Chapter 9

Concluding Remarks

This chapter concludes the thesis. A summary of the main findings is presented in Section 9.1, while Section 9.2 contains suggestions for future research.

9.1 Summary of the Main Conclusions

This thesis has largely focused on the development and behavior of outlier robust procedures for detecting random walk like behavior in time series. This section summarizes the main conclusions. The introductory Chapters 1 and 2 are excluded from the discussion, because they do not contain new research material.

Chapter 3 dealt with the properties of the pseudo maximum likelihood (PML) estimator based on the Student t likelihood in the simple location/scale model. In contrast to what is commonly believed, this estimator is, in general, not robust to outliers if the degrees of freedom parameter is estimated using the Student t based PML estimator. However, it appeared from a small simulation experiment that the effect of estimating the degrees of freedom parameter on the robustness properties of the location parameter was negligible in situations of practical interest. Therefore, two main conclusions follow from Chapter 3. First, one should carefully define the parameters of interest. Estimators and inference procedures for these parameters should be robust, while estimators for the nuisance parameters may, in certain cases, be nonrobust. Second, statements about the robustness of statistical procedures should be as detailed as possible. For example, the unboundedness of the influence function and of the change of variance function of the Student t PML estimator is only a qualitative result. This result has to be supplemented with additional information pertaining to the robustness of the statistical procedure. Such information can be provided in the form of, e.g., a Monte-Carlo simulation experiment. Alternatively, the rate of divergence of the influence function and of the change-of-variance function can be provided. In the particular setting studied in Chapter 3, the influence functions of the estimators for the scale and the degrees of freedom parameter were unbounded, but they only diverged at the rate $\ln(y)$ for large outliers y. This suggests that the effect of outliers

on, e.g., the MLT scale estimator is much smaller than the effect of similar outliers on nonrobust estimators of scale, like the traditional sample standard deviation.

Chapters 4 through 6 dealt with the unit root testing problem in the univariate context. Using robust estimates of the parameters of an autoregressive time series model, outlier robust unit root tests were constructed. It was shown that these tests perform better than the tests based on the ordinary least-squares (OLS) estimator for time series with outliers. The improvement is twofold: 1. the level of the robust test is more stable that that of the OLS based test if outliers are added to the sample; 2. the outlier robust tests are more powerful than the OLS based tests if the errors in the model are leptokurtic. As an important side result of Chapters 4 through 6, it was established that part of the nonrobustness of the OLS based unit root tests can be removed by using heteroskedasticity consistent standard errors.

From the asymptotic distribution theory presented in Chapter 6 it appeared that the use of alternative estimation techniques in a nonstationary context is not innocuous. The limiting distributions of the OLS based unit root test and of the outlier robust unit root tests differ. This contrasts with the results obtained in a stationary setting. The difference between the asymptotic distribution of the OLS based and the robust unit root tests crucially depends on one parameter, namely the (long run) correlation between the errors in the autoregressive model representation of the series, and the (pseudo) score of these errors. New critical values had to be obtained for the outlier robust test. These values were approximated by means of Monte-Carlo simulation in Chapters 4 through 6. Alternatively, the critical values of the asymptotic distribution can be obtained by numerical integration using the results of Chapter 6 and of Abadir (1992). The usefulness of such critical values for small to moderate sample sizes, however, is an important topic for future research (see the discussion of Chapter 8, below).

Also in a Bayesian context, outliers were shown to be a serious problem for unit root analysis. As in the classical context, the sensitivity of Bayesian unit root inference is closely linked to the chosen specification of the likelihood. If a Gaussian likelihood is employed, posterior results are, in general, not robust. By choosing a different likelihood, namely the Student t, this sensitivity problem is partly solved.

The empirical examples in Chapters 4 and 5 again supported the claim that the robust unit root tests are less sensitive to outlying observations. Moreover, it was illustrated that robust estimators can be used to signal certain types of model failure. Robust estimators provide the user with weights for the observations in the sample (see Chapters 1 and 2). These weights contain valuable information, revealing which observations fit into the model and which observations show a discordant behavior. This information can be usefully exploited in empirical work (see also Franses and Lucas (1995)). Finally, robust estimators were shown to provide satisfactory results for model selection in a context with outliers (see Chapter 5).

Chapters 7 and 8 dealt with the unit root testing problem in a multivariate context, also called cointegration testing. Chapter 7 discussed cointegration tests based on the pseudo likelihood ratio principle, while Chapter 8 focused on the Wald and LM cointegration tests based on outlier robust regression estimators. It was shown that outlier robust cointegration tests can be constructed rather straightforwardly. Moreover, in cases where the error process in a VAR representation of the time series is leptokurtic, these outlier robust cointegration tests have a much better power performance than the commonly used, nonrobust test of Johansen (1988, 1991) and than the OLS based test of Kleibergen and van Dijk (1994). This was illustrated in Chapters 7 and 8 by means of simulations.

The limiting distributions of the robust cointegration tests were derived. As in the univariate case, these limiting distributions differ from the limiting distributions of the OLS based test statistics. The difference is caused by the presence of nuisance parameters in the limiting distribution of the robust tests, namely the canonical correlations between certain linear combinations of the innovations and their pseudo scores. For the pseudo likelihood ratio test, an additional set of nuisance parameters enters the limiting distribution due to the (possible) discrepancy between the true likelihood and the postulated pseudo likelihood. These additional parameters do not enter the limiting distribution of the Wald and Lagrange multiplier (LM) test (compare White (1982) for a similar result in the stationary context).

Based on a mean squared error criterion, the optimal choice of the pseudo score was discussed in Chapter 7. It turned out that the optimal pseudo score is a linear combination of the true likelihood score and the Gaussian pseudo likelihood score. This result extends the findings of Cox and Llatas (1991) to the multivariate setting. Using a near non-cointegration framework, the relative weight of the Gaussian pseudo score in the optimal score function was computed for a simple example. It turned out that: 1. the more leptokurtic the innovations, the larger the share of the true likelihood score in the optimal score function; and 2. the further one is away from the null hypothesis of no cointegration, the larger the share of the true likelihood score.

Chapter 7 also discussed the construction of a Bartlett type correction for the robust cointegration tests. The motivation for the correction was to avoid the need of simulating new critical values of the cointegration test for every new choice of the pseudo likelihood. The corrected tests seemed to meet this objective in some circumstances, but not in others. Therefore, further research in this area is needed.

In Chapter 8, much attention was devoted to the presence of drift terms in both the data generating process and the fitted regression model. The limiting distributions of the cointegration tests turned out to be extremely sensitive to the presence of either type of drift term.

Chapter 8 focused on the Wald cointegration test of Kleibergen and van Dijk (1994). An outlier robust variant of this test was developed. It appeared that the Wald test is extremely sensitive to the ordering of the variables in the

system. A simple bivariate example was constructed where the actual size of the test was as high as 75% at a nominal level of 5%. This result was only due to the incorrect ordering of the variables. In order to avoid these large size distortions, the possibilities were investigated for constructing a pretest procedure in order to check the ordering of the variables. Disappointingly, it turned out that the simple pretest investigated in Chapter 8 could not be used. Therefore, as a general conclusion from Chapter 8, it follows that the Wald cointegration test can, as yet, not be safely used for inference.

A Lagrange Multiplier (LM) test for cointegration was also briefly discussed in Chapter 8. It was shown that the limiting distribution of this test statistic has as few nuisance parameters in it as that of the Wald test. Moreover, as with the pseudo likelihood ratio test of Chapter 7, the limiting distribution of the LM test turned out to be invariant to the ordering of the variables. These two properties make the LM test the most promising candidate for constructing outlier robust cointegration tests. Further research into the properties of the outlier robust LM test for cointegration is, therefore, warranted.

9.2   Directions for Future Research

Section 9.1 already provided two topics for future research. First, one can further investigate the possibilities for correcting robust testing procedures such that standard critical values apply. A first step in this direction was taken in Chapter 7 by the construction of a first order Bartlett type correction. Second, the properties of the LM test have to be studied in more detail. As the LM test seemed to be the most promising candidate for constructing robust cointegration tests, its properties have to be evaluated under a variety of circumstances.

As appeared from Chapter 8, the asymptotic distribution of robust cointegration tests can be a bad approximation to the finite sample distribution of these tests. It would, therefore, be a valuable contribution if the results of Abadir and Larsson (1994) could be extended to estimators based on non-Gaussian pseudo likelihoods. This, however, is far from trivial.

Another possibility for contributing to the existing literature would be to develop outlier robust procedures for more complicated econometric models. One can think of nonlinear models, but also of models in a panel data context (see, e.g., Lucas et al. (1994) and van Dijk et al. (1994)). Alternatively, empirical applications of the techniques described in this thesis would be useful to further illustrate the use of outlier robust procedures in settings of practical interest (see, e.g., Franses and Lucas (1995)).

Related to the subject of empirical applications is the development of efficient algorithms for computing robust estimators. For simple regression models, standard algorithms can be used (see, e.g., Hampel et al. (1986)). In other contexts, e.g., nonlinearity in the parameters (cointegration), convergent algorithms are much more difficult to obtain. The availability of such algorithms

is a first requirement for the robust procedures in this thesis to be applied in practice.[1]

Finally, more research has to be put into the development of outlier robust model selection and model evaluation procedures. Diagnostic tests to assess the adequacy of robustly estimated regression models seem an important ingredient of serious econometric model building. Of course, the development of such diagnostic tests is somewhat in contrast to the original idea of robust estimation. The original idea of robust estimation was to protect the user from possible violations of assumptions underlying the model or the estimation method (see Chapter 1). In practice, however, applied researchers do not only want to be *protected*, but they also want to *detect* the type of model failure they have to deal with. For the nonrobust OLS estimator, diagnostic tests are amply available. It is an interesting topic for future research to find out whether these traditional diagnostic tests can, in some way, also be used in a context of robust estimators.

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[1]For univariate, single-equation models, I have written an interactive program, RIPE, that can compute several robust estimators. The program has some graphical possibilities, a data interface, and is intended to be reasonably user-friendly and interactive. It can be obtained upon request. A Gauss routine for performing robust cointegration tests a Student t pseudo likelihood is also available upon request.