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OPMERKINGEN EN AANTEKENINGEN – COMMUNICATIONS

MEASURING WELFARE OF PRODUCTIVE CONSUMERS*

1 *Scope of Article*

In the last few decades an increasing number of economists have contributed to new methods of welfare (or utility) measurement. As set out in this journal (Tinbergen, 1985) three groups of economists have been active in this field since 1968, initially relatively independently; an American, a British and a Dutch group, of which the leading economists were Dale W. Jorgenson (Harvard), George W. McKenzie (Cambridge, UK) and Bernard M.S. van Praag (Erasmus). Additional imaginative contributions have been made since by several other economists, mentioned in my 1985 note.[1] In that same note I mentioned a lacuna in the Anglo-Saxon method: it considers utility derived from consumption but not utility (positive or negative) from work or from risk taking. The empirical research by the Dutch group implies all sources of satisfaction (in this article a third word for utility). The present article is an attempt to fill part of the lacuna stated, but simplified to the extreme, with the intention to clarify the essence of the additional aspect.

Among the simplifications one must be mentioned in advance: the model submitted is static. This may be a disadvantage to some (or many) readers; and it may be avoided. Some remarks about a dynamic model will be made. Since the static version already introduces a number of new concepts the present author tentatively starts with the static version.

2 *One Individual's Utility from Consumption*

As usual we first consider one individual and the maximization of her or his satisfaction from consumption. Since another source of satisfaction will be added we push our simplification so far as possible and consider only one consumer good of which a quantity x is consumed, omitting at this stage the individual's suffix n. The utility caused by x is assumed to be $ln\,(x + 1)$. This function has the advantage of showing decreasing mar-

* I am grateful to Professors Joop Hartog and Simon K. Kuipers for valuable comments on an earlier version of this communication. Remaining errors of course are mine.

1 A serious lacuna in my 1985 note was that the work by the well-known French economist Maurice Allais was not mentioned. His most recent contribution to the subject of measuring utility known to me is Allais (1984). The concepts used were developed from 1953 on, *e.g.* in La Psychologie de l'Homme Rationnel devant le Risque, la Théorie et l'Expérience, *Journal de la Société de Statistique de Paris*, Janvier-Mars 1953, pp. 47–73.

ginal utility $1/(x+1)$ and in the extensive empirical material collected by Van Praag *et al.* gave an equally good fit as the function chosen by them. Instead of my previous option for $ln\ x$, I now prefer to add the $+\ 1$ term since that implies that utility of $x = 0$ is zero. The difference may be very small if the unit of x is small, but we have good reasons for preferring large units, as will be set out in section 3.

3 *One Individual's Satisfaction from Work*

As another simplification we consider only one productive source of satisfaction, work, and neglect the possibility of contributing to production by supplying physical capital. We assume that the individual considered has a job and that his job can be characterized by a certain quantity of *schooling required*, s. The person will be characterized by a quantity of *human capital*, v, resulting from her or his *formal schooling*, e, completed and her or his *innate abilities*, u. Schooling required is based on some assumption about average innate abilities of the group of individuals from which the occupants of the job in question are usually recruited, but these assumptions are not explicity stated. The ability offered, v, results from u and e and we choose our functional forms and our units of measurement so as to make:

$$v = u + e \tag{3.1}$$

These units will be called *years of education*, mainly because that is the concept on which a considerable quantity of statistical data is available. A more realistic approach, as proposed by De Wolff and Van Slijpe (1973), would be to give different weights to earlier and later years of schooling, but this will require still more preparatory research than the proposal under discussion and will not be pursued further. In addition to the concepts so far introduced one more will be introduced: the individual's *maximum absorption capacity of formal schooling*, $\bar{e}$. It is a – as yet unknown – function of innate capabilities, u. All schooling variables – s, v, e and $\bar{e}$ – will be measured in years of schooling. So will x and y.

In reality, which must be described by a dynamic model, the individual first chooses an education (or formal schooling process) perhaps, but not necessarily already, based on a job desired, but the application for a job as a rule will follow education and be codetermined by the education completed. During the individual's career, training on the job will add to her or his ability, and series of consecutive jobs occupied as well. Society as a whole comprises individuals in each of the consecutive situations and innate ability classes. The single individual now studied is sort of an average of all and hence derives utility, positive or negative, from all the consecutive stituations.

To the utility derived from consumption we add two additional terms:

$$\varepsilon\ ln\left(1 - \frac{e}{\bar{e}}\right) - \tfrac{1}{2}\sigma(s - v)^2;\ \varepsilon > 0,\ \sigma > 0 \tag{3,2}$$

The first consitutes the utility from the formal schooling process. The expression $ln\ (1 - e/\bar{e})$ is zero for the start and tends to $-\infty$ when e approaches the individual's eduction absorption capacity. An individual who likes learning will have positive constant satisfaction in addition, but this is irrelevant for the process of utility maximization and so need not be mentioned. Such an individual will often have a high $\bar{e}$ and choose a high e.

The second additional term represents satisfaction from work. Again a positive constant has been omitted. The variable part has a maximum for $s = v$, where v matches s. Deviations on both sides are negative, and constitute the 'tension theory' this author adheres to: people don't like to have a job below their ability, but don't like one above their ability either. Ability, in this oversimplified model, consists of schooling ability. All other relevant abilities, such as character or leadership, have been neglected.

Both additional terms to the utility function are characterized by a coefficient, ε and σ respectively, expressing their intensity in comparison to the consumption term. *Coefficients* are assumed to be the same for all individuals. Their individual characteristics are expressed by *parameters*, of which our examples are $\bar{e}$ and u.

4 *Optimizing One Individual's Welfare; The Production Function*

The welfare optimum for one individual is found by maximizing welfare or utility under some restrictions. In the present case one restriction is the relation (3.1). The other is that total production, say y, is used for either consumption x or education e. If we measure the latter as well as y in the same units, this implies

$$y = x + e \tag{4.0}$$

Production will be the result of the job(s) held and will depend on both s and v. In an attempt to stick to the simplest approach possible I assume that production is rising with rising s and rising v:

$$y = \alpha s + \beta v \tag{4.1}$$

As long as simplicity does not 'simplify away' the essence of our problem or, in a later stage, fits measured results, we stick to it.[2] The optimization process then reduces to maximize under two restrictions:

$$\omega = ln(x + 1) + \varepsilon ln\left(1 - \frac{e}{\bar{e}}\right) - \tfrac{1}{2}\sigma(s - v)^2 + \lambda(\alpha s + \beta v - x - e) + \mu(e - v + u) \tag{4.2}$$

where λ and μ are Lagrange multipliers. The optimum conditions are that the derivatives of ω with regard to the unknowns x, e, s and v vanish and the two restrictions apply. This means:

$$\partial\omega/\partial x = 1/(x + 1) - \lambda = 0 \tag{4.3}$$

$$\partial\omega/\partial e = -\varepsilon/\bar{e}(1 - e/\bar{e}) - \lambda + \mu = 0 \tag{4.4}$$

$$\partial\omega/\partial s = -\sigma(s - v) + \alpha\lambda = 0 \tag{4.5}$$

$$\partial\omega/\partial v = \sigma(s - v) + \beta\lambda - \mu = 0 \tag{4.6}$$

2 Actually, in a recent publication Joop Hartog (1986) does find an interaction between s and v in a production function. Addition of a term in sv may introduce such interaction.

Elimination of λ can be done by taking α (4.3) + (4.5):

$$\alpha/(x+1) - \sigma(s-v) = 0 \tag{4.7}$$

and elimination of μ by (4.4) + (4.6) with $\lambda = 1/(x+1)$:

$$-\varepsilon/(\bar{e}-e) + (-1+\alpha+\beta)/(x+1) = 0 \text{ or}$$

$$\varepsilon/(\bar{e}-e) = (1-\alpha-\beta)/(x+1) \tag{4.8}$$

Writing (4.7) and (4.8) in a more appropriate form we get:

$$x + 1 = \alpha/\{\sigma(s-v)\} \tag{4.9}$$

$$\bar{e} - e + \varepsilon(x+1)/(1-\alpha-\beta) \tag{4.10}$$

Our procedure can only be valid if $s > v$ and $\alpha + \beta < 1$. The conditions $s > v$ means that capacities are scarce, which applies widely. Whether $\alpha + \beta < 1$ has wide validity, is less certain. If $\alpha + \beta > 1$ there may be no flat maximum of ω. Empirical research is needed anyway; our example is kept simple, since it is an illustration only. The restrictions add:

$$\alpha s + \beta v = x + e \tag{4.11}$$

$$e = v - u \tag{4.12}$$

The solution of the last four equations for given α, β, ε and $\bar{e}$ is easy. Even for a very crude statistical check we lack data on $\bar{e}$, and may have to disregard differences between e and v or assume them to be proportional with $e < v$ in order to leave for u their difference. This means that at least some statistical programs are suggested. Valuable data for s in the United States have been calculated by Rumberger (1981).

5 *Optimal Welfare of a Population*

From the micromodel of section 4 we may now derive a macromodel[3] for a population (the total population of an area or a sample of such a total population). This means that we must aggregate the variables and parameters used for the description of an individual to variables or parameters of the population considered. Coefficients will remain the same, since we made the assumption that coefficients are identical for all individuals. The aggregated variables will be indicated by capital letters. Aggregation poses no problem for linear equations. This is clearest for equations (4.1) and (4.2), which in the macro-model can be written:

$$Y = X + E \tag{5.1}$$

3 A macromodel is what most of the authors quoted are aiming at, for instance to focus on inequalities and the effect of income redistribution policies. If all relationships are linear the totals of the variables are simply N times the per capita values.

and

$$Y = \alpha S + \beta V \tag{5.2}$$

where $Y = \sum_n y_n$, $X = \sum_n x_n$, $E = \sum_n e_n$, $S = \sum_n s_n$, and $V = \sum_n v_n$.

Similarly (3.1) becomes, in the macromodel:

$$V = U + E \tag{5.3}$$

where U may be called the innate ability of the population.

The most important macro-equations must be derived from (4.9) and (4.10). The latter equation, being linear as well, becomes

$$\bar{E} - E = \varepsilon(X + N)/(1 - \alpha - \beta) \tag{5.4}$$

where N is the highest n and the size of the population. Equation (4.9) had better be rewritten $s - v = \frac{\alpha}{\sigma}/(x + 1)$ and the macro-shape of the left-hand side $S - V$. Aggregation of the right-hand side requires the introduction of another macro-variable $X' = \sum_n \frac{1}{x_n + 1} \neq \frac{1}{X + 1}$ and the equation becomes:

$$S - V = \frac{\alpha}{\sigma} X' \tag{5.5}$$

If the dispersion of the x_n is modest, X' may be approximated by $1/(X + 1)$, though.

The macro-formulae shown enable us to estimate the coefficients. Equation (5.2) may be used to estimate α and β, (5.4) to estimate ε and (5.5) to estimate σ. With their aid each individual's welfare may be estimated and the population's welfare Ω by aggregating individual welfare.

$$\Omega = \sum_n ln(x_n + 1) + \varepsilon \sum_n ln(1 - e_n/\bar{e}_n) - \tfrac{1}{2}\sigma \sum_n (s_n - v_n)^2 \tag{5.6}$$

6 *Summary*

In this note an attempt is made to show, with the aid of the simplest example conceivable, how welfare estimates as made by Jorgenson *et al.* (see Jorgenson and Slesnick, 1984, 1986) can be extended to include welfare derived from productive effort. It appears that notwithstanding the model's oversimplification the data are lacking which are needed to make numerical estimates. The missing data refer to $\bar{e}$, the formal schooling absorption capacity, and the information needed to make a distinction between formal schooling e and total schooling v. In addition, of course, relevant other productive abilities should be included and the corresponding terms in the welfare function added. Finally, both the production and the welfare function may have to be chosen differently to obtain sufficient fit with observed values of the variables included. These aspects and several others have been given full attention by Jorgenson *et al.*

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