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The power of weather

Some empirical evidence on predicting day-ahead power prices through weather forecasts*

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Abstract

In the literature the effects of weather on electricity sales are well-documented. However, studies that have investigated the impact of weather on electricity prices are still scarce (e.g. Knittel and Roberts, 2005), partly because the wholesale power markets have only recently been deregulated. We introduce the weather factor into well-known forecasting models to study its impact. We find that weather has explanatory power for the day-ahead power spot price. Using weather forecasts improves the forecast accuracy, and in particular new models with power transformations of weather forecast variables are significantly better in term of out-of-sample statistics than popular mean reverting models. For different power markets, such as Norway, Eastern Denmark and the Netherlands, we build specific models. The dissimilarity among these models indicates that weather forecasts influence not only the demand of electricity but also the supply side according to different electricity producing methods.

Key words: Electricity prices, forecasting, GARCH models, weather forecasts.

JEL Classification Code: C53, G15, Q40.

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1 Introduction

The heatwave in Europe during August 2003 (the warmest summer in Europe since 1500), resulted in extremely high prices in several power markets, as France, Germany and the Netherlands, see for example Figure 1. The fact that not the 15000 casualties due to the heatwave but the technical problems of electricity supply experienced by Électricité de France (EDF), the main power supplier in France, were on the top of the agenda of the French Cabinet meeting held on August 11, 2003, illustrates the tremendous importance of the functioning of the power system to our society.

A decade ago, the weather effects would only affect electricity sales, which is documented in various studies (see for example the special issue of Journal of Econometrics 1979). Back then, the electricity industry was vertically integrated, prices were regulated and reflected the short-term marginal (production) costs. Hence, the demand curve that is a function of temporal effects such as seasonality, weather and business activity did not affect the price at that time. But this all changed when many governments started reforming their electricity industry as of the 1990s. Various market places were created with complementary investment horizons to trade electricity on spot or forward (hour-ahead, day-ahead, month-ahead), and power prices were based on the economic law of demand and supply. Market participants are now exposed to the volatile market conditions that stem from the non-storability of electricity; and the absence of inventories makes that supply and demand of power must be balanced on every precise moment in time. Hence, the change to a market-based electricity price system implies that temporal and regional effects such as seasonality, time-varying volatility and extreme price shocks explain the observed price behavior rather well.

Many studies have documented these stylized facts from examining the prices observed at day-ahead markets[1], which are by far the most liquid power wholesale markets, see Escribano, Pena, and Villaplana (2002), Lucia and Schwartz (2002) and Koopman, Ooms, and Carnero (2007). Bunn and Karakatsani (2003), provide a thorough review of the stochastic price models presented in these studies and classify these into three groups, being random walk models, basic mean-reversion models, and extended mean-reversion models that incorporate time-varying parameters (to control for seasonality and volatility patterns). They conclude that the idiosyncratic price structure has not been accurately described. Furthermore, the results reported in these studies are often obtained from in-sample tests, hence they do not resolve the issue of the out-of-sample predictive value of power models.

However, only few studies have recognized the need for modelling weather directly and addressed some interesting issues. Knittel and Roberts (2005) test stochastic price models on hourly hour-ahead power prices obtained from the California market and find that the forecasting performance is superior for price models which account for seasonal patterns and temperature effects.

In this study we attempt to shed more light on the issue of forecasting perfor-

[1] On these markets, hourly prices are quoted for delivery of electricity on certain hours on the next day.

mance of stochastic day-ahead price models. We examine six stochastic price models to forecast day-ahead prices of the two most active power exchanges in the world: the Nordic Power Exchange and the Amsterdam Power Exchange, see Geman (2005). Three of these forecasting models extend Knittel and Roberts (2005) by including new weather variables as price factors. Firstly, considering that operators make decisions today on tomorrow's electricity, the real weather of tomorrow is unknown at that moment, and the only available information of weather comes from the weather forecasts. Therefore, we use weather forecasts as predictors, which are more appropriate than real weather. The empirical study agrees with this intuition. Secondly, for specific weather variables, we consider temperature, total precipitation and wind speed, which may capture significant and interpretable supply and demand effects. Thirdly, since we find that the influence of the weather forecasts on the electricity prices is non-linear, we use non-linear transformations of the weather forecasts in our new models. Finally, we implement specific models for different power markets due to their heterogeneity in weather conditions and production plants.

We find that an extended ARMA model, which includes power transformations of next-day weather forecasts, yields the best forecasting results for predicting one day-ahead power prices. This model has some predictability power to anticipate prices jumps. Intuitively, adverse climate conditions often lead to sharp increases in demand resulting in supply shortages in electricity. We investigate carefully the relation between prices and weather. We find that the weather forecasts influence the electricity prices via the demand as well as the supply side, and when production is less related to weather, which is the case for Amsterdam Power Exchange, the weather forecasts play only a minor role. We also show that a GARCH specification extended with weather forecast variables provides accurate forecasts. This result contradicts with earlier findings that 'standard' GARCH models would predict electricity prices poorly [2].

The remainder of the paper is structured as follows. Section 2 introduces the day-ahead power markets. Section 3 presents the data. Section 4 describes the forecasting models. Section 5 discusses the empirical results. Section 6 concludes.

2 Day-ahead power markets

On 1 January 1991, the Norwegian government imposed a deregulation process on its electricity industry that resulted in the establishment of the first national power market for short-term delivery of power (real-time and day-ahead[3]) in the world, the Nordic Power Exchange (NPX). Two years later, in 1993, the range of products was extended with forward and futures contracts that had longer maturity horizons. Another few years later, Sweden joined the NPX (1996), soon followed by Finland (1998), West-Denmark (1999) and East-Denmark (2000). From 2003 all customers of Scandinavian electricity markets may trade freely in the market. The NPX,

[2]See for example Knittel and Roberts (2005).

[3]We remember from section 1 that day-ahead means that prices are quoted at day t for delivery of electricity on certain hours on the day $t + 1$.

now also named Nord Pool ASA, is considered as the most liquid wholesale market worldwide. Nord Pool ASA constitutes of a day-ahead market (Elspot), a financial market (Elbas), and a clearing service. In the remainder, we mainly focus on the Elspot market. For more details on Nord Pool ASA we refer to NordPool (2004).

Another country that liberalized its power industry at an early stage onwards, is the Netherlands. In 1999, here the second electronic power exchange was founded, being the Amsterdam Power Exchange (APX). The APX is also composed by a day-ahead market and a financial market. For more details on APX we refer to www.apxgroup.com.

In Figure 2 some descriptive statistics of these two markets are listed. The Nord Pool market is largely dependent on electricity that is generated by renewable sources. In particular, hydro-plants, which use water stored in reservoirs or lakes, are dominant in Norway and partly Sweden; wind-plants, which use wind to produce electricity, are dominant in Denmark. In the APX market oil, coal, gas or a combination of these fuels is used to generate electricity.

Electricity prices are affected by regional and temporal influences due to the transportation and transmission limits of electricity. This statement is particular important in the Nord Pool market. For instance, when a power plant falls out in the eastern part of Sweden this only affects the power supply in the surrounding region. Hence, this will not affect power supply in the western part of Sweden and the rest of the market. Similarly, rainfall in the southern part of Norway, will potentially affect the regional demand and/or supply curve, but not the bidding curves in other regions. Nord Pool faces the problem by allowing to split the market in several bidding and prices areas. Therefore, we take into account the Nord Pool bidding area prices separately, rather than examining the Elspot system price (which is a weighted average of the bidding prices in all Nord Pool bidding areas). We examine two out of the eleven bidding areas in the Nord Pool, being the Oslo area and Eastern Denmark area. It is interesting to note that these areas are the most densely populated areas in Scandinavia.

3 Data

3.1 Electricity prices

The data set used in this study consists of day-ahead prices in EUR/MWh for Oslo, Eastern Denmark and the Netherlands from the period December 24, 2003 to March 14, 2006. Oslo and Eastern Denmark are two bidding areas of Nord Pool market; Dutch electricity prices are obtained from APX market[4]. Nord Pool provides bidding area prices both in the local currency and in EUR. We choose EUR to compare directly to APX prices. Daily prices are computed as the arithmetic mean of the available 24 hourly prices series on the physical market of each country.

Figure 4 plots the time series, the log transformations and the histograms of the

[4]Electricity prices may be available for a longer sample, but weather forecasts are available to us only for this sample.

daily day-ahead electricity prices; Table 1 gives some important descriptive statistics. As in Wilkinson and Winsen (2002) and Lucia and Schwartz (2002) we start from a statistical analysis of the data we have[5]. A first casual look reveals a quite erratic behavior of the prices. The series follow a small positive increasing trend with several spikes. Interesting, prices in Oslo have more negative spikes than positive high spikes. This may indicate that the supply was often higher than the demand and may support Geman (2005) conclusion that prices in hydropower markets are less subjects to jumps and more similar to other commodity prices than prices of thermal-based electricity. Eastern Denmark and the Netherlands prices are sensitively higher, in particular the Netherlands ones. Higher spikes are more frequent. The maximum prices in the sample are 235.71 EUR/MWh and 250.69 EUR/MWh for Eastern Denmark and the Netherlands respectively, which are more than 7 and 5 times higher than the average prices, 32.85 EUR/MWh and 44.85 EUR/MWh respectively. The histograms provide similar evidence. Eastern Denmark and the Netherlands prices are highly non-normally distributed; their volatility is very high such as the kurtosis; their skewness is positive. Oslo has a more regular distribution, but a Jarque-Bera test rejects the null hypothesis of normality for each of the three series. The series are characterized by a weekly pattern: Table 1 reports prices are lower on weekend than working days. Yearly patterns, well documented in other studies, are more difficult to notice since the series are not very long, but differences among seasons in Figure 3 may be drawn. Electricity prices are very persistent and possible close to non-stationary. We do not investigate the hypothesis of non-stationary for reasons which we discuss in Section 4.1. Table 1 shows that the sample autocorrelations are high up to 14-day lags. The last stylized fact we notice in Figure 4 is volatility clustering. Dramatic spikes tend to occur in clusters, mainly as result of consecutively exceeding the system capacity.

In our application we use log prices and not the level. The log transformation reduces the spike behavior of the prices and makes moments of the distribution of electricity prices more similar to standard distributions, in particular for Eastern Denmark and the Netherlands log prices.

3.2 Weather forecasts

We continue the data analysis by focusing on weather forecasts. Forecasts on the daily average temperature in degrees Celsius, total precipitation in mm, and wind power in m/s are applied. Data are obtained from the EHAMFORE index, which is provided by Meteorlogix (www.meteorlogix.com)[6]. We assume that market operators use the weather forecasts provided by Meteorlogix in their decisions. We think that this assumption is quite realistic considering the market share of Meteorlogix in providing real-time information services in the agriculture, energy, and commodity trading markets, and Bloomberg in providing data to operators. Weather forecasts refer to a square area around the measurement station, which implies that series

[5]We briefly discuss some stylized facts; we refer for a more detailed analysis, for example, to Lucia and Schwartz (2002) and Pilipovic (1997).

[6]Data from the EHAMFORE index are available in Bloomberg.

that cover all the lands of the markets we consider do not exist. The combination of different stations might be applied, but we exclude it because it might be difficult to collect data from minor cities, the weather forecast errors might arise introducing further noise in the forecasting process, and the country/areas that we study are quite small and quite homogenous in term of weather. Therefore, we only use weather forecasts for Oslo, Copenhagen and Amsterdam. The weather around Oslo may well approximate the weather in the area on the south of Oslo along the sea cost where most of the electricity for south-east Norway is produced. The weather in the area of Copenhagen may be a proxy for the weather of Zealand, the main island in Eastern Denmark. Finally, Amsterdam is located in the middle of the Netherlands.

Figures 5-7 plot the three variables for each country. Temperatures have highly seasonal patterns, with lower values for Oslo and higher for the Netherlands. Precipitations are higher in Oslo and the Netherlands than in Eastern Denmark. The wind is particular strong in Eastern Denmark and the Netherlands. The wind forecasts on all the three countries have a quite stable pattern in the initial months of 2004, because the meteorologic institute applies a different forecasting model on those months. We decide to keep these forecasts to extend, as much as we can, the sample period, meanwhile it is what operators got as information for the weather. Some graphical relations between the forecasted weather variables and electricity prices may be identified. For example, high precipitation in Oslo at the end of May 2004 or in October 2004 corresponds to low prices; few days of very low temperature in Oslo in February 2005 correspond to high prices; strong wind in Eastern Denmark at the end of 2004 and beginning of 2006 is associated to low prices. However, even if the real weather was the weather forecasts, a graphical analysis would not be satisfactory because the relation between weather variables and electricity prices is possibly highly nonlinear as we will discuss in Section 5.1. Therefore, we try to find specific models to interpret the weather influences.

4 Forecasting models

Knittel and Roberts (2005) shows that traditional time series approaches as ARMA models provide more accurate results in forecasting electricity prices than their continuous counterparts. Starting from these findings we built several models that may cope with the stylized facts of electricity prices.

4.1 Model 1: ARMA

The first model is a traditional time series approach to model electricity prices, the autoregressive moving average (ARMA) model (Hamilton (1994)). The ARMA(p, q) model implies that the current value of the investigated process (say, the log price) P_t is expressed linearly in terms of its past p values (autoregressive part) and in terms of the q previous values of the process ϵ_t (moving average part):

$$\phi(L)P_t = \theta(L)\epsilon_t \tag{1}$$

where $\phi(L)$ and $\theta(L)$ are the autoregressive and moving average polynomials in the lag operator L respectively, defined as:

$$\phi(L) = 1 - \phi_1 L - \phi_2 L^2 - ... - \phi_p L^p \tag{2}$$

$$\theta(L) = 1 - \theta_1 L - \theta_2 L^2 - ... - \theta_p L^p \tag{3}$$

and where ϵ_t is an independent and identically distributed (iid) noise process with zero mean and finite variance σ. The motivation of an ARMA process follows from the correlogram. Table 1 shows high correlation between the current price and the previous days' prices.

The ARMA modelling approaches assume that the time series under study is (weakly) stationary. If it is not, a transformation of the series to stationarity is necessary, such as first differentiating. The resulting model is known as the autoregressive integrated moving-average model (ARIMA). We do not work with first difference prices for several reasons. Firstly, the Dickey Fuller test on the series rejects the null hypothesis of non-stationary. Secondly, we think more appropriate working on the levels since the final object of the study is modelling and predicting the pattern of electricity prices and the first difference transformation might drop out same important characteristics, such as the trend of the series. Thirdly, evidence in literature are almost unique in favor of the level of prices. For example, Lucia and Schwartz (2002) find that models based on levels and log levels provide more accurate results than models based on first differences and log first differences in forecasting Nord Pool electricity prices. And Weron and Misiorek (2005) find that ARMA models do better in term of out-of-sample statistics than ARIMA models in forecasting California electricity prices.

4.2 Model 2: ARMAX

The second model is an extension of model 1. ARMA models apply information related to the past of the process and do not use information contained in other pertinent time series. However, as the data analysis shows, electricity prices are generally governed by various fundamental factors, such as seasonality and load profiles. The ARMAX(p, q) can be written as:

$$\phi(L)(P_t - X_t) = \theta(L)\epsilon_t \tag{4}$$

where $X_t = \sum_{i=1}^{k} \psi_i x_{i,t}$, where $x_t = (x_1, x_2, ...x_k)'$ is the $(k \times 1)$ vector of explanatory variables at time t, and where $\psi = (\psi_1, \psi_2, ..., \psi_k)'$ is a $(k \times 1)$ vector of coefficients. Following Lucia and Schwartz (2002) we use three explanatory variables: a dummy with values 0 on working days and 1 on holidays, a seasonal dummy given by the combination of the two variables $sin(2\pi t/365.25)$ and $cos(2\pi t/365.25)$. These dummy variables may be interpreted as proxy for load profiles (higher demand on working days), and proxy for weather effect (higher demand on cold and warm seasons).

In the empirical application, the ARMAX model will be our benchmark.

4.3 Model 3: ARMAXW

Averse weather conditions may change the demand for electricity, and may also affect the production. Low amount of precipitation and low wind may cause reduction on the supply of energy, in particular in electricity markets which depend on renewable producer plants, such as Norway and Denmark. Furthermore, producer plants may study future weather conditions to estimate demand and plan their supply optimally.

The third model is an extension of model (4) and is built following the previous reasoning. Forecasts on the average daily temperature in degrees Celsius, precipitation in mm and wind speed in m/s are applied as further explanatory variables. The model is:

$$\phi(L)(P_t - X_t - W_t) = \theta(L)\epsilon_t \tag{5}$$

where $W_t = \sum_{j=1}^{l} \varphi_j w_{j,t}$, where $w_t = (w_{1,t}, w_{2,t}, ..., w_{l,t})'$ is the $(l \times 1)$ vector of weather forecast variables at time t, and where $\varphi = (\varphi_1, \varphi_2, ..., \varphi_l)'$ is a $(l \times 1)$ vector of coefficients. This model includes deterministic components that account for genuine regularities in the behavior of electricity prices and stochastic components that comes from weather shocks.

Knittel and Roberts (2005) apply a similar model for forecasting California electricity prices, where the set of weather variables is composed by the level, the square and the cubic of realized temperature. We think that the weather of tomorrow is more important of the weather of today to forecast the price of tomorrow. Therefore, we use weather forecasts and not realized values. Moreover, we add wind speed since it may play a role both in the feeling of the people - people feel colder with stronger wind - and in the supply of wind power plants. We also use precipitation since the variable may be appropriate to approximate the supply of hydroelectric plants. As in Knittel and Roberts (2005) we allow nonlinearity in the relation between prices and weather variables by including the level, the square and the cubic of the temperature forecasts, and the level and the square of the precipitation and wind forecasts[7].

Some objections may be argued for hydroelectric power generator. The water reservoir is often more important to plan production than the amount of precipitation, see e.g. Koopman, Ooms, and Carnero (2007) and Deng (2004). But water reservoir is not known in advance and may not be forecasted. Furthermore, we think that hydroelectric plants incorporate forecasted future precipitations in their strategic decisions of the amount of water to store.

4.4 Model 4: ARMAX-GARCH

ARMA models assume homoscedasticity, i.e. constant variance and covariance function, but the preliminary data analysis has revealed that electricity prices exhibit volatility clustering. The fourth model extends model 2 by assuming a time varying conditional variance of the noise term. The heteroskedasticity is modelled by a generalized autoregressive conditional heteroskedastic GARCH(p, q) model (Bollerslev

[7]Precipitation and wind forecasts are always positive, therefore we do not consider useful to include the cubic transformation.

(1986)). Relaxing the assumption of homoscedasticity may change the parameter estimates of model 2, and consequently the out-of-sample forecast of the investigated process.

The model is:

$$\phi(L)(P_t - X_t) = \theta(L)\epsilon_t \tag{6}$$

$$\epsilon_t = \nu_t h_t^{1/2} \quad with \;\; h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-j}^2 + \sum_{j=1}^{p} \beta_j h_{t-1} \tag{7}$$

where ϵ_t is an independent and identically distributed (iid) noise process with zero mean and conditional time varying variance h_t, and the coefficients have to satisfy $\alpha_i \geq 0$ for $1 \leq i \leq q$, $\beta_j \geq 0$ for $1 \leq j \leq p$, and $\alpha_0 > 0$ to ensure that the conditional variance is strictly positive.

4.5 Model 5: ARMAXW-GARCH

Following the same reasoning for model (6)-(7), model 3 can be extended by assuming a noise process with a time varying conditional variance.

Model 5 is:

$$\phi(L)(P_t - X_t - W_t) = \theta(L)\epsilon_t \tag{8}$$

$$\epsilon_t = \nu_t h_t^{1/2} \quad with \;\; h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-j}^2 + \sum_{j=1}^{p} \beta_j h_{t-1} \tag{9}$$

4.6 Model 6: ARMAXW-GARCHW

Koopman, Ooms, and Carnero (2007) find that seasonal factors and other fixed effects in the variance equation are also important to estimate electricity prices. The fifth model extends model 4 by reformulating model 3 and 4 to incorporate Koopman, Ooms, and Carnero (2007) results. The conditional variance of the noise term in model 3 is assumed to be time-varying and modelled with a GARCH expression where some explanatory variables are added to the ARMA form of equation (7). The model looks as:

$$\phi(L)(P_t - X_t - W_t) = \theta(L)\epsilon_t \tag{10}$$

$$\epsilon_t = \nu_t h_t^{1/2} \quad with \;\; h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-j}^2 + \sum_{j=1}^{p} \beta_j h_{t-1} + \sum_{f=1}^{k+l} \varrho_f z_{f,t} \tag{11}$$

where $z_t = [x_t', w_t']'$, and where $\varrho = (\varrho_1, \varrho_2, ..., \varrho_{k+l})'$ is a $((k+l) \times 1)$ vector of coefficients. Despite the fact that Koopman, Ooms, and Carnero (2007) assume autoregressive fractionally integrated moving average noises which we do not consider, an important difference with model 6 is the set of (weather) explanatory variables. Koopman, Ooms, and Carnero (2007) include water reservoir and consumption, which we think less adequate to forecast future prices.

5 Empirical Results

We apply the models described in Section 4 to our data set, try to figure out which is the best on forecasting. Before the out-of-sample forecast exercise, we estimate the set of models using the complete sample to have an ex-post predictability idea.

We describe some assumptions. In estimation we apply nonlinear ordinary least square (NLS) estimator (Davidson and MacKinnon (1993)) for ARMA type models and approximate maximum likelihood (QML) estimator (Davidson and MacKinnon (1993) and Greene (1993)) for GARCH family models.

We restrict our ARMA type models to be ARMA(7,0), where only lags 1 and 7 are considered. We do the same for the level equation of the GARCH models. Autocorrelation analysis and in-sample criteria would suggest more complex ARMA forms. However, the risk of over-parametrization and previous studies, for example Lucia and Schwartz (2002) show that an ARMA(7,0) provide optimal forecasts on daily day-ahead electricity prices, convince us to restrict the models to the aforementioned specification. Following the same reasoning we choose a GARCH(1,1) specification for the variance equation of models 4, 5, and 6.

The inclusion of the weather variables follows from statistical evidence. We allow different transformations of the weather forecast variables on the three markets to incorporate the fact that the weather may affect only the supply of electricity, which is different in the three markets. Precisely, since the influence of the weather variables appears to be non-linear as shown later, the generic initially unrestricted model in all the three exercises includes the level, the square and the cubic of the temperature forecasts, and the level and the square of the precipitation and wind forecasts. We use selection criteria, as the the adjust R-square and Akaike information criteria, and parameter statistical significance to specify the model.

5.1 In-sample analysis: Oslo Case

The in-sample analysis is based on the overall sample, from December 24, 2003 to March 14, 2006. We start with the ARMAX model which is considered to be a very accurate forecasting model. The ARMAX model in Lucia and Schwartz (2002) is the follows:

$$P_t = X_t + \phi_1(P_{t-1} - X_{t-1}) + \phi_7(P_{t-7} - X_{t-7}) + \epsilon_t \tag{12}$$

where

$$X_t = c + d_1 D_{hol,t} + d_2 sin(2\pi t/365.25) + d_3 cos(2\pi t/365.25),$$

where P_t is the log of the price at day t, and where $D_{hol,t}$ is a dummy variable with value 0 if day t is a working day or 1 if day t is not a working day. Taking a close look at the errors of the ARMAX model, as shown in Figure 8, the errors have non-linear relations with the daily average temperature and the total precipitation, however, a linear-like relation with the wind speed. This suggests us to introduce the weather variables as the the level, the square and the cubic of the temperature forecasts, and the level and the square of the precipitation and wind forecasts. By the selecting procedure mentioned above, the reduced specific ARMAXW model for Oslo data is the follows:

$$P_t = X_t + W_t + \phi_1(P_{t-1} - X_{t-1} - W_{t-1}) + \phi_7(P_{t-7} - X_{t-7} - W_{t-7}) + \epsilon_t \qquad (13)$$

where

$$X_t = c + d_1 D_{hol,t} + d_2 sin(2\pi t/365.25) + d_3 cos(2\pi t/365.25),$$

$$W_t = a_1 Temp_t + a_2 Temp_t^3 + b_1 Prec_t + b_2 Prec_t^2 + \gamma Wind_t,$$

where $Temp_t$, $Prec_t$ and $Wind_t$ are the forecasts on daily average temperature, total precipitation and wind speed, respectively, on day t. The estimation procedure indicates that the square of the temperature and the square of the wind can be excluded. The square of the temperature does not take into account the difference between very low and very high temperature, which is a serious limitation. The wind seems to have a direct linear relation with prices. These empirical findings agree with the above graphical analysis.

Table 2 gives the results of the estimation of model (13) with the chosen W_t over the complete sample. We discuss the estimated coefficients for the temperature forecasts, a_1 and a_2. This hopefully explains the nonlinearity in the relation of prices and weathers, and support our views to introduce them. The temperature forecasts affect the day-ahead electricity price via the following function:

$$f(Temp_t) = a_1 Temp_t + a_2 Temp_t^3$$

Taking the first order derivative, we get

$$\frac{\mathrm{d}f(Temp_t)}{\mathrm{d}Temp_t} = a_1 + 3a_2 Temp_t^2.$$

By substituting in the previous equation a_1 and a_2 with their empirical estimates, and solving $\frac{\mathrm{d}f(Temp_t)}{\mathrm{d}Temp_t} = 0$, we find the roots as ± 15. From our data set, the minimum observed temperature is -15. So the only switch point is $Temp^* = 15$. When the temperature is lower than the switch point, it is negatively influenced, i.e. the lower forecasted temperature, the higher electricity price. On the other hand, when the temperature forecast is above the switch point, it is positively influenced, i.e. the higher the forecasted temperature, the higher the electricity price. Intuitively, it reflects the fact that when temperature forecast is relatively higher or lower, the consumption of the electricity will arise. Meanwhile the difficulty of producing electricity is also increased when it is extremely hot or cold. This suggest that the weather forecasts can influence both the demand and supply side of the power. For further discussion on which side the weather forecast really affects, see next section.

Comparing to the ARMAX, the improvement of introducing the weather forecast variables is not impressive for the in-sample analysis as shown in Table 2. The inclusion of weather variables seems also appropriate in the GARCH specification. The parameters of the GARCHW equation are less persistent than the GARCH counterpart and model 5 has the lowest Akaike information criteria.

The estimate of autoregressive terms and of the constant term may suggest the presence of unit root. As we explained, we do not take this hypothesis into account.

But we notice that we could have spurious regression. Therefore we relax the analysis of the goodness of fit and proceed with the out-of-sample analysis with the chosen model.

5.2 Out-of-sample analysis: Oslo Case

The object of the out-of-sample analysis is to forecast the electricity price from January 1, 2005 to March 14, 2006. We repeat the selection procedure in Section 5.1 over the initial in-sample period, from December 24, 2003 to December 31, 2004. The reduced specific model remains the same as in (13). In forecasting, the model is re-estimated to make any new forecast, but it is not re-specified. An expanding window is used, which means that, to forecast the price of one day, all the previous data are applied.

Two criteria (typically used in the electricity forecasting literature, see e.g. Conejo, Contreras, Espinola, and Plazas (2005), Knittel and Roberts (2005), Shahidehpur, Yamin, and Li (2002), Weron (2006)) are computed to compare the models. The first one is the Root Mean Square Prediction Error (RMSPE), defined as

$$RMSPE = \sqrt{\frac{1}{n}\Sigma_{s=1}^{n}(P_{T+s} - \hat{P}_{T+s})^2}$$

where P_{T+s} is the log price at time $T+s$, where $\hat{P}_{T+s}$ is the forecasted log price at time $T+s$, where $n = 438$ is the number of days being forecasted. The alternative criterion is the Mean Absolute Percentage Prediction Error (MAPE), see for example Misiorek, Trueck, and Weron (2006). It is defined as

$$MAPE = \frac{1}{n}\Sigma_{s=1}^{n}\frac{|p_{T+s} - \widehat{p}_{T+s}|}{p_{T+s}}$$

We apply all five models in section 3 to forecast the daily price for Oslo data, and calculate the RMSPE and MAPE statistics. For description, we also report results for the Random Walk (RW) model. The results are given in Table 3.

All the models provide quite superior statistics than the RW model, implying predictability in the electricity prices. It is also clear that the model 4, ARMAXW, is the best under both the criteria. For example, for the RMSPE statistics, comparing to model 3, ARMAX, which is the best among the non-weather models, the improvement is 3.8%.

We test whether the difference between two forecasting methods is significant in order to show precisely how large is the improvement of the new weather forecast model. We choose the Diebold-Mariano test (Diebold and Mariano (1995)) with loss function the mean square prediction error (MSPE). The null hypothesis is

$$H_0 : \text{The square of the forecast errors are equal.}$$

Statistics are in Table 4. The p-value of this test is $p = 0.0024$. We conclude that the ARMAXW model is significantly improving in the sense of out-of-sample forecasting.

Figure 9 shows the 60-day average RMSPE for the ARMAX model and the ARMAXW model. From the graph, we find that, when the error of model 3 is in a relatively lower level, the errors of two models are similar; but when there is a higher error from model 3 due to possible jumps, our weather forecast model often predicts better. Price jumps are mainly due to problems of inelasticity of the demand, and of non-storability of electricity with consequent shortage in the supply. These problems often arise when the weather conditions are adverse. Empirical results confirm the theoretical intuition that the weather forecasts help in predicting high prices or jumps, possible related to extreme adverse climate situations.

Adding weather forecast variables in a GARCH model is also very beneficial. Forecasts from the model 6, ARMAXW-GARCHW, gives accurate forecasts and quite similar to model 3, even if marginally lower than it. On contrary, model 4, ARMAX-GARCH, gives very poor forecasts, and extending the mean equation with weather variable, as model 5, ARMAXW-GARCH, is not enough. To sum up, a 'classical' GARCH specification is not adequate to predict electricity prices, but adding weather variables as shock indicators improve enormously the performance.

Although the weather forecast models show improvement on predicting the day-ahead prices, it is still mysterious whether this kind of influence is via the demand of the electricity or the supply. One way to verify this is to introduce the volume variable into a forecasting model. In principle, the volume indicates the demand of the electricity. Then, if the weather only influence the consumption of the power, introducing the volume at time $T+s$ to forecast the electricity price at time $T+s$ into the ARMAX model will lead to similar results as the ARMAXW model. We stress that the volume at time $T+s$ is not known in advance, but previous literature form Engle, Granger, Ramanathan, and Andersen (1979) finds that it may be forecasted accurately, then we assume to know it. The model with volume (ARMAXV) is given as

$$P_t = X_t + V_t + \phi_1(P_{t-1} - X_{t-1} - V_{t-1}) + \phi_7(P_{t-7} - X_{t-7} - V_{t-7}) + \epsilon_t$$

$$X_t = c + d_1 D_{hol,t} + d_2 sin(2\pi t/365.25) + d_3 cos(2\pi t/365.25)$$

where V_t is the volume at time t. The calculated RMSPE is 0.0509, the improvement with respect to the ARMAX is 0.04%. We also apply the Diebold-Mariano test, the p-value is 0.8161. The comparison shows that introducing the volume on forecasting the day-ahead price is not comparable with the weather forecasts, even if the future (unknown in practice) volume is applied. In Oslo electricity market, the weather influence is not only via the demand of the electricity, but even more via the production of the electricity. This reflects to the producing method in Oslo, hydropower.

5.3 Further Application: Eastern Denmark Case

With the estimation in the in-sample period, we find that the specified model for the Eastern Denmark data only depends on the temperature and wind speed as follows

$$P_t = X_t + W_t + \phi_1(P_{t-1} - X_{t-1} - W_{t-1}) + \phi_7(P_{t-7} - X_{t-7} - W_{t-7}) + \epsilon_t \qquad (14)$$

$$X_t = c + d_1 D_{hol,t} + d_2 sin(2\pi t/365.25) + d_3 cos(2\pi t/365.25)$$

$$W_t = a_1 Temp_t + a_2 Temp_t^3 + \gamma Wind_t$$

The comparison of the six models is given in Table 3. It is still the case that the model 3, ARMAXW, is the best among them, from both of the criteria. In particular, for the RMSPE, the weather forecast model improves 3.89% from the ARMAX model. This improvement can also be tested by the Diebold-Mariano test, the corresponding p-value is 0.00002, which is significant.

The result for the Eastern Denmark market is also related to the electricity producing method in that area. The wind power is a non-trivial part of the production, which reasonably illustrate our empirical result. As in Oslo case, Figure 9 shows that the ARMAXW predicts especially better when the errors for other models are relatively higher and jumps are observed.

The ARMAXW-GARCHW still provides reasonable accurate statistics, but they are always marginally higher than the statistics of the ARMAXW model and not statistically different. However, results of the ARMAXW-GARCHW are more accurate than results of the other two GARCH models, confirming again the role of weather forecasts in the GARCH specification.

5.4 Different Story: The Netherlands Case

By in-sample analysis, we figured out that the chosen model for W_t is the same as for the Eastern Denmark market in equation (14). Therefore, forecasts on temperature, cubic of the temperature and wind speed are inserted in the regression model. This sounds realistic considering that the Netherlands is the country of windmills.

The ARMAXW does not provide longer the most accuracy tests under any criterion. Compare to the model 3 the improvement of the ARMAXW is only 1.32%, which is not only relatively small in percentage, but also not sufficient large to pass the Diebold-Mariano test at 10%.

Model 5, ARMAXW-GARCH, provide the lowest statistics, but also the ARMAX-GARCH and the ARMAXW-GARCHW forecast accurately and similar to model 5. In term of Diebold-Mariano test the ARMAX-GARCH is the best model. We interpret the results as an evidence that the GARCH specification improves forecast accuracy and the weather gives only a marginal contribution.

Intuitively, the power in the Netherlands is mainly thermal-based, which is less related to the weather. In sense of this, to introduce weather forecasts in predicting the electricity price is less efficient. On the other hand, it is also an evidence that the weather forecasts influence the electricity price via both demand and supply side: when the production is less related to weather, the weather forecasts play a minor role.

6 Conclusion

Electricity prices depend on several well-known temporal and regional price effects. Lucia and Schwartz (2002) have shown that including deterministic components that account for genuine regularities in the behavior of electricity prices give superior out-of-sample forecasts. However, more recent studies (see Bunn and Karakatsani (2003) and Knittel and Roberts (2005)) have found that the idiosyncratic price structure is not accurately described by Lucia and Schwartz (2002) model. In this paper we develop a set of models that add in previous ARMA and ARMA-GARCH specifications a new price factor: the weather forecasts.

Our empirical results suggest that the weather forecast variables play a central role in forecasting the day-ahead prices in different markets. In particular, the weather forecasts give relevant information to predict shocks in the prices. Intuitively, weather forecasts anticipate adverse weather conditions, which are often the cause of sharpen increase in the demand of electricity such as possible shortages in the supply. By studying carefully this statement, we find that the weather has high predictability power when the production plants of the market are related to the weather. This indicates that price jumps in those markets depend more on shortages in or strategic decisions of the supply than increase in the demand.

The idea of weather forecast as new price factor also revaluates the GARCH class of models in forecasting electricity prices. Extending Koopman, Ooms, and Carnero (2007) we show that a GARCH process with weather forecasts predicts day-ahead prices successfully.

There are several topics for further research. Firstly, the set of weather forecasts might include other weather-related variables, such as water reservoir, which to our knowledge are not modelled and forecasted at this time. Secondly, model 6 might be generalized by allowing seasonal variation in the parameters such as in periodic time series models and in periodic GARCH models, see Franses and Paap (2000). Weather forecasts might also be included in other nonlinear models, such as Markov regime-switching models, see Misiorek, Trueck, and Weron (2006), or jump models. Finally, models based on weather forecasts might be used in derivative contracts. The reported evidence that weather has predictive power on the underlying day-ahead price process, could imply that this price factor might be reflected in the price of derivative instruments on day-ahead electricity contracts as well. Results might be extremely important since in most of the power derivative markets derivative contracts (e.g. callable options) are commonly traded.

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Table 1: Descriptive statistics

	Oslo	Eastern Denmark	The Netherlands
Mean	30.43	32.85	44.74
St dev	4.965	12.74	22.91
Min	17.16	8.390	10.52
Max	52.45	235.7	250.7
Skewness	1.319	6.295	2.312
Kurtosis	0.306	79.13	7.865
Working days	30.90	34.81	49.68
No working days	29.35	28.40	33.28
ρ_1	0.913	0.766	0.754
ρ_7	0.736	0.629	0.774
ρ_{14}	0.544	0.490	0.700

The table reports descriptive statistics on electricity prices in Oslo, Eastern Denmark and the Netherlands. Lines working days and no working days give the sample average prices on working days and no working days (weekends and holidays) respectively. Lines ρ_1, ρ_7 and ρ_{14} give the 1^{st}, 7^{th} and 14^{th} sample autocorrelation.

Table 2: In-sample estimation: Oslo

Models	ARMAX	ARMAXW	ARMAX - GARCH	ARMAXW - GARCH	ARMAXW - GARCHW
ϕ_1	0.811	0.782	0.886	0.874	0.887
	[0.022]	[0.0224]	[0.018]	[0.019]	[0.019]
ϕ_7	0.174	0.2040	0.118	0.121	0.112
	[0.023]	[0.023]	[0.020]	[0.021]	[0.021]
c	3.4907	3.5410	2.871	2.927	6.055
	[0.122]	[0.127]	[0.970]	[0.973]	[3.205]
d_1	-0.0491	-0.0500	-0.030	-0.030	-0.031
	[0.004]	[0.004]	[0.002]	[0.002]	[0.002]
d_2	0.0412	0.011	-0.095	-0.129	-0.139
	[0.060]	[0.055]	[0.056]	[0.052]	[0.063]
d_3	-0.046	-0.098	-0.089	-0.105	-0.138
	[0.064]	[0.058]	[0.062]	[0.058]	[0.066]
a_1	-	-0.008	-	-0.004	-0.004
		[0.0013]		[0.001]	[0.001]
a_2	-	1.15E-05	-	3.06E-06	2.36E-06
		[3.84E-06]		[2.96E-06]	[2.80E-05]
b_1	-	-0.014	-	-0.007	-0.011
		[0.009]		[0.006]	[0.007]
b_2	-	0.006	-	0.002	0.004
		[0.005]		[0.003]	[0.004]
γ	-	-0.003	-	-0.001	-0.001
		[0.002]		[0.001]	[0.001]
α_0	-	-	8.71E-05	8.27E-05	5.28E-05
			[1.75E-05]	[1.45E-05]	[7.26E-05]
α_1	-	-	0.304	0.382	0.411
			[0.036]	[0.044]	[0.050]
β_1	-	-	0.696	0.651	0.584
			[0.026]	[0.027]	[0.033]
ϱ_1	-	-	-	-	2.35E-04
					[7.26E-05]
ϱ_2	-	-	-	-	9.38E-06
					[3.99E-05]
ϱ_3	-	-	-	-	2.92E-05
					[5.53E-05]
ϱ_4	-	-	-	-	4.08E-06
					[6.16E-06]
ϱ_5	-	-	-	-	1.48E-08
					[1.65E-08]
ϱ_6	-	-	-	-	-1.66E-04
					[1.67E-04]
ϱ_7	-	-	-	-	3.79E-04
					[1.69E-04]
ϱ_8	-	-	-	-	-2.25E-06
					[1.09E-05]
R-squared	0.909	0.917	0.907	0.909	0.909
Adj. R-squared	0.908	0.915	0.906	0.907	0.907
AIC	-3.286	-3.344	-3.661	-3.731	-3.731

The table reports the coefficient estimates (and their standard errors between square brackets), and selection criteria tests of the models 2-6 with Oslo data.

Table 3: Out-of-sample forecasting results

	Oslo		Eastern Denmark		The Netherlands	
Model	RMSPE	MAPE	RMSPE	MAPE	RMSPE	MAPE
RW	0.0602	0.0117	0.2160	0.0372	0.3054	0.0528
ARMA	0.0561	0.0116	0.1915	0.0333	0.2313	0.0400
ARMAX	0.0509	0.0105	0.1774	0.0309	0.2149	0.0377
ARMAXW	0.0490	0.0100	0.1705	0.0306	0.2120	0.0370
ARMAX-GARCH	0.0524	0.0105	0.1839	0.0313	0.2114	0.0368
ARMAXW-GARCH	0.0510	0.0101	0.1784	0.0307	0.2076	0.0358
ARMAXW-GARCHW	0.0496	0.0100	0.1764	0.0306	0.2110	0.0365

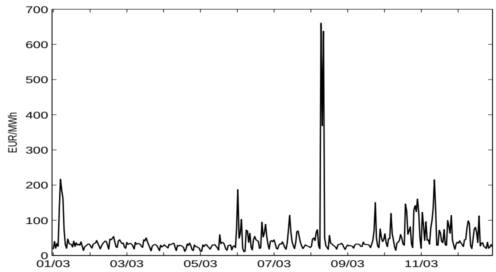
The table reports forecasting statistics of the alternative models in the three electricity markets.

Table 4: Out-of-sample accuracy comparisons

	Oslo	Eastern Denmark	The Netherlands
RW	-4.001***	-5.184***	-6.522***
ARMA	-4.872***	-5.182***	-3.130***
ARMAXW	2.816***	4.063***	1.554
ARMAX-GARCH	-1.915*	-2.197**	4.152***
ARMAXW-GARCH	-0.171	-0.402	3.986***
ARMAXW-GARCHW	2.075**	0.499	1.928*

The table reports Diebold-Mariano forecast accuracy comparison tests of the given models against those of the ARMAX model. The null hypothesis is that the two forecasts have the same mean square error. Positive values indicate superiority of the given models, one asterisk denotes significance relative to the asymptotic null hypothesis at 10%, two asterisks denote significance relative to the asymptotic null hypothesis at 5%, and three asterisks denote significance relative to the asymptotic null hypothesis at 1%.

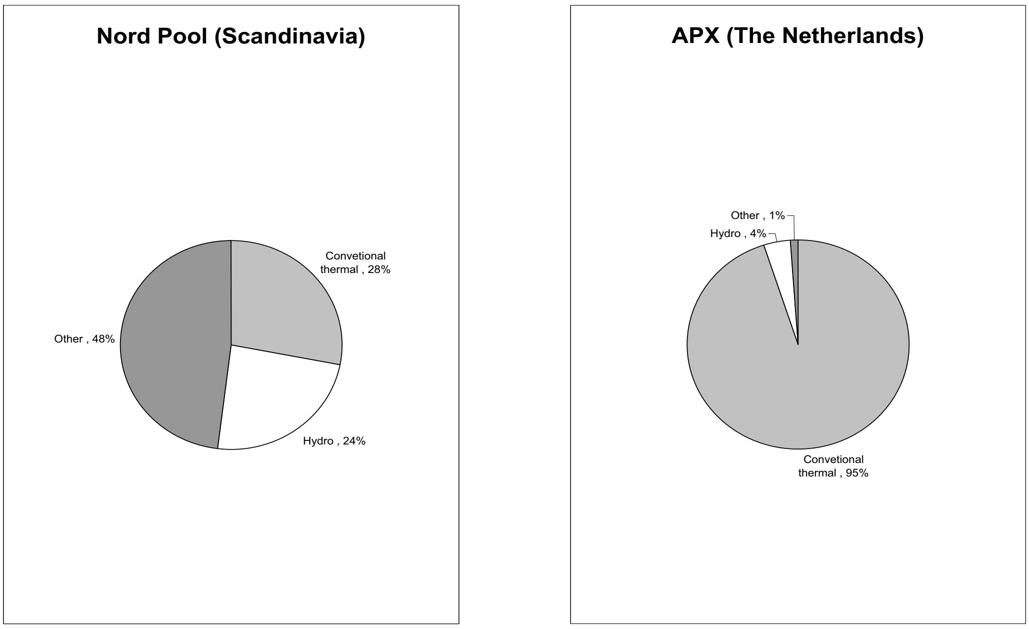
Figure 1: APX 2003 Prices



(a)

Note: The figure presents the daily electricity prices in the APX market over 2003.

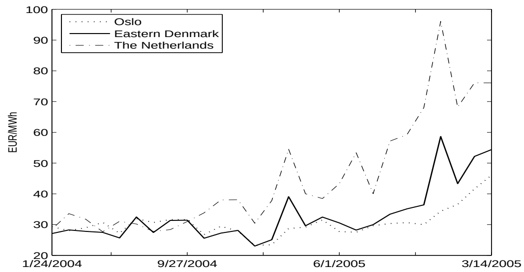
Figure 2: Producer plants



(a)

Note: The figure presents capacity figure of EU countries with most active wholesale power markets.
Conventional thermal fuelled capacity: oil, gas, coal.
Hydro fuelled capacity: reservoir, river.
Other fuelled capacity: wind, solar.

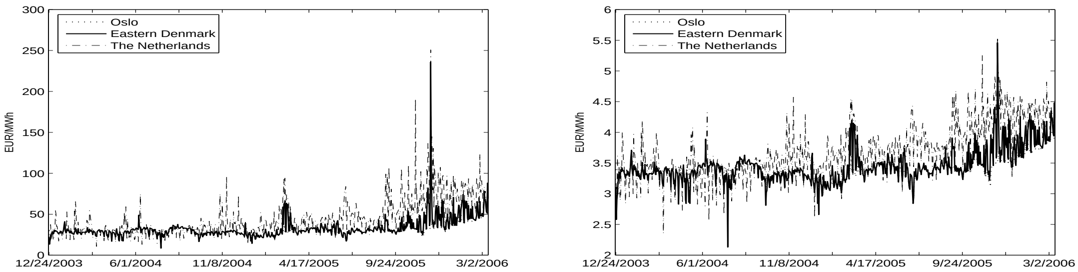
Figure 3: Monthly average prices



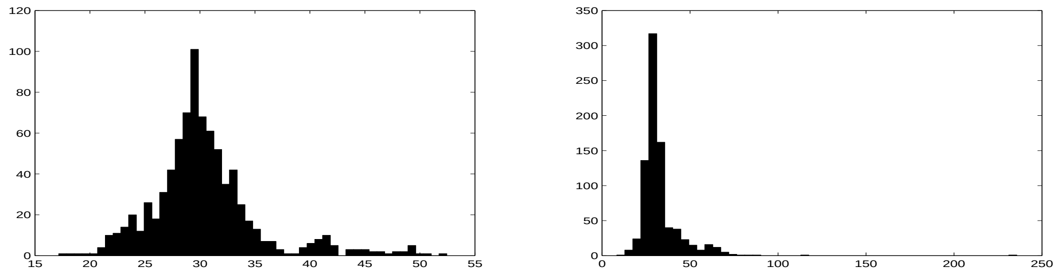
(a)

Note: The figure presents the monthly average electricity prices in Oslo, Eastern Denmark and the Netherlands.

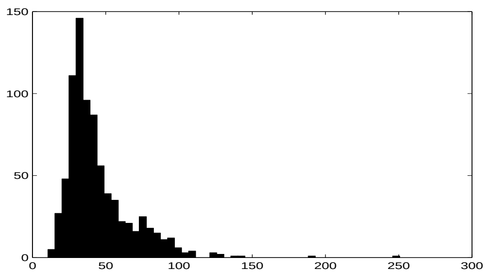
Figure 4: Prices



(a)



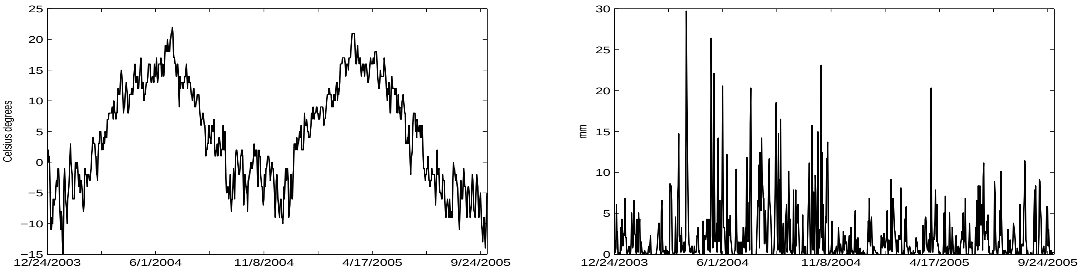
(b)



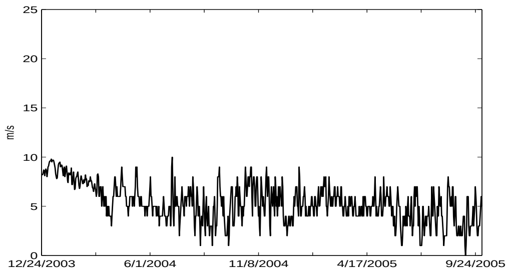
(c)

Note: The graphs in this figure present in Panel a) prices (in the left panel) and log prices (in the right panel) of daily electricity prices in Oslo, Eastern Denmark and the Netherlands; in Panel b) histograms of daily electricity prices in Oslo (in the left panel) and Eastern Denmark (in the right panel) markets; and in Panel c) histograms of daily electricity prices in the Netherlands market.

Figure 5: Weather variables: Oslo



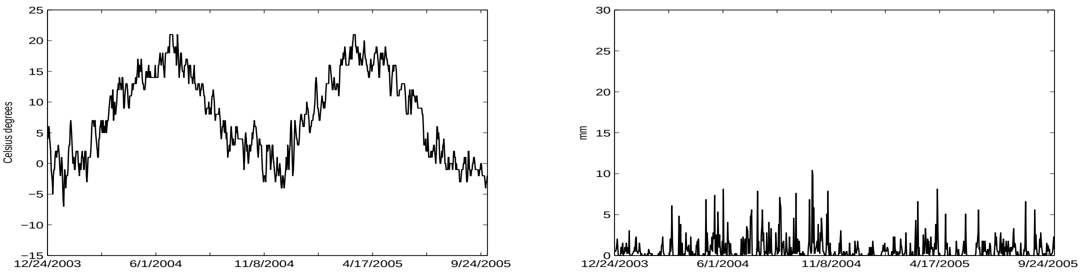
(a)



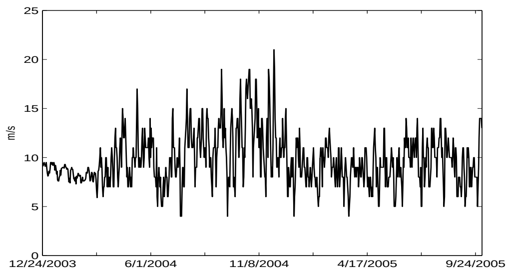
(b)

Note: The graphs in this figure present in Panel a) the forecasts on the daily average temperature (in the left panel), and total precipitation (in the right panel); in Panel b) the forecasts on wind speed in Oslo.

Figure 6: Weather variables: Eastern Denmark



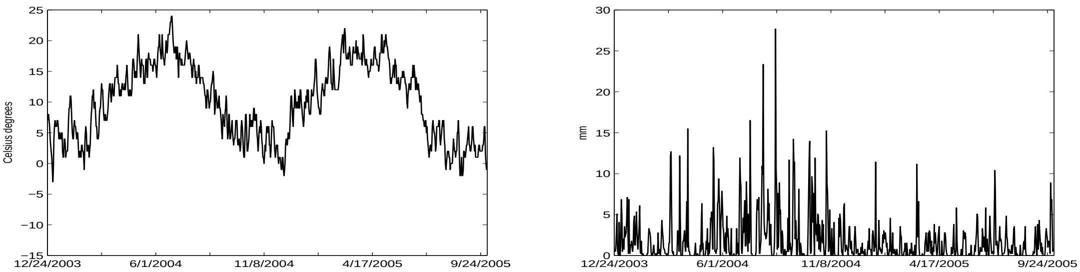
(a)



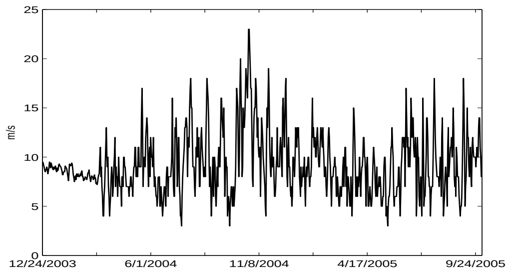
(b)

Note: The graphs in this figure present in Panel a) the forecasts on the daily average temperature (in the left panel), and total precipitation (in the right panel); in Panel b) the forecasts on wind speed in Copenhagen.

Figure 7: Weather variables: the Netherlands



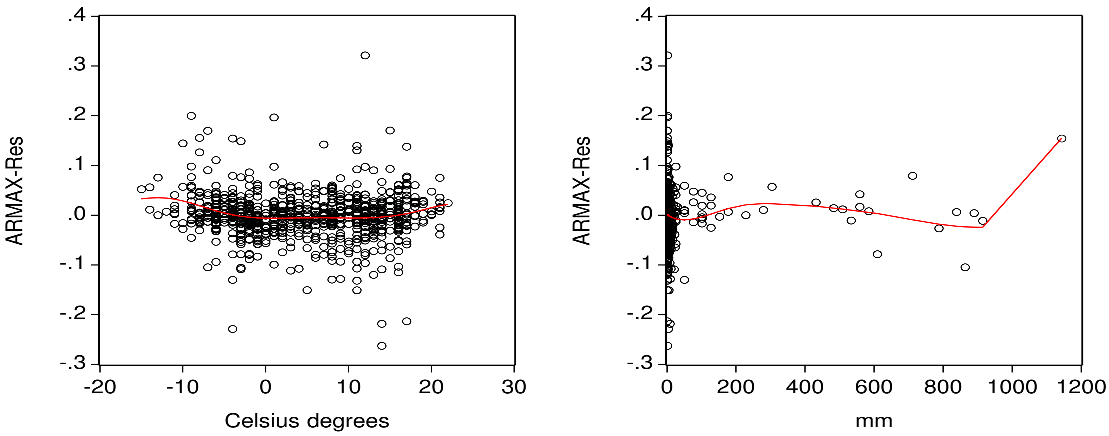
(a)



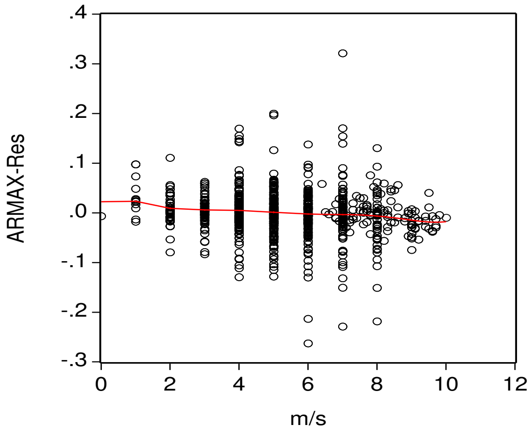
(b)

Note: The graphs in this figure present in Panel a) the forecasts on the daily average temperature (in the left panel), and total precipitation (in the right panel); in Panel b) the forecasts on wind speed in Amsterdam.

Figure 8: Scatter plot: Oslo



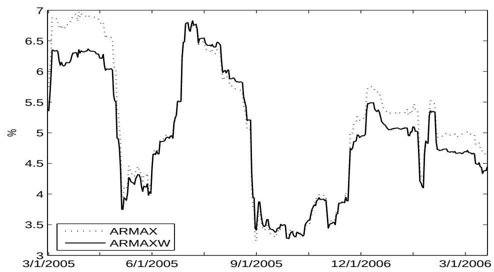
(a)



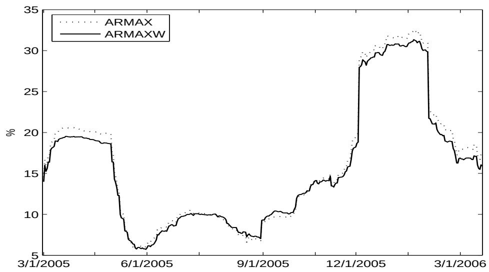
(b)

Note: The graphs in this figure present in Panel a) the scatter plot of the errors of ARMAX model against the daily average temperature (in the left panel), and total precipitation (in the right panel); in Panel b) the scatter plot of the errors of ARMAX model against the wind speed in Oslo. The trend line is also provided in each figure.

Figure 9: OSLO: 60 days average RMSPE



(a)



(b)

Note: The graphs in this figure present in Panel a) the 60 days moving average RMSPE given the ARMAX and ARMAXW models in forecasting Oslo log electricity prices; in Panel b) the 60 days moving average RMSPE given the ARMAX and ARMAXW models in forecasting Eastern Denmark log electricity prices.