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# Tax Rates as Strategic Substitutes

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# Tax Rates as Strategic Substitutes

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**Abstract**

This paper analytically derives the conditions under which the slope of the tax reaction function is negative in a classical tax competition model. If countries maximize welfare, we show that a negative slope (reflecting strategic substitutability) occurs under relatively mild conditions. Simulations suggest that strategic substitutability occurs under plausible parameter configurations. The strategic tax response is crucial for understanding tax competition games, as well as for assessing the welfare effects of partial tax unions (whereby a subset of countries coordinate their tax rates). Indeed, contrary to earlier findings that have assumed strategic complementarity in tax rates, we show that partial tax unions might reduce welfare under strategic substitutability.

**JEL codes**: E62, F21, H25, H77

**Keywords**: Strategic Substitutes; Asymmetry; Strategic Tax Response; Tax Coordination.

## 1 Introduction

The welfare effects from tax coordination are crucially determined by the slope of the tax reaction function in tax competition models. For example, Konrad and Schjelderup (1999)[1] explore the impact of tax coordination whereby some countries opt in and others opt out of a tax agreement (henceforth: tax unions). They show that a tax union is unambiguously welfare improving if tax rates are strategic complements. Numerous studies do indeed start from this presumption that tax rates are strategic complements by considering governments that maximize tax revenues rather than welfare (see a.o. Kanbur and Keen, 1993). Others rely on a linear utility function in which public goods are valued more than private goods

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[1]Burbidge, de Pater, Myers and Senqupta (1997) have shown in a classical paper that it might not be feasible to design a cooperative policy that offers all countries a higher payoff compared to their best deviation strategy. This underscores the relevance of studying enhanced cooperation agreements.

(see a.o. Devereux, Lockwood and Redoano, 2008, and Bucovetsky, 2009). Both assumptions rule out a negatively sloped tax reaction function a priori.

This paper analytically derives the conditions for the slope of the tax reaction function to be negative (or: strategic substitutes) in a classical capital tax competition model whereby governments maximize welfare, using a more generally specified utility function (see Zodrow and Mieskowsky, 1986; Wilson, 1986; and Wildasin, 1988).[2] We find that the conditions for strategic substitutes are, in fact, rather mild. Hence, its case should not be ruled out as is typically done. We also run simulations, showing that strategic substitutes occur under a wide range of reasonable parameter values. Our results put serious doubts on the merits of tax unions, which are based on the assumption of strategic complementarity. The analysis in this paper reveals what circumstances make strategic substitutability more or less likely to occur.

Strategic substitutability seems at odds with empirical findings on strategic tax setting. For example, Devereux et al. (2008) find that, on average, countries respond by increasing their tax rate in response to an increase in the average level of taxation in neighboring countries. However, this average response by no means rules out that some individual countries will respond in an opposite manner. Furthermore, we have not yet seen anything as large as the formation of a tax union. It can be questioned whether recent empirical findings shed light on the long-run policy responses to such a large shock.

A very limited number of papers in the tax competition literature explicitly address the case for strategic substitutes. Wildasin (1988), Bucovetsky (1991) and Wilson (1991) explore tax competition in which countries can influence world market prices. While this introduces an interesting strategic tax element, none of them explores the slope of the tax reaction function (see also Brueckner, 2003).[3] Brueckner and Saavedra (2001) are, to our knowledge, the first to explicitly identify strategic substitutes for asymmetric countries. They, however, assume

[2]See a.o. Bulow, Geanakoplos, and Klemperer (1985) for a classical reference on the discussion of strategic complements and substitutes applied to industrial organization.

[3]Only Wilson (1991) makes a brief remark about the slope of the reaction function, saying that it depends on the utility function and the production function, including 'hard-to-interpret'properties like third derivatives of the production functions (Wilson, 1991, p. 440, footnote 13). Laussel and Le Bretton (1998) study the existence of Nash-Equilibria in tax competition models and also mention that reaction curves can be non-linear (concave). Their model, however, is considerably more stylized than ours.

a linear utility function, which is a very special case under which strategic substitutability is rare. We generalize their result, which substantially broadens the scope for strategic substitutability.

The rest of the paper is organized as follows. Section 2 introduces the tax-competition model. Section 3 derives linearized tax reaction functions analytically and discusses conditions for tax rates to be strategic substitutes. Section 4 simulates tax reaction functions and illustrates the conditions derived. Section 5 discusses an application to illustrate the welfare gains from the formation of tax unions in case of strategic substitutes. Section 6 concludes.

## 2 A model of tax-competition

Consider $n \geq 2$ countries, that are potentially asymmetric. We use $i, j \in \{1, ..., n\}$ with $i \neq j$ as country indices. Country $i$ is populated by a fixed number of $N_i$ immobile households. Population size relative to world population ($N$) will be denoted by $s_i = N_i/N$. Each household in country i has a capital endowment ($e_i$) and a labor endowment ($l = 1$), which they supply inelastically. Hence, labor constitutes a fixed factor in production. Capital is perfectly mobile internationally.

### 2.1 Firms and Capital Market Equilibrium

A representative firm in each country produces a single good using a stock of capital ($K_i$) and effort from labor ($N_i$). There is perfect competition in the output market. In each country, the production function $F(K_i, N_i)$ is homogeneous of degree one, so it can be written in intensive form: $N_i f(k_i)$ where $k_i = K_i/N_i$ denotes the capital-labor ratio employed in country $i$. $F(.)$ is concave in its two inputs and twice continuously differentiable. Hence: $f'(k_i) > 0$, $f''(k_i) < 0$. Profit maximizing firms set the marginal product of capital equal to its price: the tax-inclusive cost of capital. Firms face a distortionary source-based unit-specific tax on capital ($t_i$).[4]

[4] Lockwood (2004) studies the case for an ad-valorem tax rate. Results are comparable, although tax-competition is more intense under ad-valorem tax rates as price changes magnify the impact of taxes. This does not affect our results though.

The first-order condition for profit maximization yields for all $i$

$$f_i'(k_i) = t_i + \rho. \tag{1}$$

The after-tax rate-of-return on capital ($\rho$) is equal across countries due to the international mobility of capital. That is, $\rho$ is endogenously determined on the world's capital market such that Eqs. 1 hold and the resource constraint

$$\sum_{i=1}^{n} s_i e_i = \sum_{i=1}^{n} s_i k_i \equiv \frac{K}{N} \tag{2}$$

is satisfied, where K denotes the fixed world capital stock.[5] An increase in the tax rate of country $i$ ($t_i$) reduces the capital stock in country $i$

$$\frac{\partial k_i}{\partial t_i} = \frac{1}{f_i''(.)}\left[1 + \frac{\partial \rho}{\partial t_i}\right] < 0, \tag{3}$$

which is obtained by partially differentiating Eq. 1, holding constant $t_j$ for $j \neq i$. Eq. 3 can be signed using $f_i''(.) < 0$ and the sign of the final term (see Appendix A for a derivation)

$$-1 < \frac{\partial \rho}{\partial t_i} = \frac{-s_i(1/f_i''(.))}{\sum_{j=1}^{n} s_j(1/f_j''(.))} < 0. \tag{4}$$

When all countries are small relative to the capital market, then we have $\partial\rho/\partial t_i \approx 0$. This case is studied in Zodrow and Mieskowsky (1986) and Wilson (1986). However, $\rho$ unambiguously decreases in $t_i$ if the number of countries is small and at least some countries are large compared to the world capital market (see a.o. Wildasin, 1989; Bucovetsky, 1991 and Wilson, 1991). An increase in $t_i$ lowers the net marginal product in country $i$ and causes capital to relocate towards the remaining countries. If the capital flow is large compared to the world's capital market, this reduces the marginal product of capital abroad. The larger country $i$ is relative to the world's capital market, the stronger its market power and larger is this effect.

[5]We assume (and make sure in our simulation analysis) that $\rho > 0$, ruling out the possibility that part of the capital stock is not used.

## 2.2 Consumers

A representative consumer features a twice-continuously differentiable, monotonously increasing utility function of the form: $U_i(g_i, c_i)$, where $g_i$ and $c_i$ denote, respectively public and private consumption. Household private consumption is subject to a household budget constraint, given by

$$c_i = [f_i(.) - f_i'(.)k_i] + \rho e_i. \tag{5}$$

Hence, private consumption equals the return to labor (the wage), reflected by the term in between square brackets on the right-hand side of Eq. (5), plus interest income from the capital endowment ($\rho e_i$).

## 2.3 Government

The government maximizes welfare, which is determined by the utility of the representative household, by choosing the tax rate $t_i$. Thereby, it takes into account the government budget constraint, which restricts public consumption to tax revenues

$$g_i = t_i k_i, \tag{6}$$

and the tax rates decided on by the government of other countries: $t_j$ for all $j \neq i$. This latter assumption implies that we study Nash-equilibria. For each country, the optimum satisfies the following condition

$$\frac{\partial U_i / \partial t_i}{u_{c,i}} = \left[\frac{\partial c_i}{\partial t_i}\right] + \frac{u_{g,i}}{u_{c,i}} \left[\frac{\partial g_i}{\partial t_i}\right] = 0, \tag{7}$$

where $u_{g,i}/u_{c,i} > 0$ is short-hand notation for $u_{g,i}(c_i, g_i)/u_{c,i}(c_i, g_i)$ denoting the marginal rate of substitution (MRS) between public and private goods. The right-hand side of Eq. (7) measures the welfare effect of the tax via, respectively, changes in private consumption and public consumption, which are obtained by taking the partial derivatives of (5) and (6)

$$\frac{\partial c_i}{\partial t_i} = -f_i''(.)\frac{\partial k_i}{\partial t_i}k_i + \frac{\partial \rho}{\partial t_i}e_i < 0, \tag{8}$$

$$\frac{\partial g_i}{\partial t_i} = k_i\left[1 + \frac{t_i}{k_i}\frac{\partial k_i}{\partial t_i}\right] > 0. \tag{9}$$

Eq. (8) shows that a higher tax rate reduces private consumption for two reasons. First, a higher tax will cause an outflow of capital. The smaller capital stock reduces labor productivity and, therefore, the wage and private consumption. Second, the higher tax reduces the world net-of-tax return on capital and, therefore, interest income. This magnifies the reduction in private consumption.

Eq. (9) shows that the effect of a higher tax on public consumption depends on where we are on the Laffer curve. The first term on the right-hand side of Eq. (9) shows that a higher tax raises revenue over the existing tax base. The second term indicates that a higher tax causes an erosion of the tax base to the extent that it reduces the domestic capital stock. This reduces tax revenue, especially when the initial tax rate is high. From Eq. (8) and (7), it follows that a utility maximizing tax rate requires that Eq. (9) is positive. That is, the utility maximizing tax rate is always on the upward sloping part of the Laffer curve.

Now define $\eta_{k,i} \equiv -\frac{\partial k_i}{\partial t_i} \geq 0$ as minus the tax coefficient of capital and $\eta_{r,i} \equiv -\frac{\partial \rho}{\partial t_i} \geq 0$ as minus the tax coefficient of the interest rate. Together with Eq. (8), (9) and (3), we can rewrite Eq. (7) as

$$\frac{u_{g,i}}{u_{c,i}} = \frac{k_i + \eta_{r,i}\left[e_i - k_i\right]}{k_i - t_i\eta_{k,i}} \equiv MCF, \tag{10}$$

reflecting the modified Samuelson rule. It shows that the marginal rate of substitution between public and private goods on the left-hand side is equal to the marginal cost of public funds (henceforth MCF) times the marginal rate of transformation (which equals unity in our model). Eq. (10) shows that the MCF rises in the tax coefficient of capital $\eta_{k,i}$ as this increases the erosion of the tax base induced by the tax. Furthermore, the MCF in Eq. (10) increases in $\eta_{r,i}$ if country $i$ is a net capital exporter ($e_i > k_i$). It decreases if it is a net capital importer ($e_i < k_i$). Intuitively, a net capital exporter is a net receiver of interest vis-a-vis the rest of the world. Therefore, it suffers from a welfare loss if the interest rate drops. This makes public goods more expensive as higher taxes reduce the interest rate. This implies that for capital exporting countries we unambiguously have $u_{g,i}/u_{c,i} = MCF > 1$. This is the standard tax competition result: the public good is undersupplied due to positive spillovers from taxation. For a net capital importer, the lower interest rate is a net benefit because the country pays less to foreign capital owners. This reduces the MCF. In principle, this might even cause $u_{g,i}/u_{c,i} = MCF_i < 1$ if the effect is large enough, implying that the public good is oversupplied.

### 2.4 Equilibrium

Equilibrium is defined as a set of Nash-Equilibrium tax rates ($t_i$ for all $i$), capital stocks ($k_i$ for all $i$) and an interest rate $\rho$ that simultaneously satisfy for each country both the modified Samuelson rule in Eq. (10) and the demand for capital in Eq. (1), and for all countries together the world resource constraint.[6] For an equilibrium to be welfare improving, we need that the second-order condition for a welfare maximum is negative

$$\frac{\partial u_{g,i}/u_{c,i}}{\partial t_i} - \frac{\partial MCF_i}{\partial t_i} < 0.$$

With respect to the first term, we follow a.o. Bucovetsky (1991) and Taugourdeau and Ziad (2011) by assuming that both $g_i$ and $c_i$ are normal goods. Together with the tax being on the upward-sloping part of the Laffer curve, this implies that: $\partial u_{g,i}/u_{c,i}/\partial t_i \leq 0$, i.e. choosing a higher tax rate leads to a reduction in the marginal valuation of the public good.

The difficulty lies in proving that $\partial MCF_i/\partial t_i > 0$. Bucovetsky (1991) shows that this is the case for a quadratic production function, which has subsequently been used by a.o. Bucovetsky (2009) and Devereux et al. (2008) (while Parry, 2003, assumes that $\partial k_i/\partial t_i$ is linear in the relevant range). This assumption ensures that the tax base elasticity, $\epsilon_{k,i}$, is unambiguously decreasing in the capital stock and that the Laffer curve of country $i$ is

[6] Some papers have proven the existence of a Nash-equilibrium in models more stylized than ours, see e.g. Laussel and Le Bretton (1998) and Bayindir-Upmann and Ziad (2005) , but for the model used here existence has not yet been proven.

unambiguously shifted outwards if capital flows in. In the next section, we will also use the quadratic production function to avoid complications.[7] However, our results continue to hold under a more general production function that obeys the assumptions laid down in Taugourdea and Ziad, see appendix B.

## 3 Strategic Tax Responses

We now explore the slope of the tax reaction function. If the slope is positive, then a tax increase abroad will induce a country to also increase its own tax rate and we speak of 'strategic complements'. If the slope is negative, then a higher tax abroad will trigger a decrease in the country's own tax rate as a response and we refer to this as 'strategic substitutes'.

In general, tax reaction functions take the form: $t_i = V_i(t_j, t_{-j})$, where $V_i(t_j, t_{-j})$ gives the best response of country $i$ to the tax rates chosen by country $j$ and all remaining countries (where we denote the set of all countries not including countries $i$ and $j$ by $-j$). Because there are no closed form expressions for the tax rates in general, we linearize the tax reaction around an initial equilibrium. This yields analytical expressions for the tax change by country $i$ in response to tax changes in (one of) the other countries. The linearized tax responses reflect optimal marginal tax responses, relative to an initial equilibrium, to an assumed exogenous marginal change in the tax rate of country $j$ (which might represent a group of countries that uniformly raise their tax). Appendix B derives the linearized tax reaction functions, assuming a CES utility function and a quadratic production function, $f(k) = b(a-k)k$. Denoting a percentage change in variable $x$ as: $\partial x/x = \partial \ln(x) = \widetilde{x}$, the reaction function looks as follows

[7] For a more general production function, Taugourdea and Ziad (2011) show that a second-order locally consistent equilibrium exists in case of a positive third derivative of the production function and the prescription that the demand for capital should not be increasing in capital ($\partial \ln f_i'/\partial \ln k_i \leq 0$). These conditions hold for a wide range of production functions commonly used in the economic literature, such as: i) Cobb-Douglas, ii) Quadratic, iii) Logarithmic, iv) Exponential, v) Logistic, and vi) CES production function in case the capital share in production, and/or the substitution elasticity between capital and the fixed factor, is not too large. However, the result also requires that all capital is owned by individuals living outside the countries considered ($e_i = 0$).

$$\tilde{t}_i = \frac{\left[\left(\frac{u_{g,i}}{u_{c,i}} + \eta_{r,i} - 1\right) - \left(\frac{\gamma_i}{\sigma}\right)\left(1 + 2b\frac{e_i - k_i}{c_i}k_i\right)\right]\epsilon'_{k,j}}{\left(2\frac{u_{g,i}}{u_{c,i}} + \eta_{r,i} - 1\right)\epsilon_{k,i} + \frac{\gamma_i}{\sigma}\left(1 + \frac{g_i}{c_i}\right) + \frac{\gamma_i}{\sigma}\frac{e_i - k_i}{c_i}t_i\eta_{r,i} - \frac{\gamma_i}{\sigma}\epsilon_{k,i}}\tilde{t}_j, \tag{11}$$

where $\sigma \equiv dlog(c_i/g_i)/dlog(MRS_i) > 0$ denotes the elasticity of substitution between public and private goods. If $\sigma$ is large, public and private goods are close substitutes, so that the MRS is not strongly affected by changes in the ratio of private-to-public consumption. In the limit $\sigma \to \infty$ we approach a constant MRS. For other parameters, $0 < \epsilon_{k,i} \equiv t_i\eta_{k,i}/k_i \leq 1$ denotes the *own* capital stock elasticity, $\epsilon'_{k,j} = (\partial k_i/\partial t_j)(t_j/k_i) = -t_j/(k_i f''_i)\eta_{r,j} > 0$ indicates a cross-elasticity and $\gamma_i = -(\partial c_i/\partial t_i)/k_i = 1 + (e_i/k_i - 1)\eta_{r,i}$ reflects the reduction in private income following a tax increase relative to the initial capital stock.

The denominator on the right-hand side of Eq. (11) is positive by assumption as it equals (minus) the second-order condition for a welfare maximum. Eq. (11) gives the marginal slope of the reaction function of country $i$: if the coefficient on the right-hand side of (11) is positive, then tax rates are strategic complements; otherwise, they are strategic substitutes. Our focus will be on the prevalence of strategic substitutability, i.e. on cases under which the coefficient is negative. To interpret the slope coefficient, we first discuss two special cases that each highlight one particular channel. We then elaborate on the general case.

### 3.1 Special Case 1: constant MRS

First, assume a constant MRS equal to $u_g/u_c$. This would be the case if the utility function were linear, which results in $\sigma \to \infty$. This assumption is adopted by e.g. Devereux et al. (2008), and Bucovetsky (2009). Note that if $u_g/u_c \to \infty$, welfare maximization coincides with revenue maximization by the government, this is assumed by e.g. Kanbur and Keen (1993). When assuming a constant MRS Eq. (11) reads as follows

$$\tilde{t}_i = \frac{\left(\frac{u_g}{u_c} + \eta_{r,i} - 1\right)\epsilon'_{k,j}}{\left(2\frac{u_g}{u_c} + \eta_{r,i} - 1\right)\epsilon_{k,i}}\tilde{t}_j \quad \text{for} \quad i \neq j. \tag{12}$$

From this we can derive the following proposition

**Proposition 1.** *In case of a constant MRS, the tax reaction function is always positively sloped as long as $u_g/u_c > 1$: tax rates are strategic complements. The slope of the reaction function is steeper, the larger is the marginal valuation of public goods relative to private goods ($u_g/u_c$).*

To understand Eq. (12), note that a higher tax in country j will cause an inflow of capital to country i, which is measured by $\epsilon'_{k,j} > 0$. This boosts both private consumption, due to a positive impact of the capital inflow on the wage rate, and public consumption as the broadening of the domestic tax base raises public revenue if the tax rate is kept unchanged. The optimal response in the tax rate depends on how consumers value public and private consumption. The term $u_g/u_c - 1$ measures the extent to which the MRS between public and private goods exceeds the marginal rate of transformation (MRT). According to modified Samuelson rule (10), this occurs if the MCF exceeds unity in which case public goods are scarcer than private goods due to distortionary taxation. Ignoring the term $\eta_{r,i}$, Eq. (12) suggests that this would make it optimal for the government to raise public goods supply. Intuitively, the exogenous inflow of capital on account of the higher tax rate abroad reduces the MCF, making it less costly to supply public goods and allowing for a higher tax rate. Accordingly, tax rates are strategic complements. [8]

The tax response depends also on the impact of the change in $t_i$ on the interest rate, which is measured by $\eta_{r,i}$. The extra inflow of capital as a result of a higher foreign tax rate is cheaper if the interest rate that needs to be paid to foreign capital owners is lower. As an increase in the domestic tax rate indeed reduces the interest rate by $\eta_{r,i}$, this encourages country $i$ to increase its domestic tax rate. This channel critically depends on country size, however. Indeed, when $n \to \infty$ or if country $i$ is very small relative to the rest of the world, this channel becomes irrelevant, i.e. $\eta_{r,i}$ is close to zero.

[8] Undersupply of the public good ($u_g/u_c > 1$) is a feature of standard tax competition models, where tax competition leads countries to choose inefficiently low tax rates (Zodrow and Mieszkowsky, 1986; and Wilson, 1986). The importance of $u_g/u_c > 1$ was also stressed by Saavedra and Brueckner (2001), who argued that countries with a low valuation of public goods ($u_g < u_c$) might feature a negatively sloped tax reaction function. In our model, this is the case only if the MCF would be smaller than unity.

### 3.2 Special Case 2: Endogenous MRS with Symmetric Countries

The second special case we consider is when the MRS is no longer constant, but countries are symmetric. This would imply $s_i = 1/n$, $e_i = k_i = e$, $U_i(c, g) = U(c, g)$ for given $c$, $g$ and all $i \in \{1 : n\}$. Also note that, $\gamma_i = -(\partial c_i/\partial t_i)/k_i = 1$, under symmetry. The marginal slope of the reaction function is now given by

$$\tilde{t}_i = \frac{\left(\frac{u_g}{u_c} + \eta_{r,i} - 1 - \frac{1}{\sigma}\right)\epsilon'_{k,j}}{\left(2\frac{u_g}{u_c} + \eta_{r,i} - 1 - \frac{1}{\sigma}\right)\epsilon_{k,i} + \left(1 + \frac{g}{c}\right)\frac{1}{\sigma}}\tilde{t}_j, \tag{13}$$

As the denominator is positive by assumption, the slope of the reaction function in Eq. (13) is determined by the sign of the numerator, which depends on two terms. The term $(u_g/u_c + \eta_{r,i} - 1)$ is the same as in Eq. (12). The other term in the numerator of Eq. (13) is reflected in the following proposition.

**Proposition 2.** *With symmetric countries, the slope of the tax reaction function might be negative (strategic substitutes), even if $u_g/u_c > 1$. At a given $u_g/u_c$, the likelihood of strategic substitutability declines in $\sigma$: the willingness to substitute private for public goods.*

As noted before, the higher tax rate in country $j$ causes an inflow of capital in country $i$. The broader tax base raises public funds and yields more public goods. If public and private goods are close substitutes (a large value for $\sigma$), then there is little reason to reduce the tax rate in order to replace public by private consumption. However, if public and private goods are close complements (a small value for $\sigma$), then the government will find it optimal to cut the tax rate so as to boost private consumption along with public consumption. This may cause tax rates to be strategic substitutes. Indeed, even if $u_g/u_c > 1$, we could have a negatively sloped reaction curve if $\sigma$ is sufficiently small. Interestingly, the case for strategic substitutes is strengthened when $n$ increases, leading to a lower value of $\eta_{r,i}$.

### 3.3 General Case

Eq. (11) allows for asymmetry so that countries can be either importers or exporters of capital. This affects the slope of the tax reaction function, which is reflected in the following proposition.

**Proposition 3.** *With asymmetric countries, ceteris paribus, the prevalence of strategic substitutes is more likely when country $i$ is a capital exporter.*

In what follows, we will focus on a country that is a capital exporter in equilibrium, in which case $e_i > k_i$ such that $\gamma_i > 1$. The reverse would hold in the case of a capital importer. Given a positive denominator, the numerator in Eq. (11) reveals that a capital exporter is more likely to have a downward sloping tax reaction function via two terms. The first is captured by the term $2bk_i(e_i - k_i)/c_i$, which is positive only for a capital exporter. The second is captured by the parameter $\gamma_i = (\partial c_i/\partial t_i)/k_i = 1 + \eta_{r,i}(e_i/k_i - 1) > 1$ for $e_i > k_i$ and which increases in the amount of capital exports. Both terms magnify the substitution effect from public to private consumption, discussed in the previous subsection. Intuitively, a capital exporter faces a larger MCF, as seen in Eq. (10) since taxes are more distortionary. The inflow of capital from abroad mitigates this distortion and induces a larger substitution effect at the margin if the initial equilibrium is more distorted. The size of the effect still critically depends on the value for $\sigma$. Indeed, if public and private consumption are close substitutes, there is little reason for the government to reduce the tax rate in order to boost private consumption along with public consumption. Thus, a negatively sloped reaction curve will be less likely. In fact, if $\sigma \to \infty$, we saw in the first special case that the third effect disappears, irrespective of whether a country is exporting or importing capital. If public and private consumption are complementary, however, strategic substitutability is more likely, especially for capital exporting countries.

Proposition 3 holds only ceteris paribus. In fact, country characteristics reflected by model parameters may simultaneously affect the different components of the initial equilibrium in Eq. (11) and, therefore, the prevalence of strategic substitutes. For example, consider country size. Bigger countries exert more power on international capital markets so that $\partial\eta_{r,i}/\partial s_i > 0$.

This makes strategic complementarity more likely for them. At the same time, however, large countries will set higher tax rates because they face a lower MCF (i.e. $\partial MCF/\partial s_i < 0$). This makes large countries more likely to be capital exporters which, as long as $\sigma$ is small enough, makes strategic substitutability more likely. Also preference parameters in the utility function could simultaneously affect the ratio $u_g/u_c$ and the likelihood of being capital exporter. Moreover, rich countries with a relatively large capital endowment are more likely to export capital, making strategic substitutability more likely. At the same time, however, capital exporters have more private income and, hence, value public goods more at the margin (higher $u_g/u_c$) for a given tax rate. This can make strategic complementarity more likely. In the next section, these interactions are important for the simulations.

## 4 Simulated Tax Reaction Functions

While linearization offers insight in the parameters determining the slope of the tax reaction function, it offers only insight in the local not in the global properties of the tax reaction functions. This subsection illustrate the global properties by simulating tax reaction curves for a range of tax rates. The simulations suggest that the results from propositions 1, 2 and 3 indeed hold globally.

In performing the simulations, we make a number of assumptions. Throughout the exercises, we use a quadratic production function: $f(k) = b(a - k_i)k_i$ and assume three countries ($n = 3$). In the first two simulations, countries are symmetric ($e = k = 1$) and of equal size ($s = 1/3$). This gives simple expressions for public and private consumption, as well as for the MCF: $c = b(a - 1) - t$; $g = t$ and $MCF = \frac{1}{1-t/(2b)(2/3)}$. In the calibration, we set $t = 1/2$ and $b = 1/2$, implying that $MCF = 3/2$. We consider two utility functions. In the first simulation, we use a linear utility function, $U = c + (u_g/u_c)g$, with a constant $u_g/u_c$ and then vary the value of $u_g/u_c$. In the second and third simulation, we adopt a CES utility function $U = (\omega c^{1-1/\sigma} + (1 - \omega)g^{1-1/\sigma})^{\sigma/(\sigma-1)}$, whereby we set $\omega = 1/2$. To satisfy the modified

Samuelson rule, the equilibrium must satisfy (using $b = t = 1/2$)

$$\frac{u_g}{u_c} = \frac{1-\omega}{\omega}\left(\frac{c}{g}\right)^{1/\sigma} = (a-2)^{1/\sigma}. \tag{14}$$

To allow for variation in $\sigma$ to illustrate proposition 2, we adjust the parameter $a$ along with $\sigma$ to hold constant $u_g/u_c$ in Eq. (14)

$$a = \frac{3((u_g/u_c)^\sigma + 1)(u_g/u_c - 1)}{u_g/u_c} + 1 = (3/2)^\sigma + 2 \tag{15}$$

In the third simulation, we consider asymmetry across countries. In particular, we assume that country $i$ is more capital abundant. Thereby, we set $\sigma = 1$ and $a = 3.5$ to ensure $u_g/u_c = 3/2$. Figures 1, 2 and 3 show the tax reaction curves for country $i$ together with the 45 degrees line (dotted).

[Figure 1 about here.]

In the first simulation with a linear utility function and symmetric countries, we set $a = 5$ such that $f(.) = 2$. We consider three different values of $u_g/u_c \in \{2/3, 3/2, 3\}$. Figure 1 shows that the tax rate set by country $i$ rises in the MRS, reflecting a higher marginal value of public goods, relative to private goods. Consistent with proposition 1, the slope of the tax reaction curve is steeper for higher values of $u_g/u_c$. For $u_g/u_c = 2/3$, we see that the slope of the reaction curve is precisely zero. Indeed, for $s_i = 1/3$ we have $\eta_{r,i} = 1/3$ so that $u_g/u_c + \eta_{r,i} - 1 = 0$.[9]

[Figure 2 about here.]

In the second simulation, we vary $\sigma \in \{\infty, 5, 1, 0.2\}$, while simultaneously adjusting $a$ to keep $u_g/u_c = 3/2$.[10] Figure 2 shows, consistent with proposition 2, that the slope of the tax reaction function is increasing in $\sigma$. For a Cobb-Douglas utility function ($\sigma = 1$), the

[9]For this latter case, we decreased $e_i$ such that country $i$ is a capital importer to make sure it chooses a positive tax rate. Note that this does not affect the slope of the reaction function as defined in Eq. (12) for the case of linear utility function.

[10]Wildasin (1989) and Parry (2003) study a range of [0.2 - 1] and [0.3 - 1] for $\sigma$, respectively, and stress that scarce empirical evidence points to an inelastic demand for public goods (see also Rubinfeld, 1987).

tax reaction function is downward sloping but rather flat. For lower values of $\sigma$, the slope becomes more negative.

[Figure 3 about here.]

Figure 3 sets $\sigma = 1$ and varies the capital endowment of country i: $e_i \in \{1, 3/2, 5/2\}$. The larger its capital endowment, the more negative is the net foreign asset position of the country. Consistent with proposition 3, Figure 3 shows that the slope of the tax reaction function becomes more negative when country 3 exports more capital.

## 5 Application: Coalition Formation in Capital Taxation

Tax reaction functions are important for the analysis of tax competition. From a policy perspective, they are also critical when studying the welfare effects of a partial tax union (see Konrad and Schjelderup, 1999). To illustrate, this section performs simulations of the welfare effects when a subgroup of countries forms a tax union. The section highlights the importance of tax reactions for the likelihood of a partial tax union, as well as the welfare effects.

In the simulations, we again use the three-country version of the model and adopt the same calibration as in the previous section. In analyzing a partial tax union, we assume that countries 1 and 2 harmonize ($h$) their tax system and form a tax union (henceforth the union countries). Country 3 remains outside the union. The union countries are assumed symmetric throughout the analysis, where $s = s_1 = s_2$ denotes their share in the world population. The union countries choose a joint tax rate $t_h$ that maximizes the sum of welfare in the two countries: $sU_1(.) + sU_2(.) = 2sU_h(.)$, given the tax rate chosen by country 3.[11] The government of country 3 chooses $t_3$ to maximize the welfare of its citizens, given the tax rate of the union countries. We allow country 3 to be either smaller or larger than countries

[11] Assuming symmetry between countries 1 and 2 implies that we side-step the complications that arise in case of a union between asymmetric countries. When union members are asymmetric, both their preference for the optimal union policy and the payoff from cooperation might differ. Our approach is familiar in the literature on coalitions, see for example Kennan and Riezman (1990).

1 and 2 in terms of population. Furthermore, we allow country 3 to have a larger capital endowment. Preferences are the same in all countries.

In exploring the impact of a tax union on tax rates and welfare, we compare the decentralized equilibrium (indicated by superscript "$D$"), where all three countries choose their tax rate independently in a Nash setting, with the equilibrium where countries 1 and 2 have formed a tax union (indicated by superscript "$H$").

## 5.1 Predictions

We first explore comparative statics to understand how a tax union is expected to affect tax rates. For country 3, there is no direct impact of a tax union. Indeed, the parameters $\eta_{k,3}^D$ and $\eta_{r,3}^D$ do not change due to the formation of a tax union by the other countries. Country 3 is, however, affected through the change in tax policy in the union countries to the extent that this modifies the allocation of capital $k_3$ and the interest rate $\rho$. For countries 1 and 2, the parameters $\eta_{k,i}^D$ and $\eta_{r,i}^D$ do change through the formation of a tax union. This will affect the optimal choices regarding tax rates and, therefore, outcomes. First, Appendix A derives the elasticities under a tax union and finds that $\eta_{k,h}^H < \eta_{k,1}^D = \eta_{k,2}^D$. The reason is that the joint policy response by countries 1 and 2 eliminates spillovers upon each other. Accordingly, evaluated at the decentralized equilibrium values of $t_i$ and $k_i$, the formation of a tax union will generally reduce the MCF. This gives the union countries an incentive to raise their tax rate. Second, Appendix A also shows that $\eta_{r,h}^H = 2\eta_{r,1}^D = 2\eta_{r,2}^D$. As the union countries together are twice the size of a single country, the interest rate response to the tax is twice as large. This larger tax coefficient of the interest rate further reduces the $MCF_h^H$ if the union countries are net capital importers. However, a larger interest coefficient mitigates the reduction in the $MCF_h^H$ when union countries are net capital exporters, and could in principle even dominate the reduction in $\eta_{k,i}$.

Following Konrad, Schjelderup (1999, p. 161), we derive the marginal change in welfare for the union countries in response to a marginal increase in the joint tax rate

$$\left.\frac{\partial U_h}{\partial t_h}\frac{1}{u_{c,h}}\right|_{t_h=t_1^D} = \left[(k_h - e_h) + \frac{(u_{g,h}/u_{c,h})t_h}{2b}\right] s\left(1 + \frac{(1-2s)}{s}\frac{\partial t_3}{\partial t_h}\right) \tag{16}$$

where we use: $\eta_{r,i} = s_i$ (under a quadratic production function). The first term between square brackets is unambiguously positive if the union countries are capital importers ($k_h > e_h$). The higher tax raises welfare, both due to an increase in public consumption and because the reduction in the interest rate raises welfare. If the union countries are net capital exporters, however, the latter effect can be opposite as the higher tax lowers net interest income received from abroad.

In Eq. (16), the slope of the tax reaction function enters via the term between round brackets. This term is always positive under strategic complementary as the outside country will adopt a tax response of the same sign as that chosen by the union countries. However, the term may become negative under strategic substitutability. Under symmetry ($k_h = e_h$), we would need $\partial t_3/\partial t_h < -1$ for union countries to experience a welfare loss from marginally increasing their tax rate. This would be an unusual case, which we may exclude.[12] If country 3 gets larger compared to countries 1 and 2, however, welfare losses in the union countries may occur under weaker conditions. In particular, the strength the strategic substitutability should be such that: $\partial t_3/\partial t_h < -s/(1-2s)$.

The change in welfare in country 3 (before it has adjusted its tax rate) is given by

$$\left.\frac{\partial U_3}{\partial t_h}\frac{1}{u_{c,3}}\right|_{t_3=t_3^D} = \left[(k_3 - e_3) + \frac{\gamma_3 t_3}{2b}\right] 2s.$$

[12]For $n$ symmetric countries the condition for a welfare loss to occur is $\partial t_3/\partial t_h < -1/(n-2)$. While recognizing that two countries change their tax rate: $2\epsilon'_{k,j} = (1/bn)$ we can rewrite Eq. 13 as $\partial t_3/\partial t_h = A/(A+B)(1/2)(n-1)]$, with $A \equiv (u_g/u_c+1/n-1-1/\sigma)$, $B \equiv (1+g/c)(1/\sigma)u_g/(u_g-u_c)+u_g/u_c > 0$ (using the definition of MCF). When $A < 0$, the second-order condition for a welfare maximum requires that $A+B > 0$, or: $A/B > -1$. The condition for a welfare loss to occur can be reformulated into: $A/B < -(n-1)/(n-3) < -1$, which contradicts with the second-order condition for a welfare maximum.

Welfare in country 3 always rises if the tax union increases its tax rate, as it benefits from an inflow of capital. Only if country 3 exports a lot of capital, a reduction in net interest income may imply that welfare could decrease.

### 5.2 Simulations

Table (1) presents the simulation results under alternative parameter values. The calibration follows Section 4, whereby the rows in the Table indicate how the parameters are varied. In particular, we consider several combinations whereby we consider two values for MRS (3/2 and 3), two values for $\sigma$ (1 and 0.2), and three values for the size of the countries that form a tax union ($s = 2/5$, 1/3 and 1/6). Furthermore, we allow country 3 to have a relative large capital endowment: $e_3 = 1/(1-2s) - 2s/(1-2s)e_h$, affecting its capital exports.

Columns (5) and (6) show the decentralized equilibrium tax rates. In the symmetric equilibrium, we have set $t = 0.5$. We see that larger countries set higher tax rates, as in Bucovetsky (1991) and Wilson (1991). A larger capital endowment has an ambiguous effect on the equilibrium tax rate.[13] A higher MRS ($u_g/u_c$) comes along with a higher tax rate since public goods are valued more.

[Table 1 about here.]

Column (7) shows the *percentage change* in the equilibrium tax rates by countries 1 and 2, after a tax union is formed. A positive sign reflects an increased tax rate, while a negative sign reflects a reduction. We see that the union countries always increase their tax rates ($t_h^H > t_h^D$) after forming a tax union. The tax increase by the union countries is increasing in their size. Intuitively, spillover between two larger countries are large, relative to spillovers vis-a-vis a small third country. When union countries import capital ($e_u < 1$), the tax increase is also generally larger, since the lower interest rate that a tax increase would induce benefits net capital importers at the expense of capital exporters.

[13] A larger capital endowment affects both sides of the modified Samuelson rule in Eq. (10). First, as discussed by Peralta and Van Ypersele (2005, 2006), a larger capital endowment causes a larger MCF such that 'rich' countries have an incentive to choose a lower equilibrium tax rate compared to 'poor'countries. On the other hand, more interest income causes more private incomes which increases the relative marginal valuation of public goods (MRS increases), giving an incentive to raise the tax rate.

Column (8) shows that the response by the non-union country varies, reflecting either strategic complementarity or strategic substitutability. In fact, country 3 reduces its tax rate in most cases reported in Table 1 consistent with strategic substitutability. Hence, strategic substitutability does not require unreasonable parameters and might in fact be quite a reasonable case in tax competition models when governments maximize welfare. If union countries increase their tax rate by more, then we see that also the change by country 3 gets larger.

The last two columns present the welfare effects measured by the compensating variation ($CV$) as a percentage of total production of a country: $f(k_i) = (a-bk_i)k_i$. The $CV_i$ represents the increase in private consumption that is required under the tax union equilibrium to make the citizens of country $i$ equally well off as compared to the decentralized equilibrium. A negative value therefore indicates a welfare gain from the formation of a tax union; a positive value reflects a welfare loss. We see that in most cases both countries gain from the formation of a tax union. This gain is bigger when large countries form a tax union and when tax rates are strategic complements. Country 3 only looses from the tax union between countries 1 and 2 when it is a capital exporter. The union countries suffer a welfare loss when (i) the union countries are small compared to the non-union country; and (ii) the non-union country responds to the tax increase by the union countries by reducing its own tax rate.[14]

## 6 Conclusion

This paper analytically derives the conditions for tax rates to be strategic substitutes in an asymmetric tax-competition model. These conditions appear to be rather weak as long as governments maximize welfare and utility is generally specified. Simulations further suggest that strategic substitutability might hold under plausible parameter configurations. This has important implications for the welfare effects of tax unions. For instance, earlier papers have either explicitly or implicity assumed that tax rates are strategic complements and found

[14]A welfare loss for the countries that form a tax union can be avoided when assuming a Stackelberg-leader game, where the union countries act as the Stackelberg leader. In this case, strategic substitutes still leads to a welfare loss, but now all is on account of country 3, as the union countries foresee the 'aggressive' response by country 3 (see de Mooij and Vrijburg, 2010, for a discussion).

that the formation of tax unions is unambiguously welfare improving for the participating countries. In the presence of strategic substitutes, however, we show that the formation of a tax union might actually reduce welfare for the union countries since the adverse response in the outside country may offset the benefits of forming the union.

Our paper leaves room for several extensions. First, one may want to allow for a more general class of government objectives, including the Leviathan government, as in Edwards and Keen (1996). Second, endogenous coalition formation along the lines of Kempf and Rota Grasiosi (2010) may shed new light on strategic tax interactions. Which tax unions will be formed, and how does this related to existence of strategic substitutes? We leave these topics for future research.

## Appendix A: Deriving Elasticities

This appendix derives the tax coefficients $\eta_{k,i}$ and $\eta_{r,i}$ for different taxation regimes considered in the text. It considers the case of decentralization and the partial tax union. We first differentiate Eq. (1) to show how a change in the tax rate in country $i$ affects the capital stock in countries $i$ and $j$

$$\frac{\partial k_i}{\partial t_i} = \frac{1}{f_i''(.)}\left[1 + \frac{\partial \rho}{\partial t_i}\right], \tag{A.1}$$

$$\frac{\partial k_j}{\partial t_i} = \frac{1}{f_j''(.)}\left[\frac{\partial \rho}{\partial t_i}\right] \quad \text{for} \quad j \neq i. \tag{A.2}$$

From Eq. (2) it follows that the total size of the capital stock is fixed, therefore

$$s_i \frac{\partial k_i}{\partial t_i} = -\sum_{j=1, j\neq i}^{n} s_j \frac{\partial k_j}{\partial t_i}. \tag{A.3}$$

Combining Eq. (A.1), (A.2) and (A.3) we obtain

$$0 > \frac{\partial \rho}{\partial t_i} = \frac{-s_i/f_i''(.)}{\sum_{j=1}^{n} s_j/f_j''(.)} = -\eta_{r,i}^{D} > -1. \tag{A.4}$$

filling this in Eq. (A.1) gives

$$\frac{\partial k_i}{\partial t_i} = \frac{1}{f_i''(.)}\left[\frac{\sum_{j=1, j\neq i}^{n} s_j/f_j''(.)}{\sum_{j=1}^{n} s_j/f_j''(.)}\right] = -\eta_{k,i}^{D} < 0, \tag{A.5}$$

When $m$ countries harmonize their taxes ($h$), we obtain: $k_i = k_h$, $f_i''(.) = f_h''(.)$ and $t_i = t_h$ for $i \in M$ where $M$ denotes the set of cooperating countries. Note, we assume $s_i = s$ for $i \in M$. The following two equations describe the capital market

$$f_i''(.)\frac{\partial k_i}{\partial t_h} = \frac{1}{f_i''(.)}\left[1 + \frac{\partial \rho}{\partial t_h}\right] \quad \text{for} \quad i \in M, \tag{A.6}$$

$$f_j''(.)\frac{\partial k_j}{\partial t_h} = \frac{1}{f_j''(.)}\frac{\partial \rho}{\partial t_h} \quad \text{for} \quad j \notin M, \tag{A.7}$$

Combining Eq. (A.6), (A.7) and (A.3) we obtain

$$0 > -\eta_{r_i}^D > \frac{\partial \rho}{\partial t_h} = \frac{-ms/f_h''(.)}{\sum_{j=1}^n s_j/f_j''(.)} = -\eta_{r,i}^H > -1 \quad \text{for} \quad i \in M \tag{A.8}$$

filling this into Eq. (A.6) gives

$$\frac{\partial k_i}{\partial t_h} = \frac{1}{f_i''(.)}\left[\frac{\sum_{j=1,j\notin M}^n s_j/f_j''(.)}{\sum_{j=1}^n s_j/f_j''(.)}\right] = -\eta_{k,i}^H > -\eta_{k_i}^D \quad \text{for} \quad i \in M. \tag{A.9}$$

# Appendix B: Derviation of the linearized tax reaction function

We linearize the tax reaction function around an initial equilibrium. We do this by linearizing Eq. (10), which equates the MRS between public and private goods and the MCF. Thus, we first linearize the MRS and then the MCF. We denote a percentage change in variable $x$ as: $\partial x/x = \partial \ln(x) = \widetilde{x}$.

### Linearizing the MRS

We linearize the MRS on the left-hand side of Eq. (10) for a Constant-Elasticity-of-Substitution (CES) utility function[15]

$$U = \left[\omega_i c_i^{(\sigma-1)/\sigma} + (1-\omega_i) g_i^{(\sigma-1)/\sigma}\right]^{\frac{\sigma}{\sigma-1}}, \tag{B.1}$$

yielding

$$\widetilde{MRS}_i \equiv \partial \ln\left(\frac{u_{g,i}}{u_{c,i}}\right) = \frac{\widetilde{c}_i - \widetilde{g}_i}{\sigma}, \tag{B.2}$$

where $\sigma \equiv dlog(c_i/g_i)/dlog(MRS_i) > 0$ denotes the elasticity of substitution between public and private goods. If $\sigma$ is large, public and private goods are close substitutes, so that the MRS is not much affected by changes in the ratio of private-to-public consumption. In the limit $\sigma \to \infty$ we approach a constant MRS.

[15]Note, for our main argument we need a utility function that is characterized by a decreasing marginal valuation of public goods (or tax revenue), for example a log-linear utility function: $U_i = c_i + \gamma log(g_i)$. We restrict our attention to CES utility function to facilitate easy interpretation.

We substitute Eq. (1) into the household budget constraint Eq. (5) to eliminate $f_i'$ and obtain for private consumption: $c_i = f_i(.) + \rho(e - k_i) - t_i k_i$. Linearizing this expression and combining it with the government budget constraint in Eq. (6), we arrive at an expression for the linearized ratio of private-to-public consumption

$$\tilde{c}_i - \tilde{g}_i = -\tilde{k}_i - \left(1 + \frac{g_i}{c_i}\right)\tilde{t}_i + \frac{(e_i - k_i)\rho}{c_i}\tilde{\rho}. \tag{B.3}$$

As a final step towards a linearized MRS as function of tax rates only, we linearize the capital market equilibrium in Eq. (2) and the first-order condition for firms in Eq. (1). Combining yields

$$\tilde{k}_i = -\epsilon_{k,i}\tilde{t}_i + \epsilon'_{k,j}\tilde{t}_j, \tag{B.4}$$

$$\tilde{\rho} = -\frac{t_i}{\rho}\eta_{r,i}\tilde{t}_i - \frac{t_j}{\rho}\eta_{r,j}\tilde{t}_j, \tag{B.5}$$

where $0 < \epsilon_{k,i} \equiv t_i \eta_{k,i}/k_i \leq 1$ denotes the *own* capital stock elasticity, and $\epsilon'_{k,j} = (\partial k_i/\partial t_j)(t_j/k_i) = -t_j/(k_i f_i'')\eta_{r,j} > 0$ represents a cross-elasticity. Substituting Eqs. (B.3), (B.4, and (B.5) into Eq. (B.2) leads to

$$\widetilde{MRS}_i = -\frac{1}{\sigma}\left(1 + \frac{g_i}{c_i} + \frac{(e_i - k_i)t_i}{c_i}\eta_{r,i} - \epsilon_{k,i}\right)\tilde{t}_i - \frac{1}{\sigma}\left(\epsilon'_{k,j} + \frac{(e_i - k_i)t_j}{c_i}\eta_{r,j}\right)\tilde{t}_j. \tag{B.6}$$

The first term shows that the MRS is either increasing or decreasing in the own tax. Intuitively, as the optimal tax rate is always on the upward sloping part of the Laffer curve, a higher tax implies more tax revenues and, hence, a lower marginal valuation of extra tax revenues. On the other hand, private consumption might both increase or decrease, depending on whether the country is a capital importer or exporter, making the whole term ambiguous. The second term shows that the MRS decreases following an increased foreign tax when country $i$ does not import too much capital. The inflow of capital causes a relative large increase in tax revenues relative to private consumption causing a lower relative marginal valuation of tax revenues. For large capital importers, the reduction in the interest rate implies a relative large increase in private consumption which reduces the relative marginal valuation of private income.

### Linearizing the MCF

Next, we use Eq. (10) to linearized the MCF

$$\widetilde{MCF}_i = \frac{1}{\gamma_i}\left[\frac{u_{g,i}}{u_{c,i}}\epsilon_{k,i}\widetilde{t}_i - \left(\frac{u_{g,i}}{u_{c,i}} - 1 + \eta_{r,i}\right)\widetilde{k}_i + \frac{u_{g,i}}{u_{c,i}}\epsilon_{k,i}\widetilde{\eta}_{k,i} + \left(\frac{e_i - k_i}{k_i}\right)\eta_{r,i}\widetilde{\eta}_{r,i}\right], \qquad \text{(B.7)}$$

with $\gamma_i = -(\partial c_i/\partial t_i)/k_i = 1 + (e_i/k_i - 1)\eta_{r,i}$. The final two terms disappear when using a quadratic production function: $f(k) = b(a-k)k$. Using this function, and Eqs. (3)-(4), we obtain simple expressions for the coefficients $\eta_{k,i}$ and $\eta_{r,i}$

$$\eta_{k,i} = \frac{1-s_i}{2b}, \quad \eta_{r,i} = s_i$$

such that $\widetilde{\eta}_{k,i} = 0$ and $\widetilde{\eta}_{r,i} = 0$. This quadratic production function, being the standard in the literature, will be used throughout the main analyzes in this paper, yielding[16]

$$\widetilde{MCF}_i = \frac{1}{\gamma_i}\left[\frac{u_{g,i}}{u_{c,i}}\epsilon_{k,i}\widetilde{t}_i - \left(\frac{u_{g,i}}{u_{c,i}} - 1 + \eta_{r,i}\right)\widetilde{k}_i\right]. \qquad \text{(B.8)}$$

The first term on the right-hand side of Eq. (B.8) shows that the MCF is increasing in the own tax rate $t_i$. This effect is stronger the larger the capital outflow, $\epsilon_{k,i}$. Second, an increase in the capital stock ($\widetilde{k}_i > 0$) reduces the MCF when public goods are under-supplied ($u_{g,i} > u_{c,i}$).

### Linearized Reaction Function

Now substitute Eq. (B.4) into Eq. (B.8), and note that, in equilibrium, the relative change in the MRS given in Eq. (B.6) should equal the relative change in the MCF given in Eq. (B.8):

[16] A quadratic production function is used by amongst others Bucovetsky (1991, 2009), Devereux et al. (2008), Parry (2003) and Wilson (1991). It is very useful as it ensures that the tax base elasticity, $\epsilon_{k,i}$, is unambiguously decreasing in the capital stock. This implies that the Laffer curve of country $i$ is unambiguously shifted outwards upon a capital inflow. In doing so, it rules out strategic substitutes occurring through an increase in the MCF. This does not necessarily hold for a more general production function, which we discuss briefly in Appendix B.

$\widetilde{MRS}_i = \widetilde{MCF}_i$. After rewriting, this yields

$$\tilde{t}_i = \frac{\left[\left(\frac{u_{g,i}}{u_{c,i}} + \eta_{r,i} - 1\right) - \left(\frac{\gamma_i}{\sigma}\right)\left(1 + 2b\frac{e_i - k_i}{c_i}k_i\right)\right]\epsilon'_{k,j}}{\left(2\frac{u_{g,i}}{u_{c,i}} + \eta_{r,i} - 1\right)\epsilon_{k,i} + \frac{\gamma_i}{\sigma}\left(1 + \frac{g_i}{c_i}\right) + \frac{\gamma_i}{\sigma}\frac{e_i - k_i}{c_i}t_i\eta_{r,i} - \frac{\gamma_i}{\sigma}\epsilon_{k,i}}\tilde{t}_j, \tag{B.9}$$

where the denominator is positive by assumption as it equals (minus) the second-order condition for a welfare maximum. Eq. (B.9) gives the marginal slope of the reaction function of country $i$.

**General production function**

Next, we consider a linearization of $\eta_{k,i}$ and $\eta_{r,i}$ for a more general production function. We simplify the analysis by focusing on the three country case $(i, j, k)$. First, note that from Eqs. 1 it follows that: $f''(.)\partial k_j = f''(.)\partial k_k$, such that from Eq. 2 it follows that

$$\frac{\partial k_j}{\partial k_i} = \frac{-s_i/f''_j(.)}{s_j/f''_j(.) + s_k/f''_k(.)}, \quad \frac{\partial k_k}{\partial k_i} = \frac{-s_i/f''_k(.)}{s_j/f''_j(.) + s_k/f''_k(.)}. \tag{B.10}$$

Using this we can differentiate Eq. (A.8)

$$\frac{\partial\rho/\partial t_i}{\partial k_i} = \frac{s_i}{(\Delta)^2}\left[\frac{f'''_i(.)}{(f''_i(.))^2}\Delta - \frac{1}{f''_i(.)}\left(\frac{s_i f'''_i(.)}{(f''_i(.))^2} + \frac{s_j f'''_j(.)}{(f''_j(.))^2}\frac{\partial k_j}{\partial k_i} + \frac{s_k f'''_k(.)}{(f''_k(.))^2}\frac{\partial k_k}{\partial k_i}\right)\right], \tag{B.11}$$

with $\Delta = s_i(1/f''_i(.)) + s_j(1/f''_j(.)) + s_k(1/f''_k(.))$. Which can be rewritten as

$$\begin{aligned}\frac{\partial\rho/\partial t_i}{\partial k_i} = \frac{\eta_{r,i}}{k_i}\frac{k_i f''_i}{\Delta}&\left[\frac{s_i f'''_i(.)}{(f''_i(.))^3}\left(\frac{s_j/f''_j(.) + s_k/f''_k(.)}{s_i/f''_i(.)}\right)\right.\\ &\left.+\left(\frac{s_j f'''_j(.)}{(f''_j(.))^3} + \frac{s_k f'''_k(.)}{(f''_k(.))^3}\right)\left(\frac{s_i/f''_i(.)}{s_j/f''_j(.) + s_k/f''_k(.)}\right)\right] = -\frac{\eta_{r,i}\delta_i}{k_i} < 0\end{aligned} \tag{B.12}$$

such that $\delta_i > 0$. The sign follows from the assumption $f'''_i > 0$. An increase in the capital stock increases the influence of country $i$ on the interest rate, $\eta_{r,i}$ increases. Now use Eq. (A.5) to derive

$$\frac{\partial k_i/\partial t_i}{\partial k_i} = \frac{1}{f''_i(.)}\left(\frac{\partial\rho/\partial t_i}{\partial k_i}\right) - \frac{f'''_i(.)}{f''_i(.)}\left(\frac{\partial k_i}{\partial t_i}\right). \tag{B.13}$$

An ambiguity arises, more market power (the first term in Equation (B.13)) leads to a reduction in $\eta_{k,i}$. On the other hand, the inflow of capital makes the marginal return more sensitive to changes in the capital stock, causing a negative second term. Summarizing

$$\widetilde{\eta}_{r,i} = \delta_i \widetilde{k}_i, \quad \widetilde{\eta}_{k,i} = -\left[\frac{f_i'''k_i}{f_i''} + \frac{\eta_{r,i}\delta_i}{1-\eta_{r,i}}\right]\widetilde{k}_i$$

When inspecting Eq. (B.7) for the case of symmetric countries ($e_i/k_i - 1 = 0$), Eq. (B.13) implies that a general production function might either cause the MCF to decrease further through a capital inflow (in case the second term in Eq. (B.13) dominates) or raise it (in case the first term in Eq. (B.13) dominates). The former case strengthens strategic complements, the latter case strengthens strategic substitutes. In general, a quadratic production function causes an unambiguous reduction in the MCF through an inflow of capital. It rules out strategic substitutes occurring through an increase in the MCF. A more general production function, allows for this.

When inspecting Eq. (B.9), we see that for a capital exporter ($e_i > k_i$) a general production function causes a capital inflow to increase the MCF relative to the case with a quadratic production function due to an increase in $\eta_{r,i}$. Ceteris paribus, this strengthens the case of strategic substitutes and strengthens proposition 3.

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Figure 1: Tax Reaction Function Country $i$ for constant $u_g/u_c$

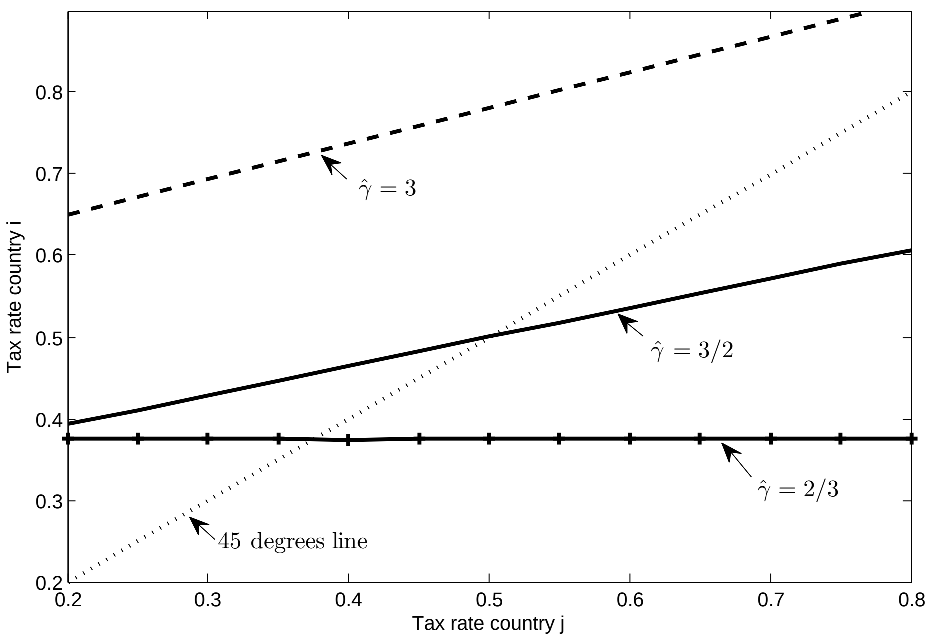

Figure 2: Tax Reaction Function Country $i$ for endogenous $u_g/u_c$

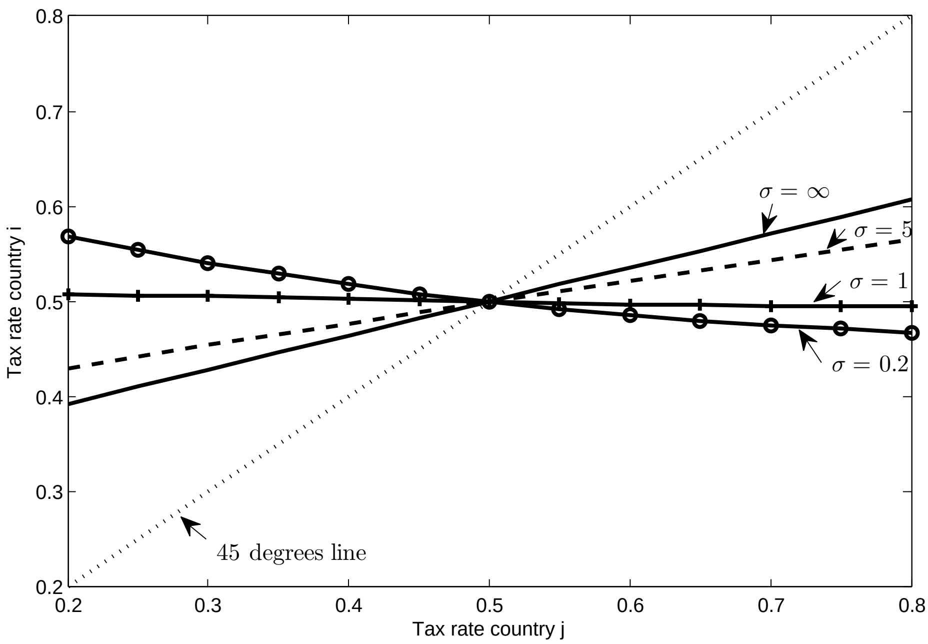

Figure 3: Tax Reaction Function Country $i$ under asymmetric capital positions $e_i \neq k_i$

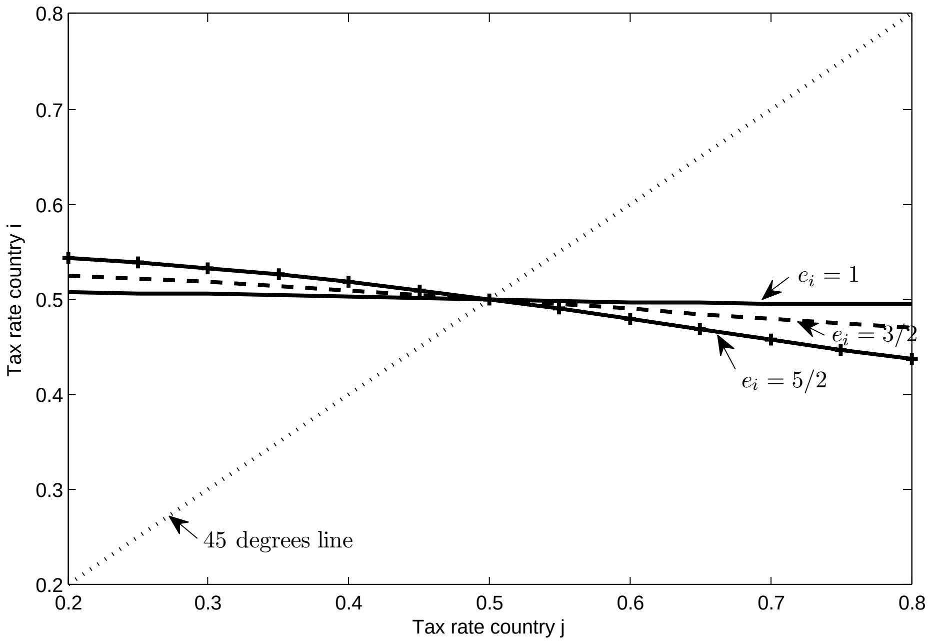

Table 1: Simulation Results

| $\frac{u_g}{u_c}$ | $\sigma$ | $s$ | $e_h$ | $t_1^D$ | $t_3^D$ | $\Delta t_h$ | $\Delta t_3$ | $CV_h$ | $CV_3$ |
|---|---|---|---|---|---|---|---|---|---|
| 3/2 | 1 | 2/5 | 1 | 0.51 | 0.48 | 12.7 % | -0.5 % | -0.008 | -0.036 |
| 3/2 | 1 | 2/5 | 3/4 | 0.52 | 0.50 | 18.5 % | -4.5 % | -0.012 | 0.002 |
| 3/2 | 1 | 1/3 | 1 | 0.50 | 0.50 | 10.9 % | -0.3 % | -0.005 | -0.024 |
| 3/2 | 1 | 1/3 | 3/4 | 0.50 | 0.50 | 15.5 % | -1.8 % | -0.008 | -0.018 |
| 3/2 | 1 | 1/6 | 1 | 0.47 | 0.56 | 5.8 % | -0.1 % | -0.001 | -0.005 |
| 3/2 | 1 | 1/6 | 3/4 | 0.46 | 0.55 | 7.8 % | -0.2 % | -0.002 | -0.006 |
| 3/2 | 1/5 | 2/5 | 1 | 0.50 | 0.49 | 2.7 % | -0.5 % | -0.001 | -0.008 |
| 3/2 | 1/5 | 2/5 | 3/4 | 0.50 | 0.50 | 4.3 % | -2.4 % | -0.002 | 0.000 |
| 3/2 | 1/5 | 1/3 | 1 | 0.50 | 0.50 | 2.4 % | -0.4 % | -0.001 | -0.006 |
| 3/2 | 1/5 | 1/3 | 3/4 | 0.50 | 0.50 | 3.7 % | -1.2 % | -0.001 | -0.005 |
| 3/2 | 1/5 | 1/6 | 1 | 0.49 | 0.51 | 1.5 % | -0.1 % | 0.000 | -0.002 |
| 3/2 | 1/5 | 1/6 | 3/4 | 0.49 | 0.51 | 2.1 % | -0.2 % | 0.000 | -0.002 |
| 3 | 1 | 2/5 | 1 | 0.53 | 0.46 | 33.7 % | 8.9 % | -0.068 | -0.223 |
| 3 | 1 | 2/5 | 3/4 | 0.51 | 0.50 | 36.0 % | 13.5 % | -0.065 | -0.330 |
| 3 | 1 | 1/3 | 1 | 0.50 | 0.50 | 26.5 % | 6.4 % | -0.049 | -0.150 |
| 3 | 1 | 1/3 | 3/4 | 0.48 | 0.52 | 28.3 % | 8.1 % | -0.047 | -0.201 |
| 3 | 1 | 1/6 | 1 | 0.47 | 0.67 | 10.9 % | 1.1 % | -0.010 | -0.031 |
| 3 | 1 | 1/6 | 3/4 | 0.45 | 0.67 | 11.8 % | 1.1 % | -0.010 | -0.035 |
| 3 | 1/5 | 2/5 | 1 | 0.51 | 0.48 | 8.0 % | -1.5 % | -0.012 | -0.070 |
| 3 | 1/5 | 2/5 | 3/4 | 0.47 | 0.54 | 8.4 % | 2.2 % | -0.013 | -0.280 |
| 3 | 1/5 | 1/3 | 1 | 0.50 | 0.50 | 8.1 % | -1.7 % | -0.008 | -0.057 |
| 3 | 1/5 | 1/3 | 3/4 | 0.46 | 0.56 | 7.7 % | -0.4 % | -0.008 | -0.122 |
| 3 | 1/5 | 1/6 | 1 | 0.47 | 0.55 | 4.8 % | -0.7 % | 0.001 | -0.013 |
| 3 | 1/5 | 1/6 | 3/4 | 0.43 | 0.57 | 4.3 % | -0.7 % | 0.002 | -0.012 |

*Notes*: For all simulations, we choose $\omega = 1/2$. The parameter $b$ in the production function $f = b(a-k)k$, is calibrate on the symmetric equilibrium, using $t = 1/2$, such that $b$ follows from the choice of $u_g/u_c = MCF$. $a$ follows from the choice of $\sigma$ such that the MRS equals the desired value of $u_g/u_c$. See Section 4 for a discussion.