5. INTERACTIVE MULTIPLE OBJECTIVE PROGRAMMING METHODS

Each time a new method is proposed, one has to judge its contribution to the set of already existing comparable methods. Because we are advocating a new interactive variant of goal programming in our study, we have to expose its relationships both with other interactive methods and with goal programming. As mentioned in Chapter 4, interactive methods may circumvent the considerable information requirements that make goal programming sometimes less valuable. A characterization, together with an overview of the main features of interactive methods is given in Section 5.2. A more detailed typology of interactive methods (based on the framework sketched in Chapter 3), is presented in Section 5.3. Because the majority of the existing interactive methods lack some important advantages of goal programming, it is worthwhile seeking an interactive variant of goal programming. In Section 5.4 we discuss a sample of such interactive variants of goal programming. Our main conclusions can be found in Section 5.5.

5.1. Features of an Interactive Approach

If in a decision situation involving multiple goals, the set of alternative solutions has been clearly described, and if the preferences of the decision maker have been expressed explicitly in terms of the instrumental variables, the multiple criteria decision problem can be reduced to an exclusively mathematical (optimization) problem. Very often, however, and especially if the number of decision alternatives increases, a situation arises in which the decision maker is unable or unwilling to provide all required *a priori* preference information. In such cases, interactive procedures might offer much help.

The interactive approach assumes that the decision maker is at

least capable of defining which goal variables influence his preferences. Furthermore, it assumes that the decision maker is able to provide local preference information, i.e. information with respect to a given solution (or a given set of solutions) which is known to exist and to be feasible. The kind of local preference information required varies for each interactive procedure. In some methods the decision maker has to give his local trade-offs with respect to the goal levels of the solution concerned. In other methods, the decision maker has to indicate whether a given solution is acceptable or not, and if not which of the goal values should be changed.

Typically, in the interactive approach, the decision maker has to express his local preferences with respect to a series of solutions, which are presented to him in a stepwise manner and are partly the result of his previous answers. This process of presenting solutions and expressing the local preferences with respect to these solutions is embedded in an interactive framework. The interaction takes place between the decision maker, an analyst and a (computer) model of the decision problem. The model (designed by the analyst in consultation with the decision maker) describes the set of feasible decision alternatives, the set of goal variables and the relationships between decision alternatives and goal variables. In most interactive procedures, the analyst (using the model) proposes a starting solution to the decision maker. The latter gives his preference information with respect to this solution to the analyst, who transforms this information into a new solution, again using the model and guided by the requirements of the interactive procedure at hand. This new solution is presented to the decision maker, who expresses his preferences, and so on, until a final solution is reached which is judged satisfactory. In consideration of errors and learning effects, the decision maker may obviously wish to repeat the whole process. Moreover, because of new insights obtained, it may become necessary to revise the model. In Figure 5.1 a sketch of the interactive approach is given.

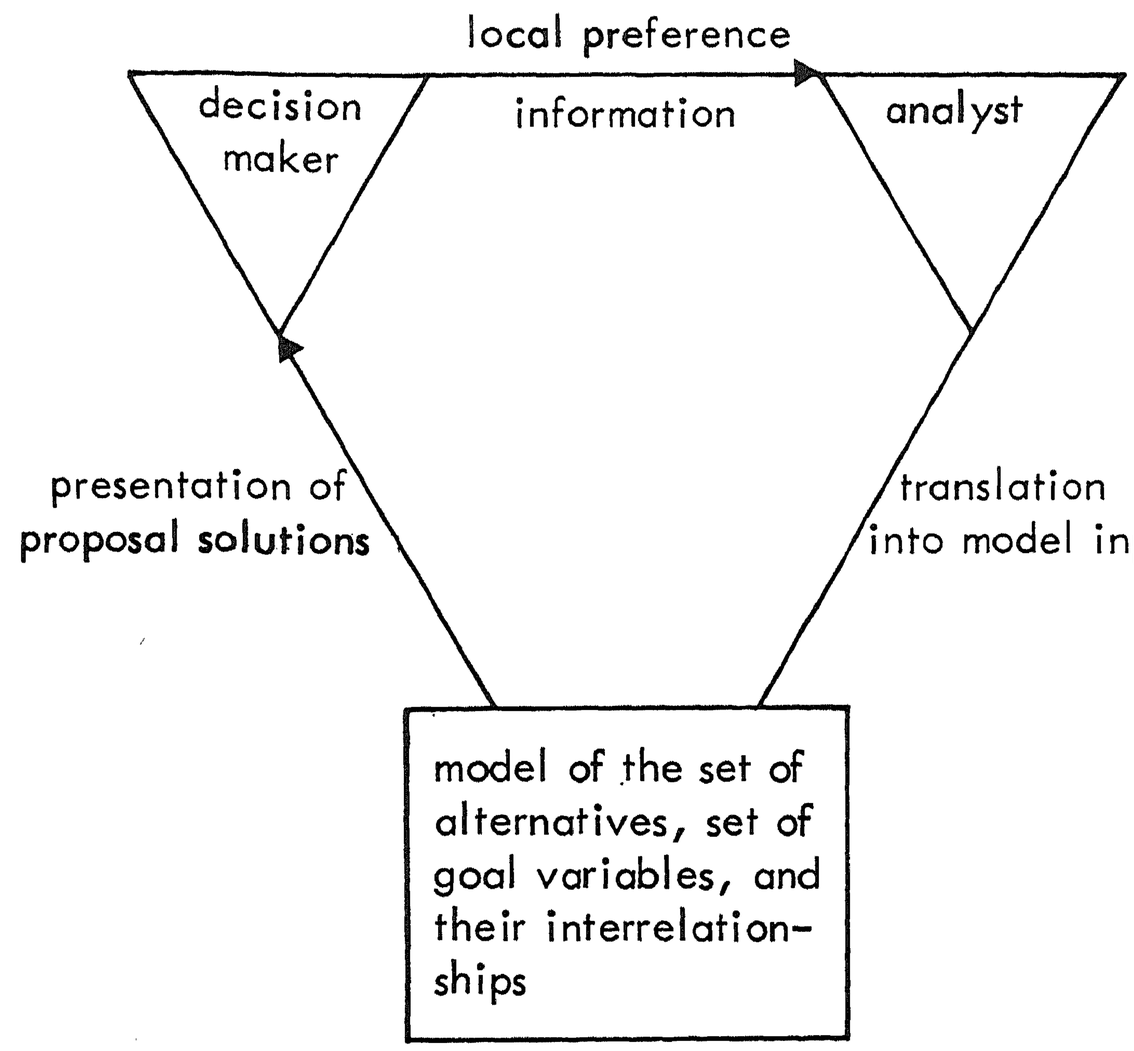

Figure 5.1. The interactive approach

Generally, the role of the analyst during the interactive process is more passive than in the preparatory phase. This is because the translation from the decision maker's preference information into a new solution can mostly be computerized in a straightforward manner. Nevertheless, the analyst still has to be involved in the interactive process: to instruct and reinstruct the decision maker about the properties of the interactive procedure at hand, to help analyze the model results and if necessary to prepare possible model revisions.

Advantages and Disadvantages of the Interactive Approach

An obvious advantage of the interactive approach is the limited amount of preference information required from the decision maker, as compared to methods in which the decision maker has to give his

preference information on an *a priori* basis. In the latter case, the decision maker has to consider all kinds of choices and trade-off questions which might be relevant. However, because this articulation of preferences takes place before knowing whether the alternatives influenced by these preferences are feasible or not, and if feasible, whether they are dominated, this kind of preference information may clearly be redundant. Furthermore, this preference information can only be obtained by means of hypothetical questions such as 'What would you prefer if you could choose between alternatives A and B?' This is in contrast with the interactive approach, in which the decision maker has to express his preferences based on a well-defined solution which is known to exist and to be feasible. Questions to be answered now become quite concrete, such as 'Do you accept this solution or not?', and 'Which goal value of the given solution is too low?'.

The assumptions related to the preference structure of the decision maker are less restrictive in the interactive approach than in procedures aiming at the *a priori* articulation of preferences. Also, because the decision maker is closely involved in the solution process, the solution finally chosen has a better chance of being implemented (cf. also Hwang and Masud [1979]).

By means of interactive decision procedures the decision maker may become more closely involved in the process of solving his decision problem, thus obtaining more insight into the trade-offs among different goal variables. The feed-back process inherent in interactive decision procedures leads to a closer cooperation between decision maker and analyst. In a sense, the interactive approach can be regarded as an operational application of learning theory (cf. also Atkinson et al. [1965], Golledge [1969], and Hilgard and Bower [1969]). One of the learning effects induced by the interactive approach may be that the decision maker's perception of the decision situation changes during the interactive process. This in turn may necessitate restructuring the model describing the decision problem, e.g. by changing a goal variable or by adding an instrumental

variable. In addition, the decision maker's preferences may change because of new insights into the decision situation. In addition, one should take the occurrence of possible errors made by the decision maker into account. Therefore, it is wise to repeat the whole interactive process several times to ensure that a final solution is found which is as close to the 'optimum' as possible.

Of course, the characteristics of the learning process started by the interactive approach depend among other things on the very interactive procedure chosen, and - given a particular procedure - on the starting point. It is virtually impossible to test the influences of different interactive procedures and different starting points on the learning process empirically. This makes the choice of a particular procedure and of a particular starting solution somewhat arbitrary, although on theoretical grounds several criteria can be formulated which may influence the choice of such a procedure (cf. Section 5.2).

Traditionally, the entrepreneurial preference structure was considered by such academic fields as economics, operations research and management science to be a given. As described in Chapter 2, this given mostly consisted of some profit maximization or cost minimization assumption. Given the recognition of the existence of multiple goals, no straightforward assumption on the decision maker's preference structure can, in a general sense, be made. The only solution is to model the decision maker's preferences in a more direct way, e.g. by interviewing techniques (cf. also Chapter 3).

In interviewing techniques the same phenomenon may arise as in interactive procedures. The very way of interviewing may also induce a learning process. As long as this is clear to the decision maker, there are no objections to this situation.

To summarize, we can state that the interactive approach offers a powerful tool in decision situations in which the decision maker's preferences are not known *a priori*. It is able to include and to benefit from learning effects, it involves a closer co-operation of the decision maker in the solution process and moreover, the

information to be given by the decision maker concerns real instead of hypothetical choices. In the next section we list a number of elements which might be useful for evaluating interactive procedures.

5.2. Elements of Interactive Methods

In the preceding section we argued that interactive methods may be regarded as a learning process in which the decision maker reacts upon the questions posed to him. From this viewpoint, the specific method of obtaining information from a decision maker may influence his answers. Therefore, one should pay heed to the behavioural assumptions made in a particular interactive method.

The convergence of a particular interactive method might often be accelerated by adding a few behavioural assumptions. This, however, would in turn affect the class of problems which can be tackled by this method.

These points show that the evaluation of an interactive decision method itself can be regarded as a multiple criteria problem. In order to illustrate this statement, we shall propose a set of elements which may be important in evaluating interactive decision methods. Some of these elements have been adopted from studies of Cohon and Marks [1975], Roy [1971,1976ab], Wallenius [1975] and Wallenius and Zionts [1976].

Because we have already (in Section 3.2) listed a number of characteristics of decision problems to be used for the evaluation of multiple criteria decision methods in general, we limit ourselves here to the more specific elements of interactive methods, subdivided into the following groups:

I. The communication process between decision maker and decision model

II. Technical properties of the interactive process

These features will be described in more detail below.

The Communication Process between Decision Maker and Decision Model

Clearly, the way in which the process of interaction between decision maker and model (possibly guided by an analyst) has been structured is of crucial importance for the acceptance of the interactive method by the decision maker. For instance, when only limited information is provided to the decision maker, he may feel that some information is being withheld from him. Likewise, he will require some freedom to influence the direction of the interactive process.

To characterize the communication process, we suggest the following qualifications.

a. <u>The questions posed to the decision maker</u>
   - Is he requested to answer 'zero-one' questions (e.g., 'Do you prefer this solution to the preceding one?') or does he have to specify cardinal information (e.g., the specification of a marginal rate of substitution between two goal variables)?
   - Is he requested to answer these questions with respect to weights, with respect to achievement levels attached to the goal variables, or both?
   - How many questions are to be answered at each iteration and in total?

To stress the importance of including these qualifications, we quote Cohon and Marks [1975, p. 209]: 'It is futile, and antithetical to the essence of planning, to complicate the analysis with all sorts of esoteric terms and terminology. Yet, some of the multiobjective techniques rely on the collection of abstruse, sometimes exotic data from the decision maker, thereby producing meaningless results'.

b. <u>The information for the decision maker</u>
   - Is the decision maker confronted with a fixed amount of information or does he have some options to select from the available information those items which he believes to be interesting?

- Does the model allow for the possibility of sensitivity analysis; at each stage of the problem?
- Is the decision maker given one or more solutions at a time?

c. Options available to the decision maker to control the interactive process
   - What are the decision maker's degrees of freedom to change the direction of the search process for a consecutive solution?
   - If the decision maker changes his mind during the interactive procedure, can he return to an earlier solution?

d. Perception of the method by the decision maker

The evaluation of an interactive method also presupposes some idea of how the method is perceived by the decision maker. Wallenius [1975] carried out a laboratory experiment in order to compare the performance of three interactive methods from a human decision maker's point of view. From this experiment we adopt the following measures, which are obviously of a more subjective nature than the preceding ones.

- The decision maker's confidence in the best compromise.
- The ease of using the method.
- The ease of understanding the logic of the method.
- The use of the information provided to aid the decision maker.
- The claim on the decision maker's time.

In the next section we will briefly describe some of the experimental results with respect to the decision maker's perception of interactive methods.

Technical Properties of Interactive Methods

Interactive methods vary from very simple rules of thumb to sophisticated mathematical programming methods. We therefore list some technical characteristics:

a. The specific solution (optimization) procedure(s) which have to be used in the computational phases of the process

b. Computer time per iteration

This is not only important for the acceptance of the method by the decision maker (see I.d), but also as a cost factor. Of course, this measure depends on the problem to be solved, the solution (optimization) procedure chosen, the efficiency of the computer program and last but not least, the type of computer used.

c. Convergency properties

- Does the procedure converge to a final solution?
- What are the minimum and maximum number of iterations needed to approach such a final solution close enough? (This depends on the accuracy chosen, but also on the number of goal variables, the preference structure and the properties of the set of possible actions).
- If the procedure converges, is there a unique final solution or not?

Many of the elements of interactive methods mentioned in this section, as well as some of the characteristics described in Section 3.2 will be discussed further in the following sections, in which we will give a classification of interactive methods.

5.3. The Need of an Interactive Variant of Goal Programming

During the last decade, the number of multiple objective programming methods grew considerably. Besides, a wide variety of literature on the efficiency concept and on vector maximization has emerged. Goal programming techniques, methods that maximize one goal variable subject to *a priori* fixed levels of the other goal variables, and methods which generate all efficient solutions (see for instance Gal [1977]) have been proposed to deal with the multiple objective programming problem. The first two procedures need a considerable amount of *a priori* information on the decision maker's preferences (cf. Chapter 4). An important disadvantage of the third kind of methods is that they often generate a very large number of

efficient solutions. Although preselection of efficient solutions seems to simplify the decision problem, the efficient set is normally practically unmanagable for the decision maker.

Many interactive procedures have been designed to overcome the above-mentioned difficulties. Recent overviews are given a.o. by Cohon [1978], Hwang and Masud [1979], Isermann [1979], and Winkels [1979]. All interactive procedures progress from one solution (or set of solutions) to another, guided by the desires of the decision maker, which must also be expressed iteratively, i.e. each time a new (set of) solution(s) is defined. With respect to the nature of the preference information to be given at each iteration by the decision maker, the set of available interactive procedures can be subdivided as follows.

(a) Methods in which the decision maker has to determine trade-offs among the goal variables at each iteration, given the goal values in the current solution.

(b) Methods in which the decision maker has to choose the 'best' solution from a limited (discrete) set of (generally efficient) solutions at each iteration.

(c) Methods in which the decision maker at each iteration has to define minimum or maximum values for one or more of the goal variables, which in most methods are translated into restrictions reducing the feasible region.

The union of type (a), (b) and (c) methods will be denoted hereafter as the class of 'interactive multiple objective programming' methods. Methods of type (c) will be classified as 'interactive goal programming' procedures. This classification of interactive programming methods is illustrated in Figure 5.2 (see p. 124). The methods in each class differ with respect to the accuracy required from the decision maker expressing his preferences. Some methods need 'exact' weights to be assigned to the goal variables, other merely require a range of weights (or present a series of weights from which the decision maker can choose). In some methods only the best one from a set of solutions must be selected, in others it suffices to select

a couple of 'best and 'near best' solutions. Similarly, certain methods need the exact amount of maximally tolerable relaxation of a goal value (given some proposal solution), others only need a lower goal value which does not necessarily need to be exact.

Below, we will give a very short description and evaluation of type (a) and type (b) methods, followed in Section 5.4 by a more detailed description of type (c) methods.

Type (a) Methods

Type (a) methods provide a mechanism to find trade-offs among the goal variables by interacting with the decision maker. A well-known method of this type is described by Geoffrion et al. [1972], who assume that the decision maker's preferences can be described by a concave and differentiable utility function. The utility function as such is unknown to the decision maker, but he is assumed to be able to provide information on all possible trade-offs among the goal variables. The optimization procedure used, in this case the Frank-Wolfe algorithm (cf. Frank and Wolfe [1956]), determines the kind of information required. This algorithm proceeds from solution to solution via the 'steepest ascent' direction, i.e. the direction with maximum marginal increase in the overall utility function. It also requires knowledge about the maximum allowable step size in this direction. Both kind of data are to be fixed with the help of the decision maker. Dyer [1972] links the above procedure to the one-sided goal programming model (cf. Chapter 4), thus presenting the first interactive multiple objective programming procedure based on goal programming. Because in Dyer's method the goal levels themselves are considered to be determined *a priori* by the decision maker, this method does not fall within our definition of interactive goal programming. Contrary to Dyer, we doubt whether the goal levels may be considered as given and fixed. Goals are being formulated in practice, but this does not necessarily mean that the decision maker is capable of formulating *a priori* any goal that seems to be relevant in a particular decision problem.

Furthermore, goals formulated by the decision maker may change during the process of formulating and solving the problem. The decision maker may change his mind because of information obtained during the process. We believe that goals *a priori* defined by the decision maker may form a rich source of information on his preference structure. However, the decision maker should have the explicit option to change these goals during the interactive process.

This type of approach is very intriguing, because it relates interactive procedures to gradient methods. Moreover, in the linear case, this approach only generates extreme points (cf. Hwang and Masud [1979, p.121]), where an extreme point is a vector in the feasible set R which cannot be expressed as a linear combination of two other, mutually different vectors of the feasible set. Furthermore, the precise nature of the interactions with the decision maker is rather obscure.

Type (b) Methods

For the class of type (b) methods a very attractive procedure has been provided by Zionts and Wallenius [1976]. They also assume an implicit utility function, on the basis of which the decision maker gives his answers. Given a starting solution, which is (arbitrarily) chosen from the efficient set, together with a set of neighbouring efficient (corner) solutions, the decision maker has to compare his preference for the starting solution with his preference for each of its neighbours. From this preference information, a new solution is derived, which again with its neighbours is presented to the decision maker, and so forth.

In this method, the demand on the decision maker's ability to express his preferences is rather small, which is of course a very important advantage. On the other hand, the method may sometimes need a considerable amount of iterations. Also, there is a possibility of suboptimization if a certain proposal solution is surrounded by efficient corner solutions with approximately the same preference level as the proposal solution. The fact that only corner solutions

are generated may in itself be a disadvantage. Nevertheless, we consider this procedure to be one of the most promising and viable interactive procedures.

Another interesting representative of this type of methods is given by Steuer [1977]. At each iteration the decision maker is confronted with a limited set of efficient corner solutions, from which he has to select the 'best'. Given his choice, a new set is calculated and presented to the decision maker. This procedure has a number of attractive properties, e.g. the kind of answers needed from the decision maker does not seem to be very difficult, and it converges within a finite number of steps. A problem with this procedure is that the set of efficient corner solutions is often very large. Therefore, it is necessary to obtain at least some *a priori* information about the decision maker's preferences.For this reason, Steuer (Ibid.) asks the decision maker to specify lower and upper bounds on the 'weights' of the individual goal variables.

An approach which only generates efficient extreme points might sometimes be inconvenient (cf. Vincke [1976]). This is also valid in cases where the number of efficient corner solutions is few. Then the appproximation of an optimal (non-corner) solution by a neighbouring corner solution may be too inaccurate.

In the next section, we will present a sample of type (c) procedures which are rather well suited for the kind of problems we are dealing with in this study. Therefore, and for reasons which will become clear in the next section, we dismiss the methods of types (a) and (b). It should be noted, however, that other decision problems may exist, in which type (a) and type (b) methods should be preferred to those of type (c).

What do the decision makers themselves say about the types of methods that can be used to express their preferences? Some empirical evidence exists. Dyer [1973] asked a number of students to compare the Geoffrion approach (see above) with a trial-and-error procedure. All students found the trial-and-error procedure equally or more

difficult than the Geoffrion approach. Moreover, the students' confidence in the results provided by the latter approach, was as high as the confidence in the results obtained by trial-and-error. Wallenius [1975] experimented with the help of business students and managers. He compared the STEP-method, the Geoffrion approach, and also a trial-and-error procedure and found that the first two procedures were not judged to be better - and in some cases even worse - than the trial-and-error procedure. To make the empirical evidence even more inconclusive, Benson [1975] found that type (c) methods are generally preferred to the other types.

In our opinion, a more detailed comparative study of interactive procedures is highly necessary. Such a project has been recently proposed by Despontin and Spronk [1979].

5.4. Interactive Goal Programming Methods

In this section the interactive multiple objective programming methods which we labeled 'interactive goal programming' (type (c)) methods in the preceding section are considered in more detail.

One broad subclass of interactive goal programming methods which we have found in literature (type (c1)) is based on the calculation of a compromise solution which has a minimum distance (given a particular distance measure) to the ideal solution. Given such a compromise, the decision maker has to define a set of goal levels (or relaxations of the levels in the compromise solution), which are then translated into restrictions and added to the underlying programming model. Next a new ideal and a new compromise solution are calculated, and so forth. Another subclass of interactive goal programming methods found, type (c2), consists of procedures which first find an 'optimal' value for one goal variable. Then, given this optimal value (which is formulated as a restriction), an optimal value for the second goal variable is strived for, and so on. As will become clear in this and the next section, there are several reasons to look for an interactive goal programming procedure outside these two subclasses (see Figure 5.2).

Successive Relaxation of Compromise Goal Values: Type (c1) Methods

Quite a number of interactive procedures are based on the STEM procedure developed by Benayoun et al. [1971]. This procedure consists of a calculation phase and a decision phase. Given the goal variables $g_j(\underline{x})$, $j = 1, \ldots, m$; the ideal solution is calculated (as defined in Chapter 3), yielding goal-values

$$(5.1) \qquad g_j^* = \text{Max}\{g_j(\underline{x}) \mid \underline{x} \in R\} \qquad \text{for } j = 1, \ldots, m.$$

Then, the distance to this ideal point is minimized by using the following minimax formulation (cf. also Section 4.3).

$$(5.2) \qquad \begin{aligned} &\text{Min } v, \text{ s.t.} \\ &v \geq w_j(g_j^* - g_j(\underline{x})) \qquad \text{for } j = 1, \ldots, m; \\ &v \geq 0, \text{ and} \\ &\underline{x} \in R \end{aligned}$$

The relative weights w_j are calculated in a rather mechanical way by means of

$$(5.3) \qquad w_j = \frac{p_j}{\sum_{j'=1}^{m} p_{j'}} \quad \text{and} \quad p_j = \frac{g_j^* - g_j^{\min}}{g_j^* \Big/ \left\{ \sum_{i=1}^{m} (c_i^j)^2 \right\}^{\frac{1}{2}}},$$

where $g_j^{\min}$ is the minimum feasible value of $g_j(\underline{x})$ during the successive optimizations of all m separate goal variables, and c_i^j denotes the coefficient of the instrumental variable x_i in $g_j(\underline{x})$.

The compromise solutions g_j^o of (5.2) are proposed to the decision maker. If some g_j^o values are satisfactory and others not, the decision maker has to accept (and to define) a certain amount of relaxation Δg_k for goal variables k, the value of which is already

satisfactory. Then the restrictions

$$g_k(\underline{x}) \geq g_k^o - \Delta g_k, \text{ and}$$

$$g_\ell(\underline{x}) \geq g_\ell^o \tag{5.4}$$

where ℓ refers to the set of goal variables of which the value was judged to be unsatisfactory are introduced in the next calculatory phase and so on.

Instead of this rather straightforward way of determining the weights w_j, other procedures can be followed. Fandel [1972] and Nijkamp and Rietveld [1976] aimed at identifying an optimal solution while taking account of trade-offs between goal variables. The essential idea of their methods is that each efficient solution is associated with a set of weights for the set of goal variables (cf. Nijkamp and Rietveld [1976, p.7]). As in STEM, they then calculate efficient compromise solutions on the basis of which the decision maker has to determine a set of relaxations as in (5.4). Basically they use the same 'pay-off' information as in (5.1). From such a pay-off table, a vector of weights can be derived which links the solutions in this table to a parametric program. These weights are then used to calculate a new compromise solution.

Fichefet [1976] proposes a combination of STEM and goal programming, which he labeled GPSTEM, to deal with the situation in which there are m decision makers, each having formulated one goal variable. In the first step of GPSTEM, each decision maker has to specify in advance a satisfactory level $\bar{g}_j$ for the goal variable with which he is concerned. In order to find an initial compromise, the following program is solved.

$$\text{Min } v = \sum_{j=1}^{m} (z_j^+ + z_j^-), \text{ s.t.}$$

$$\underline{x} \in R$$

$$g_j(\underline{x}) - z^+ + z^- = \bar{g}_j \qquad \text{for } j = 1, \ldots, m; \tag{5.5}$$

$$\text{and } z_j^+ . z_j^- = 0 \qquad \text{for } j = 1, \ldots, m.$$

On the basis of the solution of this program a set of parametric programs is solved (one for each decision maker). The solutions are used to construct a goal value matrix B which is confronted in a bimatrix game with the usual STEM-type pay-off matrix A (see (5.1)). The equilibrium solutions of this game are finally used to derive an efficient compromise solution, on the basis of which the decision makers have to determine a set of relaxations as in (5.4).

The essence of the STEM-type methods is that by using the pay-off table information, an efficient compromise solution is calculated on the basis of which the decision maker has to define a set of relaxations. Although these methods are rather elegant, it is not clear why they use such fairly mechanical and even complicated procedures to calculate the compromise solutions, which only serve as a point of reference for the decision maker, i.e. to help him to formulate his set of relaxations. Moreover, the decision maker has no opportunity to revise the relaxations if he does not like the consequences, other than by repeating the entire interactive procedure.

The concept of ideal solutions is obviously very valuable. A number of very interesting contributions to the 'theory of ideal solutions' has been delivered by Yu and Zeleny (see e.g. Yu [1973] and Zeleny [1976]). The latter proposes an interactive procedure based on the adaptive displacement of ideal solutions, which is in fact an interactive multiple objective programming procedure of type (a), as discussed in the preceding section.

Sequential Optimization: Type (c2) Methods

Several methods proceed by searching for an 'optimal' value for one goal variable. Then, with the former optimal goal value imposed as a constraint, an 'optimal' value for the second goal variable is found, and so on.

Van Delft and Nijkamp [1977] discuss hierarchical optimization methods which are based on the assumption that the set of goal variables can be ranked in an ordinal way as 'most important',

'next most important', etc. After such a hierarchical rank order has been established, a series of constrained programming problems has to be solved. First, the most important goal variable, $g_1(\underline{x})$ is maximized by

$$\text{(5.6a)} \qquad \begin{aligned} &\text{Max } g_1(\underline{x}) \\ &\text{s.t. } \underline{x} \in R \end{aligned}$$

Then, given the optimal value, g_1^*, a tolerance limit is defined, which is formulated as an inequality constraint added to the already existing set of constraints. Then, the next most important goal variable $g_2(\underline{x})$, is maximized, subject to the new set of constraints by

$$\text{(5.6b)} \qquad \begin{aligned} &\text{Max } g_2(\underline{x}) \\ &\text{s.t. } \underline{x} \in R, \text{ and} \\ &g_1(\underline{x}) \geq \beta_1 \cdot g_1^*, \end{aligned}$$

where $\beta_1 (0 \leq \beta_1 \leq 1)$ is the tolerance parameter associated with g_1^*. For the derived maximum g_2^*, the tolerance parameter β_2 is defined, which is followed by the constrained maximization of $g_3(\underline{x})$, as described below.

$$\text{(5.6c)} \qquad \begin{aligned} &\text{Max } g_3(\underline{x}) \\ &\text{s.t. } \underline{x} \in R, \\ &g_1(\underline{x}) \geq \beta_1 \cdot g_1^*, \text{ and} \\ &g_2(\underline{x}) \geq \beta_2 \cdot g_2^* \end{aligned}$$

This procedure continues up to the point where all goal variables have been dealt with. If all tolerance parameters are set equal to 1, the procedure is quite similar to the sequential method described by (4.4).

An unambiguous ranking of the goal variables is necessary to have the procedure work properly. In our opinion, it is very hard if not impossible to obtain an appropriate ranking. Is one goal variable considered more important than another if (a) the first goal variable 'should be handled first', or (b) if the first goal variable has pre-emptive priority over the other (why then tolerance parameters?), or (c) if the first goal variable has a higher 'weight' than the second? In order to use these hierarchical methods properly, one should be sure about the decision maker's notion of 'one goal variable being more important than another'.

Another problem is the assessment of the tolerance parameters β_j, $j = 1, \ldots, m$. As shown in van Delft and Nijkamp [1977], the parameters can be assessed by means of an interactive procedure, in which the 'optimal' values of the tolerance parameters are calculated one by one, in hierarchical order. Such a procedure implicitly assumes the decision maker to be able to define trade-offs between more and less important goal variables, which must be valid in the optimal solution, although the optimal solution is unknown at the time of defining these trade-offs. Obviously, in the case of two goal variables, this problem does not exist because then the tolerance limit of the most important goal variable can be traded-off directly against the value of the less important goal variable, again in an interactive way.

Benson [1975] proposes a procedure which starts by asking the decision maker to specify a set of minimally acceptable goal levels. Next, the decision maker is asked to identify which of the specified goal levels is the least satisfactory. Then the goal variable concerned is maximized subject to the constraints on the other goal variables. If this maximum value is judged to be unsatisfactory, some of the goal levels specified for the other goal variables must be relaxed. If satisfactory, the obtained solution is either optimal or the value of the former least satisfactory goal variable is now too high. In the latter case, the decision maker is asked to specify the maximal amount of relaxation which is acceptable for this goal

variable. Then the procedure is repeated by again asking the decision maker what is now the least satisfactory goal value, and so on.

Also in this method, it is hard to specify an 'optimal' value for one goal variable without knowing the optimal values for the others. Besides, there is very little help available to the decision maker to specify the optimal value (i.e. the allowable relaxation) of the formerly least satisfactory goal variable. The notion of a set of minimum goal levels to start with is very attractive, although in this case, by letting the decision maker specify these minimum levels, it may be hard and time consuming to find an initial set which is feasible. Also, these minimum values may have to be adapted in a downward direction during later stages of the procedure. These problems can be circumvented by defining (and specifying) these minimum goal values in a slightly different way (cf. Chapter 6). A clear advantage of this method is that, in principle at least, it can also solve non-linear models.

Monarchi et al. [1973] developed a procedure in which for every goal variable $g_j(\underline{x})$, $j = 1, \ldots, m$; an aspiration level g_j, $j = 1, \ldots, m$; is defined by the decision maker. The aspiration levels may be desired upper bounds, lower bounds, razor-edge targets (cf. Chapter 4), or bounds of intervals within which the most preferred values of the goal variable concerned are contained, or, alternatively, are not contained. For each goal defined in this way, a dimensionless indicator of attainment d_j is defined, which in general is a non-linear function of a goal variable which in its turn may be non-linear in the instrumental variables. At each iteration, one principal problem and m_i auxiliary problems are solved, where m_i is the number of goal variables the value of which has not yet been restricted in earlier iterations ($m_1=m$, $m_2=m-1$, $m_3=m-2,\ldots$). At the first iteration, the principal problem is to

$$\text{Min} \sum_{j=1}^{m} d_j \qquad (5.7)$$

$$\text{s.t. } \underline{x} \in R,$$

and the auxiliary problems are

$$\text{Min} \sum_{\substack{j=1 \\ j \neq j'}}^{m} d_j \qquad (5.8)$$

$$\text{s.t. } \underline{x} \in R, \text{ and}$$

$$g_{j'}(\underline{x}) \geq \bar{g}_{j'},$$

for $j' = 1, \ldots, m$. These problems are solved by means of a non-linear goal programming routine. The results are presented to the decision maker, who, on the basis of this information, revises one of the aspiration levels, which is then added as a constraint to problems (5.7) and (5.8). After dropping the indicator $d_{j'}$ (which belongs to the restricted goal variable) from the objective functions, these problems are solved again. On the basis of the results, the decision maker again adds a restriction to the problems, and so forth.

The method described has the attractive properties that it can incorporate goal variables which neither have to be maximized nor minimized, and that it can solve non-linear problems. Disadvantages are that linear problems are translated into non-linear problems (which demand a lot of computer time), the difficulty to specify aspiration levels on the basis of the data generated by the method, and the possibility of finding inconsistent constraint sets when solving auxiliary problems (cf. also Hwang and Masud [1979]).

As is evident from the above overview, there are quite a few interactive goal programming methods available. All of them share the advantage that the decision maker can express his preferences in terms of goal values. This seems to offer a close correspondence to everyday practice, in which decision makers are accustomed to thinking in terms of 'goals' and 'targets' (cf. also Chapters 2 and 4).

Interactive goal programming methods are not limited to efficient extreme points, which was one of the disadvantages of the procedures

described in the preceding section. Moreover, aspired levels of goal variables which are non-linear in the instruments can be formulated in a straightforward manner by introducing restrictions (e.g. goal chance constraints or target ratio values,(cf. also Chapter 8).

Another advantage of interactive goal programming methods is that the feasible region is reduced step by step. This may simplify the computational efforts (which in themselves are not very cumbersome in these types of methods)[1]. Moreover, each iteration reduces the size of the decision problem, i.e. the number of alternatives still to be evaluated. In general, this is not true for the interactive procedures of types (a) and (b).

There are two sides to every coin. Methods of type (c) also have their disadvantages. In the case of the STEM-based methods it is not clear why more or less complicated procedures are used for the calculation of successive compromise solutions, which only serve as a point of reference for the decision maker. Moreover, the decision maker has complete freedom in defining the goal level relaxations which are required at each iteration. This seems attractive, but may in fact lead to inconsistencies, even more so as the relaxations cannot be revised after they have been formulated. As will be shown in the next chapter it is possible to guide the decision maker in his choice of the aspired goal levels without affecting his freedom of choice. The main disadvantages of the sequential optimization methods are again the lack of assistance for the decision maker determining aspired values of the goal variables, and the fact that the optimal value of one goal variable must be determined without knowing (often not even approximately) what the values of the other goal variables will be. Interactive Multiple Goal Programming (IMGP),

1) For instance because some of the constraints become redundant if certain goal levels have to be met (see Spronk and Telgen [1979] and Telgen [1979]). On the other hand, the introduction of goal levels as constraints increases the computational efforts.

the method described in Chapter 6, tries to circumvent this problem by starting with a set of minimum goal levels which are satisfied by a whole set of alternatives. The decision maker is then asked to raise these goal values step by step, whereby it is not necessary to first drive one goal variable to its optimum, then the next, and so on. It is not necessary to define 'razor-edge' goal values. Instead minimum values are to be given for which (in the decision maker's opinion) it is certain that they do not exclude the optimal solution. In defining these values the decision maker can use several sources of information, such as the displacement of the ideal solution resulting from the definition of new minimum goal values. The latter may even be revised in later iterations.

5.5. Conclusion

In this chapter we have shown that interactive procedures may be helpful in cases where it is too difficult or too costly to obtain all the *a priori* information necessary to solve a given multiple objective programming problem. We have divided these interactive multiple objective programming methods into three types, depending on the kind of information to be delivered by the decision maker:

(a) determination of trade-offs among the goal variables, given a particular solution
(b) determination of the *subset of 'best'* from among a given set of efficient solutions
(c) specification of aspired *goal values*, given a particular solution.

Within each type, methods differ a.o. with respect to the required accuracy of the information to be given by the decision maker.

In our opinion, each of the three types of methods has a right of existence, given the fact that different decision situations may need different methods. On rather theoretical grounds, we have made a plea for type (c) methods. This type can be further subdivided, for instance, into methods which successively relax the goal values

in subsequent compromise solutions, and into sequential optimization methods. Our classification of interactive programming methods has been illustrated in Figure 5.2. In the next chapter we describe a new method, which tries to avoid the disadvantages and to combine the advantages of the already available interactive goal programming methods.

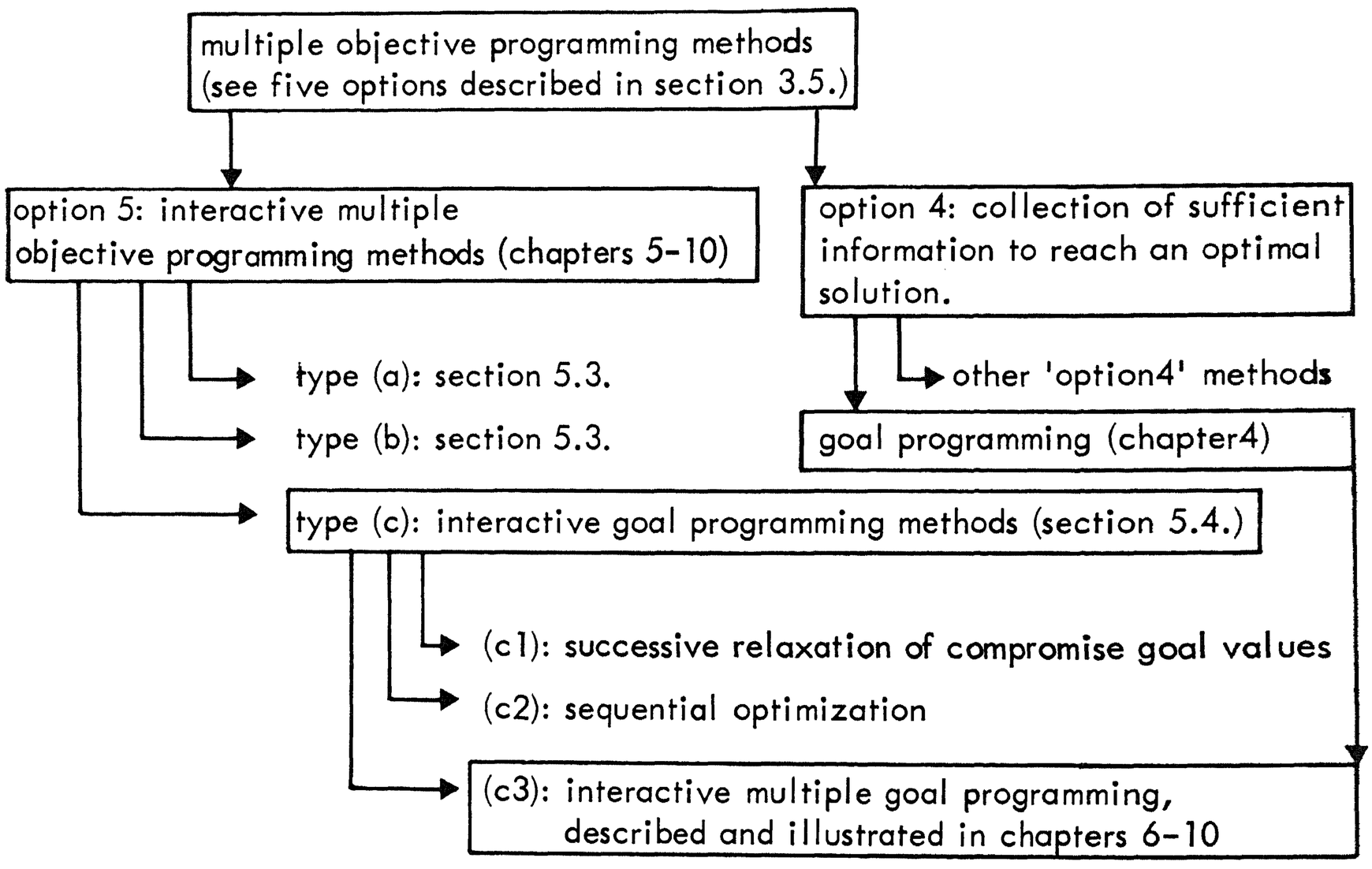

Figure 5.2. Classification of interactive multiple objective programming methods

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