opportunity Sets, is the set of all possible portfolios made of a given number of individual stocks.

\(^6\)Benedetto Matarazzo, Salvatore Greco and Jaap Spronk. This is an unpublished study that we were able to explore thanks to the generosity of its authors. The concept of Pretension is introduced by the authors and is defined as the degree of concentration of weight
Chapter 3. Concentration and Specialization

in a portfolio and a published paper by Simonelli (2005) on the indeterminacy in portfolio selection. Important portfolio approaches were made by Mussard and Terraza (2004) by arguing that the concentration of returns of a particular security around a certain value is a measure of the risk of this security, since it defines the dispersion\(^7\) of the values of returns around a central value (the average) and hence their standard deviation. Mussard et al. used the Gini index and its decomposition (suggested in the central work of Camilo Dagum) into concentration within a subgroup of the set, the concentration of the whole set and the distance between the concentrations between each subgroup.

An interesting development in portfolio concentration measures was presented by Brands et al. (2004) suggesting the usage of a divergence index\(^8\) to measure the amount of divergence between the concentration of the portfolio compared to the market concentration, which is used as a benchmark. From his side, Ogryczak (2007) dealt with allocating resources among competitive activities using inequality measures approach. This suggests using the same logic to allocate the wealth in different securities to form a portfolio.

The concentration measures used and researched in the past literature focus primarily on equality which suggests a natural wealth allocation of 1/n within a portfolio. But logically this is not the case in real life investment situation. In fact investors have subjective as well as objective preferences of investing in some stocks more than in some others or to invest in a certain sector or sectors that they believe would maximize their return for the risk they accept to take, which automatically suggests a bias to the 1/n allocation strategy.

of a stock within a portfolio. The more weight an investor puts on a particular stock, the more “pretension” he has that this stock will outperform the others. If an investor has no pretension on any particular stock, he will use a 1/n weight distribution, a situation of total equality of weights or of minimum concentration, and hence the authors defined this as a “zero pretension level”. The concept of pretension level reflects the proportion of wealth an investor “bets” on one stock over the others and reflects the pretension he has that those stocks, subject to his preference and hence where he put his bet, are going to outperform. The pretension level they defined is measured with the HHI index.

\(^7\)The statistical dispersion of returns is defined as the asset-weighted standard deviation of individual stocks’ returns within a portfolio.

\(^8\)The Divergence index is the sum of the squared difference between the individual weights of a portfolio and a benchmark portfolio, taken as a reference (usually the Market portfolio).

\[ D = \sum (w_{i\text{portfolio}} - w_{i\text{benchmark}})^2 \]
3.1. Concentration and Specialization Measures in the Literature

3.1.2 Measures of Specialization.

The development of the field of geographic economics has produced a clear interest in creating indicators to reflect the geographic industrial output or production concentration or what the geographic economist call: The regional agglomeration of industrial production, i.e. the regional concentration of the production and the industries specialization that produces this industrial concentrated output. The work of (Krugman, 1991a) and (Krugman, 1991b), while studying the relationship between transportation costs and regional industry specialization, presented a new index of specialization: The Krugman Specialization Index.

A parallel approach to defining concentration and specialization in national industrial sectors was elaborated by Ceaparz, (Ceapraz, 2008). In his study of the productive Romanian industrial sectors, he discusses the competitiveness of the Romanian industries within the European Union, by using both the Hirschman-Herfindahl index (HHI) of concentration and the Gini index at the same time. The use of both indexes allows Ceapraz to assess and describe the economic integration at a national level as well as to explain and describe the regional and geographical industry concentration in the country. However, the research on industrial specialization shows a certain differentiation between absolute and relative specialization.

Absolute and relative specialization measures The related literature focuses mainly on two different group of indexes to assess and measure the specialization or regional industrial localization as reflected in the works of (Bickenbach et al., 2013), (Palan, 2010) and also (Bickenbach and Bode, 2008). (Aiginger and Davies, 2004) describe the absolute specialization measure as reflecting the volume of output share of the country relative to the number of industries in that country. This will permit saying, for the sake of illustration, that Italy is specialized in textiles and Spain is specialized in olive oil.

9In our research we have detected a philosophical dispute between the concept of “Geographic Economics” and “Economics Geography”, which does not seem to be settled properly. Far from taking sides in such a philosophical and epistemological dispute, we mean by Geographic Economics the economics of regional or geographic industrial output or production. In this respect, our literature review shows that the subject of Economic Geography has triggered an interest in measuring industrial agglomeration and specialization like (Krugman, 1991b), (Ellison and Glaeser, 1997), (Midlefart-Knarvik et al., 2000) and (Hallet, 2000). The majority of these indexes or measures are based on, or are variations of the Gini index.

10The Krugman Specialization Index is a widely used for measuring specialization. Basically, it is the standard error of industry shares, i.e. it calculates the share of employment which would have to be relocated to achieve an industry structure equivalent to the average structure of the reference group.

11In this thesis we will use indexes and/or indices as the plural of the world index: Both indexes and indices are acceptable plurals for “index” in English—and in that order of preference today (Authority: Oxford English Dictionary and Merriam-Webster Dictionary.)
production. This measure is considered absolute because it does not compare the sample at hand with any reference group (like comparing Spain to the EU output for example). The relative measure of specialization compares the reference data (industrial output, employment, production ... etc.) of a geographic area (the EU for example) with the regional data under study. Hence we can fairly conclude that the absolute measure of specialization compares the reference region to a hypothetical $1/n$ distribution or to an equally weighted hypothetical reference data whereas the relative measurement of specialization compares the region under study with a certain reference entity that is or is not related to the sample under consideration.

Empirical and theoretical studies state that the Gini index is widely used to measure the absolute specialization of a region as well as the relative specialization with contradictory results as explained by (Bickenbach et al., 2013, p. 1) and (Palan, 2010, p. 22).

3.2 General Properties of Concentration Measures

As discussed earlier, inequality is an expression of a situation where concentration is present: the closer a set distribution is to total equality the smaller the concentration level of this distribution. Concentration is a measure of inequality. This idea stems from the fact that if most of the values of an attribute, within the set under study, are concentrated around one value, then less equality exists between the elements of the set. It follows that if a set’s attribute value is the same for all of its elements, hence total equality distribution, the concentration is minimum.

As Coulter (1989) defined it in his book, concentration is a measure of inequality. In essence, it is the degree of difference between the shares of the elements or components of the system. This is the same as saying that an inequality measure determines the amount of units to be transferred among elements of a system to reach perfect equality. However, concentration is concerned with locating this surplus among elements of a system.

Variance is a measure of distance between the elements of a set and the average value, expressed in terms of the square of the difference. Yet, in our study, variance is not a useful measure of concentration. In fact were we to double the value of the attribute under study, for example to double the value of money allocated in each stock of the portfolio, the variance would quadruple and the mean would double. However the shape of the Lorenz

\hspace{1cm}^{12}\text{If entries with infinitesimal weights are added to the sample, the specialization of the sample is altered drastically because the reference}\ \frac{1}{n}\ \text{has changed.}

\hspace{1cm}^{13}\text{In fact, Coulter (1989) specifies that a typical inequality index gives the same weight to numerous tiny differences as it does to one or two large differences among elements of a system, all other things being equal. The key difference, as he states, between inequality and concentration is the location of the surplus (and deficit) created by inequality.}

\hspace{1cm}^{14}\text{It is to be noted that, however the standard deviation, as the square root of the variance, would remain in scale with the value of dispersion in the data, we will not use it as a measure}
curve related to this portfolio will remain unchanged. Therefore it is essential, in introducing
the concept of concentration in an investment portfolio, to determine the following factors:

- What concentration measure or measures will be used and why? What are the
desirable characteristics for a measure of concentration?

- How will these measures describe the portfolio? Applying the concentration index or
indexes chosen must add value in describing the portfolio at hand. What dimension
of the portfolio will be measured (i.e. what concentration are we measuring)?

3.2.1 Technical characteristics and desirable properties of concentration measures.

In this respect, we point out the existence of many concentration measures applied in
economic research, social welfare, industrial specialization and even chemistry and physics.
As stated by Bickenbach and Bode (2008), choosing between measures of concentration is
indeed choosing between the different definitions of concentration that were applied to derive
such measures. Any inference or conclusion drawn from the results of a concentration or
specialization measure we will use or decide upon in our thesis is interpreted with regards
to the definition of the concentration and specialization we decided to establish in order
to describe an investment portfolio, since the measures we decide to choose will reflect
our definitions that we determined of concentration and specialization in an investment
portfolio.

A variety of criteria are used to choose between the various concentration measures
available. However, we believe the measure of choice should be at least a) relevant to the
attribute or dimension to be measured, b) it should contain enough “information” to draw
conclusions, c) it must be simple and sufficiently easy to use, d) it should have an acceptable
interpretability i.e. it should have a defined upper and lower bound (like 0 and 1 and not 0
and ∞ for example) and finally e) it should be a ratio rather then an interval measurement.
In what follows, we will shed more light on each characteristic we mention in this paragraph.

Relevance For every system containing \( N \) elements and with a known distribution of the
values of an attribute \( A \) to be measured there should be one and only one value for
the index used. Therefor, indexes with a zero denominator are non-relevant since
they have an undefined or infinite value at that point. Indexes using mode or median
as part of its components would yield non-relevant results in bimodal or uniform
distributions.

of concentration because of its dependability on the scale of measure, as outlined in the
text. If, as expressed, the amount of income perceived by each unit of the sample doubles
the inequality among the units remains the same, but the measure of concentration (here
variance as suggested) will quadruple, while the concentration remains unchanged.
Information  The index of choice should provide the maximum possible information about
the distribution it is describing. This means that the index chosen must reflect the
rank and influence of proportionally small elements as well as proportionally big
elements. If the choice of the index requires that the index reflects or points out the
big or the small elements then the index would be conveying the maximum possible
information required.

Simplicity  Although computations are simple today using computers, an index is prefer-
ably simple to use, easy to calculate and straightforward to interpret. Indeed among
all possible and existing concentration indexes, the simpler ones are usually prefer-
able, ceteris paribus.

Interpretation  Interpretability is related to the upper and lower boundaries of the index,
or its maximum and minimum points. Some measures are not bounded and can
vary from $-\infty$ to $+\infty$ or most commonly from 0 to $+\infty$ (Theil, 1967). It is a
common agreement among most of the literature reviewed that the lower boundary
of an index should be zero, indicating perfect equality i.e. when each element has
equal share and hence, minimum concentration. It is also agreed that a positive
increase in the measurement value should indicate a deviation from perfect equality
towards maximum concentration. Additionally, most of the literature indicates that a
measurement or an index tends towards its maximum value of unity when one element
of the system possesses all the available shares and the remaining elements possess
none of the shares. This is very useful when comparing various sets with different
numbers of elements, bounding the values of the inequality index between zero and
one. In our choice between indexes of concentration we will use bounded indexes and
will apply a normalization procedure to make unbounded measures standardized and
bounded. It is should be noted that Cowell (2009) does not agree on the bounding
of a measure from zero to one, deeming it as a “superficial attractiveness”.

Scale of measurement  the scale of measurement can be nominal, ordinal, interval or
ratio scales. Most cases would require a nominal scale to separate the attributes (ex:
north, south, east and west regions of a country) and then consider the application of
a proportion measurement to qualify the concentration of a certain attribute. As an
example, we might want to study the concentration or inequality of age distribution
in the 4 regions of a country. We would divide the country in four nominal classes
and later apply a proportion measurement to describe the age concentration in each
region. In our case, we want to classify the stocks within a portfolio into their
industrial sector (pharmaceutics, services, oil and gas, utility, bank and financial
institution etc...) and later choose a concentration measure to compare the amount
of wealth invested in one sub-sector with respect to the others and describe where is
the investment concentrated. The investor usually requires that the stocks admitted
in his portfolio have a maximum allowable standard deviation value of $x\%$ where this
percentage reflects his risk appetite. In this case we are interested in dividing the standard deviation values into intervals so that we can classify each stock within its corresponding interval. (Please refer to the tables 2.2 and 2.3 for illustration on the interval subdivision for some attributes.). So ultimately we prefer a ratio scale of measurement for its ease of comparison and relative formulation.

Coulter (1989), Cowell (2009) and many others researching on topics related to social welfare and distribution of income theory referred, at one time or another, to concentration and inequality measures and derived most of the concentration indexes utilized today. See e.g. (Pigou, 1920) and (Dalton, 1920) for the Pigou-Dalton principle of transfer, (Kolm, 1976a) and (Kolm, 1976b), (Allison, 1978) and (Schwartz and Winship, 1979) or (Rae, 1981) as well as (Nagel, 1984). Some of the criteria derived in the social welfare and income inequality studies are useful in describing an investment portfolio (Examples are the Transferability Principle and the Scale Invariance Principle discussed hereafter). Some other criteria or principles like the Constant Addition Principle, are not relevant to describing portfolios.

The criteria and principles found in the literature that we believe our concentration index(es) must satisfy are:

a) The Principle of Transfers\(^\text{16}\). The Principle of Transfers states that concentration is diminished if units are transferred from a larger share element to a smaller share element. Cowell (2009) describes two different states of this Principle, namely the Weak Transfer Principle and the Strong Transfer Principle.

The Weak Transfer Principle states that for a given distribution \( S \), with \( w_i = \frac{x_i}{\sum x_i} \), being the fraction of attribute (weights in case of a portfolio) allocated to each element \( s_i \in S \) with \( \sum w_i = 1 \) as the budget constraint, any hypothetical (positive) transfer of weight \( \epsilon \) (in our particular case) from a bigger weight \( w_1 \) to a smaller weight \( w_2 \) \((w_1 > w_2)\) should reduce the measure of concentration.

More explicitly, consider \( w_1 > w_2 \) with \( w_1 = w_2 + \delta \), then any transfer of weight

\(^{15}\) Constant Addition Principle states that concentration diminishes when a positive constant is added to all elements of a set. In income distribution this might present an important use due to the fact that some poorer members of a society might receive a constant aid from governments or other supra-national entities under some poverty eradication plan. In investment portfolios however, this addition is seldom envisaged since the dynamic allocation of the wealth invested among the chosen stocks would keep the budget constraint stable at a certain initial amount invested. If the portfolio produces profit during one period, the profit is seldom re-allocated evenly among all stocks in the portfolio but rather according to a different allocation criteria that would alter the share of each stock in the investment portfolio and hence change the value of concentration.

\(^{16}\) The Principle of Transfers is related, in our analysis to dynamic re-allocation within the portfolio, in time. The measures of specialization and concentration we propose will detect the transfer effected in the allocation between stocks of the portfolio. For a broad comment on the Principle of Transfer please see (Hao and Naiman, 2010),(ch.4, page 44–64).
from $w_1$ to $w_2$, namely $\epsilon = \Delta w_1$, with $\Delta w_1 < 2\delta$ (if not, we will be simply swapping between the weights of $w_1$ and $w_2$), will tend towards reducing inequality within the universe and hence reduces concentration. This is the Weak Transfer Principle, it is called so because it does not quantify the amount of reduction, it just confirms the reduction in concentration.

The Strong Transfer Principle states that in the case of a similar transfer of weight from one heavier element to a lighter one, as in the weak transfer principle, the decrease in concentration depends exclusively on the distance between the weights of both elements, i.e. it should depend only on $\epsilon = \delta w_1$. In other words, the amount of reduction in the concentration index depends only on the distance between the donor and the receiver, regardless of their relative rank within the population under study. It is clear that the index not depending on the ranking of the elements in the mathematical formula will obey the Strong Transfer Principle, whereas those indexes incorporating a ranking multiplicative factor within their analytical expression will not obey the Strong Transfer Principle, since even with the condition $\epsilon = \delta w_1$, with $\Delta w_1 < 2\delta$ satisfied, the amount of reduction in the index does not depend uniquely on $\epsilon$, but rather on a product involving a proportion related to the ranking $i$ of $w_i$. In fact, the size of the change in the value of the concentration index, when a redistribution of weights occurs within a portfolio (re-balancing of wealth distribution or reinvestment of dividends, etc ...), will depend on the relative position of the individual stocks among which the weight transfer occurred and not on the absolute value of their weights. Hence, the transfer of weight between the $1^{st}$ to the $2^{nd}$ stock in ranking does not have the same effect on the index as the transfer of the same amount from the $20^{th}$ to the $21^{st}$ stock in the universe, for example.

b) The Principle of Scale Invariance. This principle requires that if each and every share $w_i$ of the universe under study is multiplied by the same positive constant, the concentration measure should not change and must remain the same. On the social economic level, not all social scientists agree on the fact that inequality should not change if all the elements of the society’s wealth is increased by the same positive index. This feature is important when we consider that the multiplier could be less then one, and is assimilated to have a similar effect as a sudden devaluation event or an inflationary spree; it can also reflect the effect of taxation on the income of a society. In fact many social scientists are also concerned with the “distance” between the wealth of one individual and the other as well as to the minimum line of poverty. In this line of thought, it is not the same to have individual A with 1000 units and B with 5000 units of wealth and the same individual A with 10,000 units and B with 50,000 units. The distance in the second case is much bigger and the social scientists argue that inequality is more pronounced with a difference of 40,000 units between A and B than when the difference or distance was only 4,000 units, although the
proportion is the same. But in our case, since concentration is our basic concern rather than inequality, doubling the wealth invested in each and every stock of the portfolio will not change the description of the portfolio vis-a-vis its concentration level, since the share of each stock with respect to the total wealth invested remains the same. In fact, and from the standpoint of an investor, doubling the investment in each and every stock of the portfolio is similar to creating another portfolio of investment exactly identical to the initial one, and hence no investment opportunity is created. We believe, from a pure investment standpoint, that the concentration index of the investor’s chosen portfolio and the initial index are the same, and hence our choice of an index should focus on a scale invariant index. It is also logical to opt for an index which is scale invariant because a change of currency or an equivalent value of the portfolio in another currency should yield the same concentration value.

c) The Principle of Constant Addition. This Principle states that if a constant amount is added to each and every share of the elements of a universe under study, the concentration must diminish (because the inequality diminishes). The rationale behind this conclusion is easily seen if we envisage the constant amount added to be substantial relative to the amounts or values of each element of the universe, creating a dilution effect on the total concentration.

In other words, when adding the amount \( a \) to each share of the system, \( w_i + a \) with \( a >> w_i \), we can confidently assume that \( w_i + a \simeq a \) for all \( w_i \). This will eventually produce a system whose elements are all approximately equal and hence reduce the concentration to its minimum: all the shares tend to have “more or less” the same value and hence the set tends to get near total equality when the added constant grows to a proportionally large amount. This feature of a concentration index is sought after in social sciences and in plans dealing with poverty levels and a minimum poverty line. The amount added to all the elements of a universe (a fixed sum of money given to all members of a society) will positively remove many people if not all (depending on the amount added to the income of each defined poor citizen) from the state of poverty determined by the minimum poverty line and hence, a social scientist would argue that, poverty being diminished, more equality is achieved among the citizens and hence less wealth is concentrated in the hands of a few. This feature is not relevant in an investment portfolio because, as mentioned earlier, any additional amount that is produced as dividend will be dynamically allocated according to an investment criteria rather than blindly added at equal amounts to all the elements of the portfolio.


i. Symmetry of population. This principle requires that the concentration measure should not depend on the number of the population \( n \), and it should remain constant remains constant if we merge two populations with the same
number of elements \( n \) and with the same inequality of a certain attribute. If we measure inequality in a particular economy with \( n \) people in it and then merge the economy with another identical one, we get a combined economy with a population of \( 2n \), and with the same proportion of the population receiving any given income. If measured inequality is the same for any such replication of the economy, then the inequality measure satisfies the principle of population. This is not the case if we are applying the index to an investment portfolio. In fact, merging two investment portfolios each with \( n \) stocks and with a certain level of weight concentration will result in a new third portfolio where concentration might not be the same if both portfolios contain at least one identical stock or when each portfolio has a different specialization\(^{17}\). If both portfolios contain different stocks then the combined portfolio will result in a \( 2n \) stocks portfolio with the different specialization level and maybe different weight concentration measure. Additionally, assuming we have two portfolios of \( n \) stocks each, and assuming no stock is existing in both portfolios at the same time, combining both portfolios will not necessarily result in a portfolio with the same concentration and specialization levels, because the stocks of each portfolio might not have exactly the same attributes (location, industrial sector, level of expected return, etc.).

As an illustration, consider the following vector of screening attributes that will produce a portfolio \( F_1 \) with certain desired attributes, and \( n \) stocks with an concentration index of \( I_1 \):

\[
\vec{p}_1 = \{ \text{USA stocks, non-financial Co., } P/E \geq 15, \text{ Debt/Assets} \leq 33.3\% \}. 
\]

Now assume we create another investment portfolio \( F_2 \) of exactly the same number of stocks \( n \) and resulting in the same concentration index value \( I_1 = I_2 \), changing one screening criteria from USA market to EUROPEAN (EUR) market stocks and keeping all other screening criteria unchanged:

\[
\vec{p}_2 = \{ \text{EUR stocks, non-financial Co., } P/E \geq 15, \text{ Debt/Assets} \leq 33.3\% \}. 
\]

It is clear that \( F_1 \) and \( F_2 \) do not have any common stock existing in both portfolios at the same time. Combining both portfolios will result in a new portfolio \( F_3 \) with \( 2n \) number of stocks and with \( I_3 = I_1 \oplus I_2 \)\(^{18}\) concentration index, but with less concentration on USA stock market since the \( F_2 \) contains European stock market. Hence the Specialization of portfolio \( F_3 \) is not the same as \( F_1 \) and \( F_2 \). In fact \( F_3 \) is the result of the following screening criteria:

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\(^{17}\)This is similar to saying that, if we add the additional dimension, from the social science perspective, of gender or race, then the concentration relative to these aspects will change. Just as, in the merged portfolios, the nature of the stock (its attributes), is at the heart of the concentration we are trying to measure.

\(^{18}\)the symbol \( \oplus \) is used here instead of the symbol + to avoid implying tacit additive properties of the concentration index.
$\vec{p}_3 = \{\text{USA + EUR stocks, non-financial Co., } P/E \geq 15, \text{Debt/Assets} \leq 33.3\%\}.$

This screening vector yields a less specialization in USA or European stocks alone, but it is rather specialized in BOTH markets and hence is different from $F_1$ and from $F_2$.

ii. Sensitivity to the size of population. This principle differentiates two different types of indexes: those sensitive to the size of the population and those which are not. Coulter (1989), defines those indexes whose values are independent from the number of components as *relative* measures of inequality, and those whose measures are sensitive or dependent on the number of components as *absolute* measures of inequality, as discussed by Waldman (1976). Each category has its proper usage and caveats. This principle in particular is very important in political sciences, voting systems and parties. The research work of Taagapera (1979), related to political parties and voting systems, sheds an important light on the number of components of the population and the political ostracism or concentration of power in the hands of few individuals or one party. The concept of null components (in the case of Taagapera, the individuals who did not vote) represents a serious issue in political sciences and might have its reflection in finance and investment as we shall see.

- Relative indexes or those indexes not affected by the number of the population are indexes whose extremums (minimum and maximum) do not depend nor vary with the number of components $n$. As an example, if we have a population of five individuals where 90% of the wealth is concentrated in the hands of two of the five persons, then a relative index should yield the same value if the population consists of 100 individuals and 90% of the wealth is concentrated in the hands of the same two persons. This appears to be inappropriate since, intuitively, we know that the second population of $n = 100$ appears to be more unequal than the first, if we consider social welfare as the criteria for comparison and judgment.

- Absolute indexes are those indexes depending on the number $n$ of the population and whose extreme values depend on $n$. Waldman (September, 1977) defines various types of absolute indexes depending on the inclusion of null and non-null components. It is of an importance to note the difference in concentration, for example, in a society where a candidate in District A won 90% of the votes with half of the potential voters not voting (null entries) and a candidate in District B won with 90% of the votes with no null entries registered, all other things being equal. A relative index, not sensitive to the number of potential voters $n$, would yield the same result for districts A and B, whereas an absolute index would result in different readings, showing more inequality in district A.
Chapter 3. Concentration and Specialization

(Taagapera, 1979).

The Principle of Sensitivity to the size of population is controversial and admits subjective points of view. Consider the following example:

In a population of 100 industries, 5 industries control 99% of the industrial output and the remaining 1% is shared equally among the remaining 95 industries. Many an index would reflect a pronounced concentration in such a population.

Now assume that the 5 controlling industries buy out the remaining minor 95 industries and divide the 1% equally among each other, a relative index would show perfect equality (all industries are perfectly divided among 5 players) whereas an Absolute index would tend to reflect gross inequality, because the count of null entries is taken into consideration. In measuring specialization in an investment portfolio, we want the index to reflect gross inequality, i.e. we want our index to detect that, even if 100% of the weight is distributed equally among 5 stocks of a particular industry sector of a 100 stocks portfolio, this portfolio is specialized in this industry sector.

This means that the index should take the null entries into consideration and therefore, as per the principle of sensitivity, the index of choice to measure specialization should be an Absolute index.

e) The Principle of Decomposability. This Principle states that, within a population, the concentration measure should be decomposable into the concentrations of the subgroups conforming the population. This essentially refers to the determination of the contribution of each subgroup to the total population under study. More precisely, the overall concentration of a population should be expressed as a function of the concentration within each of its subgroups and as a function of the concentration among or between those subgroups. Particular attention is given to this characteristic when economic and social welfare is concerned. In fact, the distribution of wealth is not even between regions of a country, or between subgroups of a population like blacks and whites, males and females, ethnic and religious subgroups etc.. In our case, we will consider the decomposability of our index of choice according to the relevance of such a measure when we come to discuss it in full. It is worth anticipating that the decomposition of a concentration measure does not indicate the location of the concentration or, in other terms, does not identify which subgroup is more concentrated then the other but rather will indicate the contribution of the subgroup to the overall concentration of the sample, like is the case in the Gini decomposition that we will consider later in this research.

Having analyzed the various characteristics of the concentration measures at large, the following section will further elaborate on applying the concept of measuring concentration within an investment portfolio. The ultimate goal is to choose one or more indexes to apply them to investment portfolio that will add a descriptive value in the tool kit available to
the investor. We need to define what we are interested in measuring and how we will be measuring it.

3.3 Concentration Measures for Investment Portfolios

The conceptualization of the term “concentration” is derived, as mentioned earlier, from a wider and more generalized situation of inequality. Concentration is a dimension or a proxy of inequality, as stated by Coulter (1989). We want to measure the degree of inequality between the elements of an investment portfolio, when the weights of the individual stocks are not all the same i.e. when \( w_i \neq 1/n \). In fact, if all weights are not equal then some stocks are attributed more weight then others. It is within this background that we interpret concentration as a form of inequality and we want to quantify it. However, the word concentration bares in its meaning the sense of location. If the portfolio is concentrated then it is intuitive to say it is concentrated somewhere. Whereas, inequality is a more generic term: we say the weights of the portfolio are not all equal and hence there is inequality within the portfolio, with no sense of location or cluster of weights.

The measures of concentration that we are studying are considered anonymous measures: They will tell us that the portfolio is concentrated but they cannot possibly point out to the individual stocks which have more weight then the others. When we group the stocks into attribute groups like industry or geographical groups for example, then, as we shall see later, by decomposing the particular index of concentration, we will be able to detect which group “contributes” to the overall concentration of the portfolio. This does not mean that the decomposition will show the “heavier” subgroup. It will only calculate the contribution of the subgroup to the overall concentration. If, for example, a subgroup represents 90% of the total weight (i.e. the portfolio is concentrated in one subgroup but the elements within the subgroup are equally weighted, then the decomposed index shall not detect the concentration IN the subgroup, and will attribute the concentration to the remaining 10% subgroups. This is the contradictory results of the Gini decomposition that we mentioned earlier, which renders the usage of the decomposition technique in our particular case useless).

In this thesis we are introducing an additional measure that will describe better an investment portfolio, along with the already existing descriptors like expected return \( E(r) \), standard deviation of the historical returns \( \sigma(r) \) as well as all other available moments of those returns. We want to look at the stocks of the portfolio and be able to assess whether the portfolio is concentrated, i.e. how much deviated from a neutral \( 1/n \) allocation position it is; at the same time we want to know if the portfolio at hand is specialized in certain
attributes compared to others\(^{19}\).

The approach we suggest hereafter, is to describe the portfolio by looking at the concentration that the wealth allocation exhibits and at the specialization of the given portfolio according to the attributes (i.e. groups within the portfolio) specified by the analyst or the investor. However, because the measures of concentration\(^{20}\) are numerous and each one reflects a certain aspect of the inequality with respect to the elements of the analysis, we need to define and specify very clearly what we want to measure and what will this measure add to our information on the portfolio at hand, before we venture into choosing the appropriate measure(s) available in the research.

In practice, we are looking to assess and quantify the following additional information related to an investment portfolio:

1. Concentration of wealth or wealth distribution profile: We need to assess if, within the portfolio, the investor’s wealth is concentrated in a small number of stocks in detriment of the remaining stocks. This will measure the concentration of the portfolio i.e. it will describe the distribution of the amount of wealth allocated within it. This concentration of wealth, when existing, can reflect the attitude of the investor towards those stocks where the wealth is concentrated: The investor is betting more chips on some stocks rather than on others within the portfolio (i.e. among the stocks that he already decided to include in his portfolio)\(^{21}\). We are seeking to describe the allocation strategy within the portfolio through using a concentration measure.

2. Absolute Specialization of the portfolio: We need to assess in which attributes is the wealth allocated. This information will describe the amount of concentration of the wealth in a given attribute compared to others, hence showing a dominance of this attribute over the others with respect to its wealth allocation. This will describe the specialization of the portfolio by measuring the concentration of the attributes or groups within the portfolio\(^{22}\). The concentration of the portfolio discussed above in 1. is not enough to give the analyst or the investor a clear picture of the description of the portfolio. In fact, some stocks move in and out of the Investment portfolio with time (e.g. transferring the wealth of a stock from Japan that is excluded from

\(^{19}\)As discussed earlier in this thesis those attributes are subjective to the choices and preferences of the analyst or the investor. One given stock can have as many attributes as the analyst desires, and hence the measure of specialization relates directly to those attributes deliberately chosen by the analyst. In theory, a stock can have a very large set of attributes.

\(^{20}\)Almost entirely derived from the concept of inequality of income distribution within a population

\(^{21}\)The attitude of the investor towards placing more wealth on some stocks rather than on others, and hence creating a deviation from the \(1/n\) neutral allocation strategy was defined as the “Pretension Level” of the investor in Appendix B of this thesis.

\(^{22}\)Attributes like industry sectors or geographic location or family owned business, for example. Please refer to Section 2.1.2, page 17.
the portfolio to a replacing stock from the USA, will not alter the concentration but will alter the geographic specialization). Similarly, the dynamic allocation by shifting some of the wealth from a stock $S_i$ to stock $S_j$ will alter not only the concentration but also the specialization of the portfolio. This transfer of wealth from $S_j$ to $S_i$ could have increased the specialization in one attribute of $S_j$ while decreasing the specialization in the attribute of $S_i$. The measure we choose should be able to give information on the contribution of the concentration of each attribute to the total specialization of the portfolio.

3. Relative specialization of the portfolio: We need to compare the specialization of the portfolio under study with the initial Investment Universe. Assume the investor chooses 20 stocks from the top 500 stocks of the USA market. The specialization of his portfolio is not necessarily the same as the initial Investment Universe from where he picked his stocks. This comparison will allow the analyst to relate the behavior or the performance of his portfolio to some specialization differences or similarities to the initial Investment Universe. For example, in reference to the USA stock market, with approximately 67 industrial sectors listed, and if the portfolio contains only 10 sectors chosen out of this Investment Universe, then this choice exhibits a certain specialization that the index $I_s$ must reflect, when comparing the the specialization of the Investment Universe against the portfolio at hand.

23Consider that at time $t_0$ we have a given portfolio with a given wealth allocation matrix. Assume that, at time $t_1$, we transfer a small amount of the wealth allocated to stock $S_1$ (this amount $\epsilon$ described in Subsection 3.2.1, a) The Principle of Transfer), which is a metallurgic company to stock $S_2$ which is a retail company. We can confidently say that the portfolio became more specialized in retail and less specialized in metallurgic industrial sectors at time $t_1$ after the reallocation was executed.

24We cannot conclude or infer that there is a causal relationship or a relevant correlation between the performance of the portfolio and its specialization and/or concentration measures. Our study does not deal with this interesting issue that we believe is the subject of more thorough research in the future. This particular direction of studying the correlation between performance and concentration and specialization falls outside the scope of our thesis.

25Using classified sub-sectors according to a commonly used classification system like the GICS or the NAICS (GICS stands for Global Industrial Classification Standard and is an eight digit classification number that is used in the CRSP/COMPUSTAT market data base). A common practice is choosing the industrial sectors using GICS (sometimes appearing as GIND and SUBGIND in the database mentioned) to a six-digit filtering, resulting in approximately 67 different industrial sectors listed in the CRSP/COMPUSTAT database. NAICS stands for North American Industrial Classification Standard, and is a similar classification system used mainly by banks and governments to classify the industrial activities of companies.
3.4 Choosing the proper indexes to measure Concentration and Specialization

We have at hand an investment portfolio consisting of stocks with a weight allocated to each stock and an investor-defined attributes that will subdivide the portfolio into attributes grouping. Based on a hypothetical portfolio shown in Table 2.1, we created consequently the weight-attribute matrix in Table 2.2 and the subsequent impact matrix in Table 2.3, pages 24 and 25 consecutively. The data we have at hand is: the individual stocks, their weights and their attributes. When we group the stocks with respect to their attributes as in the weight-attribute table mentioned, we can have additional data: number (count) of stocks per group, individual stock’s and total weight per group. This available data will be used to determine the concentration and the specialization of the portfolio. The indexes or measures we choose to produce the results must obey to a set of criteria that we define hereafter, so that the index chosen reflects the best the scalar we want to measure and use in representation of measuring the concentration and the specialization of the portfolio.

3.4.1 Determinants of choice for a concentration measure applied to investment portfolios.

Earlier in this chapter, we gave a general overview of the selection criteria and the desired principles for concentration measures. In this subsection, we will narrow down the choice of measures available by deciding upon which principles and criteria are most related and applicable to an investment portfolio.

- We require to measure the concentration of the weights of $n$ stocks composing an investment portfolio without any consideration to the attributes grouping involved. Hence the decomposability of the index $I_c$ is not a relevant feature for the chosen concentration measure.

- We require to assess to which degree is the distribution of the wealth allocation deviant from an equally weighted portfolio with all its weights $w_i, i = 1, \ldots, n = 1/n$. The biggest a particular weight is, more deviation from the $1/n$ allocation exists, and hence, more concentration is exhibited by this distribution. We need an index that emphasizes the effect of “big” weights. It appears that an index with an exponent on the weights in its formula composition will amplifies the effect of this deviation from $1/n$.

- Another relevant criteria is that we require the concentration measure $I_c$ to obey the weak transfer principle discussed in Section 3.2.1, i.e. the concentration measure is...
of our choice should diminish if a transfer of weight occurs from a higher share to a lower one. This feature is desirable because it will reflect, in a time panel data, the changes in dynamic allocation within the existing stocks in one single measure. Additionally, the concentration should diminish more if the distance between the “donor” and the “receiver” is bigger. This is the strong transfer principle. This means that, the more the difference between the higher share and the lower share, the more the impact on concentration. This will reflect the nature of the reallocation, once it happens and will add to the relevance of the measure we are choosing: Not only the concentration diminishes when a transfer occurs from a higher to a lower share, but also it diminishes even more when the distance between the higher and the lower share is bigger. This in fact proxies the rank of the stock within the portfolio: The concentration must diminish more if the difference in rank between the larger and smaller stocks is bigger. In a portfolio of \( n \) stocks, ranked in a descendant sorting, from the highest \( S_1 \) to lowest weight \( S_n \), a transfer of \( \epsilon \) from \( S_1 \) to \( S_n \) diminishes the concentration more then the same transfer of \( \epsilon \) from \( S_1 \) to \( S_m \) with \( m > n \).

- The measure of choice should be scale invariant. In fact, if the investor decides to double his stakes in all the stocks, the measure of concentration should yield the same result. However, if the investor decides to add to his stakes a constant amount to all the stocks, then intuitively the measure of choice should reflect a diminishing in the concentration. At the limit, if the constant amount \( W_i \) added to each and every stock is very big compared to the wealth \( w_i \) already allocated to each, i.e. \( W_i \gg w_i \), this addition tends to smooth out the differences between the stocks because the distribution of weights will tend to an equally weighted distribution with \( W_i + w_i \approx W_i \) when \( W_i \gg W_i \).

- The measure of concentration \( I_C \) we require should not necessarily be sensitive to the size of the population involved nor to the symmetry of the population. The set of data we have at hand is a finite set, consisting of the weights of the stocks included in a portfolio of investment: \( w_i \) with \( i = 1, 2, \ldots, n \) and where \( n \), the population size, is the number of stocks in the portfolio. We also know by definition, that the concentration level is minimum when the stocks are all equally weighted, i.e. if the budget constraint is 1 as per definition, then this condition is met when each individual weight is equal to \( w_i = 1/n \) for all \( i \).

The determinants of choice for a proper concentration index to be applied to an investment index are not necessarily exhaustive nor exclusive. As stated earlier, besides being a matter of choice of the analyst to decide upon which existing index to use, the index...

\(^{27}\) The measure of concentration will not detect the reallocation of wealth from a stock that is excluded from the portfolio to a new stock. If the weight reallocation is the same, i.e. if the weight distribution is the same, the concentration measure will remain unchanged, although one stock was replaced with another.
must represent as much as possible the definition of the scalar to be measured and hence convey, as much as possible, the maximum amount of relevant information to the analyst to be able to draw proper conclusions. The quality and relevance of these conclusions are directly related to the quality and relevance of the information that the index of choice is conveying. Please refer to Table 3.1, page 53 for a summary of the points above.

3.4.2 Determinants of choice for a specialization measure applied to investment portfolios.

We measure specialization with an inequality measurement that quantify the amount of inequality between the weights of the attributes of an investment portfolio. This inequality is measured using a concentration measure. Additionally, Specialization and concentration are widely interrelated in an investment portfolio: The presence of a concentration in a portfolio indicates the existence of specialization: more weight is allocated to some stocks in detriment to some others and, therefore, more importance is given to some attributes against some others, introducing specialization. Since the input to our measurements are weights, then we can conclude that specialization is a concentration of weights in some attributes against some others related to the investment portfolio at hand.

This leads us to conclude that we require to measure the specialization of the portfolio vis-à-vis the attributes of its stocks, as determined by the investor or the analyst, as specified in Section 3.3 above. This specialization measure, which will be denoted as $I_s$ for the time being, would describe the portfolio with respect to its attributes as reflected in the weight-attribute matrix depicted in Table 2.2. Some of the features of this measure of specialization are the following:

- The measure $I_s$ being a measure of concentration as well, should have the same properties of the concentration measure that we discussed above in Section 3.4.1, in addition to the decomposability that is required when measuring the concentration of a set divided into groups.

- The decomposability of the index of choice $I_s$ is required when the specialization is measured within the portfolio, and the analyst needs to assess the contribution of each attribute to the total specialization of the portfolio. If the portfolio exhibits concentration therefore it is specialized in certain attributes against some others. The decomposed index will help determine the concentration within each group (each attribute defines a group) and its contribution to the overall index. But, as mentioned earlier, this decomposition will not locate in which subgroup is the specialization. We conclude that the decomposition is not relevant in the specialization measure.

- We prefer that the specialization measure $I_s$ to obey the transfer principle, although not required when measuring the absolute specialization. This will allow us to detect and assert that in case the specialization of the portfolio in a given sector has
diminished in the next observation time, then a transfer of weight from a heavier to a lighter stock has occurred.

Those features and characteristics are summarized in Table 3.1.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>$I_s$</th>
<th>$I_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Application concentration</td>
<td>Big weights</td>
<td>Ranking</td>
</tr>
<tr>
<td>Concentration measure sensitivity</td>
<td>Yes</td>
<td>Not important</td>
</tr>
<tr>
<td>Population size sensitivity</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Population symmetry</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Transfer principle</td>
<td>weak and strong</td>
<td>Weak</td>
</tr>
<tr>
<td>Scale invariance</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Decomposability relevance</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Maximum value</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Minimum value</td>
<td>$1/n \to 0$</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3.1: Summary of some of the most relevant determinants of choice for a concentration index $I_c$ and a specialization index $I_s$.

From the Table 3.1, it is apparent that we need more than one index to respond to all the desired characteristics for our study.

For the concentration of the portfolio, we need to find an index that, in addition to what is listed above as characteristics of an investment portfolio descriptor, has the possibility of giving a sense of size to the portfolio in question. The concentration measure must provide to the investor a certain sense of an equivalent portfolio with an allocation of $1/n_{eq}$ among $n_{eq}$ stocks equivalent to the resulting concentration of the allocation of the same wealth in the $n$ available stocks. For example, if we have chosen 20 stocks to allocate our wealth, the resulting measure of concentration must inform the investor that this wealth distribution that was allocated to those 20 stocks is equivalent to allocating the same wealth equally in 4 stocks for example. This will give the investor a sense of volume or shape of his allocation.

The next section will discuss in detail each of these two indexes and will conclude with a table exhibiting the characteristics of each one in what pertains to measuring inequality or concentration in an investment portfolio with a view to interpret this measurement as an additional descriptor of this portfolio.

### 3.5 The Hirschman-Herfindahl and the Gini indexes.

The previous section presented a discussion of the determinants of choice for an index to measure concentration and an index to measure specialization in a portfolio. The desired
Chapter 3. Concentration and Specialization

characteristics that influenced the choice were oriented mainly towards the need to find the indexes that convey as much information as possible concerning the concentration and the specialization of an investment portfolio. As we shall see in this section, although our indexes of choice do not use all the information available in the investment portfolio, namely the null entries, we shall be able to remedy this issue by virtue of the interpretation of the results that are obtained rather then by the philosophical interpretation of the numbers themselves.

The indexes chosen, namely the Hirschman-Herfindahl Index or $HHI$ to measure concentration and the Gini index or $G$ to measure specialization of an investment portfolio do not take into account the null entries in the subset of data under consideration. This means that the measures will compute the inequality (whether we call it concentration of the weights of the portfolio or specialization in some attributes of the stocks of the portfolio) among non null and positive elements of the data set, in our case being the weights of the stocks within the portfolio. This issue will be discussed fully in the present section.

Notwithstanding the exclusion of null entries as a criticism to the choice of our indexes, the relevance of the results and their interpretability make up for this rather technical characteristic. The $HHI$ and the $G$ indexes are properly bound, as we shall see, which makes the interpretation of the results as well the comparison of the indexes among several portfolio very useful and information rich.

In what follows we shall explore the features and characteristics of the $HHI$ and the $Gini$ indexes as applied to an investment portfolio.

3.5.1 $HHI$, the Hirschman-Herfindahl Index

The Hirschman-Herfindahl Index\(^\text{28}\), or $HHI$, is defined as the sum of the square of the weights of each element of the universe where the weights are taken as a percentage of the total.

In a portfolio of $n$ stocks, we assign the value $x_i$ to the wealth invested in stock $i$, with $x_i > 0\(^\text{29}\), \forall i = 1, 2, \ldots, n$, then we can define $w_i$ as the weight of stock $i$ relative to the universe of stocks:

$$w_i = \frac{x_i}{\sum_{i=1}^{n} x_i}, \text{ with } \sum_{i=1}^{n} w_i = 1 \text{ (budget constraint).}$$

\(^{28}\)The Hirschman-Herfindahl Index or $HHI$ (also known as Herfindahl Index) is essentially used to measure the size of firms in relation to the industry and as an indicator of the amount of competition among them. Named after economists Orris C. Herfindahl (1950) and Albert O. Hirschman (1945).

\(^{29}\)Throughout this thesis and by assumption, the wealth invested in each stock is strictly positive, representing always long position on the stock. By initial assumption, no shorting of stocks is allowed.
3.5. The Hirschman-Herfindahl and the Gini indexes.

We define the \( HHI \) as the sum of the squares of the weights, i.e.

\[
HHI = \sum_{i=1}^{n} \left( \frac{x_i}{\sum_{i} x_i} \right)^2 = \sum_{i=1}^{n} w_i^2.
\] (3.2)

Notice that the \( HHI \) represents the concentration level of the portfolio chosen. In general, the \( HHI \) can be used to calculate the concentration of any descriptor of the portfolio such as returns or standard deviation, depending on the choice and definition of \( x_i \). The maximum value possible for the \( HHI \) is one; this occurs when all of the weights except one are equal to zero. The minimum possible value of \( HHI \) in a portfolio occurs when all weights are equal, representing an equally weighted portfolio that we shall define later as the neutral portfolio with,

\[
w_1 = w_2 = \cdots = w_n = \frac{1}{n}.
\]

Hence \( HHI_{\text{min}} = n \times \frac{1}{n^2} = \frac{n}{n^2} = \frac{1}{n} \).

The \( HHI \) cannot be lower than this value for a given portfolio of \( n \) stocks. This extreme case would represent the situation of no concentration or total equality; all stocks are equally weighted. In this case, the greater \( n \) is, the lower \( HHI_{\text{min}} \) and theoretically

\[
HHI_{\text{min}} = \lim_{n \to \infty} \frac{1}{n} = 0.
\]

The \( HHI \) is sensitive to large weight values in a portfolio. The bigger some weights are, then the smaller the effect of the small weights on the index. In short, \( HHI \) is an index that emphasizes the presence of stocks with relatively big weights since the smaller amounts when squared become smaller and have a reduced effect on the index. Also important to note is that the \( HHI \) omits the null entries and only considers entries different from zero.

As an example consider Table 3.2, where the effect of moving 0.02 from \( w_5 \) to \( w_4 \) in portfolio P2 (resulting in a \( HHI = 0.2278 \)) is less striking than the effect of moving the same amount 0.02 from \( w_2 \) to the larger “player”, \( w_1 \) in portfolio P3 (resulting in \( HHI = 0.231 \)).

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_1 )</td>
<td>0.32</td>
<td>0.32</td>
<td>0.34</td>
</tr>
<tr>
<td>( w_2 )</td>
<td>0.23</td>
<td>0.23</td>
<td>0.21</td>
</tr>
<tr>
<td>( w_3 )</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>( w_4 )</td>
<td>0.13</td>
<td>0.15</td>
<td>0.13</td>
</tr>
<tr>
<td>( w_5 )</td>
<td>0.12</td>
<td>0.1</td>
<td>0.12</td>
</tr>
<tr>
<td>HHI</td>
<td>0.2266</td>
<td>0.2278</td>
<td>0.231</td>
</tr>
</tbody>
</table>

Table 3.2: The effect on \( HHI \) of moving 0.02 from \( w_5 \) to \( w_4 \) (P1 to P2, with a \( \Delta(HHI) = 0.0012 \)) is lower in comparison with moving the same 0.02 from \( w_2 \) to \( w_1 \) (P2 to P3, with a \( \Delta(HHI) = 0.0032 \)). This shows that \( HHI \) emphasizes more the presence of larger elements against smaller ones.
An interesting variation of the $HHI$ is the reverse $HHI$ or $rHHI$. The $rHHI$ represents the number of equally weighted stocks $m$ which would generate the same value of the index $HHI$. This is derived from the boundary conditions of the $HHI$ discussed above. When $HHI = 1$, it means that the concentration is on one stock among all the others possible stocks; in turn, an $rHHI$ of one says that the equivalent universe of 1 stock is representative of the universe under study. Consequently, when the $HHI$ is at its minimum value of $HHI = \frac{1}{n}$, the $rHHI$ equals $n$ which indicates that this concentration can only be achieved with an equivalent number of stocks “of equal weights”. Additionally, the $HHI$ being the square of the weights of the stocks of a portfolio, the small weights are ignored and the resulting equivalent $m$ stock universe will reflect the number of the higher weighted stocks.\(^3\)

The $HHI$ has an upper bound of 1 when total concentration is achieved and a lower bound of $1/n$ when minimum concentration (total equality) is achieved. As noted, the lower limit is sensitive to the number of components $n$. It is however possible to bound the $HHI$ from 0 to 1 instead of $1/n$ to 1 resulting in a Corrected $HHI$:

$$Corrected\; HHI = cHHI = 1 - \frac{1 - HHI}{1 - \frac{1}{n}}$$

(3.3)

The $cHHI$ will have a value of zero (instead of $1/n$) when total equality is achieved and will remain with 1 as its upper bound when maximum concentration is reached.

We now review the $HHI$ in reference to the desired characteristics of a measure of concentration. These characteristics are summarized in Table 3.5 for ease of reference.

\(\beta\) defines a family of information curves based on the “distance” concept or inequality aversion value between members of the set. A particular value of this measure is reached when $\beta = 1$. Substituting $\beta = 1$, (and noting that $\sum s_i = 1$ as the budget constraint), in the generalized inequality measure above we reach

$$\frac{1}{2} \left[ \sum s_i^2 - \frac{1}{n} \right] = \frac{1}{2} \left[ HHI - \frac{1}{n} \right]$$

which shows that the $HHI$ is an information index. For a complete derivation of this formula please consult (Cowell, 2009) pp. 56 to 60.

\(^3\)It is worth noting that $HHI$ is a special case of generalized entropy index, i.e. it carries an “information” on the disorder within the system, namely inequality or concentration. Cowell in his book “Measuring Inequality” (Cowell, 2009) derives an inequality measure based on entropy. He considers the shares of every entry with respect to the total amount of value measured $s_i$ as similar to the probability $p_i$ of an event to occur within the system. He bases his derivation on the work of Theil (Theil, 1967) to reach a generalized inequality measure.
3.5. The Hirschman-Herfindahl and the Gini indexes.

Characteristics of $HHI$

**Concentration measure.** Given its emphasis on big players rather than on small players, $HHI$ indicates where the big shares are and hence, by definition, reflects concentration.

**Population size sensitivity.** $HHI$ is slightly Dependant on the size of the population. In fact the lower boundary of $HHI = 1/n$, where $n$ is the population size. The upper boundary, in contrast, does not depend on the population size. However, we can fairly assume that when measuring concentration in an investment portfolio where the number of stocks is fairly high (a usual portfolio will have around 30 stocks and over), the lower bound of $HHI$ tends to the minimum of $1/n \approx 0$. It is to be noted that, $cHHI$, the corrected version of $HHI$ in Eq. 3.3 cancels out the effect of $n$ and allows the comparison of two portfolios with relatively small population sizes, since $0 \leq cHHI \leq 1$, $\forall n$.

**Transfer principle.** As discussed in Section 3.2.1, $HHI$ clearly obeys the weak as well as the strong transfer principle. In fact a transfer of wealth from one higher wealth allocated stock to a lower one will not only reduce inequality (the weak transfer principle), but is also dependent on the “distance” between the donor and the recipient. This is because $HHI$ emphasizes the concentration of allocation on bigger players and tends to under estimate the smaller ones. See Cowell (2009)(pages 150-153) for a calculation of the effect of a transfer of an infinitesimal amount from stock $j$ to stock $i$ for $HHI$.

**Scale invariance.** $HHI$ is scale invariant because a proportionate increase in the size of each share will not alter the value of $HHI$.

**Population Symmetry.** $HHI$ violates the principle of population symmetry. If two portfolios with an identical $HHI$ but with different stocks are merged the resulting $HHI$ is halved because $n$ is doubled.

**Decomposability.** The $HHI$ is easily decomposable. In fact assume we have a population of $n$ elements with $HHI = H_T$. Assume this population is split into two different groups with no overlapping or common elements, group 1 with $n_1$ elements and a share of the initial universe of $\alpha_1 = \frac{n_1}{n}$ and $HHI = H_1$ and group 2 with $n_2$ elements and a share of the initial universe of $\alpha_2 = \frac{n_2}{n}$ and $HHI = H_2$, with $n = n_1 + n_2$.

It can be shown that $H_T = \alpha_1 H_1 + \alpha_2 H_2$.

However, this decomposition does not show any inter-subgroup concentration measure as is the case of the Gini index that is the topic of Section 3.5.2. But since

31In fact, $H_T = \sum \frac{w_i^2}{n}$, $H_1 = \sum \frac{w_i^2}{n_1}$ and $H_2 = \sum \frac{w_i^2}{n_2}$, with $1 \leq i \leq n$, $1 \leq j \leq n_1$ and $n_1 + 1 \leq k \leq n$. Noting that $\sum w_i^2 = \sum w_j^2 + \sum w_k^2$ we can write $H_T = \sum \frac{w_i^2}{n} = \sum \frac{w_j^2}{n_1} + \sum \frac{w_k^2}{n_2}$. Substituting the values of $\sum w_j^2 = n_1 H_1$ and $\sum w_k^2 = n_2 H_2$ we get $H_T = \alpha_1 H_1 + \alpha_2 H_2$. 

we will use the HHI to measure the concentration of the resulting portfolio, the decomposability feature is of low relevance to us, as shown in Table 3.5.

**Simplicity.** HHI is a simple and easily calculated index. It is also widely used and popular.

### 3.5.2 The Gini Index

The Gini index\(^{32}\) (hereafter, the Gini Index, Gini and the symbol \(G\) are used interchangeably) is widely used in the field of concentration measurement. It is closely related to the Lorenz curve (Figure 3.1), where the cumulative share of each element of the universe is plotted against the rank of each share, sorted from the smallest to the largest. If all stocks have the same market share, then the Lorenz curve is a straight diagonal line, called the line of equality (minimum possible concentration, i.e. a wealth distribution of \(\frac{1}{n}\)). If there is any inequality, then the Lorenz curve falls below the line of equality showing concentration. The Gini index measures this deviation from the total equality line as seen in Figure 3.1. This deviation from the total equality line, is the concentration level that the Gini index measures.

The Gini index is usually defined mathematically based on the Lorenz curve. Geometrically, the index is defined as equal to twice the area between the 45 deg line (marking total equality or the total equality line) and the Lorenz curve and can be formally written as:

\[
G = 1 - 2 \int_0^1 L(\pi) d\pi.
\]

Where \(L(\pi) = M\) is the Lorenz curve corresponding to a distribution \(w_i\) with \(\pi = F(w_i)\) as the corresponding first moment distribution of \(w_i\) (Savaglio and Vanucci, 2008).

There exists numerous computational formula for implementing the Gini index. Probably the most revealing one is that used by Dagum (Dagum, 1997b,a) and Mussard (Mussard and Terraza, 2004):

\[
G = \frac{1}{2n^2 \mu} \sum_{i=1}^{n} \sum_{j=1}^{n} |w_i - w_j|
\]

(3.4)

Where \(n\) is the number of observations, \(i\) is the rank of the observation in an ascending series, and \(w_i\) is the weight vector with \(w_i = \frac{x_i}{\sum x_i}\) and \(\sum w_i = 1\) as the budget constraint, and \(\mu\) as the average value of the attribute under measurement. The expression of Gini above stated demonstrates that the Gini index measures the weighted average of pairwise differences between individual elements in the sample. It is the overall pairwise mean differences that matters regardless of the subgroups clustering the elements.

---

\(^{32}\)The Gini index (also known as the Gini coefficient or Gini ratio) is a measure of statistical dispersion developed by the Italian statistician and sociologist Corrado Gini and published in his 1912 paper “Variability and Mutability” (Italian: Variabilità e Mutabilità).
If the sample is finite and hence the sorting of its elements is possible, then a practical approximation to the Gini index $G$ can be written as:

$$G = \frac{1}{n} \sum_{i=1}^{n} (2i - n - 1)w_i.$$  \hspace{1cm} (3.5)

The summation core of the Gini expression is the rank of its components ordered in descending order as observed in Equation 3.5 above, which in fact emphasizes and amplifies the values in the middle range or in the modal category of the distribution rather than on the extremities.

A generalized form of the pairwise combination expression of the Gini index can be expressed as follows, and is referred to as the Generalized Gini index, $G^{(\alpha)}$, of order $\alpha$ where $\alpha > 0$:

$$G^{(\alpha)} = \frac{1}{2n^2 \mu^\alpha} \sum_{i=1}^{n} \sum_{j=1}^{n} |w_i - w_j|^{\alpha}.$$ \hspace{1cm} (3.6)

The generalized Gini index of order $\alpha = 1$ is the Gini index $G$ that we are proposing to use (Chameni Nembua, 2005).

As $G$ tends towards one, the distribution of $w_i$ is unequal; when $G$ tends towards zero, the distribution of $w_i$ is equal. Consider a portfolio with $n$ stocks where $w_1$ equals one and all remaining weights are equal to zero. In this case, we have the maximum concentration possible and the Gini index becomes: $G = (n - 1)/n$ (while the $G^{(\alpha)} = \frac{n-1}{n} n^{\alpha-1}$), and when $n$ is big enough then $\lim_{n \to \infty} G = 1$.

The series of generalized Gini index for the first values of $\alpha$ are as follows:

$$\alpha = 1 , \quad G^{(1)} = G = \frac{1}{2n^2 \mu} \sum_{i=1}^{n} \sum_{j=1}^{n} |w_i - w_j|$$

$$\alpha = 2 , \quad G^{(2)} = \frac{1}{2n^2 \mu^2} \sum_{i=1}^{n} \sum_{j=1}^{n} |w_i - w_j|^2$$

$$\alpha = 3 , \quad G^{(3)} = \frac{1}{2n^2 \mu^3} \sum_{i=1}^{n} \sum_{j=1}^{n} |w_i - w_j|^3$$

In our thesis, and for portfolio concentration measurement purposes we will be using the first order Gini index, $G^{(\alpha=1)} = G$.

Consider Table 3.3 which illustrates the Gini index.
Chapter 3. Concentration and Specialization

### Table 3.3: Gini index

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>0.3</td>
<td>0.46</td>
<td>0.9</td>
</tr>
<tr>
<td>$w_2$</td>
<td>0.23</td>
<td>0.46</td>
<td>0.03</td>
</tr>
<tr>
<td>$w_3$</td>
<td>0.2</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>$w_4$</td>
<td>0.14</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>$w_5$</td>
<td>0.13</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>HHI</td>
<td>0.2194</td>
<td>0.4254</td>
<td>0.8126</td>
</tr>
<tr>
<td>rHHI</td>
<td>4.5578</td>
<td>2.3507</td>
<td>1.2306</td>
</tr>
<tr>
<td>Gini</td>
<td>0.172</td>
<td>0.524</td>
<td>0.708</td>
</tr>
</tbody>
</table>

Note that the expression of maximum concentration, namely $G=(n - 1)/n$, with $n = 5$ in this case, gives $G_{\text{max}} = 4/5 = 0.8$. In Table 3.3, we see that Portfolio P3 is highly concentrated in the first stock, yielding a Gini index of 0.708, which is very close to $G_{\text{max}} = 0.8$ attainable in this case.

In contrast, consider a portfolio with $n$ equally weighted stocks, i.e. $w_i = \frac{1}{n}$ for all $i$. In this case we have minimum concentration or total equality. Intuitively, since the straight line in the Lorenz curve represents total equality, the area between the curve and the equality line becomes zero and we should expect the Gini index to become zero. Unlike the $HHI$ that emphasizes the presence of stocks with relatively big weights, the Gini index is sensitive to small transfers of weights between the small and middle range stocks. In other words, $HHI$ is less sensitive to changes in weights among the small weighted elements, whereas the Gini index senses this change.

To illustrate this characteristic consider Table 3.4. Note the effect on the $HHI$ of transferring 0.02 from $w_5$ to $w_4$ (small sized stocks) in P2 and from $w_2$ to $w_1$ (big sized stocks) in P3. Strikingly $HHI$ is higher when the same amount is transferred to a “big player”. In contrast, the Gini index does not sense the same transfer when it comes to big weighted elements (Gini = 0.208 for both Portfolio 2 and Portfolio 3). Rather, the Gini index senses the transfer when it happens between small weighted elements, hence Gini moves from 0.2 to 0.208 between Portfolio 1 and Portfolio 2, when the smallest weighted stock is altered, as shown.
3.5. The Hirschman-Herfindahl and the Gini indexes.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>0.32</td>
<td>0.32</td>
<td>0.34</td>
</tr>
<tr>
<td>$w_2$</td>
<td>0.23</td>
<td>0.23</td>
<td>0.21</td>
</tr>
<tr>
<td>$w_3$</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>$w_4$</td>
<td>0.13</td>
<td>0.15</td>
<td>0.13</td>
</tr>
<tr>
<td>$w_5$</td>
<td>0.12</td>
<td>0.1</td>
<td>0.12</td>
</tr>
<tr>
<td>HHI</td>
<td>0.2266</td>
<td>0.2278</td>
<td>0.231</td>
</tr>
<tr>
<td>Gini</td>
<td>0.200</td>
<td>0.208</td>
<td>0.208</td>
</tr>
</tbody>
</table>

Table 3.4: In this table we see the effect of the same weight transfer that we effected on the three portfolios P1, P2 and P3 to test the effect on HHI earlier above in Table 3.2. We realize that Gini is not sensitive to the heavier weighted stocks when a transfer occurs but rather the opposite.

In this narrow sense it can be used to complement the HHI index which emphasizes the presence of relatively big stocks. But our usage of the Gini index will be focused on measuring the specialization of the portfolio mainly because it is sensitive to small weights differences between assets’ attributes and it has a highly relevant decomposability value\(^{33}\).

We now evaluate the Gini index against the list of desirable characteristics of a concentration measure, as summarized in Table 3.5.

**Characteristics of Gini**

*Concentration measure.* The Gini index emphasizes the rank of the elements and hence more directly captures the effect of moving one stock from one attribute category or sub-group to another, since this movement will alter the ranking of both sub-groups.

*Population size sensitivity.* The Gini index is independent of population size. In fact, for fairly large values of n, using the expression of Gini in Equation 3.5, we can observe that a first order of n is present in both the nominator and denominator of the equation, canceling out the effect of n.

*Transfer principle.* As discussed in Section 3.2.1, the Gini index clearly obeys the weak transfer principle. In fact a transfer of wealth from one higher wealth allocated stock to a lower one reduces inequality (weak transfer principle) by a value depending on the rank of the donor and the recipient, instead of depending on the distance in units between them, as is the case of HHI. It can be proven that the sensitivity to

\(^{33}\)It should be noted that, for a given value of n, and a given vector of $[w_i]$ there exists one value of HHI and one value of the Gini index.
transfers is directly and linearly a function of the number of components between both donors and recipients (i.e. a function of \( j - i \) rather then \( S_j - S_i \)). Please see Cowell (2009),(pages 150-153), for a calculation of the effect of the transfer of an infinitesimal amount from stock \( j \) to stock \( i \) for the Gini index.

**Scale invariance.** The Gini index is scale invariant and its value remains unchanged when the elements of the set are multiplied by a constant positive index because the ranking remains unaltered.

**Population symmetry** In contrast to the \( HHI \), the Gini index obeys the principle of population symmetry and its value does not change when two identical systems are combined together. This is very useful when analyzing the resulting concentration of merging two portfolios of identical concentration but of different specialization, as we shall see in subsequent sections of this thesis.

**Decomposability.** Many papers and studies relate to the decomposition of the Gini index. Three methods have been proposed to decompose the Gini index: (1) a graphical method proposed by Lambert et al. (Lambert and Aroson, 1993), (2) a covariance method proposed by Yitzhaki et al. (Yitzaki and Lerman, 1984) and later (Yitzaki and Lerman, 1985) and (3) a combination of pairwise comparison method proposed mainly by Camilo Dagum (Dagum, 1997b). The Gini decomposition methods emphasize the group overlapping and the interpretation of concentration measure related to this overlapping that Dagum coined the transvariation index. Dagum derived mathematically a decomposition of the Gini index into three additive terms, namely (i) the inequality within a group \( G_w \); (ii) The contribution of the intergroup inequality \( G_b \) to the population inequality \( G \) and (iii) the transvariation or the overlapping effect between groups \( G_t \).

Using the pairwise difference expression as in Equation 3.4, Dagum (Dagum, 1997b) decomposed the Gini index as follows:

\[
G = \frac{\sum_{i=1}^{n} \sum_{r=1}^{n} |w_i - w_r|}{2n^2 \mu} \tag{3.7}
\]

And the Gini index within a subgroup \( j \) (indicated to by the double subscript \( jj \)) of the population is expressed similarly as

\[
G_{jj} = \frac{\sum_{i=1}^{n_j} \sum_{r=1}^{n_j} |w_{ij} - w_{jr}|}{2n_j^2 \mu_j} \tag{3.8}
\]

The Gini index measuring inequality between groups \( j \) and \( h \) is expressed as follows

\[
G_{jh} = \frac{\sum_{i=1}^{n_j} \sum_{r=1}^{n_h} |w_{ij} - w_{rh}|}{2n_jn_h(\mu_j + \mu_h)} \tag{3.9}
\]

**Simplicity.** The Gini is not as simple to calculate as the \( HHI \), but its graphical relation to the Lorenz curve makes it very attractive to use.
The characteristics and properties of the HHI and Gini indexes and discussed above are those that we judged most related to their application in portfolio description. Many other indexes can be used to describe the concentration and specialization levels of a portfolio, as indeed is the case of the Theil index or the Bourguignon index.

The reason we decided to use the HHI and the Gini indexes instead are summarized in Table 3.5. Notwithstanding that the \( HHI \) and Gini both have the desired characteristics depicted in the table, the decision of choosing them among many other indexes that fit the requirement, was based on their ease of computation and widespread popularity amongst practitioners\(^{34}\). Chapter 5 will explore the utilization of these indexes in measuring the concentration and specialization of an investment portfolio.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>HHI</th>
<th>Gini</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concentration measure sensitivity</td>
<td>Big weights</td>
<td>Ranking</td>
</tr>
<tr>
<td>Population size sensitivity</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Population symmetry</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Transfer principle</td>
<td>weak and strong</td>
<td>Weak</td>
</tr>
<tr>
<td>Scale invariance</td>
<td>Invariant</td>
<td>Invariant</td>
</tr>
<tr>
<td>Decomposability relevance</td>
<td>Fairly</td>
<td>Greatly</td>
</tr>
<tr>
<td>Simplicity</td>
<td>Greatly</td>
<td>Fairly</td>
</tr>
<tr>
<td>Maximum value</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Minimum value</td>
<td>1/n</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3.5: Summary of the HHI and Gini indexes characteristics.

3.6 Concluding Remarks

The process of choice for the investor’s portfolio of stocks has two basic dimensions:

1. Screening the initial Investment Universe (by definition including all the available listed stocks in the world) to focus his preference on some chosen attributes that the stock must have. We defined this process as the specialization of the portfolio.

2. Once he chooses the stocks out of the available universe, he will decide on a wealth allocation strategy, putting more of his wealth on some stocks rather than on others. We define this process as the concentration of the portfolio.

The Gini and the HHI indexes were chosen to measure the concentration of the portfolio and its specialization. For the concentration, we will use both the Gini and the \( HHI \) (as

\(^{34}\)This is a subjective criteria that is supported by a lifetime of professional experience in the field of banking and finance.
well as its reverse $rHHI$) and we shall use the ratio $p_i$ for the specialization. The reasons behind our choice rests primarily on the characteristics that were presented. The literature further supported the usage of some indexes over some others according to their application. Not only simplicity and ease of computation were the choice modulators but also the interpretive potential and relevance of both indexes in conveying a proper descriptor to the portfolio constitution. The application of the $HHI$ and the Gini indexes to measure the concentration and the specialization of an investment portfolio is discussed in Chapter 5, where a practical insight will be presented as a methodology that we are proposing to be applied to describe investment portfolios in general. The idea behind the practical result we draw from those indexes is to present to the investors and the analysts an additional tool to measure and describe the characteristics of the investment portfolio at hand without incurring in estimation risk, inherent to the nature of the statistical moments that is suggested by modern portfolio theories with Markowitz. In fact the estimators used in analyzing an investment portfolio hinges primarily on deciding upon a portfolio that produces the maximum possible return for a specified level of risk, measured by the standard deviation of the observed returns over a certain period of time. Those statistical estimators are sensitive to the panel data chosen and carry an amount of error due to the estimation process.

In the coming chapter, we will focus on the issue of estimation risk and estimation errors involved in portfolio management, when using statistical estimators. The next chapter will also discuss the current methodologies applied to portfolio management and will focus on the myriad of existing research related to the subject from Markowitz (1952) to Taleb (2012).
Chapter 4

Estimation Risk and Investment Portfolios

Because of the success of science there is a kind of pseudo-science, social science is an example, which is not a science. They follow the forms, they gather data and so forth, but they don’t get any laws, they haven’t found anything, they haven’t got anywhere (yet) . . . Maybe I am wrong, maybe they do know but I don’t think so, I have the advantage of having found out how hard it is to really get to know something, how careful you have to be about checking the experiments, how easy it is to make mistakes. I know what it means to know something and therefore I see how they get their information and I can’t believe that they have done the works necessary, and the checks necessary, and the care necessary. I have a great suspicion, that they don’t know and that they are intimidating people. I don’t know the world very well, but that’s what I think.

Richard Feynman, Interview with the BBC program Horizons, 1981

Feynman quote above risks to ruffle some feathers of many a respected quantitative professional. In the realm of finance and, more precisely, in the processes of portfolio selection, the reliance on quantitative methods has been the trend for the past decades, with the appearance of well supported approaches relying on sophisticated computer algorithms and formulas. From Markowitz (1952) to Taleb (2012), both standing on opposite side of the assessment and impact of volatility in the investment process, quantitative and heuristic
Chapter 4. Estimation Risk and Investment Portfolios

approaches are gathering momentum and adepts. The market being potentially fragile due to risk-based investing techniques, the assumptions underlying those techniques are also responsible for the repeated financial crisis that we have witnessed over the past decades. Many assumption used in the estimation process involve a low probability tail distribution which have lead to terrible shocks and financial surprises. It is in this respect that we understand Feynman quote stated above and our proposition is to complement the existing quantitative methodologies with an estimation-risk-free index that can improve the quality of the decisions taken by investors.

The purpose of this chapter is to review the various investment allocation strategies and optimization processes. We shall see that these processes are mainly based on approximation algorithms and time series related measures, which introduce estimation risk. Furthermore, we shall see that the concentration and specialization measures, as descriptors of an investment portfolio, are currently non-existent in the toolbox of today’s investors. This chapter will conclude by suggesting further potential research subjects to enhance the allocation, monitoring and managing an investment portfolio, using the suggested measures.

4.1 Optimization Processes: From Mean-Variance to Risk Parity.

Modern portfolio theory dates back to 1952, to the days of Markowitz publishing his paper “Portfolio Selection” (Markowitz, 1952). In this theory, Harry Markowitz derives the concept of “efficient frontier” and later, a simplified form of this theory was derived by Sharpe (1963), referred to as the “single-index model”. Both methods are widely used to allocate investments among asset classes and stocks.

Many investors have the misguided view that risk is proportionately reduced with each additional stock in a portfolio, when in fact this is not true. There is evidence that risk is reduced to a certain point after which there is no further benefit from diversification.

True diversification means that the portfolio must include stocks that are different from each other whether by company size, industry, sector, country, etc.. This means that the portfolio must contain stocks that are uncorrelated, or whose correlation is weak, i.e. stocks that move in different directions during different times under the same economic context.

The asset allocation process is in fact an optimization exercise whose output is usually the allocation matrix of weights \( \vec{w} \) and whose input can be statistics estimates as well as market data. This process involves many approximations, some basic assumptions that

---

1 The efficient frontier is a curve, drawn on an a risk-return coordinates axis, representing the set of optimal portfolios that will offer the maximum return for a given level of risk (represented by the standard deviation of the portfolio) or, it is the set of portfolios that, for a given level of risk chosen by the investor, the portfolios on the frontier will yield the maximum possible return.
simplify the optimization approach (like assuming that returns are normally distributed, the choice of the sample size to calculate the average return and its standard deviation as well as the time period used for calculation whether daily, monthly or quarterly data, that the correlation between stocks remains the same throughout the entire period) and the investor’s preference for risk, horizon of investment and expected target return. Many optimization models today try to maximize returns for a given level of risk, or try to invest on a risk based budget.

(Scherer, 2011) provides us with a connection between risk-based investing and the expected-return-based Fama and French model (Fama and French, 2004). Specifically, Scherer (2011) regresses returns to a minimum variance portfolio\(^2\) onto the size and value factors showing that 83% of the variation of the minimum variance portfolio’s excess returns (relative to a CAPM alternative) can be attributed to the Fama and French factors. He argues that investors can achieve a higher Sharpe ratio than the minimum variance portfolio by directly identifying the risk based pricing variations that the minimum variance portfolio draws upon. In this author’s view, the minimization of risk is, on its own, a meaningless objective: There must be a link between returns of a minimum variance portfolio and factor premium.

While, on the other hand, Maillard et al. (2010) and also Lee (2011) discuss the theoretical properties of risk parity portfolios and provide a comparison with other risk control techniques, (Roncalli, 2013), (Leote de Carvalho et al., 2012) and (Goldberg and Mahmoud, 2013) each provide a risk-only solution to the allocation process, with interesting comparison with the \(1/n\) portfolio, the minimum variance portfolio and the market portfolio. All those models use some statistical prior as their input, which introduces an additional factor of uncertainty: the estimation error involved.

Within the financial mainstream today, lies the tendency of asserting that the quantitative approaches to investment are driving the system to repeated crashes rather than protecting it by predicting and measuring the risks involved. Two authors lead this rather new approach, Nassim Taleb and Pablo Triana, among others. Both authors promote the use of “Rule of Thumb” rather than quantitative (hence predictive and measured) methodologies. This new vision started with Taleb (Taleb, 2007) in his book The Black Swan, where he describes rather low-probability extreme events and shocks (a black swan) big enough to destroy the system that it belongs to. He later refines his theory to generalize it to events with relatively bigger probabilities then “black swans” that he calls stressors. In his later book Antifragile (Taleb, 2012), he argues on the non-linearity of small events creating either

\(^2\)This is a portfolio with the minimum possible variance given the stocks that constitute it. The correlation between the individual stocks will reduce the overall variance of the portfolio, and there exists a unique vector of weights \(w_{MV}\) such that the resulting variance of the portfolio is the minimum possible achievable with any combination of the individual stocks chosen.
a “healthier” or robust system (this is an Antifragile system) or completely destroying it (being hence fragile). He concludes that the financial and the banking system are rendered a fragile system and he believes that investment processes are being fragilized because they rely on predictive tools that tend to overstate the upside gain while disregarding the lower probability yet devastating effects of downside slides.

Triana (Triana, 2009) goes one step further in putting examples from the real financial sectors to demonstrate Taleb’s theory. He criticizes the pricing methods of assets and shows the flaws in the Black and Scholes option pricing model. He concludes by saying that big investment companies are employing highly educated quantitative professionals to lead their client’s investment strategies by trading “off-the-cuff” instead of down-to-earth trained traders and investors. In his opinion this “over-quanting” of a rather simple and intuitive investment decision is introducing a destructive energy to the financial system. It is rendering it fragile, as per the definition of Taleb.

The statistical methods and tools utilized today to “predict” or forecast a certain future behavior become insufficient when we consider Taleb’s and Triana’s postulates. The downside or the fat tail risk, belonging to the very low probability bracket of the distribution, will occur and when it occurs its consequences are devastating, and most importantly, were unaccounted for in the original design of the system³.

4.2 Estimation Error and Estimation Risk

In asset pricing, estimation risk refers to the investor’s uncertainty about the parameters of the return or cash-flow process of an asset. Estimation risk renders the observable properties of prices and returns significantly different from the properties perceived by rational investors, normally derived from estimation techniques and statistical formulation. Estimation error results from endogenous and exogenous reasons. The estimation process involves various assumptions that induce model distortion in the estimation process: assumptions like normal distribution of returns or time-constant variance of time series are typical examples of endogenous estimation error causes. External or exogenous unpredictable variations and signals of the socio-economic and political environment of the asset can affect the quality of the statistical prior like exchange rates, interest rates, legal environment and geopolitical

³Consider the example of an investor willing to buy a factory for a US$1,000 and able to choose between 1000 factories available at US$1,000 each. However he is informed that there is a 1 chance of 1000 (0.1% probability) That the factory he purchases will burn to ashes and destroyed. His expected value is clearly \( \bar{E}(\epsilon) = 0.1\% \times 0 + 99.90\% \times 1000 = \$999 \) yet, considering the fat tail risk of his chosen factory (investment portfolio consisting of the chosen factory) being reduced to ashes drops his observed value to \( -\$1,000 \) or total loss. This “Black Swan” event introduces a doubt on the estimation tool used in the beginning (i.e. the expected value of the investment given the probability density).
4.2 Estimation Error and Estimation Risk

Investors and managers alike tend to oversimplify the investment process when they rely on the mean-variance model (MVM). The MV objective function is given by:

$$w' r - \frac{\lambda}{2} w' \Sigma w$$

where $w$ is the $N \times 1$ vector of portfolio weights and $w'$ its transpose, $r$ is the $N \times 1$ vector of expected returns, $\Sigma$ is the $N \times N$ covariance matrix of returns, and $\lambda$ is a scalar representing risk aversion.

In each period, the investor trades off expected portfolio return, $w' r$, versus portfolio variance $w' \Sigma w$. He chooses his portfolio $\vec{w}$ to maximize the value of the objective function above. The minimum variance frontier comprises all portfolios that have minimum variance for a given level of expected return.

The inputs to the MV objective function above are all statistical estimates (except for the risk aversion $\lambda$ which is determined by the investor) and hence carry an inherent estimation error. The expected mean future returns for each asset, the expected volatility of returns around the future expected means and the matrix of expected correlations of all returns are all estimates and statistically derived from ex-ante time series of returns of the risky asset. This estimation process induces an additional risk namely estimation risk resulting from the statistically inherent errors in the estimation process. So an asset’s total risk is composed of two components: intrinsic risk and estimation risk.

Additionally, the MV objective function’s result is usually a concentrated portfolio in a limited number of assets of the wider Investment Universe. But, most importantly, the MV solution is very sensitive to its input parameters. Small changes in those parameters, mainly in expected returns, will lead to significant variations in the weight vector $\vec{w}$, as discussed by Merton (1980). This situation is considered as unreasonable by most investors: it seems unwise to discard some assets because their performance in the past is weak and was not spotted by the Markowitz optimization process. The argument against the optimization MV method is that the past performance of an asset (whether weak or strong) does not necessarily constitute an evident projection in the future: this performance is based on a probabilistic distribution model that is flawed with estimation errors. Additionally, the instability of the covariance matrix and average return in time, imposes on the allocation process an upper and a lower bound for each asset. In our opinion, this corresponds to allocating the target portfolio according to the investor’s information and not according to the results of the optimization process itself because this will include most assets at the minimum allocation lower bound in the portfolio although ruled out by the process itself. The same applies to the upper bound limit, where the investor will shunt down any asset’s weight.

These signals are received through disclosures such as earning announcements, dividend initiations or shares repurchase decisions, and mergers and acquisitions related developments (Loughran and Vijh, 1997). The signals could also be received analysts whose recommendation or projections can be considered informative (Brav and Lehavy, 2003) and (Sorescu and Subrahmanyam, 2006).
allocation above the maximum limit allowed regardless of the results of the weight vector produced by the MV optimization approach.

Recent development in asset allocation techniques involve a more heuristic approach\(^5\). One prominent tendency in the mainstream financial market today is to use risk parity or risk budgeting allocation, which is a weight allocation technique that balances and equalizes the contribution of each asset’s risk to the total risk or volatility of the portfolio. In his white paper, Qian sets the standards for such technique as early as 2005 (Qian, 2005).

Although a wider agreement is reached on the less beneficial application of the Markowitz MV objective function to reach a weight allocation vector \(\vec{w}\), more and more investors and managers are opting for better techniques that minimize the effect of estimation error\(^6\). Similarly, we seldom find a consensus in mainstream financial markets for the adoption of the risk parity or risk budgeting techniques. Asness et al. (2012) document the empirical out-performance of a risk parity strategy over a market cap weighted portfolio and refers to the leverage aversion effect to explain this out performance. In contrast, (Anderson et al., 2012) review and refute the empirical evidence provided by (Asness et al., 2012).

Another heuristic approach to asset allocation is the \(1/n\) or equally weighted portfolio. (DeMiguel et al., 2009) refer to this strategy as the “naïve diversification strategy”. (Kirby and Ostdiek, 2012) suggest an active portfolio strategy with time related dynamic re-allocation that outperforms the \(1/n\) allocation strategy. The idea is to rely only on risk assessment and not on returns to solve for the allocation vector. The reason is that the impact of the estimation error of expected returns has a bigger effect on the resulting weight vector than the impact of the same on variance\(^7\). It is clear that the effect of estimation risk alters the weight vector dramatically, and hence the concentration and specialization levels of the portfolio are affected. (Best and Grauer, 1993) show that a small increase in the mean of one asset in a portfolio drives half of the securities from the portfolio. This tells us, beyond any doubt, that the effect of estimation error is directly affecting the concentration

\(^5\)Heuristic solutions are usually rule-of-thumb, experience based techniques to find solutions to problems relying on simple and intuitive methods rather then on complicated algorithms.

\(^6\)When based on sample means and covariances, MV optimized portfolios are highly concentrated. They show sudden shifts in allocations along the efficient frontier and are also very unstable across time. As (Michaud, 1998) pointed out, this is due to those statistical optimizers being “estimation error maximizers”. MV optimizers overweight those assets that have large estimated expected returns, low estimated variances and low estimated correlations to other assets. These assets are the ones most likely to have large estimation errors.

\(^7\)Chopra et al., (1993) find that errors in means are about ten times as important as errors in variances, and errors in variances are about twice as important as errors in covariances. While (Best and Grauer, 1993) show that optimal portfolios are very sensitive to the level of expected returns. They note that “a surprisingly small increase in the mean of just one asset drives half the securities from the portfolio. Yet the portfolio’s expected return and standard deviation are virtually unchanged”. 
and the specialization of the portfolio.

Many empirical studies have been conducted recently to show the superior performance of allocation strategies. The central results of a study presented by (Herold and Maurer, 2006) show that all of the approaches that operate under the IID (Independent and Identically Distributed random variables) assumption, whether they account for estimation risk or not, are not superior to simple investment strategies like holding the market portfolio, the equally-weighted portfolio or the minimum-variance portfolio which refrains from estimating expected returns at all.

The minimum variance portfolio is a specific portfolio on the efficient frontier and is usually highly concentrated in few assets. The equally weighted portfolio \(\frac{1}{n}\) avoids concentrated positions in wealth allocation and implies an exposure to the small cap assets as well because even the smallest market cap assets are allocated a \(\frac{1}{n}\) weight, in contrast with the market portfolio which is loaded towards large caps, by virtue of its construction. Note that when all assets have the same volatility and when all pairwise correlations are the same then the equally weighted portfolio becomes the same as the minimum variance portfolio.

Another approach to eliminate the effect of estimation risk was presented by (Choueifaty and Coignard, 2008), proposing a maximum diversification portfolio, using a diversification ratio based on the standard deviation of the assets available, as follows:

\[
\text{Diversification ratio} = \frac{\sum_i w_i \sigma_i}{\sigma_p} = \frac{\sum_i w_i \sigma_i}{\sum_i w_i \sigma_i \rho_{ip}}
\]

Where \(w_i\) denotes the weight of asset \(i\), \(\sigma_i\) represents the standard deviation of the asset \(i\) and \(\sigma_p\) the standard deviation of the portfolio, while \(\rho_{ip}\) is the correlation between the expected returns of asset \(i\) and the portfolio. By representing the diversification ratio with respect to the weights of the individual assets of the portfolio, the optimization process is reduced to assuming that the diversification ratio is maximized for the best performance resulting in an optimal weight vector accordingly.

An interesting comparison between \(\frac{1}{n}\), minimum variance portfolio MVP, the maximum diversification portfolio, the equal risk contribution portfolio, the inverse volatility portfolio and the maximum Sharpe ratio portfolio is presented in (Hallerbach, 2013) and tabulated according to various criteria of comparison with very attractive conclusions. The author concludes that risk control strategies provide a sensible starting point in portfolio optimization when there is considerable uncertainty about the required inputs (estimation error or high estimation risk), i.e. mainly the expected return and the variance.

The application of any of the methods available today to reach a final allocation vector (weight vector \(\vec{w}\)) will modulate or alter the concentration and specialization of the portfolio. The introduction of additional constraints will further affect \(\vec{w}\) and hence a further alteration on specialization and concentration is introduced. It seems only obvious to explore further this aspect of the optimization process in the light of the specialization and...
concentration that we are proposing as additional descriptors of an investment portfolio.

4.3 Effects of Portfolio Constraints on Concentration and Specialization.

By definition, a portfolio of minimum concentration is a portfolio consisting of equally weighted assets, and hence if the portfolio contains \( n \) assets, each asset’s weight is hence \( \frac{1}{n} \). Any deviation from the \( \frac{1}{n} \) weight will produce a concentration of the portfolio. The maximum concentration that could be attained in a portfolio, assuming no short selling, is where all the wealth is allocated to one single asset, i.e. a \( \frac{n}{n} = 1 \) weight to one specific asset.

In the investment process, the investor’s choices impose on the universe of available assets certain constraints and conditions that meet the specific preferences of the investor, from risk appetite, to geographic location and any other particular property of the asset itself, as it is shown in the stages A and B in the figure below:

![Diagram](image)

Figure 4.1: Stages of the Investment process.

It is important to note that, on an absolute scale, an investment universe might be naturally specialized. Far from considering the market indexes available, the universe of available assets itself can be skewed towards some particular attributes against some others. Consider, for example, the USA listed stock market of the year 2008 as the Investment Universe. If we consider creating the a \( \frac{1}{n} \) portfolio consisting of all the stocks included in the aforementioned Investment Universe, the total number of stocks included in the portfolio
4.3. Effects of Portfolio Constraints on Concentration and Specialization.

will be \( n = 9983^a \). Since we are building an equally weighted portfolio regardless of any particular attribute specialization, all the stocks included in the listing will be considered, regardless of the market value of the companies involved. As a result, we realize that the resulting 2008 US market \( 1/n \) is specialized in companies providing financial services and related activities. Table 4.1 shows a primary form of statistics of the 2008 USA stock market industrial sectors.

<table>
<thead>
<tr>
<th>Industrial Sector</th>
<th>% Top 10</th>
<th>Industrial Sector</th>
<th>% Bottom 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Div. Financial Serv.</td>
<td>8.12%</td>
<td>Tires &amp; Rubber</td>
<td>0.06%</td>
</tr>
<tr>
<td>Specialized Finance</td>
<td>4.75%</td>
<td>HyperMrkts &amp; SMkt</td>
<td>0.06%</td>
</tr>
<tr>
<td>Regional Banks</td>
<td>4.49%</td>
<td>Home Retail</td>
<td>0.06%</td>
</tr>
<tr>
<td>Thrift &amp; Mortgage Fin</td>
<td>4.26%</td>
<td>Catalog Retail</td>
<td>0.06%</td>
</tr>
<tr>
<td>Oil &amp; Gas expl.</td>
<td>3.62%</td>
<td>Photographic Prdcts</td>
<td>0.04%</td>
</tr>
<tr>
<td>Asset Mngmt</td>
<td>3.44%</td>
<td>Consumer Finance</td>
<td>0.04%</td>
</tr>
<tr>
<td>Genetic R&amp;D</td>
<td>3.13%</td>
<td>Marine Ports &amp; Srvc</td>
<td>0.03%</td>
</tr>
<tr>
<td>Metal &amp; Mining</td>
<td>2.33%</td>
<td>Motorcycle Mngmt</td>
<td>0.02%</td>
</tr>
<tr>
<td>Application Software</td>
<td>1.97%</td>
<td>Highways &amp; Rail tracks</td>
<td>0.02%</td>
</tr>
<tr>
<td>Manuf. Health eqp.</td>
<td>1.96%</td>
<td>Real Estate Mngmt</td>
<td>0.01%</td>
</tr>
</tbody>
</table>

Table 4.1: Percentage Count of 2008 US stock market listed companies totaling 9983 stocks. The table depicts the top and bottom 10 industrial sectors in count of listed companies. Note that the first top four entries are all companies in the financial sectors, which shows clearly that the 2008 USA stock market “Universe” is highly specialized in financial sector’s companies with a total count of more than 25.06%. (figures calculated by author).

We note from the Table 4.1 that the top 10 companies represent 38.06% in count of the total 2008 universe, or 3800 companies from the total of 9983 available in this 2008 USA listed stocks Universe under consideration. Also noticeable the specialization in financial services companies representing 25.05% of the total listed 2008 USA companies, with the top 4 being financial services representing 21.62%. We can detect the specialization of this particular universe by using the top and bottom percentiles only.

This leads us to conclude that the \( 1/n \) portfolio is not necessarily a diversified portfolio at all, but rather it reflects the exact count and elements of all the industrial sectors included in the chosen Investment Universe. The Gini index of the 2008 USA stock market universe, on the count of its stocks is \( G = 0.58 \) which represents a rather highly concentrated

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*The dataset used in this thesis was provided by CRSP/COMPSTAT Merged Database–Fundamentals Annual. The dataset contains all the US traded equity from 1987 until 2009 inclusive.*
Chapter 4. Estimation Risk and Investment Portfolios

(specialized) data set. This results in a counterintuitive conclusion, where a naive investment strategy of equally weighted distribution would intuitively, but wrongly, minimize the concentration and the specialization of a portfolio in general.

Another possible approach is to achieve a $1/n$ portfolio but with minimum specialization, that is to choose from the Investment Universe, 1 stock per each available industrial sector and include it in the portfolio at equal weight criteria. The question remains: which stock to choose from each industrial sector? Is the highest market cap stock chosen? the lowest? the middle? or the stock that represents the average value of market cap of each sector? what about choosing the stock, in each industrial sector, that represents the company least leveraged, or more leveraged, or the one whose debt/assets ratio equals a certain predetermined level? This is where specialization and concentration indexes play an important role in shaping or rather translating the preference of the investor in his portfolio selection. Earlier in this chapter we have seen several methodology of portfolio selection based on risk criteria, risk-return considerations, efficient frontier related portfolio and also heuristic $1/n$ selection.

What we propose is the inclusion of the specialization and the concentration measures to help fine tune the selection process and later the monitoring and dynamic re-allocation of wealth within the portfolio, in conjunction with the quantitative methods used by the investors. The specialization and concentration indexes we propose are estimation-risk-free in the sense that they are not based on any a priori assumption or on the distribution or on any approximation of the volatility of the priors. In fact, the ideal situation would be to conjugate various optimization approaches with the inclusion of our proposed scalars, to fine tune the final selection process and to optimize the re-allocation and monitoring processes. It is also necessary to explore the limitations and usage of the measures we introduce and analyze their correlation with other measures utilized in investment portfolio quantitative analysis. This leads us to suggest some interesting future research projects and interesting further studies in the following topics:

- A portfolio optimization algorithm utilizing concentration and specialization measures.
- Study the relationship between the concentration and specialization of a portfolio, from one side, and the portfolio future risk and return measurements.
- Study the relationship between concentration and specialization measures in a portfolio to correlations among the returns of individual stocks.
- Analyze covariates of the different concentration and specialization measures, to obtain a better idea of why and when these measures go up or down.
- An empirical study analyzing the the return and risk characteristics of a portfolio generated in part based on the concentration and specialization measures.
- Generate a concentration and specialization benchmark or index based on allocation
strategies and portfolio investment techniques and compare them in relation to the measures introduced.

- Relate the time-series variation and important changes to financial and economic variables or events concomitant with the observed variation.

While very necessary to understand better the use and limits of the measures introduced, this research is oriented towards introducing the concentration and specialization measures as portfolio descriptors. It is left to future research initiatives to introduce these descriptors in allocation and re-allocation strategies as well as performance monitoring and benchmark.

The next chapter will deal in more details with what is discussed here, with a proposition of a toolbox to assist the investor in his final choices. The set of constraints that the investor imposes on his portfolio will introduce various specialization and concentration criteria and it will prove important to include these two characteristics in the description of the investment portfolio.
Chapter 5

Application of Concentration Measures in Investment Portfolios

Wide diversification is only required when investors do not understand what they are doing.

Warren Buffett

What is intuitively understood from Warren Buffet’s “wide diversification” is contrasting the word “wide” with the word “narrow” or “restricted”. In this sense, we can argue that wide diversification is the inclusion of most of the available stocks in the Investment Universe within the portfolio. This quote also suggests an inherent understanding of the concept of “wide” in the level of diversification which implies that the investor can perceive differences in the level of diversification.

This chapter answers partially research question 4. It will define the usage of the $HHI$ and the Gini indexes applied to an investment portfolio. The numerical application of this method will be illustrated in the next chapter.

This brings us back to the $1/n$ portfolio strategy or what is called the “naïve diversification strategy” by Kirby and Ostdiek (2012). As shown in Table 4.1, page 73, this $1/n$ strategy is naïve but not diversified at all. In fact it results in a rather highly specialized portfolio in financial services. Hence, applying a $1/n$ heuristic approach to stock selection does not result in “naïve diversification” but it is rather an “uneducated guess strategy” re-
fecting no pretension on the quality of the bets placed in this investment\textsuperscript{1}. We understand by “wide diversification”, the inclusion of most of the possible available attributes of the stocks, in order to render the portfolio the least specialized possible, and later to split the available wealth evenly among the chosen stocks. This approach results in a portfolio with minimized specialization or minimized concentration (or both), not necessarily reflecting the market 1/n portfolio. We shall see later in this chapter, that minimizing the concentration of the portfolio is rather an easy task: a necessary and sufficient condition to obtain a portfolio with the least concentration possible is to tend to allocate the wealth equally among all chosen stocks in the portfolio\textsuperscript{2}. However, this is not the case for the specialization. In fact the least specialized portfolio is obtained with respect to the attributes chosen, and it is usually not possible to have a portfolio least specialized in all possible attributes. We shall discuss this particular case later in this chapter.

5.1 Portfolio Characteristics: Specialization and Concentration

When making investment decisions and allocation strategies, the investor relies on various inputs, among which we state (i) the market data available, (ii) his interpretation of this data and (iii) his risk appetite or preference. The investor’s choice is the product of these input modulators and hence the process is predominantly a subjective one. The investor’s final decision can be based upon his own perception of the best configuration of his portfolio. Assuming that we have an investor who has no market data, who is risk averse and who is presented with all the possible combinations of POS\textsuperscript{3} as defined in Definition 4 (Page 6) available for investment, it seems natural that his choice will be to allocate an equal amount of wealth in each and every stock forming an equally weighted portfolio, with $w_i = \frac{1}{n}$, $\forall i$ where $1 \leq i \leq n$. This 1/n strategy of allocation does not necessarily represent the best allocation strategy that will maximize the return of the portfolio under a predefined risk preference, as discussed by DeMiguel et al. (2009), where the authors prove that the 1/n allocation strategy is inefficient and suboptimal. In fact it was shown in the previous chapter

\textsuperscript{1}The investor “Pretension Level” is a concept narrowly related to HHI and concentration measures of a portfolio and discussed in Appendix B. I owe to Professors B. Matarazzo, S. Greco and J. Spronk the gratitude of extending to me an unfinished and unpublished white paper to this respect. The “Pretension Level” was coined by the aforementioned professors and my contribution is related to the inclusion of such definition in the broader specialization and concentration descriptors of an investment portfolio.

\textsuperscript{2}In fact, if all $w_i = \frac{1}{n}$ for all $1 \leq i \leq n$ then the concentration is minimum.

\textsuperscript{3}Portfolio Opportunity Set or POS is defined as the set of all possible compositions of a portfolio given the set of assets one could invest in, the investment opportunity set, and the constraints that a portfolio manager must obey. This concept was developed by Hallerbach and Spronk (1997), Hallerbach et al. (2004) and Pouchkarev et al. (2006).
that this strategy can sometimes yield a highly specialized portfolio.

However, the $1/n$ allocation strategy neutralizes all possible pretensions of the investor: the selection of all existing stocks implies no preference of one stock’s attributes over another and allows the level of specialization to be dictated by the Investment Universe components. The allocation of equal wealth in all existing stocks (equal weight for all stocks, i.e. no concentration) can be described as a “neutral investment position”.

**Definition 5. Neutral Portfolio:**
A neutral portfolio is a portfolio derived from an Investment Universe of $n$ stocks, with $w_i = \frac{1}{n}$, $\forall i$ where $1 \leq i \leq n$, where $w_i$ is the respective weight of each stock in the portfolio. A neutral portfolio results from a neutral investment position.

We can deduce from Definition 5 that a neutral portfolio is not concentrated since all the wealth is equally divided among its components. We cannot however ascertain the nature of the specialization of this equally weighted portfolio, because, by choosing all the stocks available in the Investment Universe the same specialization of this Universe is replicated into the portfolio without any pretension from the investor. Hence an equally weighted portfolio is not concentrated but it may or may not be specialized.

To illustrate this point of view, let us consider the following hypothetical equally weighted portfolio, composed of the 10 stocks, as illustrated in Table 5.1. The portfolio in question is not concentrated but is highly specialized.

<table>
<thead>
<tr>
<th>Stock $S_i$</th>
<th>Weight $w_i$</th>
<th>Industry</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>0.1</td>
<td>Retail</td>
</tr>
<tr>
<td>$S_2$</td>
<td>0.1</td>
<td>Oil&amp;Gas</td>
</tr>
<tr>
<td>$S_3$</td>
<td>0.1</td>
<td>Mining</td>
</tr>
<tr>
<td>$S_4$</td>
<td>0.1</td>
<td>Mining</td>
</tr>
<tr>
<td>$S_5$</td>
<td>0.1</td>
<td>Mining</td>
</tr>
<tr>
<td>$S_6$</td>
<td>0.1</td>
<td>Mining</td>
</tr>
<tr>
<td>$S_7$</td>
<td>0.1</td>
<td>Mining</td>
</tr>
<tr>
<td>$S_8$</td>
<td>0.1</td>
<td>Mining</td>
</tr>
<tr>
<td>$S_9$</td>
<td>0.1</td>
<td>Mining</td>
</tr>
<tr>
<td>$S_{10}$</td>
<td>0.1</td>
<td>Mining</td>
</tr>
</tbody>
</table>

Table 5.1: Hypothetical equally weighted 10-stocks portfolio. While it is very clear that this is a neutral portfolio, as per Definition 5, we observe that 80% of the weight is allocated to the industry group “Mining”. This portfolio is not concentrated but highly specialized. We conclude that an equally weighted portfolio is not concentrated but not necessarily not specialized.
Chapter 5. Application of Concentration Measures in Investment Portfolios

5.1.1 Specialization

Consider a hypothetical investment universe consisting of two different stocks, $S_1$ and $S_2$, represented by a non-negative variable $w_1$ and $w_2$ respectively. Initially, a combination of those two assets creates a portfolio that can be specialized if the weight of either one of the two stocks is 0 and the other is 1, as discussed by Hadar et al. (1977). Defining a random portfolio $P(k)$ created from a combination of the two stocks as:

$$P(k) = kw_1 + (1-k)w_2,$$

with $0 \leq k \leq 1$

The authors define a specialized portfolio as a portfolio with $k = 0$ or $k = 1$. In the case where $k = 0$, $P(k) = P(0) = w_2$, which is a portfolio specialized in asset $S_2$; in the case where $k = 1$, $P(k) = P(1) = w_1$, which is a portfolio specialized in asset $S_1$. In the cited article, a portfolio is considered diversified if and only if $0 < k < 1$.

In our opinion, this definition is rather one dimensional in the sense that it understates the attributes of an asset that we chose to consider as clearly depicted in Figure 2.2 (Page 20). As apparent in this illustration, the specialization vector carries various dimensions and hence we need to specify a different definition approach to specialization, other than the one used by (Hadar et al., 1977).

To further illustrate our point in multidimensional specialization, consider the example illustrated in Table 5.1. In fact the stocks $S_i$ included in this portfolio, have the industry attribute as depicted, but also have the geographical location attribute as shown in the following Table 5.2, where an additional attribute “Geographic Location” is added:

---

4 (Hadar et al., 1977) define specialization and diversification as subject to and relative to a non-negative variable pertaining to an asset $S$. This will prove to be one dimensional considering that we describe the portfolio according to several attributes and hence one asset could be described not only by a one non-negative variable or attribute but rather with many attributes some of which not necessarily numerical, as we saw in Chapter 3, e.g. an asset can be Japanese, family owned, petroleum industry sector, etc., which introduces a multidimensional variability in defining specialization, as we shall see. The definition involves positive variables representing the weights allocated to each asset in the portfolio, which by construction of the same admits no short selling and hence only long positions are considered. Please refer to Section 3.1.1, page 32.
5.1. Portfolio Characteristics: Specialization and Concentration

<table>
<thead>
<tr>
<th>Stock $S_i$</th>
<th>Weight $w_i$</th>
<th>Industry</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>0.1</td>
<td>Retail</td>
<td>USA</td>
</tr>
<tr>
<td>$S_2$</td>
<td>0.1</td>
<td>Oil&amp;Gas</td>
<td>Japan</td>
</tr>
<tr>
<td>$S_3$</td>
<td>0.1</td>
<td>Mining</td>
<td>USA</td>
</tr>
<tr>
<td>$S_4$</td>
<td>0.1</td>
<td>Mining</td>
<td>USA</td>
</tr>
<tr>
<td>$S_5$</td>
<td>0.1</td>
<td>Mining</td>
<td>USA</td>
</tr>
<tr>
<td>$S_6$</td>
<td>0.1</td>
<td>Mining</td>
<td>USA</td>
</tr>
<tr>
<td>$S_7$</td>
<td>0.1</td>
<td>Mining</td>
<td>Japan</td>
</tr>
<tr>
<td>$S_8$</td>
<td>0.1</td>
<td>Mining</td>
<td>Japan</td>
</tr>
<tr>
<td>$S_9$</td>
<td>0.1</td>
<td>Mining</td>
<td>Japan</td>
</tr>
<tr>
<td>$S_{10}$</td>
<td>0.1</td>
<td>Mining</td>
<td>Japan</td>
</tr>
</tbody>
</table>

Table 5.2: Hypothetical equally weighted 10-stocks portfolio, showing two attributes: Industry Group and Geographic Location. We observe that 80% of the weight is allocated to the industry group “Mining”, while the “Geographic Location” weights are split 50-50 among USA and Japan. This portfolio is specialized in Mining and not specialized in Geographic location, or alternatively equally specialized in Japan and USA stocks, while being not concentrated (i.e. equally weighted among all stocks).

Hence, Specialization is relative to the attributes of the stocks that the analyst or the investor have chosen to include in the portfolio. The portfolio can be specialized in one of these attributes while not specialized in another one, or more specialized in one particular attribute and less specialized in another one. We conclude that the term specialization must be preceded by the attribute it measured: Geographic location Specialization, Industry sector Specialization, etc..

Let us consider a second example in which we highlight the meaning of specialization relative to multiple attributes.

Assume that we have a demonstration of 20,000 persons, 50 demonstrators are picked at random. We want to describe this sample with respect to some attributes chosen, or in other words, the specialization of this set of 50 demonstrators with respect to the following chosen attributes:\footnote{The choice of attributes appears to be subjective in the present example and rather generic; attributes like gender seem to be an obvious choice with easy categories. But when the chosen attribute is age group an interval grouping is required or else a continuous scale or open scale of measurement is used. It is our choice to simplify the choice of interval in this example and in the financial example that we discuss later in this chapter which is summarized in Tables 5.4 and 5.5 (pages 91 and 92), where the choice of the interval, albeit subjective, is not random but obeys a certain financial criteria that the analyst is interested in.}
Chapter 5. Application of Concentration Measures in Investment Portfolios

- Gender: Male or Female. [40 M, 10 F]
- Education level: Illiterate, High school, University graduate. [16 I, 16 HS, 18 U]
- Age: Age group: 1,2,3,4. [5 G1, 5 G2, 20 G3, 20 G4]
- Region of origin: North, South, East or West. [9 N, 0 S, 0 E, 41 W]

We can calculate the specialization indexes of the demonstrators group for each attribute chosen: specialization of gender, of educational level, of age and of region. The result will be a set of indexes corresponding to the attributes chosen. Note, however, in contrast to a portfolio, the concept of “weight” is irrelevant as each protester is ultimately one person, one voice. The weight distribution per element is 1 among 50, i.e. \( \frac{1}{50} \). Therefore the set of 50 demonstrators is a specialized set but its concentration is minimal, because it is equally distributed among its constituent elements. The concentration of the demonstrators subset can be assessed by considering the share in weight of each attribute subdivision. It is clear that the demonstrators subset is specialized in Males, equally specialized in G3 and G4 age groups, specialized in people from the Western region and not specialized in any particular education level or equally specialized in all education levels. We shall use the total weight per attribute to determine the specialization of non overlapping set of elements (investment portfolios in our case).

It is logical therefore, in an attempt to define a specialized portfolio, to introduce along with the attributes of the asset its weight also. In the second example above, the elements of the set under study (the demonstrators in the sample chosen) are individuals and hence the concept of weight per element is reduced to \( \frac{1}{50} \) as stated above. The examples used to illustrate the specialization related to an attribute both represent an equally weighted allocation. But even when the set is concentrated (i.e. not equally weighted) the specialization measure can be proxied by the sum of weights (as in the case of the portfolio in Table 5.2), or as the sum of individuals as in the case of the demonstrators example.

On the other hand, we believe that the specialization of the portfolio at hand must be related to the initial Investment Universe from which the investor decided to choose his stocks. If the investor decided to choose his stocks from the USA market, and if he chooses “Geographic Location” as an attribute, it is very logical that the portfolio will include only stocks from the USA and it is logical to say that the portfolio is highly specialized in USA stocks, because the investor had no other geographic location to choose from when he decided to invest in the USA market. The following example illustrates graphically this particular situation:

Let \( U_1 \) be an Investment Universe containing only USA companies, from which we pick one stock \( S_1 \), and let \( U_2 \) be a different Investment Universe containing European and Asian companies in equal proportion from which we pick a stock \( S_2 \). Let \( P_1 \) and \( P_2 \) be two different
5.1. Portfolio Characteristics: Specialization and Concentration

portfolios containing $S_1$ and $S_2$ respectively. Hence, $P_1$ can contain a stock from the USA only and $P_2$ can contain a stock from either Asia or Europe, as shown in Figure 5.1.

Figure 5.1: Both portfolios, $P_1$ and $P_2$ have the same Geographic Location specialization index ($I_{P_1} = I_{P_2}$) since they both contain exactly one stock, while their initial Universes ($U_1$ and $U_2$) have different Geographic Location specialization level, because $U_1$ is more specialized then $U_2$ ($I_{U_1} > I_{U_2}$).

What we can say about the Specialization of $P_1$ and $P_2$ related to the geographic location attribute is that both $P_1$ and $P_2$ contain only 1 geographic attribute: $P_1$ is specialized in USA stocks with a Specialization index$^6$ $I_{P_1} = 1$, and $P_2$ is also specialized in 1 geographic location (Asia or Europe). Thus, both $P_1$ and $P_2$ have the same specialization level of 1. However, a structural difference exists between $P_1$ and $P_2$: $P_1$’s Specialization index reflects that this portfolio is specialized in one geographic location as well as $P_2$’s, $P_1$ nevertheless reflects the exact specialization of its universe $U_1$, namely specialized in USA companies, while $P_2$ introduces a different notation. In fact, $U_2$ is specialized in two geographic locations (EU and Asian companies) while $P_2$ is specialized in one single geographic location (in this case Asia). In other words $U_2$ is less specialized then $P_2$ and this tells us a lot about $P_2$. We conclude that in order to be able to correctly describe the Specialization level of a portfolio we must consider as well the specialization level of its initial Universe to be able to compare. It is to be noted that we can describe $P_2$ with sufficient relevance if we say “$P_2$

---

$^6$We assume that the specialization index $I$ is included in the interval $I \in [0, 1]$ where $I = 0$ represents the minimum specialization possible and $I = 1$ represents the maximum specialization possible. Later in this chapter we will return to explore the meaning of each value and we will specify the proper index to be used to represent the specialization level in a portfolio.
is specialized in one geographic location" but our description is more informative if we say "P₂ is specialized in one geographic location, increasing the specialization in this attribute comparing to its initial Universe U₂." This concept of relative specialization describes the specialization of a portfolio with respect to its investment universe. This becomes important when the investment strategy itself presents constraints on the investors in form of investment principles or investment compliance, like socially responsible investment, environment friendly investment or investment principle boycotting some sectors of the economy or even some countries.

In other words, considering a Specialization index scalar I, the following is a valid:

\[ I_{U_1} > I_{U_2} \quad \text{while} \quad I_{P_1} = I_{P_2} = 1, \]

which describes P₁ and P₂ with respect to the attribute of geographic location, as depicted in Figure 5.1. We can say the following with respect to the attribute, “geographic location = USA stocks”:

\[ I_{P_1}^{US} = 1, \quad \text{and} \quad I_{P_2}^{US} = 0 \]

specialization level in USA stocks.

Similarly we can also say the following with respect to the attribute, “geographic location (Asia + EU) stocks”:

\[ I_{P_1}^{Asia+EU} = 0, \quad \text{and} \quad I_{P_2}^{Asia+EU} = 1 \]

related to the specialization level of P₁ and P₂ in (Asia + EU) stocks.

Specialization is therefore a descriptor relative to an attribute and it can be also be measured as a relative specialization as compared to an initial Universe of population. The examples discussed in this present section will help us define the Specialization in an investment portfolio.

Let P denote the portfolio with n stocks. The elements of P are the stocks, Sᵢ, where i = 1, 2, 3, \ldots, n.

Furthermore, let \( \vec{\rho} = A_1, A_2, \ldots, A_k \) denote the vector of k desired attributes Aₖ that the investor would like his portfolio to contain. Thus, the portfolio contains n stocks each with k attributes. Each attribute Aᵢ with j = 1, 2, 3, \ldots, k has a size nⱼ. Therefore the portfolio P is partitioned into k attributes, and each attribute consists of nⱼ subgroups. This subdivision is illustrated in Table 5.5, page 92, which refers to our hypothetical portfolio

\footnote{This comparison between the Initial Investment Universe and the portfolio can be referred to as the relative specialization. This relative attribute specialization remains however theoretical: In practice the investor does not start with a defined Investment Universe, but rather with a pretension on some markets. We are not therefore always sure what is the initial universe that the investor started from. This is the case when we analyze an investment portfolio without knowing the initial pretensions of the investors and hence without a sufficient information and knowledge of the initial attributes he had in mind. In this case we can define the attributes of analysis in a subjective manner and the relative specialization will be irrelevant in this case.}
5.1. Portfolio Characteristics: Specialization and Concentration

used for illustration throughout this thesis.

**Definition 6. Specialization of a portfolio:**

We define the absolute specialization of an investment portfolio within defined attribute \( A_k \) as the measure of concentration of its consisting subgroups. The portfolio will have as much specialization indexes \( I_k \) as it has attributes \( A_K \). It is clear that the scalar being used in the measurement is the weight of each stock, as subdivided into attributes and subgroups\(^8\).

The analyst can decide to analyze the specialization levels of various attributes \( k \), where each attribute specialization, that we denote \( I_k \) for now, is such that \( I_k \in [0, 1] \). Hence if our interval of measurement assumes a maximum value of 1, then a specialized portfolio is a portfolio such that \( I_k \to 1 \). The utilization of a concentration index to describe the portfolio at hand is more eloquent and will give more relevant information when we compare the concentration indexes of the same attribute \( k \) of both the Investment Universe and the portfolio. This pairwise comparison gives a broader image, vis-a-vis the attributes we have chosen \( k \), on the description of the constituents of the portfolio.

In this sense, the specialization vector that we established in Subsection 2.1.3 which is applied to an investment universe \( U \) to “extract” or filter out an investment portfolio\(^9\) introduces the element of specialization in the resulting filtered set. This specialization vector \( \vec{p} \) includes \( k = 4 \) filtering criteria, and hence the corresponding sub-universe \( S = P \) (which is in our case the portfolio that includes only those stocks that obey to the filtering criteria defined by the investor) is a specialized portfolio. This specialization vector represents the stock choice process (those stocks that the investor will invest his wealth in) or Stage A we discussed earlier, which is the “specialization box” re-depicted in Figure 5.2 below for ease of reference.

\(^8\)It is clear from this discussion that the index of choice should have a decomposability feature in order to show the concentration within each subgroup and between the subgroups. The Gini index, that we chose to measure specialization, has this particularity as shown in Appendix A.

\(^9\)\( \vec{p} = \{ \text{USA stocks, non-fincl Co., } P/E \geq 15, \text{ Debt/Assets} \leq 33.3\% \} \)
5.1.2 Concentration

As shown in Figure 5.2 above, once the stock choice process or specialization is achieved, the investor moves to the wealth allocation stage, which is the concentration stage. This stage consists of dividing the available wealth among the stocks chosen. In theory, and assuming no short selling is allowed, the allocated wealth proportion $w_i$ to stock $i$ can take any value between 0 and 1, i.e. $0 \leq w_i \leq 1$ creating an infinite number of possible portfolios or POS (portfolio opportunity sets), provided the budget constraint is always met. Some particular portfolios can be singled out like, for example:
1. A one stock portfolio which is the maximum concentration level possible where all the wealth is placed in one single stock \( S \): \( w_s = 1 \) and \( w_i = 0 \), for all \( 1 < i < n \) with \( i \neq s \).

2. An equally weighted portfolio or the neutral portfolio as defined in Definition 5 above, is a portfolio where the available wealth is divided equally among all stocks \( n \); hence \( w_i = \frac{1}{n} \), \( \forall i = 1, 2, \ldots, n \). We can have only one and unique equally weighted portfolio for a given choice of investment universe.

**Definition 7. Concentration of a portfolio:**

We define the concentration of a portfolio as the measure of inequality relative to the weight allocation of the available wealth among the available stocks. A portfolio where equal wealth is allocated to each and every stock chosen initially by the investor, i.e. \( w_1 = w_2 = w_3 = \ldots w_n = \frac{1}{n} \), with no preference of one stock over the other is called an equally weighted portfolio and hence is the “minimum concentration portfolio”. This portfolio is equally weighted, and by definition, this portfolio is not concentrated. A portfolio with all the wealth invested in one single stock is, therefore, a maximum concentrated portfolio.

The more the investor deviates from the \( 1/n \) wealth allocation strategy, the more he invests in some stocks against the others within the \( n \) stocks universe. This will deviate his portfolio, consequently, from an equally weighted, minimum concentration portfolio towards a maximum concentrated portfolio, according to the weight allocation per stock \(^{10}\).

### 5.2 Specialization and Concentration Measurement in an Investment Portfolio

For the purpose of illustration, we recall in this section the hypothetical example we used in Chapter 2, which is a portfolio built up using the following screening vector applied to a hypothetical investment universe:

\[
\bar{p}_1 = \{E_r > 7\%, \sigma < 15\%, PE > 12, USA + Jap, Pharm + Oil + Retail + Metl + Mining\}
\]

In the resulting portfolio \( P \), there is a specialization in the US and Japanese markets only, and a specialization in pharmaceutical and oil and retail and metallurgical and mining companies, while excluding all other sectors. Besides, among the chosen geographical and

\(^{10}\)The concentration index of a portfolio tells us something very relevant about the investor’s decision to choose this particular strategy. If he decides on an equally weighted allocation strategy, and hence his bet is on a neutral portfolio (see Definition 5) this tells us that he does not have any particular preference of one stock over any other. This could be described as the investor’s pretension level. If he opts for a higher concentration level in his portfolio this also tells us about his pretension concerning the chosen stocks over the others. The pretension level will be discussed in more detail in Appendix B.
industrial sector specialization, the filtering vector will admit only those companies with
expected return $E_r > 7\%$ and whose standard deviation does not exceed 15\% while also
admitting only those stocks whose Price earning ratio is above 12, as shown in the graphical
representation, Figure 5.3 and its corresponding Table 5.3 below.

Figure 5.3: Graphical representation of the specialization vector $\vec{p}_1 = \{E_r > 7\%, \sigma < 15\%, PE > 12, USA + Jap, Phrm + Oil + Retl + Metl + Mining\}$. 
5.2 Specialization and Concentration Measurement in an Investment Portfolio

<table>
<thead>
<tr>
<th>$S_i$</th>
<th>$w_i$</th>
<th>$E_{r_i}$, %</th>
<th>$\sigma_{i}$, %</th>
<th>$P/E_i$</th>
<th>Location</th>
<th>Industry</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0.07</td>
<td>14</td>
<td>8</td>
<td>12.5</td>
<td>Japan</td>
<td>Pharma</td>
</tr>
<tr>
<td>S2</td>
<td>0.06</td>
<td>13.5</td>
<td>12</td>
<td>15</td>
<td>USA</td>
<td>Oil-Gas</td>
</tr>
<tr>
<td>S3</td>
<td>0.07</td>
<td>16</td>
<td>9.5</td>
<td>13</td>
<td>Japan</td>
<td>Oil-Gas</td>
</tr>
<tr>
<td>S4</td>
<td>0.08</td>
<td>11</td>
<td>14</td>
<td>14.5</td>
<td>Japan</td>
<td>Pharma</td>
</tr>
<tr>
<td>S5</td>
<td>0.09</td>
<td>8</td>
<td>13</td>
<td>17</td>
<td>Japan</td>
<td>Retail</td>
</tr>
<tr>
<td>S6</td>
<td>0.11</td>
<td>7.2</td>
<td>9</td>
<td>19</td>
<td>USA</td>
<td>Pharma</td>
</tr>
<tr>
<td>S7</td>
<td>0.14</td>
<td>12</td>
<td>9</td>
<td>12.5</td>
<td>USA</td>
<td>Metal</td>
</tr>
<tr>
<td>S8</td>
<td>0.18</td>
<td>15</td>
<td>14</td>
<td>14.7</td>
<td>Japan</td>
<td>Metal</td>
</tr>
<tr>
<td>S9</td>
<td>0.09</td>
<td>12.5</td>
<td>14.5</td>
<td>19</td>
<td>Japan</td>
<td>Mining</td>
</tr>
<tr>
<td>S10</td>
<td>0.11</td>
<td>9</td>
<td>13.9</td>
<td>15</td>
<td>USA</td>
<td>Pharma</td>
</tr>
</tbody>
</table>

Table 5.3: Hypothetical 10-stocks portfolio resulting from the screening vector: $\vec{p}_1 = \{E_{r} > 7\%, \sigma < 15\%, PE > 12, USA+Jap, Pharma+Oil+Retail+Metal+Mining\}$. This portfolio will be used to illustrate the usage of the toolbox in this chapter.

Accordingly, the final Portfolio $P$ shown in Figure 5.3, after applying the screening filters $\vec{p}$ can be expressed as the result of the intersection (and not the subtraction) of each individual criteria-attribute within the chosen Investment universe and is represented as follows:

$$P = S_{(USA+Jap)} \cap S_{(E_{r} > 7\%)} \cap \sigma < 15\% \cap S_{(P/E > 12)} \cap S_{(Pharma+Oil+Retail+Metal+Mining)}$$

The reduction in number of included stocks can be very drastic according to the number of filters and the level of cut-offs desired\textsuperscript{11}.

Our portfolio analysis methodology consists of identifying, therefore, the screening attributes

\textsuperscript{11}A typical vector of screens used to produce an Islamic Investment Universe out of the USA market could reduce the initial Universe from 10,000 stocks to 1,500 eligible stocks, or a reduction in count by 85% (Chammas and Spronk, 2009).
vector, and, after applying it to the selected Investment Universe, we end up with the resulting “chosen” stocks to be included in the investment portfolio as shown in Table 5.3. We then construct the weight-attribute matrix, shown in Table 5.4, where we reflect the choices made in the filtering vectors and map those choices against the resulting chosen stocks $S_i$. The weight-attribute matrix affected by the wealth distribution allocated to each stock will result in the impact matrix, shown in Table 5.5, where the impact of the weight vector on each attribute belonging to the filtering vector is shown.

In summary, the hypothetical portfolio consists of 10 stocks. The chosen attributes and their respective subgroups are:

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Subgroup 1</th>
<th>Subgroup 2</th>
<th>Subgroup 3</th>
<th>Subgroup 4</th>
<th>Subgroup 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_r &gt; 7%$</td>
<td>[7%, 10%]</td>
<td>[10%, 13%]</td>
<td>[13%, +]</td>
<td>[0, 10%]</td>
<td>[10%, 15%]</td>
</tr>
<tr>
<td>$\sigma &lt; 15%$</td>
<td>[0, 10%]</td>
<td>[10%, 15%]</td>
<td>[10%, 15%]</td>
<td>[10%, 15%]</td>
<td>[10%, 15%]</td>
</tr>
<tr>
<td>$PE &gt; 12$</td>
<td>[12, 14]</td>
<td>[14, 16]</td>
<td>[16, +]</td>
<td>Japan</td>
<td>USA</td>
</tr>
<tr>
<td>Location</td>
<td>Pharmaceutical</td>
<td>Oil&amp;Gas</td>
<td>Retail</td>
<td>Metallurgy</td>
<td>Mining</td>
</tr>
</tbody>
</table>

It is interesting to note that the attributes and their subgroups are subdivided at the discretion of the analyst. For example, the expected return attribute $E_r$ was divided into 3 subgroups, as shown above, but it could as well be divided into 2 subgroups or rather 4 subgroups. This subgrouping within the attribute reflects the view of the analyst and his information requirements within the target of his analysis.
5.2 Specialization and Concentration Measurement in an Investment Portfolio

The table below outlines a hypothetical specialization vector \( \vec{p} \) based on the criteria of \( E_r > 7\% \), \( \sigma < 15\% \), and \( PE > 12 \). The vector is categorized under Location (USA, Japan, Pharma, Oil, Retail, Metal, Mining), Industrial Sector (S1, S2, S3, ..., S10), and Sij, where each entry represents the weight or value associated with each criterion.

<table>
<thead>
<tr>
<th>Location</th>
<th>Industrial Sector</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>S6</th>
<th>S7</th>
<th>S8</th>
<th>S9</th>
<th>S10</th>
<th>Tot</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>10</td>
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<td>Japan</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>Pharmaceutical</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>Oil</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>Retail</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>Metal</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<td>10</td>
</tr>
<tr>
<td>Mining</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 5.4: Detailed Weight-Criteria matrix of a hypothetical specialization vector \( \vec{p} = \{E_r > 7\%, \sigma < 15\%, PE > 12\} \).
The table below demonstrates the detailed impact matrix for the hypothetical specialization vector $\vec{p}_1 = \{E_r > 7\%, \sigma < 15\%, PE > 12, USA + Jap, Pharma + Oil + Retail + Metal + Mining\}$.

<table>
<thead>
<tr>
<th>$S_i$</th>
<th>$w_i$</th>
<th>$[7, 10]$</th>
<th>$[10, 13]$</th>
<th>$[13, +]$</th>
<th>$[0, 10]$</th>
<th>$[10, 15]$</th>
<th>$[12, 14]$</th>
<th>$[14, 16]$</th>
<th>$[16, +]$</th>
<th>Jap</th>
<th>USA</th>
<th>Phr</th>
<th>Oil</th>
<th>Ret</th>
<th>Met</th>
<th>Mng</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0.07</td>
<td>0</td>
<td>0</td>
<td>0.07</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.07</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>S2</td>
<td>0.06</td>
<td>0</td>
<td>0</td>
<td>0.06</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.06</td>
<td>0.06</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>S3</td>
<td>0.07</td>
<td>0</td>
<td>0</td>
<td>0.07</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>S4</td>
<td>0.08</td>
<td>0</td>
<td>0</td>
<td>0.08</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0.08</td>
<td>0.08</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>S5</td>
<td>0.09</td>
<td>0.09</td>
<td>0</td>
<td>0</td>
<td>0.09</td>
<td>0</td>
<td>0</td>
<td>0.09</td>
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<td>0.09</td>
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<td></td>
</tr>
<tr>
<td>S6</td>
<td>0.11</td>
<td>0.11</td>
<td>0</td>
<td>0</td>
<td>0.11</td>
<td>0</td>
<td>0</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>S7</td>
<td>0.14</td>
<td>0</td>
<td>0.14</td>
<td>0.14</td>
<td>0</td>
<td>0.14</td>
<td>0</td>
<td>0</td>
<td>0.14</td>
<td>0.14</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.14</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>S8</td>
<td>0.18</td>
<td>0</td>
<td>0</td>
<td>0.18</td>
<td>0</td>
<td>0.18</td>
<td>0</td>
<td>0.18</td>
<td>0.18</td>
<td>0.18</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.18</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>S9</td>
<td>0.09</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.09</td>
<td>0.09</td>
<td>0.09</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td>S10</td>
<td>0.11</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.11</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.11</td>
<td>0.11</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>tot</td>
<td>1.00</td>
<td>0.31</td>
<td>0.31</td>
<td>0.38</td>
<td>0.39</td>
<td>0.61</td>
<td>0.28</td>
<td>0.43</td>
<td>0.29</td>
<td>0.58</td>
<td>0.42</td>
<td>0.37</td>
<td>0.13</td>
<td>0.09</td>
<td>0.32</td>
<td></td>
</tr>
</tbody>
</table>

|       | 1.00  | 1.00      | 1.00      | 1.00      | 1.00      | 1.00      | 1.00      | 1.00      | 1.00      | 1.00| 1.00| 1.00| 1.00| 1.00| 1.00|

Table 5.5: Detailed Impact Matrix of the hypothetical specialization vector $\vec{p}_1 = \{E_r > 7\%, \sigma < 15\%, PE > 12, USA + Jap, Pharma + Oil + Retail + Metal + Mining\}$. 
5.2 Specialization and Concentration Measurement in an Investment Portfolio

5.2.1 Measuring Specialization: The Gini Index of each subgroup and its total relative weight.

In a previous chapter, we have discussed the properties of the Gini index and its adequacy to measure the specialization of a portfolio. Our definition of specialization (Definition 6) refers to the concentration of the subgroups related to the attributes of the securities included in a portfolio. In Appendix A we present an extended view on the Gini index and its decomposition. The results of the formulas involved using the Gini index as a measure of concentration are summarized in Table 5.6.

The decomposition of the Gini index, as proposed by Dagum (1987) and detailed in Appendix A, is applied when a population or group is divided into subpopulations or subgroups with overlapping. In our case, a chosen portfolio attribute would divide it into subgroups and we are interested in measuring the Gini or $G$ index of each subgroup to describe the specialization level related to this particular attribute. For this matter, $G$ is decomposed into three main components, so that $G = G_w + G_{nb} + G_t$ as shown in Equation A.9a, and where:

- $G_w$ is the weighted contribution to $G$ of the Gini within the subgroup,
- $G_{nb}$ is the net contribution to $G$ of the Gini between subgroups,
- $G_t$ is the contribution to $G$ of the transvariation between subgroups,
- $G_{gb} = G_{nb} + G_t$ is the gross contribution to $G$ of the Gini between subgroups.

We have also introduced the $G_{jnull}$ which measures the Gini index within a subgroup taking into consideration the null entries in the subgroup. If, within a portfolio of 10 stocks, the subgroup “pharmaceutical industries” is represented by four of the ten stocks, then $G_{Pharmnull}$ will be computed taking into consideration $n = 10$ and not $n = 4$, i.e. the null entries are taken into consideration. This will enable us to compare specialization of portfolios of different sizes (i.e. different $n$).

We are measuring the specialization of the portfolio in one attribute using a concentration measure, namely the Gini index and its decomposition. Hence the more concentration index a subgroup of an attribute exhibits, the more specialized the portfolio is in this attribute’s subgroup.

<table>
<thead>
<tr>
<th>Index</th>
<th>$E_r$</th>
<th>$\sigma$</th>
<th>PE ratio</th>
<th>Geo.Loc.</th>
<th>Industry Gr</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G$, Gini index</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G_{jnull}$</td>
<td>0.046</td>
<td>0.11</td>
<td>0.1</td>
<td>0.046</td>
<td>0.316</td>
</tr>
<tr>
<td>$G_{gb}$</td>
<td>0.134</td>
<td>0.092</td>
<td>0.129</td>
<td>0.1</td>
<td>0.166</td>
</tr>
<tr>
<td>$G_{nb}$</td>
<td>0.02</td>
<td>0.01</td>
<td>0.033</td>
<td>0.02</td>
<td>0.154</td>
</tr>
<tr>
<td>$G_t$</td>
<td>0.114</td>
<td>0.082</td>
<td>0.096</td>
<td>0.08</td>
<td>0.012</td>
</tr>
<tr>
<td>$G_w$</td>
<td>0.052</td>
<td>0.094</td>
<td>0.057</td>
<td>0.086</td>
<td>0.02</td>
</tr>
<tr>
<td>$HHI$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$HHI = 0.112$</td>
</tr>
<tr>
<td>$rHHI$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$rHHI = 8.91$</td>
</tr>
</tbody>
</table>

Table 5.6: Summary of the Gini decomposition formulas related to the hypothetical 10-stocks portfolio of our example and its $HHI$. 
It is necessary to point out that, for each subgroup we considered and within each subgroup, the sum of all partial weights within each subgroup is always 1, i.e. \( \sum p_i = 1 \) where \( p_i = \frac{n_i}{n} \). This means that no overlap between the subgroup exists and hence we can conclude that, since no overlapping is present, each attribute is a structural inequality factor, that is the partial weights of its element constitute a basis for the calculation of the specialization. This method will be applied to our 500 stocks portfolio in Chapter 6 with satisfactory results.

To illustrate our point, please consider the following table, which analyzes the attribute “industrial sector” related to our hypothetical portfolio:

<table>
<thead>
<tr>
<th>id</th>
<th>Industry Groups</th>
<th>All Groups</th>
<th>Oil-Gas</th>
<th>Mining</th>
<th>Retail</th>
<th>Pharma</th>
<th>Metal</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>Size of group</td>
<td>10</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>1b</td>
<td>Total weight (q_j)</td>
<td>1</td>
<td>0.130</td>
<td>0.09</td>
<td>0.09</td>
<td>0.37</td>
<td>0.32</td>
</tr>
<tr>
<td>1c</td>
<td>Mean weight (m_j)</td>
<td>0.1</td>
<td>0.005</td>
<td>0.09</td>
<td>0.09</td>
<td>0.0925</td>
<td>0.16</td>
</tr>
<tr>
<td>1d</td>
<td>Share of the group/Tot.Count (p_j)</td>
<td>1</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td>0.4</td>
<td>0.2</td>
</tr>
</tbody>
</table>

| 7  | Gini Index \( (G = G_{oc} + G_{ob} + G_t) \) | 0.186 |
| 8  | HHI                | 0.112  | 0.5    | 1      | 1      | 0.26   | 0.54  |
| 9  | rHHI (reverse HHI) | 8.91   | 1.99   | 1      | 1      | 3.86   | 1.97  |
| 10 | rHHI/n_j           | 0.891  | 0.99   | 1      | 1      | 0.96   | 0.98  |

Table 5.7: Industry group attribute: Gini (Specialization index) and HHI (Concentration index) toolbox output.

It is clear that the hypothetical portfolio at hand is specialized in Pharmaceuticals and metallurgy, by considering the relative weight of each industry. This will also be applied to a larger portfolio in Chapter 6.

5.2.2 Measuring Concentration: The HHI index.

The \( HHI \) measures the concentration level of the portfolio. The hypothetical portfolio under study exhibits an \( HHI = 0.112 \) or a \( rHHI = 8.91 \). The minimum possible \( HHI \) in the case of a portfolio of 10 stocks is \( HHI_{min} = \frac{1}{n} = \frac{1}{10} = 0.1 \) and hence a \( rHHI = 10 \). The portfolio at hand is not concentrated at all and its \( HHI = 0.112 \) is very near the minimum concentration level possible of 0.1. We can fairly conclude that the hypothetical portfolio under study is not a concentrated portfolio. The \( rHHI = 8.91 \approx 9 \) shows that the portfolio’s concentration level is identical to a 9 stock equally weighted portfolio. This conclusion is corroborated by the similar derivations of the \( HHI \), where the corrected \( HHI \) or the \( cHHI = 0.0135 \) while the minimum \( cHHI \) is equal to zero.

One important measure we derived is the \( \frac{rHHI}{n} \) which is the per unit concentration of a group. The lower this indicator is, the higher the absolute concentration of the subgroup under study. In fact, since \( rHHI \) indicates the equivalent number of equally weighted stocks which have a similar \( HHI \) then the initial portfolio, then the per unit concentration \( \frac{rHHI}{n} \)
5.3. Concluding remarks

This chapter introduced the measurement of concentration and specialization in an investment portfolio using $HHI$, Gini and the relative weight of each element of the subgroups. The relative ease of computation makes these measures attractive and accessible for the user. We also note that the Gini index decomposition which involves a pairwise comparison between the elements of the portfolio, and a pairwise comparison between the elements of the subgroups, taken each two at a time, is not required nor necessary. It should be noted that this decomposition can render the spreadsheet very bulky and difficult to read when the

will allow the analyst to compare the concentration of portfolios with different $n$.

Table ?? (rows 9 and 11) shows the minimum reached per unit $rHHI$ or $\frac{HHI}{n} = 0.85$ for the maximum reached $cHHI = 0.06$ for the subgroup of $PE$ ratio group between 14 and 16 or G2. This is the most concentrated subgroup in the portfolio where 75% of the wealth allocated to this subgroup is concentrated in 2 out of the 4 stocks available in this subgroup.

5.2.3 Description of the Hypothetical Investment Portfolio.

From the above discussion we can fairly say that our hypothetical portfolio consisting of 10 stocks is not concentrated, with a $rHHI = 8.91$ i.e. it is similar to a portfolio with almost nine equally weighted stocks.

As for the specialization of the portfolio in each of the attributes chosen, we outline the following:

- Expected Return Attribute: The portfolio is not specialized in any subgroup of the Expected return attribute.
- Standard Deviation Attribute: The portfolio is specialized in stocks whose standard deviation $\sigma$, where $10\% < \sigma < 15\%$.
- Price-Earning Ratio Attribute: A slight specialization of stocks whose PE ratio is between 14 and 16 is shown.
- Geographic Location Attribute: The portfolio is specialized in stocks from Japan more than stocks from the USA.
- Industry Groups attribute: The portfolio shows a specialization in pharmaceutical industry stocks and metallurgy stocks among the other industries included in the portfolio. We also noted that, within the pharmaceutics subgroup, there is no concentration of one stock against the others since the weights within this subgroup is nearly evenly distributed amongst the four stocks consisting of the pharmaceutics subgroup.

5.3 Concluding remarks

This chapter introduced the measurement of concentration and specialization in an investment portfolio using $HHI$, Gini and the relative weight of each element of the subgroups. The relative ease of computation makes these measures attractive and accessible for the user. We also note that the Gini index decomposition which involves a pairwise comparison between the elements of the portfolio, and a pairwise comparison between the elements of the subgroups, taken each two at a time, is not required nor necessary. It should be noted that this decomposition can render the spreadsheet very bulky and difficult to read when the
portfolio subgroups become numerous, regardless of the number of stocks in the portfolio. A further illustration in the next chapter of our suggested method and measures will prove elegant, handy and accessible to common desktop calculation tools.

Within this context, several questions arise:

- Are these measures, as defined in this research, accepted or “liked” by an investor? In other words, would an investor judge that, measuring the concentration and specialization of his investment portfolio is a required or necessary information for his investment? Consequently, are these numbers and tables useful?

- Can the investor change the concentration and the specialization of his portfolio using the technical insight that we are proposing? Can the investor “fill in the gap” in his portfolio if the description of this portfolio shows a certain specialization in some sectors against some others that he wants to include in his portfolio?

- Eventually, will the description of the portfolio, vis-à-vis its concentration and specialization be a useful addition to the usual return and volatility information that is commonly used today in investment decisions?

The application of our indexes to investment portfolios will be shown in the next chapter. We will apply the HHI and the Gini index to measure the concentration specialization of various 500 stocks portfolios and we will show that the description provided by these indexes is useful and relevant to the investor and the financial market place at large.
Chapter 6

Concentration and Specialization of the US listed Stocks

6.1 Introduction

In previous chapters we have derived the formulas for measuring specialization and concentration in an investment portfolio as well as commented on technical insights concerning these aforementioned measures when applied to an investment portfolio. This resulted in tables that depict the results related to our ten-stocks hypothetical portfolio. We chose the Hirschman-Herfindahl index, $HHI$, and its derivations to measure the concentration within a portfolio of investment along with $G$, the Gini index. We also chose $G$ to describe the specialization of the portfolios.

In this chapter, we propose a numerical application of the toolbox, as stated in the second part of research question 4.

We will apply our toolbox formulas to two series of quarterly portfolios\(^1\) composed of stocks listed in the United States. We will choose the top 500 stocks per capitalization in each period and we will form two portfolios. The first one consists of a market-capitalization weighted portfolio $P_{mcap}$ and the second one an equally weighted portfolio $P_{eq}$. Again, our approach to choose only the top 500 stocks and not other number or to utilize the United

\(^1\)The quarterly portfolios that are formed for the purpose of our numerical study consist of the top 500 stocks listed at the end each quarter, i.e. last dealing day of month 3,6,9,12. The choice of these intervals is purely subjective and it could have been any other cut-off date provided the intervals are equal.
Chapter 6. Concentration and Specialization of the US listed Stocks

States listed stocks rather than the Japanese or the French listed stock is purely subjective. We believe that our choice was mainly oriented by the ease of obtaining the data and the abundance of listed companies in the United States. The choice to include 500 top market capitalization stocks in a given trading day was to include as many stocks as possible by the limited computational resources we have available.

Throughout the time interval of our consideration, from January 1993 until September 2013 included, our analysis will be two-fold:

- Describe the trends and the variation over time of the concentration and the specialization of the quarterly portfolios described in the methodology section, and
- Describe the concentration and the specialization of equally weighted and market capitalization portfolios of three particular quarters that we believe are outstanding examples of the methodology we wish to convey in this thesis. This means that we will study thoroughly 6 portfolios pertaining to 3 different quarters and constructed in two different ways.

Our data consists of around 82 consecutive quarters spanning over 21 years of American Stock exchange listed stocks, as described in the following sub-section.

6.1.1 Data Description and Methodology.

The data was downloaded from the CRSP/COMPSTAT database. It comprises stocks listed in the United States from the date: January 1993 until September 2013 included. We downloaded monthly end-of-the-month closing prices along with the basic fundamentals listed, mainly capitalization values, price in base currency, number of shares, trading volume, industry identification code, price in base currency, exchange rate, local currency, country identification, book value-to-price ratio along with the usual stock identifiers like GIND code and CUSIP code. The data was analyzed and its robustness was tested. Some minor corrections were made to eliminate obvious stray values or to extrapolate between previous and next month to replace a monthly fundamental which is missing. In total only 25 minor corrections were made. The data is robust and the results we obtained look solid and consistent with previous published data on returns and volatility.

For the purpose of our analysis, the data we used represents the USA stock exchange market from Q1 of 1993 inclusive until Q3 of 2013 inclusive, for a total of 82 quarters data. Therefore our study includes 82 quarterly portfolios with market capitalization weights allocation $P_{cap}$ and 82 quarterly portfolios with equal weights allocation $P_{eq}$.

On every end of month, we calculated the market capitalization of each stock and sorted the results by descending order. Every month, the first 500 stocks were included in our dataset.

We choose the attribute of “industry group” to conduct our analysis of concentration and specialization. We have detected 44 different industry subgroups that are specified by
the database\textsuperscript{2} we downloaded. We have noted the creation of new subgroups that were not
existing at the initial date of our data but were introduced later like the subgroup “REIT”
(Real Estate Investment Trust,) or “Cellular and Wireless”.

**The Methodology**  The analysis is two-fold and consists first of an overview of
the 21 years period of quarterly portfolios that we will create, with respect to the variations
and trends in their respective concentration and specialization. Second, the analysis will
focus of 3 particular quarters that we will choose which exhibit maximum and relative min-
imum concentrations, describing each particular portfolio with respect to its concentration
and specialization. The purpose of the first analysis is to observe the variations of the port-
folios’ concentration during the observed period and to compare the variation of the equally
weighted to the cap weighted strategy. The purpose of the second analysis is to explain the
observed remarkable or important variations (drastic jumps or declines in concentration,
absolute extreme values) and try to examine their particular specialization. It is worth
noting that the analysis will not attempt to explain the reasons behind the variations or
the plausible causes of the change in specialization.

We are particularly interested in exploring and describing equally weighted portfolios
and market capitalization weighted portfolios in function of their concentration and spe-
cialization. It is worth noting that we have decided to choose the top market capitalization
500 stocks from the available investment universe for ease of data manipulation and for
maximum relevance to our analysis.

Accordingly, two quarterly portfolios are first created as follows:

(a) A cap-weighted portfolio, $P_{\text{cap}}$, comprising the first 500 market capitalization val-
ues, calculated at the end of each month, where the weight $w_i$ for Stock $S_i$, $i = 1, 2, 3, \ldots, 500$ is equal to:

$$w_i = \frac{\text{Cap}_i}{\sum_{i}^{500} \text{Cap}_i}$$

where $\text{Cap}_i$ is the market capitalization of stock $S_i$ during the period in consideration.

\textsuperscript{2}The complete list of industry subgroups is the following: Agriculture, food, beverage;
Beer, liquor, and tobacco; Basic minerals and metals; Oil and coal resources; Integrated
oil companies; Oil drilling and services; Oil distribution; Construction materials; Forest
products, paper; Construction, home-building; Chemicals, rubber; Metal products, machin-
ery; Instruments; Mainframe & minicomputers; Photo-optical, micros, office machinery;
Textiles and apparel; Drugs & pharmaceuticals; Soaps & cosmetics; Furniture, household
items; Consumer durables; Cars and trucks; Commercial aircraft, components; Govt. air-
craft & defense; Land and water transportation; Airlines; Electric utilities; Gas and other
public utilities; Communications utilities; Publishing, broadcasting, cinema; Restaurants,
hotels, theaters; Retail; Health care & hospital; Information, services; Real estate develop-
ment; Financial investments; Banks & credit institutions; Miscellaneous finance; Insurance;
Biotechnology; Software; Cellular & wireless; IT hardware; Reits; Wholesale; Trading com-
pany.
An equally weighted portfolio, $P_{eq}$, comprising the first 500 market capitalization values, calculated at the end of each month, where the weight $w_i$ for Stock $S_i$, $i = 1, 2, 3, \ldots, 500$ is equal to:

$$w_1 = w_2 = \cdots = w_i = \frac{1}{500} = 0.002.$$ 

In both portfolios we assume no short selling is allowed and hence the budget constraint in both portfolios, $\sum_{i=1}^{500} w_i = 1$, is always met.

We will consider the attribute of “industry group” to subdivide our portfolios into subgroups, one subgroup being the industry group: for example subgroups like “Furniture, Household Items”, “Communication Utilities”, or “Soaps & Cosmetics” are actual examples of subgroups in our portfolios.

We will then calculate for each quarterly portfolio the concentration and the specialization measures, as suggested in Chapter 5. We will compile the results in tables and use them to describe the portfolios at hand. We will show that using the concentration and specialization measure is useful and gives the investor an additional insight on his investment, with no estimation errors involved.

The results we obtained show clear patterns of concentration and specialization over time, and we will highlight some important dates and describe the portfolio around these dates. We will show that the specialization of the portfolio changes over time and that the investor is capable of detecting these changes and taking the decision to modify the content of the portfolio with respect to its concentration and specialization.

Additionally, our results will give the investor an insight on how the economy is changing over time, as well as an insight to the state of his portfolio at a particular time of observation (in our case quarterly observations).

The methodology of using the concentration and specialization measure to describe an investment portfolio comprises the following calculation levels:

1. **At the individual stock level:** Given the 500 stocks of the portfolio, each with its weight $w_i$, we will calculate the $HHI$ and $G$ at the level of the individual weights. In this case the number of entries in the universe is $n = 500$. This process is applied to all 82 quarterly portfolios.

2. **At the weight of industry sectors level:** Given a quarterly portfolio, we observe that its 500 stocks are clustered in the possible 44 industry subgroups available in the initial data. We will calculate the $HHI$ and $G$ of the sum of weight of the stocks per each subgroup, i.e. the sum of weights per industry subgroup or cluster. This will give us the industry concentration of the portfolio. In this case the number of

---

3It is worth noting that, due to the initial constraint we have imposed by including only the top 500 market capitalization stocks in both the $P_{cap}$ and $P_{eq}$, some industry subgroups will eventually not be represented in some quarterly portfolios because they fall outside the top 500 stocks chosen.
6.2 Results: Concentration and Specialization of \( P_{eq} \) and \( P_{cap} \).

It follows that the results obtained are divided into two parts: Part I will describe the evolution and trend of concentration and specialization from 1993 until 2013 in the United States stock market and, Part II will describe the concentration and specialization of the 3 particular portfolios chosen, as summarized in Table 6.1. We shall point out to some particular and specific economic and market events and crisis that occurred within the time span of the particular 3 portfolios we are considering, to further show that our approach offers a particular insight into the investment situation per se, without establishing any inferential conclusion or a causal relationship between the concentration level and the economic event.

Our approach provides a different angle to look at the market and its dynamics, and this view focuses on concentration and specialization. We describe the evolution and trends of concentration and specialization of \( P_{eq} \) and \( P_{cap} \) throughout the entire period of data available (i.e. from 1993 Q1 to 2013 Q3 both inclusive). In this section we will explore the results we obtained and present an in-depth commentary on the same in view of i) describing the portfolios at hand, ii) use the concentration and specialization measures as a detection tool for possible market patterns or tendencies that will eventually help in investment decision making and iii) propose our measures as a possible foundation to monitor investment portfolios\(^4\). Table 6.1 shows the content of the results Part I and Part II for clarity.

\(^4\)The present section contains graphs whose labels will follow the following abbreviations:
Chapter 6. Concentration and Specialization of the US listed Stocks

### PART I

All Portfolios

<table>
<thead>
<tr>
<th>Measurement</th>
<th>( P_{\text{cap}} )</th>
<th>( P_{\text{eq}} )</th>
<th>( P_{\text{eq}} )</th>
<th>( P_{\text{eq}} )</th>
<th>( P_{\text{eq}} )</th>
<th>( P_{\text{eq}} )</th>
<th>( P_{\text{eq}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concentration: at Stock Level</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Concentration: at Industry Level</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Specialization: at Industry Level</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

Table 6.1: Mapping of the content of Part I and II analysis that the results will reflect in this subsection

#### 6.2.1 Results Part I: Variation of Concentration of \( P_{\text{cap}} \) and \( P_{\text{eq}} \).

When the equally weighted portfolio, \( P_{\text{eq}} \) is considered, some obvious results can be deduced intuitively, even prior to calculating its concentration levels:

- First, by virtue of its design, an equally weighted portfolio exhibits no concentration at the level of individual stocks included. Each stock of the 500 has a weight of:
  \[
  w_1 = w_2 = \cdots = w_{500} = \frac{1}{500} = 0.002,
  \]
  obeying to the budget constraint
  \[
  \sum_{i=1}^{500} w_i = 1.
  \]
  This leads us to deduce that \( HHI_{P_{\text{eq}}} = \frac{1}{n} = 0.002 \), and
  \( G_{P_{\text{eq}}} = 0 \)

- Second, when considering the industry groups clustering, the equally weighted portfolio is not at its minimum concentration level. The 500 stocks are clustered among 44 industry groups, and hence this clustering is not equally weighted. Our Table 6.4 below will show the values of the industry clustering concentration for each quarterly portfolio.

- Third, it is intuitive to observe that the specialization of the equally weighted portfolio follows the concentration of the industry clusters. In fact, since the individual weights are all equal to 0.002, hence the total weight per cluster of industries is:
  \[
  w_{\text{ind}_j} = 0.002 \times S_j, \text{ where } S_j \text{ is the number of stocks included in the industry cluster } j.
  \]
  It follows that the bigger the \( S_j \) of a given industry cluster, the more specialized the portfolio in this particular cluster of industry \( j \). We can generalize this conclusion to any given attribute \( m \), where, for an equally weighted portfolio, the count of the

“Gini eq stk” stands for The Gini index for \( P_{\text{eq}} \) at the stock level. “Gini cap stk” stands for the Gini index for \( P_{\text{cap}} \) at the stock level. “Gini eq ind” stands for the Gini index for \( P_{\text{eq}} \) at the industry cluster level and “Gini cap ind” stands for the Gini index for \( P_{\text{cap}} \) at the industry cluster level. Similarly for “HHI eq stk” which represents the HHI index for \( P_{\text{eq}} \) at the stock level.
individual stocks clustered in an attribute \( m \), defined by the investor, represents the specialization of the portfolio in this particular attribute.

**Concentration measures of \( P_{eq} \) and \( P_{cap} \) at the level of individual weights of stocks.**

As discussed earlier, the \( HHI \) and the Gini index were measured at the 500 stocks constituting each portfolio, at quarterly intervals. The results are depicted in Figure 6.1. The straight line confounded with the \( x \)-axis, represents a value of \( Gini = 0 \) for the equally weighted portfolio, whereas its corresponding \( HHI = 0.002 \) is depicted in the red dashed line directly above it. Please note that \( HHI \) is plotted against the secondary axis to the right of the figure.

![Figure 6.1: Gini and \( HHI \) indexes of quarterly portfolios \( P_{eq} \) and \( P_{cap} \) consisting of 500 stocks each. \( P_{eq} \) concentrations measures are represented by a straight line ( \( Gini = 0 \) and \( HHI = \frac{1}{n} = \frac{1}{500} = 0.002 \)). Note that \( HHI \) is plotted against the secondary axis to the right of the graph.](image)

The concentration of the \( P_{cap} \) portfolio is not constant over time. We can observe some local peaks, namely during the year 2000 where a \( G_{max} = 0.6349 \) was reached in Q2 of 2000 (corresponding to \( HHI = 0.0096 \)), followed by a steady decline in concentration and then another prominent peak in the last quarter of 2008 and first half of 2009.
Chapter 6. Concentration and Specialization of the US listed Stocks

Table 6.2: Summary of the top and bottom 10 concentration values, measured at the stocks level, related to $P_{cap}$.

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Gini</th>
<th>HHI</th>
<th>rHII</th>
<th>Quarter</th>
<th>Gini</th>
<th>HHI</th>
<th>rHII</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000 Q2</td>
<td>0.634</td>
<td>0.0096</td>
<td>103</td>
<td>1993 Q3</td>
<td>0.487</td>
<td>0.0053</td>
<td>186</td>
</tr>
<tr>
<td>2000 Q1</td>
<td>0.625</td>
<td>0.0098</td>
<td>101</td>
<td>1993 Q4</td>
<td>0.489</td>
<td>0.0054</td>
<td>185</td>
</tr>
<tr>
<td>2000 Q3</td>
<td>0.609</td>
<td>0.0086</td>
<td>115</td>
<td>1993 Q2</td>
<td>0.499</td>
<td>0.0056</td>
<td>176</td>
</tr>
<tr>
<td>1999 Q4</td>
<td>0.624</td>
<td>0.0098</td>
<td>101</td>
<td>1993 Q1</td>
<td>0.501</td>
<td>0.0057</td>
<td>174</td>
</tr>
<tr>
<td>1999 Q1</td>
<td>0.618</td>
<td>0.0087</td>
<td>114</td>
<td>1994 Q2</td>
<td>0.497</td>
<td>0.0054</td>
<td>183</td>
</tr>
<tr>
<td>1999 Q3</td>
<td>0.617</td>
<td>0.0090</td>
<td>110</td>
<td>1994 Q1</td>
<td>0.489</td>
<td>0.0054</td>
<td>183</td>
</tr>
<tr>
<td>2001 Q4</td>
<td>0.614</td>
<td>0.0091</td>
<td>109</td>
<td>1994 Q3</td>
<td>0.498</td>
<td>0.0054</td>
<td>182</td>
</tr>
<tr>
<td>2001 Q3</td>
<td>0.625</td>
<td>0.0092</td>
<td>107</td>
<td>1994 Q4</td>
<td>0.507</td>
<td>0.0057</td>
<td>175</td>
</tr>
<tr>
<td>2001 Q1</td>
<td>0.608</td>
<td>0.0085</td>
<td>116</td>
<td>1995 Q1</td>
<td>0.505</td>
<td>0.0056</td>
<td>177</td>
</tr>
<tr>
<td>2001 Q2</td>
<td>0.607</td>
<td>0.0092</td>
<td>107</td>
<td>1995 Q3</td>
<td>0.509</td>
<td>0.0056</td>
<td>175</td>
</tr>
</tbody>
</table>

It is noticeable that the $rHII$, representing an equivalent number of equally weighted stocks portfolio varies between $rHII = 101$ and $rHII = 186$ out of the 500 stocks of the quarterly portfolios. This indicates a fairly high level of concentration in the market capitalization portfolio $P_{cap}$, as seen in Figure 6.2

![Figure 6.2: $rHII$ for both $P_{eq}$ and $P_{cap}$](image)

The $rHII$ of $P_{cap}$, during the observed period of 82 quarters, varied between 101 and 186. The mean value of $rHII$ is 139 and the median value is 136. However the histogram of $rHII$ reveals a tendency towards more concentrated portfolios, as shown in Fig 6.3
6.2. Results: Concentration and Specialization of Peq and Pcap.

Figure 6.3: Histogram of $rHHI$ (at stock level) for $P_{cap}$. The histogram shows more cumulative frequency on the left side of the figure, indicating a rather more concentrated portfolios over the 82 quarters of observation.

The $P_{cap}$ is fairly concentrated over the observation period and reached a maximum concentration level in the Q1-Q2 of the year 2000, while the minimum observed concentration was in Q3 of year 1993. It is noticeable that the concentration level we are measuring goes from its minimum to its maximum observable value in just 9 months (from 1999 Q3 to 2000 Q2), within the bracket of time we are considering. We know that during this period, the DotCom bubble crisis occurred, but we cannot ascertain a causal relationship between this crisis and the sudden change in concentration. We shall however shed a light on the specialization of the portfolios during this same period and try to relate the results in a non-conclusive manner.

In summary, the concentration levels of the quarterly portfolios from 1993 Q1 till 2013 Q3, exhibit the following characteristics at the individual stocks level:

1. $P_{eq}$, by virtue of its construction, exhibits a constant $G = 0$ and a constant $HHI = \frac{1}{n} = 0.002$.

2. $P_{cap}$ shows an absolute maximum value of $G_{\text{max,stk}} = 0.634$ in Q2 2000 and an absolute minimum value of $G_{\text{min,stk}} = 0.487$ in Q3 1993.

3. In terms of $rHHI$, the 500 stocks included in $P_{cap}$ did not exceed the value of $rHHI_{\text{max,stk}} = 186$ in 1993 Q3 and an absolute minimum value of $rHHI_{\text{min,stk}} = 101$ both in 1999 Q4 and 2000 Q1, where the concentration reached its maximum. This means that, at its most concentrated level, the 2000 Q1 portfolio was equivalent to a virtual portfolio of only 101 stocks out of the 500 available (1:5 compression). At its best, the least concentrated portfolio, at the start of the observation period, is represented by a virtual equivalent portfolio of 187 stocks out of the 500 available
Concentration measures of $P_{eq}$ and $P_{cap}$ at the level of industry sectors.

When we consider clustering the stocks of each portfolio, namely of $P_{cap}$ and $P_{eq}$ in the stocks’ respective industry subgroup, we obtain a different levels of concentration, and the equally weighted portfolio starts exhibiting concentration levels different from the $HHI_{min} = 0.002$ and $G_{min} = 0$. This is due to the fact that we have 500 stocks in the portfolio but only 44 industry sectors related to these stocks. Therefore, when we consider the concentration level of the portfolios under study with respect to the industry sectors we will obtain a different set of concentration levels, as is apparent in Figure 6.4.

The graph in Figure 6.4 shows the same peak in concentration at around the period from 1999 Q3 to 2000 Q2 that was singled out in the previous subsection above, but with a sharper slope. If we plot in the same graph the concentration measures of both portfolios with respect to the individual stocks (Figure 6.1) and industry clusters (Figure 6.4), we are able to compare the respective concentration values per quarter, as shown in Figure 6.5.
6.2. Results: Concentration and Specialization of \( P_{eq} \) and \( P_{cap} \).

Table 6.3: Summary of the top and bottom 10 concentration values, measured at the industry clusters level, related to \( P_{cap} \).
Chapter 6. Concentration and Specialization of the US listed Stocks

Table 6.4: Summary of the top and bottom 10 concentration values, measured at the industry clusters level, related to $P_{eq}$.

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Gini</th>
<th>HHI</th>
<th>rHHI</th>
<th>Quarter</th>
<th>Gini</th>
<th>HHI</th>
<th>rHHI</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000 Q1</td>
<td>0.542</td>
<td>0.0524</td>
<td>19</td>
<td>2005 Q2</td>
<td>0.414</td>
<td>0.0375</td>
<td>26</td>
</tr>
<tr>
<td>2000 Q2</td>
<td>0.540</td>
<td>0.0519</td>
<td>19</td>
<td>2005 Q3</td>
<td>0.417</td>
<td>0.0370</td>
<td>26</td>
</tr>
<tr>
<td>2000 Q3</td>
<td>0.538</td>
<td>0.0506</td>
<td>19</td>
<td>2005 Q4</td>
<td>0.419</td>
<td>0.0378</td>
<td>26</td>
</tr>
<tr>
<td>1999 Q4</td>
<td>0.530</td>
<td>0.0507</td>
<td>19</td>
<td>2006 Q2</td>
<td>0.424</td>
<td>0.0367</td>
<td>27</td>
</tr>
<tr>
<td>1999 Q3</td>
<td>0.505</td>
<td>0.0465</td>
<td>21</td>
<td>2005 Q4</td>
<td>0.427</td>
<td>0.0380</td>
<td>26</td>
</tr>
<tr>
<td>1999 Q1</td>
<td>0.502</td>
<td>0.0460</td>
<td>21</td>
<td>2006 Q4</td>
<td>0.429</td>
<td>0.0379</td>
<td>26</td>
</tr>
<tr>
<td>1999 Q2</td>
<td>0.499</td>
<td>0.0458</td>
<td>21</td>
<td>2006 Q1</td>
<td>0.430</td>
<td>0.0377</td>
<td>26</td>
</tr>
<tr>
<td>2000 Q4</td>
<td>0.498</td>
<td>0.0449</td>
<td>22</td>
<td>2004 Q3</td>
<td>0.432</td>
<td>0.0396</td>
<td>25</td>
</tr>
<tr>
<td>1998 Q4</td>
<td>0.494</td>
<td>0.0460</td>
<td>21</td>
<td>2004 Q4</td>
<td>0.432</td>
<td>0.0390</td>
<td>25</td>
</tr>
<tr>
<td>1995 Q4</td>
<td>0.491</td>
<td>0.0454</td>
<td>21</td>
<td>2008 Q3</td>
<td>0.434</td>
<td>0.0368</td>
<td>27</td>
</tr>
</tbody>
</table>

We observe that the industry clusters’ concentration is usually less than that of the individual stocks except in the periods of 1993 Q4 (one quarter only) and, more notoriously, in the period between 1993 Q3 and 2000 Q2. In this latter period, the industry clusters concentration measure peaks to $G = 0.665$ in 2000 Q3, which is the highest attainable concentration during the period under study. The concentration measure for the $P_{eq}$ at the industry level (line labeled ‘Gini eq ind’ in Figure 6.5) is always less than $P_{cap}$ at all time during the period under study.
6.2. Results: Concentration and Specialization of \( P_{eq} \) and \( P_{cap} \).

Figure 6.5: A comparative graph of Gini concentration at the levels of both Industry cluster’s and individual stocks. Note that during the period of 1999 Q3 to 2000 Q2 we observe that the industry clusters’ concentration is higher then the individual stocks concentration.

In summary, the concentration levels of the quarterly portfolios from 1993 Q1 till 2013 Q3, exhibit the following characteristics at the industry clusters level:

1. \( P_{eq, ind} \) exhibits pronounced concentration levels, specially around the same period we detected earlier, i.e. 1999 Q3,Q4 till 2000 Q3. It is to be noted that \( G_{P_{eq, ind}} \) reaches 0.542 at its maximum value compared to \( G_{P_{cap, ind}} \) maximum of 0.665, as shown in tables 6.3 and 6.4. This corresponds to a \( rHHI \) of 19 and 13 respectively, for a compression level of 4:10 against 2.8:10 respectively. The market cap portfolios are more concentrated then the equally weighted portfolios when the industry clusters are considered.

2. As seen in Figure 6.4, a relative minimum or ‘dip’ in the Gini curves is noticeable at around 2005 Q2 and Q3, where \( G_{P_{eq, ind}} \) reaches 0.414 (\( rHHI = 26 \)) at its absolute minimum value compared to \( G_{P_{cap, ind}} \) relative minimum of 0.488 corresponding to a \( rHHI = 23 \). In our PART II analysis, we will analyze this portfolio among two others.

3. In Figure 6.5 we observe that the curve of \( G_{P_{cap, ind}} \) (in solid line) is always below the curve of \( G_{P_{eq, stk}} \) (in dotted line) except in the short period of 1999 Q3 to 2000 Q2. The individual stocks concentration exhibiting a higher concentration level is counter intuitive. In fact, the intuitive rationale is that the more scattered a sample is the less concentrated it is, and the more clustered and grouped the more concentrated it is. However, in the case of portfolio concentration, the clustering or grouping of individual stocks around their respective industry sector will relatively smooth.
out the differences between individual stocks weights, in the absence of an absolute single industry class heavy weight. This is further corroborated when we observe the $G_{P_{eq\text{ind}}}$ (in double line) which falls below both above mentioned curves. 2000 Q1 will be considered individually in the PART II analysis to further explore its specialization particularity.

**Specialization of $P_{cap}$ and $P_{eq}$ at the level of industry sectors.**

$P_{cap}$ The graphs and figures of this paragraph are depicted in Appendix C, Page 145 for better clarity. Each graph will be plotted on a separate page for ease of reading.

In order to determine the specialization in industry sectors and its variation over all the 83 quarters’ portfolios that we formed, we will consider the ranking in weight of each industry sector in each portfolio (i.e. in each quarter). This ranking of each industry is not constant over time and will vary from quarter to other and hence the specialization of the portfolios is not the same and will accordingly vary from a quarter to another.

The ranking involves the 43 industries and hence one industry will be ranked first, being the heaviest in a given portfolio among the 43 industry sectors existing. So it is natural to conclude that the portfolio is not only specialized in the first in rank industry sector, but also among other following industry sectors as well. The issue is where to draw the line and decide that we shall consider the first 5 sectors or the first 10 in weight ranking. What could be a consistent criteria to follow in order to produce comparative and consistent results over different portfolios? One possible criteria is to use the Pareto 80-20 principle, drawing the line at the 20% of the industry sectors (i.e. after the first $43 \times 0.2 = 8.6$, the 8th or 9th ranked industry). However, the idea of using Pareto rule with 80% of the weight concentrated in 20% of the industry sectors seems to be far fetched. In fact, in a well planned and scientifically allocated portfolio, diversification will impose a lower weight percentage at the 20% cut-off line for which we decided to opt out of the Pareto rule.

Nevertheless, and in order to determine an intuitive approach to the minimum or adequate number of industry sectors determining the specialization of the portfolio (the cut-off line), we drew the following graph, represented in Figure 6.6, depicting the number of industries that achieve a 10, 25, 50, 75 and 90% of the total weight of the portfolio.
6.2. Results: Concentration and Specialization of $P_{eq}$ and $P_{cap}$.

Figure 6.6: Number of industry sectors achieving 10, 25, 50, 75 and 90% of total weight of $P_{cap}$. Note also the brackets indicating the special portfolios that will be considered later.

The green curve in Figure 6.6 (third curve from above) represents 50% of the weights related to the first top 9 ranked industry sectors. This green curve represents the 80-20 Pareto principle since 80-20 of weights-sectors is indeed 50% of the weight represented by 9 sectors. We think that choosing the first 7 to 9 industry sectors to decide on the specialization of the portfolio is fair and will reflect the purpose of our study, knowing that the first 7 to 9 top ranked industry sectors are representative of at least 50% of the total weight of the portfolio.

Adopting the top ranked industries by weight to determine the specialization of the portfolio can be misleading or, to say the least, incomplete. Mathematically, the top $m$ industries by weight could be all equally weighted or nearly equally weighted, and hence the conclusion that the portfolio is specialized in any of these top $m$ industries is not conclusively correct. We must explore the dispersion of the weights of industries within the top $m$ industries and hence, a Gini index of the higher ranking industries must be calculated and the degree of specialization in a given industry rather than in others can be correctly assessed. We shall explore this aspect of the specialization in more details later in this chapter.

Figure 6.7 comprises four stacked individual graphs, dividing the 83 portfolios under study into four groups for ease of reading. Each graph is also depicted alone in

From this stacked graph we realize that the “Ind 28” is more often ranked among the first 3 industries through all the period under study. Table 6.5 summarizes the top industries in each period under study. We observe the exit of “Agriculture, Foods, Beverages” in Period 2, 3 and 4. In fact, Figure C.2 shows that this industry exits from the top ranked sectors in 1998 Q3. A complete histogram of the frequencies of rank per each industry is depicted in Figures C.6 to C.14, Pages 152 to 160. The histograms and the relative frequency plots show clearly the “history” of the rank frequency of each industry subgroup throughout the portfolios under studies. These bar graphed histograms are very useful when studying the rank of a given industry throughout a panel of time portfolios. For example, if we consider “Ind 7” in Figure C.6 we can see that this industry (Oil Distribution) has never been ranked among the top 10, and the best ranking reached is Rank 20, and the most frequent rank occupied was rank 41. Furthermore, considering “Ind 17” (Drugs and Pharmaceuticals) in Fig C.10, Page 156, we can see that the histogram shows that this industry has always been one of top 10 industries, during the period under study, mostly occupying rank 3, 4 or 5, as shown, while “Ind 36”, (Banks and Credit Institutions) in Figure C.13, shows that this industry sector has almost always occupied the first, second or third rank.
### 6.2. Results: Concentration and Specialization of $P_{eq}$ and $P_{cap}$.

#### Table 6.5: $P_{cap}$: Summary of the specialization per industry sectors of the cap-weighted portfolios, $P_{cap}$, covering the portfolios under study. The industry sectors in this table are those which appeared more frequently then others in each rank-period.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
<th>Period 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Communication</td>
<td>Communication</td>
<td>Banks &amp; Credit</td>
<td>Banks &amp; Credit</td>
</tr>
<tr>
<td></td>
<td>Utilities</td>
<td>Utilities</td>
<td>Instit.</td>
<td>Instit.</td>
</tr>
<tr>
<td>2</td>
<td>Banks &amp; Credit</td>
<td>Banks &amp; Credit</td>
<td>Retail</td>
<td>Retail</td>
</tr>
<tr>
<td></td>
<td>Instit.</td>
<td>Instit.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Retail</td>
<td>Retail</td>
<td>Software</td>
<td>Drugs &amp; Pharmaceutics</td>
</tr>
<tr>
<td>4</td>
<td>Drugs &amp; Pharmaceutics</td>
<td>Drugs &amp; Pharmaceutics</td>
<td>Insurance</td>
<td>Software</td>
</tr>
<tr>
<td>5</td>
<td>Insurance</td>
<td>Software</td>
<td>IT Hardware</td>
<td>Insurance</td>
</tr>
<tr>
<td>6</td>
<td>IT hardware</td>
<td>Insurance</td>
<td>Integrated oil</td>
<td>Integrated oil</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Companies</td>
<td>Companies</td>
</tr>
<tr>
<td>7</td>
<td>Agriculture,</td>
<td>IT Hardware</td>
<td>Misc. Finance</td>
<td>Communication</td>
</tr>
<tr>
<td></td>
<td>Foods, Beverages</td>
<td></td>
<td></td>
<td>Utilities</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>From</th>
<th>1993 Q1</th>
<th>1999 Q1</th>
<th>2005 Q1</th>
<th>2011 Q1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Till</td>
<td>1998 Q4</td>
<td>2004 Q4</td>
<td>2010 Q4</td>
<td>2013 Q3</td>
</tr>
</tbody>
</table>
Figure 6.7: Specialization graphs of $P_{\text{cap}}$. The dotted lines represent the Gini index of the depicted top ranked industries. The $G$ measures the degree of concentration among the top ranked industries and hence it will reflect the distribution of the specialization among them. The brackets indicate the three special portfolios selected for the analysis in PART II of the results.
6.2. Results: Concentration and Specialization of Peq and Pcap.

The analysis below will describe the portfolios we are analyzing with respect to their concentration and specialization. The implications of the description for the investor are the inclusion of additional estimation-risk free measures in his toolbox to modulate his allocation strategies and monitoring and re-allocation decisions.

From Table 6.5 we can conclude that the market capitalization weighted quarterly portfolios, from 1993 Q1 to 2013 Q3, are either specialized in Communication Utilities or in Banks and Credit institutions. It is important to note the entry of “Ind 38” (Insurance) in 2001 Q2 while at the same time “Ind 28” (Communications Utilities) exited the top ranking to re-enter in 2011 Q1 with the exit of “Ind 5” (Integrated Oil Companies). Equally noticeable is the exit of “Ind 1” (Agriculture, Food, Beverages) in 1998 Q3 to never re-enter again in the top 7 industries, during the period of our study.

It is important to note that, while ranking the top 7 or 8 industries per weight gives an outlook on the specialization of the portfolio, this ranking remains incomplete unless the concentration of these top ranked weights is considered. This is obviously the case if the top 7 industries are equally weighted, among themselves. In this case, the portfolio is equally specialized in each and every industry of the top 7 list. This particular distribution of the weights of the top ranked industries in a portfolio could be determined by considering the \( G \) of those top ranked weights. The \( G \) is graphed in dotted lines along the top ranked industries (against the right hand vertical axis) in the Figure 6.7 and also in Appendix C, Figures C.2, C.3, C.4 and C.5 in Pages 148 to 151.

Examining the Gini index of the top ranked industries in \( P_{\text{cap}} \) shows the values of \( G \) between 0.046 \( \leq G \leq 0.2155 \), the maximum concentration occurring in 2000 Q1 (our second special portfolio chosen for analysis in PHASE II). A low \( G \) value indicates low concentration, and hence, a quarter where the \( G \) value shows low concentration among its top ranked industries indicates an equal specialization in each of the top ranked industries. This is how we can fairly conclude that, for example, in the period between 2005 Q1 and 2009 Q3 (please refer to Fig C.4, Page 150), where \( G \) is low (0.046 \( \leq G \leq 0.095 \) the specialization is shared among the top ranked industries like Banks and credit institutions (Ind 36), Retail (Ind 31), Software (Ind 40), Insurance (Ind 38) and IT hardware (Ind 42). In contrast, during the period between 1996 Q1 and 1998 Q1 (please refer to Fig C.2, Page 148) where the Gini index \( G \) shows higher concentration (0.16 \( \leq G \leq 0.2 \)), the portfolios are more specialized in Communication utilities (Ind 28), Banks and credit institutions (Ind 36) and Drugs and pharmaceuticals (Ind17) rather then in the other top ranked industries in this same period like Agriculture food and beverage (Ind 1) or IT hardware (Ind 42) and Insurance (Ind 38).

In the case of \( P_{\text{eq}} \), we realize that the low \( G_{\text{eq}} \) of the top ranked industries in
Chapter 6. Concentration and Specialization of the US listed Stocks

most of its portfolios, requires the analysis of the specialization within the top 7 or 8 industries as we concluded in this subsection. However, while analyzing the specialization of $P_{eq}$ in the next paragraph, the $G_{eq}$ is much higher the $G_{cap}$, showing that the specialization can be fairly assessed by considering only the top 3 or 4 industry clusters.

$P_{eq}$ The graphs and figures of $P_{eq}$ are depicted in Appendix C, Page 145 for better clarity. Each graph is plotted on a separate page for ease of reading. (For $P_{eq}$ graphs, Please refer to Figures C.15, to C.19, from Page 161 to 165.)

The most striking characteristic of the specialization of $P_{eq}$ is that the $G_{eq}$ of its top ranked industries is fairly higher than that of $P_{cap}$. From Figure C.15 we see that $G_{eq}$ averages around 0.23 and is included in the bracket $0.1776 \leq G_{eq} \leq 0.303$ whereas the average of $G_{cap}$ is 0.128 and is included in the bracket $0.04 \leq G_{cap} \leq 0.215$, as shown in Figure 6.8. This indicates that the specialization of the equally weighted portfolios $P_{eq}$, in general, is concentrated on fewer industries as compared to the specialization of the market capitalization portfolios, $P_{cap}$. We also observe that both the absolute minimum and maximum values of $G_{eq}$ and $G_{cap}$ occur at around the same period: The absolute minimum value of $G_{eq}$ and $G_{cap}$ occur in 2008 Q2, and the absolute maximum value of $G_{eq}$ and $G_{cap}$ occur in 2000 Q1, which is one of our three chosen portfolios to be analyzed individually in the next PHASE II.
6.2. Results: Concentration and Specialization of $P_{eq}$ and $P_{cap}$.

![Gini index comparison](image)

Figure 6.8: Comparative Gini index of $P_{cap}$ and $P_{eq}$, showing a more concentrated profile of the $P_{eq}$ among its top ranked industries, which indicates specialization in fewer industries as compared to $P_{cap}$.

However, this does not mean that $P_{cap}$ and $P_{eq}$ exhibit the same concentration behavior, between its top ranked industries. The correlation between both Gini indexes is around 0.44 and we can observe that, while the absolute extrema happen at the same time, it is not true for the relative extrema. In fact 1993 Q1 show a relative maximum for $P_{eq}$ while $P_{cap}$ is at a relative minimum. The same occurs in 2010 Q1 with an absolute minimum in $G_{cap}$ and an absolute maximum in $G_{eq}$.

The swings around the average value of $G_{cap}$ are greater than those of $G_{eq}$ and more frequent, among the top ranked industries. In fact, the weights allocation strategy in $P_{eq}$ is based on dividing the budget among the $n$ existing stocks, that are clustered around industry sectors. We expect the concentration at the level of individual stocks to be at its minimum. But when the concentration is calculated at the industry cluster level, we expect the concentration to be different from the minimum, i.e. the industry weights are not equal among each other. However, since each individual stock’s weight is $\frac{1}{n}$, the concentration of the portfolio at the industry clusters level is nothing but the concentration of the number of stocks in each industry cluster $^5$.

---

$^5$The scalar does not affect the concentration as discussed in Chapter 3, the concentration does not change if we multiply the whole population under study by a constant $c$. In the case of the concentration of $P_{eq}$, an industry sector having $m$ stocks will have a total weight of $m \times \frac{1}{n}$, where $\frac{1}{n}$ is the scalar, and hence the concentration of the equally weighted portfolio at the industry level is the same as the concentration of the number of stocks included in
This is why, the concentration of the industry groups or clusters in the case of an equally weighted portfolio are smooth and less frequent then the concentration when a market capitalization portfolio is concerned. There is no reflection of the price volatility in the case of an equally weighted portfolio and hence the changes within the industry groups involving equally weighted stocks reflect actually the changes in the market structure and the market constituents. In other words, given an attribute \( m \) (in this case the industry group of each stock), the concentration of the a portfolio of equally weighted stocks calculated at the level of the attribute \( m \), describes the status of this attribute in the market under study. Additionally, the variation in the concentration index of an equally weighted portfolio calculated around an attribute \( m \), describes the variation in the number of members of each group of the attribute \( m \) (in our case industry groups).

Another interesting observation related to Figure 6.8 is that, when we consider the \( G \) of the top ranked industries only, we observe that, during of the period of the study, it is always true that \( G_{eq} > G_{cap} \). However, and referring to Fig 6.4, Page 107 (and detailed in Figures C.16, C.17, C.18, C.19 in Pages 162 to 165), which depicts the concentration indexes \( G \) and \( HHI \) of the entire portfolios \( P_{eq} \) and \( P_{cap} \) at the industry level (and not the top ranked industries only) we observe that the concentration of the equally weighted portfolio around its industry sectors is ALWAYS smaller the that of the market capitalization portfolios, i.e. \( G_{eq} < G_{cap} \), during the period of our study.

As for the specialization of the portfolios \( P_{eq} \), the ranking graphs depicted in Figure 6.10 stacking all 4 periods of study, as well as the complete specialization graph \( P_{eq} \) in Figure C.15, Page 161 reflect clearly from one side the higher level of concentration among top ranked industries \( G_{eq} \) (represented by the dotted line), and a fairly stable specialization of the portfolios over time, in the sense that we observe that the industries in the graphs “remain in the same rank” for a longer time then is the case with \( P_{cap} \), from the other. In fact if we consider the histogram of “Ind 36” (Banks and Credit Institutions) which appears to be among the first ranks during all the period of study, we observe that this industry remained in Rank #1 for 55 quarters, as shown in Figure 6.9.
6.2. Results: Concentration and Specialization of $P_{eq}$ and $P_{cap}$.

This high frequency was never attained in the portfolios of market capitalization, and the highest frequency achieved by any of the top 7 industries considered was 15. The “Ind 36, Banks and Credit Institutions” dominates the specialization of the $P_{eq}$, throughout the period of study. It remained ranked #1 from 1993 Q1 until 2007 Q4. By the beginning of 2008 it was ranked #7 until 2012 Q4. By the end of our period of study in 2013 Q3, the rank of this industry improved to #4. It is noticeable that, as argued earlier in this section, that our specialization measure in this case is based on the number of stocks or companies included in this industry sector, and hence our measure describes the industry of Banks and Credit Institutions as being the industry with most listed companies until the beginning of 2008. The interpretation of this rank variation and its correlation with financial and economic events remains to be researched and explained and it is beyond the scope of this work.

The Software industry, “Ind 40”, enters the top ranked industries by 1998 and reaches rank#1 for the first time in 2009 Q4 to remain later and until 2013 Q3 among the top 5 ranked industries. The same happens with Insurance, “Ind 38”, which starts with a solid rank#3 until 2004 Q4 where it takes the lead in the subsequent quarters and specially when the financial crisis of the sub-primes in 2008 hits the markets where we witness a leading rank#1 takeover from Banks and Credit Institutions, “Ind 36”. By the last quarters of our study, Insurance sector alternates with “Ind 31, Retail” which was always ranked among the top 5 industries in $P_{eq}$. 

Figure 6.9: Histogram of “Ind 36, Banks and Credit Institutions”, in $P_{eq}$, showing the frequency of its rank. This industry was ranked as #1 for 55 quarters out of the 83 $P_{eq}$ portfolios we are studying.
Chapter 6. Concentration and Specialization of the US listed Stocks

<table>
<thead>
<tr>
<th>Rank 1</th>
<th>Period 1 From</th>
<th>Period 2 Till</th>
<th>Period 3 From</th>
<th>Period 4 Till</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank 1</td>
<td>Banks &amp;Credit</td>
<td>Banks &amp;Credit</td>
<td>Banks &amp;Credit</td>
<td>Retail</td>
</tr>
<tr>
<td>Rank 2</td>
<td>Electric Utili-</td>
<td>IT Hardware</td>
<td>Retail</td>
<td>Software</td>
</tr>
<tr>
<td>Rank 3</td>
<td>Insurance</td>
<td>not defined</td>
<td>Insurance</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.6: $P_{eq}$: Summary of the specialization per industry sectors of the Equally weighted portfolios, $P_{eq}$, covering the portfolios under study. The Equally weighted portfolio exhibits a high $G$ among the top ranked industries and hence the dominance of the first 3 ranked ones define better the specialization of these portfolios.
6.2. Results: Concentration and Specialization of $P_{eq}$ and $P_{cap}$.

Figure 6.10: Specialization graphs of $P_{eq}$. The dotted lines represent the Gini index of the depicted top ranked industries.
In practice, the specialization study of $P_{eq}$ is a study of the market industry sectors and the market stocks during the period of study. The information conveyed by describing the investment portfolio (like we did, applying our method to $P_{eq}$ and $P_{cap}$) is rich, substantial and different from what is conveyed by using the classic market metrics of the average return and its moments.

6.2.2 Results Part II: Concentration of the chosen 3 portfolios.

As we mentioned earlier in this chapter, and in order to complement our study of the variation of concentration and specialization of the 83 portfolios included in our study (from 1993 Q1 to 2013 Q3 both inclusive) we chose three particular portfolios to analyze: i) the portfolio of absolute maximum concentration 2000 Q1, ii) the portfolio of relative minimum concentration occurring directly after it 2005 Q2 and iii) the portfolio of absolute minimum concentration 1994 Q3, within the time bracket of our study. Our analysis will include both the equally weighted portfolio allocation as well as the market capitalization allocated portfolios related to the same dates chosen.

As outlined in Section 2.2, Page 20, we are suggesting the creation of a weight-attribute matrix to analyze the concentration and the specialization of a portfolio as an additional information describing an investment portfolio at large. In the case of our three chosen portfolios, the impact matrix as detailed in Table 2.3 Page 25 will not be depicted here because it is very big\(^6\). However the impact matrix is elaborated separately and the results are depicted in the subsequent tables for comparison.

---

\(^6\)In fact the impact matrix for any of these 3 portfolios with 500 stocks included and 43 industry groups will be a $500 \times 43$ matrix that is very big to fit in this thesis.
### 6.2. Results: Concentration and Specialization of \( P_{eq} \) and \( P_{cap} \)

Table 6.7: Descriptive summary of the three chosen portfolios: Concentrations at the individual stock and at the industry levels and the specialization.

<table>
<thead>
<tr>
<th></th>
<th>( P_{cap} )</th>
<th>( P_{eq} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( HHI_{stk} ), stock level</td>
<td>0.0054</td>
<td>0.098</td>
</tr>
<tr>
<td>( rHHI_{stk} ), stock level</td>
<td>182</td>
<td>101</td>
</tr>
<tr>
<td>( HHI_{ind} ), industry level</td>
<td>0.0424</td>
<td>0.0755</td>
</tr>
<tr>
<td>( rHHI_{ind} ), industry level</td>
<td>23</td>
<td>13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>( P_{cap} )</th>
<th>( P_{eq} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G ) top ranked Industries</td>
<td>0.11</td>
<td>0.24</td>
</tr>
<tr>
<td>Rank 1</td>
<td>Comm. Utilities</td>
<td>IT hardware</td>
</tr>
<tr>
<td>Rank 2</td>
<td>Banks &amp; Credit Instits.</td>
<td>Comm. Utilities</td>
</tr>
<tr>
<td>Rank 3</td>
<td>Agric, Food, Beverage</td>
<td>Software</td>
</tr>
<tr>
<td>Rank 4</td>
<td>Drugs, Pharma.</td>
<td>Banks &amp; Credit Instits</td>
</tr>
<tr>
<td>Rank 5</td>
<td>Retail</td>
<td>Drugs, Pharma.</td>
</tr>
</tbody>
</table>

Table 6.7 summarizes the concentration of the three portfolios at the individual stock level as well as at the industry group’s level. It also shows the specialization of each portfolio, listing the top 7 ranked industries per portfolio. The most concentrated portfolio at the
Chapter 6. Concentration and Specialization of the US listed Stocks

stock and industry levels alike is 2000 Q1. The portfolio least concentrated at the stock and industry levels is 1994 Q3. A particular result is shown in the relative minimum concentration portfolio 2005 Q2 which exhibits a relative minimum concentration of $HHI = 0.0071$ while having the same concentration at the industry level as the portfolio with absolute minimum concentration 1994 Q3, both with $HHI_{ind} = 0.042$.

Gini index, derived from the Lorenz curve, indicates the same tendency shown by the $HHI$ above. It is interesting to note that the market capitalization portfolios exhibit the lowest concentration at the industry level, graphically apparent if the Lorenz curves are drawn for each of those portfolios, as shown in Figure 6.11.

The following is a comprehensive description of the portfolios at hand:

- 1994 Q3 Market capitalization allocation: This portfolio has a relatively low concentration measure at the levels of individual stocks as well as industry groups, with $HHI = 0.0054$ and $HHI = 0.0424$ respectively. It consists of 500 stocks, with a
budget constraint of 1, not allowing short sale allocations. Its \( r_{HHI} = 182 \) at the stock level, i.e. a concentration ratio of \( \frac{182}{500} = 0.36 \). The stocks are listed in the USA capital market, and they are grouped in 43 industrial sectors. When the stocks are grouped according to their industrial groups, the portfolio exhibits a concentration level of \( HHI = 0.0424 \) equivalent to a \( r_{HHI} = 23 \) or a concentration ratio of \( \frac{23}{43} = 0.53 \) at the industry groups level, which is a relatively high concentration. This portfolio is specialized in Communications utilities, Banks and Credit institutions, Agriculture, Food and Beverages, Drugs and Pharmaceuticals, Retail and Insurance. The Gini index of the top ranked industries is low, indicating an equal specialization among all the top ranked industries. Its heaviest stock weighs 0.0232 (2.32%) and it belongs to the industry of communication utilities.

- 2000 Q1 Market capitalization allocation: This portfolio has a relatively high concentration measure at the levels of individual stocks as well as industry groups, with \( HHI = 0.0098 \) and \( HHI = 0.0755 \) respectively. It consists of 500 stocks, with a budget constraint of 1, not allowing short sale allocations. Its \( r_{HHI} = 101 \) at the stock level, i.e. a concentration ratio of \( \frac{101}{500} = 0.20 \). The stocks are listed in the USA capital market, and they are grouped in 43 industrial sectors. When the stocks are grouped according to their industrial groups, the portfolio exhibits a concentration level of \( HHI = 0.0755 \) equivalent to a \( r_{HHI} = 13 \) or a concentration ratio of \( \frac{23}{43} = 0.30 \) at the industry groups level, which is a relatively very high concentration. This portfolio is specialized in IT Hardware, Communications utilities, Software, Banks and Credit institutions, Drugs and Pharmaceuticals, Retail and Photopptical Micros and office Machinery. The Gini index of the top ranked industries is high, indicating more specialization among all the first top ranked industries. Its heaviest stock weighs 0.0387 (3.87%) and it belongs to the industry of Software.

- 2005 Q2 Market capitalization allocation: This portfolio has a relatively medium high concentration measure at the levels of individual stocks and a low concentration at the level of industry groups, with \( HHI = 0.0071 \) and \( HHI = 0.0429 \) respectively. It consists of 500 stocks, with a budget constraint of 1, not allowing short sale allocations. Its \( r_{HHI} = 139 \) at the stock level, i.e. a concentration ratio of \( \frac{139}{500} = 0.28 \). The stocks are listed in the USA capital market, and they are grouped in 43 industrial sectors. When the stocks are grouped according to their industrial groups, the portfolio exhibits a low concentration level of \( HHI = 0.0429 \) equivalent to a \( r_{HHI} = 23 \) or a concentration ratio of \( \frac{23}{43} = 0.53 \) at the industry groups level, which is a relatively a low concentration. This portfolio is specialized in Banks and Credit institutions, Retail, Insurance, Software, IT Hardware, Miscellaneous Finance and Integrated Oil Companies. The Gini index of the top ranked industries is very low at \( G_{top} = 0.09 \), indicating equal specialization among all the top ranked industries. Its heaviest stock weighs 0.0309 (3.09%) and it belongs to the industry of Financial
Investments.

6.3 concluding remarks

This chapter is an example of the application of the methodology of describing an investment portfolio that we are proposing. The descriptors are concentration measures, which are estimation risk free because they are not based on any forecasting assumption. The description we propose can be applied to a period of time, where a certain back analysis is required or when the analyst and the investors alike require an ext post description of the market dynamic rather then the market performance. The methodology also can be applied to an individual portfolio, where an ex ante description is required free of any estimation risk. The methodology does not give any forecast on the available moments of the asset, according to its historical data, but rather is an instantaneous snapshot on the nature of the portfolio with respect to its individual stocks and the attribute related to the stocks, according to the investors’ choice. In our example this attribute of choice was the industry group.

The approach we are proposing shows clearly the trends over a past period of time of the market players, the industries relative weights in the portfolios, but also the trends in specialization or, similarly, de-specialization. The graphs we exposed in this chapter showed clearly the entry of new sectors at the detriment of the exit of others, like Communication entering and food and beverages exiting, as an example.

The approach we are proposing can be the basis of an investment monitoring approach, tracking the changes in the concentrations of the portfolios at hand and the tendencies of specialization along the time axis. The monitoring according to the concentration and specialization of the portfolios will add valuable information to the investors’ community and will enhance the forecasting capabilities of the investors and managers alike.
Chapter 7

Conclusion

The study set out to explore the concept of concentration and specialization in investment portfolios and has identified the nature and form of concentration measures applied in economics and welfare studies, their application in investment portfolios, the quality of information they would provide to the investor and the characteristics and limitations of such measures in describing an investment portfolio. The study has also sought to know whether the new suggested measures of concentration and specialization introduce any estimation risk into the description. The conclusion is that the suggested measures are free of estimation risk because they do not rely on any forecasting statistical calculation but are rather a direct representation of the data parameters at hand.

General theoretical literature on the subject of measuring concentration and specialization in investment portfolios is rare and somehow nonexistent, since the suggested measures are usually applied in the domain of poverty, welfare and wealth distribution. However, the literature reviewed reflected the existence of a multitude of measures, each with a different set of characteristics suited for some particular application in social and political economics.

The application of concentration measures to investment portfolios, as presented in our research, is a new approach to describing an investment portfolio and this innovation should see its application with investors and investment managers alike. Indeed, two different portfolios can have the same statistical measures and yet differ substantially in concentration and specialization. The techniques and measures suggested in this study are an additional tool for monitoring and managing an investment portfolio.

The study explores the investment portfolio selection process and sheds some light on two particular aspects of the resulting investment universe, namely i) the concentration of the portfolio, i.e. how is the wealth distributed among the individual stocks chosen, and ii) the specialization of the portfolio, i.e. what are the attributes related to the chosen stocks that are more prominently present than others. Indeed, in seeking to maximize his profit
and the liquidity of his portfolio, the investor places his “bet” on some particular stocks among some others based on his view of the future performance of the portfolio. This future view requires the investor to tap into available information related to his investment and which will help him allocate to his wealth. The final portfolio can therefore be described ex-ante and ex-post using historical results and observed performance, respectively. The set of descriptors used by the investor are prone to an estimation error since they are related to a historical data-based forecast. The study proposes to include the descriptors of concentration and specialization as additional information to assist the investor in his initial choice of the portfolio and also to monitor the performance of his investment. The descriptors we introduce are free of estimation risk since they are a direct measure and not a statistical approximation of a data time series.

The study sought to answer some particular questions related to the descriptors free of estimation error that it is proposing:

1. What additional information is provided by determining the concentration and the specialization of an investment portfolio?
2. What is or are the most adequate measures of concentration that best meet the requirement of the additional information proposed?
3. Once the adequate measures are chosen, how are the concentration and the specialization of an investment portfolio measured and how will theses measures describe a constrained portfolio?
4. What added value do these proposed measures present to an investor?

The theoretical implications of the relationship between a concentrated portfolio and its diversification potential remain an open question to be researched further. It is not necessarily true that a less concentrated portfolio is diversified, and inversely, a diversified portfolio is necessarily a less concentrated one. The research does not reach a conclusive verdict on the diversification and the specialization of the portfolio.

The thesis therefore concludes with a theoretical position on the neutrality of the investment portfolio at large. The concluding conjecture states that, apart from special cases\(^1\), no portfolio is neutral, even the equally weighted portfolio, because there is at least one attribute that is concentrated with respect to the others. Hence, every portfolio has a bet or a “pretension” measured by its pretension level and its specialization, and hence every portfolio has a bias, especially the indexes, as is shown in the research.

It is also conjectured that no portfolio is diversified enough. In fact, no portfolio can be diversified in all possible attributes of its individual stocks.

\(^{1}\)In general, it is assumed that the number of attributes \(m\) is greater than the number of stocks \(n\) i.e. \(m > n\), because it is logical to think that the number of stocks is limited and the number of attributes can be very big. However, in constrained portfolios, the number of attributes \(m_c\) can be reduced and hence \(m_c < n_c\). In this case, a constrained portfolio might be less biased than an unconstrained one.
The dynamic allocation of wealth in an investment portfolio is based quite often on the modern theory of portfolios, led by the seminal work of Markowitz (Markowitz, 1952). The most recent development in this field lies in the theory of risk weighting or allocation with respect to the risk of the asset, which is prone to estimation error. The research is suggesting a sustainable framework in relation with wealth allocation relying on a scalar free of estimation error: the concentration of the portfolio. It is in this direction that the research defines the pretension level of an investor, which gives clear and unambiguous information on the nature and constitution of his invested portfolio. Indeed, the pretension level being derived from the Hirschman-Herfindahl concentration measure reflects the amount of wealth allocated to each asset (or stock) and hence reflects, in a sense, the view of the investor about the market: where he puts his “bets” and how he distributes his wealth gives clear information on his expectation of performance and hence his pretension to the future results of his portfolio.

The findings of the study do not challenge pre-existing views and descriptors of a portfolio in general. It contributes to adding a new error-free descriptor that will increase and improve the level and quality of the initial information that the investors normally seeks in order to compose his portfolio. hence the proposed method in the study contributes positively to the decision making process during the phases of the investment, i.e. before the portfolio is composed and later, during the life of the portfolio to monitor and re-allocate dynamically the wealth invested. It is left to future researchers to ascertain or refute the causal correlation between the descriptor we suggest in our study and the performance of the portfolio.

Our research utilizes a measure of concentration and specialization that provides a description of an investment irrespective of past values. The actual measures used in the financial mainstream are distribution measures providing averages and higher moments of the historical data. Our measures reflect the changes in the investment portfolio over time, with respect to other attributes then the periodical performance. We can express every single investment and also every single portfolio in terms of its attributes. Our research shows how portfolio can change over time in terms of its individual attributes and this can be done for all the possible attributes that the investor decides to consider. So in fact we are providing the investor an additional information that can be of an interest to him, other then the historical changes of price over time and its statistical derivations. Additionally, the investor usually wants to change the composition of his portfolio by adding some new shares or stocks and removing some others, and hence the investor not only changes the fundamental statistics of his portfolio, but also he alters the concentration and the specialization of his portfolio: by adding and removing some stocks he also adds and removes those stock’s attributes. Our method measures this effect and provides the investor with a valuable information: what is the new measure of concentration and specialization of the new re-allocated portfolio. This information is very valuable to the investor and gives
an additional insight on his investment. Our research provides a quantified measure of his allocation strategy. By defining his “pretension” our descriptors quantify the status of his portfolio in terms of its concentration and specialization.

The practical or empirical implications of our proposed approach is better observed when it is applied to an index, which was simulated in the research by applying it to several portfolio composed of the top 500 stocks listed in the United States. Many investors who follow the indexes may have ignored many other stocks with interesting properties and attributes that the indexes do not include. The research shows the concentration and the specialization of such possible indexes and the changes over time in terms of the new proposed descriptors rather then in terms of the average of the price and its volatility. The new information provided by measuring the concentration and the specialization of the portfolio adds vital insight to the investors along with the usual return-risk and other statistical measures. The investor knows what additional attributes are included and excluded from the index he is following and this additional information will modulate his decision making process and probably his dynamic management techniques and approach: the investor will also think in terms of the new attributes (industry sector, geographical location, ownership structures etc.) included or excluded from his investment as a result of his allocation decisions. In fact the research shows that many attributes were not included in the index due to the market capitalization allocation strategy and hence it shows a pronounced specialization in few industrial sectors at the detriments of others.

There has been various empirical methods showing results that beat the market index by several basis points, all based on “alternative” allocation strategies. The research does suggests the utilization and application of the measures of concentration and specialization as an additional information to the investor: Not only he needs to know what is the historical performance of such or such stocks but also what are their attributes and how do these individual attributes influence and alter the overall concentration and specialization of the portfolio at hand.

The research suggests that the stocks’ attributes like the industry sector, the geographical location, the ownership structure or the number of employees and their respective concentration measures do have a meaning to the investor in addition to the statistics derived from the historical stocks’ quotes.

The popular proverb “Learn from the past but live in the present” can be existential and somewhat reflects a philosophical perception of Time. However, it shines with a new light on the approach to investment that this research is suggesting. Past quotes are useful to give the investor an idea on their volatility but this statistical approximation has its drawbacks: it is prone to estimation error. Concentration and specialization measures are “present time” measures and do not rely on historical data and therefore are free of estimation error. It is for the wise to combine both past and present dimensions of information in the investment decision making process.
Appendix A

Toolbox formulas and assumptions

A.1 The Gini Index and its Decomposition

In the article “Decomposition and Interpretation of Gini and the Generalized Entropy inequality Measures”, (Dagum, 1997a) Camilo Dagum decomposes the Gini index and compares it to Theil, Hirschman-Herfindahl and Bourguignon indexes. Dagum’s aim was to introduce a three components decomposition method with $G_w$ (Gini index within the group), $G_b$ (also $G_{nb}$, the net contribution to the Gini index between the groups) and $G_t$ (the between group distance or transvariation).

Given a population $Q$, with $n$ units with attributes $w_i$ with $i = 1, \ldots, n$ and a probability density $f(w)$, a cumulative function $F(w)$, and mean $\mu$. Assume the population $Q$ is subdivided into $k$ groups or sub-populations, resulting in $Q_j$, $f_j(w)$, $F_j(w)$ and mean $\mu_j$ with $j = 1, \ldots, k$. Assume also that that each sub-population $Q_j$ (or subgroup) has a size $n_j$, $j = 1, 2, 3, \ldots, k$ with $\sum_{j=1}^{k} n_j = n$. Therefore, the vector of attributes of the partitioned initial universe $Q$ can be expressed as:

$$(w_1, w_2, \ldots, w_k) = ((w_{11}, \ldots, w_{1n_1}), \ldots, (w_{1j}, \ldots, w_{jn_j}), (w_{1k}, \ldots, w_{kn_k}))$$

(A.1)

Where the subindex $(nk)$ denotes the elements of subgroup $k$. We also define:

$$p_j = \frac{n_j}{n}, \quad q_j = \frac{n_j\mu_j}{n\mu}, \quad j = 1, \ldots, k$$

(A.2)

Where $p_j$ is the $j^{th}$ group’s population share and $q_j$ is the $j^{th}$ group attribute share$^1$.

$^1$Please note that $\sum p_j = \sum q_j = 1$ and that $\sum_{j=1}^{k} \sum_{r=1}^{k} p_j q_r = 1$. 

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Appendix A. Toolbox formulas and assumptions

Gini Coefficient.
Gini (1914) defined the Gini index for a population of \(n\) units as follows:

\[
G = \frac{\Delta}{2\mu} = \frac{1}{2n^2\mu} \sum_{i=1}^{n} \sum_{r=1}^{n} |w_i - w_r| 
\]  
(A.3)

Where \(\Delta\) is the total Gini mean difference, i.e.:

\[
\Delta = E[w_i - w_r] = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{r=1}^{n} |w_i - w_r| 
\]  
(A.4)

If, however, we consider the population \(Q\) as subdivided into \(k\) sub-populations, then Equation A.3 can be written as:

\[
G = \frac{1}{2n^2\mu} \sum_{j=1}^{k} \sum_{h=1}^{k} \sum_{i=1}^{n_j} \sum_{r=1}^{n_h} |w_{ji} - w_{hr}| 
\]  
(A.5)

Gini within a subpopulation \(j\).
Consider the sub-populations of \(Q\), as per Equation A.1 above, and let \(Q_j, j = 1, \ldots, k\) be one subpopulation of \(Q\). Then we define the Gini index \(G_{jj}\) within the sub-population \(j\) as:

\[
G_{jj} = \frac{\Delta_{jj}}{2\mu_j} = \frac{1}{2n_j^2\mu_j} \sum_{i=1}^{n_j} \sum_{r=1}^{n_j} |w_{ji} - w_{jr}| 
\]  
(A.6)

Gini between two subpopulations.
Consider two sub-populations of \(Q, Q_j\) and \(Q_h\) with \(j, h = 1, \ldots, k\), then we define the Gini index between the \(j\)-th and \(h\)-th sub-population (Dagum, 1987) as:

\[
G_{jh} = \frac{\Delta_{jh}}{(\mu_j + \mu_h)} = \frac{1}{n_j n_h (\mu_j + \mu_h)} \sum_{i=1}^{n_j} \sum_{r=1}^{n_h} |w_{ji} - w_{hr}| 
\]  
(A.7)

Relative and Gross economic affluence.
In the same paper, Dagum (1987) introduces the definition of gross economic affluence \(d_{jh}\) between two sub-populations as well as the directional economic distance ration or relative economic affluence \(D_{jh}\) between two sub-populations as follows:

\[
d_{jh} = \frac{\Delta_{jh} - p_{jh}}{\Delta_{jh}} 
\]  
(A.8a)

\[
D_{jh} = \frac{(d_{jh} - p_{jh})}{\Delta_{jh}} = 1 - \frac{2p_{jh}}{\Delta_{jh}} 
\]  
(A.8b)
A.1. The Gini Index and its Decomposition

Gini index decomposition by Dagum.
In this paragraph we refer to Dagum (1987), Theorem 3 (P.524), stating that the total Gini index of a population of size \(n\) partitioned in \(k\) sub-populations, as per A.1 can be decomposed into three components, as follows:

\[
G = G_w + G_{nb} + G_t, \quad \text{(A.9a)}
\]

\[
G_{gb} = G_{nb} + G_t \quad \text{(A.9b)}
\]

Where:
- \(G_w\) is the weighted contribution to \(G\) of the Gini within the sub-population,
- \(G_{nb}\) is the net contribution to \(G\) of the Gini between sub-populations,
- \(G_{gb}\) is the gross contribution to \(G\) of the Gini between sub-populations,
- \(G_t\) is the contribution to \(G\) of the transvariation between sub-populations, and such that,

\[
G_w = \sum_{i=1}^{k} p_i q_i G_{jj} \quad \text{(A.10a)}
\]

\[
G_{nb} = \sum_{j=2}^{k} \sum_{h=1}^{j-1} (p_j q_h + p_h q_j) D_{jh} G_{jh} \quad \text{(A.10b)}
\]

\[
G_t = \sum_{j=2}^{k} \sum_{h=1}^{j-1} (p_j q_h + p_h q_j)(1 - D_{jh}) G_{jh} \quad \text{(A.10c)}
\]

\[
G_{gb} = \sum_{j=2}^{k} \sum_{h=1}^{j-1} (p_j q_h + p_h q_j) G_{jh} = G_{nb} + G_t \quad \text{(A.10d)}
\]

Gini decomposition: Particular cases to note.
The following two cases that we note hereafter are proven in Dagum (1987) paper, P.526, under Corollary 3, and Corollary 5 respectively.

**Case 1:** \(\mu_j = \mu\). In the assumption of equal means between the entire group \(\mu\) and the sub-populations \(\mu_j\) then \(q_j = \frac{n_j}{n} = p_j\), with \(j = 1, 2, \ldots, k\).
The corollary 3 (Dagum, 1987) proves that, iff \(\mu_j = \mu\), \(D_{jh} = 0\) and

\[
G_w = \sum_{j=1}^{k} p_j^2 G_{jj} \quad \text{(A.11a)}
\]

\[
G_t = 2 \sum_{j=2}^{k} \sum_{h=1}^{j-1} p_j p_h G_{jh} \quad \text{(A.11b)}
\]

\[
G_{nb} = 0 \quad \text{(A.11c)}
\]
Case 2: $k$ sub-populations do not overlap. In the assumption that the measured attribute density functions (in our case the weights of the stocks) of the $k$ populations do not overlap, (Dagum, 1987) Corollary shows that $p_{jh} = 0$ and $d_{jh} = \Delta_{jh}$ leading to $D_{jh} = 1$ and

\[ G = G_w + G_{nb} \quad \text{since:} \quad G_t = 0 \quad \text{and} \quad G_{nb} = G_{gb} \quad (A.12) \]

Case 3: Initial Universe is equally weighted. This is an assumption which is particular to investment portfolio rather then to population income or social welfare. In the case the initial universe, in our case the portfolio, is equally weighted, i.e. $w_i = \frac{1}{n}$ for all $i = 1, 2, 3, \ldots, n$ then, intuitively and by definition $G = 0$, reflecting a total equality within the portfolio.

If we divide the portfolio into subgroups, according to certain attributes of choice, then for any subpopulation $Q_j$ with $j = 1, 2, 3, \ldots, k$ where $k$ is the number of subgroups, then $w_{j1} = w_{j2} = \cdots = w_{jn_j} = \frac{1}{n}$, and consequently $\Delta_{jj} = 0$ (Equation A.4) and $G_{jj} = 0$ (Equation A.6) as well as $G_{jh} = 0$ (Equation A.7). Consequently, the decomposed Gini index becomes:

\[ G = G_w = G_{nb} = G_t = G_{gb} = 0 \quad (Equation A.9a) \]

The particularity of the equally weighted portfolio vis-à-vis its specialization resides in the fact that, although the decomposed $G$ index indicates no specialization, the portfolio might eventually be specialized when the attributes are taken into consideration. Suppose that our hypothetical portfolio of 10 stocks was equally weighted, with $w_1 = w_2 = w_3 = \cdots = w_{10} = \frac{1}{10} = 0.1$. Its $G = 0$ as well as all its decompositions, but, from Table 5.3 Page 89 we can see that the portfolio is specialized in Pharmaceutical companies (four companies out of the ten available), when the attribute “industry group” is described.

Hence, we conclude that the method we are introducing to measure the specialization within a portfolio using Gini index and its decomposition is not applicable when the portfolio is equally weighted. A possible simple measure of specialization in this case would be the ratio of $\frac{n_j}{n}$ where $n_j$ is the number of stocks within a subgroup (corresponding to a particular attribute) and $n$ is the total number of stocks in the portfolio.\(^2\)

Gini index including null entries: $G_{null}$.

The Gini index including null entries $G_{null}$ is a new measure we introduce in this thesis to reflect the exact situation of the specialization within a sub-population. It is defined as the Gini index of the whole sub-population, including the null entries. The $G_{null}$ is not related to the decomposed Gini index we discuss above. It is an additional measure to further assess the specialization of a portfolio of investment.

\(^2\)All weights being equal, this measure of $\frac{n_j}{n}$ proxies the average $\mu_j$ of the subgroup. In fact, $\mu_j = \frac{\sum_n (w_i)}{n} = \frac{n_j}{n}$.
A sub-population $Q_j, j = 1, \ldots, k$ with its entries as follows: $(w_{j1}, w_{j2}, \ldots, w_{jn})$ which was referred to throughout our thesis, assumes no null entries within the sub-population, since, in the measurement of income concentration where all these formulas are applied, there is no coherent meaning to a null entry because it reflects an individual with no income whatsoever. Hence, all the members of the sub-population $Q_j$ are not null, i.e. they represent an income to a certain given individual in the chosen sub-population. However, in our application of the Gini index to measure specialization in a portfolio, the sub-populations (or subgroups) chosen to reflect the attributes we want to describe include null entries. This is understandable given the fact that, if we are focusing on the geographic locations of the stocks as our chosen attribute, we understand that some of the stocks belong to a certain location whereas the others do not i.e. we assume a priori, that within a given portfolio and with a given attribute under study, no single stock can belong to two different subgroups at the same time, within the same attribute\textsuperscript{3}. We can fairly describe our ten-stock portfolio, vis-à-vis the geographic location as:

Subgroup 1: Six stocks belonging to Japan and four stocks not belonging to Japan, and,
Subgroup 2: Four stocks belonging to the USA and six stocks not belonging to the USA.

We are proposing to introduce a new Gini index that includes the null entries to reflect the relative specialization of one subgroup with respect to the other.

Consider, for example, the attribute “industry group” reflected in Table 5.7, which shows 1 stock in each of Mining and Retail subgroups. If we apply the A.3 above to the above subgroups, with 1 stock only present in each subgroup, we will have a $G_{jj} = 0$ (please refer to Table 5.7, Page 94, row 2b) showing total equality and lack of specialization: all the wealth is equally allocated among all the units of the subgroup. However, from an investment perspective, it is clear that the opposite is true: this subgroup exhibits a high specialization since all the wealth allocated to this subgroup is concentrated in 1 stock among all the 10. Therefore, in order to describe better this particularity in investment portfolio, we introduce the $G_{jnull}$, which is the Gini index of a subgroup $j$, taking into consideration the null entries\textsuperscript{4} as follows:

$$G_{jnull} = \frac{1}{2n^2\mu_{jnull}} \sum_{i=1}^{n} \sum_{r=1}^{n} |w_i - w_r|$$ (A.13)

Where $\mu_{jnull}$ is the mean of subgroup $j$ taking into consideration the null entries, and $n$ is the total number of stocks in the portfolio. In our example of industry group, this is

\textsuperscript{3}This is to say, if we choose the geographic location as an attribute under study, no single stock can belong to two different locations at the same time. Hence, given the ten stocks that are included in our hypothetical portfolio we observe that six out of the ten stocks are from Japan and the other four out of ten are from US

\textsuperscript{4}This concept of null entries in concentration measure is very relevant in elections analysis, where those who did not vote for a particular candidate must be taken into account in the analytical conclusions as discussed by Taagapera (1979).
reflected in Table 5.7, Page 94, row 2a. The value of $G_{jnull}$ corresponding to the “All Groups” category represents the Gini index between the total weights of each subgroup.

A high $G_{jnull}$ indicates the presence of a specialization within the subgroup because it indicates that the weights of the stocks within the subgroup are not equally distributed. A low $G_{jnull}$ (between 0 and 0.25) indicates the absence of specialization within the group (i.e. no subgroup’s weight is high compared to the other subgroups).

### A.2 The HHI or Hirschman-Herfindahl Index

**HHI Index.**

We define the $HHI$ as the sum of the squares of the weights, i.e.

$$HHI = \sum_i \left( \frac{x_i}{\sum x_i} \right)^2 = \sum w_i^2. \quad (A.14)$$

In our tables, when the HHI used reflects the “All Groups” concentration, as well as the per subgroup concentration. The values of the subgroups concentration do not add to the HHI of all groups.

**cHHI: Corrected HHI**

The $HHI$ has an upper bound of 1 when total concentration is achieved and a lower bound of $\frac{1}{n}$ when minimum concentration is achieved in an equally weighted portfolio. It is however possible to bound the $HHI$ from 0 to 1 instead of $1/n$ to 1 resulting in a Corrected $HHI$:

$$\text{Corrected } HHI = cHHI = 1 - \frac{1 - HHI}{1 - \frac{1}{n}} \quad (A.15)$$

The $cHHI$ will have a value of zero (instead of $\frac{1}{n}$) when minimum concentration is achieved and will remain with 1 as its upper bound when maximum concentration is reached.

**rHHI: Reverse HHI index.**

$$\frac{rHHI}{n} = \frac{1}{n \times HHI} \quad (A.16a)$$

$$rHHI = \frac{1}{HHI} \quad (A.16b)$$

The reverse $HHI$ is a concentration measure that indicates the number of equally weighted stocks in a portfolio that would generate the same $HHI$. This gives an indication of the “degree” of concentration of a given portfolio vis-à-vis the number of stocks included. As an illustration, if a portfolio of 10 stocks has an $HHI = 0.5$ it can be said that this portfolio is equivalent to a $rHHI = \frac{1}{5 \times 0.5} = 2$ equally weighted stocks’ portfolio.
If the portfolio of $n$ stocks is equally weighted then its $HHI = \frac{1}{n}$ and its $rHHI = n$ which is the maximum value that $rHHI$ can take, i.e.

$$1 \leq rHHI \leq n$$

It is useful to be able to compare $rHHI$ of various portfolios or subgroups within a portfolio to assess the amount or the level of concentration in terms of the total number of stocks $n$. We suggest a per unit measure of equally weighted equivalent number of stocks by dividing the $rHHI$ by $n$, $\frac{rHHI}{n}$, a measure that appears in row 11 of our Tables ??, ??, ??, ?? and 5.7.
Appendix B

Portfolio Pretension Level

B.1 The HHI and the Portfolio Pretension Level

If an investor chooses to invest equal wealth in each stock, i.e. $w_1 = w_2 = w_3 \cdots = w_n = 1/n$, then it can be assumed that he does not have any preference of one stock or sector over the other, and we define this investor attitude as a minimum pretension level of investment. The more the allocation deviates from the $1/n$ allocation strategy, the more wealth is invested in one stock against the others within the $n$ stocks chosen. This indicates a higher pretension level, in the sense that the investor assumes or pretends that the latter choice, different from the lowest possible level of pretension, will eventually outperform the $1/n$ portfolio. Although more weight is invested in some stocks and less in others, and hence creating a portfolio with a higher pretension level, the choice remains a pretension or ex-ante prognostic. Ex-post results could be different from the investor’s “pretension” and therefore a higher pretension level is definitely not a guarantee of a higher or a better performance.

Definition of pretension level

1. We define a pretentious portfolio as one with a weight matrix $[w_i]$, resulting in a concentration index $HHI$, such that:

   $$HHI \geq \frac{1}{n}$$

   i.e. the more the portfolio allocation deviates from the equally weighted allocation $1/n$ the more pretension of the investor.

2. The minimum pretension portfolio is a unique portfolio being the $1/n$ or equally weighted portfolio whose $HHI = \frac{1}{n}$.

3. A given pretension level, i.e. a given value of HHI can be generated by various
matrices \([w_i]\), hence a given pretension level does not impose a unique portfolio, but rather an isometric HHI Portfolio Opportunity Set: POS. We further define the Portfolio Pretension Set (PPS) as the POS that satisfies a unique pretension level, a chosen HHI value, or the ISO-HHI-POS.

4. From 3. above we conclude that for a given pretension level HHI, we have a variety of resulting POS (i.e. the several sets of \([w_i]\) satisfying HHI=\(h\)), and hence a given HHI value or level does not specify a unique return value\(^1\). In fact, the return of a portfolio is \(R = [w_i]^T [r_i]\), where \([r_i] = [r_1, r_2, r_3, \ldots, r_n]\) is the return vector of each individual stock included in the chosen universe. It is clear that the value of \(R\) is not unique for a given HHI, since the given HHI does not define a unique weight vector \([w_i]\).

It is practical to normalize the measure of pretension and define a pretension level of 0 as a no-pretension portfolio hence relative to an equally weighted portfolio and define a pretension level of 1 as a maximum pretension level resulting from a totally concentrated portfolio in one single stock. Hence the pretension level or pretension index PRET is bounded as follows:

\[0 \leq \text{PRET} \leq 1\],

assuming no short selling.

This requires the correction of the lower bound of HHI from \(\frac{1}{n}\) to 0. We define the pretension index or pretension level of a portfolio \(\text{PRET} = cHHI\) as being equal to the corrected HHI as follows:

\[
\text{PRET} = cHHI = 1 - \frac{1 - HHI}{1 - \frac{1}{n}}
\]

(B.1)

Where a minimum pretension portfolio, or a non-pretentious portfolio, is represented by a \(cHHI = 0\) in fact corresponding to a minimum \(HHI = 1/n\).

\(^1\)A given pretension level, i.e. a given value of HHI can be generated by various vectors \([w_i]\), hence a given pretension level does not impose a unique portfolio but rather an ISO-HHI Portfolio Opportunity Set (POS). Intuitively, and since the HHI is not a ranking sensitive index, for each value of HHI corresponding to a weight vector \([w_i] = [w_1, w_2, w_3, \ldots, w_n]\), the same value of HHI is attained with a combination of the elements of \([w_i]\), among other possibilities. For example a weight vector of \((0.3, 0.5, 0.2)\) has the same \(HHI = 0.38\) as the vector \((0.5, 0.3, 0.2)\) and the vector \((0.2, 0.5, 0.3)\). Please note that theoretically the POS set of 3 stocks satisfying a \(HHI = 0.38\) is obtained by solving the quadratic equation:

\[w_1^2 + w_2^2 + w_3^2 = 0.38\]

subject to the following constraints:

- \(w_1 + w_2 + w_3 = 1\) (the budget constraint), and
- \(w_1, w_2\) and \(w_3 > 0\) (i.e. no short selling allowed).
Towards a return–risk–pretension portfolio description

As discussed earlier, the process of deciding on an investment portfolio is a matter of subjective choices based on some defined criteria that depend on the investor. If the investor has no preference or pretension of any one stock over the others he will eventually choose an equally weighted portfolio. If however he has valuable information or market data or wants to take more risk he will move from the equally weighted portfolio to a portfolio where more weight is put on some particular stocks at the detriment of the others, assuming no short sales. In this case, the concentration level is measured in terms of budget fractions invested per one stock or equivalently by its weight fraction contribution within the portfolio \( w_i \).

The concentration level or pretension level of a portfolio is a totally objective measure not depending on any estimation error or approximation of distribution. In fact it is a measure independent from risk and return. It reflects objectively the subjective choice of the investor. The more the investor likes risk or the more information he believes he has or perceives from the market, the more concentrated the final chosen portfolio is. It is not possible to separate the investor’s perception and preference effect from of the resulting portfolio’s concentration. This relationship between concentration resulting from the investor’s choices and preferences was defined earlier as pretension. In its broad sense, the pretension level describes, in an estimation free sense, the resulting preferences and perceptions of the investor. It is an additional descriptor of the investment portfolio, error free and estimation free. Each and every portfolio is earmarked with a pretension level that is the same ex-post and ex-ante if the weights are not reallocated.

In Figure B.1 we used 3 stocks to generate 10,000 different possible portfolios or POS, hence generating 10,000 matrices \([w_1, w_2, w_3]\) including the 4 particular portfolios:

(i) the equally weighted portfolio \([1/3, 1/3, 1/3]\) and (ii) the three one-stock portfolios, \([1, 0, 0]\), \([0, 1, 0]\), \([0, 0, 1]\). From the graphs in Figure B.1 we can observe the following:

a) The 4 particular portfolios mentioned above appear very clearly in the graphs. At the 0 pretension level, tangent to the y-axis, we find the equally weighted portfolio. At the pretension level of 1 we find the 3 one-stock portfolios.

b) The portfolio of maximum return (upper right leg of return graph) is the portfolio consisting of the stock having the highest return among the three used at a pretension level of 1, with three different portfolios.

c) The equally weighted portfolio is not necessarily the highest return portfolio or the lowest risk possible portfolio.

d) The portfolio of minimum Standard Deviation = 0.0211 (minimum risk) occurs at a pretension level of \(c\text{HHI}=0.4087\) with \([w_i]=[0.27, 0.73, 0]\) and a return of .0058; this point is graphically identified as the tangent to the standard deviation graph and parallel to the x-axis.
Figure B.1: Relationship between the cHHI or pretension level (horizontal axis) and Total return and Standard deviation of a hypothetical portfolio of 3 stocks, using a generated POS of around 10,000 portfolios.

e) The particularity of the minimum variance portfolio mentioned in d) above is that it does not include stock $s_3$. This is graphically clear since the tangent line to the standard deviation graph lies to the right of the vertex of the parabola joining stocks $s_1$ and $s_2$. In fact, the extremums of the parabolas joining each 2 stocks pairwise occur at the value $cHHI = 1/4$. In general, for any portfolio of $n$ stocks the pretension level of any sub-portfolio of 2 of its stocks, equally weighted, is represented by the general weight matrix $[w_i] = [0, 0, \ldots, 0.5, \ldots, 0.5, \ldots]$ to which corresponds the value of $HHI = \frac{1}{2}$, and a $cHHI_2$ of:

$$cHHI_2 = 1 - \frac{1 - \frac{1}{n}}{1 - \frac{1}{n^2}} = 1 - \frac{1}{2(1 - \frac{1}{n})} = 1 - \frac{n}{2(n - 1)}$$

(B.2)

With $n = 3$ then $cHHI_2 = \frac{1}{4}$.

When $n \to \infty$ then $cHHI_2 = 1 - \frac{1}{2(1 - \frac{1}{n})} \to \frac{1}{2}$.

f) Building on the results of e) above and using the same derived relationship, $cHHI_2$, we can conclude that the $(cHHI_2 - \epsilon)$ point is the maximum value of $cHHI$ that a portfolio can reach, i.e. $cHHI < cHHI_2$, if the investor wants to include each and every available stock in his portfolio. Hence, when $n$ is big enough a pretension level $cHHI < 0.5$ will ensure that all available stocks are included in the portfolio.

g) We observe that the minimum standard deviation value, $\sigma_{min} = 0.0211$ occurs at a point lower than the lowest standard deviation of one single stock (in our example it is 0.0224). This clearly reflects the effect of correlation among the 3 stocks. Indeed the effect of the correlation on the cHHI vs Standard deviation curve is to tilt it
B.2. Towards a return–risk–pretension portfolio description

slightly counter clockwise, allowing for a lower point the lowest possible single stock
standard deviation value.

h) For example, let us fix a pretension value at 0.2. As discussed earlier, this pretension
value is generated by an infinitely many possible vectors of weights. However, there is
only one vector which will have this pretension level and yield the maximum possible
return. This can be found by solving the following set of equations:

\[ w_1^2 + w_2^2 + w_3^2 = 0.2 \]

s.t. \( w_1 + w_2 + w_3 = 1 \) (budget constraint)

\[
\text{max } R_p = [w_1r_1 + w_2r_2 + w_3r_3]
\]

Solving this set of equations will result in the following weight vector: \([w_{0.2}] = [0.63, 0.21, 0.16]\) with a corresponding \(R_{max_{0.2}} = 0.0072\) and a \(\sigma_{0.2} = 0.025\), which
is clearly greater then \(\sigma_{min} = 0.0211\). This shows that the graphs of cHHI vs.
Return,(RET), and Standard deviation,(STDDEV), are consistent with the efficient
frontier theory.

i) The fact that the same return level can be achieved by more then one weight vector,
implies that the pretension level can be used to monitor the changes in a portfolio’s weights across time. This means that if the investor dynamically reallocates
(reallocation = change of weight = change in pretension level) the weights to adjust
for a desired return or desired risk level, then the pretension level would depict this
change. This is similar to drawing a horizontal line on Figure B.1 above, hence fixing
a level for return and \(\sigma_{desired}\). This in turn yields, at least graphically, the bracket
of pretension level and hence concentration level where the portfolio reallocation will lead.
Appendix C

Tables and Graphs Related to $P_{eq}$ and $P_{cap}$.

This Appendix includes all the graphs related to the specialization analysis of the portfolios $P_{eq}$ and $P_{cap}$. The analysis corresponds to our initial choice of Industry sector attribute, therefore the graphs appearing in this Appendix are those related to the industry sectors.

As mentioned in Chapter 6, PART I, the analysis is related to specifying the specialization variation of two portfolios, the equally weighted and the market capitalization portfolio.

We have identified 44 industry sectors using the GICS\textsuperscript{1} code that were downloaded from CRSP/COMPUESTAT. In our analysis we shall use a simple reference system to designate the available 44 industry sectors from Ind 1 to Ind 44, as it will appear on the graphs. Table C.1 is a mapping of the simplified coding we are using in the analysis.

\textsuperscript{1}GICS, The Global Industry Classification Standard (GICS) is an industry taxonomy developed in 1999 by MSCI and Standard & Poor’s (S&P) for use by the global financial community. In the database we downloaded from CRSP/COMPUESTAT, it is referred to as GIND and SUBGIND (Wikepedia definition).
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Table C.1: Reference system to designate the available 44 industry sectors referred to in the analysis
Figure C.1: A complete specialization graph of $P_{cap}$. The dotted line represents the Gini of the top ranked industries. For clarity and ease of reading this graph will be divided into 4 graphs following, Graph No.1,2,3 and 4.
Appendix C. \( Peq \) and \( Pcap \) Related Tables and Graphs

Figure C.2: Specialization graph No. 1/4 of \( Pcap \).
Figure C.3: Specialization graph No.2/4 of $P_{cap}$. 
Appendix C. $P_{eq}$ and $P_{cap}$ Related Tables and Graphs

Figure C.4: Specialization graph No.3/4 of $P_{cap}$. 

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GINI of top ranked industries

RANK

Pcap - Specialization (3 of 4)
Figure C.5: Specialization graph No.4/4 of $P_{cap}$.
Figure C.6: Histogram No.1/9 of the frequency and rank of all the industry sectors comprised in $P_{cap}$. 
Figure C.7: Histogram No.2/9 of the frequency and rank of all the industry sectors comprised in $P_{cap}$. 
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$P_{eq}$ - Specialization (2 of 4)

Gini of top ranked industries
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Summary

The information available to the investor to build up his portfolio can be of financial as well as non-financial nature. Past performance, volatility and price to earning ration, among other financial data are readily estimated and widely available. Non-financial information like geographic location, industry sector, ownership structure or carbon emission levels are also available and constitute some ex-ante modulators in the investment decision making process.

However, two different sample portfolios with different stocks can have the same expected return and the same standard deviation. In this research we introduce an additional descriptor class for investment portfolios that is estimation error free, namely the concentration and the specialization of an investment portfolio. This information provides the investor with a tool for monitoring and managing his portfolio, as well as an additional insight on the composition of the same.

We define a concentrated and a specialized portfolio and we suggest the Hirschman-Herfindahl index and the Gini index as practical measures for concentration and specialization. We create two quarterly portfolios with the top listed 500 stocks of the US market, one equally weighted and the other weighted by the market capitalization of each stock over 21 consecutive years and we describe the variation of the concentration and specialization of each quarterly portfolio over the years.

The results obtained show the relevance of the descriptors we introduce in understanding the nature and the constitution of each portfolio with respect to its concentration and specialization. We conclude with a conjecture on the absence of a neutral portfolio when there are no constraints on the attributes of the stocks constituting, and hence on the importance of the new descriptors in monitoring and managing an investment portfolio.
Samenvatting (Summary in Dutch)

De informatie die de belegger ter beschikking staat bij de samenstelling van zijn portefeuille kan zowel van financiële als van niet-financiële aard zijn. Resultaten uit het verleden, volatiliteit en koers/winst ratio’s bijvoorbeeld zijn eenvoudig te schatten en ruim voorhanden financiële kenmerken. Niet-financiële informatie zoals geografische locatie, industriële sector, eigendomsstructuur of CO2 emissieniveaus zijn ook beschikbaar en vormen ex ante modulators in het proces van beleggingsbeschikvorming.

Twee portefeuilles met verschillende aandelen kunnen in principe hetzelfde rendement en dezelfde standaardafwijking vertonen. In dit onderzoek introduceren wij een additionele categorie van descriptoren voor beleggingsportefeuilles die vrij is van schattingsrisico, namelijk de concentratie en de specialisatie van een beleggingsportefeuille. Deze informatie verschaft de belegger een instrument om zijn portefeuille te monitoren en te managen, en daarbij nieuwe inzicht in de samenstelling van die portefeuille.

We definiëren concentratie en specialisatie van portefeuilles en we stellen de Hirschman-Herfindahl en de Gini index voor als praktische maatstaven om concentratie en specialisatie van portefeuille te meten. Gebruik makend van de koersen van de 500 grootste genoteerde ondernemingen in de VS berekenen we op kwartaalbasis over 21 jaren twee portefeuilles: een gelijk gewogen portefeuille en een op basis van marktwaarden gewogen portefeuille. We beschrijven en vergelijken het verloop van beide portefeuilles over de jaren op basis van hun concentratie en specialisatie.

De verkregen resultaten tonen de relevantie van de geïntroduceerde concentratie en specialisatie maatstaven om de veranderende samenstelling van portefeuilles te beschrijven en te begrijpen. We sluiten af met een constatering over de afwezigheid van een neutrale portefeuille als er geen beperkingen gesteld worden aan het aantal kenmerken in termen waarvan een portefeuille gespecialiseerd kan zijn. Waarmee het belang van het beschrijven en monitoren van concentratie en specialisatie niveau van portefeuilles nogmaals onderstreept wordt.
About the author

Ghassan Chammas is a banking and investment advisor and a university lecturer. He holds a Masters degree in Finance from Ecole Supérieure de Commerce de Paris (ESCP-EAP), France, A Masters degree in International Finance and Treasury from Ecole Supérieure des Affaires (ESA) Beirut, Lebanon, a Masters degree in International Marketing from EAFIT University, Colombia and an Electrical Engineering degree from the American University of Beirut, Lebanon.

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