Model-based forecast adjustment;
With an illustration to inflation

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This paper introduces the idea to adjust forecasts from a linear time series model where the adjustment relies on the assumption that this linear model is an approximation of for example a nonlinear time series model. This way to create forecasts can be convenient when inference for the nonlinear model is impossible, complicated or unreliable in small samples. The size of the forecast adjustment can be based on the estimation results for the linear model and on other data properties like the first few moments or autocorrelations. An illustration is given for an ARMA(1,1) model which is known to approximate a first order diagonal bilinear time series model. For this case, the forecast adjustment is easy to derive, which is convenient as the particular bilinear model is indeed cumbersome to analyze. An application to a range of inflation series for low income countries shows that such adjustment can lead to improved forecasts, although the gain is not large nor frequent.

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1. Introduction

Forecasts from econometric time series models are frequently adjusted by experts who have domain knowledge, see Franses (2014) and the many studies cited therein. There are various reasons why such econometric model-based forecasts are adjusted. The observation at the forecast origin may be an outlier or an explanatory variable suffers from measurement error. It can be believed that parameters will change in the future, or one may know that there will be a structural shift in the forecast sample. These are all a few examples of possible reasons. There are also various ways of expert adjustment of forecasts. One may simply add or subtract a number of the given quote, one may change an estimated parameter into another value, one may multiply the quote with some number, one may change the observation of an explanatory variable into another value, or one may replace the observation at the forecast origin by another observation.

In this paper I propose yet another reason to adjust a model-based forecast. The forecast is believed to be based on an incorrectly specified model, while it is assumed known what the correct specification should be, but where the data do not allow that potentially appropriate model to be analyzed. In fact, here the idea is to generate a forecast from a linear time series model, and to adjust this forecast based on the assumption that a specific nonlinear time series model would be a more appropriate specification. There are many nonlinear time series models around, see de Gooijer (2017) for a recent extensive survey, but typically proper parameter estimation for these models requires quite a number of observations, potentially with a high frequency. Also, for some nonlinear time series models asymptotic theory is missing and there can be problems with the likelihood function.

The present paper specifically addresses one-step-ahead forecasts for annually or monthly observed inflation rates for low income countries. For many countries in Africa, typically, data are not collected at a higher frequency than per year, and the available samples typically cover five or six decades at the very maximum. Inflation rates in low income countries once in a while can show periods of hyperinflation, while at other times inflation rates can be moderate. Inflation rates data seem a good candidate for nonlinear time series models.

The econometric linear time series model to be addressed is an autoregressive moving average model of order (1,1), in short an ARMA(1,1). Now inflation typically shows patterns that have periods with high inflation interchanging with periods with lower inflation. One may view such patterns as reflecting recurring structural shifts, see for example Ariz et al. (2005) and Castle et al. (2014). Alternatively, one may see such longer periods with higher or lower inflation as a reflection of long memory, perhaps to be modelled by a fractionally integrated time series model, see for example Bos et al. (2002).

In the present paper, and also for illustrative purposes, I assume that the proper model for annual inflation rates would be a so-called diagonal bilinear time series model, see Granger and Anderson (1978). Inference for this bilinear model is notoriously difficult, and also for many bilinear models the asymptotic properties of the parameter estimators are unknown. For point-forecasting purposes, the latter properties may be viewed as less relevant, as long as one gets the proper estimates of the parameters.

The outline of this paper is as follows. In Section 2 the focus is on the ARMA(1,1) model and how it relates to a first order diagonal bilinear time series model. First, the linear model can be viewed as a proper linear approximation of this bilinear model. Second, given the expressions for the expected values of the levels and the squared levels of the data, the parameters in the bilinear model could be identified and a methods-of-moments estimator could be used, although it will be shown that then the data should have rather peculiar properties. Note that a useful by-product of this exercise is thus that a simple diagnostic method can be implemented which can be used to see if it would be worthwhile to try to estimate the parameters in a bilinear model in the first place. This diagnostic method is based on the difference between the expected values of the levels and the squares. Luckily, to model-based adjust the forecasts from a linear ARMA(1,1) model, we will see that a combination of the key parameters will do. This combination can simply be retrieved from the first moment of the data. In Section 3, I illustrate the potential merits of such adjustment for data on annual inflation for 41 countries on the African continent and for 11 monthly inflation series for Suriname (a country which recently experienced a period of very high inflation). For all series the ARMA(1,1) model is fitted. Looking at the quality of the in-sample one-step-ahead forecasts, it can be learned that for 8 of the 41 countries, the adjusted forecasts lead to improvement. For the remaining 33 countries, the adjusted forecasts are less accurate, and sometimes much less accurate. For the 11 sector-specific inflation rates, there is moderate forecast improvement for 3 series. So, there can be forecast gains of model-based adjustment, and the assumption of a diagonal bilinear time series model. Section 4 concludes with ideas for further research.

2. The main idea

In this section I outline the main idea of model-based forecast adjustment. First, I discuss the linear model and the nonlinear model. Next, the expression for the adjusted forecast will be presented.

Linear and bilinear models

Consider a time series y_t and suppose that a reasonable model for this time series is an ARMA(1,1) model, that is,

$$y_t = \tau + \alpha y_{t-1} + u_t + \theta u_{t-1}$$

with $|\alpha| < 1$ and $|\theta| < 1$. Typical values in case the model is fitted to for example inflation data are that α is in between 0.5 and 0.95, while θ can be positive or negative. For the 41 African series, to be analyzed in more detail later on, the median value of α is 0.620. With such parameter values, various shapes of the autocorrelation function can be obtained. The first order autocorrelation of the ARMA(1,1) model is

$$\rho_1 = \frac{(1+\alpha\theta)(\alpha+\theta)}{1+2\alpha\theta+\theta^2}$$

The next autocorrelations obey the scheme

$$\rho_k = \alpha\rho_{k-1}$$

for *k* = 2, 3, .. Simple algebra gives that

$$\rho_1 - \alpha = \frac{\theta(1-\alpha^2)}{1+2\alpha\theta+\theta^2}$$

This shows for $\alpha > 0$ that $\rho_1 > \alpha$ when $\theta > 0$ and that $\rho_1 < \alpha$ when $\theta < 0$. Hence, with a positive value of θ, there is more persistence in the process.

The one-step-ahead forecast from origin T for an ARMA(1,1) model is based on

$$y_{T+1|T} = \tau + \alpha y_T + \theta u_T \quad (1)$$

where in practice of course the parameters are replaced by estimated values. The forecast error is $u_{T+1} = y_{T+1} - y_{T+1|T}$. So, too low a forecast means a positive forecast error, and when $\theta < 0$ there is a tendency to revert to the mean. In case of inflation, this is perhaps an unwanted effect as typically inflation can peak for a few periods in a row. That is, high initial inflation levels can spur a period with again high inflation.

To make a link with a bilinear model, one may now want to replace θ by a function of y_T to mitigate any mean-reverting effect, that is, one may want to replace the ARMA(1,1) forecast by

$$y_{T+1|T} = \tau + \alpha y_T + \beta y_T u_T \quad (2)$$

That is, the θ in (1) is replaced by βy_T in (2). This forecast function corresponds with a so-called first-order diagonal bilinear time series model

$$y_t = \alpha y_{t-1} + \beta y_{t-1}\varepsilon_{t-1} + \varepsilon_t \quad (3)$$

where the notation for α is kept the same for a reason to become clear below. There is no need to include an intercept, as we will see below. Naturally, the u_t in the ARMA(1,1) model is not the same as the ε_t in (3). This first-order diagonal bilinear model was introduced in Granger and Andersen (1978, page 56 and further).

This model has acquired quite some attention in the literature. Basrak et al. (1999) examine the sample autocorrelation function of (3). Bibi and Oyet (2004) extend the model to allow for time-varying coefficients. Brunner and Hess (1995) discuss the potential problems with the likelihood function. Charemza et al (2005) study (3) in case $\alpha = 1$. Guegan and Pham (1989) discuss the estimation of the parameters using the least squares method. A method of moments estimator for this diagonal model is considered by Kim et al (1990). Pham and Tran (1981) discuss various other properties of this first order bilinear time series model. Sesay and Subba Rao (1988) look into estimation methods using higher order moments, and Subba Rao (1981) provides a general theory of bilinear models. Amongst the few studies where bilinear models in general are considered for forecasting are Poskitt and Tremayne (1986) and Weiss (1986), where it is found for a few cases that bilinear models can slightly improve on linear models. Finally, Turkman and Turkman (1997) derive the properties of the extremal observations corresponding to bilinear time series models.

That a bilinear time series model can be associated with extremal observations can also be seen from the following. For the bilinear model in (3) it can be derived that

$$\mu = E(y_t) = \frac{\beta\sigma_\varepsilon^2}{1-\alpha}$$

$$\omega = E(y_t^2) = \frac{\sigma_\varepsilon^2(1 + 2\beta^2\sigma_\varepsilon^2 + 4\alpha\beta\mu)}{1 - \alpha^2 - \beta^2\sigma_\varepsilon^2}$$

see Granger and Andersen (1978). It can also be derived that the autocorrelation function of the first-order diagonal bilinear time series model in (3) is the same as that of an ARMA(1,1) model like

$$y_t = \tau + \alpha y_{t-1} + u_t + \theta u_{t-1}$$

with exactly the same α, see Granger and Andersen (1978).

Now, one could now think that with an expression for α and the expressions for the first and second moments, one can design separate estimators for β and σ_ε^2. However, in the Appendix it is shown that this method-of-moments-type method is quite unlikely to be successful for empirical data. In short, the reason is that ω should be more than (about) 8 times as large as μ^2, or ω should be very small relative to μ. For the inflation data in Africa, to be analyzed later on, this occurs only for Chad and the Democratic Republic of Congo. For the data on Suriname this does not happen at all. This shows that, as such, the first-order diagonal bilinear time series model may not be successfully analyzed in practice, and this may also explain the relatively small number of empirical applications of this model.

Model-based forecast adjustment

Fortunately, to create the model-adjusted forecast

$$y_{T+1|T} = \tau + \alpha y_T + \beta y_T u_T$$

there appears to be no need to estimate β and σ_ε^2 separately. This can be seen as follows. When the first-order diagonal bilinear time series model is the data generating process, and we fit an ARMA(1,1) model to these data, then the estimated variance σ_u^2 for the ARMA model is not an estimator for σ_ε^2. Hence, the model-adjusted forecast should correct for the difference between the two, and to properly scale the added term, like

$$y_{T+1|T} = \tau + \alpha y_T + \beta y_T \left(\frac{\sigma_\varepsilon^2}{\sigma_u^2}\right) u_T \tag{4}$$

Given an estimator for the variance σ_u^2 for the ARMA model, and given the observable value y_T, we thus need to know $\beta\sigma_\varepsilon^2$. This last term can simply be found from the first moment, that is,

$$\mu(1-\alpha) = \beta\sigma_\varepsilon^2$$

All in all, we now have quite a simple way of finding a forecast based on a first-order diagonal bilinear time series model, without having to estimate its model parameters.

3. Empirical application

This section deals with a comparison of the forecasts from an ARMA(1,1) model and from a model-based forecast adjustment, where it is presumed that the first-order diagonal model in (3) could have generated the data. Fitting the model to the data is unlikely going to work, and therefore I chose for the forecast adjustment approach.

41 countries in Africa

The first set of data concern annual inflation rates for 41 African countries, ranging from 1960 to 2015. The data source is Franses and Janssens (2017). Table 1 presents the estimates of $\alpha, \mu, \beta, \sigma_\varepsilon^2$, and $\frac{\beta\sigma_\varepsilon^2}{\sigma_u^2}$, where this latter term will be used in (4) to create the adjusted forecast. The estimates of that term have a maximum value of 0.402 (Botswana) and a minimum of 2.14E-05 (the Democratic Republic of Congo).

Table 2 presents the results on measures of forecast accuracy, where here it is chosen to use the Median Absolute Forecast Error (MAFE). The forecasts are all one-step-ahead forecast errors, within the sample, where the estimates are obtained for the full sample. Much more refined forecast evaluation methods can be considered, but it is believed that the overall qualitative outcome will be about the same. In italics are those cases where the model-based adjusted forecasts give a lower MAFE than those from the linear ARMA model. Clearly, there are only 8 cases with some slight improvement. On the other hand, for some of the 33 other cases, the adjusted forecasts can be very poor.

11 categories in Suriname

Figure 1 presents the monthly inflation rates for the South-American country Suriname. The inflation rates concern the percentage differences between prices in a current month and that same month the year before. The prices data range from 2013 January to 2017 December and are obtained from Statistics Suriname (http://www.statistics-suriname.org/), and hence the inflation rates data start in 2014 January. Clearly, there were months with exceptionally high inflation levels.

Table 3 presents similar estimation results for Suriname as were given in Table 1 for Africa. The estimates for $\frac{\beta\sigma_\varepsilon^2}{\sigma_u^2}$ range from 0.007967 (category Housing, Utilities) to 0.132 (category Food away from home). Table 4 presents the MAFE results, also again for the one-step-ahead in-sample forecasts. We see that for 3 categories there can be some slight forecast improvement.

4. Conclusion

"I think bilinear models are not going to have much future. I do not see much evidence of them helping forecasting, for example"

Clive W.J. Granger in Phillips (1997, page 277)

The results in this study seem to provide some support for this quote by Clive Granger, although there were some cases with forecast improvement. Instead, the reason that bilinear time series models were not overly successful in the last years is perhaps the fact that these models are very difficult to apply in practice. This is due to all kinds of peculiar properties that these models have, in terms of moments, correlations and extremal observations. No doubt that other nonlinear time series models suffer from similar problems, which make them less often used in practice.

In this paper I therefore proposed an alternative approach, which does rely on an assumption of a nonlinear data generating process, but which does not require parameter estimation and asymptotic inference. This approach simply estimates a linear time series model, and then modifies the forecast using properties of the data that associate with the nonlinear data generating process. For 11 of the in total 41+11=52 cases, it was found that some forecast improvement is possible.

Further work on this approach can consider various other nonlinear models. For example, consider the bilinear model

$$y_t = \beta y_{t-2}\varepsilon_{t-1} + \varepsilon_t$$

which is the focus of Ling et al (2015). The expected value of y_t is zero, and also the autocorrelations are zero. This means that the linear model would simply be $y_t = u_t$, where $\sigma_u^2 = \sigma_y^2 \neq \sigma_\varepsilon^2$. An adjusted forecast for T+1 would then be

$$y_{T+1|T} = \beta y_{T-1} u_T \frac{\sigma_\varepsilon^2}{\sigma_u^2}$$

Grahn (1995) shows that

$$E(y_t y_{t-1}) = \beta\sigma_\varepsilon^2 y_{t-2}$$

and hence, also for this model we can obtain an estimate of $\beta\sigma_\varepsilon^2$. This makes it possible to create the rather simple model-based-adjusted forecast equal to

$$y_{T+1|T} = \beta y_{T-1} u_T \frac{\sigma_\varepsilon^2}{\sigma_y^2} = \beta\sigma_\varepsilon^2 \frac{y_{T-1} y_T}{\sigma_y^2}$$

Appendix

To show that the first order diagonal bilinear model is difficult to handle in practice, consider the case where $\alpha = 0$ (to save notation), that is, consider

$$y_t = \beta y_{t-1}\varepsilon_{t-1} + \varepsilon_t$$

The first and second moments are

$$\mu = E(y_t) = \beta\sigma_\varepsilon^2$$

and

$$\omega = E(y_t^2) = \frac{\sigma_\varepsilon^2(1 + 2\beta^2\sigma_\varepsilon^2)}{1 - \beta^2\sigma_\varepsilon^2}$$

This last equation can be written as

$$(1 - \beta^2\sigma_\varepsilon^2)\omega = \sigma_\varepsilon^2(1 + 2\beta^2\sigma_\varepsilon^2)$$

Replacing σ_ε^2 by $\frac{\mu}{\beta}$ and rearranging gives a second order equation for β:

$$-\mu\omega\beta^2 + (\omega - 2\mu^2)\beta - \mu = 0$$

To solve for β, the determinant is

$$D_\beta = \omega^2 - 8\mu^2\omega + 4\mu^4$$

To see when D_β is positive, solve $D_\beta = 0$ for ω, to get the determinant

$$D_\omega = 48\mu^4$$

The solutions for ω are $(4+2\sqrt{3})\mu^2$ and $(4-2\sqrt{3})\mu^2$. So, to find estimates based on a methods of moments estimator for σ_ε^2 and β, it should hold that

$$\omega > (4+2\sqrt{3})\mu^2$$

or that

$$\omega < (4-2\sqrt{3})\mu^2$$

Both conditions are very rare for empirical data. For the African countries the first condition occurs twice, and for the Suriname data, neither one of the conditions occur.

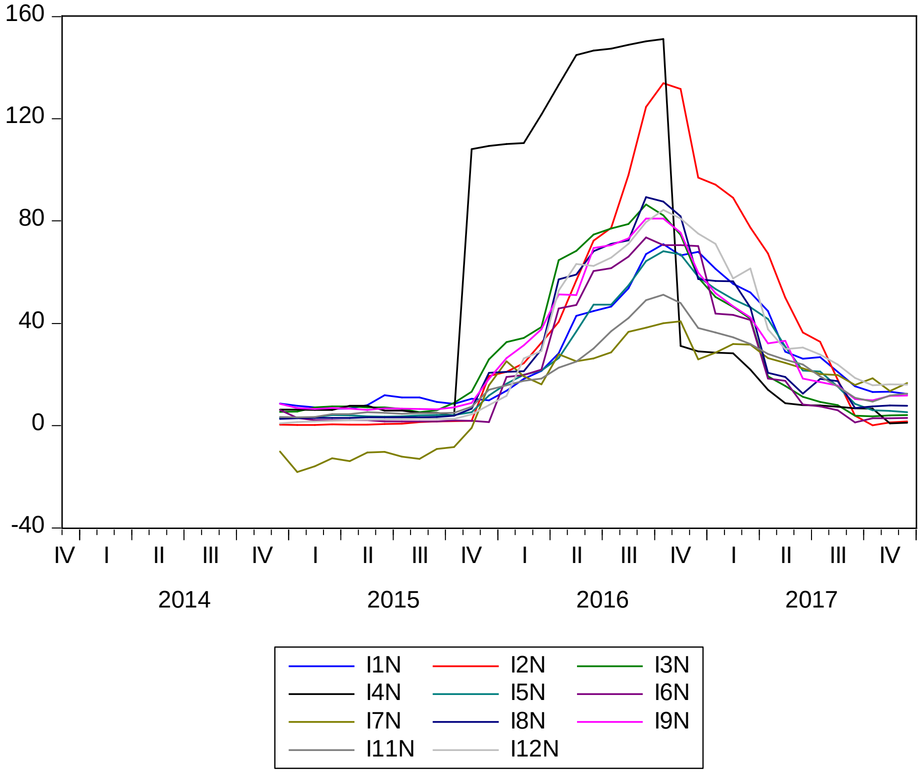

Figure 1: Monthly inflation in Suriname, 2014 January to 2017 December

Table 1: Estimation results for annual inflation in Africa

Country	α	μ	$\beta\sigma_\varepsilon^2$	$\frac{\beta\sigma_\varepsilon^2}{\sigma_u^2}$
Algeria	0.620	8.957	3.404	0.151
Angola	0.123	339.36	297.62	9.12E-04
Benin	0.924	7.343	0.558	0.016
Botswana	0.839	9.754	1.570	0.402
Burkina Faso	0.647	4.577	1.616	0.033
Burundi	-0.293	9.892	12.790	0.227
Cape Verde	0.816	4.504	0.829	0.078
Central African Republic	-0.615	4.132	6.673	0.170
Chad	-0.092	4.789	5.230	0.058
Republic of Congo	0.021	10.614	10.391	0.048
DR of Congo	0.644	642.67	228.79	2.14E-05
Egypt	0.862	9.264	1.278	0.068
Equatorial Guinea	0.126	3.596	3.143	0.119
Ethiopia	-0.259	8.614	10.845	0.108
Gabon	0.167	4.995	4.161	0.084
Gambia	0.666	8.041	2.686	0.046
Guinea Bissau	0.977	31.213	0.718	0.005
Ivory Coast	-0.081	5.586	6.038	0.196
Kenya	0.373	10.271	6.440	0.138
Libya	0.731	5.303	1.427	0.038
Madagascar	0.367	11.725	7.422	0.119
Malawi	0.732	26.179	7.016	0.021
Mali	-0.212	3.180	3.854	0.271
Mauritius	0.450	7.407	4.074	0.111
Morocco	0.893	4.454	0.477	0.058
Mozambique	0.811	18.741	3.542	0.042
Niger	0.233	4.502	3.498	0.061
Nigeria	0.337	15.948	10.574	0.075
Rwanda	0.173	7.734	6.396	0.205
Senegal	0.533	5.104	2.384	0.050
Seychelles	0.075	6.959	6.437	0.185
Sierra Leone	0.892	23.770	2.567	0.004
Somalia	0.689	23.171	7.206	0.008
South Africa	0.827	8.195	1.418	0.361
Sudan	0.886	28.486	3.247	0.008
Swaziland	0.843	9.554	1.500	0.077
Tanzania	0.860	16.145	2.260	0.067
Togo	0.387	5.380	3.298	0.067
Tunisia	0.863	5.521	0.756	0.041
Uganda	0.712	30.964	8.918	0.011
Zambia	0.598	36.616	14.720	0.023

Table 2: the Median Absolute Forecast Error (MAFE), based on in-sample one-step-ahead forecasts, Africa

Country	ARMA	Model-based adjusted
Algeria	2.728	2.852
Angola	126.00	143.99
Benin	*2.746*	*2.061*
Botswana	1.125	3.144
Burkina Faso	3.460	3.569
Burundi	4.126	6.689
Cape Verde	1.711	2.330
Central African Republic	3.781	5.824
Chad	5.013	6.710
Republic of Congo	6.408	7.655
DR of Congo	*498.37*	*235.61*
Egypt	2.502	2.843
Equatorial Guinea	1.786	1.900
Ethiopia	4.747	8.738
Gabon	2.260	2.713
Gambia	1.829	2.261
Guinea Bissau	2.073	2.389
Ivory Coast	2.297	2.652
Kenya	4.096	5.172
Libya	*3.072*	*2.928*
Madagascar	3.990	4.548
Malawi	*7.011*	*5.315*
Mali	0.841	1.207
Mauritius	2.398	2.713
Morocco	1.179	1.814
Mozambique	2.255	3.554
Niger	4.560	5.196
Nigeria	5.020	6.807
Rwanda	3.626	4.240
Senegal	*2.947*	*2.361*
Seychelles	2.677	3.042
Sierra Leone	*46.169*	*31.360*
Somalia	*18.197*	*6.804*
South Africa	1.163	2.725
Sudan	44.366	50.824
Swaziland	2.557	3.539
Tanzania	3.043	3.476
Togo	2.901	3.423
Tunisia	*1.312*	*1.033*
Uganda	30.390	42.712
Zambia	41.664	48.906

Table 3: Estimation results for monthly inflation in Suriname

Variable

Category	α	μ	$\beta\sigma_\varepsilon^2$	$\frac{\beta\sigma_\varepsilon^2}{\sigma_u^2}$
Food, Non-Alcohol	0.939	27.82	1.697	0.077
Alcohol, Tobacco	0.942	38.15	2.213	0.022
Clothing, Footwear	0.947	29.75	1.577	0.030
Housing, Utilities	0.882	49.95	5.894	0.008
Household Furnishing	0.956	24.07	1.059	0.069
Health Care	0.942	23.03	1.336	0.018
Transportation	0.918	13.53	1.109	0.035
Communication	0.924	28.52	2.168	0.027
Recreation, Education	0.952	30.03	1.441	0.034
Food away from home	0.943	19.57	1.115	0.132
Other	0.943	30.90	1.761	0.033

Table 4: the Median Absolute Forecast Error (MAFE), based on in-sample one-step-ahead forecasts, Suirname

Category	ARMA	Model-based adjusted
Food, Non-Alcohol	2.179	2.254
Alcohol, Tobacco	3.866	4.879
Clothing, Footwear	*2.147*	*1.749*
Housing, Utilities	*5.951*	*5.126*
Household Furnishing	1.715	2.206
Health Care	1.567	2.316
Transportation	2.866	4.153
Communication	2.178	6.873
Recreation, Education	*2.639*	*2.469*
Food away from home	1.504	2.853
Other	1.767	2.599

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