OPMERKINGEN EN AANTEKENINGEN – COMMUNICATIONS

MEASURABILITY OF UTILITY (OR WELFARE)

After the Pivogian era the tendency of the majority of economists was to adhere to Pareto's view that utility is not measurable. (In this note the words utility and welfare will be considered as synonymous). Recently in the Netherlands measurability has been defended again, by Bernard M.S. van Praag and his school (including A. Kapteyn, A. Kouwenhoven, Th. Goedhart, T.J. Wansbeek, F.G. van Herwaarden, J. Buyze, H v.d. Stadt, and others) and by this author. Van Praag and his pupils made considerable contributions to both theory and measurement; the present author concentrated on the application only. The essence of their *empirical* work consists of direct interviewing of very large numbers of European consumers. The question consists of indicating the income intervals in which the interviewee would feel resp. 'excellent,' 'good,' 'amply sufficient,' *etc.* ... down to 'very bad' (9 intervals).

There appears to be a lack of communication among proponents of measurable utility, however: among three groups of economists working in this field. The main Anglo-Saxon authors who worked on the subject have not, as far as I am aware, been quoted by Van Praag and his school.[1] An American group to which Dale W. Jorgenson, Laurits R. Christensen and Lawrence J. Lau belong did not quote Van Praag *et al.* and only occasionally an English group, George W. McKenzie and I.F. Pearce, who again did not, to my knowledge, quote either Van Praag *et al.* or Jorgenson *et al.* Jorgenson *et al.* published preparatory work in 1975, McKenzie and Pearce in 1976 and Van Praag in 1968, and empirical work from 1971 on. This lack of communication may have obscured the fact until now that a reversal from the Paretian to the Pigovian way of thinking seems well under way. It is the intention of this note to draw the attention of all interested parties to this new trend and to present some brief comments on the essence of the work by all three groups. It is my hope that a more penetrating analysis can be offered later.

To begin with, an extremely simple *example* of the Anglo-Saxon work may be given in order to *characterize the main procedure.* An individual is assumed to live in a world with two commodities, 1 and 2, only, whose prices are p_1 and p_2. The individual spends his income y on these two goods and buys quantities x_1 and x_2, respectively, so as to maximize his utility $U(x_1, x_2)$. It is assumed that, to begin with, this utility can be approximated by the equation:

$$U(x_1, x_2) = a_1 \ln x_1 + a_2 \ln x_2 \tag{1}$$

1 A. Kapteyn did, however, mention Jorgenson and Lau in his recent inaugural address in Tilburg (I owe this information to T. Wansbeek).

This will be maximized under the restriction that the budget equation limits the total expenditure to Y:

$$x_1 p_1 + x_2 p_2 = Y \tag{2}$$

This will be attained when $a_1 \ln x_1 + a_2 \ln x_2 + \lambda(Y - x_1 p_1 - x_2 p_2)$ is maximized, where λ is a Lagrangian multiplier; and the conditions are:

$$a_1/x_1 - \lambda p_1 = 0 \tag{3}$$

$$a_2/x_2 - \lambda p_2 = 0 \tag{4}$$

Eliminating λ we find

$$a_1 x_2/a_2 x_1 = p_1/p_2 \text{ or } x_2 = x_1 p_1 a_2/p_2 a_1 \tag{5}$$

Substituting this into (2) we can solve for x_1:

$$x_1 = \frac{a_1}{a_1 + a_2} \frac{Y}{p_1} \tag{6}$$

from which we can find x_2:

$$x_2 = \frac{a_2}{a_1 + a_2} \frac{Y}{p_2} \tag{7}$$

Equations (6) and (7) are the *demand equations* for goods 1 and 2, respectively. (the same demand functions could have been obtained from a monotonically transformed utility function.) These can be observed and if the equations do not fit the observations a utility function of another mathematical shape than (1) can be tried out. In fact, Jorgenson chose a *translog utility function* (one quadratic in the logarithms) and one *varying over time*. For an illustration of the principle we may stick, however, to our simplest example.

If now we want to express our utility function numerically we have to determine a_1 and a_2. Since Y and the x and p are all observable, we may try (6) and (7) to determine the numerical values of a_1 and a_2; we find

$$a_1/(a_1 + a_2) = x_1 p_1/Y = w_1 \tag{8}$$

$$a_2/(a_1 + a_2) = x_2 p_2/Y = w_2 \tag{9}$$

where w_1 and w_2 are the shares of the budget spent on good 1 and 2, respectively.

From this result we see that no absolute values of a_1 and a_2 can be found; only their ratio. Since any absolute level of utility is acceptable (only the shares matter in the demand equations (6) and (7)) we choose $a_1 + a_2 = 1$, or $a_2 = 1 - a_1$. The utility function becomes

$$a_1 \ln x_1 + (1 - a_1) \ln x_2 = U(x_1, x_2) \tag{10}$$

This means that utility can be *derived from an observed system of demand equations.*

As observed already, this example is much too simple. Not only is another mathematical shape required, but also more variables and, in addition, a number of *parameters,* that is, characteristics of the individuals or groups of individuals considered. In the most extensive and impressive empirical study so far (Jorgenson and Slesnick, 1983) five consumer goods were introduced, seven family sizes, six age-of-head groups, four regions of residence, two race groups and two type-of residence (urban and rural) groups were distinguished, hence 21 parameters.

This enables Jorgenson and Slesnick to transform the income and consumption variables into utility (or welfare) estimates for the population of the United States and for each of the years 1958 through 1978.

Other combinations of the American author group had started in 1975 with what could now be called preparatory work. Thus, the translog utility function had been introduced by Christensen, Jorgenson and Lau (1975) and by Jorgenson and Lau (1975), using three groups of goods and services and time. In these studies the problems considered were the *validity of demand theory* – called 'inconsistent with the evidence' in the first article – and a number of characteristics of demand equations. This may explain why these studies were not understood to contribute to the measurement of utility and so were not quoted by Van Praag's school. There were close contacts, however, with the Rotterdam econometricians working on systems of demand equations.

The studies mentioned above, as well as those by McKenzie and Pearce (1976), did not present *empirical* results for the measurement of utility. Nor did their 1982 article and McKenzie's 1983 book. McKenzie and Pearce explicitly aimed at presenting a '*money metric*' of utility – in formulae, not in figures. They deviate from Jorgenson *et al.* by rejecting a restriction to closed forms for the preference and cost of utility functions, in which they see too strong a restriction. All Anglo-Saxon authors so far discussed added the concept of an *indirect utility function,* however, and this may be illustrated with the aid of our oversimplified example. By an indirect utility function they understand a utility function expressed in terms of Y and the p's instead of the x's. We obtain it by substituting (6) and (7) into (1) and so it is:

$$U = a_1 \ln \frac{a_1 Y}{(a_1 + a_2)p_1} + a_2 \ln \frac{a_2 Y}{(a_1 + a_2)p_2} \tag{11}$$

McKenzie's and Pearce's money metric requires the solution of the indirect utility function for Y. Writing, as before, $1 - a_1$ for a_2, we obtain:

$$\ln Y = \ln U - a_1 \ln a_1 - (1 - a_1) \ln (1 - a_1) + a_1 \ln p_1 + (1 - a_1) \ln p_2 \tag{12}$$

This equation is also called *cost of utility function* (at least in its nonlogarithmic form).

In addition it must be stated that all Anglo-Saxon authors deal with a variety of other problems, among which integrability and various aspects of the mathematical shapes of the direct and indirect utility functions. As far as I can see these subjects are not essential for the points raised in this note. Also, the Dutch authors deal with additional problems, including several very interesting ones.

A different point I want to raise is whether or not the attempts to measure utility or welfare discussed so far suffer from an *omission of satisfaction derived from the*

individual's productive activities (job, or work). Income Y is supposed to be given, but the way it is obtained is not considered; and the same income may be obtained from work with a positive, and other work with a negative, satisfaction. There may be scope to introduce not only measurements of job satisfaction, but also of the ways in which this may be influenced by two types of choices, the *choice of occupation* aimed at and the *choice of education* made. This is not the place to elaborate on this aspect. I may be permitted to refer to some attempts I made to tackle these problems (1975, 1984, 1985).

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