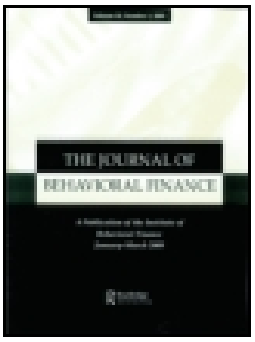

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Do Survey Probabilities Match Financial Market Beliefs?

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ABSTRACT
This article considers whether survey respondents' views regarding the likelihood of stock index returns exceeding specific thresholds are comparable to market views indicated by index options with strikes at analogous thresholds. It is motivated by the observation that the wording used to elicit subjective beliefs in surveys about expected future returns resembles the question a purchaser of a call option might ask. Building on this association, the authors document a similarity between the views of survey respondents and those of financial market participants as measured through call options, although the association is not 1-for-1. They find a closer association for those demonstrating a better understanding of the laws of probability, suggesting that numeracy affects the accuracy of an elicited response.

KEYWORDS
RAND American Life Panel (ALP); Subjective response; Probabilistic assessment; Survey expectations; Option-implied probabilities; Focal points

Introduction

In this article we compare people's beliefs about future stock returns as elicited through surveys with those derived from option prices. In performing this comparison, we contribute to the literature that has documented differences between respondents' stated probabilities and behavioral probabilities in the context of stock returns (e.g., Hurd and Rohwedder [2010], Dominitz and Manski [2011], Barone-Adesi, Mancini, and Shefrin [2013]). When we compare the beliefs of survey respondents with the behavior of financial market participants as revealed through option prices, we find that survey expectations of the general population do seem to reflect the beliefs embedded in option prices and that this similarity is stronger for those with greater probabilistic understanding.

Our results also contribute to the research on the information content of subjective (survey) expectations, the tendency of survey respondents to report focal points (clustering around rounded numbers) when asked probabilistic questions, and the role numeracy and cognition plays in the elicitation of survey responses. Underlying this research is the question of whether surveys provide meaningful information, and there are large literatures that have considered this question in a variety of contexts. Although these areas have all been studied previously in the context of stock returns, no study (to the best of our knowledge) has drawn the connection between subjective expectations of a specified return threshold and corresponding option strike levels, as we do in this article. In considering this connection, we are particularly motivated by the literature on question framing. Question framing has been shown to influence survey responses to even the most basic questions (in the context of a simple 5-point self-assessed health question, see Bowling and Windsor [2008], Jürges, Avendano, and Mackenbach [2008], Lumsdaine and Exterkate [2013]) as well as consumer behavior (e.g., Juster [1966], Schweitzer [1995], Brown, Kapteyn, and Mitchell [2016]). In addition, the role of framing in communication is garnering increasing attention (regarding communication of pension funds to their clients, see Keren [2012]). We therefore consider whether the probabilities reported by survey participants in response to an option-framed question resemble those implied by actual option prices. Evidence of similarity would indicate that respondents seem to understand the question being asked, and would lend validity to the use of surveys to elicit probabilities about financial topics.

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Color versions of one or more of the figures in the article can be found online at www.tandfonline.com/hbhf.

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The article proceeds as follows. The second section contains a brief review of related literature. In the third section we detail the data construction and descriptive statistics of the main variables. In the fourth section we describe the model used in our analyses. In the fifth section we present results. In the final section we conclude. Supplemental material containing information on the sample construction, the construction of financial market beliefs, detailed descriptive statistics, the derivation of the likelihood function, and additional estimation results is provided in a series of Online Appendices.

Background

We are interested in the determinants of people's survey responses regarding the probability of future stock returns and whether these responses resemble the probabilities that are observed in financial markets via option prices, acknowledging that there are 2 opposing views a priori as to what we might find. To some a link would seem obvious; they might argue that (1) financial market fluctuations are merely the aggregate result of individual investor decisions, and (2) management decisions of publicly traded firms reflect the views of the general (shareholder) population. For example, Barone-Adesi et al. [2013] provided a theoretical foundation for this link, noting that both investors' beliefs and option prices can be described by the same underlying pricing kernel with the link between them akin to a change of measure. Even beyond such efficient markets explanations, feedback and herding models in behavioral finance would suggest that people's beliefs are influenced by what happens in the financial markets (e.g., Hirshleifer [2001], Daniel, Hirshleifer, and Teoh [2002], Shiller [2003]). In contrast, to others a link would be surprising, given evidence of low financial literacy rates (Lusardi and Mitchell [2011]), the link between survey measurement and cognition in general (Schwarz [2007]) and specifically evidence that the ability to form probabilities about stock returns is related to cognitive ability (Binswanger and Salm [2014]), the fact that few Americans hold stocks outside of a retirement portfolio (Poterba and Samwick [1995]) and the reliance on heuristics in forming responses to survey questions (Krosnick [1991]).

As a result, for the most part, many who have documented a link between survey expectations and returns emphasize the importance of eliciting expectations from financial market participants rather than the general population as the two populations may differ (see Mankiw, Reis, and Wolfers [2004], Bacchetta, Mertens, and van Wincoop [2009], Hoffman, Post and Pennings [2013, 2015]). But even among the subset of the population that is active in financial markets, others have found evidence that not all participants are informed (e.g., De Long, Shleifer, Summers, and Waldmann [1990]) and that for a variety of reasons, returns of subgroups of investors often differ systematically (e.g., Barber and Odean [2000], Coval, Hirshleifer, and Shumway [2005]). So in extending our inquiry to survey expectations from a representative sample of the U.S. population it is not at all apparent whether their responses will reflect financial market participants' views as measured by financial prices, and if so, to what extent. Additionally, most comparisons between beliefs about future events elicited through surveys of the general population have been with historical stock returns (e.g., Dominitz and Manski [2011], Hurd and Rohwedder [2012]), which by their nature are backward looking.[1] We instead use option prices for comparison, as by construction they are forward-looking. Therefore, they may prove to have a stronger link to survey beliefs than that previously found using returns.[2]

Researchers using survey data to elicit expectations about future equity returns find substantial heterogeneity across individuals (Brennan, Cao, Strong, and Xu [2005], Ben-David, Graham, and Harvey [2010], Dominitz and Manski [2011], Hudomiet, Kézdi, and Willis [2011]), whether those surveys cover professional forecasters or members of the general population. This heterogeneity has been linked to a variety of demographic characteristics (e.g., race, gender, education), financial knowledge or experience, differences in susceptibility to behavioral biases such as probability weighting, (over)optimism and the disposition effect (Shefrin and Statman [1985], Weber and Camerer [1998], Odean [1998], Hens and Vlcek [2011]), and many other explanations. Heterogeneity in expectations has in turn been used to explain heterogeneous equity investment decisions (Kézdi and Willis [2003, 2011]).

There is also substantial evidence in nonfinance contexts that survey responses do not exactly align with true probabilities (Viscusi and Hakes [2003])—for example, due to large clusters of responses occurring at focal points of the response distribution (e.g., Dominitz and Manski [1997], Hurd, McFadden, and Gan [1998], Kleinjans and Van Soest [2010], Manski and Molinari [2010])—and that adjustments to survey data to account for such aspects are necessary to improve inference (Bassett and Lumsdaine [2000], Lillard and Willis [2001]).

Beyond studying the relationship between survey expectations and subsequent realizations, a number of researchers consider the inclusion of survey expectations in models of economic behavior (for a survey of this literature, see Manski [2004]) and demonstrate that including probabilistic expectations can improve inference about economic behavior relative to models using only data on economic choices (revealed preference models).

As noted previously, in the context of equity returns, most research using survey expectations has focused on the views of "informed" investors (i.e., those that are active in the financial markets) and hence are less likely to suffer from psychological biases in the evaluation of probabilities. Three important exceptions are Hurd and Rohwedder [2012], who used the same data we consider to identify correlations between survey expectations and subsequent equity returns, and Dominitz and Manski [2011] and Greenwood and Shleifer [2014], both of whom used the University of Michigan survey of consumers (the latter also consider a number of investor-based surveys). To the best of our knowledge, none of the previous literature has used option prices in conjunction with survey responses to compare these views.

Data

We use the American Life Panel (ALP) for our analysis, an internet panel administered by the RAND Corporation. Since its start in 2003, it has expanded from about 500 to over 6,000 panel members, drawn from the University of Michigan Monthly Survey and other sources. The ALP contains around 500 researcher-contributed survey modules, the responses of which are publicly available. Demographic characteristics of respondents are available through the Household information module, which is updated on a quarterly basis. Sampling weights are assigned such that the weighted distribution is representative of the U.S. population with regard to sociodemographic variables.[3] Throughout this article, sampling weights are used when reporting descriptive statistics and regression results.[4]

While some of the survey modules are stand-alone, others belong to periodically repeated series (waves) on the same topic. This article uses responses obtained from modules designed by Michael Hurd and Susann Rohwedder to investigate the effects of the financial crisis on American households, gathered from November 5, 2008 until March 10, 2011, corresponding to 25 waves of information. For each module, participants are invited to fill out the survey within a certain time frame.

Hurd and Rohwedder [2010] provided a detailed description of this series of modules; it is briefly summarized here. The first wave asks respondents about a wide range of topics such as labor force status, stock ownership, mortgage payments, and expectations about the future. Through linkage with the Household Information module, each module also contains demographic control variables such as age, race, gender, marital status, and education. The final sample (after adjustments for, e.g., missing observations) consists of 47,488 surveys from 2,652 respondents (94.9% of the total number of surveys and 98.3% of the total number of respondents) gathered over 364 survey days. The sample construction is further detailed in Supplemental Online Appendix A.

Survey expectations about stock market returns

The ALP elicits expectations about stock market returns from survey participants via a series of questions, the first of which is the following:

> "We are interested in how well you think the economy will do in the future. On a scale from 0% to 100% where 0 means that you think there is absolutely no chance, and 100 means that you think the event is absolutely sure to happen, what are the chances that by next year at this time mutual fund shares invested in blue chip stocks like those in the Dow Jones Industrial Average will be worth more than they are today?"

Respondents can give an answer ranging from 0 to 100 (the answer need not be an integer) to indicate the percentage chance of the event happening, or they can leave the response blank.

The same structure is repeated for two additional questions, asking respondents to assess the chances of a greater than 20% return and a greater than −20% return.[5] For expositional ease, the questions referring to the probability of a positive return, a more than 20% return, and a more than −20% return will be referred to as *PositiveReturn*, *>Plus20*, and *>Minus20*, respectively. Using all 3 questions (when available) from the 47,488 surveys yields a total sample size for studying survey-generated probabilities of 139,327 observations.

The phrasing of these questions may lead to differences in respondents' interpretation and hence the answers they give, since there is an implicit subjectivity associated with respondents' understanding of "mutual fund shares" or "blue chip stocks like those in the Dow Jones Industrial Average." For the purposes of this article, however, we assume that the responses given represent respondents' subjective probability that the nominal (not inflation-adjusted) level of the Dow Jones Industrial Average (DJIA) in 1 year will have increased (similarly, will have increased by more than 20% or more than −20%) versus the current level of the DJIA. For each respondent, the current level of the index is assumed to be the closing level on the most recent business date before the date of interview, so that the response is assured to chronologically follow the information on which the financial market prices are based.

Figure 1 shows a histogram of the frequency of specific responses to each of the three probabilistic questions individually, as well as all three combined ("Aggregate"). Most of the responses are integers—only 41 of 139,327 are non-integers. That respondents appear to favor round numbers, leading to clusters of responses at certain values, is a

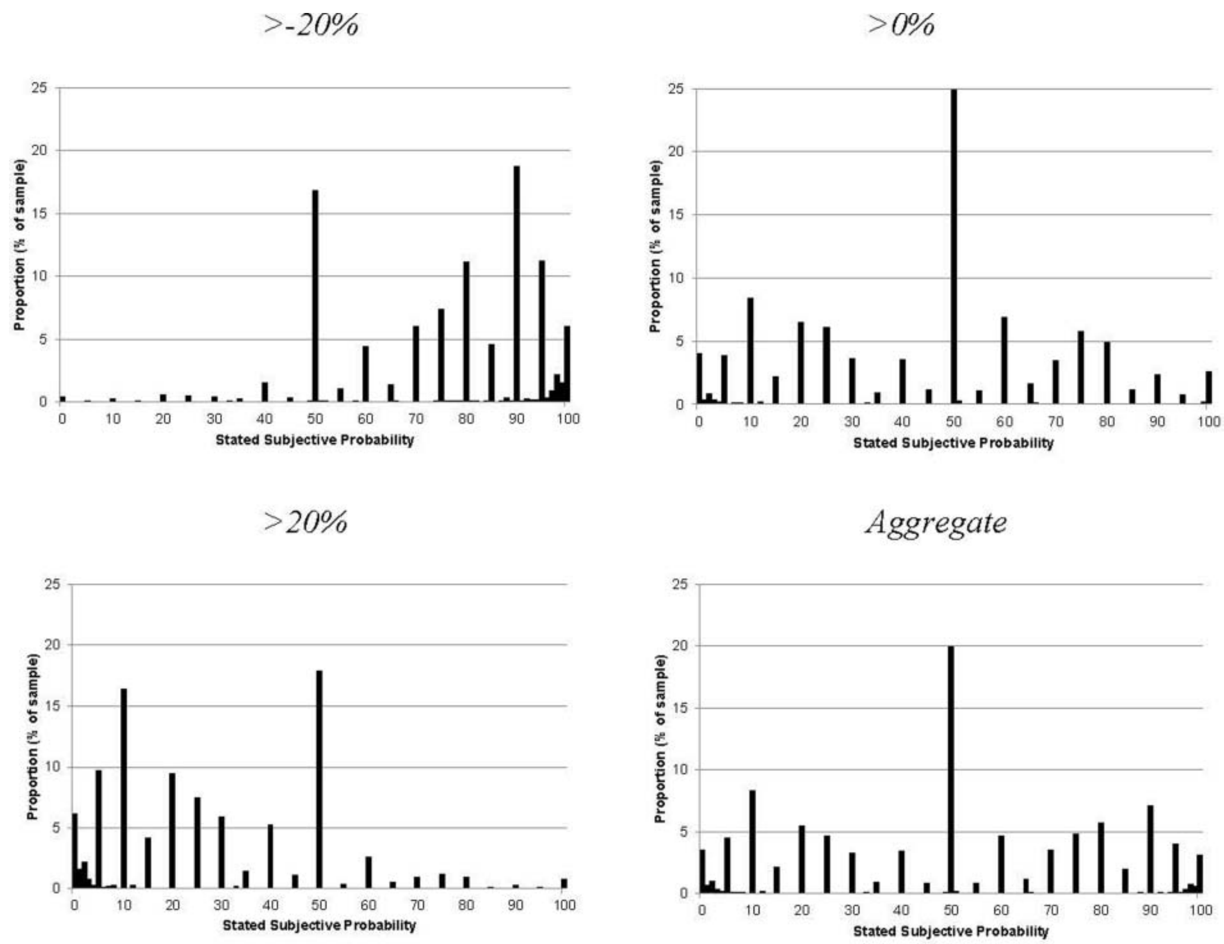

Figure 1. Frequency of responses to probabilistic questions. *Note*: These figures contain histograms of the responses to the 3 questions that ask respondents to consider the probability of a more than −20% return, a positive return, and a more than 20% return, as well as for all 3 questions combined ("Aggregate"). The responses to the 3 questions are called >*Minus20*, *PositiveReturn*, and >*Plus20*, respectively, and together comprise the dependent variable. The figures document the large pile-up of responses at focal points, particularly around the response of "50," motivating the econometric model. Observations in these figures are unweighted.

common occurrence in responses to probabilistic subjective questions.[6] For the 3 questions in this article, 93.8% of person-wave responses are a multiple of 5 and 68.0% are a multiple of 10. A response of 50 occurs 19.9% of the time; in addition, 3.5% are 0 and 3.1% are 100. In addition, 63.0% of the 8,701 responses that are not multiples of 5 are between 0 and 5 or between 95 and 100.

Calculating Option-Implied Probabilities

The wording and return thresholds (–20%, 0%, 20%) given in the 3 ALP questions correspond to strike price levels of a European call option with 1-year maturity, namely the 20% in-the-money, at-the-money, and 20% out-of-the-money thresholds. For each day that a survey was answered (364 days in total), we therefore use Bloomberg to derive analogous probabilities from option prices for comparison to those reported by the respondents in the ALP. The details of this construction are explained extensively in Supplemental Online Appendix B and hence only summarized briefly here. While we recognize that there are numerous ways to derive such probabilities, in this article we adopt a fairly basic approach so as not to obscure the main question of interest (the degree of relationship between the two sets of beliefs).[7] In the standard Black-Scholes model, the price at time t of a European call option with a strike price of K and an expiry date of T > t is given by:

$$\Phi(d_1)S_t e^{-q(T-t)} - \Phi(d_2)Ke^{-r(T-t)} \tag{1}$$

where

$$d_1 = \frac{\ln\left({}^{S_t}/_{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}, \quad d_2 = d_1 - \sigma\sqrt{T-t} \tag{2}$$

The interest rate (r) is the (continuous) U.S. dollar swap rate over the period $[t,T]$ (which is the default rate for option price calculations in Bloomberg), the dividend rate (q) is the Bloomberg forecast for the DJIA dividend rate during the same period, the volatility (σ) is the implied volatility corresponding to each specified strike price (relative to the spot price S_t) and a time to expiration $T - t$.

For each survey day t, the daily implied volatilities are determined by Bloomberg based on option prices assuming a Black-Scholes option pricing kernel. For those strike prices and times to expiration for which options on a particular asset are available (i.e., traded), corresponding implied volatilities can be derived. From these, implied volatilities for other combinations of strike prices and times to expiration can be estimated (details are given in Online Appendix B).[8] In particular, implied volatilities for a time to expiration *(T-t)* of approximately 1 year (i.e., ranging from exactly 1 year in the case that the option is traded and as short as 336 days—that is, 1 year minus 29 days—for the day after an option is traded) and strike prices of 80%, 100%, and 120% of the level of the index at time t were constructed, consistent with the time horizon and return categories articulated in the ALP survey questions and corresponding to the questions *>Minus20*, *PositiveReturn*, and *>Plus20*, respectively.

Comparing survey responses to option price information

To compare the ALP survey responses to the information contained in the option prices, each of the 139,327 survey observations is first assigned a corresponding Option-Implied Probability (OIP) associated with the date before the day the interview was conducted.[9] Specifically, for a given option threshold (i.e., $> -20\%$, $>0\%$, or $> 20\%$) all individuals that were interviewed on a given day are assigned the same OIP, and the number of OIPs assigned to a specific person corresponds to the number of waves in which the person provided a survey response at that threshold. The time series showing the average daily survey responses and the corresponding OIP for each of the three thresholds is shown in Figure 2. In each panel, the 2 pairs of series broadly move together. In general the survey respondents predict a lower probability of a $> -20\%$ return and a higher probability of a $> 20\%$ return than is implied from option prices. Table 1 contains summary statistics for the three sets of probabilities, aggregated across all observations. Not surprisingly, the average probability associated with *>Plus20* is lower than the average probability associated with *PositiveReturn*, which in turn is lower than that associated with *>Minus20*.

For the upper return threshold on the DJIA (the row labeled "*> 20%*"), there is a relatively close correspondence between the average OIP (24.0%) and the average survey probability (27.1%) regarding the expectation that the 1-year return on the DJIA will exceed 20%. There is more of a difference when considering the probability that the DJIA will increase (the row labeled "*> 0%*"), with an average 40.4% survey probability of a positive return versus an OIP average of 57.1% from the corresponding OIPs. There is also a divide between the 2 measures when it comes to the probability of a more than -20% return in the DJIA (the row labeled "*> −20%*"), with the survey responses markedly more pessimistic (the mean of *>Minus20* is 75.8%) than the average OIP (83.0%). If market probabilities are considered objective probabilities, then the survey probabilities are consistent with the overweighting of extreme events (returns), or the overestimation of small probabilities and underestimation of large ones that are often found among respondents (e.g., Hakes and Viscusi [2004]). Further, survey respondents' overall beliefs are more pessimistic than those of the market.

Looking solely at the means across all three strike levels is of course not sufficient to draw conclusions as to whether survey respondents' and the market's beliefs coincide, despite similar patterns that show respondents assigning relatively higher probabilities to large changes in the level of the index. Comparing standard deviations, the survey probabilities inherently have greater variation than can be explained by the variation in OIPs alone, further motivating the need for a formal model that incorporates additional covariates. Standard deviations for the survey responses (20.0%, 26.8%, and 21.6% for *>Minus20, PositiveReturn*, and *>Plus20*, respectively) are very large both as a proportion of the bounded range of 0–100% and in comparison to those derived from option prices (4.9%, 2.9%, and 4.3%, respectively). This is partly a result of the analytical design, as all participants on the same day are assigned the same OIP so that there is no within-day variation in the option price sample.

Model

We use a generalized linear model (for an extensive description, see McCullagh and Nelder [1989]) to model the 3 survey probabilities. Let X represent the matrix of data (covariates) available to the statistician. Respondents' unobserved true belief p^* is assumed to be related to a linear combination of a subset of covariates $X_1 \in X$, through the logistic function,

$$p^* = f^{-1}(X_1\beta_1) = \frac{1}{1 + \exp(-X_1\beta_1)} \qquad (3)$$

where β_i is a vector of parameters corresponding to the columns of X_i, i = 1, 2 (X_2 will be introduced in Equation 4). This function is the most commonly

Figure 2a: *Average daily survey and financial market probability of a >-20% return in one year for the DJIA*

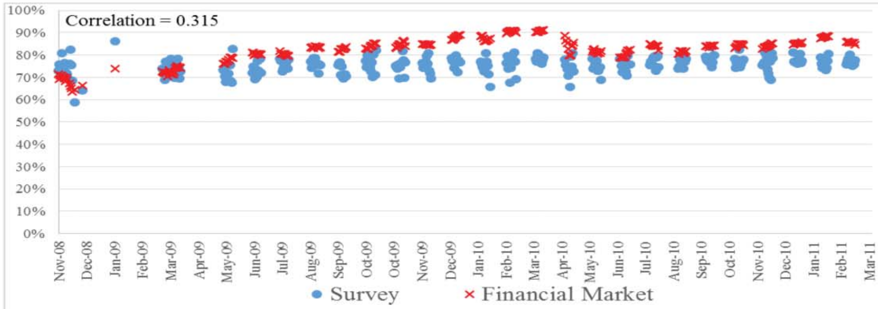

Figure 2b: *Average daily survey and financial market probability of a positive return in one year for the DJIA*

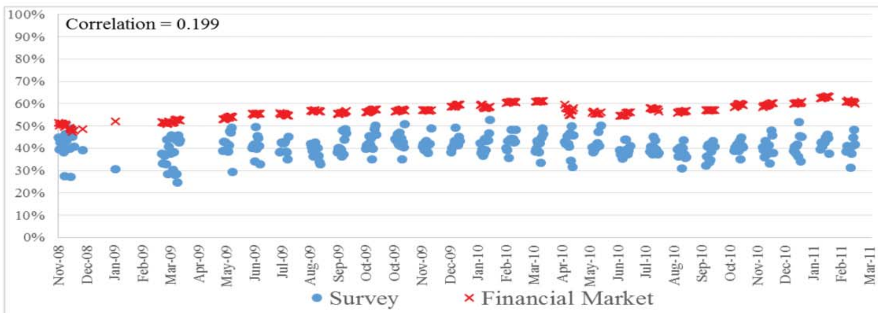

Figure 2c: *Average daily survey and financial market probability of a >20% return in one year for the DJIA*

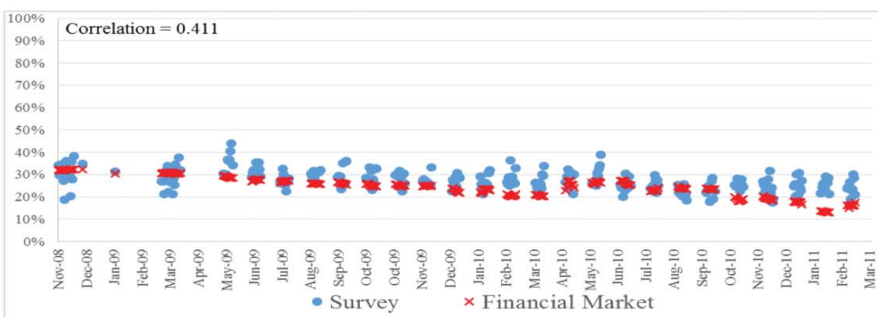

Figure 2. Average daily survey responses and corresponding option-implied probabilities. *Note*: These figures contain scatter plots of the time series of the average daily survey response to each of the 3 main questions, and the corresponding option-implied probability. For each day on which more than 10 survey responses were given, we compute the weighted average of these responses—these are represented by circles in the figures. The corresponding option-implied probability for that day is calculated as described in the Calculating OIPs section and the Supplemental Online Appendix. The 3 figures show the (survey and market) probabilities of a greater than −20% return (>*Minus20*), a positive return (*PositiveReturn*), and a greater than 20% return (>*Plus20*).

Table 1. Descriptive statistics of stated and option-implied probabilities.

Probability of Return	Observations	Financial Market Mean	Financial Market Std. Dev.	Survey Mean	Survey Std. Dev.
> −20%	46,232	0.830	0.049	0.758	0.200
>0%	47,438	0.571	0.029	0.404	0.268
> 20%	45,657	0.240	0.043	0.271	0.216

Note. Summary statistics for the aggregate sample, computed over all person-wave observations. The rows show means and standard deviations of the 3 probabilities used: the probability of a more than −20% return ("*> −20%*"), a positive return ("*>0%*"), and a greater than 20% return ("*> 20%")*. The second column shows the number of person-wave observations, the third and fourth show the means and standard deviations of the financial market beliefs and the last 2 columns show these statistics for the survey beliefs.

used link function for binary data (see Albert and Chib [1993]).

Due to evidence suggesting that survey respondents report their belief with error, we assume that their response is a beta-distributed random variable, $\tilde{p}$, for which $E[\tilde{p}] = p^*$ holds. The beta distribution is well suited for describing probabilities or proportions because it is defined on the unit interval and has a flexible functional form that allows for a wide variety of shapes (e.g., Viscusi and Hakes [2003], Law and Kelton [1982]). It has been used to model survey responses in Bruine de Bruin, Fischbeck, Stiber, and Fischhoff [2002] and earlier by Winkler [1967].

In addition, as noted previously, a substantial proportion of responses are clustered at particular values corresponding to round numbers. We choose to explicitly model the probability of a response of 0, 50, or 100 as a function of observable covariates, for the following reasons. First, a number of articles (e.g., Fischhoff and Bruine de Bruin [1999]) have noted that a response of 50 should be considered as distinct from other responses as it can indicate uncertainty or a lack of probabilistic sophistication of the respondent rather than a true subjective probability of 50%. In addition, a response of 50% may indicate partition priming (e.g., Fox and Rottenstreich [2003], Fox and Clemen [2005]) where open-ended questions such as the ones we study result in a twofold case partition (e.g., the probability the event will happen versus the probability that it will not), inducing the respondent to assign equal probabilities to both the event and its complement. Further, responses of 0 or 100 are special in that they indicate pure certainty of the survey respondent but cannot equal the true probability of an uncertain event occurring. These three responses might therefore reflect a lesser ability on the part of the respondent to express oneself in probabilistic terms. We refer to these responses as focal responses henceforth, and use the approach formulated in Hurd et al. [1998] to model the propensity of giving a focal response via a latent variable w^*:

$$w^* = X_2\beta_2 + \eta \tag{4}$$

with η a normally distributed error term with mean zero and variance 1. A nonfocal answer is given if and only if $w^* < 0$. When the latent variable $w^* \geq 0$, rather than reporting their actual actual $\tilde{p}$ respondents instead give a focal response of zero, 50, or one hundred, depending on where $\tilde{p}$ lies relative to fixed thresholds:

$$\begin{aligned} p &= 0 && \text{if} \quad \tilde{p} \leq \psi_0 \\ p &= 0.5 && \text{if} \quad \psi_0 < \tilde{p} < \psi_1 \\ p &= 1 && \text{if} \quad \tilde{p} \geq \psi_1 \end{aligned} \tag{5}$$

The complete model therefore consists of a system of two equations; we estimate it via maximum likelihood (details of the parameterization of the beta distribution and the likelihood calculation are contained in Supplemental Online Appendix C). It is assumed that the sets of covariates (X_1 and X_2) determining both an individual's true belief and their propensity to give a focal response are the same for all three subjective probability thresholds. We therefore group all three responses together in our estimation but include dummy variables for each response in recognition of the fact that the average response corresponding to each of the three thresholds is different.

Results

We estimate both equations jointly; the results for Equation 3 are reported in Table 2. Because the highly nonlinear nature of the model makes it difficult to interpret the estimated coefficients, we follow the convention for these models and instead report marginal effects and their associated standard errors.[10] In addition, unless otherwise noted, our discussion of the results is with reference to statistical significance at the 95% level of confidence.

The two columns of Table 2 contain estimates associated with the subjective probability assessment p^* (from Equation 3), with the second column containing results of a regression with additional covariates added to the X_1 matrix to account for the level of probabilistic understanding of the respondent (discussed subsequently). The main variable of interest is the OIP, representing the beliefs of financial market participants (described in the Calculating OIPs section). The OIP enters the model in logit form ($f(.)$, the inverse of the logistic function in Equation 3) to be consistent with our treatment of the subjective response (i.e., the dependent variable is also in logit form). Both equations also include demographic controls (i.e., gender, age, race, education, and marital status), dummy variables for the 3 questions and proxies for stock market knowledge (i.e., two self-reported assessments: how closely the respondent follows the stock market and his or her understanding of it). The estimation additionally includes dummy variables for whether the respondent owns stocks or has a retirement account (as proxies for wealth and financial wellbeing); we assume such ownership has no bearing on the likelihood of a focal response.

Subjective probability results

The coefficient on our main variable of interest, the OIP, is statistically significant, suggesting that the views of survey respondents indeed resemble the probabilities implied by option prices. The marginal effect of 0.173 implies that a 10-percentage-point increase in the OIP on average is associated with an increase in survey

Table 2. Regression results, Equation 3.

	Model 1	Model 2
	∂p^*	∂p^*
Option-Implied Probability	0.174	0.250
	(0.014)	(0.014)
Inconsistent		0.088
		(0.003)
Inconsistent * OIP		−0.437
		(0.008)
>*Minus20*	0.398	0.407
	(0.009)	(0.009)
Positive Return	0.077	0.079
	(0.006)	(0.006)
Stock Owner	0.024	0.025
	(0.002)	(0.002)
Has Retirement Account	0.020	0.022
	(0.002)	(0.002)
Closely Following	0.014	0.013
	(0.004)	(0.004)
Somewhat Following	0.028	0.028
	(0.002)	(0.002)
Good Understanding	0.024	0.023
	(0.004)	(0.004)
Medium Understanding	0.009	0.008
	(0.002)	(0.002)
Demographic Controls*	YES	YES
ψ_0	0.114	0.110
	(0.002)	(0.002)
ψ_1	0.831	0.828
	(0.003)	(0.003)
Observations	139,327	139,327
Log Likelihood	−70,125	−66,969

Note. Marginal effects are reported, with standard errors in parentheses. All coefficients are significant at the 5% level. All variables except the Option Implied Probability are dummy variables. Model 1 shows the baseline results; model 2 additionally considers the effect of probabilistic understanding by including a dummy variable equal to 1 if a person provided inconsistent responses and an interaction term between the option implied probability and that variable. Sampling weights are used.
*Demographic controls are gender, age, race, education, and marital status.

respondents' probability by 1.73 percentage points. Survey respondents' beliefs are thus positively related to OIPs, but on average this relation is far from 1-to-1. That the survey probabilities only partially reflect a change in OIPs is consistent with a variety of behavioral theories on the partial updating of beliefs (for a discussion of these in the context of financial markets, see Hirshleifer [2001]). The effects still may vary substantially across individuals, for example, according to an individual's level of probabilistic understanding. We consider this possibility in the next section.

To avoid multicollinearity, it is necessary to suppress one category of each set of dummy variables in the estimation. The constant (not shown) indicates the predicted subjective probability of a hypothetical person whose other covariates are equal to 0 (i.e., the person belongs to all the suppressed categories and the logit (OIP) is equal to 0).[11] In this case, the hypothetical person is predicted to give a response of 26.3% to >*Plus20*. Any other predicted response can be calculated by adding the marginal effects in the table to this percentage. For example, the predicted responses to *PositiveReturn* and >*Minus20* for the hypothetical respondent are 34.0% (26.3% + 7.7%) and 66.1%. In contrast, when the control variables are evaluated at their population averages, the predicted responses to >*Minus20*, *PositiveReturn*, and >*Plus20* are 67.2%, 34.0%, and 27.4%, respectively. Note that because the OIP is time varying, the predicted responses for any individual also will vary over time. When in addition to the control variables the OIP is also evaluated at its mean value for each threshold, the predicted responses are 76.2%, 39.1%, and 26.8%, respectively. These values are close to the descriptive statistics reported in Table 1 that do not take into account the respondents' characteristics (i.e., 75.8%, 40.4%, and 27.1%, respectively).

The results also enable us to draw inference about the relative optimism of people with various characteristics, ceteris paribus. For example, those that own stocks or have a retirement account are more optimistic about the stock market than those who do not, stating on average a 2.4 and 2.0 percentage points higher belief of the return going over a given threshold. In addition, the predicted probabilities of those that profess to have a good understanding of the stock market are on average 2.4 percentage points higher than for those who report no understanding of the stock market (the omitted category), while they are 0.9 percentage points higher for those with a medium understanding. Those that claim to be somewhat following the stock market are 2.8 percentage points more optimistic than those who do not follow it (the omitted category). Interestingly, those who claim to be closely following the stock market are less (1.4 percentage points) optimistic, perhaps reflecting their anxiety about large absolute returns in the aftermath of the financial crisis (Gherzi, Egan, Stewart, Haisley, and Ayton [2014]).

Finally, the demographic controls (not shown) indicate that men, older people, non-Hispanic Whites, non-Hispanic Blacks, and more highly educated people have more optimistic beliefs about the stock market than do the rest of the population. These results are in line with patterns documented in earlier literature (e.g., Kézdi and Willis [2003], Dominitiz and Manski [2011]).

Inconsistent responses

In addition to the statistical challenge of focal responses, addressed via the model, a further challenge is that a relatively large proportion (approximately 20%) of response sets[12] are inconsistent with the laws of probability (probabilities adding up to more than 100%), perhaps suggesting that some respondents did not fully understand the questions.[13] Ben-David et al. [2010] documented miscalibration of survey respondents with respect to probability distributions in their sample of Chief Financial Officers; Fox, Rogers, and Tversky [1996] also noted that even experienced option traders violate basic probability principles. Such miscalibration is likely to be more severe among the general population in our sample. This naturally raises the question of the extent to which the survey responses represent a respondent's true beliefs and, if they do not, whether this affects our estimated connection between survey and market beliefs.

To investigate this question, we re-estimate the model with an additional control variable that indicates whether a set of responses is inconsistent with the laws of probability; in addition we interact this control with the OIP to allow the main effect of interest to differ depending on whether or not a person provided consistent responses in a given wave. As noted previously, the estimates associated with the probability of a focal response (Equation 4) remain unchanged.

The results from including these additional covariates are displayed in the second column of Table 2. Controlling for inconsistency strengthens our earlier findings. For people with consistent answers, the effect is 50% larger than that of the baseline effect: a 10% increase in the OIP is associated with a 2.5-percentage-point increase in their beliefs. In contrast, people with inconsistent responses are less likely to provide subjective probabilities that are in line with option implied probabilities; if the OIP increases by 10%, their belief decreases by 1.9 percentage points ($0.249 - 0.437 = -0.188$).

Conclusion

Are subjective probability responses from surveys at odds with probabilities derived from financial markets data? A novel approach, comparing survey responses to probabilistic questions about future stock market performance with their corresponding OIPs, investigates 1 aspect of this question: whether financial market probabilities coincide with the views of survey respondents. We find a significant relationship between the probabilities extracted from option prices and those elicited from longitudinal survey responses. The analysis demonstrates that subjective response elicitations are useful reflections of sentiment regarding the financial markets and are not necessarily at odds with the views of financial market participants as seen through option prices.

The results further show, however, that while OIPs are linked to survey respondents' outlook, the association is far from 1 to 1. Specifically, on average a 10-percentage-point increase in the OIP that future DJIA returns will exceed a given threshold leads to a 1.73-percentage-point increase in the average beliefs of the survey respondents. This effect is larger (2.49 percentage points) for those who give responses consistent with the laws of probability, and negative (–1.88 percentage points) for those who give inconsistent responses. When considered in the context of the large literature documenting that a higher degree of financial literacy leads to better financial forecasts and decisions (e.g., Bernheim and Garrett [2003], Lusardi and Mitchel [2011]), our results provide further evidence of a link between mathematical skill and financial literacy (Lusardi [2012]). The results also demonstrate a possible way that observed inconsistencies in survey responses may prove useful for inference—suggesting caution be exercised before imposing such consistency through the survey design.

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Notes

1. We thank an anonymous referee for drawing our attention to this distinction. Barone-Adesi et al. [2013] did note, however, that by linking S&P returns to future price-earnings ratios, stock returns also can be transformed to be somewhat forward looking.
2. Throughout this article, we use the word *link* to describe an association or resemblance and emphasize that the association we document is not necessarily causal.

Therefore, our investigation is more motivated by an interest in whether two possibly distinct sources of information can yield the same underlying views, rather than identifying whether option prices drive individuals' beliefs or vice versa. Barone-Adesi et al.'s [2013] definition of sentiment that amounts to a change of measure between observed prices and investors' beliefs is 1 example of a theoretical model that would give rise to such an association but it is important to emphasize that other explanations and models are also possible.

3. Weights are determined by the RAND Corporation via an iterative (raking) process until the weighted distribution is sufficiently close to the target distribution (i.e., the Current Population Survey).
4. A referee suggested that weights should not be used in the regression analyses; results without weights are available from the authors on request. The results are qualitatively similar both with and without weights although not using weights strengthens our results, both due to an increase in the coefficient of interest and tighter standard errors, resulting in greater significance of all variables. See Solon, Haider and Wooldridge [2015] for a comprehensive discussion of the use of weights in regression analysis.
5. The exact wording of these questions is: "By next year at this time, what are the chances that mutual fund shares invested in blue-chip stocks like those in the Dow Jones Industrial Average will have increased (fallen) in value by more than 20% compared with what they are worth today?" Assuming respondents satisfy binary additivity, as most theories of subjective beliefs do and as also has been empirically demonstrated (see Windschitl [2000] and references therein), the probability of a $>$ 20% decrease in value is equal to 1 minus the probability of a more than -20% return. The response to the $>$ 20% decrease question is thus subtracted from 100% to correspond to a greater than -20% return. This alternative representation will be useful in the analysis when comparing the subjective response values to expectations inferred from option prices.
6. See, for example, Hurd et al. [1998], Bassett and Lumsdaine [2000], Lillard and Willis [2001], Hurd and McGarry [2002], Kézdi and Willis [2003], Manski [2004], Kleinjans and Van Soest [2010], Manski and Molinari [2010], and Dominitz and Manski [2011].
7. Specifically, we calculate the risk-neutral probability density by taking the second derivative of the price of a call option with regard to the strike price (Breeden and Litzenberger [1978]), and obtain the OIPs by adjusting for the 6% equity premium that corresponds to the historical equity premium over the period 1961 to 2011. Further information is provided in Supplemental Online Appendix B. As noted there, we have also estimated all results under the risk-neutral assumption (i.e., without the equity premium); the results are robust to this change.
8. For more information on the calculation of the implied volatility, see Cui and Frank [2011]. The document can be found by typing DOCS 2056700 <GO> when logged in to a Bloomberg terminal.
9. The OIP is akin to Barone-Adesi et al.'s [2013] "representative investor." In their approach, they compare the beliefs of the representative investor to beliefs derived from the historical distribution of returns (as measured by a "rational investor," that is one whose beliefs are assumed to be "objectively correct" [p. 4]) and define sentiment to be the change of measure linking the two sets of beliefs. In contrast, in our framework, we compare the beliefs of the representative investor to those drawn from the subjective response distribution, without making assumptions about what is the correct distribution.
10. The estimated coefficients and associated standard errors, as well as the estimation results for Equation 4 are available in Supplemental Online Appendix D. Although in the estimation procedure the subjective probability and focal response likelihood equations are estimated jointly, the likelihood of a focal response is invariant to the choice of covariates included in the X_1 matrix used to estimate the subjective probability and hence will not affect the estimates of the main coefficient of interest. Equation 4 estimates were relegated to the Supplemental Online Appendix at the suggestion of the referee.
11. This hypothetical person is a non-Hispanic man who is unmarried; did not attend college; is currently not working, and does not own a home, stocks, or a retirement account; professes to have a bad understanding of the stock market and claims to not follow it. Additionally, in this hypothetical situation, the noncategorical variable (the logit of the OIP) is equal to 0, which implies the OIP itself is 50%.
12. That is, the 3-tuple of a person's $>$*Minus20, PositiveReturn*, and $>$*Plus20* responses for a given wave.
13. That the sum of responses to 3 or more related probability questions often exceed 1, in contrast to binary additivity holding when there are only 2 responses elicited, is discussed in Windschitl [2000].

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