Dynamic Conditional Correlations for Asymmetric Processes*

Manabu Asai
Faculty of Economics
Soka University, Japan

Michael McAleer
Econometric Institute
Erasmus School of Economics
Erasmus University Rotterdam
and
Tinbergen Institute
The Netherlands
and
Institute of Economic Research
Kyoto University, Japan

EI 2010-76

Revised: December 2010

* For financial support, the first author acknowledges the Japan Society for the Promotion of Science and the Australian Academy of Science, and the second author wishes to acknowledge the Australian Research Council, National Science Council, Taiwan, and the Japan Society for the Promotion of Science.

Abstract

The paper develops two Dynamic Conditional Correlation (DCC) models, namely the Wishart DCC (WDCC) model and the Matrix-Exponential Conditional Correlation (MECC) model. The paper applies the WDCC approach to the exponential GARCH (EGARCH) and GJR models to propose asymmetric DCC models. We use the standardized multivariate *t*-distribution to accommodate heavy-tailed errors. The paper presents an empirical example using the trivariate data of the Nikkei 225, Hang Seng and Straits Times Indices for estimating and forecasting the WDCC-EGARCH and WDCC-GJR models, and compares the performance with the asymmetric BEKK model. The empirical results show that AIC and BIC favour the WDCC-EGARCH model to the WDCC-GJR and asymmetric BEKK models. Moreover, the empirical results indicate that the WDCC-EGARCH-t model produces reasonable VaR threshold forecasts, which are very close to the nominal 1% to 3% values.

Keywords: Dynamic conditional correlations, Matrix exponential model, Wishart process, EGARCH, GJR, asymmetric BEKK, heavy-tailed errors.

1 Introduction

The class of multivariate Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models has been used to model the co-movements of volatilities in financial assets. The various model specifications can be categorized as follows: (i) diagonal GARCH model of Bollerslev, Engle and Wooldridge (1998) and Ding and Engle (2001); (ii) BEKK (Baba, Engle, Kraft and Kroner) model of Engle and Kroner (1995), which models the conditional covariances directly; (iii) constant conditional correlation (CCC) model of Bollerslev (1990), VARMA-GARCH model of Ling and McAleer (2003), and VARMA-AGARCH model of McAleer, Hoti and Chan (2007); (iv) Engle's (2002) dynamic conditional correlation (DCC) model, Tse and Tsui's (2002) varying conditional correlation (VCC) model, and Bauwens, Laurent and Rombouts's (2006) generalized DCC model, and McAleer et al.'s (2008) Generalized Autoregressive conditional correlation (GARCC) model, which relax the assumption of constant conditional correlations and model the dynamic conditional correlations and covariances; (v) generalized orthogonal GARCH model of van der Weide (2002); and (vi) the matrix-exponential GARCH model of Kawakatsu (2006). For further details of these models, see the review papers of McAleer (2005) and Bauwens, Laurent and Rombouts (2006).

For multivariate GARCH models, the primary concerns are the positive-definiteness of the conditional covariance matrices and the large numbers of parameters. Regarding the latter issue, the number of parameters increases with the square of the dimension. One of the primary advantages of the DCC, VCC and GARCC models is that they reduce drastically the number of parameters in the time-varying structures of the conditional correlation and covariance matrices.

In the framework of univariate models, the asymmetric GARCH approach is typically modelled by using either the exponential GARCH (EGARCH) model of Nelson (1991) or the GJR (alternatively, the threshold GARCH) model of Glosten, Jagannathan and Runkle (1992), whereby positive and negative shocks or equal magnitude have different effects on conditional volatility. The GJR model uses a threshold indicator function to describe the asymmetric effects. On the other hand, one of the appealing features of the EGARCH model is that it is a discrete time approximation to the continuous time asymmetric stochastic volatility model, as shown in Nelson (1990). Although Deb (1996) showed that the absolute value function in the EGARCH model is known to lead to bias in finite samples, the problem can be avoided by either of the following two approaches: (i) approximate the absolute value function by the rectangular hyperbola rotated counterclockwise by 45 degrees; or (ii) employ two step estimation for the conditional mean and conditional variance components (see Hentschel (1995) for further details).

For multivariate models, Kroner and Ng (1998) proposed the asymmetric BEKK model, while McAleer, Hoti and Chan (2009) suggested the asymmetric VARMA-GARCH (or VARMA-AGARCH) model as a multivariate extension of the GJR model. Both of these models are multivariate generalizations of the univariate GJR model as they are based on threshold effects. Although the former is very flexible due to the BEKK specification, it suffers from the traditional large number of parameters associated with the BEKK specification. The latter model is an extension of the VARMA-GARCH model, and hence assumes constant conditional correlations. Recently, Kawakatsu (2006) suggested the matrix-exponential GARCH model, which is a multivariate extension of the EGARCH model.

The purpose of this paper is to develop alternative specifications within the DCC class. Two approaches will be developed, with one based on the Wishart distribution and the other on the matrix-exponential model of Chiu, Leonard and Tsui (1996). We employ the new DCC specification to propose two asymmetric DCC GARCH models, which are based on the EGARCH and GJR models, respectively. For the heavy-tails associated with financial returns, the standardized multivariate *t*-distribution is used. As a benchmark, we will use the asymmetric BEKK model, and also discuss the matrix-exponential GARCH model.

In the remainder of the paper, Section 2 develops the two new DCC models. Section 3 applies the theoretical results to suggest the asymmetric DCC class based on the GJR and EGARCH models. Section 4 presents an empirical example using the trivariate data of the Nikkei 225 Index, Hang Seng Index and Straits Times Index, and examines estimation of the parameters and forecasts of the VaR thresholds, based on the new class of models.

2 Alternative DCC Models

2.1 Background

Let the returns on $m\,(\geq 2)$ financial assets be given by

$$y_t = \mu_t + \varepsilon_t, \tag{1}$$

where $y_t = (y_{1t}, \ldots, y_{mt})'$, $\varepsilon_t = (\varepsilon_{1t}, \ldots, \varepsilon_{mt})'$, $\mu_t = (\mu_{1t}, \ldots, \mu_{mt})' = E(y_t \mid \Im_{t-1})$, and $\Im_t$ is the past information available at time t. It is assumed that

$$\begin{aligned} \varepsilon_t &= D_t \eta_t, \\ \eta_t \mid \Gamma_t &\sim \text{iid}(0, \Gamma_t), \end{aligned} \tag{2}$$

where Γ_t denotes the time-varying conditional correlation matrix, $D_t = \left[\text{diag}\{h_t\}\right]^{1/2}$, $h_t = (h_{1t}, \ldots, h_{mt})'$, $\text{diag}\{x\}$ for any vector x denotes a diagonal matrix with x along the diagonal, and h_{it} is the conditional variance for each asset.

It then follows that the conditional covariance matrix is given by

$$Q_t = V(y_t \mid \Im_{t-1}) = E(\varepsilon_t \varepsilon_t' \mid \Im_{t-1}) = D_t \Gamma_t D_t. \tag{3}$$

While some authors, including Bollerslev, Engle and Wooldridge (1988) and Engle and Kroner (1995), have developed multivariate GARCH specifications in order to model Q_t, Engle (2002) concentrated on modelling Γ_t, the matrix of dynamic conditional correlations.

By using the Hadamard product, Ding and Engle (2001) provided a new representation of the diagonal GARCH model that was developed by Bollerslev, Engle and Wooldridge (1988). The simplest diagonal GARCH model is given as follows:

$$Q_t = \Omega + \Psi_1 \circ \varepsilon_{t-1}\varepsilon_{t-1}' + \Psi_2 \circ Q_{t-1}, \tag{4}$$

where Ω, Ψ_1 and Ψ_2 are assumed to be positive semi-definite matrices, and '$\circ$' denotes the Hadamard product of two identically-sized matrices or vectors, which is computed simply by element-by-element multiplication. Ding and Engle (2001) argued that, if any one of Ω, Ψ_1 and Ψ_2 is positive definite, then Q_t will also be positive definite.

On the other hand, the CCC and DCC models assume that the conditional variance of each asset follows the GARCH process, that is:

$$h_t = \omega + \alpha \circ \varepsilon_{t-1} \circ \varepsilon_{t-1} + \beta \circ h_{t-1}, \tag{5}$$

where $\omega = (\omega_1, \ldots, \omega_m)'$, $\alpha = (\alpha_1, \ldots, \alpha_m)'$, and $\beta = (\beta_1, \ldots, \beta_m)'$. If we specify $\Gamma_t = \Gamma$ for all t, then we have the CCC model, as proposed by Bollerslev (1990). Engle (2002) proposed the specification of Γ_t as follows:

$$\Gamma_t = C_t^{-1} P_t C_t^{-1}, \tag{6}$$

$$C_t = \left[\text{diag}\left\{ \text{vecd}\left(P_t \right) \right\} \right]^{1/2} \tag{7}$$

$$P_t = \left(\iota_m \iota_m' - \Theta_1 - \Theta_2 \right) \circ P + \Theta_1 \circ \eta_{t-1} \eta_{t-1}' + \Theta_2 \circ P_{t-1}, \tag{8}$$

where ι_m is the $m \times 1$ unit vector, P is a positive definite matrix, and 'vecd' creates a vector by stacking the diagonal elements of a matrix. As in the diagonal GARCH model, one of Θ_1, Θ_2 and $\left(\iota_m \iota_m' - \Theta_1 - \Theta_2 \right)$ is assumed to be positive definite, and the remaining two can be positive definite or semi-definite.

Engle (2002) suggested a simpler model than in equation (8) that is based on scalar parameters, as follows:

$$P_t = \left(1 - \theta_1 - \theta_2 \right) P + \theta_1 \eta_{t-1} \eta_{t-1}' + \theta_2 P_{t-1}, \tag{9}$$

where $\theta_1 > 0$, $\theta_2 > 0$ and $\theta_1 + \theta_2 < 1$.

2.2 Wishart Approach

In order to present the basic idea of the approach to be adopted in this paper, we will begin with a Wishart variate, $\Xi \sim W_m(k, P)$, where P may be the constant part of the time-varying correlation matrix, as given in equation (8). Now consider the following process:

$$P_t = \frac{1}{k} P_{t-1}^{d/2} \Xi_t^{1-d} P_{t-1}^{d/2}, \tag{10}$$

where $k > 1$ and $|d| < 1$. The last condition is required for stationarity. Taking the log-determinant of both sides of equation (10) gives

$$\log|P_t| = -\log k + d \log|P_{t-1}| + (1-d)\log|\Xi_t|,$$

so that $\log|P_t|$ follows an AR(1) process. Clearly, $\log|P_t|$ is the weighted average of $\log|P_{t-1}|$ and $\log|k^{-1}\Xi_t|$. As the mean of Ξ_t is kP, this representation provides the motivation for the approach to be adopted in the paper.

It should be noted that the model in equation (10) is different from the Wishart Inverse Covariance (WIC) model of Asai and McAleer (2009) in the sense that $1-d$ is the exponent of Ξ_t, so that P_t in equation (10) does not have the Wishart distribution, unlike the WIC model. However, the weighted average of $\log|P_t|$ arises from the presence of $1-d$.

Based on the above structure, we now propose a new DCC model. If η_t has a normal distribution, then $\eta_t\eta_t'|P_t \sim W_m(1, P_t)$. However, as $\eta_t\eta_t'$ is positive semi-definite, it cannot be a proxy for Ξ_{t+1}. Instead, $\eta_t\eta_t' + (k-1)P$ is used in order to derive

$$P_t = \frac{1}{k} P_{t-1}^{d/2} \left[\eta_{t-1}\eta_{t-1}' + (k-1)P\right]^{1-d} P_{t-1}^{d/2}, \tag{11}$$

where $k > 1$ and $|d| < 1$. The number of parameters for the correlation structure is given by $0.5m(m-1)+2$, which is the same as for the scalar DCC model. In order to distinguish this model from Engle's DCC, we will refer to it as the Wishart DCC (WDCC) model.

2.3 Matrix Exponential Approach

By using the matrix exponential operator, we can also consider alternative specifications. For any $m \times m$ matrix A, the matrix exponential transformation is defined by the following power series expansion:

$$\mathrm{Exp}(A) = \sum_{s=0}^{\infty} (1/s!) A^S,$$

where A^0 reduces to the $m \times m$ identity matrix and A^s denotes the standard matrix multiplication of A s times. Thus, in general, the elements of $\mathrm{Exp}(A)$ do not typically exponentiate the elements of A. Note that the matrix exponential is not element-by-element exponentiation of each of its element, even for a diagonal matrix.

For convenience, we use log(.) and exp(.) as the element-by-element logarithmic and exponential operators, respectively. For any symmetric matrix *A*, Exp(*A*) is always positive definite. This property is attractive for modelling covariance and correlation structures (for further details, see Chiu, Leonard and Tsui (1996), Kawakatsu (2006), and Asai, McAleer and Yu (2006)).

Instead of the DCC process in equation (8), we suggest a new specification, as follows:

$$P_t = \mathrm{Exp}\left(S_t\right),$$

$$S_t = S + A \circ \eta_{t-1}\eta'_{t-1} + B \circ S_{t-1}, \tag{12}$$

where S, A and B are $m \times m$ symmetric matrices of parameters. If S_0 is symmetric, then the S_t are symmetric for all t, so that the P_t are positive definite, by definition. As in the case of the DCC model of Engle (2002), we suggest a scalar DCC model, which is based on the following specification:

$$S_t = S + a\eta_{t-1}\eta'_{t-1} + bS_{t-1}, \tag{13}$$

where S is an $m \times m$ symmetric matrix of parameters, and a and b are scalar parameters. For convenience, we will refer to this model as the Matrix-Exponential Conditional Correlation (MECC) model.

We also propose an intermediate model as follows:

$$S_t = S + A \circ \eta_{t-1}\eta'_{t-1} + bS_{t-1},$$

as the simple DCC model of Engle (2002) has been criticized because of the overly strong assumption that all the conditional correlations follow the same process.

Regarding the stationarity conditions, we need to check each element of S_t. For the (i,j)-element of S_t, we have

$$\begin{aligned} s_{ij,t} &= s_{ij} + a_{ij}\eta_{i,t-1}\eta_{j,t-1} + b_{ij}s_{ij,t-1} \\ &= s^*_{ij} + b_{ij}s_{ij,t-1} + v_{ij,t-1} \end{aligned}$$

where $v_{ij,t-1} = a_{ij}\left\{\eta_{i,t-1}\eta_{j,t-1} - E\left(\eta_{i,t-1}\eta_{j,t-1}\right)\right\}$ and $s^*_{ij} = s_{ij} + a_{ij}E\left(\eta_{i,t-1}\eta_{j,t-1}\right)$. Hence, $v_{ij,t}$ has mean zero and finite variance if the fourth moment of η_t exists. In this case, $s_{ij,t}$

follows an AR(1) process, so that the stationarity condition is given by $|b_{ij}| < 1$, which must be satisfied for any *i* and *j*. For the scalar and intermediate DCC-EGARCH models, the stationarity condition is $|b| < 1$.

The merit of this approach is that positive-definiteness is automatically obtained by the matrix-exponential transformation. On the other hand, a drawback of the matrix-exponential operator is that it is not easy to interpret the connection between the (*i*,*j*)-element of Exp(*A*) and the (*i*,*j*)-element of *A* itself (see also the discussion in the following section). For this reason, we will emphasize the DCC specifications based on the Wishart distribution in the reminder of the paper.

3 WDCC-EGARCH and WDCC-GJR Models

Using the DCC structure and the Wishart approach, we propose two new families of DCC models, namely the WDCC-EGARCH and WDCC-GJR models, that are based on equations (6), (7) and (11).

For the WDCC-EGARCH model, we assume that the conditional variance of each asset follows the EGARCH process, namely:

$$\log h_t = \kappa + \phi \circ \log h_{t-1} + \gamma \circ \eta_{t-1} + \delta \circ |\eta_{t-1}|, \tag{14}$$

where $\kappa = (\kappa_1, \ldots, \kappa_m)'$, $\phi = (\phi_1, \ldots, \phi_m)'$, $\gamma = (\gamma_1, \ldots, \gamma_m)'$, and $\delta = (\delta_1, \ldots, \delta_m)'$. Depending on the values of the parameters, the EGARCH model can capture asymmetry and leverage, whereby negative shocks increase volatility and positive shocks decrease volatility.

In the WDCC-GJR model, the conditional variance of each asset follows the GJR process, namely:

$$h_t = \omega + \left(\alpha^+ \circ \left\{\iota_m - d_{t-1}^-\right\} + \alpha^- \circ d_{t-1}^-\right) \circ \varepsilon_{t-1} \circ \varepsilon_{t-1} + \beta \circ h_{t-1}, \tag{15}$$

where $\alpha^{+} = \left(\alpha_1^{+}, \ldots, \alpha_m^{+}\right)'$ and $\alpha^{-} = \left(\alpha_1^{-}, \ldots, \alpha_m^{-}\right)'$. The vector $d_t^{-} = \left(d_{1t}^{-}, \ldots, d_{mt}^{-}\right)'$ denotes a set of indicator variables, and d_{it}^{-} takes the value of one if $\varepsilon_{it} < 0$, and zero otherwise.

We now consider estimation of the WDCC-EGARCH and WDCC-GJR models. Assuming normality of the conditional distribution of the standardized residuals, we can estimate the parameters by the maximum likelihood (ML) method for the DCC class of models. The conditional log-likelihood function is given by

$$L = \sum_{i=1}^{T} l_t,$$

$$l_t = -\frac{m}{2}\log\left(2\pi\right) - \log\left|D_t\right| - \frac{1}{2}\log\left|\Gamma_t\right| - \frac{1}{2}\varepsilon_t' D_t^{-1}\Gamma_t^{-1}D_t^{-1}\varepsilon_t.$$

If the assumption of normality does not hold for the standardized residuals, the procedure is defined as the quasi-maximum likelihood estimator (QMLE). For more efficient estimators using adaptive methods, see Ling and McAleer (2003).

As an alternative to the Gaussian assumption, we consider the standardized multivariate *t*-distribution for the conditional distribution. In this case, the contribution to the log-likelihood function from observation t is

$$l_t = c\left(\nu, m\right) - \log\left|D_t\right| - \frac{1}{2}\log\left|\Gamma_t\right| - \frac{\nu + m}{2}\log\left(1 + \frac{\varepsilon_t' D_t^{-1}\Gamma_t^{-1}D_t^{-1}\varepsilon_t}{\nu - 2}\right),$$

where

$$c\left(\nu, m\right) = \log\Gamma\left(\frac{\nu + m}{2}\right) - \log\Gamma\left(\frac{\nu}{2}\right) - \frac{m}{2}\log\pi - \frac{m}{2}\log\left(\nu - 2\right),$$

$\Gamma(x)$ is the complete gamma function, and ν is the degrees of freedom parameter. For the specification, ε_t has the multivariate *t*-distribution with mean zero and variance Q_t. If we consider a portfolio $\varepsilon_{p,t} = w'\varepsilon_t$ with weight vector *w*, $\varepsilon_{p,t}$ has the *t*-distribution with mean zero and variance $w'Q_t w$.

For convenience we denote WDCC-EGARCH-n and WDCC-EGARCH-*t* corresponding to the normal distribution and *t*-distributions, respectively.

Recently, Kawakatsu (2006) developed the matrix exponential GARCH model, while Asai, McAleer and Yu (2006) proposed the matrix exponential SV model. Compared with the DCC-EGARCH model that is proposed here, the other two approaches have certain drawbacks. First, these two models are based on the unconditional (or unstandardized) shocks, ε_t, instead of the standardized shocks, η_t, in order to describe the leverage and size effects. In this sense, the matrix exponential GARCH model is not a direct extension of the EGARCH model. Second, the matrix exponential GARCH and matrix exponential SV models suffer from having a large number of parameters, as in the case of the BEKK model. Although the respective authors considered alternative ways of reducing the numbers of parameters, they still exceed those of the scalar DCC-EGARCH model. Third, the interpretation of the parameters is not straightforward as matrix exponentiation is not element-by-element exponentiation. As explained in Kawakatsu (2006), it requires additional computations in order to derive the relations between the (*i*,*j*) element of the covariance matrix and the (*k*,*l*) element of the matrix-logarithmic process.

It should also be noted that the matrix exponential SV model of Asai, McAleer and Yu (2006) should perhaps be reconsidered since it is a multivariate SV model. In this regard, the superiority of univariate SV models over the GARCH and EGARCH models have frequently been shown in the volatility literature (see, for example, the review by McAleer (2005)).

Alternative asymmetric GARCH models are multivariate extensions of the GJR model. Kroner and Ng (1998) developed the asymmetric BEKK model, while McAleer, Hoti and Chan (2009) proposed the asymmetric VARMA-GARCH (or VARMA-AGARCH) model. The latter model assumes that the conditional correlations are constant. The asymmetric BEKK model is given by

$$Q_t = KK' + A\varepsilon_{t-1}\varepsilon'_{t-1}A' + BQ_{t-1}B' + C\varepsilon^*_{t-1}\varepsilon^{*\prime}_{t-1}C' , \tag{16}$$

where K is the lower triangular matrix, A, B and C are square matrices, and $\varepsilon^*_t = \min(\varepsilon_t, 0) = (1/2)\{\varepsilon_t - |\varepsilon_t|\}$.

In addition to these models, we suggest the asymmetric diagonal GARCH model, which is given by

$$Q_t = \Omega + \left(A + C \circ d^-_t d^{-\prime}_t\right) \circ \varepsilon_{t-1}\varepsilon'_{t-1} + B \circ Q_{t-1} , \tag{17}$$

where the vector $d^-_t = \left(d^-_{1t}, \ldots, d^-_{mt}\right)'$ denotes a set of indicator variables, and d^-_{it} takes the value one if ε_{it} is negative, and zero otherwise. This model is a multivariate extension of the GJR model Like the relation between the BEKK and diagonal GARCH models, diagonal specifications such that $A = \text{diag}\{a\}$, $B = \text{diag}\{b\}$ and $C = \text{diag}\{c\}$ in (16) yield

$$Q_t = KK' + (aa') \circ \varepsilon_{t-1}\varepsilon'_{t-1} + (bb') \circ Q_{t-1} + (cc') \circ \varepsilon^*_{t-1}\varepsilon^{*\prime}_{t-1} , \tag{18}$$

which is also a vector diagonal specification.

4 Empirical Results

In this section, we examine the MLE of the DCC-GARCH, DCC-EGARCH and DCC-GJR models for three sets of empirical data, namely the Nikkei 225 Index (Nikkei), Hang Seng Index (Hang Seng), and Straits Times Index (Straits Times) returns. The sample period for the three data series is 1/4/1988 to 7/17/2002, giving T = 3773 observations. Returns, y_{it}, are defined as $100 \times \{\log P_{it} - \log P_{i,t-1}\}$, where P_{it} is the closing price on day t for stock i. We use the filtered data, $\varepsilon_{it} = y_{it} - \mu_{it}$, based on the threshold AR(1) model.

Table 1 shows the MLE for two kinds of bivariate DCC-GARCH-n models; namely the DCC model of Engle (2002) and the WDCC-GARCH-n model. The data sets are from (Nikkei, Hang Seng). The estimates for the GARCH parameters are close to each other, whereas the estimates of P_{21} are significant and different from each other. With respect to the persistence of the correlation structure, $\hat{\theta}_1 + \hat{\theta}_2$ for the original DCC model is 0.999, while $\hat{d}$ for the new DCC is lower at 0.975. The AIC and BIC criteria favour the new WDCC to Engle's (2002) DCC model, implying that the WDCC model is competitive with the standard DCC version. The important point to be made is that the DCC specification of Engle (2002) is not the only approach for describing dynamic conditional correlations, and is certainly not the best approach empirically.

Table 2 presents the ML estimates for the trivariate WDCC-GARCH-n model. Compared with the results in Table 1, the estimate of *d* becomes smaller, while the estimate of *k* becomes larger. This is quite reasonable as the common components of trivariate variables are smaller than those of their bivariate counterparts.

Table 3 presents the ML results for the trivariate WDCC-GARCH-*t* model. The estimate of ν is 6.59, showing that the conditional distribution is far from a normal distribution. The likelihood ratio test rejects the null hypothesis of normality. Hence, we will employ the multivariate standardized *t*-distribution in the remainder of the paper. The estimates of correlations in *P* for Table 3 are smaller than those in Table2.

Table 4 gives the results for the trivariate WDCC-GJR-*t* model. The estimates of α_i^+ are significantly different from those of α_i^-, indicating that there are asymmetric effects in the conditional volatilities. The AIC and BIC criteria also favour the WDCC-GJR-*t* model relative to the WDCC-GARCH-*t* model. The estimates of *P*, *d* and *k* are close to those of the WDCC-GARCH model, implying that the inclusion of asymmetric effects alters slightly the dynamic conditional correlations.

Table 5 gives the ML estimates of the trivariate WDCC-EGARCH-*t* model. The estimates of δ_i are positive and significant, while those of γ_i are negative and significant, which are typical for EGARCH specifications. The AIC and BIC criteria for the WDCC-EGARCH-*t* model are smaller than those of the WDCC-GARCH-*t* model, while the estimates of *P*, *d* and *k* are close to those of the WDCC-GARCH-*t* and WDCC-GJR-*t* models. For the asymmetric models, the AIC and BIC criteria both favour the WDCC-EGARCH-*t* specification.

In order to compare the new asymmetric WDCC models, we also estimate the asymmetric BEKK model. We use the standardized multivariate *t*-distribution for the distribution of η_t. In order to reduce the number of parameters, we use diagonal specifications for *A*, *B* and *C*, and refer to the asymmetric diagonal BEKK-*t* model as AD-BEKK-*t*. It should be noted that the scalar BEKK models are not analyzed, as Engle (2002) showed the superiority of the DCC-GARCH model over the scalar BEKK model on the basis of Monte Carlo simulations.

The numbers of parameters for the DCC-GJR-*t*, DCC-EGARCH-*t* and AD-BEKK-*t* are $.5\left(m^2+7m\right)+3$, $.5\left(m^2+7m\right)+3$ and $.5\left(m^2+7m\right)+1$, respectively. For the number of parameters, the difference among the three models is 2.

Table 6 shows the ML estimates for the AD-BEKK-*t* model. The estimate of ν is 6.79, showing the rejection of the normality assumption. The estimates of c_{ii} are significant, indicating that the negative shock has a larger effect than a positive shock of similar magnitude. The AIC and BIC criteria for the AD-BEKK-*t* model are close to those of the WDCC-GJR-*t* model. Among the WDCC-GJR-*t*, WDCC-EGARCH-*t* and AD-BEKK-*t* models, the AIC and BIC criteria select the WDCC-EGARCH-*t* as the best empirically.

Next, we compare the out-of-sample forecasts for the Value-at-Risk (VaR) for the WDCC-EGARCH-*t* and AD-BEKK-*t* models. Fixing the sample size in estimation to be 500, we re-estimate the model and forecast one-step-ahead VaR thresholds for the last 500 observations, where the 1 percent VaR threshold is given by $\hat{\mu}_{t|t-1}+t^s_{0.01}\left(\nu\right)\times\sqrt{\hat{h}_{p,t|t-1}}$, where $\hat{\mu}_{t|t-1}$ and $\hat{h}_{t|t-1}$ are the one-step-ahead predictions of the mean and variance, respectively, and $t^s_{0.01}\left(\nu\right)$ is the 1 percentile of the standardized *t*-distribution with degrees-of-freedom given by ν. Note that $\hat{\mu}_{t|t-1}$ is the same for the two models, and that this setting makes the effects of each volatility forecast more clear. Table 7 gives the failure percentages for the VaR forecasts based on the WDCC-EGARCH-*t* and AD-BEKK-*t* models with respect to the true values for 1%-3%. The tail behaviour of the two models is quite similar, although the WDCC-EGARCH-*t* produces slightly more conservative results.

5 Concluding Remarks

In this paper, we proposed alternative Dynamic Conditional Correlation (DCC) models based on two approaches, namely the Wishart distribution and the matrix-exponential approaches. For a clear interpretation of the models, we chose the Wishart DCC approach in order to develop the new WDCC-EGARCH and WDCC-GJR models.

The standardized multivariate *t*-distribution was used to capture the well known heavy-tails associated with financial assets. An empirical example for the trivariate data of the Nikkei 225, Hang Seng and Straits Times Index returns showed that AIC and BIC favoured the WDCC-EGARCH-*t* model to the WDCC-GJR-*t* and asymmetric BEKK-*t* models. Moreover, the empirical results indicated that the WDCC-EGARCH-t model produced reasonable VaR threshold forecasts, which are very close to the nominal 1% to 3% values.

References

Asai, M. and M. McAleer (2008), "A Portfolio Index GARCH Model", *International Journal of Forecasting*, **24**, 449-461.

Asai, M. and M. McAleer (2009), "The Structure of Dynamic Correlations in Multivariate Stochastic Volatility Models", *Journal of Econometrics*, **150**, 182-192.

Asai, M., M. McAleer, and J. Yu (2006), "Multivariate Stochastic Volatility: A Review", *Econometric Reviews*, **25**, 145-175.

Bauwens L., S. Laurent and J.K.V. Rombouts (2006), "Multivariate GARCH Models: A Survey", *Journal of Applied Econometrics*, **21**, 79-109.

Bollerslev, T. (1990), "Modelling the Coherence in Short-run Nominal Exchange Rates: A Multivariate Generalized ARCH Approach", *Review of Economics and Statistics*, **72**, 498-505.

Bollerslev, T., R.F. Engle and J.M. Wooldridge (1988), "A Capital Asset Pricing Model with Time Varying Covariances", *Journal of Political Economy*, **96**, 116-131.

Chiu, T.Y.M, T. Leonard and K.-W. Tsui (1996), "The Matrix-Logarithmic Covariance Model", *Journal of the American Statistical Association*, **91**, 198-210.

Deb, P. (1996), "Finite Sample Properties of the Maximum Likelihood Estimator of EGARCH Models", *Econometric Reviews*, **15**, 51-68.

Ding, Z. and R.F. Engle (2001), "Large Scale Conditional Covariance Matrix Modeling, Estimation and Testing", *Academia Economic Papers*, **1**, 83-106.

Engle, R.F. (2002), "Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models", *Journal of Business and Economic Statistics*, **20**, 339-350.

Engle, R.F. and K.F. Kroner (1995), "Multivariate Simultaneous Generalized ARCH", *Econometric Theory*, **11**, 122-150.

Glosten, L., R. Jagannathan and D. Runkle (1992), "On the Relation Between the Expected Value and Volatility of Nominal Excess Returns on Stocks", *Journal of Finance*, **46**, 1779-1801.

Hentschel, L. (1995), "All in the Family: Nesting Symmetric and Asymmetric GARCH Models", *Journal of Financial Economics*, **39**, 71-104.

Kawakatsu, H., (2006), "Matrix Exponential GARCH", *Journal of Econometrics*, **134**, 95-128.

Kroner, K. and V. Ng (1998), "Modeling Asymmetric Comovements of Asset Returns", *Review of Financial Studies*, **11**, 817-844.

Ling, S. and M. McAleer (2003), "On Adaptive Estimation in Nonstationary ARMA

Models with GARCH Errors", *Annals of Statistics*, **31**, 642-674.

McAleer, M. (2005), "Automated Inference and Learning in Modeling Financial Volatility", *Econometric Theory*, **21**, 232-261.

McAleer, M., F. Chan, S. Hoti and O. Lieberman (2008), "Generalized Autoregressive Conditional Correlation", *Econometric Theory*, **24**, 1554-1583.

McAleer, M., S. Hoti, and F. Chan (2009), "Structure and Asymptotic Theory for Multivariate Asymmetric Conditional Volatility," *Econometric Reviews*, **28**, 422-440.

Nelson, D. B. (1990), "ARCH Models as Diffusion Approximations", *Journal of Econometrics*, **45**, 7–38.

Nelson, D.B. (1991), "Conditional Heteroskedasticity in Asset Returns: A New Approach", *Econometrica*, **59**, 347-370.

Tse, Y.K., and A.K.C. Tsui (2002), "A Multivariate GARCH Model with Time-Varying Correlations", *Journal of Business and Economic Statistics*, **20**, 351–362.

van der Weide, R. (2002), "GO-GARCH: A Multivariate Generalized Orthogonal GARCH Model", *Journal of Applied Econometrics*, **17**, 549-564.

Table 1 Estimates of Two DCC-GARCH-n Models

Engle's (2002) DCC Model			New DCC Model		
Parameters	Nikkei	Hang Seng	Parameters	Nikkei	Hang Seng
ω_i	0.0199	0.0662	ω_i	0.0197	0.0632
	(0.0039)	(0.0093)		(0.0038)	(0.0090)
α_i	0.0832	0.1119	α_i	0.0845	0.1116
	(0.0078)	(0.0096)		(0.0079)	(0.0096)
β_i	0.9111	0.8684	β_i	0.9094	0.8694
	(0.0080)	(0.0100)		(0.0081)	(0.0099)
P_{21}	0.5561		P_{21}	0.2741	
	(0.1276)			(0.0495)	
θ_1	0.0095		k	1.1421	
	(0.0022)			(0.0773)	
θ_2	0.9898		d	0.9753	
	(0.0026)			(0.0151)	
LogLike	-12704.6		LogLike	-12703.7	
AIC	25427.2		AIC	25425.4	
BIC	25483.3		BIC	25481.5	

Notes: Standard errors are in parentheses.

The structure of the DCC model of Engle (2002) is given in equation (9), and the DCC model of this paper is given in equation (11).

Table 2: Estimates of Trivariate DCC-GARCH-n Model

Parameters	Nikkei	Hang Seng	Straits Times
ω_i	0.0209	0.0656	0.0785
	(0.0039)	(0.0089)	(0.0082)
α_i	0.0844	0.0971	0.1443
	(0.0078)	(0.0083)	(0.0114)
β_i	0.9091	0.8782	0.81288
	(0.0080)	(0.0094)	
P_{2i}	0.3247	1	
	(0.0212)		
P_{3i}	0.3010	0.4644	1
	(0.0212)	(0.0188)	
k	1.6682		
	(0.1839)		
d	0.9036		
	(0.0210)		
LogLike	-17855.0		
AIC	35738.1		
BIC	35825.3		

Notes: Standard errors are in parentheses.

The structure of the DCC model in this paper is given in equation (11).

Table 3: Estimates of Trivariate DCC-GARCH-*t* Model

Parameters	Nikkei	Hang Seng	Straits Times
ω_i	0.0171	0.0495	0.0614
	(0.0040)	(0.0093)	(0.0098)
α_i	0.0783	0.0706	0.1323
	(0.0084)	(0.0085)	(0.0141)
β_i	0.9157	0.9033	0.8189
	(0.0085)	(0.0110)	(0.0177)
P_{2i}	0.2659	1	
	(0.0255)		
P_{3i}	0.2207	0.3671	1
	(0.0261)	(0.0251)	
k	1.6035		
	(0.1735)		
d	0.8951		
	(0.0212)		
ν	6.5881		
	(0.3557)		
LogLike	-17352.1		
AIC	34734.3		
BIC	34827.8		

Notes: Standard errors are in parentheses.

The structure of the DCC model in this paper is given in equation (11).

Table 4: Estimates of Trivariate DCC-GJR-*t* Model

Parameters	Nikkei	Hang Seng	Straits Times
ω_i	0.0215	0.0633	0.0650
	(0.0043)	(0.0105)	(0.0099)
α_i^+	0.0400	0.0401	0.0958
	(0.0099)	(0.0082)	(0.0144)
α_i^-	0.1359	0.1023	0.1749
	(0.0142)	(0.0128)	(0.0195)
β_i	0.9057	0.8956	0.8131
	(0.0103)	(0.0111)	(0.0178)
P_{2i}	0.2700	1	
	(0.0250)		
P_{3i}	0.2246	0.3686	1
	(0.0256)	(0.0248)	
k	1.6251		
	(0.1739)		
d	0.8860		
	(0.0222)		
ν	6.9074		
	(0.3863)		
LogLike	-17296.5		
AIC	34629.0		
BIC	34741.2		

Notes: Standard errors are in parentheses.

The structure of the DCC model in this paper is given in equation (11).

Table 5: Estimates of Trivariate DCC-EGARCH-*t* Model

Parameters	Nikkei	Hang Seng	Straits Times
κ_i	-0.0828	-0.0854	-0.1641
	(0.0102)	(0.0093)	(0.0147)
ϕ_i	0.9839	0.9771	0.9594
	(0.0026)	(0.0040)	(0.0068)
γ_i	-0.0820	-0.0415	-0.0413
	(0.0086)	(0.0085)	(0.0098)
δ_i	0.1234	0.1342	0.2226
	(0.0144)	(0.0137)	(0.0202)
P_{2i}	0.2662	1	
	(0.0264)		
P_{3i}	0.2191	0.3614	1
	(0.0269)	(0.0261)	
k	1.5404		
	(0.1624)		
d	0.90125		
	(0.0211)		
ν	6.8287		
	(0.3915)		
LogLike	-17281.0		
AIC	34598.0		
BIC	34710.2		

Notes: Standard errors are in parentheses.

The structure of the DCC model in this paper is given in equation (11).

Table 6: Estimates of Trivariate AD-BEKK-*t* Model

Parameters	Nikkei	Hang Seng	Straits Times
k_{1i}	0.1305	0	0
	(0.0128)		
k_{2i}	0.0380	0.0536	0
	(0.0173)	(0.0193)	
k_{3i}	0.2180	0.0594	0.2200
	(0.0198)	(0.0136)	(0.0150)
a_{ii}	0.0701	0.1986	0.2980
	(0.0219)	(0.0163)	(0.0186)
b_{ii}	0.9655	0.9549	0.9135
	(0.0034)	(0.0046)	(0.0076)
c_{ii}	0.3468	0.2236	0.2734
	(0.0189)	(0.0270)	(0.0294)
ν	6.7878		
	(0.3901)		
LogLike	-17303.2		
AIC	34638.4		
BIC	34738.1		

Note: Standard errors are in parentheses.

Table 7: VaR Forecasting Performance

Model	1%	2%	3%
WDCC-EGARCH-*t*	0.008	0.018	0.030
AD- BEKK-*t*	0.012	0.022	0.030

Note: The entries show the % violations of the VaR thresholds.