How do we pay with euro notes?

Empirical evidence from Monopoly® experiments

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Abstract

In contrast to abundant theoretical literature, there are almost no empirical studies on how individuals actually deal with cash. In this paper we analyze euro transactions, made during various games of Monopoly® (European edition), where in some games we deliberately left out one of the notes. We find that not having access to 100-euro or 10-euro notes is not as problematic for payment behavior as it is for not having 200-euro and 20-euro notes. We also find that the 50-euro note plays a crucial role in actual payments.

Keywords: euro cash, Poisson regression model, payment behavior, and experiments
1. Introduction

Euro cash consists of seven banknotes with values 500, 200, 100, 50, 20, 10 and 5, and eight coins with values, 2, 1, 0.5, 0.2, 0.1, 0.05, 0.02 and 0.01. The choice for this denominational range is guided by theoretical arguments. That is, a 1-2-5 range, like the euro has, is theoretically the denominational structure that approaches the optimal one for a payment system, see Boeschoten and Fase (1989), and Cramer (1983), among others. Most countries that now carry the euro already had a 1-2-5 range for their currency prior to the introduction of the euro. The notable exception concerned the Netherlands, which had the 1-2½-5 range for the then Dutch guilder. For some countries the introduction of the euro implied a substantial change in terms of the actual value on the notes and coins. For example, for Italy, the values went from 1000-2000-5000 to 1-2-5. For other countries, like Germany, the current euro range is close to what it was before in terms of the German mark.

Interestingly, and in marked contrast to a substantial literature on the theory of a denominational range, there are almost no empirical studies on how individuals actually deal with cash. That is, there are only a few studies, which examine whether paying individuals, in some sense, actually behave according to the theory when they make cash payments. An exception is the questionnaire-based study of van Everdingen and van Raaij (1998), which concerns stated perceptions of how individuals would face the transition from the local currency to the euro. Two other exceptions are Kippers et al. (2003) and Kippers and Franses (2003a), and in the present paper we build on these two studies.

Kippers et al. (2003) analyzed a large sample of cash payments, collected in 1998, which concerned the Dutch guilder. They construct a statistical model for this purpose and they examine whether paying individuals would be indifferent to using notes and coins, which in turn would imply that the paying public optimally uses the 1-2½-5 range. Unfortunately, the data collected at that time, 1998, did not include the wallet content, and hence Kippers et al. (2003) had to resort to simulated wallets to draw proper inference. An important finding in that study is that Dutch individuals were not at all indifferent to various coins and notes, and, in fact, the 2½-guilder coin and the 50-guilder note were used less often than would have been implied by theory. Note that this result could not have been obtained by simply observing and counting what people do when they make a payment, as each payment depends on the amount to be paid as well as on the wallet content.
Kippers and Franses (2003a) aimed to redo the analysis in Kippers et al. (2003) with two main modifications. First, the analysis now concerns the euro, and also substantial efforts were made to collect data, which now also include the wallet content prior to a payment made. The main conclusion from this second study is that Dutch individuals were indifferent to the denominational range of the euro. Hence, for the Netherlands, the introduction of the euro seems to have facilitated actual payment behavior. Unfortunately, this exercise cannot be repeated for the other euro countries, as, as far as we know, no transactions data have been collected concerning the currency prior to the introduction of the euro. Countries that still allow for such a comparison, at least, when they would decide to eventually change to the euro, are Denmark, Sweden, England, Hungary and various other countries.

In the present paper we build on the analysis of Kippers and Franses (2003a). The key features of the data in that study are (i) that individuals who were involved at the data collection stage could have had very different wallet contents, that (ii) some individuals might have faced more “difficult” amounts to pay, and that (iii) individuals were buying very different products, which could in turn involve different preferences. Additionally, the data in Kippers and Franses (2003a) were collected in a supermarket and in a hardware and home improvement store, and these stores could have used different pricing and rounding strategies. Hence, it would now be of interest to collect data in a controlled setting. Such a setting would be very difficult and expensive to realize in an actual store or supermarket. Also, to make individuals to buy the same type of products would not be easy. Therefore, we decided to collect data while observing individuals playing a game. The European edition of Monopoly® turned out to be the best kind of game, as during such a game, people face the same items to purchase, use the same amounts, and their utility functions are similar as they all want to win the game. The European edition of Monopoly® uses (toy-) notes of 500, 100, 50, 20, 10 and 5 euro, and a plastic coin of 1 euro. These notes all look very similar to the euro banknotes currently in use in reality. As there is no 200-euro note, we arranged to have copies made using a similar design, see Figure 1 for an example. Hence, in the present empirical study we will examine if the findings in Kippers and Franses (2003a) carry over to the case where individuals pay with euro notes in an experimental setting.

A second aspect of collecting data through an experiment is that we can manipulate the settings. In order to better understand how people behave, we decided to play various games (or better: have the games played, as we ourselves were not involved), while we each
time removed one of the euro notes. Hence, we study what happens when certain notes are not available. In practice this can happen when banks or stores run out of certain denominations. The theoretical results in Kippers and Franses (2003b) suggest that having no 100-euro and 10-euro notes has the smallest deteriorating effects on making payments, while the lack of 50-euro notes gives substantial problems. It seems unlikely that we can ever obtain real-life data for these situations, and therefore we resort to experiments. With these we can see how payment behavior changes in case of the unavailability of a single denomination. This can provide useful information for banks and cash counters.

The outline of our paper is as follows. In Section 2, we describe the way we collected the data. In Section 3, we discuss the statistical model used to summarize the data, and we give details of our statistical testing procedure. In Section 4, we present the empirical outcomes and in Section 5 we conclude.

2. Data collection

In this section we discuss how we collected the data. We also discuss the minor modifications we made to the rules of the game, to ensure that we would collect adequate data.

First we deal with the sample size, as data collection is rather time-consuming in our case. The simulation results in Kippers and Franses (2003a) indicate that we need something like 250 observations to have reasonably high power of the statistical tests we aim to carry out in our model to draw conclusions. It turned out that playing the game twice with 3 or 4 players for a period of about 2 hours could ensure this amount of observations. The person, who played as bank, kept records of all transactions. After one hour there was a short break so that we could write down the wallet contents, in order to allow for an intermediate check. We played games with all dominations, that is, with 500, 200, 100, 50, 20, 10, 5, and 1 and we played five additional games where each time we removed one of the denominations. As the 500-euro note and the 5-euro noted turned out to be mandatory for their use in all relevant cases, and also because the model to be discussed below needs a bottom-valued benchmark denomination, which here amounts to the 1-euro coin, we played these five games without the 200-euro note, one time without the 100-euro, without 50, without 20 and finally without the 10-euro note. Hence, in total we had 12 games played.
The individuals who participated in the games ranged in the age of 18 to 56, and there were about as many men and women. The players had different educational backgrounds, as they consisted of undergraduate students, academic colleagues and administrative assistants. In total there were 51 participants. Neither one of them was informed about what the experiments were about nor were they told that they would receive information about the purpose only as early as that this paper would have been written. In fact, we informed them that the data were collected to understand strategic behavior in games, which in fact was not really true.

As said, the European edition of the Monopoly® game uses notes of 500, 100, 50, 20, 10, and 5, and a coin of 1. As the genuine denominational range of the euro also contains 200, we had to make a new note. It turned out that nobody of the participants played this game ever before, so neither one of them was aware that our 200-euro note was new. Specifically for this purpose we had the note made, and it is depicted in Figure 1. In reality the size of this note is the same as the size of all other toy notes, and hence it is obvious that it is not a genuine note, also as it is only printed on one side. In order to make sure that we would never run out of notes of any value, we also had additional copies made of the available notes. Hence, in all situations the bank (within the game) would have an abundant availability of notes.

There were two more modifications to the game as it is outlined in the booklet of the European edition of Monopoly® and these are the following. Sometimes people play with an additional fund, where fines can be deposited. This possibility was excluded. Also, we issued wallets to all players and they were told not to inform the other players what was in the wallet. This prevented that they could help with meeting payment amounts by matching. Naturally, all players started with the same amount in the wallets.

In the end, we collected amounts of observations within the range of 210 to 280. All data on (i) wallets contents prior to each transaction, (ii) the way of payment at the transaction occasion and (iii) the money received in return were checked for correctness. The return cash was used to keep track of the wallet contents. The data were recorded in Excel and in EViews, where the latter program was also used to estimate the parameters in the model.
3. Statistical model

In this section we review various details of the statistical model. A full description appears in Kippers et al. (2003).

3.1 The model

The key feature of the model is the probability that an individual $i$ selects a combination of notes and coins, represented by a $D$-dimensional discrete random variable $Y_i$, in order to pay an amount $A_i$, given a wallet content $w_i$. $D$ corresponds to the number of denominations in the denominational range under scrutiny. The realizations of this random choice variable are denoted by $y_i$.

For modeling purposes we impose a hierarchical structure. According to this structure the choice of an individual is modeled separately for each denomination, starting from the highest denomination and ending with the lowest. The choice for a certain denomination is conditioned on previous denominations. Hence, the probability structure is

$$
\Pr[Y_i = y_i | w_i] = \Pr[Y_{D,i} = y_{D,i} | w_i] \Pr[Y_{D-1,i} = y_{D-1,i} | y_{D,i}, w_i] \ldots \Pr[Y_{1,i} = y_{1,i} | y_{2,i}, \ldots, y_{D-1,i}, w_i] \tag{1}
$$

The choice of payment with $y_{d,i}$ tokens of denomination $d$ is assumed to be distributed as truncated Poisson, where the choice set is restricted from below by a lower bound $lb_{d,i}$ and from above by an upper bound $ub_{d,i}$, denoted by an indicator function $I[\ldots]$, that is,

$$
Y_{d,i} | y_{d+1,i}, \ldots, y_{D,i}, w_i \sim POI(\exp(x'_{d,i} \beta_d)) \times I[lb_{d,i}, ub_{d,i}] \tag{2}
$$

where $\beta_d$ is a parameter vector and $x_{d,i}$ contains explanatory variables for denomination $d$.

The upper bound represents the maximum number of tokens the individual can use for payment. In principle this upper bound is determined by the wallet content. Obviously, an individual cannot pay with more tokens than are available in his or her wallet. In some cases, however, the bound is determined by another criterion. It is assumed that individuals do not pay with a token if it is unnecessary, that is, if it is expected that the same token will be
returned as change, see Cramer (1983). The upper bound is determined by one of these criteria whichever is more restrictive. The lower bound represents the minimum number of tokens of a denomination in order to make the payment. This minimum is determined by the combination of the payment amount and the availability of other denominations in the wallet. If the wallet does not contain enough of the lower denominations to pay the amount, then an individual is forced to use a minimum number of tokens of the denomination under consideration. The upper and lower bounds are defined as

\[
ub_{D,i} = \min \left( \text{ceil} \left( \frac{A_i}{v_D} \right) w_{D,i} \right)
\]

(3)

\[
ub_{d,i} = \min \left( \text{ceil} \left( \frac{A_i - \text{amount}(y_{d+1,i}, ..., y_{D,i})}{v_d} \right) w_{d,i} \right) \quad \text{for } d=D-1, ..., 1
\]

(4)

\[
lb_{D,i} = \max \left( \text{ceil} \left( \frac{A_i - \text{amount}(w_{1,i}, ..., w_{D-1,i})}{v_d} \right) 0 \right)
\]

(5)

\[
lb_{d,i} = \max \left( \text{ceil} \left( \frac{A_i - \text{amount}(y_{d+1,i}, ..., y_{D,i}) - \text{amount}(w_{1,i}, ..., w_{d-1,i})}{v_d} \right) 0 \right) \quad \text{for } d=D-1, ..., 1
\]

(6)

where \(v_d\) is the value of denomination \(d\) and where

\[
\text{amount}(x_p, ..., x_q) = \sum_{k=p}^{q} v_k x_k
\]

(7)

If \(lb_{d,i} = ub_{d,i}\), the individual has no freedom of choice and hence

\[
\Pr[Y_{d,i} = y_{d,i} = lb_{d,i} = ub_{d,i}, y_{d+1,i}, ..., y_{D,i}, w_i] = 1
\]

(8)

The main explanatory variable is given by the payment amount.

In addition to an intercept we include the explanatory variable ACORR, defined as the amount to be paid minus the value of the payments chosen for higher denominations, scaled to its face value (hence CORR of “correction”), that is,

\[
ACORR_{D,i} = \ln \left( \frac{A_i}{v_D} \right)
\]
We expect the parameter for this variable to be positive for each denomination. Also, we might expect that the parameter is statistically significantly different from zero (provided one has a large enough sample), as it means that if there is more need to use a certain note, it is also more likely to be used. For a more elaborate description of the model, as well as for an outline of the estimation method, we refer to Kippers et al. (2003). We have written a program in EViews for parameter estimation. As a courtesy to the reader, it is given in the appendix.

3.2 Statistical testing

If individuals do not reveal any preferences for any denomination, and hence are indifferent to the use of notes, the probability of choosing a denomination for payment would be equal for all denominations in similar payment situations. In our cash payment model, similar payment situations are represented by equal lower and upper bounds, and by equal values for ACORR. Therefore, indifference implies that intercept parameters are the same across notes, and also that the ACORR parameters are the same across the notes. The bottom part of the appendix displays the relevant parameter restrictions for the case where we abstain from considering the 500-euro note.

The hypothesis of equal parameters can formally be tested using a likelihood ratio (LR) test. Two models are compared, one with the restriction that the parameters are equal across denominations, and a second with unrestricted parameters. The LR statistic is defined as $-2\ln(L_{UR}/L_{R})$, where $L_{UR}$ represents the likelihood of the unrestricted model and $L_{R}$ represents the likelihood of the restricted model, and it follows a chi-square distribution with the number of restrictions as degrees of freedom. This distributional result was supported by the simulations in Kippers and Franses (2003a). Rejection of the null hypothesis suggests that the parameters significantly differ across denominations. In that case, there is no indifference towards denominations by individuals, and certain denominations are (more or less) preferred.

We use the test in two ways. First, we test whether all parameters are equal. Subsequently, we reduce the model by setting parameters equal until the LR test rejects further hypotheses of equal parameters.
As the parameter values themselves are difficult to interpret due to the high degree of non-linearity of the model, we decide to visualize the outcomes of all the final models. We can visualize the estimation results for the different denominations by plotting expected values on the basis of the estimated parameters, and for a range of values of the ACORR variable. That is, for a given value of ACORR, we compute the choice probabilities between the upper bounds and the lower bound 0 and we construct the expected value of $Y$. Various resulting curves are presented in Figure 2 and the following graphs. The curves show a stepwise pattern. The jumps correspond with an increase of the upper bound by 1. Given that the upper bound depends on the remaining amount to be paid, its value changes with the increase of ACORR. A higher expected value implies that a note is used more than other notes in similar payment situations. We will make graphs for each note, for all six games, that is, for the case with all notes and for the five games where each time a note has been removed. As such, we can see if notes are more or less used in case other notes have been removed.

For the interpretation of the empirical results we switch between LR tests and graphs. The LR-test will tell us if the hypothesis of indifference is rejected or not. The graphs will give us insights into the denominations causing differences in preferences in case of rejection.

4. *Empirical results*

In this section we first discuss the case with all notes, and next we turn to the five cases where each time a note has been removed.

4.1 *All denominations*

To give an impression of typical estimation results to be obtained from our model, we give the estimated parameters for the unrestricted model for the “all notes” case in Table 1. Almost all intercept parameters are not significantly different from zero, while all ACORR parameters are significant and within the range of 1.5 to 3.

The first row of Table 2 gives the LR statistic for the restriction that all intercepts are equal and that all ACORR parameters are equal. We give the LR test result for the case where the parameters for the 500-euro note are unrestricted. This is more relevant as the parameter values for the 500-euro note tend to be higher than the others due to the limited range of
ACORR values for which the parameter is estimated, see also Table 1. Hence, the LR test could likely suggest rejection due to the deviating parameters for the 500-euro note. We therefore focus on indifference towards all denominations while excluding the 500-euro note, and focus on the LR test of equal parameters across these denominations as presented in Table 2.

The LR statistic for the experiment with all notes included equals 19.567, and this is significant at the 5 per cent level, compared with the chi-square (10) distribution. This means that our finding for the experimental data is not in accordance with the finding in Kippers and Franses (2003a). There, it was concluded on the basis of observed euro cash payments in retail stores, that there appeared to be no preferences for using certain euro notes. While there were no significant preferences for these payments, now a significant preference does seem to be present in the payments observed in an experimental setting. Indeed, the curves in Figure 2 clearly do not coincide.

We reduce the model until a set of restrictions is found for which the LR test no longer rejects. The first panel in Table 3 presents the parameters of the reduced model for the experiment with all notes. From this, we can conclude that the use of the 50-euro note is significantly different from the use of the other notes. From Figure 2 we can tell that the 50-euro note is used far more in the upper range of ACORR values, while it is used less than the others in the lower range of ACORR values. Apparently, higher amounts in the game are paid more with 50-euro notes than one would expect.

The difference in payment behavior between the experimental setting and real-life situations should be sought in the difference of payment amounts. The median payment amount in the experiment with the full banknote range is 100 euro, while in real-life cash payments in real life usually concern lower amounts with a median around 10 euro. Although the players of the game encountered rather unfamiliar high payment amounts, they were able to pay with notes that resemble the euro notes they use in real life. Apparently, the players resort to the familiar 50-euro note to make high amount payments.
4.2  **Leaving out one note**

Table 4a to 4e give the parameter estimates for the cases where we each time leave out a single note. Table 2 gives the relevant LR tests and Figures 3 to 7 present the corresponding graphs.

We first observe from the graphs that payment behavior becomes less erratic, in the sense that lesser lines intersect, if the 100-, 50- or 10-euro note is left out. This is confirmed by the outcomes of the LR tests displayed in Table 2, as the null hypothesis of equal intercepts and ACORR parameters across the denominations cannot be rejected for these cases.

Consider the case where the range of notes in the game does not include a 200-euro note, as in the original Monopoly® game. The LR test suggests rejection, and from Figure 3 it is clear that this is caused by a preference for the 50-euro note for payment amounts higher than its nominal value. The final model for which the parameter estimates are given in the second panel of Table 3 confirms this. Apparently, the preference for a 50-euro note is not cancelled out by removing the 200-euro note from the range. In contrast, the removal of the 100-note clearly does make the 50-euro less preferred, as we can tell from Figure 4, and from the outcome of the LR test in Table 2.

In case we leave out the 50-euro note, the curves of the expected values of $Y$ in Figure 5 are remarkably close together. Indeed the LR test confirms indifference towards denominations in this experiment (see again Table 2). However, in case the 20-euro note is removed from the range, aberrant payment behavior with the 50-euro note shows up again, as it also did in the full range case. Furthermore, as we can see from the final models presented in Table 3 and the curves in Figure 6, only for a few denominations equal payment behavior emerges. Hence, removing the 20-euro note has created additional preferences for other notes.

Lastly, the experiment without the 10-euro note shows a balanced use of all remaining denominations. The curves of expected use in Figure 7 lie close together, and the LR statistic in Table 2 is 14.451, which is not significant at the 5 per cent level.
5. Conclusions

The empirical results in this paper, based on various games of the European edition of Monopoly®, suggest the following conclusions. The first is that if all notes are available, the participants of the game use the 50-euro note in another way than other denominations in similar payment situations, which is in contrast to the findings of Kippers and Franses (2003a). Hence, the settings of the Monopoly experiment apparently induce preferences that do not exist in real life payment situations. The atypical use of the 50-euro note is however not surprising. This is the highest valued note that is most familiar for Dutch individuals. Indeed, in Kippers et al. (2003) a similar preference was found for its about equally valued note in the former guilder range, that is, the 100 guilder note.

A second conclusion is that the removal of the 100-euro or 10-euro note has a positive influence on payment behavior and this is in accordance with the theoretical results in Kippers and Franses (2003b). In these cases the aberrant payment behavior related to the 50-euro note is cancelled out. This suggests that banks or cashiers may, for a short while, not issue 100-euro or 10-euro notes, perhaps without substantial problems. As a consequence, payment behavior improves and the remaining range of banknotes is used optimally. A third conclusion is that the 200-euro and 20-euro notes are the most crucial notes. They cannot be left out as payment behavior does not improve in these cases, and even worsens for the case without the 20-euro note. Finally, removing the 50-euro note itself has a rather positive effect on payment behavior, that is, the other notes are used in the way they should be in case the paying public would be indifferent to the notes. Apparently, taking out the most familiar note is caught up perfectly by the remaining notes. In sum, the 50-euro note plays a crucial role in actual payments.

We see at least four avenues for further research. Changing the Monopoly® settings, for example by changing the payment amounts, might lead to results that are more consistent results with day-to-day payments in for example retail stores. Then, the emphasis would be on the notes regularly used for payment, that is, the 50-, 20-, 10- and 5-euro notes. Also, as the euro has been introduced quite recently, it might be of interest to repeat these experiments in the next year or two, in particular to examine the consequences of not having certain notes. Next, it would be quite exciting, now we are experimenting anyway and creating our own toy notes, to see how people would react if we were to introduce 25-euro notes, 40-euro notes or
300-euro notes, to mention just a few. Finally, it would be interesting to study how individuals in other countries behave when making cash payments. Of particular interest are (i) those countries where they do not (yet?) have the euro, like Denmark, Sweden and England, (ii) those countries which use a mixture of the 1-2-5 and 1-2½-5 ranges like the US (where there are quarter dollars and 2-dollar notes) and (iii) those countries where the coins and notes substantially differ in size and shape, like in Australia where the 50-AUD coin is rather large.
Appendix: Computer code in Eviews

@logl logl1

@temp c1 c2 c3 c4 c5 c6 c7 l1 l2 l3 l4 l5 l6 l7

\[
\begin{align*}
\lambda_1 &= \exp(c(11) + c(12) \cdot \text{acorr500}) \cdot (\text{ub500} > \text{lb500}) + 3 \cdot (\text{ub500} = \text{lb500}) \\
\lambda_2 &= \exp(c(21) + c(22) \cdot \text{acorr200}) \cdot (\text{ub200} > \text{lb200}) + 3 \cdot (\text{ub200} = \text{lb200}) \\
\lambda_3 &= \exp(c(31) + c(32) \cdot \text{acorr100}) \cdot (\text{ub100} > \text{lb100}) + 3 \cdot (\text{ub100} = \text{lb100}) \\
\lambda_4 &= \exp(c(41) + c(42) \cdot \text{acorr50}) \cdot (\text{ub50} > \text{lb50}) + 3 \cdot (\text{ub50} = \text{lb50}) \\
\lambda_5 &= \exp(c(51) + c(52) \cdot \text{acorr20}) \cdot (\text{ub20} > \text{lb20}) + 3 \cdot (\text{ub20} = \text{lb20}) \\
\lambda_6 &= \exp(c(61) + c(62) \cdot \text{acorr10}) \cdot (\text{ub10} > \text{lb10}) + 3 \cdot (\text{ub2} = \text{lb10}) \\
\lambda_7 &= \exp(c(71) + c(72) \cdot \text{acorr5}) \cdot (\text{ub5} > \text{lb5}) + 3 \cdot (\text{ub5} = \text{lb5})
\end{align*}
\]

\[
\begin{align*}
c_1 &= @\text{cpoisson}(\text{ub500}, \lambda_1) - @\text{cpoisson}(\text{lb500}-1, \lambda_1) \cdot (\text{lb500} > 0) \\
c_2 &= @\text{cpoisson}(\text{ub200}, \lambda_2) - @\text{cpoisson}(\text{lb200}-1, \lambda_2) \cdot (\text{lb200} > 0) \\
c_3 &= @\text{cpoisson}(\text{ub100}, \lambda_3) - @\text{cpoisson}(\text{lb100}-1, \lambda_3) \cdot (\text{lb100} > 0) \\
c_4 &= @\text{cpoisson}(\text{ub50}, \lambda_4) - @\text{cpoisson}(\text{lb50}-1, \lambda_4) \cdot (\text{lb50} > 0) \\
c_5 &= @\text{cpoisson}(\text{ub20}, \lambda_5) - @\text{cpoisson}(\text{lb20}-1, \lambda_5) \cdot (\text{lb20} > 0) \\
c_6 &= @\text{cpoisson}(\text{ub10}, \lambda_6) - @\text{cpoisson}(\text{lb10}-1, \lambda_6) \cdot (\text{lb10} > 0) \\
c_7 &= @\text{cpoisson}(\text{ub5}, \lambda_7) - @\text{cpoisson}(\text{lb5}-1, \lambda_7) \cdot (\text{lb5} > 0)
\end{align*}
\]

\[
\begin{align*}
l_1 &= (\text{ub500} - \text{lb500} > 0) \cdot (-\lambda_1 + \text{b500} \cdot \log(\lambda_1) - \log(c_1) - \log(@\text{fact}(\text{b500}))) \\
l_2 &= (\text{ub200} - \text{lb200} > 0) \cdot (-\lambda_2 + \text{b200} \cdot \log(\lambda_2) - \log(c_2) - \log(@\text{fact}(\text{b200}))) \\
l_3 &= (\text{ub100} - \text{lb100} > 0) \cdot (-\lambda_3 + \text{b100} \cdot \log(\lambda_3) - \log(c_3) - \log(@\text{fact}(\text{b100}))) \\
l_4 &= (\text{ub50} - \text{lb50} > 0) \cdot (-\lambda_4 + \text{b50} \cdot \log(\lambda_4) - \log(c_4) - \log(@\text{fact}(\text{b50}))) \\
l_5 &= (\text{ub20} - \text{lb20} > 0) \cdot (-\lambda_5 + \text{b20} \cdot \log(\lambda_5) - \log(c_5) - \log(@\text{fact}(\text{b20}))) \\
l_6 &= (\text{ub10} - \text{lb10} > 0) \cdot (-\lambda_6 + \text{b10} \cdot \log(\lambda_6) - \log(c_6) - \log(@\text{fact}(\text{b10}))) \\
l_7 &= (\text{ub5} - \text{lb5} > 0) \cdot (-\lambda_7 + \text{b5} \cdot \log(\lambda_7) - \log(c_7) - \log(@\text{fact}(\text{b5})))
\end{align*}
\]

\[\text{logl1} = l_1 + l_2 + l_3 + l_4 + l_5 + l_6 + l_7\]

Restriction corresponding with indifference, all notes

\[
\begin{align*}
c(21) &= c(31) = c(41) = c(51) = c(61) = c(71) \\
c(22) &= c(32) = c(42) = c(52) = c(62) = c(72)
\end{align*}
\]

(Example) restriction corresponding with indifference, all notes except for one

\[
\begin{align*}
c(21) &= c(31) = c(41) = c(51) = c(61), \ c(71) \\
c(22) &= c(32) = c(42) = c(52) = c(62), \ c(72)
\end{align*}
\]
Table 1:

Parameter estimates with standard errors (Se) for the full model.

The denominations are 500, 200, 100, 50, 20, 10, 5 and 1.

<table>
<thead>
<tr>
<th>Denomination</th>
<th>Intercept Parameter</th>
<th>Se</th>
<th>ACORR Parameter</th>
<th>Se</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>-0.049</td>
<td>0.845</td>
<td>2.969 a</td>
<td>1.474</td>
</tr>
<tr>
<td>200</td>
<td>-0.200</td>
<td>0.261</td>
<td>1.846 a</td>
<td>0.438</td>
</tr>
<tr>
<td>100</td>
<td>0.038</td>
<td>0.341</td>
<td>2.237 a</td>
<td>0.399</td>
</tr>
<tr>
<td>50</td>
<td>-0.044</td>
<td>0.389</td>
<td>2.770 a</td>
<td>0.640</td>
</tr>
<tr>
<td>20</td>
<td>-0.663 a</td>
<td>0.299</td>
<td>1.640 a</td>
<td>0.705</td>
</tr>
<tr>
<td>10</td>
<td>0.105</td>
<td>0.342</td>
<td>1.574 a</td>
<td>0.538</td>
</tr>
<tr>
<td>5</td>
<td>0.311</td>
<td>0.509</td>
<td>1.817 a</td>
<td>0.905</td>
</tr>
</tbody>
</table>

a Significantly different from zero at the 5 per cent level.

The sample size of this dataset is 217.
Table 2:
Likelihood Ratio tests for equality of the intercept parameters and for the ACORR parameters, where the parameters for the 500-euro note are left unrestricted

<table>
<thead>
<tr>
<th>Denominations</th>
<th>Sample size</th>
<th>LR test</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>217</td>
<td>19.567a</td>
</tr>
<tr>
<td>No 200</td>
<td>235</td>
<td>18.314a</td>
</tr>
<tr>
<td>No 100</td>
<td>226</td>
<td>9.576</td>
</tr>
<tr>
<td>No 50</td>
<td>280</td>
<td>12.186</td>
</tr>
<tr>
<td>No 20</td>
<td>210</td>
<td>18.024a</td>
</tr>
<tr>
<td>No 10</td>
<td>210</td>
<td>14.451</td>
</tr>
</tbody>
</table>

a Significantly different from zero at the 5 per cent level.

The LR test should be evaluated using the critical values of the chi-square (10) distribution in case of all notes and of the chi-square (8) distribution in case one of the notes is excluded.
Table 3: Estimation results for final models
(The results for the 500-euro note are not reported)

<table>
<thead>
<tr>
<th>Denomination</th>
<th>Intercept Parameter</th>
<th>Intercept Se</th>
<th>ACORR Parameter</th>
<th>ACORR Se</th>
</tr>
</thead>
<tbody>
<tr>
<td>All denominations</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All but 50</td>
<td>-0.202</td>
<td>0.138</td>
<td>1.812</td>
<td>0.169</td>
</tr>
<tr>
<td>50</td>
<td>-0.044</td>
<td>0.392</td>
<td>2.770</td>
<td>0.637</td>
</tr>
<tr>
<td>All denominations except 200</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All but 50</td>
<td>-0.316</td>
<td>0.140</td>
<td>1.817</td>
<td>0.233</td>
</tr>
<tr>
<td>50</td>
<td>-0.032</td>
<td>0.380</td>
<td>2.680</td>
<td>0.275</td>
</tr>
<tr>
<td>All denominations except 100</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>-0.045</td>
<td>0.136</td>
<td>1.583</td>
<td>0.073</td>
</tr>
<tr>
<td>All denominations except 50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>-0.263</td>
<td>0.120</td>
<td>1.632</td>
<td>0.064</td>
</tr>
<tr>
<td>All denominations except 20</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200-100</td>
<td>-0.170</td>
<td>0.211</td>
<td>1.861</td>
<td>0.148</td>
</tr>
<tr>
<td>50-5</td>
<td>-0.464</td>
<td>0.307</td>
<td>2.971</td>
<td>0.639</td>
</tr>
<tr>
<td>10</td>
<td>-0.012</td>
<td>0.372</td>
<td>1.444</td>
<td>0.332</td>
</tr>
<tr>
<td>All denominations except 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>-0.138</td>
<td>0.093</td>
<td>1.646</td>
<td>0.024</td>
</tr>
</tbody>
</table>
Table 4a (no 200):
Parameter estimates with standard errors (Se) for the full model.

The denominations are 500, 100, 50, 20, 10, 5 and 1.

<table>
<thead>
<tr>
<th>Denomination</th>
<th>Intercept Parameter</th>
<th>Se</th>
<th>ACORR Parameter</th>
<th>Se</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>1.497</td>
<td>0.794</td>
<td>4.219</td>
<td>a 1.330</td>
</tr>
<tr>
<td>100</td>
<td>-0.415</td>
<td>0.284</td>
<td>1.972</td>
<td>a 0.347</td>
</tr>
<tr>
<td>50</td>
<td>-0.028</td>
<td>0.386</td>
<td>2.693</td>
<td>a 0.279</td>
</tr>
<tr>
<td>20</td>
<td>-0.394</td>
<td>0.259</td>
<td>2.053</td>
<td>a 0.675</td>
</tr>
<tr>
<td>10</td>
<td>-0.021</td>
<td>0.306</td>
<td>1.344</td>
<td>a 0.475</td>
</tr>
<tr>
<td>5</td>
<td>-0.521</td>
<td>0.373</td>
<td>2.020</td>
<td>a 0.908</td>
</tr>
</tbody>
</table>

a Significantly different from zero at the 5 per cent level.

The sample size of this dataset is 235.

Table 4b (no 100):
Parameter estimates with standard errors (Se) for the full model.

The denominations are 500, 200, 50, 20, 10, 5 and 1.

<table>
<thead>
<tr>
<th>Denomination</th>
<th>Intercept Parameter</th>
<th>Se</th>
<th>ACORR Parameter</th>
<th>Se</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>1.649</td>
<td>0.817</td>
<td>3.514</td>
<td>a 1.166</td>
</tr>
<tr>
<td>200</td>
<td>0.020</td>
<td>0.255</td>
<td>1.647</td>
<td>a 0.364</td>
</tr>
<tr>
<td>50</td>
<td>-0.136</td>
<td>0.280</td>
<td>1.534</td>
<td>a 0.140</td>
</tr>
<tr>
<td>20</td>
<td>-0.140</td>
<td>0.302</td>
<td>1.854</td>
<td>a 0.504</td>
</tr>
<tr>
<td>10</td>
<td>0.338</td>
<td>0.341</td>
<td>2.091</td>
<td>a 0.666</td>
</tr>
<tr>
<td>5</td>
<td>-0.303</td>
<td>0.398</td>
<td>1.664</td>
<td>a 0.782</td>
</tr>
</tbody>
</table>

a Significantly different from zero at the 5 per cent level.

The sample size of this dataset is 226.
Table 4c (no 50):
Parameter estimates with standard errors (Se) for the full model.
The denominations are 500, 200, 100, 20, 10, 5 and 1.

<table>
<thead>
<tr>
<th>Denomination</th>
<th>Intercept Parameter</th>
<th>Se</th>
<th>ACORR Parameter</th>
<th>Se</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>0.468</td>
<td>1.253</td>
<td>4.280</td>
<td>a</td>
</tr>
<tr>
<td>200</td>
<td>0.048</td>
<td>0.242</td>
<td>1.970</td>
<td>a</td>
</tr>
<tr>
<td>100</td>
<td>-0.244</td>
<td>0.296</td>
<td>2.295</td>
<td>a</td>
</tr>
<tr>
<td>20</td>
<td>-0.436</td>
<td>0.260</td>
<td>1.821</td>
<td>a</td>
</tr>
<tr>
<td>10</td>
<td>-0.027</td>
<td>0.248</td>
<td>1.461</td>
<td>a</td>
</tr>
<tr>
<td>5</td>
<td>-0.418</td>
<td>0.379</td>
<td>2.319</td>
<td>a</td>
</tr>
</tbody>
</table>

Significantly different from zero at the 5 per cent level.
The sample size of this dataset is 280.

Table 4d (no 20):
Parameter estimates with standard errors (Se) for the full model.
The denominations are 500, 200, 100, 50, 10, 5 and 1.

<table>
<thead>
<tr>
<th>Denomination</th>
<th>Intercept Parameter</th>
<th>Se</th>
<th>ACORR Parameter</th>
<th>Se</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>0.079</td>
<td>0.819</td>
<td>1.816</td>
<td>a</td>
</tr>
<tr>
<td>200</td>
<td>-0.130</td>
<td>0.278</td>
<td>1.809</td>
<td>a</td>
</tr>
<tr>
<td>100</td>
<td>-0.233</td>
<td>0.303</td>
<td>1.929</td>
<td>a</td>
</tr>
<tr>
<td>50</td>
<td>-0.255</td>
<td>0.348</td>
<td>2.795</td>
<td>a</td>
</tr>
<tr>
<td>10</td>
<td>-0.013</td>
<td>0.374</td>
<td>1.445</td>
<td>a</td>
</tr>
<tr>
<td>5</td>
<td>-1.114</td>
<td>0.829</td>
<td>3.723</td>
<td>a</td>
</tr>
</tbody>
</table>

Significantly different from zero at the 5 per cent level.
The sample size of this dataset is 210.
Table 4e (no 10):
Parameter estimates with standard errors (Se) for the full model.

The denominations are 500, 200, 100, 50, 20, 5 and 1.

<table>
<thead>
<tr>
<th>Denomination</th>
<th>Intercept Parameter</th>
<th>Se</th>
<th>ACORR Parameter</th>
<th>Se</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>4.551</td>
<td>4.064</td>
<td>9.650</td>
<td>7.281</td>
</tr>
<tr>
<td>200</td>
<td>-0.002</td>
<td>0.071</td>
<td>2.520</td>
<td>a</td>
</tr>
<tr>
<td>100</td>
<td>-0.125</td>
<td>0.334</td>
<td>2.341</td>
<td>a</td>
</tr>
<tr>
<td>50</td>
<td>-0.137</td>
<td>0.031</td>
<td>2.037</td>
<td>a</td>
</tr>
<tr>
<td>20</td>
<td>-0.218</td>
<td>0.306</td>
<td>1.747</td>
<td>a</td>
</tr>
<tr>
<td>5</td>
<td>0.142</td>
<td>0.435</td>
<td>1.521</td>
<td>a</td>
</tr>
</tbody>
</table>

* a  Significantly different from zero at the 5 per cent level.

The sample size of this dataset is 210.
Figure 1: The newly constructed 200-euro note
(its true size is equal to the size of the other toy notes and its color is yellow)
Figure 2  Expected values of $Y$ for experiment with all notes, max. upper bound = 3, lower bound = 0

Figure 3  Expected values of $Y$ for experiment without 200, max. upper bound = 3, lower bound = 0
Figure 4  Expected values of $Y$ for experiment without 100, 
max. upper bound = 3, lower bound = 0

Figure 5  Expected values of $Y$ for experiment without 50, 
max. upper bound = 3, lower bound = 0
Figure 6  Expected values of $Y$ for experiment without 20,
max. upper bound = 3, lower bound = 0

Figure 7  Expected values of $Y$ experiment without 10,
max. upper bound = 3, lower bound = 0
References


