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Mergers in Bidding Markets*

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Abstract. We analyze the effects of mergers in first-price sealed-bid auctions on bidders' equilibrium bidding functions and on revenue. We also study the incentives of bidders to merge given the private information they have. We develop two models, depending on how after-merger valuations are created. In the first, single-aspect model, the valuation of the merged firm is the maximum of the valuations of the two firms engaged in the merger. In the multi-aspect model, a bidder's valuation is the sum of two components and a merged firm chooses the maximum of each component of the two merging firms. In the first model, a merger creates incentives for bidders to shade their bids leading to lower revenue. In the second model, the non-merging firms do not shade their bids and revenue is actually higher. In both models, we show that all bidders have an incentive to merge.

Key Words: Mergers, first-price sealed-bid auctions.

JEL Classification: D44, D82.

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1. Introduction

Bidding markets become more and more important in many aspects of economic life. In government procurement, auctions are more and more prevalent. In business-to-business markets, auctions or other tender procedures are used increasingly often. Like in other markets, mergers do take place, and an important question, especially for competition authorities, is how to evaluate mergers in bidding markets.

One important class of such mergers occurs in public transport markets in Europe, where (local) governments have used public tender procedures to choose a private transportation company to provide regional public transportation services. One popular view on mergers in bidding markets is that due to the buyer power of the auctioneer, mergers are not anti-competitive as long as there remains one serious independent competitor in the market after a merger (see, *e.g.*, Lexecon, 1995). Theoretically, this view seems to be based on the traditional Bertrand model with homogeneous goods, where indeed one firms takes and serves the market, and the existence of one competitor is enough to create a competitive outcome. Klemperer (2006) argues against this view, stating that mergers in bidding markets must be evaluated essentially along the same lines as mergers in other markets.

In this paper, we argue that mergers in bidding markets do warrant a special treatment, but that it is certainly not the case that the presence of one serious bidder is sufficient for bidding markets to remain as competitive as without the merger. We consider a standard first-price sealed-bid auction (as most bidding markets use this auction format) and ask the question whether firms have an incentive to merge and what the effects of a merger are on equilibrium bidding functions and on revenue.

We model a merger in the following way. Without mergers, a bidding market is presented as a static first-price sealed-bid auction where bidders have private information concerning their valuations. After knowing his own private valuation, but without knowing the private valuation of the other players, one of the players offers a proposal to merge to another player. Merging implies acting as a single bidder and submitting together one bid in the auction. The bidder receiving the offer to merge, knowing his private valuation, but not knowing the private valuation of the bidder that makes the offer, has to decide whether to accept the offer or not. If the offer is not accepted, everyone makes separate bids. If the offer is accepted, all other bidders know a merger has taken place, and that the merged entity has access to all the information of both merging bidders. Note that private information of the other merged bidder becomes only available to both merging bidders after the merger is accepted and implemented.

Two modeling issues are important in this set-up. The first issue is how to model the valuation of the merged entity. The second issue is how to deal with the possibility that the process of the merger, *i.e.*, whether to propose a merger and whether to accept it, reveals (a part of) private information about the type of the proposing and/or accepting bidders.

On the first issue, we consider two variations. In the basic set-up, we assume that the valuation of the merged entity is the maximum of the valuation of both merging bidders. After accessing private information of both bidders, the merged entity realizes who has the highest value and then bids according to this highest value. In the more advanced set-up, the valuation of a bidder is built up from more basic processes within the firm. In public transportation, for instance, a firms' after-auction profits, which is the valuation in the auction, depends on its marketing abilities, the ability to manage the logistic processes, its potential to bargain low prices for the coaches or trains it is using, *etc*. For simplicity, we assume that the bidder's value is the sum of such partial valuations, which we call aspects and model these partial valuations as independent random variables. Thus, a bidder's type in this advanced set-up is multi-dimensional. After the merger, the merged entity has access to the best aspects of the merging firms, and chooses to combine the largest partial valuations.[1] Accordingly, the basic set-up is called the single-aspect model, while the advanced set-up is called the multi-aspect model. In the discussion section at the end of the paper, we discuss to what extent our multi-aspect model should be regarded as modeling efficiencies in bidding markets (in a similar vein to the cost efficiencies introduced by Perry and Porter, 1985, building on Williamson, 1968).

Concerning the issue whether the actions to propose or accept a merger reveals private information, we consider the incentives of different types of bidders to merge. If all types of bidders have an incentive to propose and accept a merger, then these actions do not reveal firms' private information. This is, obviously, the simplest situation to analyze; otherwise the merger proposal and acceptance strategies, along with auction bidding, are to be analyzed as a signaling game where both the proposer and the receiver of the proposal needs to take into account which types are willing to propose and which types are willing to accept the merger.

The analysis of mergers in bidding markets requires analyzing asymmetric first-price sealed-bid auctions. Even if the auction is symmetric before a merger, it transforms into an asymmetric auction after the merger in both the single aspect and the multi-aspect model. The asymmetry arises as after the merger the valuation of the merged entity follows a different distribution function than the valuation of the non-merging bidders. Asymmetric first-price sealed-bid auctions are inherently difficult to analyze analytically. Marshall *et al* (1994) have developed numerical techniques to solve asymmetric first-price auctions, and Lebrun (2006) has proved, under fairly general conditions, that an equilibrium exists and is unique. Thus, the analysis in this paper is also largely based on numerical methods. We do, however, prove some results on the effect of mergers by comparing revenues in symmetric auctions with four firms with revenues in symmetric auctions with two firms, where the

[1] Note that in most of the auction literature, it is not interesting to consider the composition of a bidder's valuation. In the end, it is a bidder's overall valuation that determines his bid. With mergers, this is no longer true because in the multi-aspect model, the valuation of the merged entity can be strictly higher than the maximum of the two valuations of the merging bidders.

valuations of each of the two bidders are equal to the maximum of the valuations of two of the bidders in the four bidder auction. As the competition issues seem to be mainly relevant when the bidding market is already concentrated to begin with, we mainly focus on the case where the number of bidders is small.

In the single-aspect model, we arrive at the following results. After the merger, there is some bid shading, especially by the merged firm, and therefore revenue is lower than without a merger. Moreover, all types have an incentive to merge, and, therefore, the act of proposing or accepting a merger is not a signal of a firm's type. It also implies there is no merger paradox, as in Salant *et al* (1983). Note, however, that the reduction in revenue is much smaller than what could be expected from just taking one rival bidder out of the market. Note also that despite all types having an incentive to merge, it is the higher valuation types that have most incentives to merge and that actually, like in mergers in "normal markets", it is the non-merging firms that benefit the most from the merger.

These results are fairly standard in any merger analysis and can be explained as follows. For a non-merging bidder, the changes caused by the merger are, in a sense, of second-order magnitude. In a first-price sealed-bid auction, the bidding behavior of a player is driven by the distribution of the highest bid of its competitors. Since the valuation of the merged bidder is the maximum of the valuations of the two merging firms, the best response bidding strategy of non-merging bidders remains unaffected. On the other hand, facing the same bidding strategy of non-merging bidders, the merged entity shades its bid compared to the bids of the two merging bidders because it faces one competitor less. Thus, the overall merger effect on the bids of non-merged bidders is based on two effects. First, due to bid shading of the merged bidder, non-merged bidders also bid less. Second, given that the merged bidder has a better (in the sense of first-order stochastic dominance, FOSD hereinafter) distribution of the value than any of the individual bidders, these latter non-merged bidders bid more aggressively after the merger.

The results in the multi-aspect model are, however, very different. Our main result here is that mergers are often revenue increasing. Moreover, as in the singe-aspect case, all bidders' types have an incentive to merge. Finally (and interestingly), it is the *merged bidder* that benefits the most from the merger, not the non-merging firms. That is, unlike most of the merger literature where free-riding is prevalent (see, *e.g.*, Deneckere and Davidson, 1985), there is no free-riding of the non-merging firms in this context.

In order to explain these results, we point at the same two forces as in the single-aspect model. The main difference is that in the multi-aspect case, the merged bidder has a better value distribution than it has in the single-aspect case. This means that, despite the merged firm shading his bid, the bid distribution of the merged entity improves (in FOSD sense) so that all non-merged firms bid more aggressively than in the single dimension case. In short, in the multi-aspect model mergers are competition and revenue enhancing!

To the best of our knowledge there are three papers on mergers in bidding markets that are directly comparable to ours.[2] Waehrer (1999) studies coalitions of bidders in first and second price auctions in terms of the average profit a collation member makes. He numerically shows that mergers in first and second-price auctions have different effects in that in second-price auctions only the expected profit of the merging firms is affected. Waehrer and Perry (2003) derive analytic results for second-price sealed-bid auctions (as these auctions have a weakly dominant bid strategy) and find that mergers are profitable and reduce competition. They also consider how an auctioneer may affect auction outcomes by choosing a reserve price. Dalkir *et al* (2000) extensively discuss how bidding markets are relevant for the evaluation of hospital mergers in the USA where hospitals sell their service to preferred

[2] There are three other papers that deserve mentioning. Using bidders' values that are drawn from Extreme Value distributions, Tschantz *et al*. (2000) analyze first-price and second-price auctions with three bidders. Thomas (2004) studies bidding markets where firms' types are drawn from a discrete distribution, and derives analytic results for this case. Our paper and the papers mentioned in the main text consider auctions with a continuum of types. A recent paper by Gössl and Wambach (2012) studies dynamic issues in bidding markets and focuses on a setting where bidders' valuations are common knowledge. Our paper is static in nature, but does consider the private information nature of a bidder's valuation.

provider organizations (PPOs). That analysis covers the revenue and bidding aspect of what we have termed the single aspect model. Compared to this literature, we add three points to the analysis of mergers in bidding markets. First, we introduce a methodology to assess a bidder's incentives to merge given his private information. Second, we analyze the way merged bidders can combine different aspects of the merging firm's valuation. Third, we prove a result on revenues in asymmetric auctions by comparing two carefully constructed symmetric auctions.

The rest of the paper is organized as follows. Section 2 sets up the signal-aspect model, analyzes its equilibrium bidding behavior and its revenue consequences, and studies the incentive of the bidders to merge. Section 3 covers the multi-aspect model. Section 4 concludes with a discussion. The appendix contains proofs and elaborates derivations of more complicated expressions.

2. Single-aspect model

Consider a first-price sealed-bid auction where $i = 1, \ldots, N$ bidders have private valuations v_i which are independently distributed according to a distribution function F_i. Two out of these *N* firms consider to merge. After the merger, we denote the merged entity by the superscript M and the other non-merged bidders by the superscript N. Lebrun (2006) has demonstrated that under fairly general conditions, an equilibrium always exists and is unique in asymmetric first-price sealed-bid auctions. Therefore, we know that both before and after the merger equilibrium exists and is unique. In an equilibrium, bidder i bids $b_i(v_i)$. For simplicity, we assume that before the merger, individual valuations are uniformly distributed over the interval $[0,1]$. Auction revenue is denoted by R.

Before the merger takes place, we have a standard symmetric auction where the equilibrium bidding function is given by $b(x) = \frac{N-1}{N}x$ and revenue equals

$$R = E\left[\frac{N-1}{N}\max_i[v_i]\right] = \frac{N-1}{N}\int_0^1 x dx^N = \frac{N-1}{N+1}.$$

Let bidders 1 and 2 merge. The new bidder M is assumed to have a value $v^M = \max[v_1, v_2]$. The equilibrium bidding functions after a merger are given by: $b^M(v^M)$ and $b^N(v_i)$, $i = 3, \dots, N$, where the valuation of the merged bidder does not follow a uniform distribution anymore, but instead is given by the distribution of $v^M = \max[v_1, v_2]$, which is $F^M(x) = x^2$.

Profits for the merged and non-merged firms of type x, bidding as if their type were y and all others bid according to the equilibrium bidding functions, are now given by

$$\pi^M\big(x, b^M(y)\big) = \big(x - b^M(y)\big) \prod_{i=3,\dots,N} \Pr[b^N(v_i) < b^M(y)]$$

$$\pi_i^N\big(x, b^N(y)\big) = \big(x - b^N(y)\big) \Pr[b^M(v^M) < b^N(y)] \prod_{\substack{j=3,\dots,N \\ j\neq i}} \Pr\big[b^N\big(v_j\big) < b^N(y)\big]$$

If we write $\varphi^M(b) \equiv (b^M)^{-1}(b)$ and $\varphi^N(b) \equiv (b^N)^{-1}(b)$ for the inverses of the bidding functions, we can simplify the profit functions to the following expressions:

$$\pi^M(x, b) = (x - b)\big(\varphi^N(b)\big)^{N-2}$$

$$\pi_i^N(x, b) = (x - b)\big(\varphi^M(b)\big)^2\big(\varphi^N(b)\big)^{N-3}$$

In equilibrium, both merged and non-merged firms should maximize their profits, giving a system of first-order conditions that is analytically intractable:

$$\begin{cases} 0 = \dfrac{\partial \pi^M}{\partial b}(\varphi^M, b) = \Big((N-2)(\varphi^M - b)\varphi^{N'} - \varphi_i^N\Big)(\varphi^N)^{N-3} \\ 0 = \dfrac{\partial \pi_i^N}{\partial b}(\varphi^N(b), b) = \varphi^M(\varphi^N)^{N-4}\Big((\varphi^N - b)\big(2\varphi^N\varphi^{M'} + (N-3)\varphi^M\varphi^{N'}\big) - \varphi^M\varphi^N\Big) \end{cases}$$

For the numerical analysis, we fix the number of bidders to $N = 4$ and compute the Bayes-Nash equilibrium for the asymmetric auction where bidders 1 and 2 merge. The merged entity bids $\varphi^M(x)$ and other bidders $i = 3,4$ bid $b^N(v_3)$ and $b^N(v_4)$. Figure 1 represents bidding functions $b(x)$ before, and $b^M(x)$ and $b^N(x)$ after the merger.

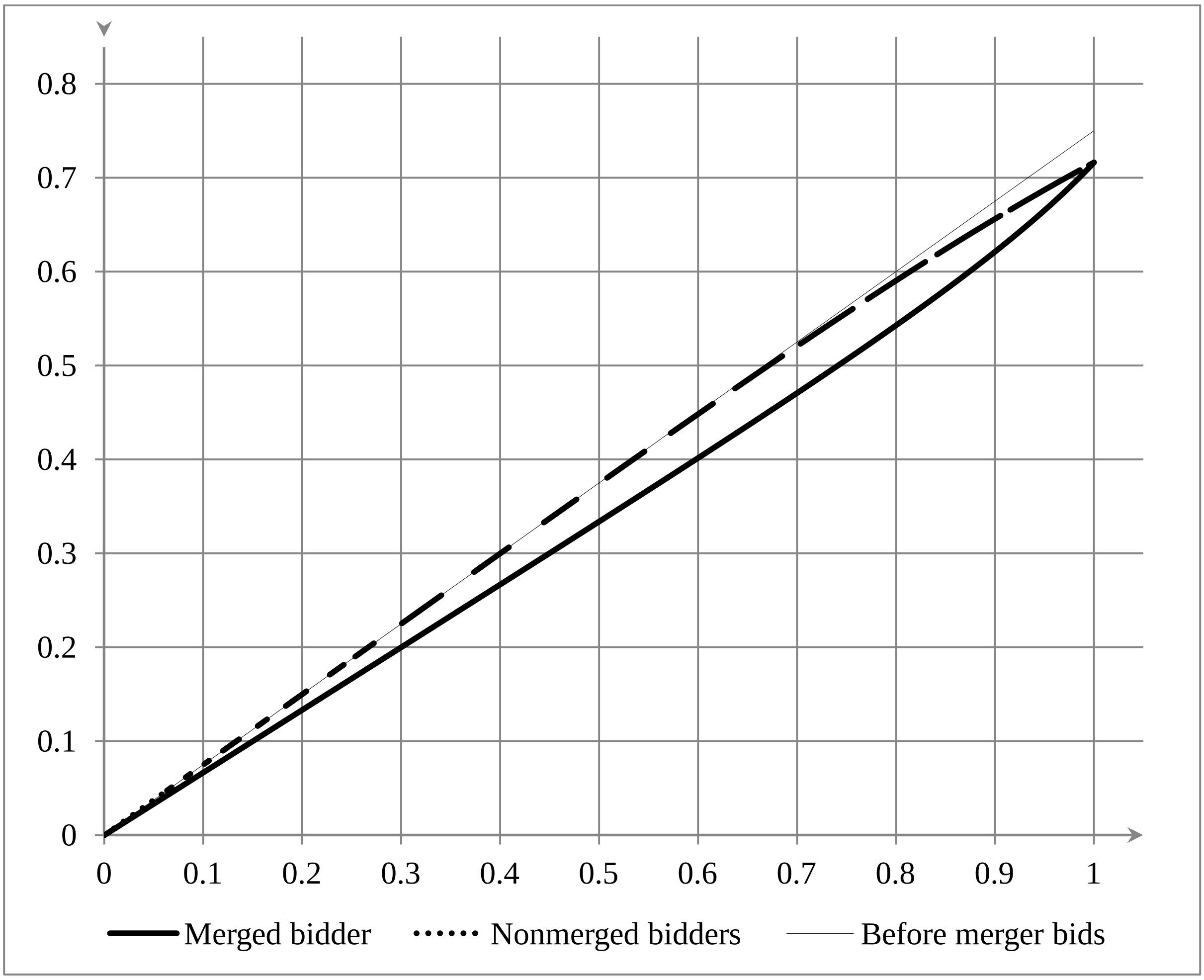

Figure 1. Bidding functions $b(x)$ before and $b^M(x)$ and $b^N(x)$ after the merger of bidders $i = 1,2$, for $N = 4$.

One can clearly see that the merged bidder shades its bid and then, in reaction, so do the non-merging bidders $i = 3,4$, especially for higher valuations. It is, therefore, not a surprise that the auction revenue drops because of the merger from $R = 0.600$ to $R^M = 0.569$. Bidders' expected *ex ante* surpluses before they know their types, are 0.050 before and 0.109 for the merged bidder and 0.060 for the remaining bidders after the merger. Thus, like in most of the merger literature, the non-merging firms benefit most from the merger, *i.e.*, there is a free-riding problem in the sense that everybody wants others to merge.

We next look into the incentives of different types to merge. Before the merger takes place, bidder $i = 1$ of type $v_1 = x$ bids $b_1 = b(x) = \frac{3}{4}x$, getting the following surplus:[3]

[3] We use the following common notation here: $\mathrm{E}[A : B] = \mathrm{E}[A|B] \cdot \Pr[B]$.

$$s_1(x) = \mathrm{E}_{v_{-1}}[x - b_1(x) : \max[v_{-1}] \le x] = \big(x - b(x)\big)\big(F(x)\big)^{N-1} = \frac{1}{4}x^4$$

From the prospective of bidder 1, bidder $i = 2$ gets an expected surplus conditional on x of

$$\bar{s_2}(x) = \mathrm{E}_{v_{-1}}[s_2 | v_1 = x] = \mathrm{E}_{v_{-1}}\big[\big(v_2 - b_2(v_2)\big) | v_1 = x\big]$$

$$= \mathrm{E}_{v_{-2}}\left[\frac{1}{4}v_2 : \max\{x, v_3, v_4\} \le v_2\right] = \frac{1}{4}\int_x^1 \left(\int_0^{v_2} v_2 d(z^2)\right) dv_2 = \frac{1}{16}(1 - x^4).$$

Thus, the joint expected profit of bidders $i = 1,2$ before the merger from the perspective of type x of bidder 1 is:

$$\overline{s_{1+2}}(x) \equiv s_1(x) + \bar{s_2}(x) = \frac{1}{16}(1 + 3x^4).$$

To investigate the surplus of the merged bidder conditional on bidder $i = 1$ having type x, we first consider the expected surplus conditional on v^M of the merged bidder $s^M(v^M)$:

$$s^M(v^M) = \mathrm{E}_{v_i^N}\left[\big(x - b^M(x)\big) : \max_{i=3,4}[b^N(v_i)] \le x\right] = \big(x - b^M(x)\big)\left(F\left(\varphi_i^N\big(b^M(x)\big)\right)\right)^2.$$

Its expectation conditional on $v_1 = x$ is given by

$$\overline{s^M}(x) \equiv \mathrm{E}_{v_2}[s^M(v^M) | v_1 = x] = \mathrm{E}_{v_2}[s^M(\max[x, v_2])]$$

$$= \Pr[v_2 < x] \times s^M(x) + \mathrm{E}_{v_2}[s^M(v_2) : v_2 > x] = F(x)s^M(x) + \int_x^1 s^M(v_2)dv_2.$$

Using numerically computed bidding functions $b^M(x)$ and $b_i^N(x)$, we compute the joint expected profit $\overline{s^M}(x)$ of bidders $i = 1,2$ after the merger. Figure 2 represents graphs of $\overline{s_{1+2}}(x)$ (thing curve), and of $\overline{s^M}(x)$ (bold curve).

As one can see, $\overline{s^M}(x) > \overline{s_{1+2}}(x)$ for all $x \in [0,1]$. This implies that bidder $i = 1$, whatever his private valuation v_1 is, always has an incentive to approach any another bidder and to propose a merger. Indeed, by doing so, bidder $i = 1$ increases his joint surplus when the proposal is accepted. Similarly, bidder $i = 2$ always has the same incentive to accept the proposal. Thus, we do not need to worry about potential signaling issues: all types have an

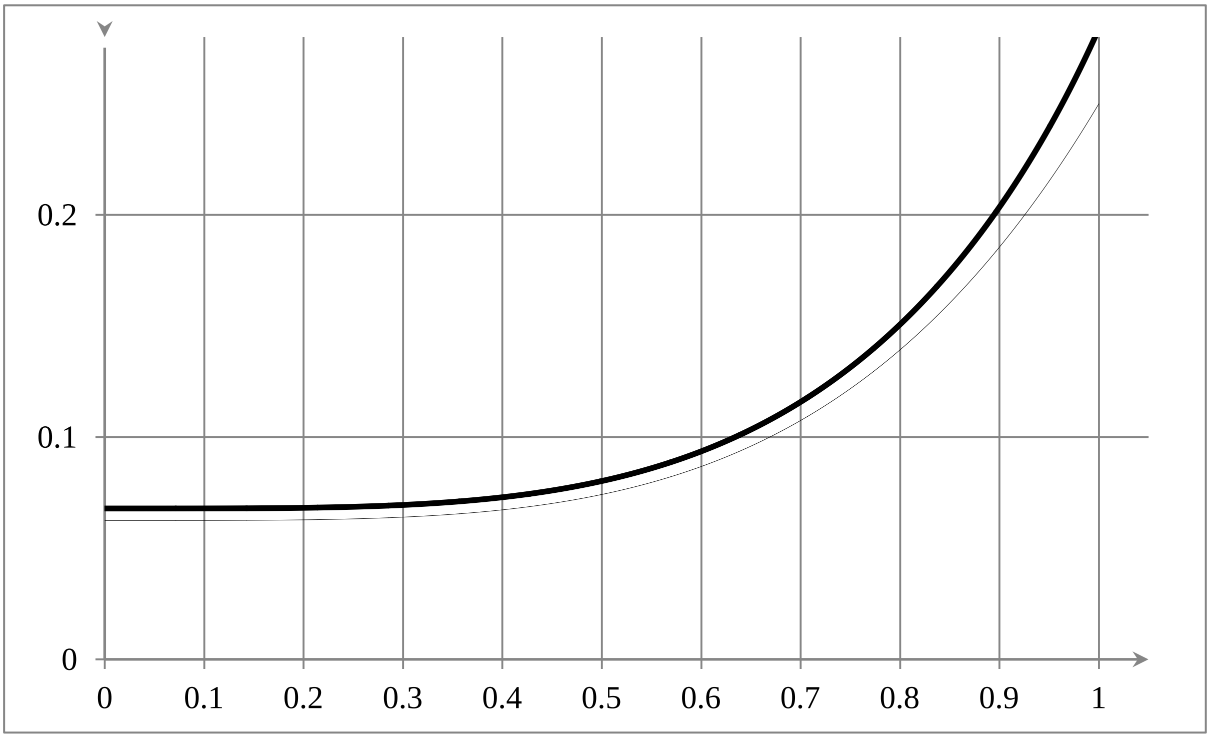

Figure 2. Joint expected surplus of bidders $i = 1,2$, before, $\overline{s_{1+2}}(x)$ (thing curve), and after the merger, $\overline{s^M}(x)$ (bold curve), for $N = 4$.

incentive to merge. One can also see that the incentive to merge of higher types are stronger than that of low types.

3. Multi-aspect model

We now consider the multi-aspect model briefly discussed in the Introduction. To simplify the exposition, we assume two aspects only, *i.e.*, the overall valuation is composed of two additive terms. In particular, let the value of bidder i be the sum of two independent random variables v_i^1 and v_i^2, *i.e.*, $v_i = v_i^1 + v_i^2$. When bidders $i = 1,2$ merge, the merged entity gets the value $v^M = \max[v_1^1, v_2^1] + \max[x_1^2, x_2^2]$. That is, the merged bidder chooses the best components of each merging bidder, and then adds the two components together to create the overall valuation.

In order to keep valuations v_i distributed over the interval $[0,1]$, we let the components v_i^1 and v_i^2 be independently and uniformly distributed over the interval $\left[0,\frac{1}{2}\right]$. The density and distributions functions of v_i are therefore no longer uniform, but given by

$$f(x) = \int_{-\infty}^{\infty} f_1(x-t) f_2(t) dt = 2 \int_{x-0.5}^{x} f_1(z) dz = \begin{cases} 4\int_0^x dz = 4x & \text{, if } x \in \left[0,\frac{1}{2}\right] \\ 4\int_{x-0.5}^{0.5} dz = 4(1-x), & \text{if } x \in \left[\frac{1}{2},1\right] \end{cases}$$

and

$$F(x) = \begin{cases} \int_0^x 4t dt = 2x^2 & \text{, if } x \in \left[0,\frac{1}{2}\right] \\ \frac{1}{2} + \int_{0.5}^{x} 4(1-t) dt = 1 - 2(1-x)^2, & \text{if } x \in \left[\frac{1}{2},1\right] \end{cases}$$

We first describe the implications of a merger in this two-aspect valuation model for the bidding functions of different bidders. As before, we are interested in the (symmetric) bidding function $b(x)$ before the merger and the bidding functions $b^M(x)$ for the merged and $b^N(x)$ for non-merged firms in the asymmetric auction after the merger.

For the numerical analysis, we compute the Bayes-Nash equilibrium before and after the merger. Figure 3 presents the results for $N = 4$. One can see that, roughly speaking, the same effects arise here as in the single-aspect model, but now with different strengths. First, the merged entity has an incentive to shade its bid (given that it has the same valuation as one of the firms before the merger). The non-merging bidders also want to shade their bids, but they know that they now compete with a stronger bidder (with a higher valuation in a FOSD sense than the valuation of the pre-merged bidders).

One can see in the graph that these two effects roughly cancel out in the present case, but interestingly, for middle range valuations, the non-merging bidders slightly increase their bids, while for higher valuations the bid-shading incentive dominates. The reason for this

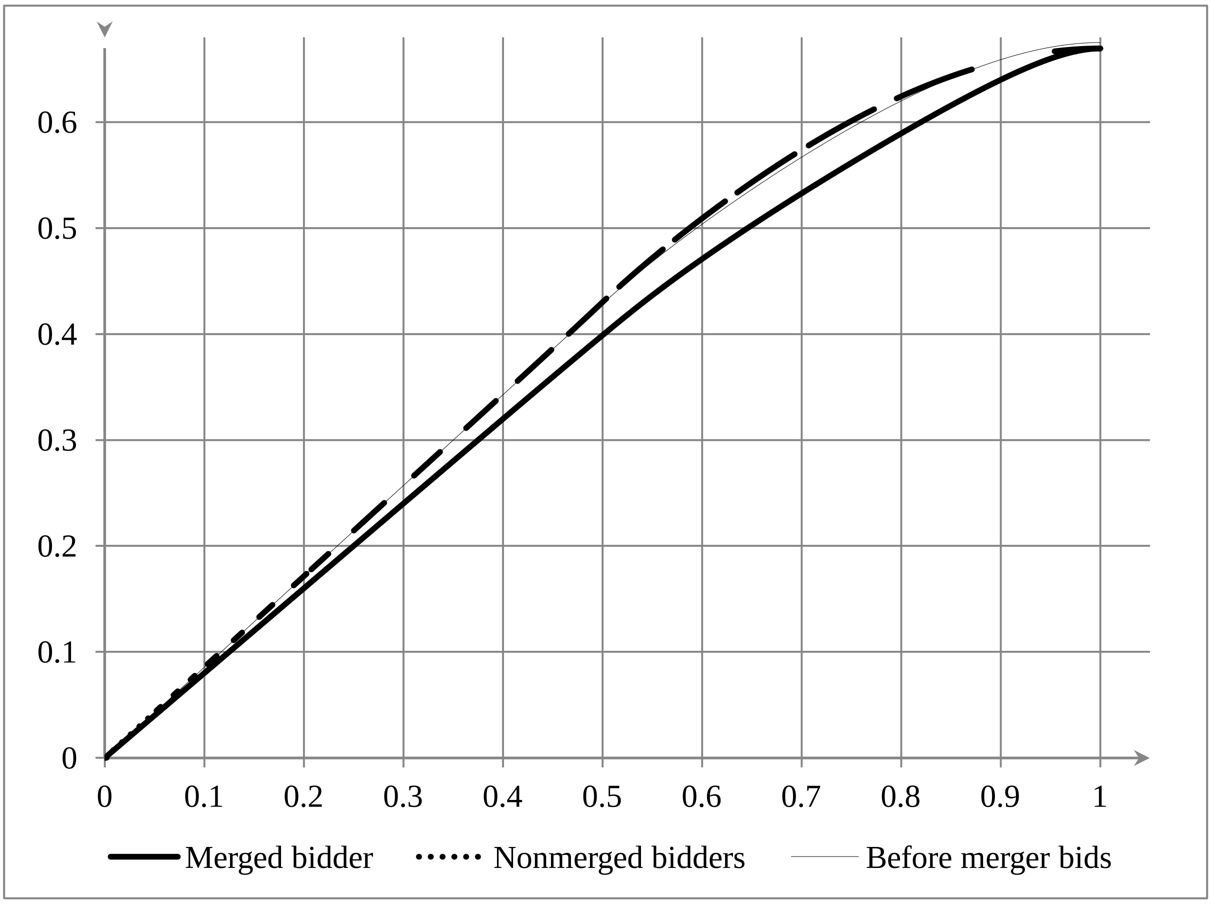

Figure 3**.** Bidding functions before and after the merger of bidders $i = 1,2$ for $N = 4$.

property is that it is much more likely that the after-merger value exceeds the maximum of the two bidders' values for the middle range valuations than for the upper range valuations.

Given these bidding functions, we can now analyze the revenue implications of a merger in the two-aspect model when N is small. We can analytically prove that when $N = 4$, at least some of the mergers that can arise in such an environment are revenue increasing.

Proposition 1. Consider a two-aspect bidding market with $N = 4$ symmetric firms. If we let any pair of firms merge sequentially, then it must be the case that in at least one of these mergers the revenue has increased.

The way Proposition 1 is proven is as follows. Before and after two sequential mergers, we have a symmetric auction with $N = 4$ and $N = 2$ firms, respectively. Each of these symmetric auctions satisfies the conditions of the revenue equivalence theorem, and we, therefore, can use it to express the revenue in both cases as the expected value of the second-highest order statistic of bidders' values. Comparing revenues before and after the merger reveals that the revenue in the auction with $N = 2$ bidders is larger than the revenue with $N = 4$ bidders. Then, as this overall change in revenue is the sum of the revenue change going first from $N = 4$ to $N = 3$ bidders, and then from $N = 3$ to $N = 2$, it must be that at least one of these mergers has resulted in higher revenues. Thus, we use results of symmetric auctions, to prove a result that involves at least one asymmetric auction.'

Using numerical methods, we can show that the revenue actually increases in both steps. With $N = 4$ bidders, the expected revenue is $R(4) = 0.5637$. When two bidders merge resulting in an auction with $N = 3$ bidders, expected revenue increases to $R(3) = 0.5653$. When the two remaining yet non-merged bidders then subsequently also merge, the auction is again symmetric, and the expected revenue is $R(2) = 0.5719$. Thus, each subsequent merger increases the expected revenue.

In a similar way, one can also analyze mergers in bidding markets starting with $N = 6$ bidders. The initial expected revenue is $R(6) = 0.6376$. With each subsequent merger, eventually leading to $N = 3$ symmetric firms, expected revenues increase to $R(5) = 0.6479$, $R(4) = 0.6602$, and, finally, to $R(3) = 0.6738$. Numerical simulations show that expected revenue goes up when we move from $N = 2k$ firms to $N = k$ firms for all even N up to at least 30. Similar statements as the one in Proposition 1 can be proven analytically for each fixed $N = 6,8$, *etc.*

We will investigate, finally, the incentives of bidders to merge. The joint expected surplus $\overline{s_{1+2}}(x)$ of bidders $i = 1,2$ before the merger can be numerically computed in a way that is similar to what we have done in the previous section. That is, from the perspective of bidder $i = 1$ with value $v_1 = x$, we have the standard surplus for bidder $i = 1$ and an expected surplus $\overline{s_2}(x)$ of bidder $i = 2$ before the merger conditional on x. Note that before the merger, the distribution of valuation over the two different aspects is unimportant as it is the overall valuation that determines a player's bid and his surplus:

$$s_1(x) = \left(x - b_1(x)\right)\left(F(x)\right)^{N-1}$$

$$\bar{s_2}(x) = \mathrm{E}_{v_{-2}}\left[\left(v_2 - b_2(v_2)\right) : \max\{x, v_{-2}\} \leq v_2\right]$$

$$= \int_x^1 \left(v_2 - b_2(v_2)\right)\left(F(v_2)\right)^{N-2} dF(v_2) = \frac{1}{N-1}\int_x^1 \left(v_2 - b_2(v_2)\right) d\left(F(v_2)\right)^{N-1}.$$

Thus, joint expected surplus $\overline{s_{1+2}}(x) \equiv s_1(x) + \bar{s_2}(x)$ is

$$\overline{s_{1+2}}(x) = \left(x - b_1(x)\right)\left(F(x)\right)^{N-1} + \frac{1}{N-1}\int_x^1 \left(v_2 - b_2(v_2)\right) d\left(F(v_2)\right)^{N-1}.$$

For the situation after the merger, it is the distribution of $v^M = \max[v_1^1, v_2^1] + \max[x_1^2, x_2^2]$ conditional on (v_1^1, v_1^2) that determines the expected surplus of the merged entity. That is, we have to evaluate

$$F^M(x|v_1^1, v_1^2) \equiv \Pr[\max[v_1^1, v_2^1] + \max[v_1^2, v_2^2] \leq x\,|v_1^1, v_1^2].$$

In the Appendix, we show the derivations of $F^M(x|v_1^1, v_1^2)$ for the different cases we have to consider. Then, we compute the expected surplus $s^M(v_1^1, v_1^2)$ of the merged bidder conditional on the private information (v_1^1, v_1^2) of bidder $i = 1$. The willingness to merge $WTM(v_1^1, v_1^2)$ is then defined as follows:

$$WTM(v_1^1, v_1^2) \equiv s_M(v_1^1, v_1^2) - \overline{s_{1+2}}(v_1^1 + v_1^2).$$

Thus, $WTM(v_1^1, v_1^2)$ is the expected gain in surplus due to the merger from the perspective of a bidder of type (v_1^1, v_1^2). Figure 4 shows the graphs of $WTM(v_1^1, v_1^2)$ as a function of v_1^1, for 11 different values of v_1^2, $v_1^2 \in \{0, 0.05,\ 0.1,\ 0.15, \dots, 0.45,\ 0.5\}$.

We conclude that (as in the single-aspect case analyzed in the previous section) all types have an incentive to merge. The graph clearly shows that almost symmetric types that are close to $(v_1^1, v_1^2) = (0,0)$ and $(1,1)$ have the lowest incentives to merge, while asymmetric types that are close to $\left(0, \frac{1}{2}\right)$ and $\left(0, \frac{1}{2}\right)$ have the largest incentives to merge. This is, of

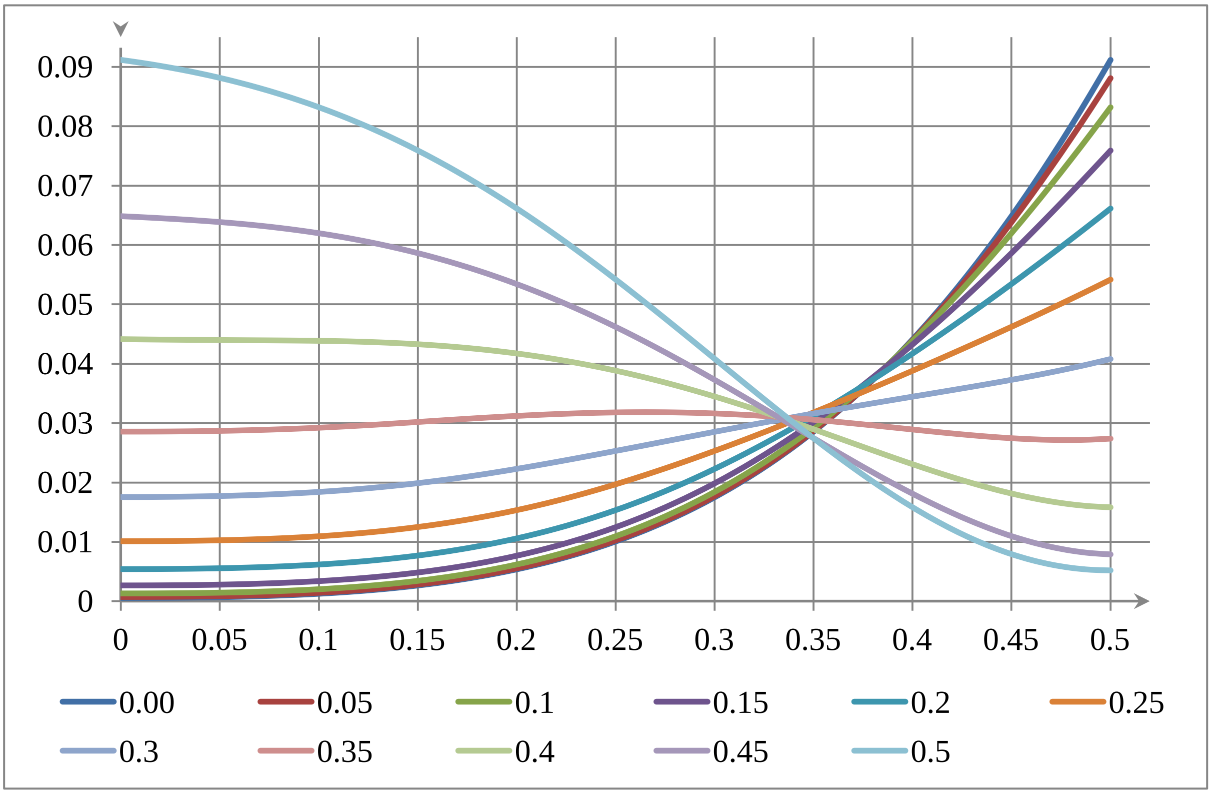

Figure 4. Willingness to merge $WTM(v_1^1, v_1^2)$, as a function of v_1^1 (horizontal axis) for different values of v_1^2 (different graphs), for $N = 4$.

course, not so surprising. Asymmetric types have the highest chance of positively contributing to the valuation of the merged firm (due to a very high value on one aspect), while also benefitting from the fact that the chances are high that the other merging firm also positively contributes (due to a very low value on the other aspect).

One final remark about this two-aspect valuation model is that the merging bidders significantly benefit from the merger. The joint *ex ante* expected surplus of both merging bidders, *i.e.*, before they know their types, increases from 0.0371 to 0.0485, while the surplus of the non-merging bidders slightly decreases. This implies that the two-aspect model overcomes the free riding issue in mergers.

4. Discussion and Conclusion

In this paper, we have analyzed symmetric and asymmetric sealed-bid auctions to study the impact of mergers in bidding markets on equilibrium bidding functions and expected revenue. We have also asked the question whether firms have an incentive to merge given the private information they have. The study of the incentives to merge in a private information context is, as far as we know, new to the literature on mergers.

In case the valuation of the merged entity is the maximum of the valuation of the merging entities, we confirm the standard economic intuition regarding mergers in "normal" markets: mergers make the auction less competitive measured by lower expected revenue for the auctioneer. We also show that bidders have an incentive to merge, no matter how small or large their valuation. Thus, there is no merger paradox. Nevertheless, there is a free-riding problem in the sense that everybody wants others to merge.

In many markets, however, the valuation of a bidder is composed of several aspects (such as marketing, organizational, production, *etc.*) and a merged entity may be able to select the best aspects of each of the merging bidders to create the overall value of the newly merged bidder. With two aspects, we show that the results of the single-aspect model summarized in the previous paragraph are overturned. In particular, bidding markets may become more competitive and expected revenue may increase after a merger. Still, because of the increase in expected value due to the merger, bidders of all types do have an incentive to merge. Moreover, the free riding issue is absent in the two-aspect model; the merging bidders benefit from the merger whereas non-merging bidders suffer from it.

One interesting issue is whether two-aspect mergers should be regarded as generating a synergy between the merged parties. One may argue that the two-aspect model assumes synergies as in expected terms the value of the merged entity is higher than the maximum of the valuations of the two merging firms.[4]

There are, however, two arguments that shed at least some doubt on this view. First, Farrell and Shapiro (1990, p. 112) argue that when the only thing that a merged firm can do after the merger is to "better allocate outputs across facilities" then the merger is said to generate *no* synergies. They call this process a rationalization of production possibilities. In the spirit of Farrell and Shapiro (1990), one could argue that the two-aspect model only allows the firm to re-allocate resources over production facilities, and, therefore, it does not generate synergies. Second, one interesting feature of the two-aspect model in our private information setting is that firms do not know in advance whether they will merge with a firm that has a comparative advantage in one of the relevant aspects. That is, even if one acknowledges that there is a feature of synergies present in the two-aspect model, the firms do not know about this before the merger materializes.

A potential drawback of the present analysis is that there are only few analytical results that can be derived. The numerical analysis, on the other hand, depends on the specific distributions of valuations one starts out with, and on the number of firms considered. The uniform distributions we have considered, however, are not prone to the critic that the analysis depends on skewness of the distributions, with many high or low types. We also considered small number of firms to start with, in order to focus on bidding markets where mergers potentially have large effects. One potential interesting issue for further research is to analyze situations where the proposal or the acceptance to merge provides information (signals) about the type of a firm. This can be achieved in the current set-up by introducing a cost of a merger that is such that some types of firms find the merger beneficial whereas other types do not have an incentive to merge. This signaling aspect affects the beliefs of merging and non-merging firms, as well as the beliefs of competition authorities concerning the types

[4] In terms of the traditional merger analysis, this means that the (expected) cost function of the merged entity is below the cost function of each of the merging firms.

of firms that do effectively merge. It would be interesting to see how the results concerning bidding function and revenue are affected by such signaling motives.

Appendix

Proof of Proposition 1. For $N = 2k$, the revenue before the merger is denoted by $R(2k)$. As the auction is symmetric, the revenue equivalence holds. Thus, $R(2k)$ is the expectation of the second highest-order statistics amongst $2k$ random variables with density $f(x)$ from section 3:

$$R(2k) = \int_0^1 x \cdot 2k(2k-1)\big(F(x)\big)^{2k-2}\big(1-F(x)\big)f(x)dx.$$

For $k = 2$ we have:

$$R(4) = 12\int_0^1 x\big(F(x)\big)^2\big(1-F(x)\big)f(x)dx$$

$$= 12\left(16\int_0^{\frac{1}{2}} x^6(1-2x^2)dx + 8\int_{\frac{1}{2}}^1 x(1-2(1-x)^2)^2(1-x)^3dx\right) = \frac{947}{1680}$$

Let now bidders $i = 1,2$ merge. Then, we define $v^{M,1} \equiv \max[v_1^1, v_2^1]$ and $v^{M,2} \equiv \max[v_1^2, v_2^2]$. Components $v^{M,1}$ and $v^{M,2}$ follow distributions:

$$F^{M,1}(x) \equiv \Pr[v^{M,1} \le x] = 4x^2,\ \ F^{M,2} = \Pr[v^{M,2} \le x] = 4x^2$$

with the densities:

$$f^{M,1}(x) = f^{M,2}(x) = 8x.$$

The value $v^M = v^{M,1} + v^{M,2}$ of the merged entity is distributed according to the following density:

$$f^M(x) = \int_{-\infty}^{\infty} f^{M,1}(x-t)f^{M,2}(t)dt = 8\int_0^{0.5} f_1(x-t)\cdot tdt = 8\int_{x-0.5}^{x} f_1(z)\cdot(x-z)dz$$

$$= \begin{cases} \frac{32}{3}x^3 & \text{, if } x \in \left[0, \frac{1}{2}\right] \\ \frac{16}{3}\left(2(1-x)^3 - 6(1-x)^2 + 3(1-x)\right) & \text{, if } x \in \left[\frac{1}{2}, 1\right] \end{cases}$$

The distribution function can be found by integrating $f^M(x)$:

$$F^M(x) = \begin{cases} \frac{8}{3}x^4 & \text{, if } x \in \left[0, \frac{1}{2}\right] \\ 1 - \frac{8}{3}((1-x)^4 - 4(1-x)^3 + 3(1-x)^2) & \text{, if } x \in \left[\frac{1}{2}, 1\right] \end{cases}$$

If all bidders form pairs, the resulting auction has $N = k$ bidders (instead of $N = 2k$) whose values are distributed according to $F^M(x)$ instead of $F(x)$. The auction is again symmetric and revenue equivalence holds. Thus, $R^M(k)$ is the expectation of the second highest-order statistics amongst k random variables with density $f^M(x)$:

$$R^M(k) = \int_0^1 x \cdot k(k-1)(F^M)^{k-2}(1-F^M)f^M dx.$$

Computing for $k = 2$ and using the notation $t = (1-x)$, we get:

$$R^M(2) = 2\int_0^1 x(1-F^M)f^M dx = \frac{64}{3}\int_0^{\frac{1}{2}} \left(1 - \frac{8}{3}x^4\right)x^4 dx +$$

$$+\frac{256}{9}\int_{\frac{1}{2}}^1 x(t^4 - 4t^3 + 3t^2)(2t^3 - 6t^2 + 3t)dx = \frac{1622}{2835}.$$

Thus, the revenue increase, when two pairs sequentially merge, is:

$$R_M(2) - R(4) = \frac{1622}{2835} - \frac{947}{1680} = \frac{383}{45360} > 0.$$

The fact that $R_M(2) - R(4) > 0$ implies that either the merger from $N = 4$ to $N = 3$ firms or the merger from $N = 3$ to $N = 2$ firms must increase profits. *Q.E.D.*

Expressions for $F^M(x|v_1^1, v_1^2)$.

One of the important ingredients in the analysis of the incentives to merge is the distribution function of the value of the merged firm $v^M = \max[v_1^1, v_2^1] + \max[x_1^2, x_2^2]$ conditional on the two aspects (v_1^1, v_1^2) of the valuations of one of the merging bidder $i = 1$, *i.e.*, $F^M(x|v_1^1, v_1^2)$. Obviously, $F^M = 0$ when $v_1^1 + v_1^2 < x$. Then, F^M has a mass point at $x = v_1^1 + v_1^2$:

$$F^M(v_1^1 + v_1^2|v_1^1, v_1^2) \triangleq \Pr[v^M \leq v_1^1 + v_1^2|v_1^1, v_1^2] = \Pr[v_2^1 \leq v_1^1, v_2^2 \leq v_1^2|v_1^1, v_1^2] = v_1^1 v_1^2.$$

Suppose that $x > v_1^1 + v_1^2$. Then $F^M(x|v_1^1, v_1^2) \triangleq \Pr[v^M \leq x|v_1^1, v_1^2]$ is:

$$\begin{aligned}
F^M &= \Pr[v^M \leq x : v_2^1 > v_1^1, v_2^2 < v_1^2] + \Pr[v^M \leq x : v_2^1 < v_1^1, v_2^2 > v_1^2] \\
&\quad + \Pr[v^M \leq x : v_2^1 > v_1^1, v_2^2 > v_1^2] + \Pr[v^M \leq x : v_2^1 < v_1^1, v_2^2 < v_1^2] \\
&= \Pr[v_2^1 \leq x - v_1^2 : v_2^1 > v_1^1, v_2^2 < v_1^2] + \Pr[v_2^2 \leq x - v_1^1 : v_2^1 < v_1^1, v_2^2 > v_1^2] \\
&\quad + \Pr[v_2^1 + v_2^2 \leq x : v_2^1 > v_1^1, v_2^2 > v_1^2] + \Pr[v_1^1 + v_1^2 \leq x : v_2^1 < v_1^1, v_2^2 < v_1^2] \\
&= 4\big((x - v_1^2 - v_1^1)v_1^2 + (x - v_1^1 - v_1^2)v_1^1 + v_1^1 v_1^2\big) + \int_{v_1^1}^{0.5} \left(\int_{v_1^2}^{x - v_2^1} dF(v_2^2) \right) dF(v_2^1).
\end{aligned}$$

Considering four cases depending on relations between x, v_1^1, and v_1^2 yields:

- Case $x - v_1^1 \leq 0.5$, $x - v_1^2 \leq 0.5$:

$$\begin{aligned}
F^M(x|v_1^1, v_1^2) &= 4\left(\big(x - (v_1^1 + v_1^2)\big)(v_1^1 + v_1^2) + v_1^1 v_1^2\right) + 2\big(x - (v_1^1 + v_1^2)\big)^2 \\
&= 2(x^2 - (v_1^1 + v_1^2)^2 + 2v_1^1 v_1^2) = 2(x^2 - (v_1^1)^2 - (v_1^2)^2).
\end{aligned}$$

- Case $x - v_1^1 \leq 0.5$, $x - v_1^2 \geq 0.5$:

$$F^M(x|v_1^1, v_1^2) = 2\left(x^2 - \left(x - \frac{1}{2}\right)^2 - (v_1^1)^2\right) = 2x - \frac{1}{2} - 2(v_1^1)^2.$$

- Case $x - v_1^1 \geq 0.5$, $x - v_1^2 \leq 0.5$:

$$F^M(x|v_1^1, v_1^2) = 2\left(x^2 - \left(x - \frac{1}{2}\right)^2 - (v_1^2)^2\right) = 2x - \frac{1}{2} - 2(v_1^2)^2.$$

- Case $x - v_1^1 \geq 0.5$, $x - v_1^2 \geq 0.5$:

$$F^M(x|v_1^1, v_1^2) = 2\left(x^2 - 2\left(x - \frac{1}{2}\right)^2\right) = 4x - 2x^2 - 1 = 1 - 2(1 - x)^2.$$

This ends the description of all relevant cases.

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