The Role of Prior Beliefs in Decisions from Experience

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March, 2017

Abstract

This paper points to the importance of prior beliefs in understanding the gap between decisions from experience and decisions from description. It puts forward a two-stage account that effectively incorporates prior beliefs into the examination of decisions from experience. The two-stage account assumes that (1) the subjective probabilities are estimated in a Bayesian manner, combining prior beliefs with observations, and (2) the estimated probabilities are transformed by probability weighting. The first stage provides a Bayesian explanation for the commonly found overestimation of infrequent outcomes, and an empirically appealing way to deal with always – or never – observed outcomes. A source dependent probability weighting in the second stage captures deviations from Bayesian rationality under experienced uncertainty. The two-stage model is tested by reanalyzing the data sets in Glöckner et al. (2016), as well as the famous Technion Prediction Competition data set of Erev et al. (2010). Model comparisons using BIC scores indicate that the two-stage model performs better than the single stage model approximating subjective probabilities with observed relative frequencies. The estimation results show that the two-stage model can accommodate both the classic and the reversed description – experience gap.

Introduction

Early studies of decisions from experience (henceforth, DFE) suggested that people make choices as if they underweight the impact of rare outcomes. This empirical observation is inconsistent with the findings from traditional decisions from description (henceforth, DFD) and with the predictions of prospect theory (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992), the most prominent theory for risk and uncertainty. Since the influential studies by Barron & Erev (2003) and Hertwig et al. (2004) introducing the intriguing DFD-DFE gap, an ever-growing DFE literature has clarified two main factors underlying the gap.

First, under-observation of the rare outcomes in small samples, also known as the sampling error, is a major factor underlying the underweighting. This implies that observed relative frequencies of outcomes rather than the objective probabilities, which are unknown to the decision maker, should count in DFE (Fox & Hadar, 2006). Controlling for the sampling error, the DFD-DFE gap still amounts to relatively less overweighting, if not underweighting as it was claimed originally (Ungemach et al., 2009; Hau et al., 2009; Camilleri et al., 2009).

The second factor concerns the information asymmetry between DFD and DFE (Hadar & Fox, 2009). Whereas DFD involves risk (known probabilities), DFE involves ambiguity due to incomplete information about the set of possible outcomes and probabilities. Importantly, the decision maker also lacks a priori knowledge about certainty or possibility of outcomes, which is relevant in the presence of always – or never – sampled outcomes. Several studies have shown that the gap is reduced or even reversed if the information asymmetry is reduced by providing information about the possible outcomes in prospects or by the absence of sure outcomes in choice problems (Abdellaoui, L'Haridon, & Paraschiv, 2011; Glöckner, Hilbig, Henninger, & Fiedler, 2016; Hadar & Fox, 2009; Kemel & Travers, 2016; Kellen, Pachur, & Hertwig, 2016). The recently found reversed DFD-DFE gap, implying even more pronounced overweighting of rare outcomes under DFE, is consistent with the previous literature on ambiguity (Abdellaoui, Vossmann, & Weber, 2005; Abdellaoui, Baillon, Placido, & Wakker, 2011; Fox & Tversky, 1998; Fox, Rogers, & Tversky, 1996; Tversky & Fox, 1995; Tversky & Wakker, 1995). The decreased likelihood sensitivity under ambiguity is commonly attributed to the overestimation of infrequent outcomes due to sub-additive subjective beliefs or regression to the mean effects in probability estimations (Erev, Wallsten, & Budescu, 1994; Fiedler, Unkelbach, & Freytag, 2009; Fiedler & Unkelbach, 2014; Rottenstreich & Tversky, 1997; Tversky & Koehler, 1994).

This paper points to the role of prior beliefs as another important factor in the DFD-DFE gap. Except a few studies eliciting introspective judged probabilities (Hau et al. 2008; Camilleri & Newell 2009; Ungemach et al. 2009), the previous studies usually approximate subjective probabilities with observed relative frequencies neglecting the role of prior beliefs. However, the importance of the subjective prior beliefs is particularly evident in the face of the ambiguous nature of DFE because every subject brings his own prior expectations about the experimental setting into the laboratory. For example, even though not specified explicitly, a subject can reasonably anticipate the range of possible outcomes, and predict that extreme losses or gains do not occur very frequently due to ethical reasons or the budgetary constraints of the experimenter. The ecological rationality account of Pleskac & Hertwig (2014) illustrates the importance of such intuitions under ambiguous situations. If prior beliefs are not incorporated into the analysis of subjective probabilities, then the estimations of probability weighting may be confounded because the impact of prior beliefs will incorrectly be modeled through probability weighting.

This study puts forward a more complete account of subjective probabilities under DFE, which involves a combination of prior beliefs with observed relative frequencies. As a working hypothesis, the present account proposes a Bayesian updating method for the estimation of subjective probabilities. Notably, the Bayesian updating of an ignorance prior will estimate the probability of an infrequent outcome higher than its observed relative frequency. This gives a rational basis for the regressions to the mean effects in probability estimations, also documented in the previous DFE studies eliciting judged probabilities.

Hence, I introduce a two-stage decision model for DFE according to which (1) subjective probabilities are estimated using a Bayesian updating method developed by Rudolf Carnap (1952); (2) and the estimated probabilities are transformed using prospect theory's rank- and sign-dependent probability weighting (Tversky & Kahneman, 1992). Besides being a normative method for belief updating, Carnap's method, introduced in the next section, is also psychologically natural. Following the source method of Abdellaoui et al. (2011), the probability weighting in the second stage captures deviations from Bayesian rationality, and it is assumed to be source dependent. This means that having revealed the subjective probabilities, the model allows for different attitudes towards described and experienced probabilities observable through different probability weightings. It should be noted that the current model differs from the two-stage model of Tversky & Fox (1995) and Fox & Tversky (1998), which attributes ambiguity attitudes to sub-additive beliefs under uncertainty. Another distinguishing feature of the current account is that it adheres to the revealed preference approach of (behavioral) economics by relying on choice-based probabilities rather than introspective probability judgments.

The two-stage model is empirically tested by reanalyzing the data sets of Glöckner et al. (2016), as well as the Technion Prediction Competition data set of Erev et al. (2010). As will be illustrated later, the model successfully disentangles the role of beliefs from preferences in DFE. Accordingly, the reversed DFD-DFE gap in probability weighting is estimated to be considerably smaller when prior beliefs are controlled for. Moreover, the classic DFD-DFE gap is also reduced, or even reversed, under some plausible assumptions on subject's prior expectations about the set of possible outcomes. Overall, the robust likelihood insensitivity under DFE suggests further deviations from Bayesian rationality due to ambiguity. Lastly, model comparisons based on Bayesian Information Criteria (BIC) scores also indicate that the two-stage model performs better than the single stage approach using observed relative frequency approximation of subjective probabilities. Thus, the two-stage model provides a parsimonious way to analyze DFE by adding only one extra parameter to the preceding models.

Carnap's Updating Method and the two-stage Model

The current paper makes use of the inference method that Rudolf Carnap[1] put forward to quantify the degree of confirmation of a hypothesis stating that the next observation from a population will be the outcome x_i based on the evidence that a previous sample of N observations contains n_i observations from the outcome x_i:

$$p_i = \frac{cp_i^0 + N\frac{n_i}{N}}{c + N} .$$

Thus, as in every Bayesian approach, the method combines a prior probability p_i^0 of the outcome x_i with the observed relative frequency $\frac{n_i}{N}$. The respective weights are proportional to a constant $c > 0$ and the total number of observations N. The prior probability p_i^0 together with the constant c represent the complete prior knowledge of the decision maker. A common intuitive interpretation is that the prior knowledge of the decision maker can be thought to be roughly equivalent to a hypothetical sample consisting of c observations with relative

[1] Rudolf Carnap is a well-known philosopher of science who also contributed to the theory of probability by providing a logical definition of probability (Carnap, 1945; 1950; 1952). Briefly, his theory views probability as a logical relation between two statements, namely the degree of confirmation of a hypothesis *h* on the evidence *e* (Carnap, 1945, p. 72), which is analogous to the concept of the degree of belief in the theory of subjective probability.

frequency p_i^0. In Bayesian inference, the method is also known from the updating of the conjugate beta family and the conjugate Dirichlet family for its multinomial extension (Winkler, 1972; Wilks, 1962; Zabell, 1982).

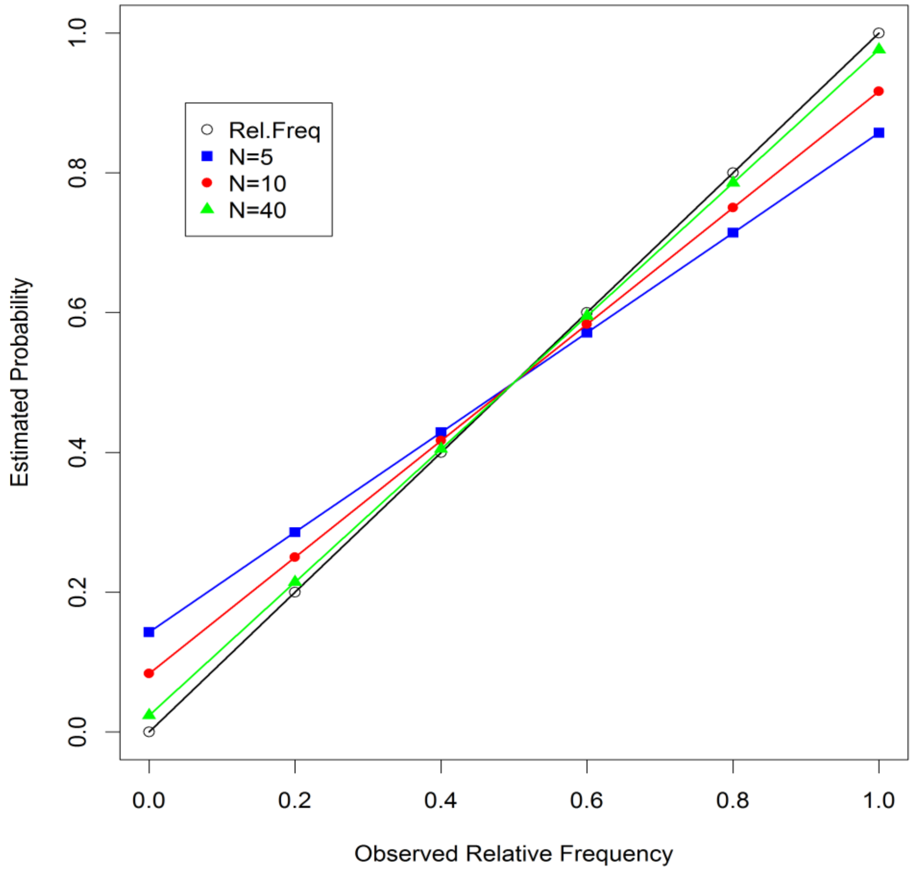
Carnap's method is empirically appealing. First, the posterior probability of an outcome always lies between the prior and the observed relative frequency. The estimation converges to the relative frequency as more and more observations are accumulated, reflecting increasing confidence in empirical probabilities. In the case where there are two possible outcomes, a "flat" prior representing ignorance is captured by $p_i^0 = \frac{1}{2}$ and $c = 2$, which turns the formula into the posterior mean of a uniform beta prior (Winkler, 1972). This case is illustrated in Figure 1. The posterior estimations tend to the 50/50 prior especially when the number of observations is small. This tendency reduces significantly as the number of observations increases from 5 to 40.

Second, the method reduces to relative frequency when c converges to 0. Carnap (1945, p. 86) points out the major problem of using relative frequencies in estimations of probabilities concerning always – or never – observed outcomes. This problem is also commonly encountered in DFE experiments. In particular, assigning 1 or 0 probability to these outcomes may be implausible. A famous historical example of this issue is Laplace's (1825) sunrise problem, asking the likelihood of the sun rising tomorrow. Laplace's rule of succession for dealing with the problem, $p_i = \frac{1+N}{2+N}$, is simply the restricted version of Carnap's method under ignorance illustrated in Figure 1.[2] For example, the method results in $p_i = \frac{1+10}{2+10} = \frac{11}{12} \cong 92\%$ when the observed relative frequency is $10/10$. The posterior estimation converges to certainty when N increases.

Carnap (1952) justifies the appropriateness of the method by providing logical axioms for it. Wakker (2002) presents the axioms in a decision theoretic context, and highlights the normative status of the method. The first property is positive relatedness of the observations. It means that an extra observation from an outcome only increases its likelihood. The second property is exchangeability. It means that only the number of observations from the outcomes matters, regardless of the order of observations. The third property is disjoint causality. It means that there is no causal relationship between different outcomes. Therefore, the probability of an outcome x_i depends only on the number of observations of itself (n_i) and of not-itself ($N - n_i$), regardless of which other outcomes were observed among the ($N - n_i$) other outcomes.

[2] A historical review of the rule of succession is in Zabell (1989).

Figure 1. Posterior estimations with Carnap's method when $p_i^0 = \frac{1}{2}$ and $c = 2$



In principle, Carnap's properties are applicable to the DFE experiments, where the sample information is obtained from a fixed outcome distribution with replacement, i.e. from a stationary and independent process. It is worth noting that the properties can be violated due to subjects' unjustified beliefs about the random processes and the cognitive illusions such as the hot hand and gambler's fallacies (Tversky & Kahneman, 1971; Kahneman & Tversky, 1972; Ayton & Fischer, 2004; Sundali & Croson, 2006). However, these effects have been mainly documented in repeated settings such as in feedback paradigms of DFE (Barron & Yechiam, 2009) and in probability matching tasks (Sundali & Croson, 2006) but not in the sampling paradigm, where the observations are made only for the purpose of information acquisition.

Having constructed beliefs using Carnap's method, the two-stage model assumes that prospects are evaluated by prospect theory (Tversky & Kahneman, 1992) in the second stage. In what follows, I denote a prospect with outcomes $x_1, \dots, x_n$ with respective probabilities $p_1, \dots, p_n$ by $(p_1: x_1, \dots, p_n: y_n)$. The prospect theory value of an experienced prospect with $x_1 > \cdots > x_k > 0 > x_{k+1} > \cdots > x_n$ is

$$PT(p_1\colon x_1, \dots, p_n\colon x_n) = \sum_{i=1}^{k} u(x_i)\,[w_e^{+}(p_i + \dots + p_1) - w_e^{+}(p_{i-1} + \dots + p_1)] + \sum_{j=k+1}^{n} u(x_j)\,[w_e^{-}(p_j + \dots + p_n) - w_e^{-}(p_{j+1} + \dots + p_n)].$$ [3]

The utility $u(.)$ is strictly increasing and continuous with $u(0) = 0$. The probability weighting functions $w_e^s(.)$ for gains ($s = +$) and losses ($s = -$) are strictly increasing with $w_e^s(0) = 0$ and $w_e^s(1) = 1$. Here, the subscript e designates the experienced source of ambiguity. Specifically, $w_e^{\mp}(.)$ measures the weighting of subjective probabilities under DFE. Prospects under DFD are similarly evaluated by prospect theory, where $w_e^{\mp}(.)$ is replaced by $w^{\mp}(.)$ measuring the weighting of objective probabilities. Hence, different attitudes towards experienced ambiguity and described risk can be captured by differences between $w_e^s(.)$ and $w^s(.)$.

Testing the two-stage Model

The following sections provide an empirical test of the two-stage model by parametric estimations of the prospect theory components under the two-stage model. I use Goldstein & Einhorn's (1987) two-parameter family for probability weighting, and the commonly used power family for utility. The choice probabilities are calculated using the stochastic logit rule.

$$w^{+}(p) = \frac{\delta^{+} p^{\gamma^{+}}}{\delta^{+} p^{\gamma^{+}} + (1-p)^{\gamma^{+}}}$$

$$w^{-}(p) = \frac{\delta^{-} p^{\gamma^{-}}}{\delta^{-} p^{\gamma^{-}} + (1-p)^{\gamma^{-}}}$$

$$u(x) = \begin{cases} x^{\alpha} & if\ x \geq 0 \\ -\lambda(-x)^{\beta} & if\ x < 0 \end{cases}$$

$$p(A,B) = \frac{1}{1 + e^{-\sigma(PT(A) - PT(B))}}$$

The parameter $\delta^{\mp}$ determines the elevation of probability weighting, and measures the pessimism/optimism of the decision maker. Higher $\delta^{\mp}$ leads more elevation, and thus more optimism in the gain domain and more pessimism in the loss domain. The parameter $\gamma^{+/-}$

[3] Here $p_{i-1} + \dots + p_1 = 0$ when $i = 1$, and $p_{j+1} + \dots + p_n = 0$ when $j = n$.

determines the curvature of the probability weighting function and it captures sensitivity towards probabilities. For, $\gamma^{+/-} < 1$ the probability weighting is inverse S-shaped reflecting likelihood insensitivity. The parameter λ determines the degree of loss aversion. To avoid extra complexity, utility curvature in the gain and in the loss domain is assumed to be the same by constraining $\alpha = \beta$. This assumption avoids an identification problem in the estimation of loss aversion (Wakker, 2010, section 9.6), and it is empirically supported by previous findings (Tversky & Kahneman, 1992). For $\alpha, \beta < 1$, the utility curve is concave in the gain domain and it is convex in the loss domain. Lastly, the parameter σ in the logit formula determines the sensitivity to differences in prospect theory values of prospects.

Three different cases of subjective priors are considered in the estimations. The first case assumes symmetric prior probabilities, p_i^0, equally distributed over all the outcomes that are believed to be possible in a prospect, and the constant c in Carnap's formula is treated as a free parameter to be estimated together with the other parameters. Hereafter, this will be called the *Carnap prior* case. The second case concerns the *ignorance prior* that was already mentioned in the previous section. The *ignorance prior* is a special case of the *Carnap prior* case. It assumes that the prior knowledge of the subject is equivalent to a hypothetical sample that contains one and only one observation from each of the possible outcomes. For instance, for a prospect with k possible outcomes, the prior probability of every outcome is $\frac{1}{k}$ and $c = k$. The third case suppresses prior beliefs altogether and simply approximates subjective probabilities with observed relative frequencies. In other words, this is Carnap's method with $c = 0$. Following the Bayesian terminology, this case will be called the *diffuse prior* (Winkler, 1972, p. 178). This case has been commonly used in the previous DFE studies.

The maximum likelihood estimations are done using the estimation routine in STATA software described by Harrison (2008). Standard errors are cluster-corrected at the individual subject level. Model comparisons are based on BIC scores.

Accounting for the reversed DFE-DFD gap: A reanalysis of Glöckner et al. (2016) with the two-stage model

Glöckner et al. (2016) finds the reversed DFD-DFE gap based on an analysis of four experimental data sets. The authors mainly attribute the results to regression to the mean effects in probability estimations due to noise and reduced evaluability under uncertainty. But they also point out the possibility of an alternative explanation with updating of ignorance priors in a footnote (footnote 8, p. 490). This section tests this alternative explanation by re-examining the four data sets using the two-stage model. The data sets are made available by the authors at Open Science Framework: https://osf.io/d9f8q/.

Data

The first data set is obtained from a previous study by Glöckner, Fiedler, Hochman, Ayal, & Hilbig (2012). The second and the third data sets are based on the Experiments 1 and 2 in Glöckner et al. (2016). These experiments replicate the experiment by Glöckner et al. (2012) with slight procedural variations. The choices concern only the gain domain in these three data sets. The fourth data set is based on Experiment 3 in Glöckner et al. (2016). This data set contains choices in the gain, loss, and mixed domains. In all the data sets, either all or the majority of the problems involve a choice between two two-outcome prospects, whereas the rest involves a choice between a two-outcome prospect and a sure outcome. Subjects in DFE conditions were informed about the number of possible outcomes in prospects, except half of the subjects in Experiment 2. There were no differences observed due to the information provision in this data set. Readers are referred to Glöckner et al. (2016) for more details on the experimental design.

Results

The estimation results are in Table 1. The last column reports the estimation results with the pooled data set. The resulting probability weighting functions are in Figure 2. The estimations under DFE using the *diffuse prior* indicate a significant DFD-DFE gap with respect to the likelihood sensitivity parameters $\gamma^{+/-}$. In all data sets, there is less likelihood sensitivity in the DFE condition than in the DFD condition. The gap is reduced under the two-stage model using the *ignorance prior*. In particular, the gap turns insignificant in the estimations based on the data sets of Glöckner et al. (2012) and Experiment 1, and in the gain domain of Experiment 3. However, the gap is never fully closed quantitatively. Thus, it is also significant in the estimations with the pooled data due to increased statistical power.

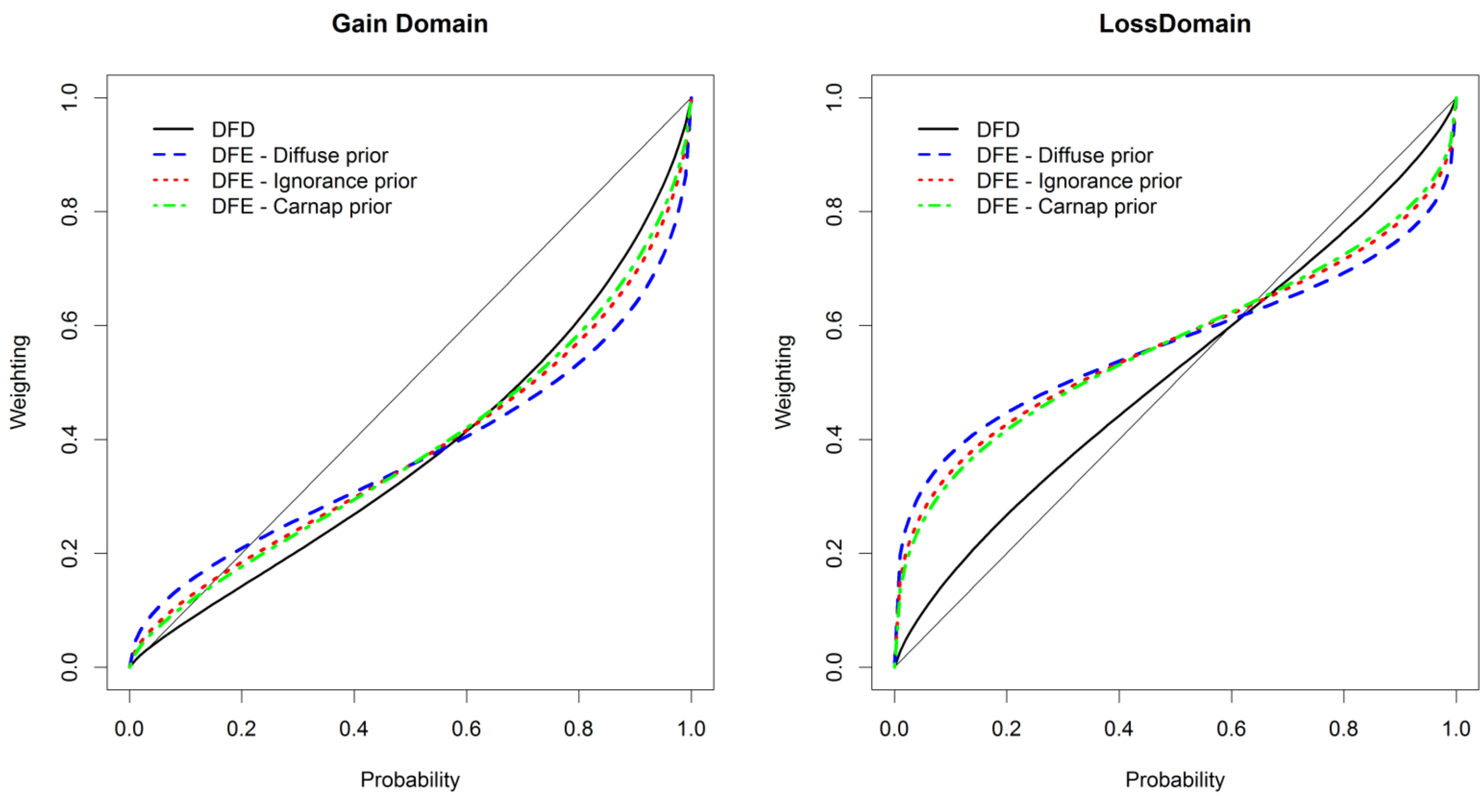
Table 1. Estimation results with the data sets in Glöckner (2016)

		Glöckner et al. (2012)	Experiment 1	Experiment 2	Experiment 3	Pooled
α	DFD	0.652	1.058	0.617	0.954	0.850
	DFE – Diffuse prior	0.642	0.853	0.925*	0.991	0.796
	DFE – Ignorance prior	0.644	0.871	0.941*	0.981	0.800
	DFE – Carnap prior	0.621	0.890	0.949**	0.972	0.801
c	DFD	-	-	-	-	-
	DFE – Diffuse prior	0	0	0	0	0
	DFE – Ignorance prior	2	2	2	2	2
	DFE – Carnap prior	-1.602***	6.456	3.603	10.117	2.768
δ^{+}	DFD	0.553	0.320	0.695	0.637	0.511
	DFE – Diffuse prior	0.561	0.392	0.482	0.661	0.541
	DFE – Ignorance prior	0.565	0.386	0.478	0.673	0.542
	DFE – Carnap prior	0.551	0.376	0.475	0.684	0.542
γ^{+}	DFD	0.732	0.736	0.961	0.559	0.810
	DFE – Diffuse prior	0.560*	0.552*	0.553***	0.423*	0.536***
	DFE – Ignorance prior	0.670	0.684	0.665***	0.502	0.645***
	DFE – Carnap prior	0.425**	0.919	0.740	0.733	0.682
δ^{-}	DFD				1.074	1.091
	DFE – Diffuse prior				1.211	1.341
	DFE – Ignorance prior				1.243	1.357
	DFE – Carnap prior				1.288	1.361
γ^{-}	DFD				0.856	0.787
	DFE – Diffuse prior				0.462***	0.371***
	DFE – Ignorance prior				0.544**	0.443**
	DFE – Carnap prior				0.793	0.467*
λ	DFD				1.001	1.237
	DFE – Diffuse prior				0.916	1.068
	DFE – Ignorance prior				0.917	1.081
	DFE – Carnap prior				0.920	1.084
σ	DFD	2.117	0.639	2.123	1.512	1.099
	DFE – Diffuse prior	2.045	0.865	0.791*	1.268	1.232
	DFE – Ignorance prior	1.997	0.825	0.758*	1.308	1.218
	DFE – Carnap prior	2.180	0.778	0.740*	1.326	1.212
N	DFD	1298	1417	3180	2484	8379
	DFE	1283	1632	2912	2585	8412
LL	DFD	-585.502	-659.720	-1547.839	-1312.611	-4225.120
	DFE – Diffuse prior	-558.414	-774.531	-1296.291	-1340.263	-4056.058
	DFE – Ignorance prior	-561.825	-772.261	-1290.621	-1335.877	-4048.111
	DFE – Carnap prior	-555.063	-771.387	-1289.894	-1332.834	-4047.754
BIC	DFD	1199.678	1348.465	3127.936	2679.945	8513.475
	DFE – Diffuse prior	1145.456	1578.653	2624.488	2735.529	8175.377
	DFE – Ignorance prior	1152.279	1574.111	2613.149	2726.755	8159.483
	DFE – Carnap prior	1145.911	1579.762	2619.67	2728.528	8167.807

Notes: Stars indicate DFD – DFE gap. $* p < 0.05$. $** p < 0.01$. $*** p < 0.001$.

The estimations under the two-stage model demonstrate further reductions of the gap when the *Carnap prior* is used. In this case, the gap disappears in all data sets except in the data set of Glöckner et al. (2012). Surprisingly, Carnap's c is found significantly negative in this data set. This result resembles to representativeness in probability updating where too much weight is assigned to the relative frequencies at the expense of prior probabilities (Grether, 1980; Griffin & Tversky, 1992). As the negative c induces underestimation of rare outcomes, the reversed gap is more pronounced here. In the rest of the estimations, the constant c is estimated positive although not significantly different than 0. The estimations with the pooled data set indicates $c > 0$ ($p = 0.08$), and the gap is accommodated in the gain domain.

Figure 2. Probability weighting functions based on the pooled data set of Glöckner et al.



Notes: Solid black lines show probability weighting under DFD. Dashed blue lines show probability weighting under DFE when the diffuse prior is used. Dotted red lines show probability weighting under DFE when the ignorance prior is used. Dot-dash green lines show probability weighting under DFE when the Carnap prior is used.

The comparisons of the BIC scores indicate that the two-stage model account for the data at least as good as the model based on the diffuse prior. In particular, the two-stage model with *ignorance priors* outperforms the models with *diffuse* and *Carnap priors*, except for the data set of Glöckner et al. (2012). Overall, the results confirm the ability of the two-stage model to explain the reversed DFD-DFE gap, although a residual gap still remains in probability weightings as shown in Figure 2.

Accounting for the classic DFE-DFE gap: A reanalysis of Erev et al. (2010) with the two-stage model

The ambiguity in DFE is augmented when the set of possible outcomes is unknown. Whereas the ambiguity due to unknown probabilities can be easily studied in a tractable manner as illustrated in the previous section, the additional ambiguity about the outcome space poses a more complex problem to deal with. In particular, the subject's prior beliefs about the ambiguous set of outcomes are not easily observable. This section first introduces some plausible assumptions about prior beliefs under such ambiguous situations to explain the classic DFD-DFE gap. Then, the assumptions are empirically tested under the two-stage model by reanalyzing the Technion choice prediction competition data set of Erev et al. (2010).

Prior beliefs over ambiguous outcome space

Here, I describe two hypothetical scenarios about possible considerations in a subject's mind. The first is called *context-dependent expectations*, and it was put forward by Glöckner et al. (2016). To illustrate, consider the following choice problem under DFD involving two options: Option A is a sure outcome 8.7 and Option B is a risky prospect with $(0.91\colon 9.6,\ 0.09\colon -6.4)$ (taken from figure 3 in Glöckner et al.). Under DFE, the subject does not know the number of possible outcomes in the options, and therefore she is not aware of the certainty of the outcome 8.7 that she observes from Option A successively. Glöckner et al. (2016) argues that while forming beliefs about an option, the subject will not only use the information that she sampled from the very same option but also the information that she gathered from the other option. Accordingly, the rare outcome observed from Option B may be projected upon Option A. Specifically, the experience of the rare outcome -6.4, along with the common outcome 9.6 in Option B, can create an expectation that a similarly bad and rare outcome also exists in Option A. Thus, her updated belief about Option A makes it less attractive, and therefore she may prefer Option B. Notably, this gives the impression that she is underweighting the small probability of -6.4 in Option B, although in fact the choice is due to her prior beliefs.

More specifically, suppose that the subject updates her beliefs according to Carnap's method, and she uses an *ignorance prior*. Furthermore, she makes 10 observations from the each option, and observes the outcome -6.4 only once. The relative frequency approximation of her subjective beliefs falsely implies that she is making a choice between the sure outcome $(\frac{10}{10}: 8.7)$ and the risky prospect $(\frac{9}{10}: 9.6,\ \frac{1}{10}: -6.4)$, whereas she is indeed making a choice between a perceived $(\frac{11}{12}: 8.7,\ \frac{1}{12}: \sim -6.4)$ and $(\frac{10}{12}: 9.6,\ \frac{2}{12}: -6.4)$ as implied by *context dependent expectations* and Carnap's updating method.

The second scenario is a natural extension of the *context-dependent expectations*. It assumes that the subject will have more comprehensive beliefs by expecting that any outcome that she is aware of within a given problem can in principle exist in both options. Hence, this scenario will be called *comprehensive expectations*. Continuing with the previous example, it leads to a choice between A: $(\frac{1}{13}: 9.6,\ \frac{11}{13}:\ 8.7,\ \frac{1}{13}: -6.4)$ and B: $(\frac{10}{13}: 9.6, \frac{1}{13}: 8.7\ , \frac{2}{13}: -6.4)$, where the probabilities are calculated based on the Carnap's method with $p_i^0 = \frac{1}{3}$ and $c = 3$. This scenario has a less clear prediction for the attractiveness of the sure prospect. In the present problem, adding good and bad outcomes, 9.6 and -6.4, to the Option A will impact its attractiveness depending on the relative steepness of the lower parts of the probability weighting curves in the gain and in the loss domains. For instance, more overweighting of $\frac{1}{13}$ in the loss domain than in the gain domain decreases the attractiveness.

In the following analysis, the perceived set of possible outcomes is constructed based on the *context-dependent* or *comprehensive expectations*. Then, the parametric estimations will be done under the two stage model by using *Carnap, ignorance* and *diffuse priors*.

Data

I focus on the DFD and DFE sampling conditions in Erev et al. (2010). The study consists of an estimation data set and a competition data set. The two data sets are pooled in the current analysis. The pooled data set contains 40 subjects in the DFD condition and 80 subjects in the DFE condition. Each subject makes 60 choices in the DFD condition and 30 choices in the DFE condition. The problems always involve a choice between a sure outcome and a two-outcome risky prospect. Prospects were equally divided into gain, loss, and mixed domains. Subjects in the DFE condition were provided with minimal information about the content of the two prospects. Importantly, unlike in Glöckner et al. (2016), they do not know the number of possible outcomes in prospects. They make a single choice after an exploratory sampling stage. The readers are referred to Erev et al. for more details on the experimental design.

Results

The estimation results are in Table 2. The parameter estimations under the DFE condition using the *diffuse prior* replicate the classic DFD-DFE gap. Specifically, the DFE condition indicates more likelihood sensitivity compared to the DFD condition. This means that the rare outcomes are less overweighted under DFE than under DFD. It should be noted that the underweighting of rare events claimed in the early DFE studies is not found here. This happens mainly because of the correction of the sampling error by using observed relative frequencies rather than unknown objective probabilities. There is no DFD-DFE gap with respect to other prospect theory parameters.

Table 2. Estimation results with the data set of Erev et al. (2010)

	DFD	DFE	DFE		DFE	
			Ignorance Prior		Carnap Prior	
		Diffuse Prior	Context-Dependent Expectations	Comprehensive Expectations	Context-Dependent Expectations	Comprehensive Expectations
α	0.932	0.917	0.720**	0.734**	0.859	0.815
λ	1.111	1.158	1.179	1.145	1.283	1.123
c	-	0	2	3	0.202	1.038*
δ^{+}	0.779	0.803	0.670	0.501	0.790	0.512
γ^{+}	0.599	0.891*	0.925*	0.461	0.795	0.338
δ^{-}	1.167	1.268	1.301	1.565	1.504	1.686
γ^{-}	0.589	0.924**	0.626	0.403	0.265**	0.307
σ	1.055	1.450	2.681**	3.853*	1.874	2.871
N	2400	2400	2400	2400	2400	2400
LL	-1313.621	-1202.785	-1270.405	-1204.649	-1191.344	-1189.832
BIC	2681.724	2460.053	2595.292	2463.781	2444.955	2441.929

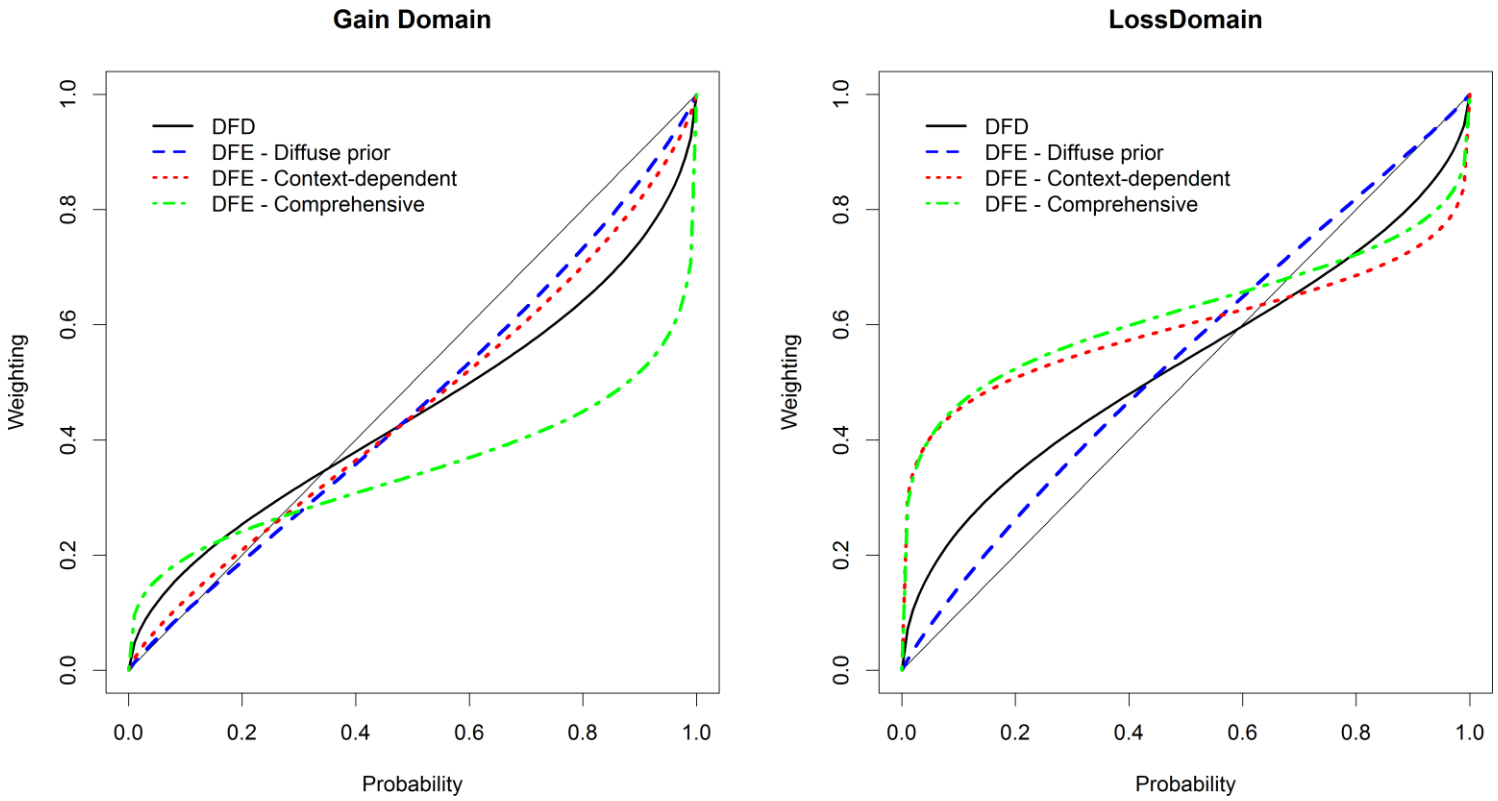
Notes: Cluster-robust standard errors are given in parenthesis. Stars indicate DFD – DFE gap. * $p < 0.05$. ** $p < 0.01$. *** $p < 0.001$.

The estimation results based on *context-dependent expectations* imply different conclusions about the gap depending on the prior assumptions. Under the assumption of *ignorance prior*, the gap is persistent in the gain domain; and insignificant in the loss domain. Under the assumption of *Carnap prior*, the gap is insignificant in the gain domain; and significantly reversed in the loss domain. Carnap's constant c is marginally different from 0 (p=0.078). The estimated $c = 0.202$ means that for the median number of 5 draws, a never-observed rare outcome receives almost 2% probability. Based on the BIC scores, the *Carnap prior* accounts for the data better than the *ignorance prior*.

The estimations based on *comprehensive expectations* consistently indicate a reversed DFD-DFE gap in both the gain and the loss domains, although these are not significant. Under the assumption of *Carnap prior,* the parameter c is different from 0 ($p = 0.012$). The estimated $c = 1.038$ means that a never-observed rare outcome receives almost 6% probability for the median sample size. As in the case of *context-dependent expectations*, the *Carnap prior* accounts for the data better than the *ignorance prior* based on BIC scores. It is also worth noting that the significant gap observed in utility functions under the models with *ignorance prior* suggests unwarranted interactions between model parameters under this assumption.

The models with the *Carnap prior* account for the data better than the model with *diffuse prior* ($BayesFactor > 10^3$ for both context-dependent and comprehensive expectations; see Wagenmakers, 2007). *Comprehensive expectations* perform slightly better than *context-dependent expectations*. The resulting probability weighting functions with the *Carnap prior* are in Figure 3.

Figure 3. Probability weighting functions from the data set of Erev et al. (2010)



Notes: Solid black lines show probability weighting under DFD. Dashed blue lines show probability weighting under DFE when the diffuse prior is used. Dotted red lines show probability weighting under DFE based context dependent expectations. Dot-dash green lines show probability weighting under DFE based on comprehensive expectations.

Discussion

DFD-DFE gap

The weighing of uncertainty under DFE concerns both the probabilistic inference and probability weighting. The aforementioned two-stage model gives a refined analysis of probability weighting under DFE by modelling probabilistic inference with a rigorous Bayesian method of updating. The findings with the two-stage model showed that the rational updating of symmetric priors accommodates the commonly found regressive probability estimations, and it explains a considerable part of the reversed DFD-DFE gap. The remaining gap is explained by the source dependent probability weighting. The reanalysis of the classic DFD-DFE gap confirmed the validity of the two-stage model by revealing the persistence of the enhanced likelihood insensitivity under DFE, which is consistent with the reversed DFD-DFE gap.

There are two possible factors that may give rise to different probability weighting under DFD and DFE: sampling experience and ambiguity. Contrary to the reversed DFD-DFE gap, previous studies by van de Kuilen & Wakker (2006), van de Kuilen (2009) and Humphrey (2006) on experienced risk; and studies by Ert & Trautmann (2014) and Kemel & Travers (2016) on experienced ambiguity report that the sampling experience reduces, rather than enhances, likelihood insensitivity. While the experienced ambiguity in the present study can explain the discrepancy with the former studies on experienced risk, a possible reason for the discrepancy with the previous studies on experienced ambiguity can be different methodologies used in these studies. Kemel & Travers (2016) uses certainty equivalents of experienced prospects, rather than binary choice data, to elicit PT parameters. Therefore, their method requires comparisons of experienced prospects with explicitly described certain outcomes. Similarly, Ert & Trautmann (2014) focuses on choices between experienced ambiguous prospects and described risky prospects. Future research can clarify the impact of sampling experience when the choice is between two experienced ambiguous prospects as in Glöckner et al. (2016) and Erev et al. (2010).

The enhanced likelihood insensitivity is a common finding in the ambiguity literature (Wakker, 2010, p. 292). This residual deviation from Bayesian rationality can be explained by perceived ambiguity in estimated probabilities (Dimmock, Kouwenberg, Mitchell, & Peijnenburg, 2015). Specifically, acknowledging the uncertainty about his probability estimation, the decision maker can consider a range of possible probabilities around his estimate. However, the range is very likely to be asymmetric around small probabilities such as 5% because there is much more room between 5% and 100% than between 0% and 5%. As a result, a decision maker who is weighting the rare outcome with an average of minimum and maximum of the perceived range of probabilities – as in the $\alpha - maxmin$ model (Hurwicz 1951; Luce & Raiffa 1957) – is likely to assign a weight higher than the small probability estimate of 5%. Such multiple prior accounts of ambiguity are common in the behavioral economics literature (Baillon, Bleichrodt, Keskin, L'Haridon, & Li , in press; Chateauneuf, Eichberger, & Grant, 2007; Ghirardato, Maccheroni, & Marinacci, 2004; Gilboa & Schmeidler, 1989; Marinacci, 2015).

The Bayesian method of updating

This paper utilizes a tractable Bayesian updating method in analyzing subjective probabilities under DFE. Despite its promising performance in accounting for the previous empirical findings, the descriptive validity of the method can be questioned. First, as in any Bayesian approach, in Carnap's method, the strength and the weight of evidence, i.e. $\frac{n_i}{N}$ and N, are combined in a way that they receive equal emphasis in evaluation. Notwithstanding this normative property, an influential study by Griffin & Tversky (1992) indicates that people systematically focus on the strength of evidence while paying insufficient attention to the credibility. This tendency results in either representativeness (overconfidence) or conservatism (underconfidence) in probability judgments. A recent study Kvam & Pleskac (2016) replicates these findings in an environment where the information is accumulated by observation similar to the sampling paradigm.

Although Carnap's method cannot differentiate the relative impact of the strength and the weight of evidence, biases similar to representativeness and conservatism can still be observed by negative values of c. In particular, $-N < c < 0$ implies too much updating in the direction of sample information resembling to representativeness, and $c < -N$ implies improper updating in the direction of prior beliefs resembling to an extreme case of conservatism.

Second, the Bayesian updating method assumes perfect memory where there is no recency effects in probability judgments. Although some early studies report recency effects in DFE, the evidence is still mixed in the literature (see the comprehensive meta-analysis by Wulff et al. 2016). Nonetheless, there are also ways to capture these effects within the Carnap's formula. One way is to simply use the sampling information from the second half of the observed sequence of outcomes as if the first half is forgotten. Such modelling of recency effects is discussed in Ashby & Rakow (2014) and in Wulff & Pachur (2016) (the sliding window model). Another way is to assign different weights to the relative frequencies observed in the first and in the second half of the sequence. For example, taking N_1 and N_2, the sample sizes of the first and second half the observations with $N_1 + N_2 = N$, one can use $N_1' = N_1$ and $N_2' = \varphi N_2$ where $\varphi < 1$ implies recency.

Conclusion

The preceding literature on DFE has extensively argued for the role of sampling error and ambiguity in the DFD-DFE gap. This study points to another important factor, being prior beliefs. The Bayesian approach, taken as a working hypothesis in this study, is shown to be useful in resolving the controversy about the gap by offering a tractable way to analyze prior beliefs. Importantly, the DFD-DFE gap is almost fully accommodated when prior beliefs are taken into account. The residual gap is explained by perceived ambiguity. Bayesian updating does remarkably well in explaining experimental data despite its normative nature. A promising topic for future research will concern more descriptive methods for analyzing beliefs under DFE.

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