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Testing for Stochastic Unit Roots
Some Monte Carlo evidence

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Abstract

This paper considers the properties of the tests of the null hypothesis of a random walk against the alternative of a stochastic unit root when the data are generated by a process related to, but not exactly equal to, the processes under the null and alternative. The extensive Monte Carlo experiments demonstrate that the test statistics are particularly sensitive to non-stationary RCAR processes with mean root less than unity, random walk processes with structural change in the variance, processes with changing persistence and trend-stationary processes with a break in the trend.

1 Introduction

This paper examines the properties of the tests for a Stochastic Unit Root [SUR] of McCabe and Tremayne (1995), Leybourne, McCabe and Treymane (1996) [LMT hereafter], and Leybourne, McCabe and Mills (1996) [LMM], when applied to time series which follow processes that do not correspond exactly to either the process under the null or alternative hypothesis. The analysis is based on an extensive set of Monte Carlo experiments. As data generating processes [DGPs] we consider general random coefficient autoregressive [RCA] processes, heteroskedastic processes, unit

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root processes with additive outliers [AOs], fractionally integrated or long-memory processes, threshold unit root [TUR] and changing persistence [CP] processes, and processes with structural breaks in the deterministic elements in the process. The first two of these are closely related to the SUR process, whereas the remaining processes have been put forward as alternatives to the standard unit root model, just like the SUR process. It is of interest to determine whether or not the tests which have been designed specifically against the SUR alternative are sensitive to other alternatives as well. We briefly discuss the SUR process and the tests against SUR in section 2. For more elaborate introductions to this topic we refer to LMT, LMM and Granger and Swanson (1997). In section 3 we describe the set-up of the Monte Carlo experiments, together with some discussion and interpretation of the results. Section 4 concludes the paper.

2 Stochastic Unit Roots: Model and Test Statistics

Consider the following first order random coefficient autoregressive [RCAR(1)] process,

$$y_t = \alpha_t y_{t-1} + \varepsilon_t, \qquad t = 1, \dots, T, \tag{1}$$

where $\alpha_t = \alpha + \delta_t$, $\delta_0 = 0$ and

$$\delta_t = \rho\delta_{t-1} + \eta_t, \qquad t = 1, \dots, T, \tag{2}$$

with $|\rho| \leq 1$. In (1), ε_t is i.i.d. $(0, \sigma^2)$, while η_t in (2) is i.i.d. $(0, \omega^2)$, independent of ε_t. If $\alpha = 1$ and $\omega^2 > 0$, y_t is an RCAR(1) process with mean unit root, which therefore is called a stochastic unit root process by LMT and Granger and Swanson (1997). If $\rho < 1$ in (2), α_t is a stationary AR(1) process with a mean of unity, while in case $\rho = 1$, α_t is a random walk. Also note that if $\omega^2 = 0$, $\alpha_t = 1$ for all t and, consequently, y_t follows a random walk.

Following LMM, we consider a generalization of (1) given by

$$y_t^* = \alpha_t y_{t-1}^* + \varepsilon_t, \qquad t = 1, \dots, T, \tag{3}$$

$$y_t^* = y_t - \lambda_t - \sum_{i=1}^{p} \phi_i y_{t-i}, \tag{4}$$

where α_t is as specified above and λ_t contains any deterministic terms such as constants and trends. The lag polynomial $\phi(L) \equiv 1 - \phi_1 L - \ldots - \phi_p L^p$ is assumed to have all roots outside the unit circle. Substituting (4) in (3) and using the fact that $\alpha_t = 1 + \delta_t$, we obtain after some manipulation

$$\Delta y_t - \Delta\lambda_t - \sum_{i=1}^{p} \phi_i \Delta y_{t-i} = \delta_t \left(y_{t-1} - \lambda_{t-1} - \sum_{i=1}^{p} \phi_i y_{t-1-i} \right) + \varepsilon_t, \tag{5}$$

where $\Delta \equiv (1 - L)$ is the first difference operator defined *via* the usual lag operator L: $L^k y_t = y_{t-k}$ for all integer k. If $\omega^2 = 0$, $\delta_t = 0$ for all t, and hence y_t is an ARIMA$(p, 1, 0)$ process.

LMM discuss four test statistics to test the null hypothesis $\mathcal{H}_0 : \omega^2 = 0$ in (5) with (2), depending on whether the alternative is (2) with $\rho < 1$ or $\rho = 1$ and the contents of the deterministic part λ_t in (5). Consider first the case where the null hypothesis is tested against (5) and (2) with $|\rho| < 1$, such that δ_t is a mean zero stationary process. Define

$$\hat{\varepsilon}_t = \Delta y_t - \Delta\hat{\lambda}_t - \sum_{i=1}^{p} \hat{\phi}_i \Delta y_{t-i}, \tag{6}$$

where a hat denotes the Ordinary Least Squares [OLS] estimates from the fitted model

$$\Delta y_t = \Delta\lambda_t + \sum_{i=1}^{p} \phi_i \Delta y_{t-i} + \varepsilon_t, \tag{7}$$

with $\lambda_t = \beta + \gamma t + \xi t(t+1)/2$ or $\lambda_t = \beta + \gamma t$. The test statistic, denoted Z_1 and Z_2 for these respective choices of λ_t, then is computed as

$$Z_. = \frac{1}{T^{3/2}\hat{\sigma}^2\hat{\kappa}} \sum_{t=1}^{T} \left(\sum_{j=1}^{t-1} \hat{\varepsilon}_j \right)^2 \left(\hat{\varepsilon}_t^2 - \hat{\sigma}^2 \right), \tag{8}$$

where $\hat{\sigma}^2 = T^{-1} \sum_{t=1}^{T} \hat{\varepsilon}_t^2$ and $\hat{\kappa}^2 = T^{-1} \sum_{t=1}^{T} \left(\hat{\varepsilon}_t^2 - \hat{\sigma}^2 \right)^2$.

In case the null hypothesis is to be tested against the alternative with $\rho = 1$ in (2), such that δ_t follows a random walk, define $\hat{\varepsilon}_t$ as above and denote the test statistics for $\lambda_t = \beta + \gamma t + \xi t(t+1)/2$ or $\lambda_t = \beta + \gamma t$ as E_1 and E_2, respectively. These are computed as

$$E_. = \frac{1}{T^3\hat{\sigma}^4} \sum_{i=1}^{T} \left\{ \left[\sum_{t=i}^{T} \hat{\varepsilon}_t \left(\sum_{j=1}^{t-1} \hat{\varepsilon}_j \right) \right]^2 - \hat{\sigma}^2 \sum_{t=i}^{T} \left(\sum_{j=1}^{t-1} \hat{\varepsilon}_j \right)^2 \right\}. \tag{9}$$

LMT and LMM derive the asymptotic distributions of the various statistics and provide finite sample critical values.

3 Monte Carlo Experiments

All experiments described below are based on 10000 replications, for sample sizes $T = 100, 250$ and 500, with the shocks ε_t taken to be i.i.d. $N(0, 1)$. All results presented and discussed here are based on a 5% nominal significance level. Results for other significance levels are qualitatively similar and are available upon request from the corresponding author.

When computing the SUR test statistics $Z.$ and/or $E.$, one has to decide upon an appropriate lag length p in (7). Various approaches can be used for this purpose, including information-based rules such as the familiar Schwarz and Akaike information criteria, and deterministic rules based solely on the sample size T. An increasingly popular approach is the 'general-to-specific' procedure advocated by Hall (1994) and Ng and Perron (1995) for use with Augmented Dickey Fuller [ADF] tests. Here one specifies an initial lag order $p_{\max}$ and then sequentially tests, using conventional t-statistics and a prespecified significance level, for the statistical significance of the highest order lag, using a backward elimination algorithm. The number of lagged first differences in (7) is then selected as the maximum lag length at which the algorithm terminates; this may be zero, although the algorithm may be programmed to terminate if the lag order attains some prespecified lower bound, say $p_{\min}$.

In order to mimic the situation of a practitioner as closely as possible, we do not assume that the correct lag length is known in the Monte Carlo experiments, but rather employ the general-to-specific approach described above to determine the lag length for each individual series. In the algorithm, we set $p_{\max} = 4$ for series of sample size $T = 100$ and $p_{\max} = 12$ for series of length $T = 250$ and 500. The minimum lag length $p_{\min}$ is set equal to zero throughout, while a 5% decision rule is used to assess the significance of the highest order lag. Results for experiments using other methods to determine the lag length and with a fixed number of lags are available upon request from the corresponding author, but are not qualitatively different from those reported below.

LMT and LMM derive the asymptotic distributions of the various statistics under the assumption that the distribution of y_t^*, as defined by (3) and (4), conditional on $\{\alpha_{s+1}, \varepsilon_s, s \leq t-1\}$ is Gaussian with mean $\alpha_t y_{t-1}^*$ and variance σ^2. These asymptotic distributions are shown to be independent of lag order p in the auxiliary regression (7). However, as noted by Cheung and Lai (1995) in the context of ADF tests, the finite sample distribution can be sensitive to the choice of lag order. Furthermore, the critical values may also depend on the manner in which this lag order is determined; cf. Taylor (1999). We therefore generate 50000 artificial series from a driftless random walk, i.e., (1) with $\alpha_t = 1$ for all t and $y_0 = 0$, for the appropriate sample sizes and calculate the test statistics $Z.$ and $E.$, after applying the Ng and Perron (1995) procedure with the same values of $p_{\max}$, $p_{\min}$ and decision rule as in the simulations. Critical values are then given by the empirical distribution functions for the different test statistics thereby obtained.

In the following we describe the different DGPs which are employed and report the results which we observe. In particular, we examine the behavior of the $Z.$ and $E.$ test statistics when applied to series generated according to a variety of stationary and non-stationary RCAR processes and heteroskedastic processes, which both are closely related to the SUR process. Furthermore, we consider random walk processes with additive outliers, long-memory processes, threshold unit root processes, changing persistence processes and processes with structural changes in the deterministic part. These DGP's have in common that all of them have been put forward or might be considered as alternatives to the standard unit root model.

3.1 Random coefficient autoregressive processes

We consider the first-order RCAR process as given in (1) and (2) with $\rho = 0$. We set $\alpha = 1, .99, .95, .9$ and $.8$ and $\omega^2 = 0, .01$ and $.10$. Throughout, δ_0, y_0 and λ_t are all fixed at zero, while $\sigma^2 = 1$. The parameterizations with $\alpha = 1$ serve to give an impression of the genuine power of the SUR test statistics. General RCAR processes are discussed extensively in Nicholls and Quinn (1982). A necessary and sufficient condition for stationarity of the RCAR(1) model is $\alpha^2 + \omega^2 < 1$. This condition is obviously violated in case $\alpha = 1$, but also for the combinations of $\alpha = 0.99$ and 0.95

with $\omega^2 = 0.10$.

- insert Table 1 about here -

Results for the RCAR processes are presented in Table 1. Some interesting features emerge. When $\alpha = 1$, all four statistics display increasing power both as ω^2 increases and as T increases, as might be hoped. It is noticeable that the E_1 statistic appears to dominate the E_2 test on power, while contrastingly the Z_1 test is dominated by the Z_2 test. In all but one case the $E.$ tests are dominated by the $Z.$ tests, as should be expected given the DGP. Turning to the cases where $\alpha < 1$, we see that the behaviour of the tests is largely dependent on the quantity $\alpha^2 + \omega^2$. Where this exceeds unity, all of the test statistics deviate above nominal size, and increasingly so as T increases, although, other things being equal, these deviations decrease as α decreases. In cases where $\alpha + \omega^2 < 1$, such that the process is stationary, McCabe and Smith (1998) demonstrate that the $Z.$ statistics will converge in probability to zero. This is confirmed by the results in Table 1, and also appears to be true of the E_2 statistic, but does not seem to hold for the E_1 statistic. For example, in cases where $\alpha < 1$ and $\omega^2 = 0$ we see that the E_1 test actually deviates further above nominal size as α decreases. This is clearly a highly undesirable property of the E_1 test, not shared by the other tests.

3.2 Heteroskedastic random walk processes

As discussed in LMT, the SUR process can also be interpreted as an heteroskedastic random walk process (see also Hansen, 1992). This follows directly by combining (1) and (2) with $\rho = 0$ into $y_t = y_{t-1} + (\eta_t y_{t-1} + \varepsilon_t)$, which shows that y_t is a random walk process with conditional variance $\omega^2 y_{t-1}^2 + \sigma^2$. Because the conditional variance of y_t is directly related to the level of y_{t-1}, LMT examine the empirical size of the Z_1 and Z_2 statistics for (I)GARCH processes and conclude that the rejection frequencies of the tests are not particularly high. Here we consider two alternative types of *unconditional* heteroskedasticity. We generate artificial time series according to the random walk process $y_t = y_{t-1} + v_t$ with $y_0 = 0$ and $v_t = \sigma_t \varepsilon_t$. In the first case, v_t

exhibits periodic heteroskedasticity, by setting

$$\sigma_t^2 = E(v_t^2) = \begin{cases} \sigma_1^2 & \text{for } t \text{ odd,} \\ \sigma_2^2 & \text{for } t \text{ even.} \end{cases} \tag{10}$$

Without loss of generality, we set σ_1 equal to 1 and take σ_2 equal to 2 or 10. In the second case, we assume that a structural change in the variance occurs at a certain point in the sample. To be precise, we set

$$\sigma_t^2 = E(v_t^2) = \begin{cases} \sigma_1^2 & \text{for } t \leq \lfloor \tau T \rfloor, \\ \sigma_2^2 & \text{for } t > \lfloor \tau T \rfloor, \end{cases} \tag{11}$$

where $0 \leq \tau < 1$ and $\lfloor \cdot \rfloor$ denotes integer part. We vary τ among .25, .5 and .75 and vary both σ_1 and σ_2 among 1, 2 and 10.

Results for the bi-period heteroskedastic random walk process are set out in Table 2. The rejection frequencies correspond very closely with the nominal significance level, the only exception being the E_1 statistic at sample size $T = 100$. In fact these results follow almost directly from the theoretical work in Burridge and Taylor (1999) who demonstrate that the ADF test for a zero frequency unit root is unaffected by periodic heteroscedasticity. The key point here is that the four statistics remain asymptotically pivotal. The simulation results therefore confirm this (large sample) property of the SUR tests, which seems also to hold even for fairly moderate sample sizes.

- insert Tables 2 and 3 about here -

Table 3 shows results for the one-time change in the variance of the process. Here it is easily seen that none of the four statistics are asymptotically pivotal, in each case depending on the nuisance parameters σ_1/σ_2 and τ. Rejection frequencies of the E_1 and E_2 statistics in this case increase dramatically when the variance increases, whereas they collapse to zero when the variance decreases. For the Z_1 and Z_2 tests, rejection frequencies increase above the nominal significance level both in case of a variance reduction and in case of variance increase, although the effect is more pronounced in the latter case. The effect of the break in variance on the tests is also strongly effected by the timing of the break, as should be expected. This again mirrors the pattern for the ADF test under (11), as discussed in Hamori and Tokihisa (1997).

3.3 Seasonal Random Walk

In this case we consider the effects of applying the tests to data which are generated as seasonal random walk processes; *viz*,

$$y_t = y_{t-4} + \varepsilon_t. \tag{12}$$

- insert Table 4 about here -

Results for this DGP are given in Table 4. No problems arise in this case - the rejection frequencies of all tests are virtually identical to the nominal significance level. Again this tallies with existing work on the ADF test for a zero frequency unit root; see Ghysels *et al.* (1994).

3.4 Random Walk with Additive Outliers

Suppose now that we observe the process y_t, which is equal to a random walk process x_t, i.e. $x_t = x_{t-1} + \varepsilon_t$, plus an additive outlier [AO] process,

$$y_t = x_t + \zeta z_t, \tag{13}$$

where $\{z_t\}$ is a stochastic contamination process, which takes non-zero values with positive probability, and where $\zeta > 0$ is a non-zero constant indicating the magnitude of the AOs. Here we only consider isolated AOs, which occur when the contamination process z_t in (13) is characterized as $P(z_t = 0) = 1 - \pi$, $P(z_t = 1) = P(z_t = -1) = \pi/2$, with $0 < \pi < 1$. We vary the absolute magnitude of the outliers ζ among $\{3, 5, 7\}$ and the probability of occurrence π among $\{.01, .05\}$.

Hoek *et al.* (1995) and Franses and Haldrup (1994) consider the effects of AOs on standard tests for unit roots. In particular, they demonstrate that in the presence of AOs, the Dickey-Fuller t-test for a unit root overrejects the null hypothesis when it is in fact true. This effect is due to the non-robustness of the OLS estimator which is used in constructing the test statistic. It might be expected that the tests against SUR suffer from this problem as well.

- insert Table 5 about here -

Results for the random walk contaminated with AOs are presented in Table 5. It is seen that all tests are biased toward the alternative if AOs are present. In virtually all cases, however, the size distortions are seen to decrease as the sample size grows larger. With few exceptions, the size distortion becomes larger as the magnitude of the outliers ζ increases. Increasing the frequency π of outliers is seen to have a rather different effect. For small outliers ($\zeta = 3$), the empirical size increases as π is increased from 0.01 to 0.05, while for larger outliers ($\zeta = 5, 7$), the empirical size *de*creases as π becomes larger.

3.5 Long memory

We now consider the case where y_t is generated according to the ARFIMA(0,d,0) or fractionally differenced noise process introduced by Granger and Joyeux (1980) and Hosking (1981),

$$(1 - L)^d y_t = \varepsilon_t, \tag{14}$$

where d is a real number. For non-integer values of d, the fractional differencing filter $(1 - L)^d$ can be expanded as

$$(1 - L)^d = 1 - dL - \frac{d(1-d)L^2}{2!} - \frac{d(1-d)(2-d)L^3}{3!} - \dots . \tag{15}$$

For $d \in (-.5, .5)$, the process y_t is both stationary and invertible. When d is positive, the process exhibits long-memory characteristics, in the sense that the spectral density at frequency zero is infinite and the autocorrelations of y_t are not absolutely summable. In our simulations, we vary the parameter d as $d \in \{.1, .25, .4, .6, .75, .9\}$, i.e. we consider a range of values which imply both stationary and non-stationary processes. An approximate AR-method is used to generate the series (i.e. we use the AR(∞) representation of the ARFIMA(0,d,0) process, truncated after $p = 200$ lags).

- insert Table 6 about here -

Table 6 displays the results for fractionally integrated series. It is seen that the E_1 statistic is quite sensitive to long memory, in the sense that the rejection frequencies are larger than the nominal significance level, although the rejection frequency diminishes as d increases and disappears as d approaches 1 (which makes sense because for $d = 1$ the process satisfies the null hypothesis). By contrast, the other tests *under*-reject for all values of d considered here. In the case where $d < 0.5$ the results in McCabe and Smith (1998) again apply (since the process is stationary in this case), telling us that the $Z.$ statistics will converge in probability to zero. This also appears to be true of the E_2 statistic, just as in the RCAR case, but again not of the E_1 statistic. Notice that, other things being equal, as d decreases the E_1 test distorts further above nominal size. Again this mirrors the behaviour of this statistic under stationary environments seen in Table 1.

3.6 Threshold unit root processes

We consider two different Self-Exciting Threshold Autoregressive [SETAR] models, where the AR model in one of the regimes contains a unit root, while the AR models in the other regimes do not. Of course, such models are similar to the SUR model, except that here the dominating root of the process fluctuates between lying on and outside the unit circle. In the SUR case the root lies on the unit circle only on average; for any particular realisation at any particular time it will either lie inside or outside the unit circle, with equal probability.

First, we adopt the Monte Carlo setup of Caner and Hansen (1998), who study testing for a unit root in the presence of thresholds (see also González and Gonzalo, 1997, Enders and Granger, 1998, and Berben and van Dijk, 1999) and the reverse problem of testing for threshold effects in the presence of unit roots. Series are generated according to the Threshold Unit Root [TUR] model

$$\Delta y_t = (\mu_1 + \phi_1 y_{t-1}) 1_{\{\Delta y_{t-1} < \gamma\}} + (\mu_2 + \phi_2 y_{t-1}) 1_{\{\Delta y_{t-1} \geq \gamma\}} + \varepsilon_t, \qquad (16)$$

where $1_{\{A\}}$ is the indicator function for the event A. Following Caner and Hansen (1997), we set $\phi_1 = 0$ and let ϕ_2 vary among $\{-.05, -.15, -.25\}$. The influence of the intercepts is examined by setting $\mu_1 = -\mu_2$ and varying $\Delta\mu \equiv \mu_1 - \mu_2$ among $\{0, .4, 1, 2\}$. The threshold γ is set equal to zero throughout.

The second process we consider is a so-called BAND-TAR process, see Balke and Fomby (1997). The series y_t follows a random walk as long as it is within a band (γ_1, γ_2), and a mean-reverting AR(1) when it moves outside. The mean of the stationary AR(1) processes in the outer regimes is such that y_t reverts to the closest edge of the band. The basic BAND-TAR model is given by

$$y_t = \begin{cases} \gamma_1(1-\rho_1) + \rho_1 y_{t-1} + \varepsilon_t & \text{if } y_{t-1} < \gamma_1, \\ y_{t-1} + \varepsilon_t & \text{if } \gamma_1 \leq y_{t-1} < \gamma_2, \\ \gamma_2(1-\rho_2) + \rho_2 y_{t-1} + \varepsilon_t & \text{if } y_{t-1} \geq \gamma_2, \end{cases} \tag{17}$$

or

$$\Delta y_t = \phi_1(y_{t-1} - \gamma_1)1(y_{t-1} < \gamma_1) + \phi_2(y_{t-1} - \gamma_2)1(y_{t-1} \geq \gamma_2) + \varepsilon_t, \tag{18}$$

where $\phi_i = \rho_i - 1, i = 1, 2$. In the simulations, we set $-\gamma_1 = \gamma_2 \equiv \gamma$, and take $\gamma = 1, 3, 5$. The AR-parameters ϕ_1 and ϕ_2 are varied among $\{-.1, -.3, -.5\}$.

- insert Tables 7 and 8 about here -

Results for series generated according to the the TUR model (16) are set out in Table 7. The E_1 statistic slightly overrejects, especially if the autoregressive parameter in the upper regime ϕ_2 becomes smaller. The remaining test statistics are conservative, except the Z_1 statistics for the smaller sample sizes $T = 100$ and 250. Notice that the rejection frequencies tend to go down as the absolute magnitude of ϕ_2 and the length of the time series T increases. Similar conclusions are drawn for BAND-TAR model from the results in Table 8. Notice that the E_1 test displays some deviation above nominal size when γ is small; that is, where more frequent regime shifts occur. In general though, the SUR statistics appear to display little or no power against threshold unit root processes.

3.7 Changing persistence processes

Kim (1997) studies the problem of detecting structural change characterized by a shift in persistence of a time series. In particular, he develops a test to detect whether a process shifts from stationarity to nonstationarity or *vice versa* at a particular point in time.

Processes with changing persistence [CP] are similar to the TUR process discussed above, the main difference being that the threshold variable Δy_{t-1} in (16) is replaced by time t; in other words, there is a single break in the dominant root of the process which occurs at a fixed point and with fixed magnitude. The basic CP process is given by

$$\Delta y_t = (\mu_1 + \phi_1 y_{t-1})1_{\{t<\lfloor \tau T \rfloor\}} + (\mu_2 + \phi_2 y_{t-1})1_{\{t \geq \lfloor \tau T \rfloor\}} + \varepsilon_t, \tag{19}$$

with $0 < \tau < 1$ and $\lfloor \cdot \rfloor$ denotes integer part as before. The fraction of the sample at which the break occurs is set equal to $\tau = .25, .5, .75$. The remaining parameters are varied as in the TUR experiments described above, i.e., we set $\phi_1 = 0$ and let ϕ_2 vary among $\{-.05, -.15, -.25\}$. Furthermore, we set $\mu_1 = 0$ and $\mu_2 = 0.5$. We experimented with other values of μ_2 but this did not alter the results; these results are available on request. The reason for restricting μ_1 to be equal to zero is that now the process changes from a drift-less random walk to a stationary process, with possible nonzero mean. If μ_1 were not equal to zero, the random walk in the first part of the sample would contain a drift as well, which would make the processes before and after the change point $t = \lfloor \tau T \rfloor$ too different from each other to resemble real-world time series.

- insert Table 9 about here -

Results for the CP process are shown in Table 9. In most cases we see that the rejection rates of the SUR statistics exceed the nominal level under changing persistence. The statistics do not appear to diverge, however; as T increases, for fixed values of τ and ϕ_2, the SUR statistics, with the exception of E_1, appear to stabilize. Concerning the influence of τ, the timing of the change, all tests appear to show a different pattern of response. For the E_1 and Z_2 statistics, the rejection frequencies decrease and increase, respectively, as the break occurs at a later point in the sample. For the E_2 and Z_1 statistics, the rejection frequencies show a hump-shaped pattern, such that rejection is smallest and largest, respectively, when the break occurs in the middle of the sample. Regarding ϕ_2, all of tests display increasingly greater rejection rates as ϕ_2 decreases, ceteris parabus. Banerjee *et al.* (1992) found that the ADF test applied to CP processes displayed similar properties to those noted above for the SUR tests.

3.8 Structural change in deterministic model part

Finally we consider linear AR processes which exhibit structural change in the deterministic part of the model at a certain point in the sample. Starting with Perron (1989), breaks in trends have been considered as an alternative to the random walk and a large literature on testing for unit roots in the presence of such trend-breaks has developed since. The basic structural change process is given by,

$$y_t = (\mu_1 + \xi_1(t - \lfloor \tau T \rfloor))1_{\{t<\lfloor \tau T \rfloor\}} + (\mu_2 + \xi_2(t - \lfloor \tau T \rfloor))1_{\{t\geq\lfloor \tau T \rfloor\}} + \rho y_{t-1} + \varepsilon_t. \quad (20)$$

We adopt part of the Monte Carlo set-up of Vogelsang (1997). In particular, we consider processes with a break in mean, by setting $\xi_1 = \xi_2 = 0$ and processes with a break in the slope of the linear deterministic trend, by setting $\mu_1 = \mu_2 = 0$. For the break-in-mean processes, we set $\mu_1 = 0$ (without loss of generality - the test only depends on the difference in means before and after the change-point and is invariant to the absolute magnitude of μ_1) and varying μ_2 among $\{1, 3, 5\}$. For the break-in-slope processes, we set $\xi_1 = 0$ (again without loss of generality) and vary ξ_2 among $\{.3, .9, 1.5\}$. As before, the fraction of the sample at which the break occurs is set equal to $\tau = .25, .5, .75$, while the AR parameter is set equal to $\rho = 0, 0.5$ and 1. Notice that for $\rho = 1$ the interpretation of the parameters $\mu_j, j = 1, 2$ is different than for $\rho < 1$. In all cases the SUR statistics, like the ADF statistic, will not be asymptotically pivotal, depending upon these nuisance parameters.

- insert Tables 10 and 11 about here -

Tables 10 and 11 display Monte Carlo results for the experiments involving processes with a structural break in the deterministic parts and show how these nuisance parameters affect the empirical size of the SUR statistics. In case of a change in the mean/drift only ($\xi_1 = \xi_2 = 0$, $\mu_1 \neq \mu_2$), the rejection frequencies of the E_1 statistic become quite large, especially for the smallest sample size $T = 100$. The other statistics are not affected or at least to a much smaller extent, see Table 10. In case of a change in the slope of the linear trend ($\mu_1 = \mu_2 = 0$, $\xi_1 \neq \xi_2$), the situation is reversed: the E_1 statistic now performs (much) better than the Z_1, E_2 and Z_2 statistics. For the Z_1 and Z_2 statistics, the rejection frequencies increase monotonically as the magnitude of the break or the sample size increases. The behavior of the E_2 statistic is rather peculiar: sometimes rejection frequencies *decrease* as ξ_1 increases, sometimes they show a hump-shaped pattern. Just as in Table 3, the timing of the break, τ, can have a profound impact on the patterns of size distortions seen in both Tables 10 and 11. Moreover, the effect of the timing of the break does not seem to be independent of ρ and ξ_2, as might be expected.

4 Conclusions

The main conclusion from our Monte Carlo exercise is fairly obvious: the tests against the SUR alternative have power against certain other deviations from the random walk null hypothesis as well. In particular, rejection rates against non-stationary RCAR processes with mean root less than unity, random walk processes with structural change in the variance, processes with changing persistence and trend-stationary processes with a break in the trend are rather high. Although the E_2 and Z_j, $j = 1, 2$ statistics all degenerate below nominal size when the process under test is stationary, the E_1 statistic does not and would thus appear to be of limited practical use. When the SUR tests are applied to empirical time series, rejection of the null hypothesis, is not fully informative concerning the appropriate alternative. Additional tests will have to be performed to distinguish between the various possibilities. This may be especially important if the SUR tests are being used as pre-tests in a stochastic co-integration exercise as in Harris *et al.* (1999).

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Table 1: Rejection frequencies of SUR tests in case of random coefficient process[1]

		$T = 100$				$T = 250$				$T = 500$			
α	ω^2	E_1	E_2	Z_1	Z_2	E_1	E_2	Z_1	Z_2	E_1	E_2	Z_1	Z_2
1.00	.00	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
	.01	0.26	0.09	0.21	0.24	0.40	0.14	0.53	0.49	0.49	0.20	0.80	0.68
	.10	0.46	0.23	0.73	0.61	0.52	0.25	0.93	0.81	0.57	0.31	0.98	0.92
0.99	.00	0.05	0.05	0.05	0.05	0.05	0.03	0.04	0.03	0.05	0.01	0.03	0.02
	.01	0.16	0.05	0.18	0.22	0.20	0.04	0.44	0.40	0.23	0.03	0.70	0.57
	.10	0.42	0.20	0.71	0.59	0.48	0.21	0.92	0.79	0.54	0.27	0.98	0.90
0.95	.00	0.06	0.01	0.04	0.02	0.06	0.00	0.01	0.00	0.08	0.00	0.00	0.00
	.01	0.07	0.01	0.11	0.09	0.07	0.00	0.13	0.06	0.08	0.00	0.12	0.03
	.10	0.29	0.09	0.61	0.46	0.35	0.09	0.84	0.62	0.44	0.11	0.95	0.72
0.90	.00	0.07	0.00	0.02	0.01	0.08	0.00	0.00	0.00	0.11	0.00	0.00	0.00
	.01	0.07	0.00	0.05	0.02	0.08	0.00	0.03	0.01	0.11	0.00	0.01	0.00
	.10	0.20	0.04	0.45	0.29	0.25	0.02	0.66	0.37	0.33	0.03	0.79	0.39
0.80	.00	0.09	0.00	0.01	0.00	0.09	0.00	0.00	0.00	0.13	0.00	0.00	0.00
	.01	0.09	0.00	0.02	0.00	0.09	0.00	0.01	0.00	0.13	0.00	0.00	0.00
	.10	0.14	0.01	0.21	0.09	0.17	0.00	0.28	0.08	0.23	0.00	0.28	0.06

[1] Rejection frequencies at 5% significance level using finite sample critical values for series generated by the random coefficient process given in (1) and (2), with $\varepsilon_t \sim$ i.i.d. $N(0,1)$, $\rho = 0$ and $\eta_t \sim$ i.i.d.$N(0,\omega^2)$. The table is based on 10000 replications.

Table 2: Rejection frequencies of SUR tests in case of random walk with periodic variance[1]

		Test			
	σ_2	E_1	E_2	Z_1	Z_2
$T = 100$					
	2	0.07	0.05	0.05	0.05
	10	0.09	0.05	0.05	0.05
$T = 250$					
	2	0.05	0.05	0.05	0.05
	10	0.05	0.05	0.05	0.05
$T = 500$					
	2	0.05	0.05	0.05	0.05
	10	0.05	0.05	0.05	0.05

[1] Rejection frequencies at 5% significance level using finite sample critical values for series generated by the random walk process with periodic variance as given in (10) with $\sigma_1 = 1$ and $\varepsilon_t \sim$ i.i.d.$N(0,1)$. The table is based on 10000 replications.

Table 3: Rejection frequencies of SUR tests in case of random walk with structural change in variance[1]

			$T=100$				$T=250$				$T=500$			
τ	σ_1	σ_2	E_1	E_2	Z_1	Z_2	E_1	E_2	Z_1	Z_2	E_1	E_2	Z_1	Z_2
.25	1	2	0.51	0.09	0.12	0.15	0.67	0.09	0.18	0.24	0.80	0.09	0.27	0.36
	2	1	0.00	0.04	0.20	0.09	0.00	0.03	0.32	0.15	0.00	0.04	0.47	0.25
	1	10	0.78	0.18	0.18	0.21	0.87	0.17	0.30	0.34	0.93	0.22	0.47	0.52
	10	1	0.00	0.03	0.68	0.36	0.00	0.02	0.85	0.53	0.00	0.03	0.94	0.62
.50	1	2	0.85	0.37	0.21	0.23	0.91	0.37	0.33	0.34	0.94	0.41	0.47	0.42
	2	1	0.00	0.03	0.20	0.19	0.00	0.03	0.34	0.31	0.00	0.03	0.49	0.41
	1	10	1.00	0.84	0.46	0.42	1.00	0.85	0.73	0.57	1.00	0.88	0.88	0.68
	10	1	0.00	0.03	0.47	0.38	0.00	0.02	0.79	0.55	0.00	0.03	0.90	0.66
.75	1	2	0.79	0.32	0.28	0.17	0.89	0.29	0.41	0.24	0.93	0.31	0.51	0.30
	2	1	0.00	0.04	0.11	0.16	0.00	0.04	0.19	0.27	0.00	0.04	0.29	0.38
	1	10	1.00	1.00	0.79	0.50	1.00	1.00	0.92	0.61	1.00	1.00	0.96	0.67
	10	1	0.00	0.04	0.19	0.24	0.00	0.04	0.36	0.42	0.00	0.04	0.50	0.55

[1] Rejection frequencies at 5% significance level using finite sample critical values for series generated by the random walk process with structural change in variance as given in (11) with $\varepsilon_t \sim$ i.i.d.$N(0,1)$. The table is based on 10000 replications.

Table 4: Rejection frequencies of SUR tests in case of a seasonal random walk[1]

	Test			
T	E_1	E_2	Z_1	Z_2
100	0.04	0.05	0.05	0.05
250	0.05	0.05	0.05	0.05
500	0.05	0.05	0.05	0.05

[1] Rejection frequencies at 5% significance level using finite sample critical values for series generated by the seasonal random walk process (12) with $\varepsilon_t \sim$ i.i.d.$N(0,1)$. The table is based on 10000 replications.

Table 5: Rejection frequencies of SUR tests in case of additive outliers[1]

	π	ζ	Test E_1	E_2	Z_1	Z_2
$T = 100$						
	.01	3	0.11	0.05	0.08	0.06
		5	0.21	0.09	0.12	0.09
		7	0.26	0.14	0.14	0.09
	.05	3	0.16	0.05	0.10	0.07
		5	0.24	0.09	0.12	0.08
		7	0.29	0.11	0.11	0.07
$T = 250$						
	.01	3	0.10	0.05	0.08	0.07
		5	0.21	0.07	0.14	0.10
		7	0.29	0.10	0.17	0.11
	.05	3	0.12	0.05	0.08	0.07
		5	0.18	0.06	0.09	0.07
		7	0.21	0.06	0.08	0.07
$T = 500$						
	.01	3	0.08	0.05	0.08	0.07
		5	0.16	0.05	0.13	0.10
		7	0.23	0.06	0.15	0.11
	.05	3	0.10	0.05	0.08	0.07
		5	0.14	0.05	0.08	0.08
		7	0.17	0.05	0.08	0.07

[1] Rejection frequencies at 5% significance level using finite sample critical values for series generated by the random walk process with additive outliers (13) with $\varepsilon_t \sim$ i.i.d.$N(0,1)$. The table is based on 10000 replications.

Table 6: Rejection frequencies of SUR tests in case of long memory[1]

	d	Test E_1	E_2	Z_1	Z_2
$T = 100$					
	.1	0.11	0.00	0.00	0.00
	.25	0.10	0.00	0.00	0.00
	.4	0.09	0.00	0.01	0.00
	.6	0.08	0.00	0.01	0.01
	.75	0.07	0.01	0.02	0.01
	.9	0.06	0.02	0.03	0.03
$T = 250$					
	.1	0.10	0.00	0.00	0.00
	.25	0.09	0.00	0.00	0.00
	.4	0.08	0.00	0.00	0.00
	.6	0.07	0.00	0.01	0.00
	.75	0.06	0.01	0.02	0.01
	.9	0.06	0.02	0.03	0.03
$T = 500$					
	.1	0.13	0.00	0.00	0.00
	.25	0.11	0.00	0.00	0.00
	.4	0.09	0.00	0.00	0.00
	.6	0.07	0.00	0.00	0.00
	.75	0.06	0.00	0.01	0.01
	.9	0.05	0.01	0.02	0.03

[1] Rejection frequencies at 5% significance level using finite sample critical values for series generated by the fractional noise process (14) with $\varepsilon_t \sim$ i.i.d.$N(0,1)$. The table is based on 10000 replications.

Table 7: Rejection frequencies of SUR tests in case of threshold unit roots[1]

		$T = 100$				$T = 250$				$T = 500$			
ϕ_2	$\Delta\mu$	E_1	E_2	Z_1	Z_2	E_1	E_2	Z_1	Z_2	E_1	E_2	Z_1	Z_2
-.05	0	0.05	0.04	0.05	0.04	0.05	0.01	0.04	0.02	0.05	0.00	0.02	0.01
	0.4	0.05	0.03	0.04	0.04	0.05	0.02	0.04	0.02	0.05	0.00	0.02	0.01
	1.0	0.05	0.04	0.05	0.05	0.05	0.02	0.05	0.03	0.05	0.00	0.03	0.01
	2.0	0.05	0.05	0.06	0.05	0.05	0.03	0.05	0.04	0.05	0.01	0.03	0.02
-.15	0	0.06	0.01	0.05	0.03	0.07	0.00	0.04	0.01	0.08	0.00	0.01	0.00
	0.4	0.06	0.01	0.04	0.02	0.06	0.00	0.03	0.01	0.08	0.00	0.01	0.00
	1.0	0.05	0.01	0.04	0.03	0.06	0.00	0.03	0.01	0.07	0.00	0.01	0.00
	2.0	0.05	0.02	0.05	0.03	0.06	0.00	0.03	0.01	0.07	0.00	0.00	0.00
-.25	0	0.07	0.00	0.06	0.02	0.08	0.00	0.04	0.01	0.11	0.00	0.02	0.00
	0.4	0.07	0.00	0.04	0.01	0.07	0.00	0.03	0.00	0.10	0.00	0.01	0.00
	1.0	0.06	0.00	0.04	0.02	0.07	0.00	0.02	0.00	0.09	0.00	0.00	0.00
	2.0	0.05	0.01	0.04	0.02	0.07	0.00	0.02	0.00	0.09	0.00	0.00	0.00

[1] Rejection frequencies at 5% significance level using finite sample critical values for series generated by the threshold unit root process (16) with $\phi_1 = 0$, $\mu_1 = -\mu_2$ and $\Delta\mu \equiv \mu_1 - \mu_2$, and $\varepsilon_t \sim$ i.i.d.$N(0,1)$. The table is based on 10000 replications.

Table 8: Rejection frequencies of SUR tests in case of BAND-TAR[1]

γ	ϕ_1	ϕ_2	$T=100$				$T=250$				$T=500$			
			E_1	E_2	Z_1	Z_2	E_1	E_2	Z_1	Z_2	E_1	E_2	Z_1	Z_2
1	$-.1$	$-.1$	0.06	0.01	0.03	0.02	0.06	0.00	0.02	0.00	0.08	0.00	0.00	0.00
		$-.3$	0.07	0.00	0.03	0.01	0.07	0.00	0.01	0.00	0.09	0.00	0.00	0.00
		$-.5$	0.07	0.00	0.03	0.01	0.08	0.00	0.01	0.00	0.10	0.00	0.00	0.00
	$-.3$	$-.3$	0.08	0.00	0.02	0.00	0.08	0.00	0.01	0.00	0.11	0.00	0.00	0.00
		$-.5$	0.09	0.00	0.02	0.00	0.09	0.00	0.00	0.00	0.12	0.00	0.00	0.00
	$-.5$	$-.5$	0.09	0.00	0.01	0.00	0.09	0.00	0.00	0.00	0.13	0.00	0.00	0.00
3	$-.1$	$-.1$	0.05	0.02	0.04	0.03	0.05	0.01	0.03	0.01	0.05	0.00	0.01	0.00
		$-.3$	0.06	0.01	0.05	0.03	0.06	0.00	0.03	0.01	0.06	0.00	0.01	0.00
		$-.5$	0.06	0.01	0.06	0.03	0.06	0.00	0.03	0.01	0.07	0.00	0.01	0.00
	$-.3$	$-.3$	0.06	0.01	0.05	0.03	0.06	0.00	0.02	0.01	0.07	0.00	0.00	0.00
		$-.5$	0.06	0.01	0.06	0.03	0.07	0.00	0.03	0.01	0.08	0.00	0.00	0.00
	$-.5$	$-.5$	0.07	0.01	0.06	0.03	0.07	0.00	0.03	0.01	0.08	0.00	0.00	0.00
5	$-.1$	$-.1$	0.05	0.03	0.05	0.04	0.05	0.01	0.04	0.03	0.05	0.00	0.02	0.01
		$-.3$	0.06	0.02	0.05	0.04	0.06	0.01	0.04	0.03	0.05	0.00	0.02	0.01
		$-.5$	0.06	0.02	0.06	0.05	0.06	0.01	0.05	0.03	0.05	0.00	0.02	0.01
	$-.3$	$-.3$	0.06	0.02	0.06	0.04	0.06	0.01	0.05	0.02	0.05	0.00	0.02	0.00
		$-.5$	0.06	0.02	0.06	0.05	0.06	0.01	0.05	0.02	0.06	0.00	0.02	0.00
	$-.5$	$-.5$	0.06	0.02	0.07	0.05	0.06	0.00	0.06	0.02	0.06	0.00	0.02	0.00

[1] Rejection frequencies at 5% significance level using finite sample critical values for series generated by the BAND-TAR process (18) with $-\gamma_1 = \gamma_2 \equiv \gamma$ and $\varepsilon_t \sim$ i.i.d.$N(0,1)$. The table is based on 10000 replications.

Table 9: Rejection frequencies of SUR tests in case of changing persistence[1]

		$T = 100$				$T = 250$				$T = 500$			
τ	ϕ_2	E_1	E_2	Z_1	Z_2	E_1	E_2	Z_1	Z_2	E_1	E_2	Z_1	Z_2
.25	$-.05$	0.12	0.05	0.10	0.05	0.23	0.03	0.09	0.04	0.36	0.04	0.08	0.03
	$-.15$	0.17	0.03	0.16	0.04	0.32	0.04	0.13	0.04	0.54	0.05	0.15	0.05
	$-.25$	0.15	0.03	0.21	0.04	0.33	0.05	0.19	0.05	0.61	0.07	0.22	0.09
.50	$-.05$	0.05	0.21	0.09	0.12	0.05	0.14	0.09	0.09	0.08	0.15	0.09	0.09
	$-.15$	0.07	0.15	0.13	0.12	0.15	0.14	0.12	0.10	0.24	0.16	0.12	0.13
	$-.25$	0.13	0.14	0.16	0.13	0.25	0.17	0.16	0.14	0.36	0.20	0.16	0.16
.75	$-.05$	0.05	0.12	0.10	0.14	0.04	0.10	0.11	0.15	0.04	0.09	0.11	0.15
	$-.15$	0.08	0.12	0.18	0.21	0.08	0.08	0.16	0.18	0.08	0.08	0.17	0.18
	$-.25$	0.15	0.13	0.22	0.24	0.16	0.09	0.20	0.21	0.19	0.09	0.20	0.21

[1] Rejection frequencies at 5% significance level using finite sample critical values for series generated by the changing persistence process (19) with $\phi_1 = 0$, $\mu_1 = 0$, $\mu_2 = .5$ and $\varepsilon_t \sim$ i.i.d.$N(0,1)$. The table is based on 10000 replications.

Table 10: Rejection frequencies of SUR tests in case of structural change in mean[1]

			$T = 100$				$T = 250$				$T = 500$			
τ	ρ	μ_2	E_1	E_2	Z_1	Z_2	E_1	E_2	Z_1	Z_2	E_1	E_2	Z_1	Z_2
.25	1.0	1	0.04	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
		3	0.02	0.04	0.07	0.04	0.03	0.04	0.06	0.05	0.04	0.05	0.06	0.06
		5	0.00	0.02	0.13	0.04	0.01	0.04	0.10	0.05	0.03	0.05	0.08	0.06
	0.5	1	0.09	0.00	0.00	0.00	0.10	0.00	0.00	0.00	0.14	0.00	0.00	0.00
		3	0.05	0.00	0.01	0.00	0.08	0.00	0.01	0.00	0.14	0.00	0.00	0.00
		5	0.01	0.00	0.06	0.00	0.05	0.00	0.09	0.00	0.11	0.00	0.03	0.00
	0.0	1	0.11	0.00	0.00	0.00	0.11	0.00	0.00	0.00	0.16	0.00	0.00	0.00
		3	0.05	0.00	0.02	0.00	0.10	0.00	0.03	0.01	0.18	0.00	0.01	0.00
		5	0.01	0.00	0.09	0.00	0.04	0.00	0.13	0.00	0.14	0.00	0.09	0.00
.5	1.0	1	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.06
		3	0.05	0.05	0.06	0.08	0.05	0.05	0.05	0.07	0.05	0.05	0.05	0.07
		5	0.05	0.05	0.10	0.12	0.05	0.05	0.08	0.10	0.05	0.05	0.06	0.09
	0.5	1	0.10	0.00	0.01	0.00	0.09	0.00	0.00	0.00	0.13	0.00	0.00	0.00
		3	0.08	0.00	0.02	0.00	0.06	0.00	0.01	0.00	0.06	0.00	0.00	0.00
		5	0.07	0.00	0.05	0.01	0.04	0.00	0.07	0.00	0.03	0.00	0.02	0.00
	0.0	1	0.11	0.00	0.00	0.00	0.08	0.00	0.00	0.00	0.10	0.00	0.00	0.00
		3	0.07	0.00	0.02	0.00	0.04	0.00	0.03	0.00	0.02	0.00	0.01	0.00
		5	0.06	0.00	0.05	0.00	0.03	0.00	0.06	0.00	0.01	0.00	0.03	0.00
.75	1.0	1	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.06
		3	0.12	0.06	0.07	0.07	0.08	0.05	0.06	0.06	0.06	0.05	0.05	0.06
		5	0.29	0.09	0.11	0.12	0.15	0.06	0.09	0.09	0.10	0.06	0.07	0.07
	0.5	1	0.10	0.00	0.00	0.00	0.09	0.00	0.00	0.00	0.13	0.00	0.00	0.00
		3	0.14	0.00	0.01	0.01	0.07	0.00	0.01	0.01	0.06	0.00	0.00	0.00
		5	0.25	0.01	0.05	0.05	0.07	0.00	0.06	0.07	0.04	0.00	0.01	0.02
	0.0	1	0.11	0.00	0.00	0.00	0.08	0.00	0.00	0.00	0.10	0.00	0.00	0.00
		3	0.13	0.00	0.02	0.02	0.04	0.00	0.02	0.03	0.02	0.00	0.00	0.01
		5	0.20	0.00	0.06	0.09	0.05	0.00	0.07	0.10	0.01	0.00	0.04	0.06

[1] Rejection frequencies at 5% significance level using finite sample critical values for series generated by the structural change process (20) with $\xi_1 = \xi_2 = \mu_1 = 0$ and $\varepsilon_t \sim$ i.i.d.$N(0,1)$. The table is based on 10000 replications.

Table 11: Rejection frequencies of SUR tests in case of structural change in trend[1]

			$T=100$				$T=250$				$T=500$			
τ	ρ	ξ_2	E_1	E_2	Z_1	Z_2	E_1	E_2	Z_1	Z_2	E_1	E_2	Z_1	Z_2
.25	1.0	0.3	0.05	0.16	0.06	0.07	0.08	0.32	0.08	0.10	0.11	0.65	0.12	0.19
		0.9	0.09	0.71	0.17	0.19	0.30	0.75	0.27	0.22	0.55	0.99	0.40	0.33
		1.5	0.19	0.71	0.34	0.24	0.39	0.51	0.43	0.24	0.54	0.91	0.53	0.36
	0.5	0.3	0.08	0.15	0.03	0.06	0.15	0.97	0.21	0.24	0.72	1.00	0.41	0.38
		0.9	0.08	0.94	0.19	0.23	0.53	0.65	0.44	0.23	0.86	0.99	0.58	0.37
		1.5	0.16	0.91	0.38	0.28	0.51	0.23	0.57	0.29	0.65	0.52	0.71	0.43
	0.0	0.3	0.07	0.56	0.08	0.14	0.44	1.00	0.35	0.29	1.00	1.00	0.55	0.40
		0.9	0.16	1.00	0.33	0.28	0.51	0.25	0.55	0.27	0.65	0.63	0.70	0.43
		1.5	0.32	0.87	0.49	0.30	0.62	0.10	0.73	0.41	0.82	0.19	0.86	0.59
.5	1.0	0.3	0.05	0.17	0.05	0.11	0.05	0.35	0.06	0.18	0.06	0.68	0.08	0.28
		0.9	0.05	0.72	0.11	0.29	0.06	0.56	0.19	0.30	0.09	0.95	0.34	0.39
		1.5	0.04	0.55	0.25	0.34	0.04	0.25	0.39	0.33	0.05	0.46	0.53	0.41
	0.5	0.3	0.08	0.11	0.02	0.14	0.02	0.97	0.12	0.33	0.01	1.00	0.35	0.42
		0.9	0.04	0.98	0.11	0.34	0.00	0.20	0.41	0.30	0.03	0.62	0.62	0.40
		1.5	0.02	0.72	0.28	0.36	0.00	0.08	0.56	0.33	0.00	0.02	0.73	0.43
	0.0	0.3	0.07	0.53	0.03	0.25	0.01	0.94	0.27	0.35	0.07	1.00	0.58	0.43
		0.9	0.02	0.99	0.25	0.36	0.00	0.07	0.54	0.31	0.00	0.03	0.72	0.42
		1.5	0.02	0.58	0.46	0.38	0.00	0.11	0.70	0.39	0.00	0.03	0.86	0.49
.75	1.0	0.3	0.04	0.06	0.06	0.09	0.03	0.08	0.08	0.13	0.03	0.18	0.12	0.22
		0.9	0.02	0.24	0.15	0.25	0.01	0.18	0.26	0.29	0.02	0.32	0.39	0.37
		1.5	0.01	0.17	0.31	0.33	0.01	0.02	0.42	0.32	0.00	0.03	0.51	0.40
	0.5	0.3	0.04	0.00	0.03	0.08	0.00	0.18	0.19	0.29	0.00	0.99	0.40	0.40
		0.9	0.00	0.14	0.18	0.30	0.00	0.02	0.41	0.30	0.01	0.01	0.56	0.40
		1.5	0.00	0.35	0.34	0.37	0.00	0.00	0.54	0.30	0.00	0.00	0.70	0.40
	0.0	0.3	0.02	0.01	0.07	0.18	0.00	0.38	0.33	0.34	0.06	1.00	0.54	0.43
		0.9	0.00	0.56	0.30	0.35	0.00	0.00	0.51	0.29	0.00	0.00	0.69	0.40
		1.5	0.00	0.12	0.45	0.38	0.01	0.00	0.71	0.31	0.00	0.00	0.85	0.39

[1] Rejection frequencies at 5% significance level using finite sample critical values for series generated by the structural change process (20) with $\mu_1 = \mu_2 = \xi_1 = 0$ and $\varepsilon_t \sim$ i.i.d.$N(0,1)$. The table is based on 10000 replications.