The Correct Regularity Condition and Interpretation of Asymmetry in EGARCH*

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Abstract

In the class of univariate conditional volatility models, the three most popular are the generalized autoregressive conditional heteroskedasticity (GARCH) model of Engle (1982) and Bollerslev (1986), the GJR (or threshold GARCH) model of Glosten, Jagannathan and Runkle (1992), and the exponential GARCH (or EGARCH) model of Nelson (1990, 1991). For purposes of deriving the mathematical regularity properties, including invertibility, to determine the likelihood function for estimation, and the statistical conditions to establish asymptotic properties, it is convenient to understand the stochastic properties underlying the three univariate models. The random coefficient autoregressive process was used to obtain GARCH by Tsay (1987), an extension of which was used by McAleer (2004) to obtain GJR. A random coefficient complex nonlinear moving average process was used by McAleer and Hafner (2014) to obtain EGARCH. These models can be used to capture asymmetry, which denotes the different effects on conditional volatility of positive and negative effects of equal magnitude, and possibly also leverage, which is the negative correlation between returns shocks and subsequent shocks to volatility (see Black 1979). McAleer (2014) showed that asymmetry was possible for GJR, but not leverage. McAleer and Hafner showed that leverage was not possible for EGARCH. Surprisingly, the conditions for asymmetry in EGARCH seem to have been ignored in the literature, or have concentrated on the incorrect conditions, with no clear explanation, and hence with associated misleading interpretations. The purpose of the paper is to derive the regularity condition for asymmetry in EGARCH to provide the correct interpretation. It is shown that, in practice, EGARCH always displays asymmetry, though not leverage.

Keywords: Conditional volatility models, random coefficient complex nonlinear moving average process, EGARCH, asymmetry, leverage, regularity condition.

JEL: C22, C52, C58, G32.

"They're digging in the wrong place!"

Indiana Jones, Raiders of the Lost Ark

1. Introduction

In the class of univariate conditional volatility models, the three most popular are the generalized autoregressive conditional heteroskedasticity (GARCH) model of Engle (1982) and Bollerslev (1986), the GJR (or threshold GARCH) model of Glosten, Jagannathan and Runkle (1992), and the exponential GARCH (or EGARCH) model of Nelson (1990, 1991).

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Surprisingly, the conditions for asymmetry in EGARCH seem to have been ignored in the literature, or have concentrated on the incorrect conditions, with no clear explanation, and hence with associated misleading interpretations. The purpose of the paper is to derive the regularity condition for asymmetry in EGARCH to provide the correct interpretation. It is shown that, in practice, EGARCH always displays asymmetry, though not leverage.

The paper is organized as follows. In Section 2, the GARCH and EGARCH models are derived from different stochastic processes, the first two from random coefficient autoregressive processes and the second from a random coefficient complex nonlinear moving average process. The correct regularity condition for asymmetry in EGARCH is derived. Some concluding comments are given in Section 3.

2. Stochastic Processes for GARCH and EGARCH

2.1 Random Coefficient Autoregressive Process and GARCH

Consider the conditional mean of financial returns, as follows:

$$y_t = E(y_t|I_{t-1}) + \varepsilon_t, \quad (1)$$

where the financial returns, $y_t = \Delta log P_t$, represent the log-difference in financial commodity prices, P_t, I_{t-1} is the information set at time *t-1*, and ε_t is a conditionally heteroskedastic error term, or returns shock. In order to derive conditional volatility specifications, it is necessary to specify the stochastic processes underlying the returns shocks, ε_t.

Now consider the random coefficient AR(1) process underlying the return shocks, ε_t:

$$\varepsilon_t = \phi_t \varepsilon_{t-1} + \eta_t \quad (2)$$

where

$\phi_t \sim iid(0, \alpha)$, $\alpha \geq 0$,

$\eta_t \sim iid(0, \omega)$, $\omega \geq 0$,

$\eta_t = \varepsilon_t/\sqrt{h_t}$ is the standardized residual, with h_t defined below.

Tsay (1987) derived the ARCH (1) model of Engle (1982) from equation (2) as:

$$h_t \equiv E(\varepsilon_t^2 | I_{t-1}) = \omega + \alpha\varepsilon_{t-1}^2 \tag{3}$$

where h_t represents conditional volatility, and I_{t-1} is the information set available at time *t-1*. A lagged dependent variable, h_{t-1}, is typically added to equation (3) to improve the sample fit:

$$h_t \equiv E(\varepsilon_t^2 | I_{t-1}) = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}. \tag{4}$$

From the specification of equation (2), it is clear that both ω and α should be positive as they are the unconditional variances of two different stochastic processes.

Given the non-normality of the returns shocks, the Quasi-Maximum Likelihood Estimators (QMLE) of the parameters have been shown to be consistent and asymptotically normal in several papers. For example, Ling and McAleer (2003) showed that the QMLE for a generalized ARCH(*p*,*q*) (or GARCH(*p*,*q*)) is consistent if the second moment is finite. A sufficient condition for the QMLE of GARCH(1,1) in equation (4) to be consistent and asymptotically normal is $\alpha + \beta < 1$.

2.2 Random Coefficient Complex Nonlinear Moving Average Process and EGARCH

A conditional volatility model that can accommodate asymmetry is the EGARCH model of Nelson (1990, 1991). McAleer and Hafner (2014) showed that EGARCH could be derived from a random coefficient complex nonlinear moving average (RCCNMA) process, as follows:

$$\varepsilon_t = \phi_t \sqrt{|\eta_{t-1}|} + \psi_t \sqrt{\eta_{t-1}} + \eta_t \tag{5}$$

where

$\phi_t \sim iid(0, \alpha)$,

$\psi_t \sim iid(0, \gamma)$,

$\eta_t \sim iid(0, \omega)$,

$\sqrt{\eta_{t-1}}$ is a complex-valued function of η_{t-1},

and $\eta_t = \varepsilon_t / \sqrt{h_t}$ is the standardized residual.

McAleer and Hafner (2014) show that the conditional variance of the squared returns shocks in equation (5) is:

$$h_t = E(\varepsilon_t^2 \mid I_{t-1}) = \omega + \alpha \mid \eta_{t-1} \mid + \gamma \eta_{t-1}, \qquad (6)$$

where it is clear from the RCCNMA process in equation (6) that all three parameters should be positive as they are the variances of three different stochastic processes.

Although the transformation of h_t in equation (6) is not logarithmic, the approximation given by:

$$\log h_t = \log(1 + (h_t - 1)) \approx h_t - 1$$

can be used to replace h_t in equation (6) with 1 + $\log h_t$ to give:

$$\log h_t = E(\varepsilon_t^2 \mid I_{t-1}) = (\omega - 1) + \alpha \mid \eta_{t-1} \mid + \gamma \eta_{t-1}, \qquad (7)$$

The use of an infinite lag for the RCCNMA process in equation (5) would yield the standard EGARCH model with lagged conditional volatility.

As EGARCH can be derived from a random coefficient complex nonlinear moving average (RCCNMA) process, it follows that there is no invertibility condition to transform the returns shocks to the standardized residuals. Therefore, there are as yet no asymptotic properties of the QMLE of the parameters of EGARCH. Recently, Martinet and McAleer (2017) showed that the EGARCH(p,q) model could be derived from a stochastic process, for which the invertibility conditions can be stated simply and explicitly. This theoretical result is likely to lead to the development of asymptotic properties for the QMLE of EGARCH.

McAleer and Hafner (2014) showed that leverage exists for EGARCH if:

Condition for Leverage for EGARCH: $\gamma < 0$ **and** $\gamma < \alpha < -\gamma$ **.**

It is clear that leverage is not possible for EGARCH as both α and γ, which are the variances of two stochastic processes, must be positive. The second parametric condition for leverage is typically omitted in the literature on EGARCH, without explanation.

For example, any version of the EViews econometric software manual can be seen to state incorrectly and without explanation that:

"The presence of leverage effects can be tested by the hypothesis that $\gamma < 0$ **. The impact is asymmetric if** $\gamma \neq 0$ **".**

In order to obtain the correct regularity condition, the derivatives of $\log h_t$ in equation (7) with respect to η_{t-1} are given as:

(1) $\alpha + \gamma$ when $\eta_{t-1} \geq 0$;

(2) $-\alpha + \gamma$ when $\eta_{t-1} < 0$.

Symmetry requires that the impacts of positive and negative shocks of similar magnitude on volatility should be the same. Therefore, symmetry exists for EGARCH if:

$\alpha + \gamma = -\alpha + \gamma$, that is, $\alpha = 0$.

It follows that the regularity condition for symmetry in EGARCH is:

Condition for Symmetry in EGARCH: $\alpha = 0$.

As can be seen from the statement in EViews above, which has been recited and repeated numerous times in the literature, the condition given in the literature for asymmetry concentrates on the incorrect parameter, γ, rather than the correct parameter, α.

In virtually every empirical example where EGARCH is estimated, the quasi maximum likelihood estimation of α is statistically significant. Therefore, in practice, EGARCH always displays asymmetry, though not leverage.

3. Conclusions

The paper was concerned with to of the most widely-used univariate conditional volatility models, namely the GARCH and EGARCH models. The EGARCH model is important in capturing asymmetry, which is the different effects on conditional volatility of positive and negative effects of equal magnitude, and possibly also leverage, which is the negative correlation between returns shocks and subsequent shocks to volatility.

As the conditions for asymmetry in EGARCH seem to have been ignored in the literature, or have concentrated on the incorrect conditions, with no clear explanation, and hence with associated misleading interpretations, the purpose of the paper was to derive the regularity condition for asymmetry in EGARCH to provide the correct interpretation.

The condition given in the literature for asymmetry in EGARCH concentrates on the incorrect parameter, γ, rather than on the correct parameter, α. In virtually every empirical example where EGARCH is estimated, the quasi maximum likelihood estimation of α is statistically significant. Therefore, in practice, EGARCH always displays asymmetry, though not leverage.

This is reminiscent of Indiana Jones who, in Raiders of the Lost Ark, exclaimed that the Nazis were “digging in the wrong place” in searching for the lost Ark of the Covenant.

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