Stationarity and Invertibility of a Dynamic Correlation Matrix*

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Abstract

One of the most widely-used multivariate conditional volatility models is the dynamic conditional correlation (or DCC) specification. However, the underlying stochastic process to derive DCC has not yet been established, which has made problematic the derivation of asymptotic properties of the Quasi-Maximum Likelihood Estimators (QMLE). To date, the statistical properties of the QMLE of the DCC parameters have purportedly been derived under highly restrictive and unverifiable regularity conditions. The paper shows that the DCC model can be obtained from a vector random coefficient moving average process, and derives the stationarity and invertibility conditions of the DCC model. The derivation of DCC from a vector random coefficient moving average process raises three important issues, as follows: (i) demonstrates that DCC is, in fact, a dynamic conditional covariance model of the returns shocks rather than a dynamic conditional correlation model; (ii) provides the motivation, which is presently missing, for standardization of the conditional covariance model to obtain the conditional correlation model; and (iii) shows that the appropriate ARCH or GARCH model for DCC is based on the standardized shocks rather than the returns shocks. The derivation of the regularity conditions, especially stationarity and invertibility, should subsequently lead to a solid statistical foundation for the estimates of the DCC parameters. Several new results are also derived for univariate models, including a novel conditional volatility model expressed in terms of standardized shocks rather than returns shocks, as well as the associated stationarity and invertibility conditions.

Keywords: Dynamic conditional correlation, dynamic conditional covariance, vector random coefficient moving average, stationarity, invertibility, asymptotic properties.

JEL classifications: C22, C52, C58, G32.

1. Introduction

Among multivariate conditional volatility models, the dynamic conditional correlation (or DCC) specification of Engle (2002) is one of the most widely used in practice. Both multivariate conditional correlations and the associated conditional covariance models, are very useful for determining optimal hedging strategies, volatility spillovers and causality in volatility among financial commodities. These issues are especially important in energy finance, where the relationships among fossil fuels, such as oil, coal and gas, and carbon emissions, are crucial for public and private policy making (see, for example, Chang and McAleer (2017), Chang, McAleer and Tansuchat (2011), and Chang, McAleer and Wang (2017)).

The two alternative models that have been used widely for estimating and forecasting multivariate conditional correlations and conditional covariances have been based on: (i) the diagonal and full BEKK models of Baba et al. (1985) and Engle and Kroner (1995), which have been derived from an m-dimensional vector random coefficient autoregressive process (see McAleer et al. (2008) and section 2 below)); and (ii) the DCC model, which was presented without an underlying stochastic specification in Engle (2002).

The basic DCC modelling approach has been as follows: (i) estimate the univariate conditional variances using the GARCH(1,1) model of Bollerslev (1986), which are based on the returns shocks; and (ii) estimate what is purported to be the conditional correlation matrix of the standardized residuals. The first step is entirely arbitrary as the conditional variances could just as easily be based on the standardized residuals themselves, as will be shown in Section 4 below.

A similar comment applies to the varying conditional correlation model of Tse and Tsui (2002), where the first stage is based on a standard GARCH(1,1) model using returns shocks. The second stage is slightly different from the DCC formulation as the conditional correlations are defined appropriately. However, no regularity conditions are presented, and hence no statistical properties are given.

The DCC model has been analyzed critically in a number of papers as its underlying stochastic process has not yet been established, which has made problematic the derivation of the asymptotic properties of the Quasi-Maximum Likelihood Estimators (QMLE). To date, the statistical properties of the QMLE of the DCC parameters have been derived under highly restrictive and unverifiable regularity conditions, which in essence amounts to proof by assumption.

This paper shows that the DCC specification can be obtained from a vector random coefficient moving average process, and derives the conditions for stationarity and invertibility of the DCC model. The derivation of regularity conditions should subsequently lead to a solid statistical foundation for the estimates of the DCC parameters.

The derivation of DCC from a vector random coefficient moving average process raises three important issues: (i) demonstrates that DCC is, in fact, a dynamic conditional covariance model of the returns shocks rather than a dynamic conditional correlation model; (ii) provides the motivation, which is presently missing, for standardization of the conditional covariance model to obtain the conditional correlation model; and (iii) shows that the appropriate ARCH or GARCH model for DCC is based on the standardized shocks rather than the returns shocks.

The remainder of the paper is organized as follows. In Section 2, the standard ARCH model is derived from a random coefficient autoregressive process to provide a background for the remainder of the paper. The multivariate counterpart of ARCH is derived from a vector random coefficient autoregressive process, which will explain intuitively how the univariate results of Marek on a random coefficient moving average process can be extended to an m-dimensional vector counterpart. In Section 3, the DCC model is presented and discussed. Section 4 presents and discusses a new vector random coefficient moving average process that will be used as an underlying stochastic process in order to derive DCC. Several new results are derived for the associated univariate models, including a novel conditional volatility model expressed in terms of standardized shocks rather than returns shocks, as well as the associated stationarity and invertibility conditions. In section 5, DCC is demonstrated to be derived from the vector random coefficient moving average process. The conditions for stationarity and invertibility of DCC are derived in Section 6. Some concluding comments are given in Section 7.

2. Random Coefficient Autoregressive Process

This section presents the underlying stochastic autoregressive processes for univariate and multivariate GARCH processes, as compared with the multivariate moving average process for the multivariate DCC process in the following section. Consider the following random coefficient autoregressive process of order one:

$$\varepsilon_t = \phi_t \varepsilon_{t-1} + \eta_t \tag{1}$$

where

$\phi_t \sim iid(0, \alpha)$,

$\eta_t \sim iid\ (0, \omega)$, independent of $\{\phi_t\}$.

The ARCH(1) model of Engle (1982) can be derived as (see Tsay (1987)):

$$h_t = E(\varepsilon_t^2 \mid I_{t-1}) = \omega + \alpha \varepsilon_{t-1}^2 . \tag{2}$$

where h_t is conditional volatility, and I_{t-1} is the information set at time *t-1*. The use of an infinite lag length for the random coefficient autoregressive process leads to the GARCH model of Bollerslev (1986).

The diagonal and full BEKK models of Baba et al. (1985) and Engle and Kroner (1995), though not the Hadamard BEKK and full BEKK models, can be derived from a vector random coefficient autoregressive process (see McAleer et al. (2008)). As the statistical properties of vector random coefficient autoregressive processes are well known, the statistical properties of the parameter estimates of the ARCH, GARCH, and diagonal BEKK models are straightforward to establish.

3. DCC Specification

This section presents the DCC model, as given in Engle (2002), which does not have an underlying stochastic specification that leads to its derivation. Let the conditional mean of financial returns be given as:

$$y_t = E(y_t \mid I_{t-1}) + \varepsilon_t \tag{3}$$

where $y_t = (y_{1t,...,} y_{mt})'$, $y_{it} = \Delta \log P_{it}$ represents the log-difference in stock prices (P_{it}), i = 1,…,m, I_{t-1} is the information set at time *t-1*, and ε_t is conditionally heteroskedastic. Without distinguishing between dynamic conditional covariances and dynamic conditional correlations, Engle (2002) presented the DCC specification as:

$$Q_t = (1-\alpha-\beta)\overline{Q} + \alpha\eta_{t-1}\eta_{t-1}^{'} + \beta Q_{t-1} \tag{4}$$

where $\overline{Q}$ is assumed to be positive definite with unit elements along the main diagonal, the scalar parameters are assumed to satisfy the stability condition, $\alpha + \beta$ < 1, the standardized shocks, $\eta_t = (\eta_{1t,...,}\eta_{mt})'$, which are not necessarily *iid*, are given as $\eta_{it} = \varepsilon_{it} / \sqrt{h_{it}}$, and D_t is a diagonal matrix with typical element $\sqrt{h_{it}}$, i = 1,…,m. If m is the number of financial assets, the multivariate definition of the relationship between ε_t and η_t is now given as $\varepsilon_t = D_t\eta_t$.

Define the conditional covariance matrix of ε_t as Q_t. As the $m \times 1$ vector, η_t, is assumed to be *iid* for all *m* elements, the conditional correlation matrix of η_t is given by Γ_t. Therefore, the conditional expectation of the covariance matrix of ε_t is defined as:

$$Q_t = D_t \Gamma_t D_t . \tag{5}$$

Equivalently, the conditional correlation matrix, Γ_t, can be defined as:

$$\Gamma_t = D_t^{-1} Q_t D_t^{-1} . \tag{6}$$

Equation (5) is useful if a model of Γ_t is available for purposes of estimating Q_t, whereas equation (6) is useful if a model of Q_t is available for purposes of estimating Γ_t.

In view of equations (5) and (6), as the matrix in equation (4) does not satisfy the definition of a correlation matrix, Engle (2002) uses the following standardization:

$$R_t = (diag(Q_t))^{-1/2} Q_t (diag(Q_t))^{-1/2} \tag{7}$$

There is no clear explanation given in Engle (2002) for the standardization in equation (7) or, more recently, in Aielli (2013), especially as it does not satisfy the definition of a correlation matrix, as given in equation (6). The standardization in equation (7) might make sense if the matrix Q_t were the conditional covariance matrix of ε_t or η_t, though this is not made clear. It is worth noting that the unconditional covariance matrix of ε_t is not analytically tractable.

Despite the title of the paper, Aielli (2013) also does not provide any stationarity conditions for the DCC model, and does not mention invertibility. Indeed, in the literature on DCC, it is not clear whether equation (4) refers to a conditional covariance or a conditional correlation matrix, although the latter is presumed. Some caveats regarding DCC are given in Caporin and McAleer (2013).

4. Vector Random Coefficient Moving Average Process

The random coefficient moving average process will be presented in its original univariate form in section 4.1, as in Marek (2005), with an extension to its multivariate counterpart in section 4.2, in order to derive the univariate and multivariate conditional volatility models, respectively.

4.1 Univariate process

In an interesting paper, Marek (2005) proposed a linear moving average model with random coefficients (RCMA), and established the conditions for stationarity and invertibility. In this section, we extend the univariate results of Marek (2005) using an m-dimensional vector random coefficient moving average process of order *p*, which is used as an underlying stochastic process to derive the DCC model, and prove the stationarity and invertibility conditions. Several new results are also derived for the associated univariate models, including a novel conditional volatility model expressed in terms of standardized shocks rather than returns shocks, as well as the associated stationarity and invertibility conditions.

Consider a univariate random coefficient moving average process given by:

$$\varepsilon_t = \theta_t \eta_{t-1} + \eta_t \tag{8}$$

where $\eta_t \sim iid\ (0,\omega)$. The sequence $\{\theta_t\}$ is supposed to be independent of $\eta_{t-1}, \eta_t, \eta_{t+1}, \ldots$, which is called the Future Independence Condition, with a mean zero and variance α. It is also assumed to be measurable with respect to I_t, where I_t is the information set generated by the random variable, { $\varepsilon_{t,}\varepsilon_{t-1,...}$ }. Furthermore, it is assumed that the process { ε_t } is stationary and invertible, such that $\eta_t \in I_t$. For further details, see Marek (2005).

Without the measurability assumption on $\{\theta_t\}$ it would be difficult to obtain results on the invertibility of the model. However, an important special case of the model arises when $\{\theta_t\}$ is *iid*, that is, not measurable with respect to I_t , in which case the conditional and unconditional expectations of ε_t are zero, and the conditional variance of ε_t is given by:

$$h_t = E(\varepsilon_t^2 \mid I_{t-1}) = \omega + \alpha\eta_{t-1}^2 \qquad (9)$$

which differs from the ARCH(1) model in equation (2) in that the returns shock is replaced by the standardized shock. This is a new result in the conditional volatility literature.

As η_t ~ *iid* $(0,\omega)$, the unconditional variance of ε_t is given as:

$$E(h_t) = (1+\alpha)\,\omega.$$

The use of an infinite lag length for the random coefficient moving average process in equation (8), with appropriate restrictions on θ_t , would lead to a generalized ARCH model that differs from the GARCH model of Bollerslev (1986) as it would replace the returns shock with a standardized shock.

The univariate ARCH(1) model in equation (9) is contained in the family of GARCH models proposed by Hentschel (1995), and the augmented GARCH model class of Duan (1997).

It can be shown from the results in Marek (2005) that a sufficient condition for stationarity is that the vector sequence $\upsilon_t = (\eta_t, \theta_t\eta_{t-1})'$ is stationary. Moreover, by Lemma 2.1 of Marek (2005), a new sufficient condition for invertibility is that:

$$E\left[\log\left|\theta_t\right|\right] < 0 . \tag{10}$$

The stationarity of $\upsilon_t = (\eta_t, \theta_t\eta_{t-1})'$ and the invertibility condition in equation (10) are new results for the univariate ARCH(1) model given in equation (9), as well as its direct extension to GARCH models.

4.2 Multivariate process

Extending the analysis given above to the multivariate case and to a vector random coefficient moving average (RCMA) model of order *p*, we can derive a special case of DCC(*p*,*q*), namely DCC(*p*,*0*), as follows:

$$\varepsilon_t = \sum_{j=1}^{p} \theta_{jt}\eta_{t-j} + \eta_t \tag{11}$$

where ε_t and η_t are both $m \times 1$ vectors and θ_{jt} , j = 1,…,*p* are random $m \times m$ matrices, independent of $\eta_{t-1}, \eta_t, \eta_{t+1}, \ldots$. Under Assumption 1, it is possible to derive the conditional covariance matrix of ε_t in equation (11):

Assumption 1:

(i) $E(\eta_t \mid I_{t-1}) = 0$, $E(\eta_t\eta_t' \mid I_{t-1}) = \Omega$.

(ii) The random coefficient matrices θ_{jt} have the following properties: For all *j=1,…,p*, *t=1,…,T*, it is assumed that: $E(\theta_{jt} \mid I_{t-1})$ = 0, $E(\theta_{jt,kl}\theta_{jt,mn}' \mid I_{t-1}) = A_{j,kl}A_{j,mn}'$, for appropriate matrices $A_{j,kl}$ and $A_{j,mn}$ that form the conditional covariance matrix of θ_{jt} , and $E(\theta_{jt,kl}\theta_{is,mn}' \mid I_{t-1}) = 0$, $i \neq j$, and/or $s \neq t$.

This is similar to Proposition 1 of McAleer et al. (2008) in that the conditional covariance matrix is given by

$$H_t = E(\varepsilon_t \varepsilon_t' | I_{t-1}) = \Omega + \sum_{j=1}^{p} A_j \eta_{t-j} \eta_{t-j}' A_j'$$

$$E(vec(H_t)) = \left(I_m + \sum_{j=1}^{p} A_j \otimes A_j \right) vec(\Omega) .$$

This approach can easily be extended to include autoregressive terms. For example, in a model analogous to GARCH(*p*,*q*), namely:

$$H_t = \Omega + \sum_{i=1}^{p} A_i \eta_{t-i} \eta_{t-i}' A_i' + \sum_{j=1}^{q} B_j H_{t-j} B_j'$$

where the parameter matrices B_j are such that the maximum eigenvalue of $\sum_{j=1}^{q} B_j \otimes B_j$ is smaller than one in modulus, it follows that:

$$E(vec(H_t)) = \left(I_m - \sum_{j=1}^{q} B_j \otimes B_j \right)^{-1} \left(I_m + \sum_{j=1}^{p} A_j \otimes A_j \right) vec(\Omega) .$$

The derivation given above shows that, as compared with the standard DCC formulation, which is not based on an underlying stochastic process that leads to its derivation, the formulation given above permits straightforward computation of the unconditional variances and covariances via the derived models in equations .

It should also be noted that in Aielli's (2013) variation of the standard DCC model, it is possible to calculate the unconditional expectation of the Q_t matrix, as in equation (4), but this is not equal to the unconditional covariance matrix of ε_t , which is analytically intractable. This is an additional advantage of using the vector random coefficient moving average process given in the above equations, as will be shown explicitly in the following section

5. One Line Derivation of DCC

In this section, the DCC model will be derived from a vector random coefficient moving average process as the underlying stochastic process. If θ_{jt} in equation (11) is given as:

$\theta_{jt} = \lambda_{jt} I_m$, with $\lambda_{jt} \sim iid(0, \alpha_j)$, $j = 1, \ldots, p$,

where λ_{jt} is a scalar random variable, then the conditional covariance matrix can be shown to be:

$$H_t = E(\varepsilon_t \varepsilon_t' \mid I_{t-1}) = \Omega + \sum_{j=1}^{p} \alpha_j \eta_{t-j} \eta_{t-j}' \ . \qquad (12)$$

The DCC model in equation (4) is obtained by letting $p \to \infty$ in equations (11) and (12), setting $\alpha_j = \alpha \beta^{j-1}$, and standardizing H_t in equation (12) to obtain a conditional correlation matrix. For the case $p = 1$ in equation (12), the appropriate univariate conditional volatility model is given in the new model in equation (9), which uses the standardized shocks, rather than standard ARCH in equation (2), which uses the returns shocks.

The derivation of DCC in equation (12) from a vector random coefficient moving average process is important as it: (i) demonstrates that DCC is, in fact, a dynamic conditional covariance model of the returns shocks rather than a dynamic conditional correlation model; (ii) provides the motivation, which is presently missing, for standardization of the conditional covariance model to obtain the conditional correlation model; and (iii) shows that the appropriate ARCH or GARCH model for DCC is be based on the standardized shocks rather than the returns shocks.

It is worth noting that the derivation of the DCC model using the underlying vector random coefficient moving average process is not argued to be unique as the latter has not been shown to be a necessary condition. However, to date there has been no derivation of the DCC model from an underlying stochastic process that leads to its derivation.

6. Derivation of Stationarity and Invertibility of DCC

The formulation of DCC given in the previous section is more natural than the standard treatment as it can be derived from an underlying stochastic process which leads to its derivation, and can be also analyzed in terms of mathematical and statistical properties, such as stationarity, invertibility, and existence of moments.

This section derives the stationarity and invertibility conditions for the DCC model in Theorem 1, based on Assumption 2:

Assumption 2. $E\left[\log\left\|\Theta_{t-k}\right\|\right] < \log\sqrt{pm}$ (13)

where $\left\|\Theta_t\right\|$ is the Frobenius norm, and Θ_t is given by:

$$\Theta_t = \begin{pmatrix} -\theta_{1t} & -\theta_{2t} & \ldots & -\theta_{pt} \\ 1 & 0 & \ldots & 0 \\ . & . & . & . \\ 0 & \ldots & 1 & 0 \end{pmatrix}$$

Theorem 1. A sufficient condition for stationarity is that the vector sequence:

$$\upsilon_t = (\eta_t, \theta_{1t}\eta_{t-1}, \ldots, \theta_{pt}\eta_{t-p})'$$

is stationary. Furthermore, under Assumption 2, the vector random coefficient moving average process, ε_t , is invertible.

Proof: The proof of stationarity is similar to that given above for the univariate random coefficient moving average process. For invertibility, note that:

$$\eta_t = \varepsilon_t - \sum_{j=1}^{p} \theta_{jt} \eta_{t-j}$$

which can be written as:

$$\tilde{\eta}_t = \Theta_t \tilde{\eta}_{t-1} + \tilde{\varepsilon}_t$$

where

$$\tilde{\eta}_t = (\eta_t, \eta_{t-1}, ..., \eta_{t-p+1})' \text{ and } \tilde{\varepsilon}_t = (\varepsilon_t, \varepsilon_{t-1}, ..., \varepsilon_{t-p+1})' .$$

Hence,

$$\tilde{\eta}_t = \sum_{j=0}^{n-1} \left(\prod_{k=1}^{j} \Theta_{t-k+1} \right) \tilde{\varepsilon}_{t-j} + \left(\prod_{k=0}^{n-1} \Theta_{t-k} \right) \tilde{\eta}_{t-n} .$$

Now let:

$$\tilde{\eta}_t^{(n)} = \sum_{j=0}^{n} \left(\prod_{k=1}^{j} \Theta_{t-k+1} \right) \tilde{\varepsilon}_{t-j} .$$

Consider

$$\frac{1}{n}\log\frac{1}{\sqrt{pm}}\|\tilde{\eta}_t-\tilde{\eta}_t^n\|=\frac{1}{n}\log\frac{1}{\sqrt{pm}}\left\|\left(\prod_{k=1}^{n-1}\Theta_{t-k}\right)\tilde{\eta}_{t-n}\right\|$$

$$\leq\frac{1}{n}\log\frac{1}{\sqrt{pm}}\left\|\prod_{k=1}^{n-1}\Theta_{t-k}\right\|+\frac{1}{n}\log\frac{1}{\sqrt{pm}}\|\tilde{\eta}_{t-n}\|$$

$$\leq\frac{1}{n}\sum_{k=1}^{n}\log\frac{1}{\sqrt{pm}}\|\Theta_{t-k}\|+\frac{1}{n}\log\frac{1}{\sqrt{pm}}\|\tilde{\eta}_{t-n}\|$$

$$\xrightarrow[a.s.]{}E\log\frac{1}{\sqrt{pm}}\|\Theta_{t-k}\|<0$$

as $E\log\|\Theta_{t-k}\|<\sqrt{pm}$, by assumption. This implies that $\eta_t-\eta_t^n\xrightarrow[a.s.]{}0$ and, hence, η_t is asymptotically measurable with respect to $\{\varepsilon_{t-1},\varepsilon_{t-2},...\}$, and ε_t is invertible. •

The derivation of the sufficient conditions for stationarity and invertibility of the DCC model in Theorem 1 makes it more viable and understandable in practice, and contributes toward a statistical analysis of the model for practical purposes, as discussed in Section 1.

Note that a sufficient condition for equation (13) is that:

$$\sum_{j=1}^{p}E\|\theta_{jt}\|^2<m \tag{14}$$

as $E\log\frac{1}{\sqrt{pm}}\|\Theta_{t-k}\|\leq\log E\frac{1}{\sqrt{pm}}\|\Theta_{t-k}\|$

$$=\log E\frac{1}{\sqrt{pm}}\sqrt{\sum_{j=1}^{p}\|\theta_{jt}\|^2+(p-1)m}$$

$$= \log E\sqrt{\frac{1}{pm}\sum_{j=1}^{p}\left\|\theta_{jt}\right\|^{2} + (p-1)/p}$$

$$\leq \log\sqrt{\frac{1}{pm}\sum_{j=1}^{p}E\left\|\theta_{jt}\right\|^{2} + (p-1)/p}$$

$$< 0 .$$

The condition given in equation (14) may be easier to check in practice than the condition given in equation (13). The simplicity and convenience of equation (13) may be important for the practical implementation of the DCC model.

For the special case $\theta_{jt} = \lambda_{jt} I_m$, with $\lambda_{jt} \sim iid(0, \alpha_j)$, $j = 1, \ldots, p$, discussed in Section 5 above, the condition in equation (14) simplifies to the well-known condition on the long-run persistence to returns shocks, namely:

$$\sum_{j=1}^{p} E\lambda_{jt}^{2} = \sum_{j=1}^{p} \alpha_j < 1 .$$

7. Conclusion

The paper was concerned with one of the most widely-used multivariate conditional volatility models, namely the dynamic conditional correlation (or DCC) specification. As the underlying stochastic process to derive the DCC model has not yet been established, this has made problematic the derivation of the asymptotic properties of the Quasi-Maximum Likelihood Estimators (QMLE). To date, the statistical properties of the QMLE of the DCC parameters have been derived under highly restrictive and unverifiable regularity conditions, in short, proof by assumption.

The paper showed that the DCC specification could be obtained from a vector random coefficient moving average process, and derived the sufficient stationarity and invertibility conditions of the DCC model. The derivation of the regularity conditions should eventually lead to a solid foundation for the statistical analysis of the QMLE estimates of the DCC parameters.

Several new results were also derived for univariate models, including a novel conditional volatility model that was derived from an underlying univariate random coefficient moving average process, and was given in terms of standardized shocks rather than returns shocks, as well as the associated stationarity and invertibility conditions.

The derivation of DCC from the underlying vector random coefficient moving average process demonstrated that DCC is, in fact, a dynamic conditional covariance model of the standardized shocks rather than a dynamic conditional correlation model based on returns shocks, as presumed in Engle (2002). Moreover, the derivation of the DCC model provided the motivation, which is presently missing, for standardizing the conditional covariance model to obtain the conditional correlation model. The standardization of the estimated DCC models in practice does not satisfy the definition of a correlation matrix, which has always been problematic in interpreting the DCC model (see, for example, Caporin and McAleer (2013)).

The derivation of the DCC model also showed that the appropriate ARCH or GARCH model for DCC is based on the standardized shocks rather than the returns shocks. The derivation of regularity conditions should subsequently lead to a solid statistical foundation for the QMLE of the appropriate univariate specifications that underlie the DCC model.

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