A Trinomial Test for Paired Data When There are Many Ties

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Abstract

This paper develops a new test, the trinomial test, for pairwise ordinal data samples to improve the power of the sign test by modifying its treatment of zero differences between observations, thereby increasing the use of sample information. Simulations demonstrate the power superiority of the proposed trinomial test statistic over the sign test in small samples in the presence of tie observations. We also show that the proposed trinomial test has substantially higher power than the sign test in large samples and also in the presence of tie observations, as the sign test ignores information from observations resulting in ties.

Keywords: Sign test, trinomial test, non-parametric test, ties, test statistics, hypothesis testing.

JEL Classifications: C12, C14, C15.

1 Introduction

Estimating the parameters of distributions is one the most important issues in statistics. Parametric tests make rather stringent assumptions regarding the nature of the population from which the observations were drawn (Siegel [15]). On the other hand, non-parametric methods are popular for practitioners as they do not require strong assumptions for their validity, as are required by their parametric counterparts. Non-parametric approaches based on signs and ranks form a substantial body of statistical techniques that provide alternatives to classical parametric methods. For example, most non-parametric tests require the assumption of a population from which subjects are obtained by random sampling, whereas for most non-parametric methods, treatments being compared are assumed to have been randomly assigned to subjects. A bibliography of non-parametric statistics by Savage [14] lists about 3,000 items. Among them, the sign test is one of the most widely used, and is regarded as the oldest non-parametric test procedure. The sign test was used in applications as early as 1710 in an article by Arbuthnott. The test derives its name from the procedure of converting data into plus and minus signs.

Dixon and Mood [7] and Mackinnon [12] have published tables of critical values for the sign test. On the other hand, Wilcoxon [17] indicates, for the first time, the possibility of using ranking methods in order to obtain a rapid approximation of the significance of the differences in experiments containing both paired and unpaired data. His paper is a milestone in the literature on non-parametric statistics.

In addition, Dixon and Mood [8] and Walse [16] have published short notes commenting on the power function of the sign test. Dixon and Mood use various sample sizes and the significance level, α , near 0.05 and 0.01 to tabulate the values of the power function. The sign test is found to have decreasing power for increasing sample size, increasing levels of significance and increasing values of the alternative. Walse [16] also comments that the sign test is approximately 95% efficient for small sample sizes when a comparison is made with the most powerful test for the case of a normal population.

It is well known that the sign test possesses poor performance in the presence of zero observations. Some attempts have been made to modify the sign test in order to increase its power in the presence of zero observations. One such attempt is to include the zero observations in a randomized treatment of zero observations, whereby zero observations are randomly distributed into plus and minus signs. However, using different theorems, Putter [13] and Hemelrijk [10] have proved that the non-randomized treatment of zero observations is always better than randomization for the sign test.

To circumvent the low power of the sign test in the presence of zero observations, in this paper we develop a new test statistic, the trinomial test, for pairwise ordinal data samples by incorporating the zeros in the sign test to improve power performance significantly. This new trinomial test is found to be more powerful than the sign test with the improvement becoming more obvious when the number of ties is large. The main result of the paper is to introduce a new test which will effectively take account of the zero differences, so that the new modified sign test will perform better. This new test is based on a trinomial relationship between the positive, negative and zero differences (observations).

In order to demonstrate the power superiority of our proposed trinomial test statistic over the sign test, we first conduct simulations to show that the proposed trinomial test is superior to the sign test in small samples in the presence of tie observations. We then prove that the proposed trinomial test is substantially superior in power to the sign test in the presence of tie observations in large samples. The poor performance of the sign test is due to the fact that it ignores the information from the observations resulting in ties.

2 Review of Methodologies

Arbuthnott [2] uses a sign test to study devine providence in the births of boys and girls while Savage [14] lists the sign test in his book. To take care of "tie" observations, Dixon and Mood [7] first recommend including half number of ties to positive observations as a *nonrandomized unconditional exact* (NUE) *test* (see, Coakley and Heise [3]):

$$S = N_{+} + N_{0} / 2 \tag{1}$$

as the test statistic. The null hypothesis, H0, that the probability of being positive is equal to the probability of being negative is rejected whenever S exceeds the critical value which can be calculated by B(N, 1/2) and is tabulated under different values of significance level by Dixon and Mood [7]. They also point out the test is a little more strict than the nominated significance level, especially for small sample size. However, since this procedure reduces the power in testing H0 when ties are present, ties are usually excluded in the sign test by many text books, see, for example, Dixon and Massey [6], in which N+ is used as test statistic and critical value is obtained from $B(N - N_0, 1/2)$.

Putter [13] proposes an *asymptotic uniformly most powerful nonrandomized* (ANU) *test* (Coakley and Heise [3]):

$$S_{1/2} = \frac{N_+ - N_-}{\sqrt{N_+ + N_-}} \tag{2}$$

and the null hypothesis H_0 is rejected if $S_{1/2} > z_\alpha$ where z_α is the $100(1-\alpha)$th percentile of a standard normal distribution. The asymptotic normal makes it easy to obtain the p-value for the statistic. To use this test, N must be sufficiently large. Some textbooks suggest that N should be greater than 10 while some say N should be greater than 25.

On the other hand, Coakley and Heise [3] propose an *improved nonrandomized unconditional* (INU) *test*:

$$S_{2/3} = N_+ + (2N_O/3) \tag{3}$$

and the null hypothesis H_0 is rejected if $S_{2/3} > \mathrm{k}(p_O)$ where $p_O = P(N_0)$. The idea is coming from the result of Irle and Klosener [11]. However, Wittkowski, Coakley, and Heise [19] points out that the INU test is a biased test and the weight 2/3 should be replaced by 1/2 which leads the INU test to the same as ANU test.

Through the normalization shown by Wittkowski, Coakley, and Heise [19], the standard nonrandomized traditional sign test can be easily seen to be the exact version of the ANU test. In addition, Wittkowski [18] examines the asymptotic UMP sign tests for different hypotheses. He points out that the procedure of dealing with ties could be more meaningful if we take deeper inspect on the causes of tied observations, which might be rounding error or the nature of the phenomenon. If the ties are due to the nature of the phenomenon, it will not give valuable information. If the ties are due to rounding error, the inclusion of ties should be considered.

3 The Trinomial Test

Despite the fact that the sign test is so simple and easy to apply, it does not usually compare favorably with other non-parametric test procedures. An obvious reason is that the sign test uses relatively less information from the testing samples when we have a significant number of zeros and tied observations. The greater is the number of zeros or tied observations, the greater is the loss of information due to a smaller size being examined. In order to reduce the loss of information, in this paper we develop a new test, the trinomial test, by modifying the original sign test. The trinomial test includes the information of zeros or tied observations effectively, so that the power of the trinomial test can be improved significantly.

Consider a random sample of n pairs $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$. Let $D_i = X_i - Y_i$ for $i = 1,2 \ldots ,n$. The random variable, D_i, can be partitioned into three different outcomes, D_+, D_0 and D_-, where D_+, D_0 and D_- are defined as the event when D_t is positive, zero and negative, respectively. Let n_k denote the number of trials resulting in outcome D_k and let $p_k = P(D_k)$ for $k = +, 0, -$. Then, we have:

$$P\left(N_+ = n_+, N_0 = n_0, N_- = n_-\right) = \frac{n!}{n_+!n_0!n_-!} p_+^{n_+} p_0^{n_0} p_-^{n_-}\ ; \tag{4}$$

in which $n_+ + n_0 + n_- = n$ and $p_+ + p_0 + p_- = 1$. It is intuitive that N_+ and N_- should be negatively related. One could easily show that the covariance $cov(N_+,N_-) = -np_+ p_-$ by considering

$$N_{+} = \sum_{r=1}^{n} I_{+}(r) \text{ and } N_{-} = \sum_{r=1}^{n} I_{-}(r)$$

in which $I_{+}(r) = 1$ if trial *r* results in outcome D_{+} and 0 otherwise and, similarly, $I_{-}(r) = 1$ if trial *r* results in outcome D_{-} and 0 otherwise.

Suppose that we want to test the hypotheses:

$$H_0 : p_{+} = p_{-} \text{ versus } H_1 : p_{+} > p_{-} . \tag{5}$$

The construction of the new test statistic involves observing, in a sample of *n* pairs of observations, the value n_d and a particular realization of the random variable ($N_{+} - N_{-}$). The expectation of this random variable is given by:

$$E(N_{+} - N_{-}) = n(p_{+} - p_{-}) .$$

Since $cov(N_{+} - N_{-}) = -np_{+}p_{-}$, the variance of the random variable is

$$V(N_{+} - N_{-}) = np_{+}(1 - p_{+}) + np_{-}(1 - p_{-}) + 2np_{+}p_{-} .$$

Therefore, under H_0 , we have:

$$E(N_{+} - N_{-}) = 0 , \; V(N_{+} - N_{-}) = 2np$$

Table 1: Critical Values for the Proposed Trinomial Test

p_0	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
C_α	6	5	5	4	4	4	3	3	2	2
$P(n_d > C_\alpha)$	.11	.034	.025	.044	.032	.021	.038	.021	.036	.008
$P(n_d \geq C_\alpha)$	.055	.064	.055	.093	.076	.057	.104	.071	.135	.059

where $p_+ = p_- = (1 - p_0)/2 = p$

The proposed test statistic is given by:

$$N_d = N_+ - N_-,$$

where N_+ and N_- are the number of positive and negative differences observed in a random sample of *n* pairs of observations, as defined in (4). H_0 is rejected if $n_d > C_\alpha$, where n_d is the realization of N_d and C_α is the critical value for α level of significance. Thereafter, one could easily show that the probability distribution of N_d is given by

$$P(N_D = n_d) = \sum_{k=0}^{\frac{n-n_d}{2}} \frac{n!}{(n_d + k)!k!(n - n_d - 2k)!} \left(\frac{1 - p_0}{2}\right)^{n_d + 2k} (p_0)^{n - n_d - 2k}$$

Here, the critical values C_α can be easily calculated. As an illustration, we display the critical values in Table 1 for the case where *n* = 10 and $\alpha = 0.05$.

In practice, when the value of *p*0 is unknown, we use the unbiased estimate n_0/n to replace p_0 to perform the trinomial test. When *n* = 10 and $\alpha = .05$, the rejection region of the trinomial test based on Table 1 (in the order (n_+, n_0, n_-)) is:

(10, 0, 0), (9, 0, 1), (9, 1, 0), (8, 1, 1), (8, 2, 0), (7, 2, 1),

(7, 3, 0), (6, 3, 1), (6, 4, 0), (5, 5, 0), (4, 6, 0).

When *n* = 10 and α = .05, the rejection region of the sign test obtained from the binomial table is:

(10, 0, 0), (9, 0, 1), (9, 1, 0), (8, 1, 1), (8, 2, 0), (7, 2, 1), (7, 3, 0), (6, 4, 0), (5, 5, 0).

Comparing the two rejection regions, we find that the points (6, 3, 1) and (4, 6, 0) are only in the rejection region of the trinomial test. Therefore, in the case of *n* = 10 and α = .05, the trinomial test is more powerful than the sign test (for any value of p_0).

One could easily show that the power function of the trinomial test is given by

$$\pi_T\ (p_+, p_0; \alpha, n) = \sum_{n_0=0}^{n} \sum_{n_+=C_\alpha(n_0/n)+1}^{n-n_0} P(\ n_+, n_0; p_+, p_0) \tag{6}$$

and the power function of the sign test is given by

$$\pi_S\ (p_+, p_0; \alpha, n) = \sum_{n_0=0}^{n} \sum_{n_+=C_\alpha^*(n-n_0)+1}^{n-n_0} P(\ n_+, n_0; p_+, p_0) \tag{7}$$

where

$$p(n_+, n_0; p_+^-, p_0) = \binom{n}{n_+ n_0 (n - n_+ - n_0)} p_+^{n_+} p_0^{n_0} (1 - p_+ - p_0)^{n-n_+-n_0}$$

Note that the critical value of $C_\alpha(n_0/n)$ of the trinomial test depends on (n_0/n), the unbiased estimate of the unknown probability p_0, whereas the critical value $C_\alpha^*(n-n_0)$ of the sign test depends on $(n-n_0)$, the number of non-zero signs. The power functions of these two tests in the case where $n = 10$ and $\alpha = .05$ are displayed in Table 2.

4 The Power Comparison

Associated with any statistical test procedure is the natural question of how to assess its performance in detecting the correct alternative. This question would be easily resolved if there existed a test that has power which was always at least as great as that of any other tests for parameters with values in the alternative region, given a fixed significance level α. We would resort to theories such as the Neyman Pearson Lemma to generate uniformly most powerful tests. However, it is seldom observed that a nonparametric distribution-free test procedure is uniformly more powerful than its competitors. Therefore, one option is to obtain expressions for the power functions of two competing test procedures for comparing the relative properties of the two test statistics.

Another option is to compute the powers of the two test statistics. Such a comparison would usually depend on: (i) the sample size *n*, (ii) the value of the alternative, and (iii) the chosen significance level α . We use this method to compare the power of the trinomial test with that of the sign test.

4.1 Power Comparison of Sign Test versus Trinomial Test in Small Samples

The power function is extensively employed by statisticians to assess the performance of a test procedure. When the sample size is large, one can use the binomial approximation and the usual sign test, even in the presence of a considerable number of ties. In the case of small samples, for example, a sample of size $n = 10$ in which we have, say, 4 ties, the usual sign test is not particularly useful. However, the proposed trinomial test is found to be useful in such situations.

In this section we compare the power of the trinomial test against that of the sign test based on 100,000 simulated samples of size 10. Here, the value of p_0 is estimated by the ratio (n_0 / n), and a significance level $\alpha = 0.05$ is used. The simulation results are displayed in Table 2.

From Table 2, it is clear that the performance of the trinomial test is superior to that of the sign test as the former takes into account the presence of ties while the latter ignores the presence of ties. Thus, we recommend the trinomial test for cases with a reasonable number of ties in small samples.

4.2 Power Comparison of Sign Test versus Trinomial Test in Large Samples

The trinomial test regards the number of zero differences, if any exist, as a random variable. The following trinomial distribution can be derived:

$$(N_+, N_0, N_-) \sim Trinomial(n, p_+, p_0, p_-) .$$

Consider the following hypothesis:

$$H_0 : p_+ = p_- \text{ versus } H_1 : p_+ > p_- .$$

Let $p_+ - p_- = \delta > 0$. Observing a sample of *n* pairs, from Section 2, the test statistic is given by

$$n_d = n_+ - n_- .$$

where n_+ and n_l are the realizations of N_+ and N_- defined in (4).

When the sample size *n* is reasonably large, we can use the normal approximation to the binomial distribution. Denoting $\delta = p_+ - p_-$, we have $p_+ = (1 - p_0 + \delta)/2$ and $p_- = (1 - p_0 - \delta)/2$. For α level of significance, one could easily derive the power of the trinomial test to be:

Table 2: Power of Sign Test versus Trinomial Test

p^+	Sign Test	Trinomial Test ($p_0 = n_0 / n$)
	($p_0 = 0.1$)	
0.450	0.019	0.022
0.500	0.039	0.044
0.550	0.076	0.084
0.600	0.135	0.146
0.650	0.222	0.238
0.700	0.372	0.750
0.517	0.540	0.352
0.800	0.708	0.730
0.850	0.896	0.912
	($p_0 = 0.2$)	
0.400	0.021	0.033
0.450	0.045	0.066
0.500	0.088	0.121
0.550	0.158	0.208
0.600	0.268	0.332
0.650	0.416	0.494
0.700	0.608	0.691
0.750	0.818	0.881
	($p_0 = 0.3$)	
0.350	0.020	0.036
0.400	0.044	0.075
0.450	0.090	0.142
0.500	0.170	0.250
0.550	0.291	0.400
0.600	0.468	0.595
0.650	0.694	0.807
	($p_0 = 0.5$)	
0.250	0.013	0.033
0.300	0.038	0.079
0.350	0.089	0.167
0.400	0.185	0.312
0.450	0.353	0.532
0.470	0.448	0.643
0.490	0.563	0.765

$$\text{power(trinomial)} = P\left[(n_+ - n_-) > z_\alpha \sqrt{2np} \middle| H_1\right]$$

$$= 1 - \Phi\left\{\frac{z_\alpha - \dfrac{\sqrt{n}}{\sqrt{1-p_0}}\delta}{\dfrac{\sqrt{(1-p_0-\delta^2)}}{\sqrt{1-p_0}}}\right\} \quad . \tag{8}$$

As is usual practice in comparing the medians of two samples, we ignore the information of zero differences when applying the sign test. To compare the performance of our proposed test with that of the sign test, in this section we derive the power of the sign test when zero differences are present in the observations.

Let $n^* = (n_+, n_0, n_-)$, the distribution of n^* is expressed as:

$$f(n^*) = \binom{n}{n_+ n_0 n_-} p_+^{n_+} p_0^{n_0} p_-^{n_-}$$

which is the same as in the trinomial case.

The conditional distribution of n^* given n_0 can then be derived as:

$$f(n^* | n_0) = \frac{f(n^*)}{f(n_0)}$$

$$= \frac{\dfrac{n!}{n_+!n_0!n_-!} p_+^{n_+} p_0^{n_0} p_-^{n_-}}{\dfrac{n!}{n_0!(n-n_0)} p_0^{n_0} (1-p_0)^{n-n_0}}$$

$$= \frac{(n-n_0)!}{n_+!n_-!}\left(\frac{p_+}{1-p_0}\right)^{n_+}\left(\frac{p_-}{1-p_0}\right)^{n_-}$$

Hence, we have

$$n_+ | n_0 \sim B\left(n - n_0, \frac{p_+}{1 - p_0} \right).$$

Consider the sign test for the following hypotheses

$$H_0: \; p'_+ = p'_- = \frac{1}{2} \text{ or } p'_+ = p'_-$$
$$H_1: \; p'_+ - p'_- = \Delta > 0. \tag{9}$$

where

$$p'_+ = P(n_+ | n_0) = \frac{p_+}{1 - p_0}$$
$$p'_- = P(n_- | n_0) = \frac{p_-}{1 - p_0}$$

Under H_0, we have

$$E(n_+ | n_0) = (n - n_0)\, p'_+ = \frac{1}{2}(n - n_0)$$

and

$$V(n_+ | n_0) = (n - n_0)\, p'_+ (1 - p'_+) = \frac{1}{4}(n - n_0)$$

Assuming that the sample size is large, one can easily obtain the size α , of the test to be:

$$P\left[n_+ > z_\alpha \sqrt{\frac{1}{4}(n - n_0)} + \frac{n - n_0}{2} \middle| H_0, n_0 \right]$$

Under H_1, we have

$$E(n_+ | n_0) = (n - n_0)\, p'_+ = (n - n_0)\left(\frac{1 + \Delta}{2} \right)$$

and

$$V(n_+ | n_0) = (n - n_0)\left(\frac{1 - \Delta_2}{4} \right)$$

Thereafter, the power of the sign test can be obtained to be:

$$P\left[z \text{ ¡Ý}\!\left(\frac{z_\alpha \sqrt{(n-n_0)} - (n-n_0)\Delta\,)}{\sqrt{(n-n_0)(1-\Delta_2\,)}} \right]\right] = 1-\Phi\left[\frac{z_\alpha - \sqrt{(n-n_0)}\Delta}{\sqrt{(1-\Delta_2\,)}} \right]$$

We compare the power of the trinomial test with that of the sign test by varying

$$\begin{aligned} \Delta &= p'_{+} - p'_{-} \\ &= \frac{p_+}{1-p_0} - \frac{p_-}{1-p_0} \\ &= \frac{\delta}{1-p_0} \end{aligned}$$

For the case when there is no zero observation (difference), we have $p_0 = 0$ and $\delta = \Delta$. Therefore, when $p_0 = 0$; $n_0 = 0$, following from (8) and (10), we have

$$\begin{aligned} \text{Power of the trinomial test} &= 1-\Phi\left[\frac{(z_\alpha - \square\tilde{a}\,\Delta)}{\sqrt{1-\Delta^2}} \right] \\ &= \text{Power of the sign test :} \end{aligned}$$

For situations in which there are zero observations, we obtain the following theorem:

Theorem 1 *If n is large,* $p_0 > 0$ *and* $z_\alpha > \sqrt{(n-n_0)}\Delta$ *,the power of the trinomial test is always greater than or equal to that of the sign test.*

Proof: Comparing expressions (8) and (10), it can be seen that it is equivalent to show that the variance of the trinomial test is greater than or equal to that of the sign test. As

$$\text{Variance(trinomial test)} = \left(\frac{1}{1-p_0} \right)[1-p_0-\delta^2\,] = 1-(1-p_0)^2\,\Delta^2$$

and the variance of the sign test is given by $(1-\Delta^2\,)$, if $p_0 > 0$, we have

$$[1-(1-p_0)\Delta^2\,] > (1-\Delta^2\,)$$

so that the assertion of the theorem holds.

It is worth noting that the probability distribution of the test statistic is a function of the nuisance parameter Δ . Although an unbiased estimator has been suggested above, this induces randomness to the probability (for more on this, see the classic papers by Davies [4, 5], and the extensions by Andrews and Ploberger [1] and Hansen [9]).

5 Conclusion

It is well known that the power performance of the sign test is poor in the presence of zero observations. Attempts have been made to modify the sign test to increase its power in the presence of zero observations, for example, through randomized treatment of the zero observations. However, this approach has not been able to improve power.

In this paper, we used an alternate approach by developing a new test, the trinomial test, for pairwise ordinal data samples to include the treatment of zero differences between observations in the test statistic. The proposed test statistic is superior to the sign test as it includes the information of zero differences, and thereby increases uses of sample information, while the sign test does not.

Simulations demonstrated the power superiority of the proposed trinomial test statistic over the sign test in small samples in the presence of zero observations. We also showed that the proposed trinomial test was substantially superior to the sign test in power in large samples in the presence of zero observations as the sign test ignores information from the observations resulting in ties.

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