7. IMGP IN PRACTICE: EXAMPLES AND EXPERIENCES

In this chapter we present some examples of (potential) applications of IMGP. These examples have been chosen deliberately from fields other than capital budgeting and financial planning (which are dealt with in more detail in Chapters 8, 9 and 10), in order to demonstrate that the application potential of IMGP is not limited to capital budgeting and financial planning. In fact, it can be used in any decision situation which meets the fairly unrestrictive requirements described in Chapter 6.

In Section 7.1 we demonstrate the use of IMGP by means of two simple illustrations. In order to simulate the use of IMGP in practice, we introduce in Section 7.2 an imaginary decision maker, facing a portfolio selection problem. Section 7.3 describes some of our experiences with the implementation of IMGP in combination with large scale linear programming models. Section 7.4 gives two examples of the use of IMGP in decision problems with discrete choice alternatives. IMGP can be used fruitfully in this case, though with some minor modifications. Our conclusions are given in Section 7.5.

7.1. Two Simple Examples

In this section we demonstrate the use of IMGP by means of two very simple examples. The first example concerns the choice of an 'optimal' production combination of two product varieties from an infinite number of alternatives. The second example deals with a professor who has to decide how to allocate his time budget.

Example 1: A Production Planning Problem

A brick factory can produce two brick varieties, but due to the limited capacity of machines, kiln and drying-room and to the limited availability of skilled personnel, these products cannot be produced in any desired combination. We show the area of feasible production combinations in Figure 7.1, where x_1 and x_2 represent the quantity produced of variety one and two, respectively (both in millions). For the planning period concerned management is not able to define a profit function in terms of x_1 and x_2, due to very uncertain conditions of the market and due to problems in the factory, where a recently installed machine causes many difficulties. Therefore management wants to consider both x_1 and x_2 as goal variables. We thus have

$$(7.1) \qquad g_1(x_1,x_2) = x_1 \text{ and } g_2(x_1,x_2) = x_2$$

Although the maximum production of variety one is equal to 18,000,000 it is the 'problem machine' that causes difficulties when the production of x_1 is raised over 14,000,000 units. In fact this machine runs best when approximately 12,000,000 units are produced with it. On the other hand, the factory has contracts to deliver 8,000,000 units of variety one. Although this variety has been estimated as less profitable than variety two, management wants to meet the contractual obligations because the customers concerned also buy a lot of variety two and offer a promising buying potential in the near future. Thus the preferences for $g_1(\underline{x})$ seem to be monotone non-decreasing for $g_1(\underline{x}) = x_1 \leq 12,000,000$ and monotone non-increasing for $g_1(\underline{x}) = x_1 \geq 12,000,000$. Therefore it is reasonable to consider both $g_1^* = 12,000,000$ and $g_1 = 8,000,000$ as the relevant aspiration levels for $g_1(\underline{x})$. There are no problems whatsoever in the production of the fairly profitable second variety. Management wants to produce as much as possible of this second variety, thus to maximize $g_2(x_1,x_2)$.

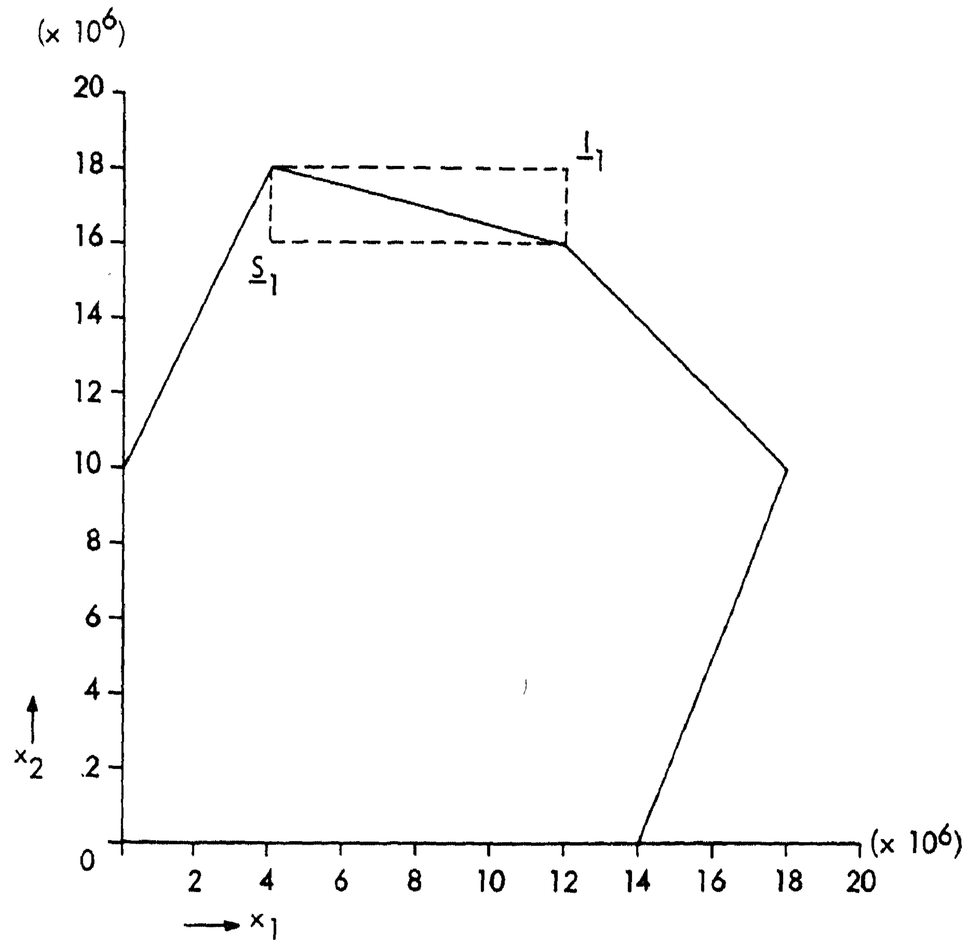

Figure 7.1. A production planning problem

When $g_2(\underline{x})$ is maximized, we have $g_2^* = 18$ and $g_1(\underline{x}) = 4$. Because this is a case of two goal variables (cf. Chapter 6), it is not necessary to accept a value of $g_1(x)$ lower than $g_1^{min} = 4$. No matter what minimal value for $g_2(\underline{x})$ is required by the decision maker (provided $\underline{x} \in R$), there is always a solution for which at the same time $g_1(\underline{x}) \geq 4$. By setting $g_1(\underline{x})$ equal to the most desired production volume $g_1(\underline{x}) = g_1^* = 12$, the value of $g_2(\underline{x})$ becomes $g_2(\underline{x}) = 16$, which is at the same time (by similar reasoning) the value of g_2^{min}. Therefore, we must find a final solution in which $4 \leq g_1(\underline{x}) \leq 12$ and $16 \leq g_2(\underline{x}) \leq 18$. Together with the information provided by the management we can list (step 2) the following aspiration levels. For $g_1(\underline{x})$ the values 4, 8 and 12; for $g_2(\underline{x})$ the values 16 and 18. The first potency matrix can be written as

$$P_1 = \begin{bmatrix} 12 & 18 \\ 4 & 16 \end{bmatrix} \tag{7.2}$$

In the third step the first solution $\underline{S}_1$ is set equal to the pessimistic solution $\underline{Q}$ and presented to the decision maker, together with the potency matrix P_1. The pessimistic starting solution $\underline{S}_1$ and the ideal solution $\underline{I}_1$ are indicated in Figure 7.1. This solution has to be evaluated by the decision maker and subsequently integrated in the model. The successive steps towards the final solution are shown in Table 7.1. and illustrated in Figure 7.2.

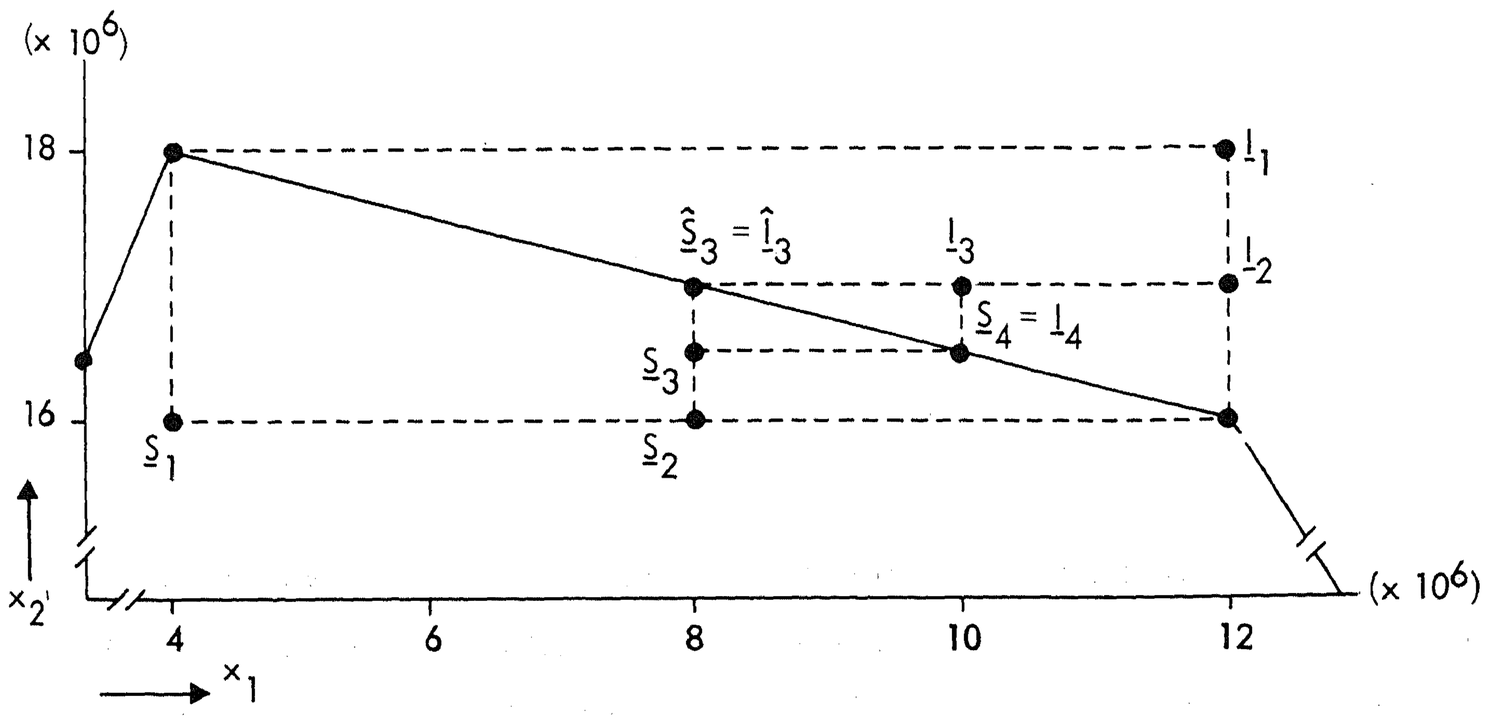

Figure 7.2. All solutions from the starting solution up to and including the final solution

In this relatively simple example we have omitted the definition and redefinition of the δ_j. To be complete we will now show how the δ_j's should have been defined during the successive iterations. At the beginning, in step 2, we have $\delta_1 = \delta_2 = 0$. The proposal solution (8,16) is accepted without modifying the proposed goal levels. Consequently, δ_1 and δ_2 remain unchanged. The proposal solution (8,17) is judged to have a too strong influence on the first goal variable. Thus δ_2 is set equal to 1. The new proposal solution (8,16.5) is subsequently accepted and δ_2 is halved to 0.5. Finally, the proposal solution (10, 16.5) is accepted without any problem, by which δ_1 remains zero. When this last proposal solution would not have been accepted directly, δ_1 would have also become positive.

Table 7.1. Successive (proposal) solutions together with the opinion of the decision maker

(proposal) solution to be evaluated	(proposal) potency matrix	steps	evaluation
$S_1 = [4,16]$	$P_1 = \begin{bmatrix} 12 & 18 \\ 4 & 16 \end{bmatrix}$	2,3,4,5	$\underline{S}_1$ not satisfactory, raise value $g_1(\underline{x})$
$\hat{S}_2 = [8,16]$	$\hat{P}_2 = \begin{bmatrix} 12 & 17 \\ 8 & 16 \end{bmatrix}$	6,7,8	shifts in P justified, set $\underline{S}_2=\hat{\underline{S}}_2$ and $P_2=\hat{P}_2$
$S_2 = [8,16]$	$P_2 = \begin{bmatrix} 12 & 17 \\ 8 & 16 \end{bmatrix}$	4,5	$\underline{S}_2$ not satisfactory, raise value $g_2(\underline{x})$
$S_3 = [8,17]$	$P_3 = \begin{bmatrix} 8 & 17 \\ 8 & 17 \end{bmatrix}$	6,7,8	shifts in P not justified, lower value $g_2(\underline{x})$
$\hat{S}_3 = [8,16.5]$	$\hat{P}_3 = \begin{bmatrix} 10 & 17 \\ 8 & 16.5 \end{bmatrix}$	9,7,8	shifts in P justified, set $\underline{S}_3=\hat{\underline{S}}_3$ and $P_3=\hat{P}_3$
$S_3 = [8,16.5]$	$P_3 = \begin{bmatrix} 10 & 17 \\ 8 & 16.5 \end{bmatrix}$	4,5	$\underline{S}_3$ not satisfactory, raise value of $g_1(\underline{x})$
$\hat{S}_4 = [10,16.5]$	$\hat{P}_4 = \begin{bmatrix} 10 & 16.5 \\ 10 & 16.5 \end{bmatrix}$	6,7,8	shifts in P justified, set $\underline{S}_4=\hat{\underline{S}}_4$ and $P_4=\hat{P}_4$
$S_4 = [10,16.5]$	$P_4 = \begin{bmatrix} 10 & 16.5 \\ 10 & 16.5 \end{bmatrix}$	4	end of procedure

Example 2: Allocation of a Time Budget

A professor has to decide how many days a week on the average he will spend on research and on teaching and related activities (meetings, preparations, exams, etc.), respectively. We assume the number of courses and the number of students to be given and fixed. Therefore time spent on teaching influences only the quality of teaching and not the formal quantity. Let us denote the decision variables by x (days of teaching) and by y (days of research). Because a week has 7 days, we have

$$x + y \leq 7 \tag{7.3}$$

If he does not want to be dismissed in the long run, he has to spend at least one day a week on teaching activities. So we have

$$x \geq 1 \tag{7.4}$$

In his opinion, three days a week spent on teaching is optimal. He does not mind spending more time on this activity, but he feels that this additional effort will not contribute significantly to the quality of his teaching. Because he wants to maximize the quality of this teaching (by simultaneously minimizing the risk of being dismissed), one of his goals is:

$$\text{Min}\{g_1 = z_1^-\} \tag{7.5}$$

$$\text{s.t. } x - z_1^+ + z_1^- = 3$$

Another desire is to minimize working during the weekends, although he does not mind working during the week. He does not have any problem in taking free days during the week.
These desires can be translated into

(7.6) $$\text{Min } \{g_2 = z_2^+\}$$

$$\text{s.t. } x + y - z_2^+ + z_2^- = 5$$

A third goal variable is his current wealth, which he wants to be as high as possible. Let us assume that each additional day spent on research contributes 6 ducats to his wealth, where a day spent on teaching contributes 2 ducats. He therefore wishes to maximize

(7.7) $$\text{Max } g_3 = 2.x + 6.y$$

A graphical representation of this problem is given in Figure 7.3. The elaboration by means of IMGP is shown in Figure 7.4.

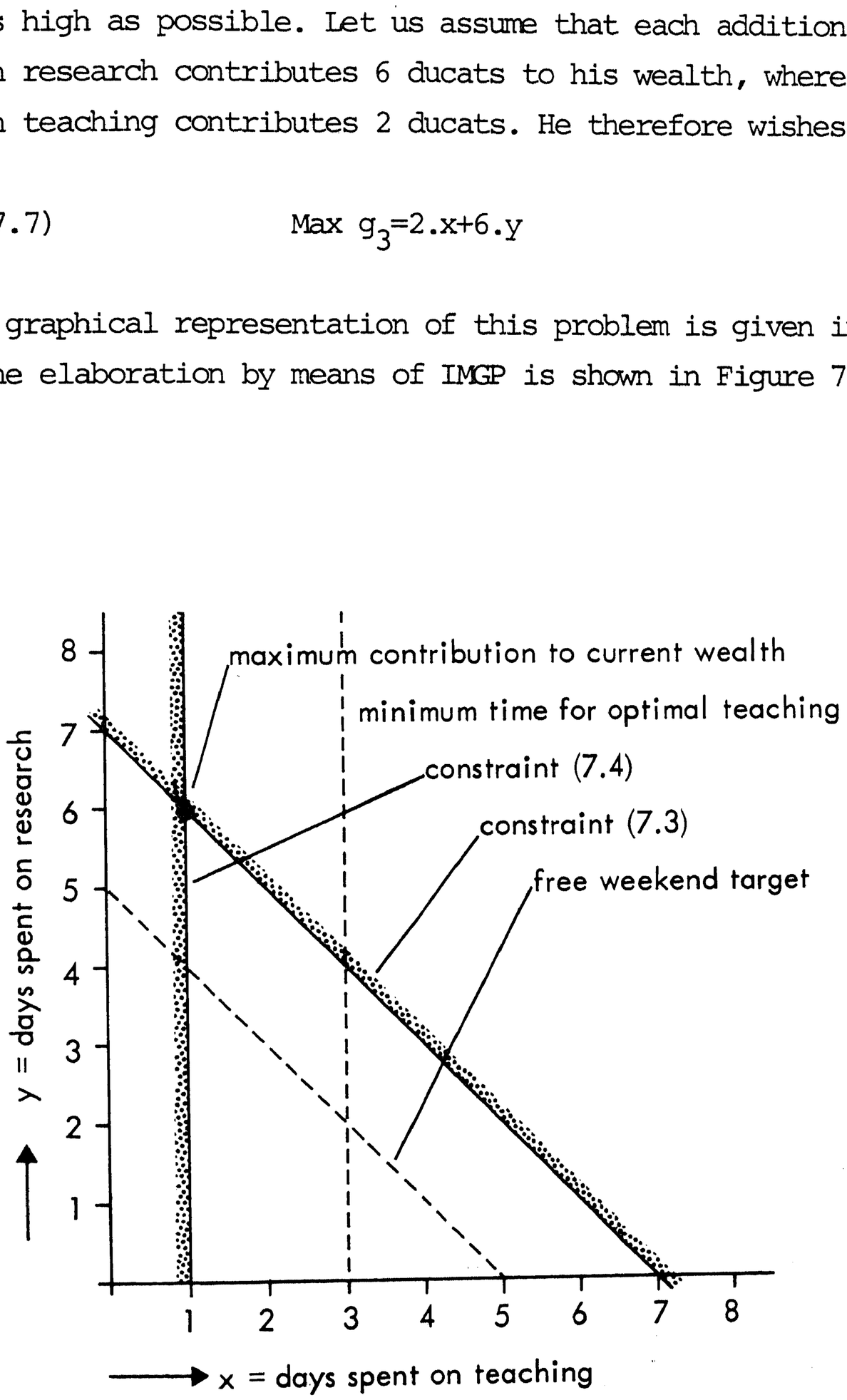

Figure 7.3. The time budget allocation problem

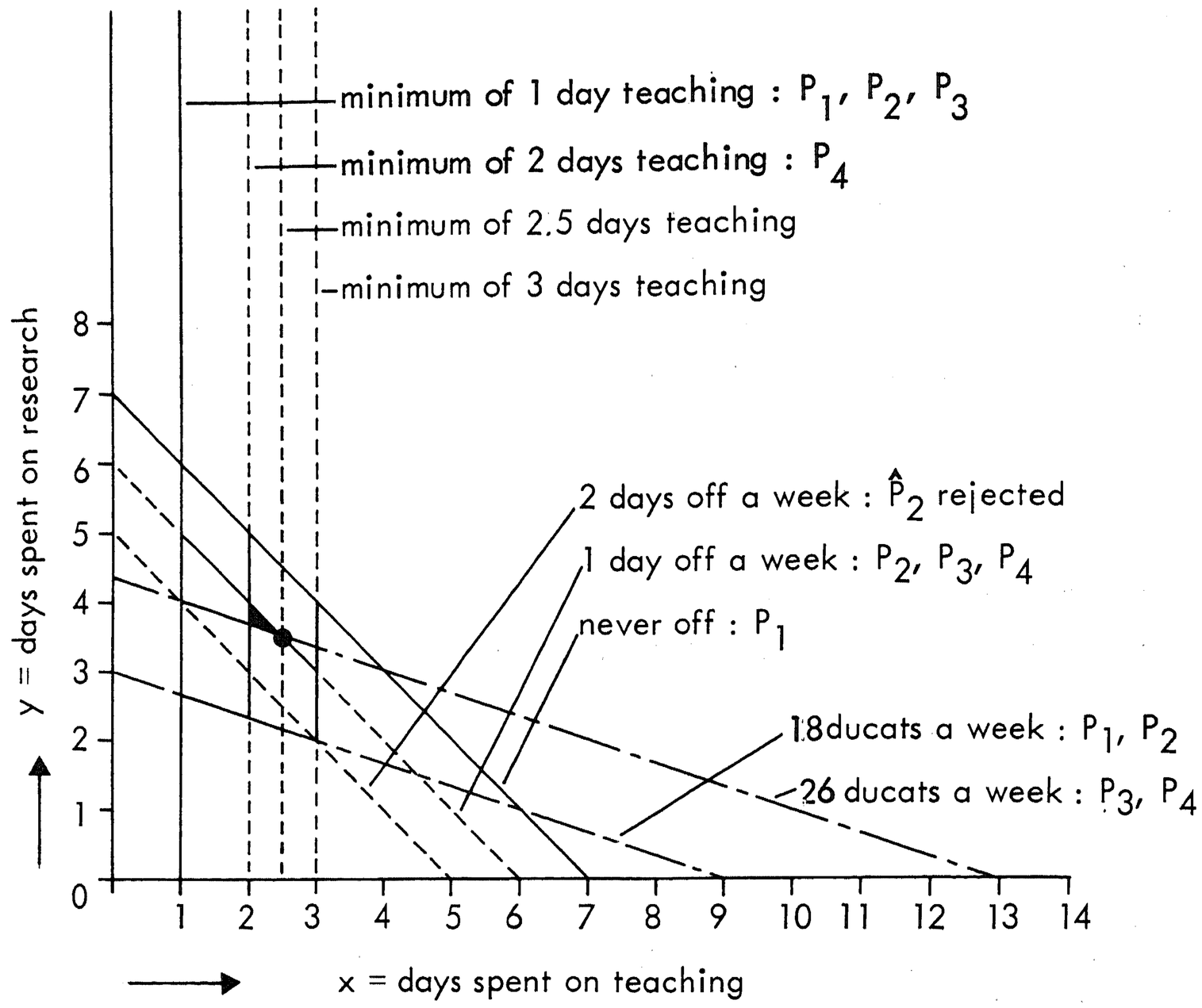

Figure 7.4. Successive IMGP steps for the time budget allocation problem

In order to discover whether (and if so, to what extent) these goal variables are conflicting, we construct (step 1) the following potency matrix P_1.

$$P_1 = \begin{bmatrix} g_1^* & g_2^* & g_3^* \\ g_1^{min} & g_2^{min} & g_3^{min} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 38 \\ 2 & 2 & 18 \end{bmatrix} \tag{7.8}$$

During the construction of this matrix, we deviated slightly from the procedure described in Chapter 6 (step 1) in order to take

account of non-unique optimum solutions [1].

Let us assume that the professor (in step 2) has defined one aspiration level for each of the goal variables, which are respectively equal to 1 for g_1, 0 for g_2 and 26 for g_3, the latter value representing his estimation of the minimum welfare needed to survive.

In the third step, the most pessimistic solution, $\underline{S}_1 = (2,2,18)$, is presented to the decision maker, together with the potency matrix P_1. We assume that he is not satisfied with $\underline{S}_1$ (step 4) and that first (step 5) he wants to have a free weekend which is as long as possible (step 6). Thus we can construct the proposal solution $\hat{\underline{S}}_2 = (2,0,18)$. In step 7 we calculate the corresponding potency matrix $\hat{P}_2$ as

$$\hat{P}_2 = \begin{bmatrix} 0 & 0 & 26 \\ 2 & 0 & 18 \end{bmatrix} \tag{7.9}$$

Comparing (in step 8) this matrix with P_1, the decision maker judges that his possibilities of increasing his wealth have critically deteriorated. We thus have to set (in step 9) a new, but less ambitious value for the second goal variable. By choosing this new value exactly between the corresponding value in $\underline{S}_1$ and the corresponding value in the rejected proposal solution, we get the new proposal solution $\hat{\underline{S}}_2 = (2,1,18)$ with the potency matrix

1) Note for instance that the minimization of $g_1 = z_1^-$ yields the complete area between points (3,0), (7,0) and (3,4). We solved this problem by calculating a new pay-off matrix for the goal variables g_2 and g_3, given the optimum value for g_1. From this, the worst values of g_2 and g_3 were taken as corresponding to the optimum for g_1.

(7.10) $$\hat{P}_2 = \begin{bmatrix} 0 & 0 & 32 \\ 2 & 1 & 18 \end{bmatrix}$$

Assuming that these shifts in the potency matrix are considered justified, we can define $\underline{S}_2 = \hat{\underline{S}}_2$ and $P_2 = \hat{P}_2$, leaving the decision maker with at least one free day a week.

Returning to step 4 and 5, the professor argues that if he cannot have a two-day weekend free he certainly has to increase his wealth by more than the current 18 ducats a week. Given this information and the list of aspiration levels, we propose $\hat{\underline{S}}_3 = (2,1,26)$ and calculate the corresponding potency matrix $\hat{P}_3$ (step 6 and 7), which is given by

(7.11) $$\hat{P}_3 = \begin{bmatrix} 0.5 & 0 & 32 \\ 2 & 1 & 26 \end{bmatrix}$$

The professor judges these shifts in the potency matrix to be acceptable. Thus we define $\underline{S}_3 = \hat{\underline{S}}_3$ and $P_3 = \hat{P}_3$. Next, in this decision maker's opinion, the quality of his teaching should be ensured. Therefore, we propose $\hat{\underline{S}}_4 = (1,1,26)$ and calculate the potency matrix

(7.12) $$\hat{P}_4 = \begin{bmatrix} 0.5 & 2/3 & 28 \\ 1 & 1 & 26 \end{bmatrix}$$

Because he judges 2 days a week on teaching (i.e. $\bar{z}_1 = 1$) a minimum to guarantee good (although not optimal) teaching, he accepts the shifts in the potency matrix implied by this requirement. Thus we have $\underline{S}_4 = \hat{\underline{S}}_4$ and $P_4 = \hat{P}_4$. Summarizing the current state of affairs, the professor has to work a maximum of six days a week, of which two days at least are spent on teaching. His increase in wealth will be at least 26 ducats a week. Given this

situation, he may continue to add requirements. For instance, he may want to maximize time spent on teaching, which becomes 2.5 days of a six-day working-week, yielding 26 ducats. Similarly, maximizing the increase in wealth yields 28 ducats a week, earned with two days spent on teaching and four days on research. Minimizing the length of the working-week implies two days of teaching and $3\frac{2}{3}$ days of research with an increase in wealth of 26 ducats a week.

However, in our opinion it is unlikely that the decision maker will choose such a razor-edge solution. It is more natural that he accepts $\underline{S}_4$ and P_4 as they are and decides, for instance, to work $5\frac{2}{3}$ - 6 days a week, of which no more than 2 days spent on teaching, earning 26 - 28 ducats a week.

7.2. Experiments with an Imaginary Decision Maker

In order to demonstrate some of the convergence properties of IMGP, we introduce in this section the concept of an imaginary decision maker, who makes the necessary managerial choices. We assume that his preference structure can be described adequately by a concave preference function exactly known to us. We assume the decision maker to be able to give his judgements concerning a solution proposed to him by the IMGP procedure, and moreover to do so in a manner which is in complete accordance with the preference function specified.

Given the *a priori* knowledge of the decision maker's preference function, we can calculate an optimal solution of the problem at hand. At the same time, we can execute the IMGP procedure by 'interacting' with the imaginary decision maker. Then, the solution obtained from the *a priori* specified preference function can be compared with the solution resulting from the interactive procedure.

In doing so we make three simplifying assumptions, each of which can be relaxed quite easily. For the moment, we assume that the

decision maker does not specify any *a priori* information. Furthermore, we assume him to give consistent answers without any errors. Finally, within the IMGP procedure, the decision maker has to indicate, given an accepted solution, which goal value should be raised. In the present case, we introduce the assumption that the decision maker first tries to drive the first goal variable to its optimal value, next the second, third and following goal variables respectively. This procedure of changing the value of a given goal variable is concluded as soon as it reaches a value which is sufficiently close to the optimal value.

The computer program for the described experiment has been run on an IBM 370/158 computer at the Technical University in Delft, The Netherlands. In the program, which has been written in PL/1, the MPSX package has been used to solve the linear programming problems (see also Appendix 7.a).

Below, we give an example of an imaginary decision maker using IMGP in order to find a solution for his portfolio selection problem.

Example 3: Selection of an Investment Portfolio

Several authors have formulated the problem of portfolio construction as a multiple goal program. Baum and Carlson [1974] showed how efficient solutions can be found. However, they did not indicate how a final solution from the set of efficient solutions should be chosen. Lee [1972], Lee and Lerro [1973], Kumar, Philippatos and Ezzell [1978] and Bronsema and van de Kieft [1978] formulated the problem as a standard multiple goal problem, requiring complete *a priori* information on the decision maker's preferences.

Lee [1972] formulated a model for selecting efficient mutual funds portfolios. Given a budget constraint, a set of legal constraints and a set of restrictions representing the maximum

allowable concentration in any industry, two goals must be strived for. The first goal is to maximize the expected return of the portfolio, E_p. The second goal is to minimize the risk of the portfolio, as measured by its market volatility. The market volatility of a portfolio consisting of n securities is represented by

$$B_p = \sum_{i=1}^{n} x_i . B_i, \tag{7.13}$$

where x_i is the fraction of the budget invested in security i, B_i is the market volatility of security i and B_p the market volatility of the portfolio. The market volatility of a security or a portfolio measures its relationship with some stock market index common to all securities. The market volatility (or beta) of a security is often called the systematic risk of a security. Besides this risk each security has its own unsystematic risk component, viewed as a random variable with zero mean and standard deviation s_i. All pairs of random variables representing the unsystematic risk are assumed to have zero covariance and to be uncorrelated with the market index. Lee wanted to account for the unsystematic risk of the portfolio explicitly. Therefore, he introduced 'current dividend yield' as an additional goal variable 'to help us compensate for having the income fund suffer some unaccounted for risk' (Ibid. pp. 227-228).

Given this framework, Lee formulated both a goal program for a growth portfolio and a goal program for an income portfolio, mutually only different with respect to their objective functions. The first objective function was formulated to maximize expected growth, to minimize market volatility and to maximize current dividend yield in this order which was moreover pre-emptive. The second objective function was also ordered by pre-emptive priority factors, but in the reverse direction.

In the IMGP approach we replaced these two objective functions by three separate objective functions, one to maximize

expected growth, one to minimize market volatility, and one to maximize current dividend. The imaginary decision maker's preference function was specified as

$$(7.14) \qquad f(E_p,B_p,D_p,) = 0.4E_p + 0.2B_p + 0.4D_p,$$

where E_p, B_p and D_p are the portfolio's expected growth, market volatility and current dividend yield respectively. For further details on this experiment, we refer to Nijkamp and Spronk [1978]. All results are presented in Table 7.2, where the solution for the *a priori* specified preference function (7.14), the solutions given by Lee, and the unconditional optimal solutions for each of the objective functions separately are also shown.

Besides the initial pessimistic solution, only nine proposal solutions had to be evaluated by the imaginary decision maker.

Given the first proposed constraint for E_p, a pessimistic solution follows in which the lower bound of D_p has also been increased. Because the optimal solution (a) is not excluded by this proposal solution, the latter has been accepted by the decision maker. The second solution has been obtained by raising E_p. This solution is unacceptable because it excludes a B_p value smaller than 0.608 and a D_p value over 0.025. The next solution is obtained by lowering the E_p value, which has been accepted. Then, the E_p value is raised again, each time with a rise of half its previous size (S_1), until the required accuracy for E_p is reached in iteration 6. In step 7 the value of B_p is lowered with $\delta_2 = 0.68$. In step 8 the value of B_p is lowered with a value which is not equal to the δ_2 in step 7 divided by 2. This is because such a value would have caused a B_p value outside the 'potency region' of B_p specified in iteration 7. In this case δ_2 has been divided by two until a feasible B_p value could be proposed in iteration 8. The same happens in iteration 9. The solution proposed in iteration 9 is the final solution, because the required accuracy for B_p has been attained and because the D_p value has been driven close enough

Table 7.2. IMPG applied to Lee's model

description of the objective function	expected growth *	expected growth **	market volatility *	market volatility **	current dividend *	current dividend **	comment
(a) solution for (7.14)	0.4975		0.5844		0.00285		
(b) growth portfolio (Lee)	0.677		1.361		0.009		
(c) income portfolio (Lee)	0.105		0.922		0.033		
(d) max. expected growth	0.677		1.361		0.009		
(e) min. market volatility	0.0		0.0		0.0		
(f) max. current dividend	0.107		0.887		0.033		
(0) initial pessimistic sol.	0.0		1.361		0.0		
(1) proposed constraints	0.338		1.361		0.0		δ_1=0.338
pessimist. solutions		0.338		1.361		0.0027	
potential solutions		0.677		0.346		0.029	accepted
(2) proposed constraints	0.507		1.361		0.0027		δ_1=0.169
pessimist. solutions		0.507		1.361		0.0029	
potential solutions		0.677		0.608		0.025	rejected
(3) proposed constraints	0.423		1.361		0.0027		δ_1=0.085
pessimist. solutions		0.507		1.361		0.0023	
potential solutions		0.677		0.462		0.028	accepted
(4) proposed constraints	0.465		1.361		0.0023		δ_1=0.043
pessimist. solutions		0.465		1.361		0.0026	
potential solutions		0.677		0.471		0.026	accepted
(5) proposed constraints	0.486		1.361		0.0026		δ_1=0.021
pessimist. solutions		0.486		1.361		0.0028	
potential solutions		0.677		0.565		0.0268	accepted
(6) proposed constraints	0.497		1.361		0.0028		δ_1=0.011
pessimist. solutions		0.497		1.361		0.0028	
potential solutions		0.677		0.584		0.025	accepted
(7) proposed constraints	0.497		0.681		0.0028		δ_2=0.681
pessimist. solutions		0.497		0.681		0.0028	
potential solutions		0.534		0.584		0.013	accepted
(8) proposed constraints	0.497		0.596		0.0028		δ_2=0.085
pessimist. solutions		0.497		0.596		0.0028	
potential solutions		0.502		0.584		0.0048	accepted
(9) proposed constraints	0.497		0.585		0.0028		δ_2=0.011
pessimist. solutions		0.497		0.585		0.0028	
potential solutions (final solution)		0.498		0.584		0.0030	accepted
(a) solution for (7.14)	0.4975		0.5844		0.00285		

* goal values belonging to optimal solutions and proposed goal constraints.

** pessimistic and potential goal values.

to its optimal value.

The relatively favourable results with the above and other examples (cf. also Nijkamp and Spronk [1978a]) cannot be generalized without some reserves. Clearly, the number of iterations needed depends on the number of goal variables, the exact shape of the preference function and the required accuracy. Furthermore, we assumed that the decision maker did not specify any *a priori* information. If this is the case, the effect on the method's convergence speed cannot be determined beforehand, because such information may either accelerate or decelerate the process. Finally, in this example we introduced an *ad hoc* hierarchical ranking of the goal variables to be changed. First, the expected growth was driven to its optimal value, then the market volatility and finally the current dividend. It may be that another order of changing the goal values may accelerate or decelerate the process.

7.3. Some Empirical Results

Just before the publication of this study, a project was started in which IMGP was used to help investigating the conflicts between a number of goal variables with respect to a large scale input-output model describing the economic-environmental system of Western Europe. First, a pilot study was carried out with a simplified version of this input-output model (cf. Hartog, Nijkamp, and Spronk [1980]). Because not all results of the main study were available at the time of publication of the present study, we confine ourselves here to a summary of the results of the pilot study. The simplified version of the input-output model consists of 160 relations in 130 structural variables describing three conventional sectors and one sector for the abatement of pollution (see for further details van Driel et al [1980]). To start with, the following goal variables have been defined by the analysts.

(1) wages

(2) consumption

(3) minimum growth of consumption

(4) maximum growth of capacity

(5) nuisance (defined as the amount of unabated pollution)

(6) production of the pollution abatement sector.

With this set of goal variables, a number of experiments have been carried out with decision makers (in this pilot study these were interested scientists being familiar with this kind of problems, both in theory and practice) trying to achieve a satisfactory solution for the problem at hand. The results of these experiments are rather encouraging. Each iteration costs some 30 seconds of CPU time (see Appendix 7.6 for details), needed to recalculate the potency matrix by means of solving 6 fairly similar linear goal programs [1)]. Although it is not strictly necessary for interactive procedures to have such a low 'return time', it can be seen as an important advantage. This is because the decision maker, at the terminal display, can easily experiment with different combinations of the goal variables, without high costs and within a relatively short period. In Table 7.3, we give a condensed description of the third session at the terminal desk of a decision maker (see Hartog et al [1980] for a more detailed description).

It is not necessary for a decision maker to continue the interactive procedure to the extent that the present decision maker did, i.e. to proceed until a unique (or nearly unique) final solution occurs. One might as well stop at an earlier iteration, being left with a number of 'scenarios' all satisfying the minimum conditions specified by the decision maker. The choice out of these scenarios

1) This fairly good performance can be ascribed to the relative simplicity of IMGP, combined with some attractive properties of the MPSX-package. Note for instance, that from iteration to iteration the goal programs change only with respect to some right-hand side values, so that each optimization can greatly benefit from earlier optimal solutions.

Table 7.3. A session with a decision maker investigating the goal conflicts within an economic-environmental system

		iteration number							
		1	2	3	4	5	6	.....	14
(1)	pess. value	793*	1500	1500	1500	1500	1696		1600
	ideal value	3292	3292	3091	2024	1993	1991		1600
(2)	pess. value	1000	1000	1000	1000	1225*	2000		1890
	ideal value	2810	2810	2643	2274	2241	2241		1890
(3)	pess. value	0	0	0	0*	5	5		10
	ideal value	20	20	19	19	19	16		10
(4)	pess. value	2751	2751*	500	500	500	500		252
	ideal value	79	79	79	79	80	87		251
(5)	pess. value	290	290	255*	30	30	30*		15
	ideal value	0	12	12	12	12	18		15
(6)	pess. value	0	0	9704	9704	9704	11316		13545
	ideal value	28189	28189	28189	19977	19042	18094		13556

* Goal value chosen to be improved

can be made by a committee or otherwise.

An examination of the detailed results associated with the final result attained by this decision maker showed that nearly all instrumental variables within the model behaved according to smooth growth paths. However, because no goal variable had been included to take care of a balanced distribution of activity over the industrial sectors, some undesired effects occurred in this respect. In fact, this was one of the learning effects mentioned in Chapters 5 and 6, which resulted in discussions and proposals for new goal variables. Other learning effects have led among others to the proposal to delete the sixth goal variable as being an improper one because its value can be raised by switching to heavily polluting sectors. Furthermore, the nuisance goal variable was proposed to be defined differently.

On the basis of the results of this pilot study, it has been decided to continue the experiments, using the full version of the abovementioned input-output model, a slightly revised set of goal variables, and with the aid of decision makers who are responsible for the kind of decisions simulated by the model (see Hartog and Spronk [1980]).

7.4. IMGP Applied to Discrete Decision Problems

With some minor modifications, IMGP can also be applied to discrete decision problems with explicitly given alternatives. In this section we give two illustrative examples. The first example concerns the selection of a single alternative from a set of mutually exclusive investment projects, each of which is described by its performance with respect to four criteria. The second example concerns an enterprise which is planning to build a new factory. There are 20 candidate locations, all of which are described in terms of seven criteria, some of which are measured on an ordinal scale.

Example 4: Selecting a Capital Investment Project

This example [1] of a multiple criteria decision problem with only a limited number of alternatives involves a laundry which is opening a new laundrette, to be equipped with 20 washing machines. For the latter, a number of alternative types is available. The decision maker believes four goal variables to be relevant in the choice from the set of alternatives, i.e. purchase price, electricity consumption, water consumption, and washing time. Each of these goal variables is to be minimized. There are 33 different types of washing machines, which are described in Table 7.4.

Although Interactive Multiple Goal Programming loses one of its attractive properties in this case (i.e. by the simple calculation of the respective optima of the goal variables no reduction of the set of alternatives can be obtained here), it can still be used as a systematic procedure to reduce the set of feasible actions by adding and tightening constraints on the values of the goal variables and showing the consequences. An 'ideal solution' is defined as before. A 'pessimistic solution' now lists, for each of the goal variables, the worst (in the present case, the highest) value amongst <u>all</u> available alternatives. The solutions are represented by row vectors with their elements representing the price of a washing-machine,

1) The case on which this example is based has been designed by Professor A. Törn. A more detailed description of this case, together with its solution by means of various procedures other than IMGP, can be found in Carlsson et al.[1980].

Table 7.4. Alternative types of washing machines

alternative number	unit price in dollars	total washing time* in minutes	electricity consumption* in kwh	water consumption* in litres	(proposal) solutions by which alternative is rejected
1	509	74	1.4	114	$\hat{S}_2$, S_2
2	425	80	1.4	110	$\hat{S}_3$, S_3
3	446	72	1.5	135	$\hat{S}_3$, S_3
4	564	65	1.5	118	$\hat{S}_2$, S_2
5	547	53	1.7	140	$\hat{S}_2$, S_2
6	450	68	1.5	135	$\hat{S}_3$, S_5, S_5
7	473	65	1.5	130	$\hat{S}_3$, final solution
8	484	56	1.6	115	$\hat{S}_4$, S_4
9	456	68	1.5	130	$\hat{S}_3$, S_6, S_6
10	488	72	1.5	114	$\hat{S}_3$, S_3
11	530	55	1.6	135	$\hat{S}_2$, S_2
12	477	76	1.4	110	$\hat{S}_3$, S_3
13	589	53	1.5	130	$\hat{S}_2$, S_2
14	534	61	1.3	122	$\hat{S}_2$, S_2
15	536	57	1.6	110	$\hat{S}_2$, S_2
16	494	72	1.4	135	$\hat{S}_3$, S_3
17	425	65	1.7	120	$\hat{S}_3$, S_4, S_4
18	555	53	1.6	125	$\hat{S}_2$, S_2
19	543	57	1.5	120	$\hat{S}_2$, S_2
20	515	68	1.4	130	$\hat{S}_2$, S_2
21	452	76	1.4	112	$\hat{S}_3$, S_3
22	547	68	1.4	120	$\hat{S}_2$, S_2
23	421	76	1.3	130	$\hat{S}_3$, S_3
24	498	68	1.5	120	$\hat{S}_2$, S_2
25	467	65	1.6	130	$\hat{S}_3$, S_4, S_4
26	595	50	1.7	135	$\hat{S}_2$, S_2
27	414	68	1.6	125	$\hat{S}_3$, S_4, S_4
28	431	66	1.6	110	$\hat{S}_3$, S_4, S_4
29	452	72	1.4	115	$\hat{S}_3$, S_3
30	408	77	1.5	119	$\hat{S}_3$, S_3
31	478	59	1.7	110	$\hat{S}_4$, S_4
32	395	76	1.4	120	$\hat{S}_3$, S_3
33	543	57	1.4	135	$\hat{S}_2$, S_2

* For the most used program

its total washing time, its electricity consumption and water consumption, respectively. The first potency matrix is given by

$$(7.15) \qquad P_1 = \begin{bmatrix} 395 & 50 & 1.3 & 110 \\ 595 & 80 & 1.7 & 140 \end{bmatrix}$$

Given the pessimistic solution, i.e. the lower row of P_1 the decision maker has to indicate which goal value should be decreased in value. Let us assume that he wants the price to be lower than $ 595 per machine but that he does not specify how much lower. We then set a proposal price of ($ 595 + $ 395) / 2 = $ 495 (all other (pessimistic) goal values remaining equal) and calculate the potency matrix associated with this proposal solution as

$$(7.16) \qquad \hat{P}_2 = \begin{bmatrix} 395 & 56 & 1.3 & 110 \\ 494 & 80 & 1.7 & 135 \end{bmatrix}$$

Thus by taking $ 495 as an upper bound for the price the decision maker is willing to pay for one machine, one has to drop the alternatives with a lower washing time than 56 minutes (i.e. alternatives 5, 11, 13, 18 and 26). On the other hand, by accepting this bound on the price level, alternatives with a higher water consumption than 135 litres (alternative 5) can also be left out of consideration. Of course, it is up to the decision maker to accept this restriction on the price level and its consequences. If he does not accept this restriction we calculate a new proposal price limit exactly between $ 595 (which was too high) and $ 495 (which appeared to be too low). However, let us assume that the decision maker accepts this solution. Besides the already mentioned alternatives, we can also drop the numbers 1, 4, 14, 15, 19, 20, 22, 24 and 33. Thus, in total, the number of alternatives is reduced from 33 to 19 (cf. Table 7.4).

Given the lastly accepted pessimistic solution, the decision maker again has to indicate which goal value should be reduced in

value. Let us assume that he wants to reduce the total washing time for the mostly used program and that, during this run, he is able to specify the desired reduction. Let the upper bound on washing time be 60 minutes, which gives

$$\hat{P}_3 = \begin{bmatrix} 478 & 56 & 1.6 & 110 \\ 484 & 59 & 1.7 & 115 \end{bmatrix} \tag{7.17}$$

Confronted with this proposal solution, the decision maker argues that the minimal electricity consumption (1.6 kwh for the most used program) is too high. This means that the imposed maximum value on washing time (60 minutes) was chosen too low. (We already know that the maximum value on washing time in the preceding accepted solution (80 minutes) was considered to be too high). Therefore, given the lastly accepted solution (i.e. the lower row of (7.16)), we propose a new solution by limiting washing time to an upper limit of 70 minutes. The implied potency matrix is given by

$$\hat{P}_3 = \begin{bmatrix} 414 & 56 & 1.5 & 110 \\ 484 & 68 & 1.7 & 135 \end{bmatrix} \tag{7.18}$$

Assuming that the decision maker accepts this proposal and its consequences (if he does not, the upper bound on washing time is further reduced to (70+80)/2 = 75 minutes), the number of alternatives remaining to be analyzed becomes 9 (10 alternatives are to be dropped: 2, 3, 10, 12, 16, 21, 23, 29, 30 and 32).

Let us next assume that the decision maker, given the accepted solution in (7.18), wants to limit electricity consumption to 1.5 kwh at most for the most intensive washing program.
This implies

$$\hat{P}_4 = \begin{bmatrix} 450 & 65 & 1.5 & 130 \\ 473 & 68 & 1.5 & 135 \end{bmatrix} \tag{7.19}$$

The decision maker accepts this proposal (so that only alternatives 6, 7 and 9 remain) and wants to reduce water consumption to at most 130 litres per washing program. We then get

$$\hat{P}_5 = \begin{bmatrix} 456 & 65 & 1.5 & 130 \\ 473 & 68 & 1.5 & 130 \end{bmatrix} \tag{7.20}$$

This proposal is also accepted. Only two alternatives, 7 and 9 are retained. The decision maker decides to choose the alternatives with the lowest electricity consumption, given by

$$S_6 = [473 \quad 65 \quad 1.5 \quad 130] \tag{7.21}$$

This is also the final solution.

Example 5: Locating a Factory

An enterprise is planning to build a new factory for the production of storage batteries. There are twenty possibilities for the location of this new factory. Each alternative has been described in terms of its contributions to the goal variables which management considers to be relevant in this situation. These goal variables are

g_1 = capacity of the factory, expressed as the annual number of units produced (in millions). Between certain limits, management wants the capacity to be as large as possible.

g_2 = costs of establishing the new factory (purchase of land and cost of construction). Of course management wants a value of this variable which is as low as possible.

g_3 = score for the quality of the facilities provided by the local government (subsidies, advice, licences). These scores are presented on an ordinal scale of increasing priority: --, -, 0, +, ++. The element -- represents a strongly negative outcome

for the local facilities concerned, whereas the element ++ represents a strongly positive outcome.

g_4 = score for the possibilities to attract skilled labour (again represented on an ordinal scale with --, -, 0, +, ++).

g_5 = score for the quality of the transportation network to be used by the factory (again represented on an ordinal scale with --, -, 0, +, ++).

g_6 = estimated size of the total local market (measured in millions of units sold per year). Of course, management prefers a more voluminous market to a smaller one.

g_7 = score for the possibilities to enter the local market (again represented on an ordinal scale with --, -, 0, +, ++).

The 'industrial profiles' (see Paelinck and Nijkamp [1976]) of the 20 possible locations are given in Table 7.5. None of the alternatives is *a priori* dominated by another. The first potency matrix can be written as

$$P_1 = \begin{bmatrix} 30 & 20 & ++ & ++ & ++ & 55 & ++ \\ 11 & 50 & -- & -- & -- & 10 & -- \end{bmatrix} \tag{7.22}$$

and the first solution as

$$\underline{S}_1 = [11,50,--,--,--,10,--] \tag{7.23}$$

Assume management's first wishes are that the quality of the local governmental facilities should not be too low ($g_3 \geq -$) and that there are not too many difficulties in attracting labour ($g_4 \geq -$). We then have

$$\hat{\underline{S}}_2 = [11,50,-,-,--,10,--] \tag{7.24}$$

As can be seen in Table 7.5, the profile numbers 2, 8, 10 and 18

Table 7.5. Profiles of the location alternatives

profile number	(scoring) values of the goal variables							(proposal) solutions by which profile is rejected
	g_1	g_2	g_3	g_4	g_5	g_6	g_7	
1	30	48	0	-	+	40	-	$\hat{\underline{S}}_3$, $\hat{\underline{S}}_4$, $\underline{S}_4$
2	29	50	-	--	++	25	++	$\hat{\underline{S}}_2$, $\underline{S}_2$
3	28	44	0	+	+	45	--	$\hat{\underline{S}}_3$, $\hat{\underline{S}}_4$, $\underline{S}_4$
4	27	40	++	++	--	20	-	$\hat{\underline{S}}_3$(two times), $\underline{S}_3$
5	26	41	-	0	+	55	+	$\hat{\underline{S}}_3$, final solution
6	25	46	0	0	++	10	++	$\hat{\underline{S}}_5$, $\underline{S}_5$
7	24	40	+	++	0	10	++	$\hat{\underline{S}}_3$, $\hat{\underline{S}}_4$, $\underline{S}_4$
8	23	43	--	+	++	10	0	$\hat{\underline{S}}_2$, $\underline{S}_2$
9	22	38	-	+	+	10	--	$\hat{\underline{S}}_3$
10	21	37	-	--	-	15	+	$\hat{\underline{S}}_2$, $\underline{S}_2$
11	20	35	0	+	0	50	++	$\hat{\underline{S}}_3$, $\underline{S}_3$
12	19	41	+	+	++	20	+	$\hat{\underline{S}}_3$, $\underline{S}_3$
13	18	36	-	+	+	25	0	$\hat{\underline{S}}_3$, $\underline{S}_3$
14	17	32	0	0	--	35	++	$\hat{\underline{S}}_3$, $\underline{S}_3$
15	16	28	0	0	+	45	+	$\hat{\underline{S}}_3$, $\underline{S}_3$
16	15	25	0	0	0	30	0	$\hat{\underline{S}}_3$, $\underline{S}_3$
17	14	26	0	++	++	15	+	$\hat{\underline{S}}_3$, $\underline{S}_3$
18	13	24	--	-	-	30	-	$\hat{\underline{S}}_2$, $\underline{S}_2$
19	12	23	+	++	+	10	+	$\hat{\underline{S}}_3$, $\underline{S}_3$
20	11	20	0	+	0	15	-	$\hat{\underline{S}}_3$, $\underline{S}_3$

do not meet these requirements. The accompanying potency matrix is

$$\hat{\underline{P}}_2 = \begin{bmatrix} 30 & 20\ ++\ ++\ ++ & 55\ ++ \\ 11 & 48\ -\ -\ -- & 10\ -- \end{bmatrix} \tag{7.25}$$

As can be read from $\hat{P}_2$ and Table 7.5 we have, by rejecting profile number 2, rejected the factory with the highest construction costs. Therefore we define

$$\underline{S}_2 = [11,48,-,-,--,10,--], \text{ and } P_2 = \hat{P}_2 \tag{7.26}$$

Management next wants a capacity of at least 20 million units per year to meet the export orders ($g_1 \geq 20$). For the same reason, it would like to have the best possible transportation facilities ($g_5 \geq ++$). We thus get

$$\hat{\underline{S}}_3 = [20,48,-,-,++,10,--] \tag{7.27}$$

As can be seen in Table 7.5, there is only one profile (number 6) left in this solution, by which the potency matrix reduces to

$$\hat{P}_3 = \begin{bmatrix} 25 & 46\ -\ -\ ++ & 10\ -- \\ 25 & 46\ -\ -\ ++ & 10\ -- \end{bmatrix} \tag{7.28}$$

Management judges this loss in potency too heavy and is interested in the results of a less perfect transportation system. The model then proposes the solution

$$\hat{\underline{S}}_3 = [20,48,-,-,0,10,--] \tag{7.29}$$

There are more profiles left in this solution, viz. the numbers 1, 3, 5, 6 and 7. The potency matrix can be written as

$$(7.30) \qquad \hat{\underline{P}}_3 = \begin{bmatrix} 30 & 40 & + & ++ & ++ & 55 & ++ \\ 24 & 48 & - & - & 0 & 10 & -- \end{bmatrix}$$

Management considers the proposal good enough to justify the shifts in the potency matrix, by which we can formulate

$$(7.31) \qquad \underline{S}_3 = [24,48,-,-,0,10,--] \text{ and } P_3 = \hat{P}_3$$

In the following step, transportation is required to be slightly better and the possibility to enter the local market must be 'not too bad' ($g_7 \geq 0$). This solution is judged to 'outweigh' the shifts in the potency matrix. Then the fourth solution is determined as

$$(7.32) \qquad \underline{S}_4 = [25,46,-,0,+,10,+],$$

for which the fifth and sixth profiles are still feasible. Finally management wants to face a local market which exceeds the 10 million units sales in $\underline{S}_4$. Consequently, the final solution becomes

$$(7.33) \qquad \underline{S}_5 = [26,41,-,0,+,55,+],$$

which is the fifth profile.

7.5. Conclusions

To illustrate the (potential) use of IMGP, we have presented some examples. Many more examples could have been given, as IMGP can be used for any linear programming problem with two or more goal variables or with one goal variable and some 'soft' constraints, and also for any other problem which meets the fairly unrestrictive requirements described in Chapter 6. In addition, IMGP can be used for some problems which do not meet these requirements (such as discrete evaluation problems), though with a minor modification.

Nevertheless, these and other examples are not well suited to draw a balanced conclusion on the merits and demerits of IMGP in practice. As is the case with most other interactive procedures, the number of experiences with IMGP in practice is still very limited. Although many practitioners appear to be very interested in interactive procedures, it will take some time before the performance of these procedures can be evaluated in practice. In the meantime, the evaluation of IMGP and other interactive procedures might be based on a theoretical framework, e.g. as provided in Section 5.1.

We are convinced that this type of interactive procedures can be used fruitfully in practice, provided the organizational setting of the decision problem and the wishes of the decision makers are taken into consideration. IMGP has a number of properties which, from a practical point of view, seem to be attractive. A detailed evaluation of IMGP based on both practical and theoretical considerations is given in Section 10.1.

At this point, it is important to note that a fully operational set of computer programs has become available for the implementation of IMGP in linear goal programs. These computer programs have been developed for a specific problem (see Section 7.3), but may be used for any other linear goal problem. The set of computer programs has been designed such that the decision maker can use a computer terminal display and thus be in conversational contact with the computer system. In this way the decision maker can evaluate the outcomes implied by his earlier choices, and can accordingly express the desired changes in the linear programs, which are then solved by the computer. A description of this set of interrelated computer programs is given in Appendix 7.b.

Appendix 7.a. Computer Program Used for the Experiments with an Imaginary Decision Maker

In this appendix, we describe [1)] the structure of the computer program used for the experiments involving an imaginary decision maker using IMGP. As mentioned in Section 7.2, we assume here that the imaginary decision maker's preference function is linear in the goal variables. The program consists of three subprograms, written in PL/I and using the MPSX-package. The first subprogram reads the data and generates the input for the MPSX-package. MPSX expects input in a certain format and in a certain order. To fulfil these requirements, we use the input generator ETMIC (cf. Lebret [1977]). The second subprogram calculates an optimal solution for the imaginary decision maker's preference function and saves the basis of their simplex solution for subsequent problems. These first two subprograms can also be used for standard linear and multiple goal programming. The third subprogram simulates the decisions made by an imaginary decision maker using IMGP.

In each of the three subprograms, the following 'standard' terms are used.

NP = number of problems, i.e. number of goal variables

DD = data set name

PP = problem name: there are always NP+1 problem names. Problem names PP||1 up to and including PP||NP correspond with the NP goal variables. Problem name PP||NP+1 corresponds with the imaginary decision maker's preference function.

TT = name of the right-hand side column

BB = name of the 'bounds' which can be used within MPSX

1) A full description of this computer program can be found in Ouwerkerk and Spronk [1978].

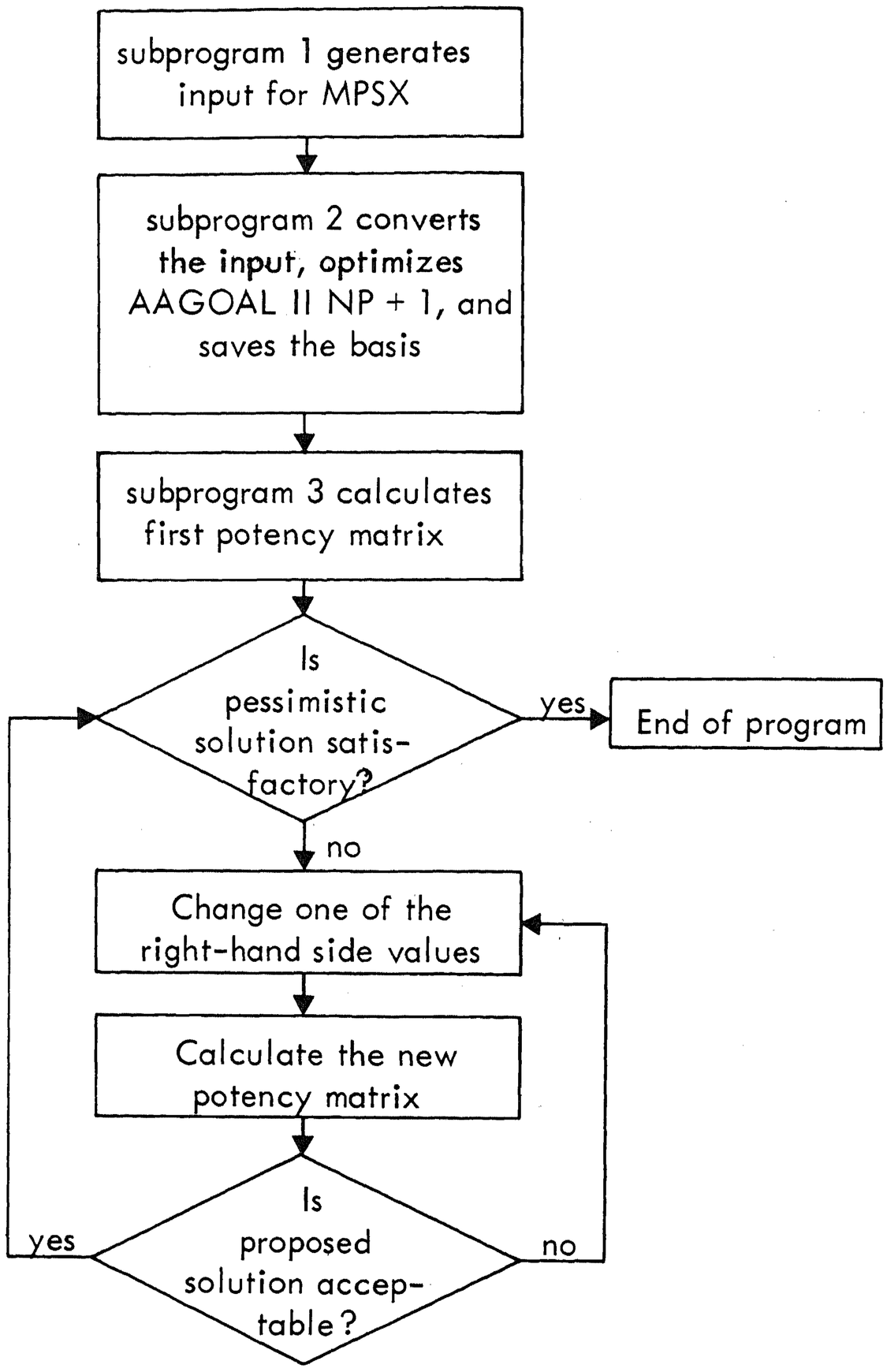

Figure 7a.1. Structure of the computer program

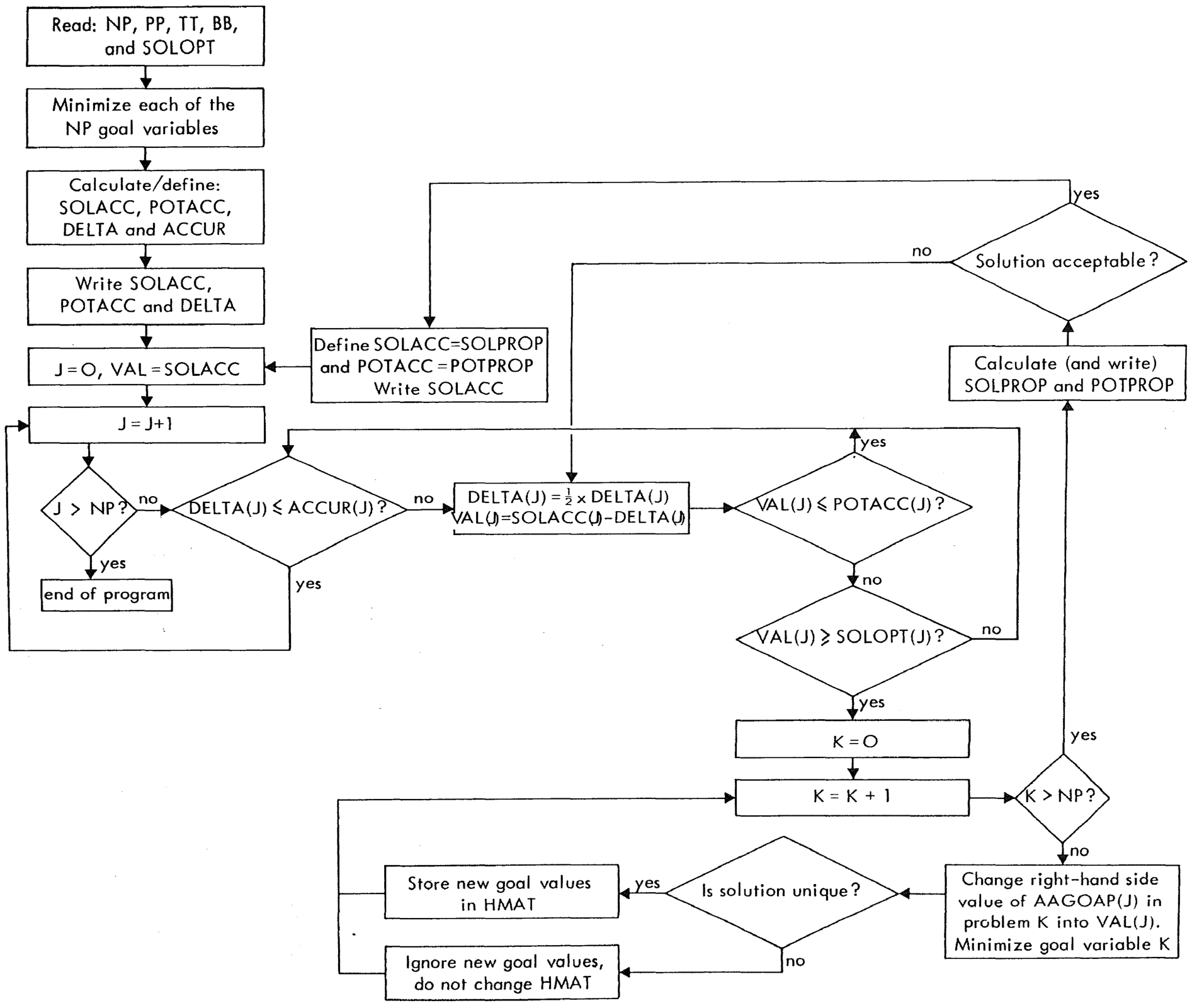

Figure 7a.2. Flow chart of the subprogram used for the experiments with an imaginary decision maker

Appendix 7.b. An Operational Computer Program for IMGP

In this appendix we give a brief description of the computer programs (and their interrelationships) which can be used for the implementation of IMGP. The design of this interrelated set of programs is such that the decision maker, using a computer terminal, is in conversational contact with the computer system (in the case of our experiments the IBM 370/158 at the University of Technology in Delft, The Netherlands). Structured programming has been used, having the advantage that parts of the program can be tested (and changed) independently of other parts. The linear programs are solved by means of calls to the IBM's MPSX/370-package [1977], embedded in PL/I computer programs. These modules are coordinated by means of command procedures.

In Figure 7b.1 we give a sketch of the system of programs. Given a new multiple criteria decision problem, the following programs have to be carried out once. The data have to be transformed into the required MPSX input format by means of a matrix generator. Then a PL/I computer program using MPSX, calculates the first potency matrices. The outcomes of the linear programs, which have to be solved in order to calculate this potency matrix, are stored in the dataset PROBFILE A. The potency matrix itself is stored in a dataset which can be displayed to the decision maker.

After these initial operations, the decision maker can choose between two command procedures, 'START' and 'RESUME', which are essentially the same, except for one point, START copies the data of the linear programs underlying the first potency matrix (stored in PROBFILE A) to the dataset PROBFILE B and displays the first potency matrix to the decision maker. RESUME does not include such a copy command, thus leaving the dataset PROBFILE B as it was after the last iteration of the preceding session. Accordingly, it displays the accompanying potency matrix to the decision maker. Clearly, START is used when a new decision maker starts tackling the problem, or when a decision maker wishes to recommence the entire interactive

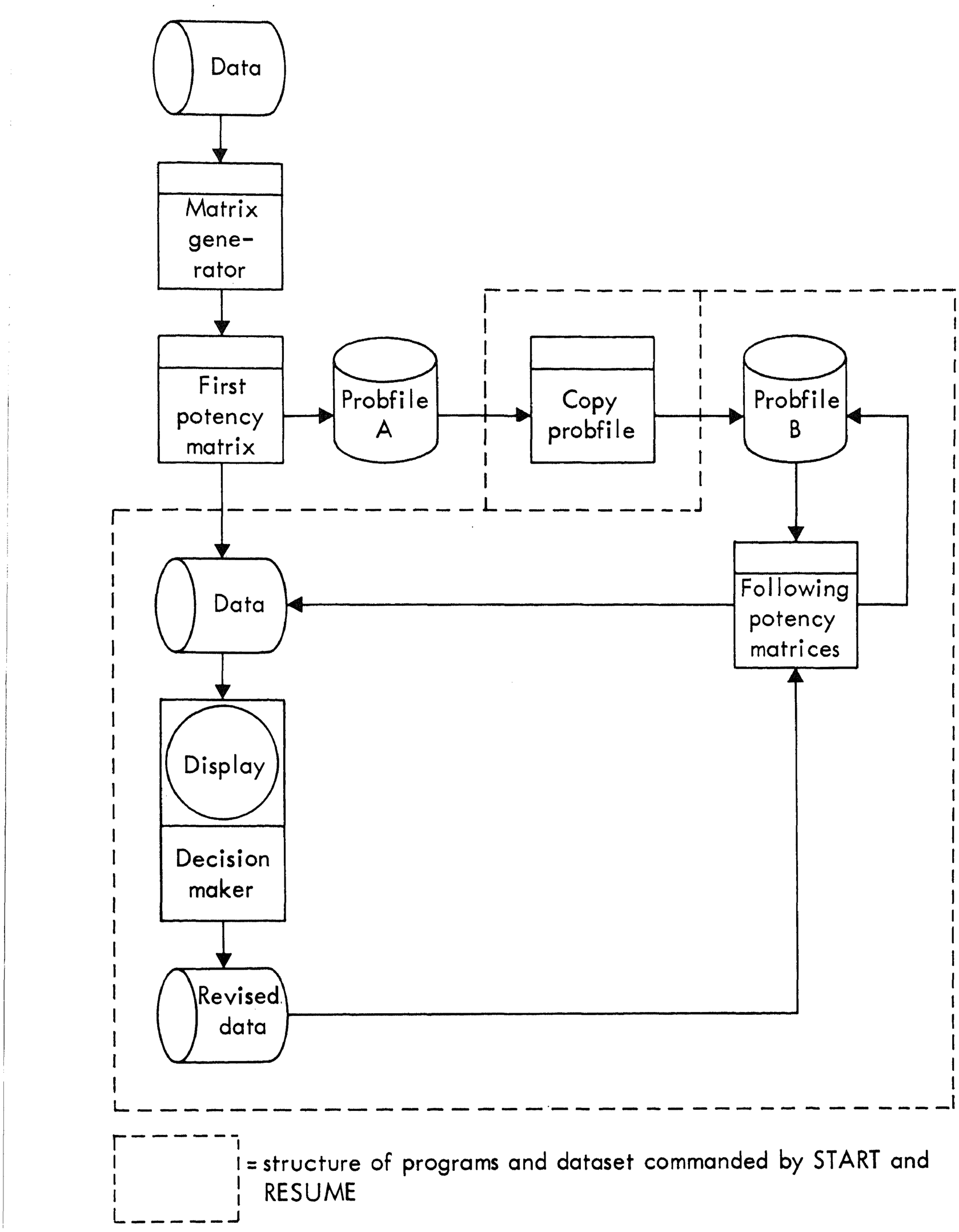

Figure 7b.1. The system of computer programs used for the implementation of IMGP

procedure. RESUME is used when a decision maker wants to continue the session after a break.

Thus both START and RESUME display a potency matrix, together with a sequence of questions which have to be answered by the decision maker. The first question is whether the presented solution is satisfactory or not. If the decision maker states it is satisfactory, he can subsequently ask for a detailed (hardcopy) description of the results. If not, he has to indicate which goal variable should be changed in value and to what amount. These data and the data of PROBFILE B are then used in a PL/I program (again calling the MPSX-package) which calculates the new potency matrix. The dataset PROBFILE B is changed. It now contains the data of the linear program underlying the last calculated potency matrix. The potency matrix itself is stored in a dataset, which again is displayed to the decision maker. The procedure terminates when the decision maker states that the presented solution is satisfactory.

References

Baum, S. and R.C. Carlson (1974), Multi-goal Optimization in Management Science, Omega, pp. 607-623.

Bronsema, H.J.J. and B.V.H. van de Kieft (1978), Enkele Risico-maatstaven en Beslissingscriteria voor het Beleggingsprobleem onder Risico: een doelprogrammeringsformulering, in Jonkhart et al. [1978].

Carlsson, C., M. Zeleny and A. Törn (1980), Three Cases in Multiple Criteria Decision Making, (forthcoming).

Despontin, M. and J. Spronk (1979), Comparison and Evaluation of Multiple Criteria Decision Models - first Results of an International Investigation, Report 7923/A, Centre for Research in Business Economics, Erasmus University Rotterdam, Rotterdam.

Driel, G.J. van, J.A. Hartog and C. van Ravenzwaaij (1980), Limits to the Welfare State, Martinus Nijhoff, Boston.

Fandel, G and T. Gal (eds) (1980), Multiple Criteria Decision Making, Theory and Application, Springer, Berlin.

Hartog, J.A. and J. Spronk (1980), Een Modelstudie naar de Relatie Milieu-Economie, WRR, Serie "Voorstudies en Achtergronden", Staatsuitgeverij, Den Haag.

Hartog, J.A., P. Nijkamp and J. Spronk (1980), Operational Multiple Goal Models for Large Economic Environmental Models, in K. Iracki, K. Malanowski, S. Walukiewicz (eds), Optimization Techniques, Part 2 ,Berlin, Springer, pp.502-512.

IBM (1977), Mathematical Programming System Extended/370 (MPSX/370) Program Reference Manual (3rd edition).

Jonkhart, M.J.L., J.W.R. Schuit and J. Spronk (eds)(1978), Financiering en Belegging, Stand van Zaken anno 1978, Stenfert Kroese, Leiden.

Khairullah, Z. and S. Zionts (1980), An Experiment with Some Approaches for Solving Problems with Multiple Criteria, in Fandel, G. and T. Gal [1980].

Kumar, P.C., G.C. Philippatos and J.R. Ezzell (1978), Goal Programming and the Selection of Portfolios by Dual-Purpose Funds, Journal of Finance, March 1978.

Lebret, M.D. (1977), EIMIC-Equation to MPS - Input Conversion, Rekencentrum T.H.- Delft, User's Guide RC-TWA-77004, Delft, April 1977.

Lee, S.M. (1972), Goal Programming for Decision Analysis, Auerbach.

Lee, S.M. and A.J. Lerro (1973), Optimizing the Portfolio Selection for Mutual Funds, Journal of Finance, pp. 1087-1101.

Nijkamp, P. and J. Spronk (1978), Interactive Multiple Goal Programming; Method and Application, Report 7812/F, Centre for Research in Business Economics, Erasmus University Rotterdam, Rotterdam.

Nijkamp, P. and J. Spronk (1980), Three Cases in Multiple Criteria Decision-Making: An Interactive Multiple Goal Programming Approach, in Carlsson et al. [1980].

Ouwerkerk, C. and J. Spronk (1978), A PL/I Computer Program for IMGP, Using the MPSX-Package, Centre for Research in Business Economics, Report 7823/A, Erasmus University Rotterdam, Rotterdam.

Paelinck, J.H. and P. Nijkamp (1976), Operational Theory and Method in Regional Economics, Saxon House, Farnborough (jointly with D.C. Heath, Lexington).