TI 2013-073/III
Tinbergen Institute Discussion Paper

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Revised: May 2013

*The authors are grateful to an Associate Editor and two anonymous referees for helpful comments and suggestions. For financial support, the first author acknowledges the Japan Society for the Promotion of Science and the Australian Academy of Science. The third author is grateful for the financial support of the Australian Research Council, National Science Council, Taiwan, and the Japan Society for the Promotion of Science.

Abstract

Most multivariate variance or volatility models suffer from a common problem, the "curse of dimensionality". For this reason, most are fitted under strong parametric restrictions that reduce the interpretation and flexibility of the models. Recently, the literature has focused on multivariate models with milder restrictions, whose purpose is to combine the need for interpretability and efficiency faced by model users with the computational problems that may emerge when the number of assets can be very large. We contribute to this strand of the literature by proposing a block-type parameterization for multivariate stochastic volatility models. The empirical analysis on stock returns on the US market shows that 1% and 5 % Value-at-Risk thresholds based on one-step-ahead forecasts of covariances by the new specification are satisfactory for the period including the Global Financial Crisis.

Keywords: block structures; multivariate stochastic volatility; curse of dimensionality; leverage effects; multi-factors; heavy-tailed distribution.

JEL classifications: C32, C51, C10.

1. Introduction

Classical portfolio allocation and management strategies are based on the assumption that risky returns series are characterized by time-invariant moments. However, the econometric literature of the last few decades has demonstrated the existence of dynamic behaviour in the variances of financial returns series. The introduction of such empirical evidence may constitute an additional source of performance for portfolio managers, as evidenced by Fleming, Kirby and Ostdiek (2001), or may be relevant for improving the market risk measurement and monitoring activities (see, for example, Hull and White (1998) and Lehar et al. (2002)). Two families of models have emerged in the literature, namely GARCH-type specifications (see Engle (2002)), and Stochastic Volatility models (see Taylor (1986) and Andersen (1994)).

However, portfolio management strategies often involve a large number of assets requiring the use of multivariate specifications. Among the possible alternative models, we cite the contributions of Bollerslev (1990), Engle and Kroner (1995), Ling and McAleer (2003), Asai and McAleer (2006, 2009a,b), and the surveys in Engle and Sheppard (2001), McAleer (2005), Bauwens, Laurent and Rombouts (2006), Asai, McAleer and Yu (2006), and Chib, Omori and Asai (2009). Most models, if not all, suffer from a common problem, the well-known “curse of dimensionality”, whereby models become empirically infeasible if fitted to a number of series of moderate size (in some cases, the models may become computationally intractable for even 5 or 6 assets). In order to match the need of introducing time-varying variances with practical computational problems, several restricted models are generally used: the diagonal VECH specifications suggested by Bollerslev, Engle and Wooldridge (1988), the scalar

VECH and BEKK models proposed by Ding and Engle (2001), the CCC model of Bollerslev (1990), the dynamic conditional correlation model of Engle (2002), and the varying conditional correlation model of Tse and Tsui (2002).

The introduction of significant and strong restrictions can reduce the interpretation and flexibility of the models, possibly affecting the purportedly improved performance they may provide and/or the appropriateness of the analysis based on their results. For example, the scalar BEKK model can reduce the number of parameters by assuming all the elements of the cross-products of the vector of past residuals have the same parameter, and the assumption can be tested by applying the asymptotic results of Hafner and Preminger (2009) if we can avoid the problem of dimensionality.

Recently, the literature has focused on multivariate models with milder restrictions, whose purpose is to combine the need for interpretability and efficiency faced by model users with the computational problems that may emerge when the number of assets is quite large. Among the contributions in this direction, we follow the approach of Billio, Caporin and Gobbo (2006). They propose specifying the parameter matrices of a general multivariate correlation model in a block form, where the blocks are associated with assets sharing some common feature, such as the economic sector. Our purpose is to adopt this block-type parameterization and adapt it to multivariate stochastic volatility models.

In general terms, Multivariate Stochastic Volatility (MSV) models have a parameter number of order $O\left(M^2\right)$, where M is the number of assets. With the introduction of block parameter matrices, we may control the number of parameters and obtain a model specification which is feasible, even for a very large number of assets. Furthermore, as in the contribution of Billio, Caporin and Gobbo (2006), the models we propose follow the spirit of sectoral-based asset allocation strategies since they will presume the existence of common dynamic behaviour within assets or financial instruments belonging to the same economic sector. This assumption is not as strong as postulating the existence of a unique factor driving all the variances and covariances, since the financial theory may suggest the existence of sector-specific risk factors (sectoral asset allocation is often followed by portfolio managers and characterized by a number of managed financial instruments).

As distinct from an extremely restricted model, we also recover part of the spillover effect between variances, which allows monitoring of the interdependence between groups of assets, an additional element that may be relevant. Within our modelling approach, the coefficients may be interpreted as sectoral specific, while the assets will be in any case characterized by a specific long term variance through the introduction of unrestricted constants in the variance equations.

For the purpose of explaining our approach, we consider a multi-component MSV model allowing leverage effects and heavy-tailed unconditional distributions, which is a multivariate extension of Chernov et al. (2003), although our approach is applicable to the factor model of Pitt and Shephard (1999) and Chib, Nardari, and Shephard (2006) and the dynamic correlation model of Asai and McAleer (2009b).

Clearly, the restrictions proposed may not necessarily be accepted by the data, as more 'complete' models will, in general, provide better results. We will show that the introduction of such restrictions provides limited losses, while yielding a significant improvement over the more restricted specifications. We also evaluate and compare the out-of-sample forecast of alternative models.

The plan of the remainder of the paper is as follows. Section 2 presents the multi-component MSV models, and discusses the differences between the MSV model and the factor specifications. Section 3 introduces the block-structure modelling approach, and addresses some estimation issues. Section 4 presents an empirical example regarding the out-of-sample forecasts, based on US stock market data for selected firms. Section 5 gives some concluding comments.

2. Multi-Component MSV Model

The block-structure model, which we will present in the next section, can be considered as a restricted specification of a general MSV model. In fact we will show how the modelling approach consists in defining a set of parametric restrictions that makes the model feasible, but without losing the interpretation of the coefficients.

We define a MSV model which contains multi-components and accommodates leverage effects. Let R_t be the *M*-dimensional vector of asset returns, and define $y_t = R_t - \mu_t$, where $\mu_t = E\left(R_t \left| \Im_{t-1}\right.\right)$ is the *M*-dimensional vector of conditional means and $\Im_t$ is the information set up to *t*. Then, the mean equation of the basic MSV model is defined by

$$y_t = D_t \varepsilon_t, \tag{1}$$

$$D_t = \text{diag}\left\{\exp\left(0.5 h_t\right)\right\}, \tag{2}$$

where h_t is the M-dimensional vector of stochastic log-volatilities, $\exp\left(x\right)$ for a vector x is the element-by-element operator of exponentiation, $\text{diag}\left\{x\right\}$ for a vector x is the operator which creates a diagonal matrix with the diagonal element corresponding to those of x, and ε_t follow the multivariate normal distribution with covariance matrix defined later. (An interesting extension which we currently do not entertain is having μ_t specified by h_t, as in Koopman and Uspensky (2002)). We consider a K-component model for the log-volatility, given by

$$\begin{aligned} h_t &= \sum_{k=1}^{K} V_t^{(k)} \\ V_{t+1}^{(k)} &= \phi^{(k)} \circ V_t^{(k)} + \xi_t^{(k)} \quad \left(k = 1, 2, \ldots, K\right), \end{aligned} \tag{3}$$

with $\phi^{(k)}$ an M-vector of parameters, $\xi_t' = \left(\xi_t^{(1)\prime}, \ldots, \xi_t^{(K)\prime}\right)$ and $\left(\varepsilon_t', \xi_t'\right)' \sim N\left(0, SPS\right)$, where S is the diagonal matrix of standard deviations, $S = \text{diag}\left\{\left(\sigma_\varepsilon', \sigma_\xi^{(1)\prime}, \ldots, \sigma_\xi^{(K)\prime}\right)'\right\}$, and P is the correlation matrix constructed by

$$P = \begin{pmatrix} P_{\varepsilon\varepsilon} & \Gamma \\ \Gamma' & P_{\xi\xi} \end{pmatrix}, \quad \Gamma = \begin{pmatrix} \Gamma_1 & \cdots & \Gamma_K \end{pmatrix}, \quad P_{\xi\xi} = \begin{pmatrix} P_{\xi\xi}^{(1)} & & O \\ & \ddots & \\ O & & P_{\xi\xi}^{(K)} \end{pmatrix}$$

with corresponding correlation matrices, $P_{\varepsilon\varepsilon}$ and $P_{\xi\xi}^{(k)}$ $\left(k = 1, 2, \ldots, K\right)$ and diagonal matrices of leverage effects, $\Gamma_k = \text{diag}\left\{\gamma^{(k)}\right\}$ $\left(k = 1, 2, \ldots, K\right)$. Here, $V_t^{(k)}$ is the k-th component of log-volatility, and it follows a restricted VAR(1) process. For convenience, we call the model the '*K*-component MSVL model'. In particular, we will denote 'MSVL1C' for $K = 1$, and so on. The number of parameters in the *K*-component MSVL model is $(3K+1)M + (K+1)M(M-1)/2$.

The *K*-component MSVL model reduces to the basic MSV model suggested by Harvey, Ruiz and Shephard (1994), when $K = 1$ and $\Gamma_1 = O$ (that is, with no leverage effect). We will explain these extensions shortly. Regarding the asymmetric effects, Asai and McAleer (2006) developed the MSVL1C model, as the multivariate extension of Harvey and Shephard (1996) (see also Danielsson (1998), Chan Kohn and Kirby (2006), Asai and McAleer (2009a), and Chib, Omori and Asai (2009)). As the leverage effects are especially observed for the individual negative correlation between an asset return and its future volatility, the above specification concentrates on the diagonal elements of Γ_k, in order to capture the correlation between the *i*-th return, ε_{it}, and the *k*-th component of future *i*-th volatility, $\xi_{it}^{(k)}$ $(i = 1, 2, \ldots, M)$. We may also consider non-diagonal matrix for Γ_k, if necessary.

Introducing an additional component for the volatility equation often yields a better fit to the data set, and is an alternative approach to cope with the fat-tails of stock return distributions, as proposed in Chernov et al. (2003). Although the models of the SV and GARCH families enable the observed series to have heavy-tailed distributions, empirical analysis has shown that assuming a Gaussian conditional distribution is insufficient to describe the tail behaviour of real data (see Liesenfeld and Jung (2000), Chib, Nardari, and Shephard (2002), and Asai (2008, 2009)). One of the contributions of Chernov et al. (2003) is to obtain a heavy-tailed return distribution by introducing a multi-component structure, without assuming heavy-tailed conditional distributions (see also Alizadeh, Brandt and Diebold (2002), Asai (2008), and Christoffersen, Jacobs and Wang (2008)). Based on the idea for the univariate model, Asai and McAleer (2009a) considered the MSVL2C model.

It is worth discussing and emphasizing the difference between the MSVL2C model and the popular factor MSV models. In the literature of MSV models, there are two major approaches for modelling factors. One is based on the volatility factor, as in Harvey, Ruiz and Shephard (1994), who introduce latent factors instead of latent volatility processes, in order to describe volatilities using a small number of factors. Calvet, Fisher, and Thompson (2006) also suggested a volatility factor MSV model with Markov switching factors. In their specification, the number of factors is not necessarily less than the dimension of y_t.

The other approach for modelling factors is the mean factor model suggested by Pitt and Shephard (1999), who assume the mean factor to have stochastic volatilities, in addition to those in the conditional distribution of y_t. Based on the mean factor model, Chib, Nardari, and Shephard (2006) allowed for jumps in the observation equation and a fat-tailed *t*-distribution, while Lopes and Carvalho (2007) suggested another general model which nests the models of Pitt and Shephard (1999) and Aguilar and West (2000). The MSVL2C model has two major advantages compared with the above factor models. First, it is not necessary to consider a heavy-tailed conditional distribution generally, and second, it can incorporate leverage effects in the factors straightforwardly. For purposes of comparison, we use the factor model of Harvey, Ruiz and Shephard (1994) in the empirical analysis.

3. Block Structure Model

Now, we develop a new specification based on a block structure of assets. We assume that the M assets are divided into B groups, with the b-th group containing m_b assets ($M = m_1 + m_2 + \cdots + m_B$). Define a block structure for the volatility by assuming that each group of assets is characterized by a common parametric behaviour in the volatility equation. Consider equation (3) with restrictions on parameters as

$$\phi^{(i)} = \begin{bmatrix} \phi_1^{(i)} \iota_{m_1} \\ \phi_2^{(i)} \iota_{m_2} \\ \vdots \\ \phi_B^{(i)} \iota_{m_B} \end{bmatrix}, \quad \sigma_\xi^{(i)} = \begin{bmatrix} \sigma_1^{(i)} \iota_{m_1} \\ \sigma_2^{(i)} \iota_{m_2} \\ \vdots \\ \sigma_B^{(i)} \iota_{m_B} \end{bmatrix}, \quad \gamma^{(i)} = \begin{bmatrix} \gamma_1^{(i)} \iota_{m_1} \\ \gamma_2^{(i)} \iota_{m_2} \\ \vdots \\ \gamma_B^{(i)} \iota_{m_B} \end{bmatrix},$$
$$P_{\xi\xi}^{(i)} = \begin{pmatrix} P_{\xi\xi,11}^{(i)} & \rho_{21}^{(i)} \iota_{m_2} \iota_{m_1}' & \cdots & \rho_{B1}^{(i)} \iota_{m_1} \iota_{m_B}' \\ \rho_{21}^{(i)} \iota_{m_2} \iota_{m_1}' & P_{\xi\xi,22}^{(i)} & \cdots & \rho_{B2}^{(i)} \iota_{m_2} \iota_{m_B}' \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{B1}^{(i)} \iota_{m_B} \iota_{m_1}' & \rho_{B2}^{(i)} \iota_{m_B} \iota_{m_2}' & \cdots & P_{\xi\xi,BB}^{(i)} \end{pmatrix}, \tag{4}$$

where ι_m is the m-dimensional vector of ones, $P_{\xi\xi,bb}^{(i)}$ are the $m_b \times m_b$ correlation matrices, and $\phi_b^{(i)}$, $\sigma_b^{(i)}$, $\gamma_b^{(i)}$ and $\rho_{b_1 b_2}^{(i)}$ $(1 \le b_2 < b_1 \le B)$ are scalar parameters. Hereafter, we refer to the model in equations (1) to (4) as the K-component Block Structure MSVL (BS-MSVL) model. The number of parameters in the BS model is $0.5M(M+1) + 3KB + 0.5K\left\{B(B-1) + \sum_{b=1}^{B} m_b (m_b - 1)\right\}$.

For practical purposes, we compare the number of parameters in the MSVL1C, MSVL2C and BS-MSVL2C models. When $M=9$ and $B=3$ ($M=50$ and $B=5$) with the same block size, the number of parameters in the BS-MSVL2C model is 87 (1775). For the MSVL1C and MSVL2C models for the case $M=9$ ($M=50$),the numbers of parameters are 108 (2650) and 171 (4025), respectively. Thus, the BS-MSV model is parsimonious in terms of the number of parameters.

In empirical analysis, the appropriate number of components is K=2 for univariate SV models, as shown by Alizadeh, Brandt and Diebold (2002) and Chernov et al. (2003). Here, we stress an interpretation of the two-factor model by Shephard (1996), who introduces an approach to deal with permanent and transitory components in stochastic volatility models, as those components in the GARCH specification by Engle and Lee (1993). In the specification, the AR(1) parameter of the permanent component is equal to one, while it is located between -1 and 1 as usual for the transitory component.

Inspired by the idea, we suggest the complete BS model for the BS-MSVL2C model, which has the first component with

$$\phi^{(1)}=\iota_M, \quad \sigma_{\xi}^{(1)}=\sigma_{*}^{(1)}\iota_M, \quad \gamma^{(1)}=\gamma_{*}^{(1)}\iota_M, \quad P_{\xi\xi}^{(1)}=I_M, \tag{5}$$

where $\sigma_{*}^{(1)}$ and $\gamma_{*}^{(1)}$ are scalar parameters. We refer to the model as the ‘CBS’ model. The number of parameters in the BS model is given as $0.5M\left(M+1\right)+3B+2+0.5\left\{B\left(B-1\right)+\sum_{b=1}^{B}m_b\left(m_b-1\right)\right\}$. When $M=9$ and $B=3$ ($M=50$ and $B=5$) with the same block size, the number of parameters in the CBS model is 68 (1527).

4. Estimation

For the estimation of the above various MSVL models, we estimate the mean and volatility equations separately. Following Harvey, Ruiz and Shephard (1994), we may work with quasi-maximum likelihood (QML) estimation based on the state space form, which is obtained by logarithmic transformation of squared returns. By using the transformation, unfortunately we will lose the information regarding the correlation between ε_t and η_t (ξ_t). Asai and McAleer (2006) suggested an approach to recover the information for the multivariate case. The QML estimator is inefficient, but it is still consistent. We may employ the Monte Carlo likelihood (MCL) approach proposed by Durbin and Koopman (1997) in order to obtain an efficient estimator, as suggested by Asai and McAleer (2009a). Since the QML estimator is consistent and fast to obtain, we will use it in the remainder of this paper. For an implementation issue, we use the sample correlation matrix for the initial value for estimating $P_{\varepsilon\varepsilon}$, which accounts for a major part of the parameters as $0.5M\left(M-1\right)$.

We present the results of a Monte Carlo study to investigate the small sample performance of the QML estimation procedure. Furthermore, we examine effects on assuming block structure. We generate R simulated time series for two kinds of bivariate MSV model; DGP1 is based on the BS-MSV1C model with parameters

$$\phi = 0.98\iota_2,\ \sigma_\xi = 0.2\iota_2,\ \gamma = -0.3\iota_2,\ \sigma_\varepsilon = \begin{pmatrix} 1.0 \\ 1.2 \end{pmatrix},\ P_{\varepsilon\varepsilon} = \begin{pmatrix} 1 & -0.3 \\ -0.3 & 1 \end{pmatrix},\ P_{\xi\xi} = \begin{pmatrix} 1 & 0.4 \\ 0.4 & 1 \end{pmatrix},$$

while DGP2 uses the MSVL1C model with the above parameters except for

$$\phi = (0.99 \quad 0.97)', \quad \sigma_\xi = (0.1 \quad 0.3)', \quad \gamma = (-0.2 \quad -0.4)'.$$

We estimate the BS-MSV1C model for the DGP1 and 2. Regarding DGP 1, we compute the sample mean, standard deviation, and root mean squared error (RMSE) and compare it with the true parameter value. For DGP2, we compare the estimated results with true parameters and with the results of DGP1.

The results given in Table 1 are for the typical sample size *T*= 1500 with the number of iterations set to R = 2000. For DGP1, Table 1 shows that most values of the standard deviations are close to those of the RMSE, indicating that the biases in finite samples are negligible. The results for DGP2 in Table 1 indicate that this blocking does not affect the covariance of ε_t, but that it affects the volatility equation and leverage effects, by construction. The blocking forces two parameters to take a single value for the cases of ϕ, σ_ξ and γ, and the means are located in the middle of two parameter values. We should note that the blocking also affects the correlation matrix of ξ_t.

5. Empirical Analysis

In this section, we estimate the MSVL, BS-MSVL1C, BS-MSVL2C and CBS models, and compare their out-of-sample forecasts. Three groups of three assets from three different sectors (*B*=3 and *M*=9) are used, namely Chemical, Banks, and Oil and Gas Producers. The companies are: Air Products and Chemicals (APD), Eastman Chemicals (EMN), and Ashland (ASH) for the chemical sector; Bank of America (BAC), JP Morgan Chase (JPM) and Wells Fargo (WFC) for the banking sector; and Chevron (CVX), Exxon Mobil (XOM) and ConocoPhillips (COP) for the oil and gas sector. The series considered are daily return indices, collected in the sample period 2 January 2000 to 31 December 2010, giving 2865 observations.

We choose two kinds of periods before/after the global financial crisis (GFC) in the following way. We fixed the sample size as T=1500 for estimation and forecasting. Then we estimate the model based on the data set for the years 2000-2005, and forecast daily covariances for the year 2006, corresponding to the period before GFC. With respect to the period which covers the GFC, we use data for the years 2004-2009 for estimating the models, and forecast daily covariances for the year 2010. We should add that our data may be influenced by the wars in Afghanistan and Iraq, and by the increasing trend in oil prices.

We estimated the conditional mean using the datasets: a set of interest rates (US Treasury bond 3 months, 6 months, 9 months, 1-3 years, 3-5 years, 5-7 years), oil prices, and two dummies (January and Monday). The model and estimation results are available from the authors upon request.

First, we estimate the MSVL model, which is given by

$$y_t = D_t \varepsilon_t, \quad D_t = \mathrm{diag}\left\{\exp\left(0.5 h_t\right)\right\}, \quad h_{t+1} = \phi \circ h_t + \eta_t, \quad \left(\varepsilon_t', \eta_t'\right)' \sim N\left(0, SPS\right),$$
$$S = \mathrm{diag}\left\{\left(\sigma_\varepsilon', \sigma_\eta'\right)'\right\}, \quad P = \begin{pmatrix} P_{\varepsilon\varepsilon} & P_{\varepsilon\eta} \\ P_{\eta\varepsilon} & P_{\eta\eta} \end{pmatrix}, \quad P_{\eta\varepsilon} = \mathrm{diag}\left\{\gamma\right\}.$$

Table 2 shows the QML estimates for the MSVL model for the two periods. In order to save space, the estimates of $P_{\varepsilon\varepsilon}$ and $P_{\eta\eta}$ are omitted. Regarding the period before GFC shown in Table 2(a), the estimates of ϕ_j are between 0.955 and 0.997, while the estimates of $\sigma_{\eta,j}$ vary from 0.067 to 0.542. These values are typical in the empirical analysis of SV and MSV models. Most of the estimates of γ_j are negative and significant, indicating the presence of a leverage effect, although some are positive and insignificant. Table 2(a) also shows that the minimum value of the estimates of γ_j is -0.166, implying that the leverage effects are weak or negligible for the data sets. Table 2(b) shows the estimation results for the period including the GFC. Compared with Table 2(a), the estimates of ϕ_j, $\sigma_{\eta,j}$ and γ_j are similar. Again, the leverage effects are minor. Unlike Table 2(a), some of the estimates of $\sigma_{\varepsilon,j}$ exceed two. All the values of the estimates of $\sigma_{\varepsilon,j}$ are larger than those for the period before the GFC, showing the increase in the unexplained factor.

Next, we examine the effects of blocking to the model parameters for the one-factor case. Table 3 shows the QML estimates for the BS-MSVL1C model, which has restrictions given by:

$$\phi = \begin{pmatrix} \phi_1 \iota_3 \\ \phi_2 \iota_3 \\ \phi_3 \iota_3 \end{pmatrix}, \quad \sigma_\xi = \begin{pmatrix} \sigma_1 \iota_3 \\ \sigma_2 \iota_3 \\ \sigma_3 \iota_3 \end{pmatrix}, \quad \gamma = \begin{pmatrix} \gamma_1 \iota_3 \\ \gamma_2 \iota_3 \\ \gamma_3 \iota_3 \end{pmatrix}, \quad P_{\xi\xi} = \begin{pmatrix} P_{\xi\xi,11} & \rho_{21}\iota_3\iota' & \rho_{31}\iota_3\iota_3' \\ \rho_{21}\iota_3\iota_3' & P_{\xi\xi,22} & \rho_{32}\iota_3\iota_3' \\ \rho_{31}\iota_3\iota_3' & \rho_{32}\iota_3\iota_3' & P_{\xi\xi,33} \end{pmatrix},$$

on the MSVL1C model above. As expected from the simulations, most of the estimates of ϕ_b and σ_b are located between the minimum and maximum of the estimates in the corresponding blocks in Table 2. For the leverage effects, the absolute values of the estimates are closer to zero than those in Table 2. Hence, imposing the restriction makes the leverage effects weaker in this empirical analysis. As explained in Section 3, we work with the block structure in order to accommodate additional latent variables controlling the number of parameters.

Now, we consider the BS-MSVL2C model with 3 blocks, given by the following equations:

$$y_t = D_t \varepsilon_t, \quad D_t = \text{diag}\left\{\exp\left(0.5h_t^{(1)} + 0.5h_t^{(2)}\right)\right\}, \quad h_{t+1}^{(i)} = \phi^{(i)} \circ h_t^{(i)} + \xi_t^{(i)} \quad (i = 1, 2),$$

$$\left(\varepsilon_t', \xi_t^{(1)\prime}, \xi_t^{(2)\prime}\right)' \sim N\left(0, SPS\right), \quad S = \text{diag}\left\{\left(\sigma_\varepsilon', \sigma_\xi^{(1)\prime}, \sigma_\xi^{(2)\prime}\right)'\right\},$$

$$P = \begin{pmatrix} P_{\varepsilon\varepsilon} & \Gamma \\ \Gamma' & P_{\xi\xi} \end{pmatrix}, \quad P_{\xi\xi} = \begin{pmatrix} P_{\xi\xi}^{(1)} & O \\ O & P_{\xi\xi}^{(2)} \end{pmatrix}, \quad \Gamma = \begin{pmatrix} \Gamma_1 & \Gamma_2 \end{pmatrix}, \quad \Gamma_i = \text{diag}\left\{\gamma^{(i)}\right\},$$

$$\phi^{(i)} = \begin{pmatrix} \phi_1^{(i)} \iota_3 \\ \phi_2^{(i)} \iota_3 \\ \phi_3^{(i)} \iota_3 \end{pmatrix}, \quad \sigma_\xi^{(i)} = \begin{pmatrix} \sigma_1^{(i)} \iota_3 \\ \sigma_2^{(i)} \iota_3 \\ \sigma_3^{(i)} \iota_3 \end{pmatrix}, \quad \gamma^{(i)} = \begin{pmatrix} \gamma_1^{(i)} \iota_3 \\ \gamma_2^{(i)} \iota_3 \\ \gamma_3^{(i)} \iota_3 \end{pmatrix}, \quad P_{\xi\xi}^{(i)} = \begin{pmatrix} P_{\xi\xi,11}^{(i)} & \rho_{21}^{(i)} \iota_3 \iota' & \rho_{31}^{(i)} \iota_3 \iota_3' \\ \rho_{21}^{(i)} \iota_3 \iota_3' & P_{\xi\xi,22}^{(i)} & \rho_{32}^{(i)} \iota_3 \iota_3' \\ \rho_{31}^{(i)} \iota_3 \iota_3' & \rho_{32}^{(i)} \iota_3 \iota_3' & P_{\xi\xi,33}^{(i)} \end{pmatrix}.$$

Table 4 gives the QML estimates for the BS-MSVL2C model. We should note that the results for the volatility part are 'block-based' by construction. With respect to the period before the GFC, the estimates of $\phi_b^{(1)}$ are close to one, while those of $\phi_b^{(2)}$ are far from one. Also, the estimates of $\sigma_b^{(1)}$ are smaller than those of $\sigma_b^{(2)}$. These results are typical in the two-component SV and MSV models, as in the empirical results of Alizadeh, Brandt and Diebold (2002), Asai (2008) and Asai and McAleer (2009a). The leverage effects by the first component are negative and significant for all three blocks, while one of the second components gives a positive value. Compared with the MSVL model, the estimates of $\sigma_{\varepsilon,b}$ are similar. Turning to the period including the GFC, the estimates in Table 4(b) are similar to Table 4(a) except for the leverage effects. In that period, the first and second components show stronger leverage effects than the period before the GFC. The estimates of $\sigma_{\varepsilon,b}$ are smaller than the estimates for the MSVL model, implying that the unexplained factor in the MSVL model for the GFC was explained by the second component to some extent.

Table 5 presents the QML estimates for the CBS models, which are specified by setting the parameters in the first component to be the same in all blocks, such that $\phi_b^{(1)} = 1$, $\sigma_b^{(1)} = \sigma_*^{(1)}$ and $\gamma_b^{(1)} = \gamma_*^{(1)}$. Table 5(a) and Table 5(b) show that the estimates of $\sigma_*^{(1)}$ are larger than the estimates given in Table 4, while the estimate of $\gamma_*^{(1)}$ in Table 3 is insignificant. According to the specification, the estimates of the second components are different from Table 4, but these values are typical in the two component SV and MSV models.

For the remainder part of this section, we calculate the forecasts of VaR thresholds as a diagnostic check. As explained above, the first period for forecasting is the year 2006, which consists of 260 observations, while the second period is the year 2010, giving 261 observations. As benckmark models, we consider the factor MSV (fHRS) model of Harvey, Ruiz and Shephard (1994) and the asymmetric dynamic conditional correlation (ADCC) model of Cappiello, Engle and Sheppard (2006). The fHRS model also reduces the number of parameters assuming a factor specification on the state space form of MSV models. Regarding the ADCC model, we work with the GJR process for each volatility equation.

We examine the characteristics of stock portfolios which are constructed based on covariance matrix forecasts from the MSVL, BS-MSVL2C, CBS, fHRS and ADCC models. As the covariance matrix is defined by $C_t = D_t P_{\varepsilon\varepsilon} D_t$, its one-step-ahead forecasts are given by $\hat{C}_t = \hat{D}_t \hat{P}_{\varepsilon\varepsilon} \hat{D}_t$, where $\hat{D}_t$ contains the forecasts of volatility in the diagonal and $\hat{P}_{\varepsilon\varepsilon}$ is the estimated correlation matrix of the conditional distribution for the return. Here, we consider the following three kinds of portfolios. The first portfolio is the minimum variance portfolio (MVP), with weights given by $w_t = \left(\iota_M' \hat{C}_t \iota_M\right)^{-1} \left(\hat{C}_t \iota_M\right)$. The second portfolio is the equally weighted portfolio (EWP), with constant weights of $w_t = M^{-1}\iota_M$. The third portfolio is the value-weighted portfolio (VWP), with time-varying weights given by $w_t = \left(\iota_M' \left(\iota_M + R_{t-1}\right)\right)^{-1} \left(w_{t-1} \circ \left(\iota_M + R_{t-1}\right)\right)$, starting with a EWP at $t = 0$.

Given the portfolio weights, w_t, we define the portfolio returns as $R_{p,t} = w' R_t$. As we assumed the conditional multivariate normal distribution, we have $R_{p,t} = \mu_{p,t} + y_{p,t}$, where $\mu_{p,t} = w_t' \mu_t$ is the conditional mean and $y_{p,t}$ has the conditional normal distribution with mean zero and variance $h_t = w_t' C_t w_t$. Fixing the sample size in estimation to be 1500, we re-estimate the model and forecast one-step-ahead VaR thresholds for the above two periods. In our analysis, we work with 1% and 5% thresholds, that is, $\hat{\mu}_t - 1.645\sqrt{\hat{h}_t}$ and $\hat{\mu}_t - 2.576\sqrt{\hat{h}_t}$, respectively. We define the hit rate as the ratio of the number of times that the portfolio return exceeds its forecast divided by the number of out-of-sample forecasts.

In addition to the three models, we consider a combined approach based on the BS-MSVL2C and CBS models, by choosing the portfolio which gives the larger forecasts of portfolio variance. It is expected to adjust the fluctuations on BS and CBS models brought by restricting the parameters of the MSVL2C model.

In order to assess the estimated VaR thresholds, the unconditional coverage and independence tests developed by Christoffersen (1998) are widely used. A drawback of the Christoffersen (1998) test for independence is that it tests against a particular alternative of first-order dependence. The duration-based approach in Christoffersen and Pelletier (2004) allows for testing against more general forms of dependence, but still requires a specific alternative. Recently, Candelon et al. (2010) have developed a more robust procedure which does not need a specific distributional assumption for the durations under the alternative. Consider the "hit sequence" of VaR violations, which takes a value of one if the loss is greater than the VaR threshold, and takes the value zero if the VaR is not violated. If we could predict the VaR violations, then that information may help to construct a better model. Hence, the hit sequence of violations should be unpredictable, and should follow an independent Bernoulli distribution with parameter *p*, indicating that the duration of the hit sequence should follow a geometric distribution.

The GMM duration-based test developed by Candelon et al. (2010) works with the J-statistic based on the moments defined by the orthonormal polynomials associated with the geometric distribution. The conditional coverage test and independence test based on q orthonormal polynomials have asymptotic χ^2_q and χ^2_{q-1} distributions under their respective null distributions. The unconditional coverage test is given as a special case of the conditional coverage test with $q = 1$. We use the 5% significance level in the following analysis.

Table 6 gives the test results for three kinds of portfolios based on the MSVL, BS-MSVL2C, CBS, fHRS and ADCC models and the combined BS+CBS approach, for the period before the GFC. The test statistics for the ADCC model for the 1% and 5% VaR thresholds are not rejected for all the cases except for one. With respect to the MSVL and fHRS models, all the test statistics for the 5% VaR thresholds are rejected for all three portfolios. The tests for the BS model are rejected for the 5% and 1% VaR thresholds for the minimum variance portfolio. All the results for CBS and BS+CBS passed the tests.

Regarding the period after the GFC, Table 7 indicates that four tests for the ADCC model are not available because it only captures one violation for the period. With respect to the fHRS model, all tests except for one case are not rejected. The MSVL model is rejected by the tests for all three portfolios. For the period, the minimum variance portfolio calculated by the CBS model gave unsatisfactory results. All the results for BS and BS+CBS passed the tests. Hence, the combined BS+CBS approach gives the best results for the forecasts before/after the GFC, and suggests that the introduction of block structures improves the forecasting performance.

6. Conclusion

In this paper we present a class of multivariate stochastic volatility models which is nested in the multi-component model with leverage effects suggested by Asai and McAleer (2009a). The distinctive feature of our model is that, contrary to fully parameterized MSV models, it remains feasible in moderate to large cross-sectional dimensions. This result is achieved by imposing a block structure on the model parameter matrices. The variables could be grouped by using some economic or financial criteria, or following data-driven classifications. In addition, by the introduction of the blocks, if these have an economic interpretation, the model we propose preserves the interpretation of coefficients, a feature which is generally lost in feasible MSV models.

We also present an empirical application where the proposed model is estimated on a set of US equities, and examine the VaR thresholds for several types of portfolios calculated by covariance forecasts. Unlike the MSV model with leverage effects, the results given by the approach based on the block structure are satisfactory.

Although the specification using the block structure makes a useful contribution in reducing the number of parameters, the conditional correlation matrix of the return vector still has many parameters. This issue is left for future research.

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Table 1: Monte Carlo Results for the QML Estimator for BS-MSVL1C Model

Param	DGP1	Mean	StdDev	RMSE	DGP2	Mean	StdDev
$\sigma_{\varepsilon 1}$	1.0	0.9994	0.1253	0.1253	1.0	0.9967	0.1242
$\sigma_{\varepsilon 2}$	1.2	1.2068	0.1585	0.1587	1.2	1.2102	0.1580
ρ_{ε}	−0.3	−0.2842	0.0954	0.0967	- 0.3	−0.2825	0.1001
ϕ	0.98	0.9766	0.0064	0.0072	(0.99,0.97)	0.9718	0.0078
σ_{ξ}	0.2	0.2052	0.0267	0.0272	(0.1,0.3)	0.2238	0.0330
ρ_{ξ}	0.4	0.3934	0.1576	0.1577	0.4	0.2754	0.1470
γ	−0.3	−0.3026	0.0835	0.0836	(−0.2,−0.4)	−0.2856	0.0966

Table 2: QML Estimates for MSVL Model

(a) Before GFC

	APD	EMN	ASH	BAC	JPM	WFC	CVX	XOM	COP
ϕ_j	0.9945	0.9888	0.9825	0.9926	0.9966	0.9925	0.9585	0.9804	0.9554
	(0.0984)	(0.0969)	(0.0969)	(0.0969)	(0.0969)	(0.0969)	(0.0235)	(0.0975)	(0.0177)
$\sigma_{\eta,j}$	0.5420	0.1018	0.0727	0.0850	0.0672	0.0995	0.1899	0.1562	0.2069
	(0.0103)	(0.0098)	(0.0050)	(0.0076)	(0.0097)	(0.0023)	(0.0258)	(0.0531)	(0.0217)
γ_j	-0.1660	0.0157	-0.1530	-0.0569	-0.0252	0.0204	-0.0523	-0.1131	0.0151
	(0.0097)	(0.0151)	(0.0542)	(0.0523)	(0.0066)	(0.0364)	(0.0010)	(0.0010)	(0.0298)
$\sigma_{\varepsilon,j}$	1.3469	1.5317	1.4156	1.4160	1.4934	1.1346	1.3024	1.2565	1.5038
	(0.1064)	(0.0925)	(0.0894)	(0.0714)	(0.0761)	(0.0715)	(0.0311)	(0.0844)	(0.0333)

(b) Middle and After GFC

	APD	EMN	ASH	BAC	JPM	WFC	CVX	XOM	COP
ϕ_j	0.9673	0.9775	0.9860	0.9905	0.9818	0.9885	0.9645	0.9727	0.9639
	(0.0981)	(0.0981)	(0.0981)	(0.0981)	(0.0981)	(0.0981)	(0.0981)	(0.0982)	(0.0910)
$\sigma_{\eta,j}$	0.2364	0.1702	0.1257	0.1452	0.2068	0.1731	0.1481	0.1413	0.1870
	(0.0108)	(0.0098)	(0.0056)	(0.0101)	(0.0124)	(0.0092)	(0.0153)	(0.0134)	(0.0111)
γ_j	-0.0998	-0.0496	-0.0636	-0.0857	-0.0095	-0.1088	-0.1019	-0.0738	0.0520
	(0.0248)	(0.0248)	(0.0251)	(0.0248)	(0.0248)	(0.0248)	(0.0249)	(0.0249)	(0.0249)
$\sigma_{\varepsilon,j}$	1.5828	1.8041	2.2625	2.3722	2.1927	2.0450	1.6192	1.5058	1.9209
	(0.4500)	(0.6056)	(0.6718)	(0.5559)	(0.7367)	(0.4855)	(0.1271)	(0.1059)	(0.0874)

Note: Standard errors are given in parentheses. The estimates of $P_{\varepsilon\varepsilon}$ and $P_{\eta\eta}$ are omitted to save space.

Table 3: QML Estimates for MSVL1C-BS Models

(a) Before GFC

	APD	EMN	ASH	BAC	JPM	WFC	CVX	XOM	COP
$\phi_b^{(1)}$		0.9832 (0.0243)			0.9934 (0.0053)			0.9736 (0.0238)	
$\sigma_b^{(1)}$		0.0932 (0.0663)			0.0826 (0.0303)			0.1267 (0.0536)	
$\gamma_b^{(1)}$		-0.0489 (0.0081)			-0.0170 (0.0035)			-0.0345 (0.0032)	
$\sigma_{\varepsilon,b}$	1.4673 (0.1111)	1.5365 (0.1158)	1.4059 (0.1065)	1.4608 (0.2243)	1.6480 (0.2515)	1.1914 (0.1817)	1.2923 (0.0892)	1.2624 (0.0867)	1.5060 (0.1054)

(b) Middle and After GFC

	APD	EMN	ASH	BAC	JPM	WFC	CVX	XOM	COP
$\phi_b^{(1)}$		0.9895 (0.0110)			0.9949 (0.0027)			0.9797 (0.0336)	
$\sigma_b^{(1)}$		0.1098 (0.0221)			0.0940 (0.0263)			0.1068 (0.0690)	
$\gamma_b^{(1)}$		-0.0152 (0.0011)			-0.0840 (0.0217)			-0.0295 (0.0032)	
$\sigma_{\varepsilon,b}$	1.2912 (0.1937)	1.4464 (0.2116)	1.7507 (0.2552)	1.5015 (0.3254)	1.5737 (0.3445)	1.3760 (0.3262)	1.4607 (0.1083)	1.3384 (0.0989)	1.6916 (0.1257)

Note: Standard errors are given in parentheses. The estimates of $P_{\varepsilon\varepsilon}$ and $P_{\xi\xi}^{(1)}$ are omitted to save space.

Table 4: QML Estimates for MSVL2C-BS Models

(a) Before GFC

	APD	EMN	ASH	BAC	JPM	WFC	CVX	XOM	COP
$\phi_b^{(1)}$	0.9942 (0.0022)			0.9971 (0.0022)			0.9836 (0.0022)		
$\sigma_b^{(1)}$	0.0460 (0.0018)			0.0576 (0.0018)			0.0977 (0.0018)		
$\gamma_b^{(1)}$	-0.0141 (0.0021)			-0.0703 (0.0021)			-0.0062 (0.0021)		
$\phi_b^{(2)}$	0.5690 (0.0022)			0.0352 (0.0022)			0.3546 (0.0022)		
$\sigma_b^{(2)}$	0.5172 (0.0020)			0.6390 (0.0021)			0.3754 (0.0025)		
$\gamma_b^{(2)}$	-0.0033 (0.0049)			0.0046 (0.0022)			-0.0389 (0.0061)		
$\sigma_{\varepsilon,b}$	1.4758 (0.2276)	1.5442 (0.4829)	1.3937 (0.2205)	1.5660 (0.2141)	1.5238 (0.2634)	1.1648 (0.1968)	1.3075 (0.2255)	1.2962 (0.2652)	1.5412 (0.3254)

(b) Middle and After GFC

	APD	EMN	ASH	BAC	JPM	WFC	CVX	XOM	COP
$\phi_b^{(1)}$	0.9941 (0.0089)			0.9954 (0.0088)			0.9822 (0.0139)		
$\sigma_b^{(1)}$	0.1129 (0.0052)			0.0927 (0.0005)			0.1036 (0.0005)		
$\gamma_b^{(1)}$	-0.1329 (0.0088)			-0.0243 (0.0025)			-0.3987 (0.0063)		
$\phi_b^{(2)}$	0.4167 (0.0023)			0.3785 (0.0056)			0.3448 (0.0050)		
$\sigma_b^{(2)}$	0.5837 (0.1438)			0.0010 (0.0004)			0.0024 (0.0006)		
$\gamma_b^{(2)}$	-0.0378 (0.0011)			-0.0782 (0.0039)			-0.0569 (0.0065)		
$\sigma_{\varepsilon,b}$	1.2065 (0.1414)	1.3868 (0.1643)	1.6350 (0.1798)	1.0506 (0.9409)	1.3507 (0.1640)	1.4329 (0.1311)	1.4919 (0.1756)	1.3398 (0.2199)	1.6691 (0.0547)

Note: Standard errors are given in parentheses. The estimates of $P_{\varepsilon\varepsilon}$, $P_{\xi\xi}^{(1)}$ and $P_{\xi\xi}^{(2)}$ are omitted to save space.

Table 5: QML Estimates for CBS Models

(a) Before GFC

	APD	EMN	ASH	BAC	JPM	WFC	CVX	XOM	COP
$\sigma_*^{(1)}$	0.3906 (0.1097)								
$\gamma_*^{(1)}$	-0.0097 (0.0361)								
$\phi_b^{(2)}$	0.7471 (0.0945)			-0.3389 (0.2151)			0.8593 (0.0467)		
$\sigma_b^{(2)}$	0.6037 (0.1259)			0.2565 (0.0561)			0.0287 (0.0038)		
$\gamma_b^{(2)}$	0.0185 (0.0280)			-0.0224 (0.0853)			-0.0076 (0.0062)		
$\sigma_{\varepsilon,b}$	1.6119 (0.1408)	1.5018 (0.1101)	1.6204 (0.1976)	1.5833 (0.1413)	1.7726 (0.1284)	1.8617 (0.1336)	1.6501 (0.1844)	1.5601 (0.1102)	1.5106 (0.1274)

(b) Middle and After GFC

	APD	EMN	ASH	BAC	JPM	WFC	CVX	XOM	COP
$\sigma_*^{(1)}$	0.1097 (0.0253)								
$\gamma_*^{(1)}$	0.0206 (0.0389)								
$\phi_b^{(2)}$	0.9841 (0.0071)			-0.0724 (0.0712)			0.9720 (0.0140)		
$\sigma_b^{(2)}$	0.7087 (0.0879)			0.1243 (0.0281)			0.0506 (0.0044)		
$\gamma_b^{(2)}$	-0.0010 (0.0023)			-0.0160 (0.0033)			-0.0047 (0.0012)		
$\sigma_{\varepsilon,b}$	1.0849 (0.1787)	1.0290 (0.1905)	1.3024 (0.1631)	0.7903 (0.1674)	1.1186 (0.1505)	0.7566 (0.1784)	0.9880 (0.1695)	0.9574 (0.1993)	1.1438 (0.1803)

Note: Standard errors are given in parentheses. The estimates of $P_{\varepsilon\varepsilon}$ and $P_{\xi\xi}^{(2)}$ are omitted to save space.

Table 6: Backtesting VaR Thresholds: Before GFC

(a) Minimum-variance portfolio

Model	VaR	% Violation	UC	ID	CC
ADCC	5%	0.1115	8.9738 [0.0027]	3.0454 [0.5503]	17.234 [0.0041]
	1%	0.0231	2.0816 [0.1491]	1.5379 [0.8199]	4.2571 [0.5130]
MSVL	5%	0.0885	4.4096 [0.0357]	0.0922 [0.9990]	5.6669 [0.3400]
	1%	0.0192	0.5834 [0.4450]	1.8953 [0.7550]	0.8961 [0.9705]
fHRS	5%	0.0923	4.7616 [0.0291]	2.0249 [0.7312]	7.9971 [0.1564]
	1%	0.0192	1.4065 [0.2356]	1.0684 [0.8992]	2.6256 [0.7575]
BS	5%	0.3885	79.765 [0.0000]	8.1793 [0.0852]	270.99 [0.0000]
	1%	0.3269	79.697 [0.0000]	6.8782 [0.1425]	360.03 [0.0000]
CBS	5%	0.0769	2.7147 [0.0994]	0.4148 [0.9813]	2.9254 [0.7115]
	1%	0.0192	0.8925 [0.3448]	4.4728 [0.3458]	2.3007 [0.8062]
BS+CBS	5%	0.0769	2.7147 [0.0994]	0.4148 [0.9813]	2.9254 [0.7115]
	1%	0.0192	0.8925 [0.3448]	4.4728 [0.3458]	2.3007 [0.8062]

(b) Equally-weighted portfolio

Model	VaR	% Violation	UC	ID	CC
ADCC	5%	0.0462	0.2931 [0.5883]	4.5270 [0.3394]	5.9745 [0.3087]
	1%	0.0077	0.9701 [0.3247]	4.6629 [0.3237]	4.5653 [0.4712]
MSVL	5%	0.0808	2.7284 [0.0986]	13.286 [0.0099]	14.178 [0.0145]
	1%	0.0192	1.7868 [0.1813]	2.4639 [0.6511]	4.1809 [0.5237]
fHRS	5%	0.0500	0.0561 [0.8127]	11.113 [0.0253]	11.113 [0.0492]
	1%	0.0154	0.9278 [0.3354]	1.3638 [0.8505]	1.2859 [0.8727]
BS	5%	0.0462	0.0691 [0.7926]	8.5580 [0.1096]	7.5484 [0.0731]
	1%	0.0077	0.9701 [0.3247]	4.6629 [0.3237]	4.5653 [0.4712]
CBS	5%	0.0538	0.0032 [0.9546]	3.3408 [0.5025]	3.1439 [0.6778]
	1%	0.0115	1.6182 [0.2033]	5.0018 [0.2871]	5.4834 [0.3598]
BS+CBS	5%	0.0462	0.0691 [0.7926]	8.5580 [0.1096]	7.5484 [0.0731]
	1%	0.0077	0.9701 [0.3247]	4.6629 [0.3237]	4.5653 [0.4712]

Note: '% Violation' is the percentage of days when returns are less than the VaR threshold. UC, IND and CC are the GMM duration-base tests for unconditional coverage, independence and conditional coverage, developed by Candelon et al. (2010). The number of orthonormal polynomials is set to 5. *P*-values are in brackets.

Table 6 (Cont.): Backtesting VaR Thresholds: Before GFC

(c) Value-weighted portfolio

Model	VaR	% Violation	UC	ID	CC
ADCC	5%	0.0465	0.2931 [0.5883]	4.5270 [0.3394]	5.9745 [0.3087]
	1%	0.0077	0.9701 [0.3247]	4.6629 [0.3237]	4.5653 [0.4712]
MSVL	5%	0.0923	4.7616 [0.0291]	22.794 [0.0001]	22.036 [0.0005]
	1%	0.0192	1.7868 [0.1813]	2.4639 [0.6511]	4.1809 [0.5237]
fHRS	5%	0.0538	0.0032 [0.9546]	14.297 [0.0064]	12.728 [0.0261]
	1%	0.0231	2.6767 [0.1018]	5.2134 [0.2661]	7.6373 [0.1774]
BS	5%	0.0462	0.0691 [0.7926]	7.5484 [0.1096]	8.5580 [0.1281]
	1%	0.0115	0.2200 [0.6390]	0.9373 [0.9192]	0.7063 [0.9826]
CBS	5%	0.0500	0.0561 [0.8127]	6.7397 [0.1503]	6.7396 [0.2407]
	1%	0.0154	0.9278 [0.3354]	1.3638 [0.8505]	1.8259 [0.8727]
BS+CBS	5%	0.0462	0.0691 [0.7926]	7.5484 [0.1096]	8.5580 [0.1281]
	1%	0.0115	0.2200 [0.6390]	0.9373 [0.9192]	0.7063 [0.9826]

Note: '% Violation' is the percentage of days when returns are less than the VaR threshold. UC, IND and CC are the GMM duration-base tests for unconditional coverage, independence and conditional coverage, developed by Candelon et al. (2010). The number of orthonormal polynomials is set to 5. *P*-values are in brackets.

Table 7: Backtesting VaR Thresholds: After GFC

(a) Minimum-variance portfolio

Model	VaR	% Violation	UC	ID	CC
ADCC	5%	0.0268	3.4741 [0.5883]	4.4665 [0.3465]	19.206 [0.0018]
	1%	0.0077	0.9900 [0.3197]	4.8862 [0.2992]	4.8520 [0.4342]
MSVL	5%	0.0881	4.3182 [0.0377]	3.1632 [0.5309]	8.4487 [0.1332]
	1%	0.0307	4.6064 [0.0319]	2.9456 [0.5670]	12.358 [0.0302]
fHRS	5%	0.0536	0.0018 [0.9660]	15.699 [0.0035]	14.595 [0.0122]
	1%	0.0077	0.8001 [0.3711]	3.0215 [0.5542]	2.5712 [0.7657]
BS	5%	0.0536	0.0032 [0.9546]	2.5191 [0.6412]	2.1080 [0.8340]
	1%	0.0153	0.0652 [0.7985]	1.1493 [0.8864]	0.1935 [0.9992]
CBS	5%	0.0996	6.2669 [0.0123]	1.9365 [0.0000]	10.605 [0.0598]
	1%	0.0383	7.2009 [0.0073]	4.8976 [0.2980]	24.196 [0.0002]
BS+CBS	5%	0.0498	0.0561 [0.8127]	2.4590 [0.6520]	2.4836 [0.7790]
	1%	0.0115	0.0656 [0.4179]	0.6120 [0.9617]	0.8500 [0.9737]

(b) Equally-weighted portfolio

Model	VaR	% Violation	UC	ID	CC
ADCC	5%	0.0038	NA	NA	NA
	1%	0.0038	NA	NA	NA
MSVL	5%	0.1149	9.8820 [0.0017]	1.7100 [0.7889]	17.198 [0.0041]
	1%	0.0460	6.6345 [0.0100]	6.2647 [0.1802]	18.468 [0.0024]
fHRS	5%	0.0575	0.1806 [0.6708]	7.4071 [0.1159]	4.8141 [0.4390]
	1%	0.0115	0.7891[0.3744]	0.8451 [0.9323]	1.1307 [0.9512]
BS	5%	0.0498	0.0178 [0.8940]	4.3026 [0.3666]	4.3922 [0.4944]
	1%	0.0115	0.7891 [0.3744]	0.8451 [0.9323]	1.1307 [0.9514]
CBS	5%	0.0575	0.1806 [0.6708]	7.4071 [0.1159]	4.8141 [0.4390]
	1%	0.0077	0.8182 [0.3657]	3.1751 [0.5290]	2.7463 [0.7390]
BS+CBS	5%	0.0460	0.2012 [0.6538]	2.2051 [0.6981]	3.8111 [0.5769]
	1%	0.0077	0.8182 [0.3657]	3.1751 [0.5290]	2.7463 [0.7390]

Note: '% Violation' is the percentage of days when returns are less than the VaR threshold. UC, IND and CC are the GMM duration-base tests for unconditional coverage, independence and conditional coverage, developed by Candelon et al. (2010). The number of orthonormal polynomials is set to 5. *P*-values are in brackets.

Table 7 (Cont.): Backtesting VaR Thresholds: After GFC

(c) Value-weighted portfolio

Model	VaR	% Violation	UC	ID	CC
ADCC	5%	0.0038	NA	NA	NA
	1%	0.0038	NA	NA	NA
MSVL	5%	0.1149	9.8820 [0.0017]	1.7100 [0.0000]	17.198 [0.0041]
	1%	0.0498	7.5968 [0.0058]	3.1102 [0.5396]	20.345 [0.0011]
fHRS	5%	0.0575	0.1806 [0.6708]	7.4071 [0.1159]	4.8141 [0.4390]
	1%	0.0115	0.7891[0.3744]	0.8451 [0.9323]	1.1307 [0.9512]
BS	5%	0.0498	0.0178 [0.8940]	4.3026 [0.3666]	4.3922 [0.4944]
	1%	0.0115	0.7891 [0.3744]	0.8451 [0.9323]	1.1307 [0.9514]
CBS	5%	0.0575	0.1806 [0.6708]	7.4071 [0.1159]	4.8141 [0.4390]
	1%	0.0077	0.8182 [0.3657]	3.1751 [0.5290]	2.7463 [0.7390]
CBS	5%	0.0460	0.2012 [0.6538]	2.2051 [0.6981]	3.8111 [0.5769]
	1%	0.0077	0.8182 [0.3657]	3.1751 [0.5290]	2.7463 [0.7390]

Note: '% Violation' is the percentage of days when returns are less than the VaR threshold. UC, IND and CC are the GMM duration-base tests for unconditional coverage, independence and conditional coverage, developed by Candelon et al. (2010). The number of orthonormal polynomials is set to 5. *P*-values are in brackets.