Monitoring time-varying parameters in an autoregression

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Abstract

We develop a sequential testing approach for a structural change in the parameters of an autoregression, which amounts to a monitoring procedure with a controlled asymptotic size as we repeat the test. Our method can be used as a general misspecification test. We apply our method to monthly US industrial production in order to investigate if its autoregressive behavior and/or its innovation variance have changed during the twentieth century.

Keywords: Structural change; autoregression; misspecification test

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1 Introduction

It is important to examine parameter stability in autoregressive time series models. For example, when shifts in the autoregressive parameters are neglected, one generates biased out-of-sample forecasts. Additionally, when such shifts in the variance are overlooked, the prediction intervals will be too wide or too narrow. Also from a descriptive point of view, it is interesting to examine if, for example, cyclical patterns (as indicated by the size of the complex roots of the AR polynomial) or persistence properties have changed over time. In this paper, we therefore develop a sequential testing approach for structural change in the autoregressive and variance parameters of an autoregression, which can be implemented as a general misspecification test. In order to obtain a method with a controlled asymptotic size as we repeat the test, we adapt the monitoring procedure as it was proposed in Chu, Stinchcombe & White (1996). We apply our method to monthly US industrial production in order to investigate if its autoregressive behavior and/or its innovation variance have changed during the twentieth century.

The outline of our paper is as follows. In section 2, we present the model and the relevant test statistic. In section 3, we outline the monitoring procedure, and we evaluate it using simulation experiments. In section 4, we report on our empirical application. In section 5, we conclude with some remarks.

2 The model and the test statistics

In this section we first consider a general autoregression and next we focus on a second order model, as this last model will be used in our empirical illustration.

2.1 Autoregression of order p

Consider a time series $\{y_t\}$, which can be described by an autoregressive process of order p [AR(p)], that is by

$$y_t = \phi_{1,t} y_{t-1} + \cdots + \phi_{p,t} y_{t-p} + \epsilon_t \ , \tag{1}$$

with $\epsilon_t|\mathcal{F}_{t-1} \sim \mathcal{N}(0, \sigma_t^2)$, and where $\mathcal{F}_{t-1}$ is the σ-field generated by $y_{1-p}, \ldots, y_0$, $\epsilon_1, \ldots, \epsilon_{t-1}$.

We are interested in examining if the parameters of the autoregressive model are stable, that is, if the parameters $\phi_t = (\phi_{1,t}, \ldots, \phi_{p,t})'$ and σ_t^2 are constant over time. More precise, it is assumed first that the model is constant up to time m: $\sigma_1^2 = \cdots = \sigma_m^2 = \sigma_0^2$, $\phi_1 = \cdots = \phi_m = \phi$, and we subsequently consider the null hypothesis

$$H_0 : (\phi_t', \sigma_t^2) = (\phi', \sigma_0^2), \; t = m+1, \ldots \tag{2}$$

against the alternative hypothesis

$$H_1 : (\phi_t', \sigma_t^2) \text{ changes at some } t \geq m+1. \tag{3}$$

Furthermore, if H_0 gets rejected, we want to have some indication as to which parameters are subject to change. For example it can be the variance of the white noise ϵ_t, the autoregressive parameters, or both. Therefore we will rely on the score process, that is, the vector of partial derivatives of the log-likelihood.

Under the null hypothesis, the time-varying parameter model (1) reduces to

$$y_t = \phi_1 y_{t-1} + \ldots + \phi_p y_{t-p} + \epsilon_t \tag{4}$$

where we assume that $\phi_1, \ldots, \phi_p$ are unknown scalars such that the zeros of the polynomial $1 - \phi_1 z - \cdots - \phi_p z^p$ lie outside the unit circle and $\epsilon_t|\mathcal{F}_{t-1} \sim \mathcal{N}(0, \sigma_0^2)$. Hence we assume covariance-stationarity.

Denote by θ the $p+1$ vector of unknown parameters, that is $\theta = (\phi', \sigma^2)'$, and denote by $l_m(\theta)$ the conditional log-likelihood at time m, that is,

$$l_m(\theta) = \sum_{t=1}^{m} \log f(y_t|\mathcal{F}_{t-1}, \theta) \; , \tag{5}$$

where,

$$\log f(y_t|\mathcal{F}_{t-1}, \theta) = -1/2 \log 2\pi - 1/2 \log \sigma^2 - (1/2\sigma^2) \; \epsilon_t^2. \tag{6}$$

Furthermore, use $\rho_t(\theta)$ to denote the conditional score function at time t, that is,

$$\rho_t(\theta) = \nabla \log f(y_t|\mathcal{F}_{t-1}, \theta) \ , \tag{7}$$

where

$$\nabla \log f(y_t|\mathcal{F}_{t-1}, \theta) = \left(\frac{\partial \log f(y_t|\mathcal{F}_{t-1}, \theta)}{\partial \theta_j} \right)_{j=1,2,\cdots,p+1}$$

is the gradient vector of $\log f(y_t|\mathcal{F}_{t-1}, \theta)$.

From the expression (6) of the log-likelihood, it is easy to see that the components of the score function at time t are given by

$$\left\{ \begin{array}{rcl} \dfrac{\partial \log f(y_t|\mathcal{F}_{t-1}, \theta)}{\partial \phi_j} & = & y_{t-j}\dfrac{\epsilon_t}{\sigma^2} \quad j = 1, \cdots, p \\ \dfrac{\partial \log f(y_t|\mathcal{F}_{t-1}, \theta)}{\partial \sigma^2} & = & \dfrac{1}{2\sigma^2}(\dfrac{\epsilon_t^2}{\sigma^2} - 1). \end{array} \right. \tag{8}$$

For further reference, denote by $\Psi_\lambda^m(\theta)$ the score process,that is

$$\Psi_\lambda^m(\theta) = m^{-1/2} \sum_{t=1}^{[m\lambda]} \rho_t(\theta) \tag{9}$$

for $\lambda \in R_+$. Our test below will be based on this score process.

Let $I(\theta)$ be the information matrix defined by

$$I(\theta) = E(\rho_t(\theta).\rho_t(\theta)') = -E\left(\frac{\partial^2 \log f(y_t|\mathcal{F}_{t-1}, \theta)}{\partial \theta_j \partial \theta_k} \right)_{j,k=1,\cdots,p+1}$$

The autocovariance function of the stationary process y_t in (4) is defined by

$$\gamma(h) = \text{cov}(y_{t+h}, y_t). \tag{10}$$

Given this function and by simple calculation, the information matrix is defined by

$$I(\theta) = \frac{1}{\sigma^2} \begin{pmatrix} \gamma(0) & \cdots & \gamma(p-1) & 0 \\ \vdots & & \vdots & \vdots \\ \gamma(p-1) & \cdots & \gamma(0) & 0 \\ 0 & \cdots & 0 & 1/2\sigma^2 \end{pmatrix} \tag{11}$$

For the sequence of processes $\{\Psi_\lambda^m(\theta)\}$, we have that

$$\Psi_\lambda^m(\theta) \Rightarrow I(\theta)^{1/2} W(\lambda) \tag{12}$$

where "$\Rightarrow$" denotes weak convergence and where W is a $p+1$-dimensional Wiener process with independent components, see Theorem VIII.3.33 in Jacod & Shiryaev (1987).

Let $\hat{\theta}$ be the conditional maximum likelihood [ML] estimator, to be found by solving the likelihood equations:

$$\sum_{t=1}^{m} \rho_t(\theta) = 0, \tag{13}$$

where $\rho_t(\theta)$ is as defined in (7). If $\hat{\theta} = (\hat{\phi}', \hat{\sigma}^2)'$, $\hat{\phi}$ is actually the least squares estimator, and $\hat{\sigma}^2 = m^{-1} \sum_{t=1}^{m} \hat{\epsilon}_t^2$, where $\hat{\epsilon}_t = y_t - \hat{\phi}_1 y_{t-1} - \cdots - \hat{\phi}_p y_{t-p}$.

The estimated score process is obtained by replacing the parameters in (9), by their estimates, resulting in

$$\hat{\Psi}_\lambda^m = m^{-1/2} \sum_{t=1}^{[m\lambda]} \rho_t(\hat{\theta}) \ . \tag{14}$$

The sequence of processes $\{\hat{\Psi}_\lambda^m\}$ converges weakly to a Gaussian process, that is,

$$\hat{\Psi}_\lambda^m \Rightarrow I(\theta)^{1/2} W^0(\lambda) \ , \tag{15}$$

where $W^0(\lambda) = W(\lambda) - \lambda W(1)$.

2.2 An autoregression of order 2

For illustration, we consider the case with $p = 2$ and compute the elements of

$$I(\theta)^{-1/2} \Psi_\lambda^m(\theta).$$

Let n be the integer part of $m\lambda$, that is, $n = [m\lambda]$. The two first components of this vector, which we denote by S^ϕ, concern the two autoregressive parameters. We have

$$S_n^\phi = \sigma \begin{pmatrix} \gamma(0) & \gamma(1) \\ \gamma(1) & \gamma(0) \end{pmatrix}^{-1/2} \sum_{t=1}^{n} \begin{bmatrix} y_{t-1}\epsilon_t/\sigma^2 \\ y_{t-2}\epsilon_t/\sigma^2 \end{bmatrix}.$$

For this case, $\gamma(0)$ and $\gamma(1)$ are given by

$$\begin{cases} \gamma(0) = \dfrac{1-\phi_2}{(1+\phi_2)[(1-\phi_2)^2 - \phi_1^2]}\,\sigma^2 \\ \gamma(1) = \dfrac{\phi_1}{(1+\phi_2)[(1-\phi_2)^2 - \phi_1^2]}\,\sigma^2 \end{cases} \tag{16}$$

see Fuller (1996), p.55. Upon using this, we can show that

$$\sigma \begin{pmatrix} \gamma(0) & \gamma(1) \\ \gamma(1) & \gamma(0) \end{pmatrix}^{-1/2} = \sqrt{1+\phi_2}\, P \begin{pmatrix} \lambda_2^{1/2} & 0 \\ 0 & \lambda_1^{1/2} \end{pmatrix} P',$$

where λ_1 and λ_2 are the eigenvalues of the matrix $\begin{pmatrix} 1-\phi_2 & \phi_1 \\ \phi_1 & 1-\phi_2 \end{pmatrix}$, that is, $\lambda_1 = 1-\phi_2-\phi_1$, $\lambda_2 = 1-\phi_2+\phi_1$ and where $P = 1/\sqrt{2}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$. Using these results gives

$$S_n^{\phi} = \sqrt{1+\phi_2}\, P \begin{pmatrix} \lambda_2^{1/2} & 0 \\ 0 & \lambda_1^{1/2} \end{pmatrix} P' \sum_{t=1}^{n} \begin{bmatrix} y_{t-1}\epsilon_t/\sigma^2 \\ y_{t-2}\epsilon_t/\sigma^2 \end{bmatrix}. \tag{17}$$

Finally, consider the variance of the error process, for which we have

$$S_n^{\sigma^2} = \frac{1}{\sqrt{2}} \sum_{t=1}^{n} \left(\frac{\epsilon^2}{\sigma^2} - 1 \right). \tag{18}$$

From (12), it follows that

$$m^{-1/2} \begin{pmatrix} S_{[m\lambda]}^{\phi} \\ S_{[m\lambda]}^{\sigma^2} \end{pmatrix} \Rightarrow W(\lambda), \tag{19}$$

and if we replace the parameters by their respective estimates, we have

$$m^{-1/2} \hat{S}_{[m\lambda]} \Rightarrow W^0(\lambda). \tag{20}$$

where

$$\hat{S}_n = \begin{pmatrix} S_n^{\hat{\phi}} \\ S_n^{\hat{\sigma}^2} \end{pmatrix} \tag{21}$$

3 The monitoring procedure

In this section we incorporate the test statistic based on the score process into a monitoring procedure. Additionally, we evaluate its empirical performance under the null hypothesis in a limited simulation experiment.

3.1 The theory

We aim to apply the score-based test in the following context. A set of m observations has been collected and the parameters are estimated from this, say, historical data set. Next, we want to check if this estimated model fits equally well to newly arriving observations. We want our test to be valid as long as we get new data, that means, we do not want the size of the procedure to increase to unity as we repeat the test. The monitoring procedure for structural change described in Chu et al. (1996) is most appropriate for these purposes. In particular, Theorem 3.4 in their article states that if a stochastic sequence, $\{S_n,\ n \in N\}$, or more precisely, the sequence of processes $X_\lambda^m \equiv \{m^{-1/2} S_{[m\lambda]}\ \lambda \in [0, \infty]\}$ converges weakly to a Wiener process, then

$$\lim_{m\to\infty} P\{|S_n| \geq \sqrt{m} g(n/m) \text{ for some } n \geq m\} = P\{|W(\lambda)| \geq g(\lambda) \text{ for some } \lambda \geq 1\},$$

when g is a suitably chosen function. This theorem works within the framework of univariate processes but it can be generalized to the multivariate case, as it is done in Section 3.3 in Chu et al. (1996). If S_n is a k-dimensional sequence and if the sequence of processes X_λ^m converges weakly to a k-dimensional Wiener process with independent components, we can deduce the probability that at least one of the components of the process $W(\lambda)$ crosses the boundary function g.

Use W to denote a k-dimensional process with independent components, such that each component W_i is a standard Wiener process. Let g be a function such that the probability that a Wiener process crosses at least once a value of this function is known, more precisely

$$P\{|W_i(\lambda)| \geq g(\lambda) \text{ for some } \lambda \geq 1\} = \alpha$$

for a certain fixed α. For this k-dimensional process we have

$$\begin{aligned} &P\{|W_i(\lambda)| \geq g(\lambda) \text{ for some } \lambda \geq 1,\ \text{for some } i = 1, \ldots, k\} \\ &\quad = 1 - P\{|W_i(\lambda)| < g(\lambda)\ \ \forall \lambda \geq 1, \forall i = 1, \cdots, k\} \\ &\quad = 1 - P\{|W_i(\lambda)| < g(\lambda)\ \ \forall \lambda \geq 1\}^k \\ &\quad = 1 - (1-\alpha)^k. \end{aligned}$$

If we take the function $g(\lambda) = [\lambda(a^2 + \log \lambda)]^{1/2}$, where a is a positive constant, the probability that a Wiener process crosses this function is

$$\alpha(a) = 2[1 - \Phi(a) - a\phi(a)],$$

where Φ and ϕ are the cdf and pdf of a standard normal distribution (see Chu et al. (1996)). Consequently, the probability that at least one of the components of the k-dimensional process W crosses the function g is

$$\tilde{\alpha}(a) = 1 - (1 - 2[1 - \Phi(a) - a\phi(a)])^k.$$

For $k = 3$, which corresponds to the AR(2) model, where we aim to evaluate the properties of the two autoregressive parameters and the variance, taking $a^2 = 10.1984$ gives $\tilde{\alpha}(a) = 0.05$ and $a^2 = 8.625$ gives $\tilde{\alpha}(a) = 0.10$. The null hypothesis in Section 2.1 will be rejected as soon as at least one of the components of the process $\hat{S}_n$ crosses a convenient boundary function. Hence our method can best be used as a general misspecification test. Using the asymptotic distribution derived in (20), we will approximate the probability of crossing under the null hypothesis by the probability that at least one of the components of the asymptotic process, W^0, crosses the boundary function somewhere, for a $\lambda \geq 1$.

The asymptotic result in (20) states that for large enough m, we can approximate the distribution of the process S_n by the distribution of the process $m^{1/2}W^0$. The distribution of the components of W_0 can easily be deduced from the distribution of a univariate Wiener process,

$$\{W_i^0(\lambda),\ \lambda \geq 1\} \stackrel{d}{=} \{(\lambda - 1)W_i(\lambda/(\lambda - 1)),\ \lambda \geq 1\}$$

Consequently, we can define a boundary function valid for W^0 from a small modification of the function g defined above. Instead of considering g, we use $h(\lambda) \equiv (\lambda - 1)g(\lambda/(\lambda - 1))$. It can be easily verified that $P\{|W_i(\lambda)| \geq g(\lambda) \text{ for some } \lambda \geq 1\} = \alpha$ implies that $P\{|W_i^0(\lambda)| \geq (\lambda - 1)g(\lambda/(\lambda - 1)) \text{ for some } \lambda \geq 1\} = \alpha$ and if S_n is a k-dimensional process such that $m^{-1/2}S_{[m\lambda]} \Rightarrow W^0$, it holds that

$$\lim_{m\to\infty} P\{|S_n^i| \geq \sqrt{m}h(n/m) \text{ for some } n \geq m \text{ and some } i = 1, \dots, k\} = \tilde{\alpha}(a). \quad (22)$$

More precisely, we have

$$\sqrt{m}h(n/m) = \sqrt{\frac{n}{m}(n - m)}\left(a^2 + \log\frac{n}{n - m}\right)^{1/2}. \quad (23)$$

3.2 Some Monte Carlo evidence

We evaluate the above monitoring procedure in a limited Monte Carlo simulation experiment. We generate $m+q$ observations from an AR(2) model with mean zero, unit variance for ϵ_t and AR parameters (1.2,-0.7) and (0.3,-0.1), respectively. Next we apply the monitoring procedure and compute the frequency of rejection of the null hypothesis at nominal size 0.05 and 0.10. The results are summarized in Tables 1 and 2.

The empirical size values in these tables indicate that the test is slightly oversized. Interestingly, it seems that the value of q does not matter much, except for very small samples. For practical work, we would recommend to set m at a reasonably large value.

To study the finite sample power of the monitoring procedure, we simulate changes in the parameters (the autoregressive parameters ϕ and the variance of the innovation process σ^2). Three kinds of change are considered. First only the autoregressive parameters change: at time $t = [m * 1.1]$, ϕ shifts from (1.2,-0.4) to (1.2,-0.7), while σ^2 remains unchanged ($\sigma^2 = 1$). The relevant results are gathered in Table 3. Next only σ^2 is subject to change. At time $t = [m * 1.1]$, σ^2 decreases from 1 to 0.5, while ϕ is constant at (1.2,-0.7). The results are in Table 4. Finally we consider the case where all parameters change. At time $t = [m * 1.1]$, σ^2 decreases from 1 to 0.5 and the autoregressive parameters shift from (1.2,-0.4) to (1.2,-0.7). The results are summarized in Table 5.

The simulation results in these table are easy to summarize. For $m \geq 200$ and $q \geq 4m$, the empirical power is close or equal to 1. The method is also quite powerful in small samples when only the AR parameters change.

4 Monthly US industrial production

In this section we apply our monitoring procedure to US industrial production. In the empirical business cycle literature, there is a substantial interest in the possible stabilization of the US economy throughout the twentieth century, see for example, Watson (1994) and the references cited therein. If an autoregression with complex roots would describe the growth rates in industrial production, stabilization would mean that AR parameters change such that the implied cycle length gets shorter. However, one may also define stability in terms of a reduced variance. Hence, significant stabilization would emerge as time-varying parameters in an autoregression.

We consider the monthly growth rate in (seasonally adjusted) US industrial production index, observed for 1919.01 to 1999.05 for which we find an AR(2) model to describe the data reasonably well. The data appear in Figure 1. Clearly, there is visual evidence of changing unconditional variance in this series, but whether this is due to changes in the AR parameters and/or the variance of the error process is not immediately obvious. Therefore, we use our monitoring procedure and we display the results, when we compare pre-war data ($m = 336$) with post-war data ($q \simeq 2m$), in Figure 2. The empirical results lead to a clear-cut conclusion: the parameters in this autoregression are not constant over time.

In a next step we calculate a Wald test for the constancy of the AR parameters and a Goldfeld-Quandt test for the constancy of the variance, where we set the break date equal to 1946.01 (which corresponds with m=336). The Wald test obtains a value 13.45 of while the second test obtains a value of 7.98. As both statistics indicate rejection of the null hypothesis, we conclude that the AR and the variance parameters both changed over time. The AR parameters changed from(0.59,-0.10) to (0.38,0.13), and the innovation variance from 7.5 10^{-4} to 9.4 10^{-5}.

It may also be of interest to restrict attention to post-war data, see Figure 3. When we compare the first 10 years of post-war growth rate (m=120) with more recent observations ($q \simeq 4m$), again upon considering an AR(2) model, we find again evidence of time-varying parameters, see Figure 4. The corresponding Wald test and Goldfeld-Quandt test for constant AR parameters now take values of 0.65 and 2.6. Here we find that the AR parameters seem constant, and that only the innovation variance has changed in the post war period. This variance changed from 1.8 10^{-4} to 7.2 10^{-5}.

In sum, we find strong evidence in favor of a reduction of the innovation variance in an autoregression for US industrial production.

5 Concluding remarks

In this paper we proposed a monitoring procedure for structural breaks in the parameters of an autoregression. The procedure was evaluated by Monte Carlo simulations, and these suggested that it has reasonable size and good power properties. We applied our method to monthly US industrial production data, for which we found evidence of stabilization, where this increased stability appeared to be mainly due to a reduction in the variance of the innovations.

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Table 1: Empirical size of the monitoring procedure: zero-mean AR(2) model with $\phi_1 = 1.2$ and $\phi_2 = -0.7$

Asymptotic size 0.05

m	50	75	100	200	500	800
q						
$2m$	19.9	17.7	16.4	13.4	12.6	11.0
$4m$	22.1	19.3	17.7	14.2	13.4	11.4
$6m$	22.8	19.9	18.1	14.6	13.6	11.6
$9m$	23.5	20.5	18.4	14.9	13.7	11.7
$19m$	24.3	20.9	18.8	15.3	13.9	11.8

Asymptotic size 0.10

q						
$2m$	23.4	21.5	20.1	17.6	16.6	15.3
$4m$	25.9	23.4	21.8	18.9	17.9	16.0
$6m$	26.8	24.1	22.3	19.5	18.2	16.3
$9m$	27.5	24.8	22.9	19.9	18.4	16.5
$19m$	28.2	25.5	23.4	20.1	18.6	16.7

Table 2: Empirical size of the monitoring procedure: zero-mean AR(2) model with $\phi_1 = 0.3$ and $\phi_2 = -0.1$

Asymptotic size 0.05

m	50	75	100	200	500	800
q						
$2m$	14.4	14.0	12.0	9.2	8.4	7.6
$4m$	15.6	15.0	12.9	9.9	9.2	8.1
$6m$	16.1	15.4	13.1	10.2	9.4	8.3
$9m$	16.6	15.8	13.3	10.4	9.5	8.4
$19m$	16.9	16.0	13.7	10.5	9.6	8.5

Asymptotic size 0.10

q						
$2m$	18.4	17.7	15.6	13.4	12.3	11.8
$4m$	19.9	19.0	16.8	14.4	13.1	12.6
$6m$	20.5	19.6	17.1	14.5	13.3	13.0
$9m$	21.0	20.1	17.7	14.7	13.5	13.2
$19m$	21.5	20.4	18.2	15.0	13.8	13.4

Table 3: Finite sample power for an AR(2) model when only the AR parameters change: $\phi_1 = (1.2, -0.4)'$ to $\phi_2 = (1.2, -0.7)'$

Asymptotic size 0.05

m	50	75	100	200	500	800
q						
$2m$	59.4	72.4	82.0	98.9	100	100
$4m$	70.8	83.5	92.1	99.9	100	100
$6m$	74.8	87.2	94.6	99.9	100	100
$9m$	78.0	89.8	96.2	100	100	100
$19m$	81.1	92.0	97.6	100	100	100

Asymptotic size 0.10

q						
$2m$	64.7	77.3	86.0	98.0	100	100
$4m$	75.8	87.8	94.6	99.7	100	100
$6m$	79.4	90.1	96.8	99.9	100	100
$9m$	82.4	92.9	97.9	100	100	100
$19m$	85.4	94.8	99.0	100	100	100

Table 4: Finite sample power for an AR(2) model when only the variance of the error process changes: σ_1^2=1 to $\sigma_2^2 = 0.5$. The parameter ϕ remains constant, $\phi = (1.2, -0.7)'$

Asymptotic size 0.05

m	50	75	100	200	500	800
q						
$2m$	9.6	13.5	28.2	90.4	100	100
$4m$	10.9	25.4	51.0	98.2	100	100
$6m$	12.2	31.9	61.1	99.1	100	100
$9m$	13.7	37.7	67.7	99.6	100	100
$19m$	15.8	43.5	74.0	99.7	100	100

Asymptotic size 0.10

q						
$2m$	12.6	25.6	46.8	95.9	100	100
$4m$	18.1	43.7	69.9	99.3	100	100
$6m$	21.8	51.8	77.3	99.7	100	100
$9m$	25.3	57.7	81.5	99.8	100	100
$19m$	29.2	63.6	85.9	99.9	100	100

Table 5: Finite sample power for an AR(2) model when both the variance of the error process and the autoregressive parameters change: σ_1^2=1 to $\sigma_2^2 = 0.5$ and $\phi_1 = (1.2, -0.4)'$ to $\phi_2 = (1.2, -0.7)'$

Asymptotic size 0.05

m	50	75	100	200	500	800
q						
$2m$	22.3	26.9	34.2	75.1	100	100
$4m$	26.7	34.0	45.2	92.0	100	100
$6m$	28.4	37.3	51.4	95.8	100	100
$9m$	30.0	40.1	55.6	97.2	100	100
$19m$	32.0	43.5	61.8	98.3	100	100

Asymptotic size 0.10

q						
$2m$	26.2	34.6	45.3	87.8	100	100
$4m$	31.9	45.5	60.6	97.5	100	100
$6m$	34.8	49.8	67.3	98.5	100	100
$9m$	37.0	53.5	72.4	99.0	100	100
$19m$	39.7	58.3	77.9	99.5	100	100

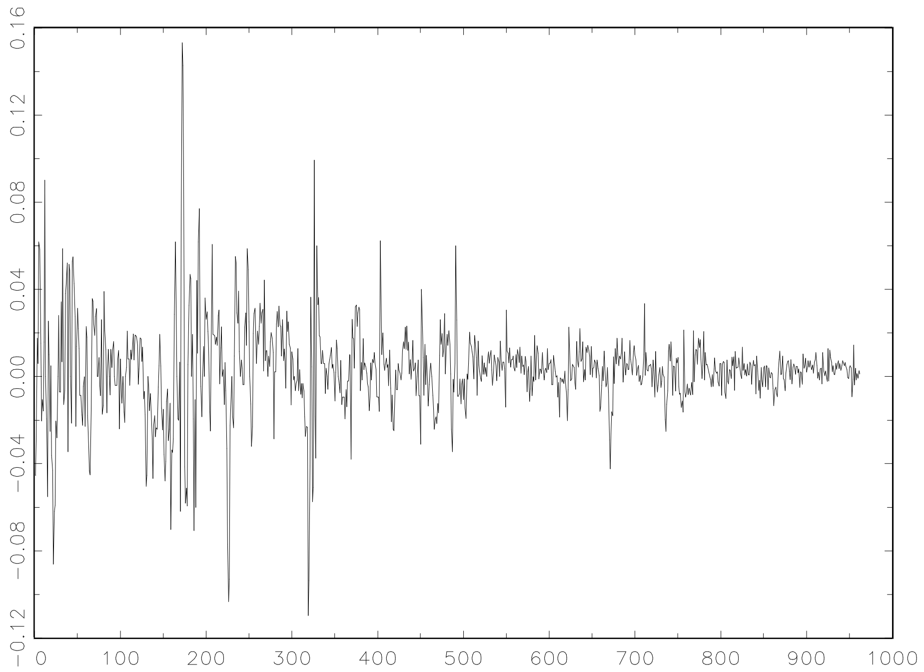

Figure 1: First differences of the (log) monthly US industrial production index (1919.01-1999.05)

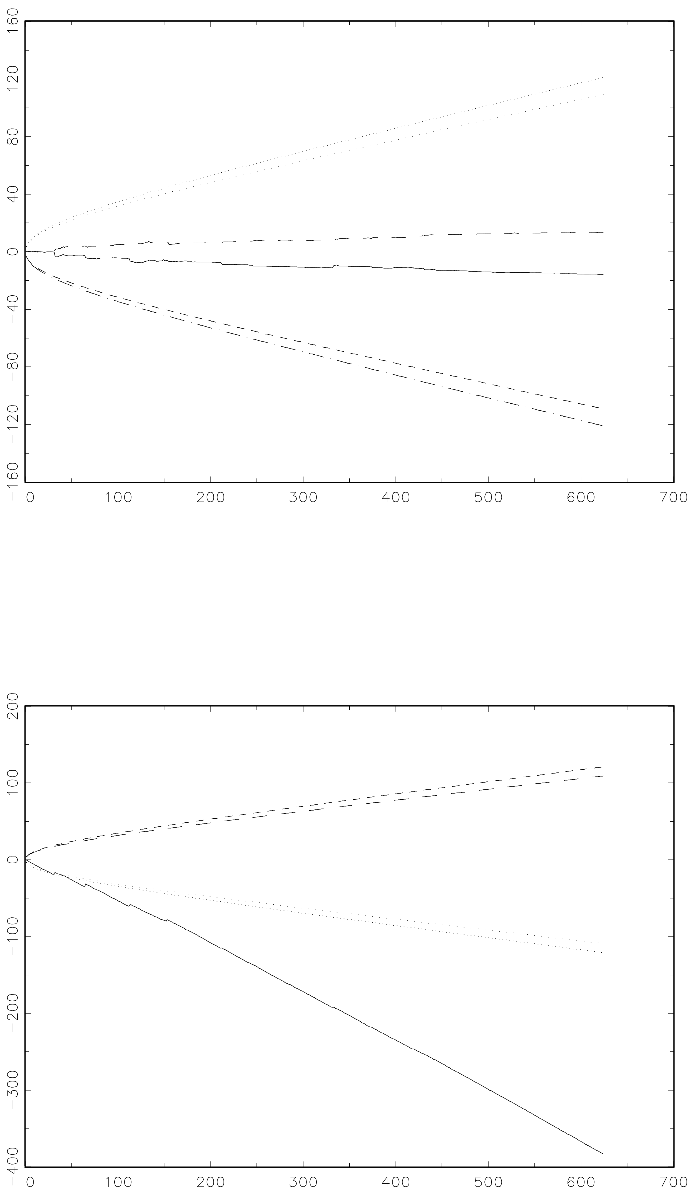

Figure 2: Comparing pre-war and post-war data (m=336).
The top panel concerns $S^{\hat{\phi}}$ and the bottom panel concerns $S^{\hat{\sigma}^2}$.

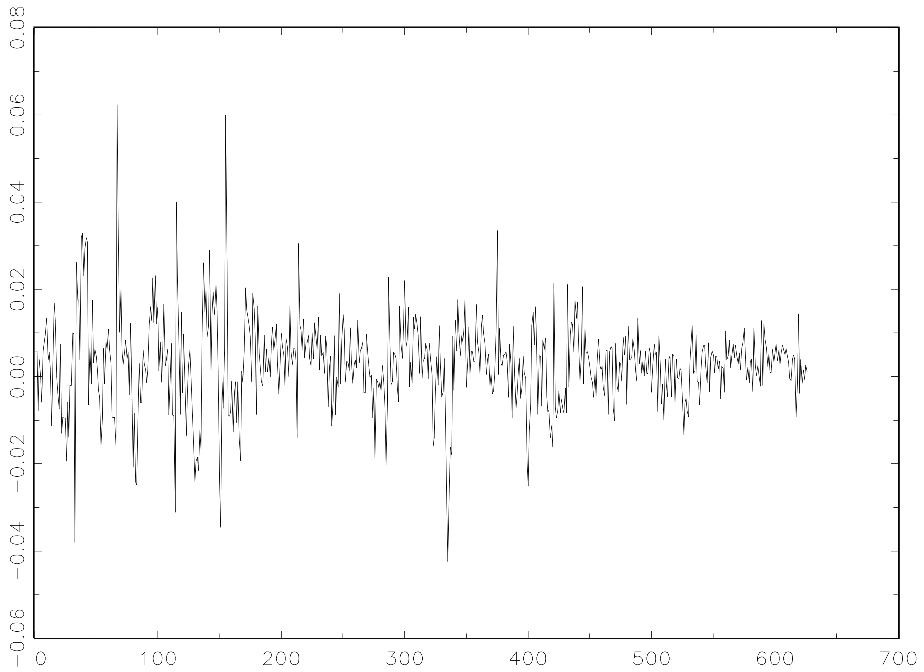

Figure 3: First differences of the (log) monthly industrial production index (1947.01-1999.05)

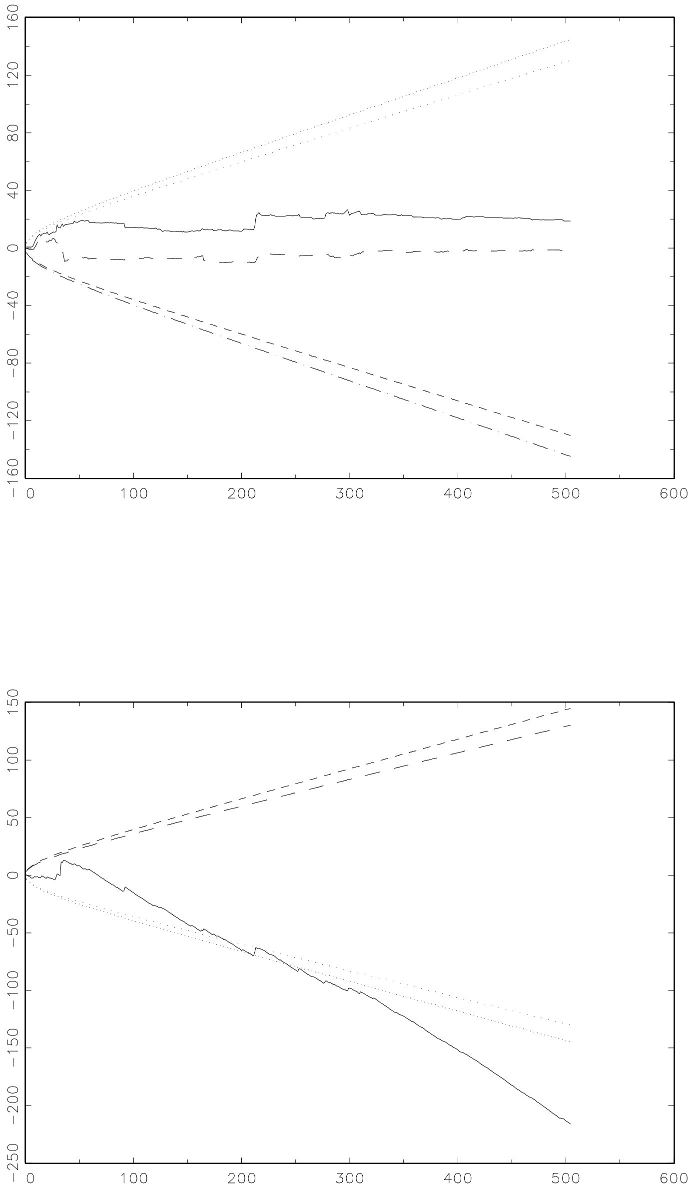

Figure 4: Is there sructural a break in post-war data? (m=120)
The top panel concerns $S^{\hat{\phi}}$ and the bottom panel concerns $S^{\hat{\sigma}^2}$.