9. USING IMGP FOR A FINANCIAL PLANNING MODEL: AN ILLUSTRATION

In this chapter we present a financial planning model including multiple goals, and we show how a financial plan might be selected by using IMGP. The financial planning model has been constructed so as to demonstrate the potential of IMGP with respect to the technical problems discussed in Chapter 8. At the same time, we have tried to include some features of financial planning, which are in practice rather usual. Nevertheless, the resulting model - as is probably the case for all financial planning models described in the literature - can be no more than an example, and certainly does not claim to be applicable to all financial planning problems. Every financial planning problem in reality has its own peculiarities. The purpose of this chapter is to show how IMGP might help to cope with some of those peculiarities occurring frequently.

In Section 9.1 we describe the assumptions underlying the present financial planning model. A detailed description of the model is given in Section 9.2. In Section 9.3 we demonstrate a session with a decision maker, using IMGP to find a solution for the above financial planning model. In this section we ignore the go/no-go nature of capital investment projects. In Section 9.4 we therefore describe how the results change when this factor is taken into account. Our conclusions are given in Section 9.5.

9.1. Introductory Remarks

In the model, management has to evaluate a set of capital investment projects, simultaneously considering a series of financing decisions. For the sake of simplicity we have assumed no equity issues and no dividend payments during the planning period. Thus the only financing decisions concern a series of debt issues. Next to these investment and financing decisions, management has to decide upon the amount of cash to be held in each period.

One of the most important goal variables in this case is assumed to be the firm's total market value. Following Myers and Pogue [1974, p.580], we adopt two propositions of modern capital market theory:

(a) The possible effect on the firm's market value, induced by the stochastic dependencies between a capital investment project and other projects (including the firm's existing assets), can be ignored by the firm.

(b) The total market value of the firm equals its unlevered value plus the market value of tax savings implied by debt financing.

By means of these two assumptions, the market value of the firm's portfolio of capital investment projects can be defined as the sum of the *unlevered present values* of the projects included plus the market value of the tax savings implied by debt financing (see Myers and Pogue [1974]). The unlevered present value of a project (assumed to be estimated directly by the management independently of the model) is defined as the market value of the project, if it were financed exclusively with equity (see also Myers [1974]). The firm's total market value is now defined as the market value of the firm's portfolio of investments plus correction terms for its cash holdings and lendings (see Section 9.2). In the absence of equity issues, the firm's total market value does not have to include a term for the costs of such issues. Similarly, the costs of possible bankruptcy have been ignored.

The financial planning model of this chapter is semi-deterministic. This means that the uncertainly which is inherent in financial planning is to be accounted for exogeneously. For instance, the unlevered present values of the projects already include the value of the risks associated with the projects. As will be shown below, other risks will be accounted for by managerial decisions with respect to the values of goal variables relating to the amount of debt issued, and relating to the liquidity position of the firm.

The choice of a deterministic financial planning model is not an essential one, but is mainly inspired by the desire to not complicate the example too much, and to limit computer time. As shown in Chapter 8, goal programming - and consequently IMGP - can handle uncertainty by means of, among other things, chance constraints.

As argued in Chapter 2, the neglicence of certain goal variables can affect the firm's total market value adversely. Instead of accounting for the goal variables either by translating them into 'cost factors' in the total market value definition, or by means of constraints on their respective values, we prefer to deal with these goal variables explicitly and separately. As such we have chosen the stability of the accounting earnings over time, the amount of debt outstanding (limited through the 'times interest earned' ratio), the minimum cash level over time and the maximum cash level.

Another goal variable adopted in this model relates to the firm's employment level. Of course, this goal variable may be selected so as to avoid possible negative effects on the firm's total market value. However, it can be equally seen as a goal variable in its own right along with the above goal variables.

In the following section we will describe how the above data can be translated into a goal program, which can be handled by IMGP.

9.2. Description of the Model

In this section we give a formal description of the financial planning model which is based on the considerations of the preceding section. First we specify some of the general features of the model. Then we succesively present the instrumental variables, the definitional equations and the goal variables.

General

As shown in Chapter 8, IMGP can handle a number of those problems often occurring in financial planning models. Because inclusion of all potential problems would make our example rather complicated (and consequently less illustrative), we illustrate the capacity of IMGP with respect to only some of the problems.

The present financial planning model covers ten periods. All data are assumed to be given and fixed. Consequently, a deterministic model can be developed. The instruments are a number of capital investment projects, the amount of debt issued in each period, the amount of cash hold in each period and the amount lent out in each period. There are no dividends and no equity issues, although each of these options can be included quite easily. The goal variables regarded as relevant in this example relate to the firm's market value, the stability of the accounting earnings over time, the amount of debt issued, the liquidity position and the employment offered by the firm.

The tax rate is assumed to be 40 percent. The interest rate assumed to be relevant for the type of debt this firm issues, is 10 percent. Moreover, this rate is assumed to be fixed over time and to be independent of the amount of debt issued.

Instruments

In this example, the firm can choose from a set of twenty capital investment projects. The relevant project effects, consisting of the present values, the per period after tax cash flows, the contribution to the per period earnings before interest and taxes, and the contribution to the firm's total employment level are given in Tables 9.1, 9.2, and 9.3. These tables also show the corresponding figures for the existing firm, if no projects were to be adopted.

In the programming model, the possibility of adopting a project is translated by means of the (0,1) variables x^n, $n = 0, \ldots, 20$; where $x^o (x^o=1)$ represents the existing firm.[1] All projects are assumed to be economically independent, both mutually and from the existing operations.

Another instrumental variable is the amount of debt issued in each period. For the sake of simplicity, only one kind of debt is assumed for which the borrowing rate is independent of the amount of debt issued and constant overtime. In consequence, the market value of debt equals its book value. Furthermore, all debt issues can be seen as one period loans D_t, representing the principal borrowed at the beginning of period t, and repaid at the end of the same period. Denoting the interest rate by r, and the tax rate by T, the market value of the tax savings due to a shift from all-equity financing to (partial) debt financing, is given by

$$(9.1) \qquad \sum_{t=o}^{10} r.T.D_t/(1+r)^t,$$

where $r.T.D_o$ represents the tax savings caused by the debt issued in the period before the first planning period, $D_o = 400$. In the model, debt to be issued in the first period after the planning horizon is limited to 1.5 times D_{10}.

Besides the option to borrow, we also assume that the firm can lend out an amount Z_t at the beginning of each period. Essentially, these lendings are assumed to be the exact counterparts of the loans D_t. Thus the market value of the lendings is equal to minus the value in (9.1), replacing D_t by Z_t. Z_o is assumed to be zero, whereas Z_{11} is limited to 1.5 times Z_{10}.

1) In general, superscripts relate to the projects. Subscripts relate to time periods.

Table 9.1. Unlevered present values and net cash flows of the projects

no. n	PV A^n	expected net cash flows after taxes from projects n in periods t = 1, ..., 10.* C_0^n	C_1^n	C_2^n	C_3^n	C_4^n	C_5^n	C_6^n	C_7^n	C_8^n	C_9^n	C_{10}^n
0	1000	130	130	130	120	120	120	120	110	110	110	110
1	10	-10	2	2	2	2	2	2	2	2	2	2
2	9		-10	2	2	2	2	2	2	2	2	2
3	8			-10	2	2	2	2	2	2	2	2
4	7				-10	2	2	2	2	2	2	2
5	6					-10	2	2	2	2	2	2
6	60	-50	10	10	10	10	10	10	10	10	10	10
7	-40	-200	18	18	18	18	18	18	18	18	18	18
8	40		-300	100	100	100	100	100				
9	10		-100	40	40	40	-60	40	40	40	40	-60
10	20			-100	50	40	30	20	10	10	10	10
11	10				-100	50	40	30	20	20	10	0
12	20	-100	50	50	50							
13	10				-100	50	50	50				
14	5							-100	50	50	50	
15	0										-100	50
16	15			-100	40	40	40	40	40	40	40	40
17	-10					-20	8	6	4	2		
18	-15						-20	8	6	4	2	
19	-20							-20	8	6	4	2
20	30		-50	10	11	12	13	14	15	15	15	15

* All cash flows are assumed to occur at the end of the period. The cash flows C_0^n are assumed to occur at the beginning of period 1.

Table 9.2. Earnings contributed by the projects

no. n	contribution to earnings before interest and taxes of projects n in periods t = 1, ..., 10.* E_0^n	E_1^n	E_2^n	E_3^n	E_4^n	E_5^n	E_6^n	E_7^n	E_8^n	E_9^n	E_{10}^n
0	100	103	105	112	108	110	115	120	125	130	135
1		1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
2			1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
3				1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
4					1.5	1.5	1.5	1.5	1.5	1.5	1.5
5						1.5	1.5	1.5	1.5	1.5	1.5
6		7	7	7	7	7	7	7	7	7	7
7		-2	-2	-2	-2	-2	-2	-2	-2	-2	-2
8			-50	70	80	90	100				
9			20	20	20	20	20	20	20	20	20
10				20	20	20	20	20	20	20	20
11					20	20	20	20	20		
12		10	15	25							
13					10	15	25				
14								10	15	25	
15											10
16				-10	0	0	10	20	30	30	20
17					0	0	0	0	0		
18						0	0	0	0	0	
19							0	0	0	0	0
20		0	2	4	6	8	10	12	14	15	15

* All earnings are assumed to occur at the end of the period. The earnings E_0^n are assumed to occur at the beginning of period 1.

Table 9.3. Projects effects on the firm's total employment

no.	contribution to the firm's total employment* of project n in period t = 1, ..., 10.**									
n	W_1^n	W_2^n	W_3^n	W_4^n	W_5^n	W_6^n	W_7^n	W_8^n	W_9^n	W_{10}^n
0	100	100	100	100	100	100	100	100	100	100
1		-1	-1	-1	-1	-1	-1	-1	-1	-1
2			-1	-1	-1	-1	-1	-1	-1	-1
3				-1	-1	-1	-1	-1	-1	-1
4					-1	-1	-1	-1	-1	-1
5						-1	-1	-1	-1	-1
6		-5	-5	-5	-5	-5	-5	-5	-5	-5
7	20	20	20	20	20	20	20	20	20	20
8		-10	-10	-10	-10	-10	-10	-10	-10	-10
9		-3	-3	-3	-3	-3	-3	-3	-3	-3
10			0	0	0	0	0	0	0	0
11				0	0	0	0	0	0	0
12	5	5	5							
13				5	5	5				
14							5	5	5	
15										5
16			0	0	0	0	0	0	0	0
17				3	3	3	3	3		
18					3	3	3	3	3	
19						3	3	3	3	3
20	5	4	2	0	-2	-2	-2	-2	-2	-2

* Measured as the average number of man-years per period.

** The total employment in period t, defined as W_t, can be found by adding the W_t^n values of all accepted projects ($W_0 = 100$).

Finally, the set of instrumental variables includes liquid assets, assumed to consist exclusively of the amounts of cash hold in each period. L_o represents the amount of cash at the beginning of period 1, L_1 is the amount hold during period one, L_2 is the amount hold during period 2, and so forth. As an investment, the holding of cash is not very lucrative. Consider a dollar, put in cash at the beginning of period t, and withdrawn at the end of the same period. The current market value of this operation is

$$-1/(1+r)^{t-1} + 1/(1+r)^{t} = -r/(1+r)^{t}, \tag{9.2}$$

where we choose r as discount rate, because the cash could have been used to repay debt. The current market value of holding the planned cash balances L_t, $t = 0,1, \ldots, 10$; is defined by

$$L_o - \sum_{t=1}^{10} r.L_t/(1+r)^{t}. \tag{9.3}$$

Obviously, the book value of L_o - the beginning cash - equals its market value. In this case, $L_o = 15$. In the model, cash to be held in the first period after the planning horizon is limited to $1.5L_{10}$.

Definitional equations

The definitional equations in this model are almost self-explanatory. E_t, the firm's total accounting earnings before interest and taxes in period t, $t = 1, \ldots, 10$; are given by

$$E_t = \sum_{n=o}^{20} E_t^n.x^n \qquad \text{for } t = 1, \ldots, 10; \tag{9.4}$$

where E_t^n is the part of the period t earnings, contributed by project n. Likewise, the total after tax cash flows are defined as

$$(9.5) \qquad C_t = \sum_{n=o}^{20} C_t^n . x^n \qquad \text{for } t = 0, \ldots, 10;$$

where C_o is the total cash flow at the beginning of the first planning period. The average number of employees in period t is defined as

$$(9.6) \qquad W_t = \sum_{n=o}^{20} W_t^n . x^n \qquad \text{for } t = 1, \ldots, 10.$$

The sources and uses of funds constraints are given by

$$(9.7) \qquad C_t + D_{t+1} + [r.T-(1+r)].D_t + L_t - L_{t+1} + [(1+r)-r.T].$$

$$Z_t - Z_{t+1} = 0. \qquad \text{for } t = 0, \ldots, 10.$$

Goal Variables

1. Current Market Value

In this example, the firm's current market value is assumed to be one of its most important goal variables. Obviously, this goal variable is to be maximized. Because no bankruptcy costs and no equity issues and thus no issue costs are assumed, the current market value can be defined as [1)]

$$(9.8) \qquad V = \sum_{n=o}^{20} A^n . x^n + \sum_{t=o}^{10} r.T.D_t/(1+r)^t - D_o - \sum_{t=1}^{10} r.L_t/(1+r)^t + L_o,$$

where A^n is the base case present value of project n, T is the

1) See the preceding section

tax rate, and r is the interest rate assumed to be relevant in this case. Note that $A^o.x^o$, $r.D_o$ and L_o are constants, which can be disregarded in the programming model. Furthermore, all effects of borrowing lending and holding cash after the planning horizon are ignored.

2. Stability of the Earnings Overtime

Although efficient capital markets should be insensible to erratic time patterns of the firm's accounting earnings, we assume here that the firm wants to smoothen the time path of their total accounting earnings before interests and taxes per period, E_t, $t = 0, 1, \ldots, 10$. Let us assume that the firm has defined the following target growth path:

(9.9) $$E^*_t = E_o.(1+g_E)^t \qquad \text{for } t = 1, \ldots, 10;$$

where E^*_t is the target for earnings in period t, and g_E is the target growth rate defined by the management ($g_E = 0.05$). In order to allow for possible deviations from this target growth path, the goal constraints

(9.10) $$E_t - e^+_t + e^-_t = E^*_t \qquad \text{for } t = 1, \ldots, 10;$$

are defined. In our example, the firm only concerns itself with negative deviations from the growth targets. Therefore, the firm has expressed the desire to minimize the maximum deviation from the growth targets. This desire can be translated by introducing the minimax constraints.

(9.11) $$e^-_{max} \geq e^-_t/(1+g_E)^t \qquad \text{for } t = 0, 1, \ldots, 10;$$

and defining e^-_{max} as a goal variable to be minimized. In (9.11), the deviations e^-_t have been divided by the factors $(1+g_E)^t$ because proportional rather than absolute deviations were thought to be relevant.

3. Debt Outstanding

If the firm's only goal would be its current market value, then ignoring bankruptcy costs, the firm should finance its operations completely with debt (see Jonkhart [1980]). However, firms in reality do not finance completely with debt, because bankruptcy costs cannot be ignored or for other reasons (see Chapter 2). Myers and Pogue [1974] formulate debt constraints such that 'the probability that the firm will get into trouble reaches an unacceptable level'. As shown in Chapter 8, IMGP can handle these kinds of chance constraints, giving the decision maker the option to define - interactively - what is an unacceptable probability of trouble. In our example we have chosen an approach, which is more familiar in practice; that is, to limit the risk of being unable to pay interests on debt out of current earnings, the *interest cover* (or *times interest earned)*, defined as the ratio of earnings before interest and taxes to the interest charges, should be maximized. Thus we have

$$(9.12) \qquad \text{Max}\{E_t/r.D_t\} \qquad \text{for } t = 1, \ldots, 10.$$

Within IMGP, these 10 goal variables might be maximized separately (see Chapter 8). However, to avoid a too large number of goal variables, we have chosen to adopt an approach suggested by Charnes and Cooper [1977]:

$$(9.13) \qquad \begin{array}{l} \text{Max } \lambda, \text{ s.t.} \\ \\ \lambda \leq E_t/(r.D_t) \end{array} \qquad \text{for } t = 1, \ldots, 10.$$

Assuming $r.D_t > 0$ for $t = 1, \ldots, 10$; the following heuristic method to maximize λ can be used. First, fix $\lambda = \bar{\lambda}$. Then solve

$$(9.14) \qquad \begin{array}{l} \text{Max } v, \text{ s.t.} \\ v \leq E_t - \bar{\lambda}.r.D_t \\ v \text{ unrestricted in sign.} \end{array} \qquad \text{for } t = 1, \ldots, 10.$$

If the maximum v* is negative, $\bar{\lambda}$ is infeasible. A lower value for $\bar{\lambda}$ should be chosen and a new v* should be calculated. If v* is positive, $\bar{\lambda}$ is feasible and can be raised. A zero value for v* corresponds with the optimal value for λ. A set of pessimistic values for the ratios $E_t/r.D_t$ is defined at the beginning as

$$E_t/r.D_t \geq 1.0 \qquad \text{for } t = 1, \ldots, 10. \tag{9.15}$$

Of course, these pessimistic values can be raised during the interactive process.

4. Minimum Cash Level

We assume that a minimum cash level of 5 per cent of the per period cash flow plus 5 per cent of the per period interest charges is desirable. Similar to the preceding goal variable, this can be formalized as

$$\begin{aligned} &\text{Min } \lambda, \text{ s.t.} \\ &L_t/(C_t+r.D_t) + \lambda \geq 0.05 \qquad \text{for } t = 1, \ldots, 10. \end{aligned} \tag{9.16}$$

Assuming that $(C_t+r.D_t) > 0$ for $t = 1, \ldots, 10$, the same heuristic method as used for the third goal variable, can again be used. Thus, fixing $\lambda = \bar{\lambda}$, and solving

$$\begin{aligned} &\text{Min } \xi, \text{ s.t.} \\ &L_t + (\bar{\lambda}-0.05).(C_t+r.D_t) + \xi \geq 0 \qquad \text{for } t = 1, \ldots, 10; \end{aligned} \tag{9.17}$$

gives either a positive or a zero value for ξ. If $\xi > 0$, the specified 'tolerance' $\bar{\lambda}$ in meeting the target ratio value 0.05 is too tight and should consequently be relaxed. On the other hand if $\xi = 0$, the tolerance measure $\bar{\lambda}$ can be tightened. Note that the best value of $\bar{\lambda}$ which can be conceived is $\bar{\lambda} = 0$, corresponding with the case that L_t equals or exceeds $0.05.(C_t+r.D_t)$ for all t. Together with this goal variable, we propose the following set of pessimistic values:

$$(9.18) \qquad L_t > \lambda^{min}.(C_t+r.D_t) \qquad \text{for } t = 1, \ldots, 10.$$

At the start of the interactive process we set $\lambda^{min} = 0$.

5. Maximum Cash Level

We assume that the amount of cash in each period must be less than or equal to 15 per cent of the cash flow plus interest charges. Furthermore, we assume that management wants the amount of cash to be as low as possible. These desires can be translated as

$$(9.19) \qquad \begin{array}{l} \text{Max} \sum_{t=1}^{10} \eta_t \\ L_t - 0.15(C_t+r.D_t) + \eta_t = 0 \qquad \text{for } t = 1, \ldots, 10. \end{array}$$

6. Employment

As an example of an organizational factor which may influence the firm's investment decisions, we have chosen the number of employees in this firm. Both to promote the conditions of employment and to avoid work disputes, management wants to minimize the maximum number of dismissals per period. This can be translated by introducing the goal constraints.

$$(9.20) \qquad W_t - w_t^+ + w_t^- = W_{t-1} \qquad \text{for } t = 1, \ldots, 10;$$

and in addition by introducing the 'minimax' constraints

$$(9.21) \qquad w_{max}^- \geq w_t^- \qquad \text{for } t = 1, \ldots, 10;$$

and defining w_{max}^- as a goal variable to be minimized. Note that in this case, the targets are defined implicitly. Furthermore, the deviations are measured in absolute rather than proportional terms. Note also that the firm in this case pays more attention to the avoidence of work disputes than to the firm's total employment level. The latter can benefit more than in the current formulation, by minimizing $\sum_{t=1}^{10} w_t^-$ or alternatively, by maximizing $\sum_{t=1}^{10} W_t$.

9.3. Selection of a Financial Plan (Continuous Case)

In this and in the following section we show how a financial plan might be selected by using IMGP. In this section we assume the variables x^n, n = 0,1, ..., 20; to be continuous. Section 9.4 will deal with the discrete case. In both cases, the decisions necessary in order to reach a final solution were - in this fictitious example - made by ourselves.

The most important results of the first iteration are presented in Tables 9.4 and 9.5. We trust that these results are self-explanatory, and therefore confine ourselves to a few remarks. First, it should be noted that the solutions which are optimal in the first iterations are mutually significantly different. Each project occurs in at least one of these solutions. Only one project (number 3) occurs in all solutions. In none of the solutions is debt issued after the fifth period. This is caused by the fact that the cash outflows exceed the cash inflows (one might say that there is a lack of new projects) and by the absence of dividend payments. In the model, the surplus of funds is lent out. The plan which gives a maximal market value might be rejected because (a) the earnings goal is not attained, (b) the interest cover is in some periods too low, (c) no cash is held, and (d) the maximal number of dismissals is too high.

Table 9.6 shows how the final solution is reached. First, minimal values for the cash hold, the interest cover and the market value are ensured, respectively. Then, it is decided to formulate the earnings goal as a restriction (iteration 5), after which the maximal number of dismissals is limited and the restriction on the interest

Table 9.4. Investment projects chosen when different goal variables are optimized, first iteration

project n	goal variable optimized: market value	underatt. min. earnings	interest cover	underatt. min. cash level	cash below upper limit	max. number discharges
0	1.0	1.0	1.0	1.0	1.0	1.0
1	1.0	1.0			1.0	
2	1.0	1.0			1.0	
3	1.0	1.0	1.0	1.0	1.0	1.0
4	1.0	1.0	1.0		1.0	
5	1.0		1.0		1.0	
6	1.0				1.0	
7	1.0			0.09	1.0	0.30
8	0.63		0.05		0.17	
9		1.0			1.0	
10	1.0	1.0		0.48	1.0	0.58
11	1.0		0.22	0.65	1.0	1.0
12	1.0	0.2			1.0	
13	1.0		1.0	1.0	1.0	0.51
14	1.0		1.0		1.0	0.51
15	1.0		1.0			0.51
16	1.0	0.72	0.11	0.47	1.0	
17				1.0	1.0	
18		1.0	0.17	1.0	1.0	
19			1.0		1.0	
20	1.0				1.0	

Table 9.5. Some outcomes when different goal variables are optimized, first iteration

goal variable	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}	D_1	D_2	D_3	D_4	D_5	D_6-D_{10}
1	120	105	205	226	242	276	215	227	233	213	639	717	685	586	120	0
2	105	130	153	153	155	167	179	184	196	194	308	292	348	120		0
3	103	103	116	129	138	155	141	151	163	152	279	201	80	113		0
4	103	105	118	142	149	169	154	158	155	155	304	190	170	215		0
5	120	143	192	208	221	249	235	247	253	223	639	677	649	555	213	0
6	102	104	124	145	150	160	157	165	155	152	340	238	173	218		0
	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	W_9	W_{10}	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6-λ_{10}
1	130	117	113	111	108	107	107	107	107	107	1.87	1.46	3.34	2.60	3.70	0.0
2	101	98	97	95	97	97	97	97	97	94	3.41	4.44	4.39	12.7		0.0
3	100	99	99	103	103	105	105	105	105	104	3.69	5.11	14.45	11.43		0.0
4	102	102	102	109	112	112	107	107	104	101	3.38	5.52	6.95	6.61		0.0
5	130	118	115	115	115	117	117	117	114	106	1.87	2.12	2.96	3.76	10.34	0.0
6	106	106	106	108	108	108	108	108	108	108	3.01	4.38	7.18	6.98		0.0
	L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	L_9	L_{10}		$\bar{E}_{max}$		$\bar{W}_{max}$		V
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		4.83		13.33		849.1
2	9.1	4.4	43.0	37.0	15.7	33.4	30.1	29.8	30.0	9.9		0.0		3.0		588.1
3	0.0	20.0	1.2	29.3	28.6	11.0	2.1	27.2	11.6	25.2		6.93		0.51		610.1
4	8.1	6.8	2.6	36.8	33.1	35.3	8.0	23.4	12.8	17.8		4.70		5.0		601.8
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		0.0		11.67		797.9
6	0.0	13.8	2.6	37.1	31.4	21.5	0.0	25.2	16.1	22.2		14.87		0.0		626.1

Table 9.6. Potential goal values during the interactive process

goal variable	potential goal values	iteration number										
		1	2	3	4	5	6	7	8	9	10	11
current market value	pess.	588.108	588.108	588.108*	700.0	700.0	700.0	700.0*	750.0*	775.0	762.5	762.5
	ideal	849.097	843.743	838.817	838.817	838.817	790.86	775.19	775.19	775.19	775.19	765.08
underattainment min. earnings	pess.	14.87	14.87	5.79	1.90*	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	ideal	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
interest cover	pess.	1.0	1.0*	2.0	2.0	2.0	2.0*	3.0	3.0	3.0	3.0*	3.25
	ideal	3.69	3.68	3.68	3.68	3.58	3.58	3.58	3.48	3.03	3.32	3.32
underattainment min. cash level	pess.	0.5*	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	ideal	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
amount of cash below upper limit	pess.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	ideal	647.15	431.43	323.32	323.32	323.32	306.44	257.93	257.93	204.13	253.74	224.4
max. number of discharges	pess.	13.33	13.0	12.15	12.15	12.15*	5.0	5.0	5.0	5.0	5.0	5.0
	ideal	0.0	0.0	0.0	0.48	0.48	0.48	0.81	2.88	4.98	3.87	4.76

* Goal value chosen to be improved.

cover is further tightened. In iterations 8 and 9, the minimally desired market value is raised. However, given the solutions in iteration 9, this minimally desired value is pushed back in step 10. In the final iteration, the interest cover is raised further, leaving us with six, in this case only slightly different solutions.

In Table 9.7 we show the solution which gives a maximum market value, given the constraints formulated in the final iteration. Both the earnings and the minimum cash goal are attained. The market value and the interest cover appear to be quite satisfactory. The maximum number of dismissals (five) occurs in one period only. This number is not considered to be too serious, if management attracts the additional labour required in period one on basis of a one-year contract.

Table 9.7. Some outcomes when the market value is maximized, final iteration

projects											
n	x^n	n	x^n	time	D_t	Z_t	L_t	C_t	E_t	W_t	λ_t
0	1.0			0	400.0		15.0	76.67		100.0	
1		11	1.0	1	338.26		5.92	84.67	109.93	105.67	3.25
2	1.0	12	0.13	2	269.93		1.34	8.93	116.10	100.67	4.30
3	1.0	13	1.0	3	295.12		2.07	11.90	142.37	97.67	4.82
4	1.0	14	1.0	4	315.34		16.49	298.24	174.10	99.0	5.52
5	1.0	15	1.0	5	34.27		14.73	291.24	184.60	96.0	53.87
6	0.8	16	0.56	6		261.04	8.61	172.24	207.16	95.0	
7		17		7		445.39	12.16	243.24	204.72	95.0	
8		18		8		715.35	12.16	243.24	216.72	95.0	
9		19		9		1007.01	6.66	133.24	218.28	95.0	
10	1.0	20	1.0	10		1197.28	10.05	201.00	202.72	95.0	

$V = 765.08$ $\qquad L_t \geq 0.05\ (C_t + r.D_t)$

$\bar{e}_{max} = 0.0$ $\qquad \bar{w}_{max} = 5.0$

$E_t \geq 3.25\ (r.D_t)$

9.4. Selection of a Financial Plan (Discrete Case)

In this section we repeat the experiment of the preceding section, while taking account of the discrete nature of the project variables x^n, $n = 0,1, \ldots, 20$. In this discrete case the amount of cash below the upper limit was not maximized anymore because, after considering the results of the continuous case, this goal variable appeared to be less relevant to us.[1)] The decision to drop this goal variable can be viewed as being based on the insights obtained by using the interactive procedure.

The first pessimistic solution is chosen to be equal to the pessimistic solution of the fifth iteration in the continuous case.[2)] This implies that the underattainment of the earnings target and of the minimal cash level target are respectively restricted to zero, the market value is at least 700 and the interest charges are covered at least two times. Table 9.8 shows the potential goal values of the fifth iteration in the continuous case and of all iterations in the discrete case. In the second iteration, the maximal number of dismissals is restricted to 5, whereas the interest cover becomes at least 3 in the third iteration. Because the potential market value has been reduced too much during these two iterations, it is decided to revise the decision made in the second iteration. That is, the maximal number of dismissals is restricted to 7 in the fourth iteration. Next, the minimally desired market value is set at 750 (iteration 5). The interest cover becomes at least 3.25 in iteration 6. In the final iteration, the maximal number of dismissals is reduced to 6.

We have found four different solutions satisfying the minimally desired goal levels formulated in the final iteration. One solution

1) Because the maximum cash level is defined as a percentage of cash flows plus interest charges (see (9.19)), the underattainment of this maximum cash level can be enlarged by simply raising the debts D_t and the loans Z_t simultaneously by equal amounts.
2) This first integer solution was found by simply rounding the continuous solution. In other problems, the identification of an integer starting solution may be more difficult.

Table 9.8. Potential goal values during the interactive process, integer case

goal variable	potential goal values	iteration number							
		5 (cont.)	1	2	3	4	5	6	7
current	pess.	700.0	700.0	700.0	700.0	700.0*	750.0	751.907	751.907
market value	ideal	838.817	824.452	759.799	724.498	771.827	771.827	760.897	760.897
underattainment	pess.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
min. earnings	ideal	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
interest	pess.	2.0	2.74	2.87*	3.0	3.0	3.0*	3.25	3.25
cover	ideal	3.58	3.31	3.26	3.26	3.29	3.29	3.29	3.29
underattainment	pess.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
min. cash level	ideal	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
max. number	pess.	12.15	10*	5	5*	7	7	7*	6
of discharges	ideal	0.48	1	1	5	5	6	6	6

* Goal value chosen to be changed.

which was found by maximizing the market value subject to the minimally desired goal levels, yields the ideal values for each of the goal variables. This solution is described in Table 9.9. Of course, it might be argued that this solution is better than the other solutions since the latter are dominated. On the other hand, if it is not certain that all goal variables important to the decision maker have been included in the analysis, it is better to present all four solutions to the decision maker.

We can be quite positive with regard to the computational aspects of IMGP in this integer case. We used MIP, the standard package for mixed integer programming offered by IBM, for solving each of the integer programs described in this section. The branch and bound procedure used in MIP offers several options which appear to be quite useful in the present case. For instance, if the value of the goal variable to be optimized is known to be bounded, such a bound can be inserted - generally simplifying the branching process. Because IMGP repeatedly shifts the minimally desired goal levels, this option is very useful. Another possibility to lighten the computational burden is to start each iteration of IMGP with an integer solution which is known to be feasible, subject to the newly added constraint. To this end, the preceding optimal integer solution of the goal variable, of which the minimum value is raised in the new iteration can be used. For instance, assume an integer solution exists for which the market value is 760. Clearly, this solution remains feasible if a constraint requiring a market value of at least 750 is added.

Using an IBM 370/158 computer, the CPU-time necessary for each iteration was on average less than one minute. In our opinion this already satisfactory performance can be improved quite considerably, because in our case we had to cope with an incidentally limited amount of real storage. Furthermore, the similarity of the optimization problems, both within and between each iteration, might offer additional possibilities to simplify the computational process.

Table 9.9. Some outcomes when the market value is maximized, final iteration, integer case

projects				time	D_t	Z_t	L_t	C_t	E_t	W_t	λ_t
n	x^n	n	x^n								
0	1			0	400.0		15.0	80	110.0	100	3.29
1		11	1	1	334.67		5.67	80	115.5	105	4.24
2	1	12		2	272.54		3.46	42	146.0	99	5.97
3	1	13	1	3	244.41		0.97	-5	174.0	96	6.25
4		14	1	4	278.29		15.19	276	184.5	98	106.46
5	1	15	1	5	17.33		13.54	269	201.5	96	
6	1	16		6		256.67	7.5	150	193.5	95	
7		17		7		418.52	11.05	221	205.5	95	
8		18		8		664.63	11.05	221	201.5	95	
9		19		9		931.0	5.55	111	191.5	95	
10	1	20	1	10		1093.36	10.05	201		95	

$V = 760.90$

$\bar{e}_{max} = 0.0$

$E_t \geq 3.29\ (r.D_t)$

$L_t \geq 0.05\ (C_t + r.D_t)$

$\bar{w}_{max} = 6$

9.5. Conclusion

The example described in this chapter suggests that IMGP can be a valuable tool for financial planning with multiple goals. Some of the technical peculiarities inherent in financial planning models can be tackled in a straightforward manner. Neither the presence of several ratio forms in the goal variables, nor the presence of integer instrumental variables appears to deliver any serious problems.

Furthermore, IMGP offers all advantages of interactive procedures. As such, the method is conceptually rather simple and thus quite comprehensible for both analyst and decision maker.

In the following chapter we will discuss some of the problems that might accompany the implementation of IMGP.

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