8. CAPITAL BUDGETING AND FINANCIAL PLANNING WITH MULTIPLE GOALS

In this chapter we first review some applications of multiple criteria decision methods in capital budgeting and financial planning, as reported in the literature. We list a number of general problems occurring in these applications. In Sections 8.2 - 8.4 these problems are discussed in more detail. It is especially shown how IMGP might help to solve these problems. Our conclusions are given in Section 8.5.

8.1. A Brief Survey of the Literature

In this section we present some examples of capital budgeting and financial planning models which explicitly incorporate multiple goals. These models have been chosen from the literature, so that we will be able to demonstrate a series of technical problems often occurring in this field of applications.

Non-interactive Models

One of the easiest ways to deal with multiple goals is to single out one of them which then has to be maximized, while requiring minimum values for the other goal variables. Such an approach was followed e.g. by Robichek et al. [1969], who extended the capital budgeting problem by imposing constraints on each period's level of earnings induced by the accepted projects. The objections to such a procedure are clear. It assumes that all goals formulated as constraints are *ex ante* equally important, and moreover, that they have absolute priority over the goal variable which is being maximized.

During the very early stages of the development of goal programming it was frequently suggested that this technique could be an important means in dealing with capital budgeting and financial planning involving multiple objectives. For instance, Ijiri et al.[1963] argue that their linear programming model for budgeting and financial planning could be combined with goal programming approaches to break-even budgeting. Indeed, a considerable number of authors have employed goal programming in financial planning and capital budgeting models (an extensive list of references can be found in Nijkamp and Spronk [1979]). There are some practical reasons to use goal programming in these fields. In this respect, Ashton and Atkins [1977] state that 'it is natural in financial planning to speak in terms of targets and goals; many of the indicators of company performance such as dividend cover, liquidity, or return on capital employed have target ratios adopted by customs and practice'. Nevertheless, the employment of goal programming is not without difficulties. Notably, its need of a considerable amount of *a priori* information to be given by the decision maker should be mentioned. This shortcoming of goal programming clearly paved the way for other procedures.

It has also been proposed that the concept of efficient solutions (see Chapter 3) be used in financial planning problems incorporating multiple goals. Sealey [1978] describes a bank financial planning model which has been formulated as a vector maximum problem. The relevant goal variables are assumed to be 'profit' and 'solvency', where the latter is defined by two distinct (although related) goal variables: (1) the capital adequacy ratio (defined as the ratio of required to actual bank capital) and (2) the risk asset to capital ratio. The instruments are the amounts to be invested in each of the six available types of assets. Furthermore, the model is subject to a number of constraints, relating to capital adequacy, diversification, required reserves, and to the balance sheet. For this example, Sealey has found a set of 17 efficient solutions, from which the decision maker has to choose a final solution.

This approach seems attractive. Nevertheless, a few important disadvantages should be mentioned. For instance, one may wonder whether for a given goal variable, more is always preferred to less (or *vice versa*). In many financial planning problems goal variables such as 'growth size', 'level of liquidity', 'amount of leverage' and several (other) ratios are considered important. For none of these goal variables is it obvious whether they should be maximized or minimized. Of course, one may maximize (or minimize) these kinds of goal variables subject to additional constraints on their range of values. As will be shown in Section 8.3, there are more flexible solutions to this problem. A more serious problem is the fact that, in general, the number of efficient solutions is very large. The identification thus requires a considerable (and sometimes even prohibitive) amount of computer storage and time. Furthermore, the number of efficient solutions easily exceeds the information processing capacity of the decision maker. This problem may be solved by means of interactive procedures.

Interactive Models

Candler and Boehlje [1971] describe a two-period capital budgeting model in which the alternative activities consist of (a) two investment projects, to be undertaken either in period 1 or in period 2, (b) the 'opportunity to put cash in the bank', (c) net tax-free cash at the end of the planning horizon, (d) the value of the assets at the end of the planning horizon, (e) dividends paid to shareholders and (f) pollution. At the same time, the latter four acitivities have been defined as goal variables, each of which has to be maximized or minimized. Furthermore, the dividends have been restricted to increase at a given (linear) rate. The outcomes of existing operations have been assumed to be given and fixed and consequently, independent of the investment projects. This problem has been formulated as a deterministic vector maximum problem. The feasible region of the acitivity vectors $\underline{x}$ is described by linear (in-) equalities in $\underline{x}$. Some of the elements of $\underline{x}$ are integer. Because the goal variables $g_j(\underline{x})$, $j = 1, \ldots, 4$; are at the same time activity variables, they can only be expressed on a linear scale. Candler and Boehlje aim at efficient (Pareto-optimal) solutions. The ultimate solution is to be found by an (unstructured) iterative and interactive approach.

Chateau [1975] gives a numerical example of a capital budgeting problem with multiple goals. The problem is to choose from a set of investment projects, some of which are indivisible. Internal capital rationing is assumed to have the highest priority. Furthermore, three other goals are assumed (an acceptable level of cash, a desired level of dividend disbursement, and a minimum target asset value).

This problem has been formulated as a deterministic, mixed-integer, goal programming model, employing pre-emptive priority factors. Chateau shows the results for a variety of objective functions, including the one originally used by Weingartner. Although Chateau finds merit in the goal programming model's flexibility, he also mentions a number of its disadvantages. In his opinion, 'the ordering and weightings on *a priori* ground and in absolute or relative terms may constitute a rigidity factor of the goal programming approach'.[1] Not surprisingly, he proposes an interactive procedure. However, for this he has chosen an approach which also requires very detailed information from the decision maker, i.e. marginal rates of substitution for multiple criteria.

With regard to multiple objective decision models, many interactive procedures have shown to be very powerful tools in the process of searching for a final (compromise) solution. However, as mentioned above, financial planners are accustomed to expressing their

1) Furthermore, Chateau seems to suggest that goal variables should be expressible in monetary terms. In our opinion, this is not always true for the goal programming formulation.

preferences in terms of goals and targets. Therefore it seems useful to search for interactive procedures which correspond to this use. An attempt in this direction was made by Ashton and Atkins [1977]. They describe an interactive procedure based on goal programming, in which both weights and targets are changed parametrically. Two important technical problems they have encountered are the choice of the distance metric in the goal program and the considerable number of goals which are being used in their financial planning model. They developed a three-stage methodology which could deal with these problems in an *ad hoc* way. In their opinion (with which we wholeheartedly agree) a specific methodology is necessary for financial planning problems involving multiple objectives. Moreover, in view of the possible applications in this field, the efforts to find such a methodology seem to be justified.

Ashton and Atkins describe an eight-period financial planning model incorporating a set of investment opportunities and financing alternatives available to the firm. Furthermore, this model includes a number of accounting variables which correspond with the U.K. tax law and accounting standards. For each of the planning periods, eight goal variables are defined (six of which are ratios). Thus the problem contains a total of 64 goal variables. Such a large number of goal variables constitutes a source of difficulties in using multiple objective programming methods.

In the above and in other financial planning and capital budgeting models, several technical problems may occur. In Section 8.2 we will pay attention to the problem of large numbers of goal variables. Section 8.3 is devoted to goal variables requiring special treatment. Notably, this is the case for goal variables defined as ratios or as chance constraints, and also for goal variables which are neither to be maximized nor minimized. In Section 8.4 we discuss the problems caused by the occurrence of (0,1) variables, which are used to represent indivisible capital investment projects.

8.2. Large Numbers of Goal Variables

In capital budgeting and financial planning, the number of goal variables can easily become unmanageable. Even if the firm considers a small number of goal variables to be important (say two or three), these goal variables may need to be formulated for each of the time periods within the planning horizon. It may also be necessary to define the same kind of goal variables separately for different divisions of the firm. It is clear that in this way, the number of goal variables becomes considerably greater than 'the magical number seven plus or minus two', which is often mentioned in the literature on multiple criteria decision making, to be the maximum number of goal variables which can be handled by the decision maker (see Miller [1956] and Tell [1978]).

As far as we can see, there are three main ways to tackle this problem:

(a) reduction of the number of goal variables, by replacing each set of analogous goal variables by some kind of aggregate.

(b) reduction of the number of goal variables by removing 'insignificant' goal variables.

(c) division of the total set of goal variables in subsets, which are considered sequentially.

We will next discuss each of these approaches in greater detail.

Replacing Sets of Goal Variables by Aggregates

A straightforward procedure to reduce the number of goal variables is to replace each set of analogous goal variables by some kind of aggregate. Most aggregation procedures have no firm theoretical basis. Nevertheless, some aggregators may be useful in practice. To mention just a few, one could use the goal variables' average, or only the goal variable defined for the end of the planning horizon, or the maximum growth rate of the goal variable, or the minimum, and so on. Of course it is necessary that the chosen aggregate has some practical meaning to the decision maker. If such an aggregate cannot be found for all sets of analogous goal variables, one has to accept the presence of non-aggregated goal variables beside goal variables which are aggregates of other goal variables.

An important objection to most aggregates is that the mutual differences between the values of the goal variables concerned are neglected. For instance, maximizing the average per period number of employees over a given planning period may result in a very erratic time-pattern of the employment. We will demonstrate an aggregate which partly meets this objection. This alternative aggregate, which is generally to be minimized, is defined as the maximum deviation from target (growth) values defined for the set of analogue goal variables concerned. As an example, let us assume a firm planning its capital investment expenditures for the periods t, $t = 1, \ldots, T$. Assume that the firm wants a growth rate of its cash flows C_t, $t = 1, \ldots, T$; of at least 10 per cent a period. The cash flow in the period before the first planning period is denoted by C_o. One way to formulate the firm's growth target is represented by

$$(8.1) \qquad C_t - y_t^+ + y_t^- = (1.1)^t . C_o \qquad \text{for } t = 1, \ldots, T.$$

The aggregate mentioned above is, in this case, defined by

$$(8.2) \qquad d = \operatorname*{Max}_{t=1}^{T} \{y_t^-\},$$

which can be calculated for any sequence of cash flows C_o, $C_1, \ldots, C_T$. In normal planning models, several sequences of cash flows are available, in which case the aggregate d might be minimized over the set of alternatives. As pointed out in Chapter 4, this in fact minimaxing procedure can also be adopted within a programming framework. In the present case, this can be achieved by adding the constraints (8.1) and furthermore

$$d \geq y_t^- \qquad \text{for } t = 1, \ldots, T; \tag{8.3}$$

after which d can be treated as a normal goal variable.

Although this formulation is rather straightforward, some slight but significant modifications can be made. To start with, the constraints (8.1) are often replaced by the constraints

$$C_t - y_t^+ + y_t^- = 1.1\ C_{t-1} \quad \text{for } t = 1, \ldots, T. \tag{8.4}$$

In (8.1), the growth targets are completely determined *a priori*, whereas in (8.4) the targets depend on the preceding cash flow.

In connection with either (8.1) or (8.4), one might consider to 'scale' the deviational variables y_t^+ and y_t^-. For instance, in the case of (8.1), this could be achieved by multiplying all variables y_t^+ and y_t^- in (8.1) with a factor $(1.1)^t$. The effect is that the deviations from the targets are measured in proportion to the target values, whereas the deviations in the unadapted form of (8.1) are measured in a unit which is not.

Finally it should be stressed that, in our example, only the negative deviations from the aspired growth levels were judged to be undesirable. Of course, other possibilities do exist. Instead of negative deviations, either positive deviations alone or both the negative and the positive deviations[1] might be used.

1) In a slightly different way, Theil [1968, pp.265-271] shows how to reduce the possibility of undesirably large fluctuations of variables over time. The procedure he proposes is to add to the objective function (to be minimized) the sum of the squares of the successive period-to-period differences of the variables.

Alternatively, the decision maker may wish to formulate two target growth levels, i.e. a minimally and a maximally desired level.

Removing insignificant goal variables

Generally speaking, goal variables specified in a decision problem are not of equal importance. On the contrary, some goal variables may be so important that they completely determine the solution of the decision problem, dominating the other goal variables. It would be of great help if these less important goal variables could be detected before solving the decision problem.

Another situation arises if all feasible solutions of a decision problem score almost equally well with respect to a given goal variable. Also in this case, it might help to eliminate this goal variable before solving the decision problem for its optimum.

These and other examples suggest that decision problems exist, in which the use of a subset of the complete set of goal variables leads to the same (or approximately the same) solution as would have been found by using the complete set of goal variables.

One approach to detect goal variables which might be omitted has been proposed by Gal and Leberling [1977] for the linear vector maximum problem. They give a procedure to identify redundant goal variables, defined as goal variables which can be omitted without changing the set of efficient solutions. The identification of redundant goal variables in linear vector maximum problems turns out to be equivalent to the identification of redundant constraints in linear systems (see Spronk and Telgen [1979]). These authors also discuss some other relationships between multiple objective programming and redundancy.[1)]

1) For an in depth treatment of redundancy and linear programs, see Telgen [1979].

For instance they show that goal constraints, either formulated *a priori* or during an interactive process, may be technically redundant. The advantage of identifying redundant goals and goal variables is that the decision maker has to provide less information about his preferences. However, it may be that the decision maker does not want to remove this 'technical' redundancy, because he may be accustomed to expressing part of his preferences in terms of these redundant entities, or even that he cannot be convinced of their redundancy.

As shown by Tell [1978], factor analysis constitutes another method for reducing the number of goal variables in a multiple criteria decision problem. In this approach all original goal variables are transformed into a smaller set of factors, which are subsequently treated as goal variables. This reduction of the number of goal variables results in a loss of information. Because the procedure provides insight into the amount of information lost (see Tell [1978]), this loss can be traded off against the reduction in the number of goal variables. However, in our opinion the amount of information lost cannot be a reliable indicator of the importance of the information which is lost.

Subdividing the set of goal variables in subsets

Another possibility for dealing with a large number of goal variables is to divide the set of goal variables in a number of subsets, each containing goal variables of approximately the same importance. Several approaches to the thus defined problem have been proposed.

Analogous to the hierarchical optimization methods (see Section 5.4), one might - as in goal programming (see Chapter 4) - try to obtain a lexicographic ordering over the subsets of goal variables. For example, it could be argued that the goal variables relating to the first planning period have pre-emptive priority over those relating to the second planning period, and so forth. If such an ordering has been obtained, the set of decision alternatives can be reduced sequentially by evaluating this set first in terms of the subset of the most important goal variables, then in terms of the second most important goal variables, and so on.

The generalized interactive procedure proposed by Spronk and Zionts [1980] offers another possibility to deal with the above-mentioned subsets of goal variables. In this procedure, minimally desired values for the goal variables in one subset are determined, subject to constraints on the values of the goal variables in the other subsets, by means of one of the usual interactive procedures. The outcomes for one subset of goal variables are defined as constraints, subject to which the desired goal values in the other subsets must be found. If no satisfactory solution can be found for one subset of goal variables, the constraints posed by one or more of the other subsets must be relaxed, and each of the subsets concerned must be reconsidered.[1)]

8.3. Goal Variables Requiring Special Treatment

In this section we deal with three types of goal variables which require special treatment. Goal variables defined as ratios of two linear functions of the instrumental variables become non-linear in the instruments, but can be handled by IMGP. Similarly, chance constraints can be treated as goal constraints being modified in an interactive manner. Finally, goal variables that are neither to be maximized nor minimized can also be handled by IMGP.

1) Although no such application has been made yet, it seems that this generalized interactive procedure is useful in solving decentralization problems.

Ratio Forms

In financial planning it is not unusual to use goal variables that are defined as ratios (see Ashton and Atkins [1977]). Well-known examples are 'return on investment','price-earnings ratio', 'quick ratio', 'debt to equity ratio', 'times interest earned' and 'inventory turnover ratio'. Ratios, being non-linear functions of the instrumental variables, can generally not be treated by means of linear multiple objective programming methods. Linear goal programming (and consequently IMGP) are exceptions, as they can deal with ratios of linear functions of the instrumental variables, be it in a less straightforward way than is often assumed. Consider for example the goal variable

$$g(x) = \frac{3x+2}{2x+4}\,, \tag{8.5}$$

and assume that a target value of $g(x) = 1$ has been defined. This might be translated by the non-linear goal restriction

$$\frac{3x+2}{2x+4} - y^+ + y^- = 1 \tag{8.6}$$

An incorrect, but often proposed formulation of the some problem would yield

$$(3x+2) - u^+ + u^- = (2x+4)\,, \tag{8.7}$$

which can be written as

$$x - u^+ + u^- = 2 \tag{8.8}$$

As shown for example by Awerbuch et al. [1976], substituting the linear formulation (8.8) for the non-linear formulation (8.6) is not, in general, correct. In our example, multiplication of both sides of (8.6) by the factor (2x+4) shows that (8.6) and (8.8) are equivalent only if $u^+ = (2x+4).y^+$ and $u^- = (2x+4).y^-$, in which case (8.8) again becomes non-linear in the instrumental variable x.

If aspired goal levels are defined as 'hard' constraints, little problems arise with the non-linearity of ratios.[1)] For instance, considering the target in the above example as a hard constraint would yield

$$(8.9) \qquad 3x+2 \geq 2x+4, \text{ or } x \geq 2$$

Problems arise only if the deviations from the target ratio are either to be maximized or minimized. In that case, the deviational variables become relevant, and the fractional form of the ratios and the deviational variables should be taken into account.

Fortunately, several procedures for dealing with fractional forms are available. Because IMGP uses a separate programming model for each goal variable, any procedure intended for the maximization of only one fractional goal variable might be considered. For instance, Charnes and Cooper [1962] transform the fractional problem into a straight linear programming problem, Joksch [1964] proposes parametric methods to solve the fractional problem, and Bitran and Novaes [1973] solve the problem by solving a series of linear programs differing with respect to the objective function only.

If there is more than one fractional goal variable to be optimized within the same program, the problem becomes more complicated. In fact, most multiple objective programming methods cannot really cope with it. Kornbluth [1973] provides a survey of the pro-

1) Provided that the denominator of the ratio assumes either positive values only or negative values only. Probably all ratios used in capital budgeting and financial planning have denominators assuming positive values only. Cases in which the denominator can become both positive and negative may give rise to disjunctive constraints.

blems and suggests some solution procedures. Kornbluth and Steuer [1980] discuss the nature of the efficient set if several, fractional goal variables are involved. They also provide an algorithm to find all weakly efficient[1)] solutions if several fractional goal variables are involved.

As mentioned above, IMGP circumvents the problem of several fractional goal variables by defining a separate goal program for each goal variable, which is to be optimized subject to hard constraints on the values of the other goal variables. Nevertheless, situations may arise in which the number of fractional goal variables becomes too large to handle. As shown by Ashton and Atkins [1980] and Charnes and Cooper [1977], the minimax metric can be used to solve this simultaneous occurrence of the ratio problem and the problem of large numbers of goal variables. This will be illustrated in Chapter 9 by means of a financial planning model.

Uncertainty and Chance Constraints

As is the case with many other planning and decision problems, financial planning is beset with uncertainty. Besides the uncertainty with respect to the technical coefficients and the availability of resources, the specification of the goal variables and the specification of the targets may also be of an uncertain nature.

A straightforward way to deal with uncertainty is to use the expected values ('best estimates') of the uncertain parameters to calculate a solution which would be 'optimal' for a risk-neutral decision maker, and to carry out a postoptimality analysis to give

1) Defined as solutions for which no other solutions can be found that are strictly better with respect to <u>all</u> goal variables. For instance, two solutions with goal value vectors (7,3,2) and (7,4,2) are both weakly efficient. However, only the first solution is efficient in the sense defined in Chapter 3.

the presumably risk-averse decision maker an idea of the risks he is running. Deshpande and Zionts [1980] and Gal [1980] among others, have paid special attention to the postoptimality analysis of the goal variable specification.

From a theoretical point of view a more elegant approach would be to treat all uncertain parameters as random variables, for which a well-defined probability distribution function can be specified. Given such information, stochastic programming methods might be used. Several research efforts have been directed towards the translation of random goal variables into stochastic goal programming models (see e.g. Charnes and Cooper [1963], Contini [1968], Lane [1970], and Leclercq [1979]). A well-known example is the use of *chance constraints*. Assume that the target value b of the goal variable $\tilde{g}(\underline{x})$ has to be reached with a probability of at least $1-\varepsilon$. We thus have

$$(8.10) \qquad P(\tilde{g}(\underline{x}) \geq b) \geq 1-\varepsilon$$

Assuming that $\tilde{g}(\underline{x})$ is normally distributed, or employing a non-parametric relationship such as Tchebysheff's inequality, this desire can be translated by means of the deterministic equivalent

$$(8.11) \qquad E(\tilde{g}(\underline{x})) + k_{\varepsilon}.\sigma(\tilde{g}(\underline{x})) \geq b$$

It is easily seen that such a formulation can be adopted within a goal programming framework, and can thus be handled by means of IMGP. Minimization of ε, i.e. minimizing the probability of not achieving the target value b, can be accomplished by repeatedly solving

$$(8.12) \qquad \text{Min}\{y^-\}, \text{ s.t.}$$

$$E(\tilde{g}(\underline{x})) + k_{\varepsilon}.\sigma(\tilde{g}(\underline{x})).+y^- \geq b,$$

while systematically changing k_ε. Obviously, if $y^- > 0$ the k_ε value should be lowered. If $y^- = 0$, k_ε might be raised.

Many other stochastic problems can, and have been formulated in goal programming. For an overview we refer to the authors mentioned above. Notwithstanding the existence of these stochastic goal programming models, very few applications have been reported in the literature thusfar. This may be explained by the fact that stochastic programming methods are relatively complicated. Furthermore, the input data required by these methods are generally hard to obtain. Notably, the decision maker has to specify a probability distribution function (or, in some cases, two or three parameter values of this function) for any uncertain entry in the goal programming model, and has to specify the interdependencies between these uncertain entries. Furthermore, he should specify his risk attitude. Therefore, despite their theoretical attractiveness, these stochastic goal programming models easily become unmanageable in practice.

Nevertheless, uncertainty should not be neglected. Depending upon organizational setting of the planning problem (time available, budget, information processing capacity of decision maker etc.), any possibility to deal with uncertainty should be considered. In some situations, the assessment of probability distribution functions may be feasible. In other cases, postoptimality analysis may be carried out fruitfully. However, it should be stressed that, in a given planning problem, the decision maker himself may have developed his 'own' measures and procedures for coping with uncertainty. A clear example in financial planning is the widespread use of several ratios as representatives of the flexibility, profitability and endurance of the financial structure. Instead of trying to convince the decision maker not to use his (perhaps) crude measures, one might as well try to translate these measures so as to incorporate them within the planning model. What is the correct procedure to follow depends largely upon the planning situation at hand, the evaluation of which will be one of the analyst's most difficult and hazardous tasks. In any case, if the 'practical' solution is chosen, one might consider adopting the decision maker's measures of risk as additional goal variables in the programming model.

More not always preferred to less and Vice Versa

Several goal variables may occur in financial planning problems, which are neither to be maximized nor minimized. Goal variables as the 'growth of accounting earnings over time', 'the amount of cash held', as well as several goal variables defined as ratios will generally not have to be maximized or minimized. Instead, the values of these goal variables will have to meet certain upper and lower limits defined by the decision maker or, alternatively will have to be as close as possible to a given target value. It may even be that the decision maker cannot define such lower limits, upper limits and target values *a priori*. A clear example of the latter type is formed by capital rationing constraints. The exact positioning of the latter constraints depends upon the evaluation and preferences of the decision maker, and thus should be considered as an output rather than an input of the decision proces.

Most interactive procedures can only deal with these situations in a fairly crude way, i.e. by maximizing a goal variable, subject to a rigid *a priori* defined constraint on the goal variable's value, or by minimizing the distance to such a constraint. Since it is based on goal programming. IMGP offers a much more flexible approach. An overview of possible objective functions in goal programming has been given in Section 4.2. In Chapter 9 we will illustrate how IMGP can handle goal variables which are neither to be maximized nor minimized.

8.4. Indivisibility of Projects

Financial planning models very often include discrete (0,1) variables to represent the yes-no options inherent in the selection of capital investment projects. As is well-known, the inclusion of

these types of variables calls for the use of special solution procedures other than the simple rounding of continuous solutions obtained after neglecting the discrete nature of the (0,1) variables. Obviously, this does not only hold for single objective programming models, but for multiple objective programming methods as well. For example, consider the case with goal variables
$g_1(\underline{x}) = x_1 + 1\frac{1}{2}.x_2 + 3x_3$, and $g_2(\underline{x}) = x_1 - x_3$. Assume that the following integer solutions are feasible:

$$\underline{x}_A = (0,0,0)$$
$$\underline{x}_B = (1,0,0)$$
$$\underline{x}_C = (0,1,0)$$
$$\underline{x}_D = (0,0,1)$$

The representation of these solutions in goal value space is given in Figure 8.1. It is obvious, that solutions B', C', and D' are

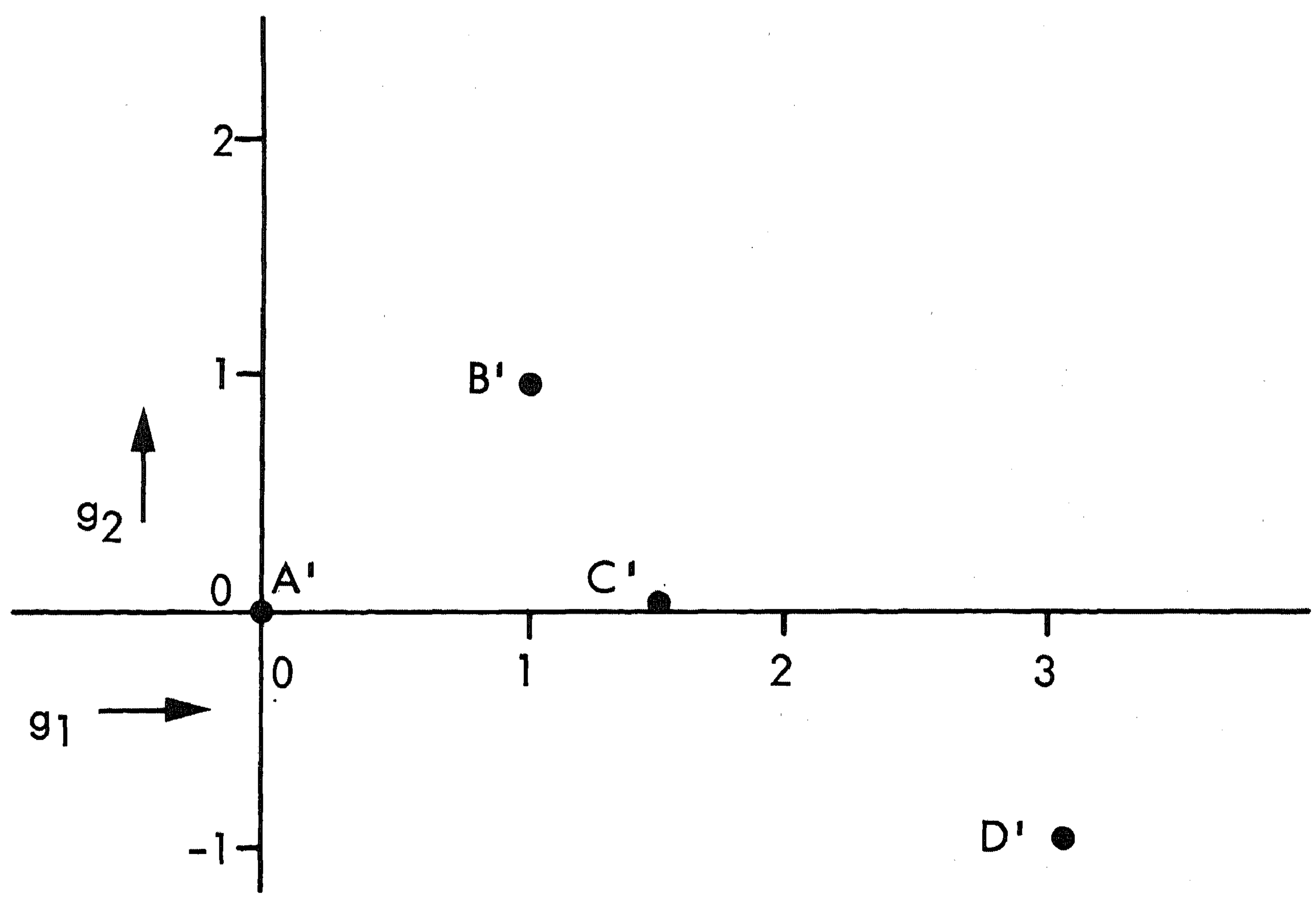

Figure 8.1. The set of feasible (0,1) solutions represented in goal value space

non-dominated. The example also shows that the set of efficient solutions cannot be obtained in the same way as in the corresponding continuous problem, i.e. by means of a parametric analysis of the weights in a linear combination of the goal variables. Such an analysis would yield the solutions B' and D', but would neglect solution C'. Procedures to generate all efficient solutions in integer multiple objective programming methods have been proposed by Bowman [1976], and by Bitran [1978] for the (0,1) integer problem. The computational experiences reported thus far are not very promising. Furthermore, the set of all efficient solutions may again be too extensive to be of much value to the decision maker (see Chapter 3).

Very few interactive multiple objective programming methods have been adapted for the integer case. For instance, Zionts [1977] has proposed an extension of the Zionts-Wallenius algorithm (see Chapter 5), in order to include integer variables. The extension consists mainly of a branch and bound procedure connected with the interactive procedure. These ideas have been developed further by Villarreal et al. [1980]. A difficulty to bear in mind in these branch and bound procedures is the implicit nature of the decision maker's preference function. However, Villarreal et al. (Ibid) suggest that after exploring a certain number of nodes, the decision maker's preference function may have been approximated precisely enough to consider the problem comparable with single objective problems.

Lee [1978] and Lee and Morris [1977] studied the integer version of goal programming including pre-emptive priorities. They presented integer goal programming methods based on the cutting plane, branch and bound, and implicit enumeration approaches. The performance of these methods in a series of test problems was measured by means of two indicators: the number of iterations and the computation time (on an IBM 370-158) required for solving a problem. On the basis of their experiments, Lee and Morris (Ibid) conclude that the branch and bound algorithm of goal programming is far more efficient than the cutting plane method, especially for the mixed integer case.

Furthermore, 'in the implicit enumeration method, the time required for solution increases dramatically as the number of variable increases'. For example, a problem with 25 constraints, 19 (0,1)-variables, and 10 pre-emptive priority factors was only solved after 919 seconds of CPU-time and 19855 iterations. For problems having more than twenty variables, Lee and Morris found it difficult ' to obtain the optimal solution within a reasonable time limit'.

Although these results are discouraging, especially for applications in large-scale financial planning and capital budgeting models, some hope does exist. That is because many variants of the above-mentioned integer programming methods do exist, which have not yet been studied for their capability to tackle goal programming with pre-emptive priorities .

After the above exposition, it will be clear that one cannot expect IMGP handling integer variables without any problems. Nevertheless, we believe that IMGP is relatively promising for the solution of integer multiple objective programming problems. Since the computational steps of IMGP consist of the solution of a series of <u>single</u> objective optimization problems, any normal integer programming method can be employed.[1)] Within these integer programming methods, the constraints on the values of the goal variables formulated during the interactive process can often be used fruitfully to facilitate the search for an optimal solution. In this respect, the results of

1) The same holds for other interactive goal programming methods of type c (cf. Section 5.4), as for instance the hierarchical procedure proposed by van Delft and Nijkamp [1976].

our experiments with a financial planning model including 21 (0,1)-variables and 163 constraints are rather encouraging (see Section 9.4). Nevertheless, more experiments are necessary in order to be able to draw more definite conclusions about the computational performance of IMGP in integer multiple objective programming problems.

As has already been indicated, the presence of integer variables also causes some difficulties in IMGP. A first problem is how to find a suitable solution to start the interactive process. As shown in Appendix 6.a, the minimal (maximal) values of the goal variables reached within the efficient set constitute a good starting point. Because the calculation of the efficient set for integer programming problems in general, and for the (0,1) programming problem in particular, is not yet technically feasible for problems of a more than very moderate format, this method of finding a starting solution should be disregarded. Instead, one might proceed by asking the decision maker to define 'least attractive' values for the goal variables. If an integer solution can be found which satisfies these very modest conditions, the vector of 'least attractive' values can be nominated as the starting solution. Alternatively, the starting solution can be defined as the minimal (maximal) values of the goal variables within the feasible region.

As argued above, the calculation of the potency matrix consisting of a series of individually optimal goal values obtained, subject to a set of minimally desired goal values, is no more difficult than the solution of the corresponding single objective integer programming problem. However, compared with the continuous case, it is often more difficult to interpret the meaning of the potency matrices, and of shifts in these matrices after a change in a minimally desired goal value. To illustrate this point, let us return to the example in Figure 8.1. Assume that the first potency matrix is given by

$$(8.13) \qquad P_1 = \begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix}$$

If the decision maker wants a value for g_1 of at least 2, the potency matrix

$$(8.14) \qquad P_2 = \begin{bmatrix} 3 & -1 \\ 3 & -1 \end{bmatrix}$$

will result. Assume next, that alternative C' in Figure 8.1 does not exist, that the same potency matrix as in (8.13) is used, and furthermore, that the decision maker again desires a value for g_1 of at least 2. In this case the same potency matrix as in (8.14) occurs! Thus from the shifts in the potency matrix it cannot be concluded that the tightening of the goal value excludes possibly valuable alternatives. This can only be verified by recalculating the potency matrix P_2 for a series of values for g_1 (e.g. using the δ procedure described in Chapter 6, yielding the g_1 values 1.5, 1.25, 1.125, etc.).

The above shows that the implementation of IMGP in integer problems appears to be technically quite simple, but is certainly not without some pitfalls. In our opinion, the integer case constitutes a fruitful area for further research.

8.5. Conclusion

In this chapter, a number of capital budgeting and financial planning models explicitly dealing with a multiplicity of goals were discussed. It appeared that a series of technical problems frequently occur in this field of application. These problems relate to the number of the goal variables, the nature of the goal variables, and to the existence of indivisible projects. It was suggested that IMGP can fruitfully be used to tackle a number of these problems. This ability will be illustrated in the following chapter.

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