

## ON COLLECTIVE AND PART-COLLECTIVE GOODS

BY

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## 1 DEFINITIONS OF COLLECTIVE GOODS

A still useful definition of a *collective good* as distinguished from an individual good is that its 'consumption' by one individual does not reduce the possibility for other individuals to 'consume' it (Samuelson, 1954). A few examples are a TV broadcast, the police and a nation's highway system (excluding toll ways). In section 4 the complete set of collective goods will be discussed. Of the three examples mentioned the TV broadcast is closest to a *purely* collective good. The other two usually are close, but not always. Extended riots may require so much of the police force that police are not available for a case of theft. On the first day of a holiday period highways may be overcrowded and not available for an additional car. In the latter two cases we will speak of part-collective goods; these will be discussed in sections 3 through 8.

The definition given may be replaced by *alternative* definitions. One is that collective goods are the goods *produced by* public authorities. Accordingly the phrase public goods has been used instead of collective goods. This definition may be combined with the statement that one of the reasons why public authorities produce these goods is that public authorities produce them more cheaply than private producers. An extreme case is that either private initiative or private capital is lacking to run the production process under discussion. Three examples may be given. In 1902 the Dutch State Mines (DSM) were established to mine coal, since private capital and initiative were too weak. Since 1930 the Dutch state runs a number of farms on newly reclaimed land for a limited number of years; experience with private farming on previously reclaimed land (Haarlemmermeer) having shown that the risks were so large that all first owners of parcels went bankrupt. The third example is the creation by the Turkish government in 1923 under Atatürk of the 'state economic enterprises' in various branches of industry: at that time private initiative to industrialize Turkey was too weak again.

Another alternative definition of collective goods is the one adhered to by

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most of today's authors on the subject, namely that they are nonexcludable and, as a consequence, nonrival. The subtleties of these qualitative expressions do not affect, however, the mathematical structure of utility functions. Rightly Samuelson, in his 1981 article, sticks to the structure of his 1954 treatment.

Alongside purely collective goods, a number of not purely collective goods has been identified by various authors. Thus, Drèze (1974) deals with *semi-collective* goods, of which an interesting example is a collective good that is only available in a restricted geographical area.

Goedhart (1975), Wolfson (1979) and several Anglo-Saxon authors, for instance Brown and Jackson (1978), introduced the concept of *quasi-collective* goods, which may be said to have a private component but are made available by public authorities at prices below cost. An important example is schooling.

The category of not purely collective goods to be discussed in the present essay is different again and constitutes another aspect already briefly set out. It has some similarity to quasi-collective goods, but cannot be identified with the latter.

## 2 SOME ADDITIONAL CHARACTERISTICS OF COLLECTIVE GOODS

The second definition of collective goods discussed in section 1 suggests that their production is required by law. Admittedly government activities are regulated by legislation. Yet at least part of them do not produce collective goods. Most state enterprises also produce individual rather than collective goods. In our view (also shared by Goedhart, 1981) collective goods are not necessarily produced by public authorities. A TV broadcast or a highway usually are produced by private associations (in the Dutch system, at least) or private firms. But public authorities do assume some responsibility for them and this is regulated by law. In what follows we will consider as the normal case that collective goods are 'ordered' by 'the government,' the latter expression being shorthand for public authorities at the various levels. Producers are paid for supplying the collective good ordered by the government out of tax revenue and taxes are collected from citizens. Purely collective goods are 'consumed' by all citizens in the same quantity, which is the implication of the definition to which we shall adhere. The tax paid for the availability of a collective good will be supposed to be such that it reflects the marginal utility of that good to the tax payer. All this will be translated into a simple mathematical model in section 8. Parliamentary decisions on the quantities of each collective good to be made available by the government are assumed to be in line with the marginal utilities of each citizen. This assumption implies that the parliamentary majority's *judgement of these marginal utilities* is substituted for the marginal utilities as experienced by the citizens themselves. This is our alternative to Wicksell's overly idealistic requirement of unanimous decisions on the volume of collective goods to be made available (*cf.* Hennipman, 1977, 1982).

In the present essay our main objective is how to introduce into economic theory the existence of part-collective goods. As stated in the beginning of this article, such goods occasionally are not available to all citizens who want them. On such occasions the responsible authorities must allocate the availability of the part-collective good under discussion. Such an allocation may consist of some sort of rationing for which they must develop a set of rules. On a crowded highway priority over other vehicles may be given to the fire brigade, to an ambulance, *etc.* In our model this allocation will also have to be expressed by a set of equations.

Situations are conceivable in which negative-valued collective 'goods' occur. The best-known example is the 'security' supplied by organized criminals who claim a contribution from shopkeepers. We shall not pursue this phenomenon.

In most nations minority groups (the parliamentary opposition) are in favour of establishing collective goods not (yet) in existence or larger (alternatively, smaller) quantities of existing collective goods. In the course of history changes in the quantities of a number of collective goods have occurred. This applies to the examples given so far, TV broadcasts, police and highways. It also applies to many of the concrete additional examples to be discussed in section 4.

This raises the general question whether collective goods should be restricted to those required by legislation or whether a more fundamental source should be substituted for legislation. In fact, legislation itself is based on more fundamental sources. It is commonplace to consider ethical or religious principles as the source of positive law.

Since this article deals with positive economics only and not with normative economics, we leave this question to a later study.

### 3 DEFINITION OF PART-COLLECTIVE GOODS

In section 1 we already briefly touched upon the possibility that there are goods where the consumption by one individual does not reduce other persons' consumption possibility by the full amount of the first person's consumption, but nevertheless somewhat reduces that possibility, given the total availability of the good considered. This category of goods will be called part-collective. Two examples have been mentioned in section 1. In contradistinction to individual or private goods, where the second person's consumption possibility – with a given supply – is reduced by the full amount and purely collective goods, where it is not reduced at all, we have to do here with a category that may be characterized by a continuous parameter related to the *degree* to which other potential consumers have to reduce their consumption.

Two further points must be made. One is that the same good may move within a zone where the parameter mentioned assumes a range of *different* values. The good is not, therefore, identified by that parameter.

The other is that *external data* affect the parameter's value. In the police example given in section 1 it is the riots which affect police availability for other tasks. In the congestion example it is the holiday period's beginning which reduces highway capacity to additional cars.

#### 4 AN ENUMERATION OF COLLECTIVE AND PART-COLLECTIVE GOODS

The main task of a positive theory of collective and part-collective goods consists of an explanation of the mechanism by which the quantities of all sorts of goods and services produced, distributed and consumed as well as their prices are determined. We will restrict ourselves to a closed economy and to the short-run statics of the mechanism. By 'goods' we understand goods and services.

One essential feature of a theory and its representation by a model is that it should be *complete* in the sense of containing all goods, all individuals and all firms which together form the economy considered. With regard to individual goods, completeness can be checked by statistics of household accounts and of production; with regard to individuals the census of population statistics and so on. In order to cover all collective goods we may lack an exhaustive source of information. If we had adhered to the definition that collective goods are those produced by public authorities their combined budgets might have been a source of information. Because of our assumption that government is responsible for the availability of collective goods we may still use that source, keeping in mind, however, that some government products show individual components. We will have to add some collective goods *not (completely) covered by legislation*, however.

From an inspection of the responsibilities of ministries and their equivalents at lower levels (down to local authorities) we derived the enumeration given below, which contains the examples given earlier.

- Internal security (police, justice, fire brigades)
- External security (military authorities' tasks, diplomatic service)
- Transportation infrastructure, water level control
- Information (statistics, libraries, news media incl. TV, radio, planning)
- School system, research and publication of its results
- Clean air, water, soil; maintenance of natural and cultural monuments
- Social security, including working time and labour conditions (cf. Hartog, 1981)
- Health services (not considered to be collective by Goedhart)
- Maintenance of a competitive production and distribution system.

If this enumeration is accepted, a few remarks have to be added.

- (i) Purely collective goods are *relatively rare*. Some types of information, working time and labour conditions are perhaps the best examples.
- (ii) As stressed in all preceding text, many of the collective goods listed are

part-collective only: this applies to police, transportation infrastructure, social security, health services, and several others.

- (iii) The 'maintenance of a competitive production and distribution system' is largely taken care of by private entrepreneurs, but up to a point only. Many markets are oligopolies!
- (iv) As stated at the end of section 2, the list may be completed by a number of collective or part-collective goods not required by legislation, but by ethical or religious principles. Some examples are: to be a good citizen, a good parent to one's children, and a good partner in marriage. Most economists today would not include these examples in their lists. Yet there may be good reasons to do so. As mentioned, the subject will be left to a later study.

#### 5 THE CONTINUUM OF PART-COLLECTIVE GOODS

As set out in the preceding sections, part-collective goods constitute a range of goods – or rather, of goods in changing environments – between the sharply defined categories of individual and purely collective goods. The essential relationship which distinguishes private from purely collective goods is the one between the supply available  $u$  and the quantities consumed  $u_i$  by individuals  $i$  ( $i = 1, \dots, I$ ). For individual goods this relationship is

$$u = \sum_i u_i \quad (5.1)$$

whereas for purely collective goods a set of equations applies

$$u_i = u \quad (i = 1, 2, \dots, I) \quad (5.2)$$

The question to be dealt with in this section is whether we can find one relation containing a parameter, say  $\rho$ , which covers the complete range of all part-collective goods with individual and purely collective goods as special cases. The answer is yes and has been inspired by our knowledge of CES production functions (*cf.* Kreijger, 1978, De Boer, 1981, where this knowledge is summarized and the initiators of this branch of economics are quoted). For our purpose a special case is needed:

$$u^{-\rho} = \sum_i u_i^{-\rho} \quad (5.3)$$

This relation becomes (5.1) for  $\rho = -1$  and it has been shown that for  $\rho \rightarrow \infty$  it converges towards the set (5.2). Its only parameter  $\rho$  is connected with the Allen elasticity of substitution  $\sigma_{ij}$  ( $i \neq j$ ) by

$$\sigma = 1/(1 + \rho) \quad (5.4)$$

Since our interpretation of (5.3) is not one of a production function, the Allen substitution elasticity does not enter into our utilization.

For the study of part-collective goods the most interesting question is the relation between the total quantity of such a good available ( $u$ ) and the sum  $\sum_i u_i$  of the quantities demanded. As an example let us take a simple case where the  $u_i$  are the 'quantity' of highway needed for one car and let all cars require the same space which we take as the unit. For a total number  $I$  of individuals we then have

$$\sum_i u_i = I \tag{5.5}$$

Using our formula (5.3) to calculate  $u$  we get:

$$u = I^{1 - 1/\rho} \tag{5.6}$$

The total number of cars which the highway network's capacity permits to circulate will be

$$I u = I^{1/\rho} \tag{5.7}$$

From an analysis of formulae (5.6) and (5.7) we may conclude that the relevant interval for  $\rho$  in this particular application is

$$-\infty < \rho < -1 \tag{5.8}$$

where  $\rho = -\infty$  corresponds to a purely collective and  $\rho = -1$  to a private good. Some values of  $I u$  are shown below and a numerical example has been added:

$\rho$	$-\infty$	$-10$	$-2$	$-1.5$	$-1$
$I u$	$I$	$I^{0.1}$	$I^{0.5}$	$I^{0.67}$	$I$
For $I =$	(Coll.)				(Priv.)
1 million:	1	4	1000	10 000	1000 000

The value  $-\infty$  corresponds with a purely collective good and  $\rho = -1$  with a private good where everybody's car may circulate. Part-collective goods are represented by the range between these two values; close to  $\rho = -1$  almost all cars and for high negative values of  $\rho$  relatively few are accommodated: there is congestion.

It is interesting that the interval of  $\rho$  that makes sense for our problem is the one excluded when (5.3) constitutes a CES production function. As set out elsewhere (Tinbergen, 1982) even then it depends on the problem studied which interval of  $\rho$ -values is relevant: it is different for problems in which 'essential' production factors only are considered from problems where nonessential production factors come in.

## 6 A POSITIVE THEORY OF FINANCING THE AVAILABILITY OF COLLECTIVE GOODS

As set out in section 2 we consider essential not the production but the 'ordering' of collective and part-collective goods by government. Since we are not dealing with a normative problem, but only with a positive (or 'analytical') treatment of the role of these goods, the question we want to answer in this section is no more than how governments are financing the collective and part-collective goods 'ordered.' A further restriction is that we only deal with a static theory. This implies that the goods ordered are paid and the total amount to be paid equals public revenue. Such revenue is collected from taxes in the widest sense, including contributions to a social security system and retributions.

Finally we make the assumption that taxes paid by enterprises (indirect taxes) are a payment for government services supplied to producers. Admittedly this assumption is hardly warranted, but in national accounting it has often been made. This reduces the budget we consider to the budget for collective and part-collective goods supplied to consumers and the revenue to direct taxes and retributions as well as consumer contributions to the social security system. For a different treatment see Samuelson (1982), where indirect taxes are the only taxes.

The main element of choice remains whether equilibrium is interpreted as the equality between total revenue and total expenditures on collective or part-collective goods, or whether some or all of the payments for the  $K$  types of such goods we shall introduce in our model are covered by particular types of revenue. In the former case there will appear *one government budget equation*, whereas in the latter there may be more, up to  $K$  such equations. The latter would apply in particular if the decision on the 'ordering' of some collective good were taken *jointly* with the decision on the source of financing. In practice this seldom occurs, to the regret of Finance ministers; but there do exist special-purpose taxes. In fact they are the best guarantee for rational decisions on public finance.

## 7 THE NEED FOR AN ALLOCATION SYSTEM FOR PART-COLLECTIVE GOODS

The last concept to be elaborated somewhat more than in section 2 (where it was first mentioned) is the concept of an allocation system needed for part-collective goods. For purely collective goods no allocation is needed. The quantity available is fully available to all citizens; our example of a TV broadcast illustrates that situation. In the first paragraphs of section 1 we mentioned the circumstances in which part-collective goods may, in contrast, not always be available. In section 2 we mentioned some allocation systems. In the present section some further elaboration is offered. In doing so we shall have in mind more examples than those of the police and the highway network. Among the collective goods

mentioned in section 4 the following provide additional examples: libraries, schools, distribution of research results, maintenance of a clean environment, such social security as takes the form of unemployment, sickness and other benefits and health services.

One method of allocation consists of the establishment of *priorities* amongst categories of different quality. Thus the police force will be used primarily for more serious categories of disaster or crime, and less serious categories may have to be left to later occasions. Water-level control will be exerted first of all in cases of floods and minor adjustments for agricultural purposes constitute a secondary concern. Groups of students admitted to universities may be categorized according to scores attained in secondary schools. A less attractive system is one of *drawing lots*.

A second method of allocation, applying to equally urgent cases within one category, consists of *rationing*. Students of equal ability may receive scholarship of equal amounts, at a level determined by total financial means available. In times of serious depression, unemployment and other social benefits may have to be lowered: another sort of rationing. Rationing of individual consumer goods in periods of extreme scarcity is a feature of many emergencies.

A third allocation system may be shifting over time of the availability of goods until circumstances have become more favourable. Another word for such shifting is *queuing* or the establishment of a *waiting list*. Libraries regularly apply this technique. For individual goods such as motor cars waiting lists are well known in Eastern Europe.

As a last case we mention the allocation system of *two-part pricing*. In order to obtain the part-collective good desired the person involved may be required to pay a fixed amount (lump sum) plus a price per unit. This is the system recommended (by Goedhart, 1981, and others) according to the so-called benefit principle. Thus, university students may be charged a teaching fee depending on later incomes: they are given a loan which they have to pay back partly only, depending on future income.

#### 8 A SIMPLE MODEL AS AN ILLUSTRATION

In this final section the interdependence of the elements discussed earlier will be shown with the aid of a simple static short-run model. In it we use the following subscripts:  $i = 1, \dots, I$  for consumers,  $h = 1, \dots, H$  for producing firms and superscripts  $j = 1, \dots, J$  for individual or private goods and  $k = 1, \dots, K$  for (part)collective goods. Since the model is meant for the short run all capital is assumed to be given; so is income  $C_i$  derived from capital by person  $i$ . The explicit version of the model assumes all production to show diminishing returns. The corresponding supply equations for constant or increasing returns indicate supply to be equal to (given) capacity or zero, so as to maximize profit or to opt for satiating production.

The *variables* entering the model and their numbers are listed below.

$x_i^j$	consumption of individual good $j$ by consumer $i$	$IJ$
$y_h^j$	production of individual good $j$ by firm $h$	$HJ$
$p^j$	price of individual good $j$	$J$
$\lambda^i$	Lagrange multiplier for consumer $i$ in maximizing utility $\omega_i$ , constrained by budget equation	$I$
$u_i^k$	use of (part)collective good $k$ by consumer $i$	$IK$
$v_h^k$	production of (part)collective good $k$ by firm $h$	$HK$
$q^k$	price of (part)collective good $k$	$K$
$q_i^k$	tax paid for (part)collective good $k$ by consumer $i$	$IK$
$l_{ix}^j$	quantity of labour supplied by individual $i$ to produce good $j$	$IJ$
$l_{hy}^j$	quantity of labour demanded by form $h$ to produce good $j$	$HI$
$w^j$	wage of labour used in production of good $j$	$J$
$l_{iu}^k$	quantity of labour supplied by individual $i$ to produce good $k$	$IK$
$l_{hv}^k$	quantity of labour demanded by form $h$ to produce good $k$	$HK$
$w^k$	wage of labour used in production of good $k$	$K$

Total number of (unknown) variables:  $I + 2J + 2K + 2HJ + 2HK + 2IJ + 3IK$

The *equations* of the model are:

Budget equation for individual  $i$ :

$$-\sum_j p^j x_i^j - \sum_k q_i^k (u_i^k) + \sum_j l_{ix}^j w^j + \sum_k l_{iu}^k w^k + C_i = 0 \quad (1) \quad I$$

$$\text{Demand by } i \text{ for } j: \frac{\partial \omega_i}{\partial x^j} = \lambda_i p^j \quad (2) \quad IJ$$

$$\text{Demand by } i \text{ for } k: \frac{\partial \omega_i}{\partial u_i^k} = \lambda_i \frac{\partial q_i^k}{\partial u_i^k} \quad (3) \quad IK$$

$$\text{Supply of labour by } i \text{ for } j: \frac{\partial \omega_i}{\partial l_{ix}^j} = -\lambda_i w^j \quad (4) \quad IJ$$

$$\text{Supply of labour by } i \text{ for } k: \frac{\partial \omega_i}{\partial l_{iu}^k} = -\lambda_i w^k \quad (5) \quad IK$$

$$\text{Budget equations for government: } \sum_i q_i^k = q^k \sum_h v_h^k \text{ (Cases 1,2)} \quad (6) \quad K$$

$$\text{Budget equation for government: } \sum_k \sum_i q^k = \sum_k q^k \sum_h v_h^k \quad (\text{Cases 3,4}) \quad (6) \quad J$$

$$\text{Supply of indiv. good } j: y_h^j = a_h^j (l_{hv}^j)^{\lambda_h^j} \quad (7) \quad HJ$$

$$\text{Supply of (part) coll. good } k: v_h^k = b_h^k (l_{hv}^k)^{\mu_h^k} \quad (8) \quad HK$$

$$\text{Demand for labour to produce good } j: \lambda_h^j a_h^j (l_{hy}^j)^{\lambda_h^j - 1} = w^j \quad (9) \quad HJ$$

$$\text{Demand for labour to produce good } k: \mu_h^k b_h^k (l_{hv}^k)^{\mu_h^k - 1} = w^k \quad (10) \quad HK$$

$$\text{Market equilibrium for good } j: \sum_i x_i^j = \sum_h y_h^j \quad (11) \quad J$$

$$\text{Availability of (purely) collective good } k: u_i^k = \sum_h v_h^k \quad (\text{Cases 1,3}) \quad (12) \quad IK$$

Availability of part-collective good  $k$ :

$$\left( \sum_h v_h^k \right)^{-\rho} = \sum_i (u_i^k)^{-\rho} \quad (\text{Cases 2,4}) \quad (13) \quad K$$

$$(-\infty < \rho < -1)$$

As will be understood, the utility functions  $u_i$ , the production functions and the tax functions  $q_i^k (u_i^k)$  are considered to be given.

Market equilibrium for labour producing good  $j$ :

$$\sum_i l_{ix}^j = \sum_h l_{hy}^j \quad (14) \quad J$$

Market equilibrium for labour producing good  $k$ :

$$\sum_i l_{iu}^k = \sum_h l_{hv}^k \quad (15) \quad K$$

As shown for equations (6) and (12) we have distinguished four cases characterized as follows: apart from the individual goods we have in:

Case 1. Only purely collective goods, each of them financed by special taxes

Case 2. Only part-collective goods, each of them financed by special taxes

Case 3. Only purely collective goods, financed out of total govt. budget

Case 4. Only part-collective goods, financed out of total govt. budget

It will be understood that 'mixed' cases can be easily modelled, by combining features of the alternative forms of equations (6) or (12) or both. In the four cases shown the numbers of equations differ as follows:

- Case 1.  $I + 2J + 2K + 2HJ + 2HK + 2IJ + 3IK$   
 Case 2.  $I + 2J + 3K + 2HJ + 2HK + 2IJ + 2IK$   
 Case 3.  $1 + I + 2J + K + 2HJ + 2HK + 2IJ + 3IK$   
 Case 4.  $1 + I + 2J + 2K + 2HJ + 2HK + 2IJ + 2IK$

Consequently in some cases a number of *degrees of freedom* results, to be found as the difference between the number of variables and of equations. They appear to be  $K(I - 1)$  in Case 2,  $K - 1$  in Case 3 and  $IK - 1$  in Case 4. Case 1 appears to be determinate. If degrees of freedom are available, they may be used by the government to impose restrictions on the taxes in order to maximize, as far as possible, some collective utility function. Such additional equations may also specify a particular *system of allocation*, supposed by parliament to be optimal.

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*Summary*

## ON COLLECTIVE AND PART-COLLECTIVE GOODS

Part-collective goods are defined as a continuum between private and purely collective goods. Their degree of collectivity is indicated by a parameter as used in a special case of the CES production function. Factor inputs in the latter play the role of quantities available to individuals of the collective good; product in the production function corresponds with total quantity available of the collective good. Parameter values range from  $-1$  (private goods) to  $-\infty$  (purely collective goods). External data determine the varying parameter values of a given partly collective good. The latter are in need of an allocation system.