Hierarchical Portfolio Management
Theory and applications

Under his own preference, how should an investor coordinate the asset managers such that his aggregated portfolio is optimized? The efficiency of each managed sub portfolio and the aggregation of all the sub portfolios are the two main underlying problems considered in this dissertation.

Contrary to popular believes, the tracking error volatility (TEV) optimization, commonly used to find the optimal active portfolio, often yields inferior portfolio choices. The results in this dissertation together with those in Jagannathan and Ma (2003) underscore how effective simple portfolio optimization techniques can be.

In aggregating all the sub portfolios, the investor’s choice is limited if the managers only report the local optimal portfolio. Since the reported portfolios are the result of a stand-alone optimization within the sub portfolio while disregarding all the rest, each reported portfolio can only be optimal locally. A rational investor should and must demand for more choices than the locally optimal choice alone.

Using simple examples in the single and multi period setting, this dissertation illustrates how significant the improvement in aggregated portfolio performance can be, both in terms of expectation as well as realization.

Given the insufficiency of the TEV optimization, the inherent question is whether the active performance measures like the information ratio still suffice in judging a manager’s performance. As it turns out, the investor should be very careful when applying the active performance measures. Preferably, the Sharpe ratio should be used to judge the added value of a manager to the aggregated portfolio.

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Promotor:
Prof.dr. J. Spronk

Overige leden:
Prof.dr. R.E. Steuer
Prof.dr. D.J.C. van Dijk
Prof.dr. M.C.J.M. Verbeek

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To mom and dad
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Preface

Life is a journey, writing a Ph.D. has been a crucial part of mine. Looking back to the days at ING Investment Management as an intern, the years spent first as a research assistant and then as a Ph.D. candidate at the finance department of Erasmus University Rotterdam have been most rewarding. More blessed than others, my supervisor and guide in this part of the journey has been great. Not only is Jaap a beacon in the darkest hours when I was lost on the ocean of knowledge, but also an incredibly patient mentor in my own development as a person. It is hard to resist his optimism and positive view on life. To state that I have learned a lot from him is an understatement. In completion of this dissertation, I am in debt to all the committee members for their helpful, critical and constructive comments. I appreciate their time and efforts as I am keenly aware that time is a scarce commodity indeed. Furthermore, I would like to thank Marcel Smith at Fortis Insurance International for all his support during the final stages of this dissertation. To finish a Ph.D. in conjunction with a day job has proven to be challenging. I am grateful to Marcel for the understanding and freedom provided to finish the dissertation in a relatively stress-free environment.

During the past four years, it was a pleasure to have shared a room with Corine Boon and Jeroen Binken. I am going to miss the conversations with Jeroen about current affairs and our debates concerning the social fabric of modern societies. From Corine I have learned just how productive strict planning and precise execution of the plans can be. Hard work and leisure do not necessarily have to be contradictory under strict self-discipline. Given her expertise it would be unwise not to ask her to be my paranimf. Moreover, I would like to thank my other paranimf Mariëlle Non for her endless patience and efforts in helping me to refine part of this dissertation making it accessible to others who does not have a background in finance. Counting one’s blessings, it must be said that I was very lucky to find myself amongst a group of Ph.D. candidates who are not only bright, but also, against all odds and prejudice, extremely social. I would like to thank the “central committee” made up by Michiel de Pooter, Francesco Ravazzolo and Chen Zhou for all their efforts to brighten up the days, and all the friends on the 9th, 14th floor, and in Rotterdam School of Management for a memorable time. At the end, I must not forget to thank Tülay for her lightning visits providing a welcomed disruption to the otherwise monotone rhythm of the working day.

Of course, no Ph.D. candidate can survive all the paperwork associated with visiting conferences and teaching without the help and support of dedicated people. Therefore, I am infinitely in debt to Betty, Hélène, Trudy and Ursula at the finance department and Tineke at Erasmus Research Institute of Management (ERIM) for all their help and support during the past years. Also, the financial support from ERIM is greatly appreciated.

At the end, I thank my family for their enduring support, especially my father for sharing his experience in writing papers and dissertation. The opportunity to be spoiled by my
mother from time to time is one privilege I seize with vigor and verve. Thanks to my brother and sister-in-law I can escape the daily routine now and then by visiting their little “paradise”. In the childish innocence of my nieces, Vivian and Tiffany, one learns to appreciate the happiness that comes from simplicity and tranquility. Last but certainly not the least, I cannot find enough words to describe my gratitude towards my girlfriend Weili. In the past seven years she has been and continues to be extremely supportive in every conceivable manner. Although short of words, I strive to explain her value via an example of reasoning. Given the growing popularity of China and imported Chinese products in every walk of life, it is only a matter of time before problems occur. If the hypothesis is that the label “made in China” is an indication of inferior quality, then Weili is certainly the counter example that falsifies this hypothesis.

As a final remark: I would like to encourage everyone who is contemplating pursuing a Ph.D. to go ahead and do it. Indeed it is an investment that is worth investing because unlike other earthly processions your knowledge can never be taken away from you!

HKN.

Management of the many is the same as management of the few: it is a question of organization!

Chapter V, The art of war, Sun Tzu (496 B.C.)

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Introduction

In portfolio management practice, large institutional investors like banks, insurers, pension funds, mutual funds and endowment funds usually invest via intermediaries and delegate management of the funds to experts. By doing so, the institutional investor has created a multi-level hierarchical portfolio selection process with at least two decision levels. Figure 1.1 gives an example of investment process with multiple decision levels.

Figure 1.1: hierarchical top-down decision process of fund allocation. In the dashed box, the investor makes the asset allocation decisions based on the information provided by, for example, a benchmark. The country, sector and security selection decisions are delegated to the lower-level managers under mandates provided by the investor.
Chapter 1                Introduction

In the top-down decision process illustrated in Figure 1.1, the investor in the overall level informs the managers of his preference as precisely as possible by using mandates. It is up to the managers in the lower level to fill in the blanks by constructing portfolios from the choice space containing all the investment opportunities under the mandated preference constraints of the investors. The aim of this dissertation is to first illustrate the problem of information loss faced by an investor after hiring multiple managers and then to propose methods to improve the asset allocation process of the investor.

This chapter serves to provide the general background information of this dissertation. Section 1.1 starts with the investment context in which the workings of a multi-level investment process in Figure 1.1 and the role of each participant in the hierarchy are defined. Also in section 1.1, a list of problems faced by the investor by employing multiple managers is provided with references to what has been done and what still lies unexplored. The research questions are formulated in section 1.2 with the aim to improve the current hierarchical portfolio selection and management performance. Section 1.3 provides the chosen decision rule for and criteria of optimality in the portfolio choice under specific constraints. The answers to the research questions are the contribution of this study to the existing literature and our understanding of hierarchical portfolio management. At the end, the potential social impact of the solution proposed in this dissertation is provided, which is followed by the roadmap of this dissertation.

1.1 The investment context

In Cambridge Advanced Learner’s Dictionary, the word hierarchy, a noun, is defined as i) a system in which people or things are arranged according to their importance or ii) the people in the upper level of an organization who control it, with hierarchical as its adjective and hierarchically as the adverb. The first definition of the word “hierarchy” best fits the context of this dissertation.
This section first gives the definitions in the hierarchical portfolio management process. Then, the information loss problem associated with the entire process is presented. Finally, I provide the current conventions and the existing solutions, which ends with the fundamental paradoxes of current conventions.

### 1.1.1 Hierarchical portfolio management (HPM)

Figure 1.1 illustrates an example of top-down hierarchical portfolio management (HPM). The managers in the lower level construct portfolios from the choice space containing all the investment opportunities under the mandated preference constraints of the investor.

**Definition 1:** an **investor** is the party to whom the invested funds belong.

An investor can be an individual. But usually, the investor is a financial institution like a bank, insurer, pension fund, mutual fund or endowment fund, as individuals often realize their financial needs via financial intermediaries. The financial institution acts on behalf of individual participants. In this dissertation, the **institutional investor** and investor are used interchangeably.

**Definition 2:** an **asset manager** is a person who is involved in managing (part of) the total funds invested.

Often, a bank also engages in asset management and some pension funds also employ in-house managers to manage part of the available funds. To avoid confusion, the term financial institution is strictly reserved for banks, insurers, pension funds, mutual funds and endowment funds that refrain from asset management activities. The asset managers of the pension funds are viewed as stand-alone entities in this dissertation to whom the wealth does not belong.

**Definition 3: hierarchical portfolio management (HPM)** is a multi-level management process to maintain and improve the portfolio composition that best fits the investor’s preferences.
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Introduction

**Definition 4:** The investment opportunity space (IOS) is a list of stand-alone investment opportunities like a stock, a bond, cash, or alternative investments.

The IOS in this dissertation is a general term used to indicate the choice space containing all the available investment opportunities. In Figure 1.1 the IOS is the space that contains all the securities in the security level.

**Definition 5:** a *sub-IOS* of a manager is the part of the total IOS assigned to that manager by the investor.

In the example given by Figure 1.1, the sub IOS of the sector manager overseeing European financials only contains 5 major banks in Europe.

**Definition 6:** the *sub portfolio opportunity space* (sub-POS) is the set containing all the portfolio combinations of the investment opportunities in the corresponding sub-IOS that fits the mandated constraints.

To the regional manager responsible for Europe in Figure 1.1, the optimized sector portfolios of “Consumer Staples”, “Financials”, “Energy” and “Health Care” form his choice space. Since each choice is a portfolio instead of a stand-alone investment opportunity like a stock, a bond, or alternatives, hence the name.

**Definition 7:** the investor IOS (IIOS) is made up of the portfolios from the sub POS of the managers that are communicated to the investor.

This dissertation accepts the limitations that exist in practice preventing the investor to have full knowledge of the entire IOS. The IIOS is a choice space limited to the portfolio combinations of all the sub-POS’ communicated to the investor. The second difference between the IOS and the IIOS is that the elements in the IIOS are not only a portfolio, but also a portfolio constructed under the constraints in the mandate of the investor.

**Definition 8:** the investor portfolio opportunity space (IPOS) is a set containing all the possible portfolio compositions given the IIOS and the constraints imposed on the choice of portfolios.
Just as with the managers, the investor must take a decision on the portfolio choice. Depending on the IIOS, the investor may have a wide choice spectrum or a narrow one. The IPOS is the collection of portfolio combinations for the investor given his IIOS.

Usually in practice, an investor’s mandate contains a benchmark for a manager to track with limited decision freedom. Section 1.2 provides a list of the benchmark tracking strategies with decision freedom currently in use and Chapter 2 provides the technical details of the strategies.

There are at least two advantages for the investor by defining a benchmark for each manager. Firstly, by fixing a benchmark the investor not only defines the sub IOS for a manager, but it also serves as an objective reference point to measure the manager’s performance. Underperformance of an active manager is difficult to accept for the investor as simple replication strategy produces the benchmark return.

Definition 9: a **benchmark** is an index with a well defined and unambiguous weighting scheme for the investment opportunities in the IOS and is generally observable to all participants in the markets: for example, the equally weighted (EW) index like the NASDAQ-100 equal weighted index or market capitalization (Mcap) weighted index like NASDAQ and S&P 500. A benchmark is part of the IPOS of the investor.

Definition 10: a **portfolio** is a vector of weights over a predefined finite collection of investment opportunities, which also constitute a benchmark.

Definition 11: a **holding portfolio** is defined as the portfolio held by a manager while the overall portfolio is the holding portfolio of the investor.

Definition 12: in the example of Figure 1.1, the **overall portfolio of the investor** is the total portfolio in the overall level, which is the aggregate of all the sub portfolios.

Definition 13: **mandates** are collections of restrictions and directives communicated from the investor to his managers, which define the risk preference of the investor. For example,
mandate may contain which benchmark to track, risk constraints, maximum exposure and social responsible investment preferences.

In the dashed box of Figure 1.1, the asset allocation decisions of the institutional investor are often based on the relevant asset class benchmarks. The country, sector and security selection decisions are delegated to the lower-level managers under mandates.

1.1.2 Problems with the HPM setup

The principal drawback of the top-down decision process with multiple managers as illustrated in Figure 1.1 is the information loss problem. Firstly, there is loss of information between the different managers, which may lead to sub optimal portfolio choices. For example, each asset class sub portfolio in Figure 1.1 is by definition only optimal conditional on the subset of the available IOS. The interdependencies between the different asset classes are not taken into account during portfolio optimization within each asset class due to the decision structure in Figure 1.1. However, these unobserved interdependencies do exist. The aggregated portfolio of the sub optimal asset class portfolios is different and probably inferior to the portfolio choice had those interdependencies been utilized in optimizing the overall portfolio choice. Secondly, there is loss of information between the investor and the managers: the investor will never discover the superior alternatives because the managers never communicated alternatives to the investor. The aggregation of the lower level sub optimal portfolios is the second component in the information loss problem.

The secondary concern is the principal-agent relation problem of controlling and steering the managers to realize the investor’s goals. As the investor and his managers are different individuals, they may have different investment goals. For instance, if an investor is investing for his retirement, then he may be very risk-averse and look for long-term relatively safe investments. However, the managers may be driven by the short-term remuneration scheme that is typically on an annual basis. Thus, the manager may be inclined to invest in short-term risky investment that bears a huge pay-off because in case
of success the manager is likely to be well compensated and his reputation greatly enhanced.

The focus of this dissertation is to solve the primary problem of information loss in HPM. The secondary principal-agent problem is indirectly mitigated by the proposed solution. Given the possible economic loss due to information loss and the executive complexity introduced by HPM, it seems that HPM with multiple managers lacks its raison d’être. Yet, not only is HPM with multiple managers wide spread amongst institutional investors, HPM with multiple managers also exists in one form or another amongst brokerage houses and other professional asset management companies. According to Sharpe (1981), the rationale of employing more than one manager lies in the investor's desire to i) exploit each manager's specialized skills and ii) to diversify amongst the managers to reduce choice risk. Barry and Starks (1984) add that by employing multiple managers is also optimal under the risk sharing considerations.

1.1.3 Current conventions in HPM

Two types of mutually contradictory investment strategy can be clearly distinguished in the current asset management practice.

The first is passive portfolio management, which comes in many forms. The easiest form is to replicate a chosen benchmark one-on-one and then hold it indefinitely. Rebalancing only occurs when the composition of the benchmark index is updated. In practice, perfect replication may be too expensive to follow for large indexes like the S&P 500. To reduce cost, there are index tracking strategies that mimic the benchmark return with fewer stocks (see e.g. Yao et al. (2006)). In this paper we refer to the benchmark replication strategy as the passive portfolio management strategy.

1 See e.g. the website of the Dutch Civil servant pension fund asset management: www.abp.nl/abp/vermogensbeheer/key_navigatie/resultaten/portefeuille/default.asp?menu=3 and the investment strategy 2007 of PGGM, the Dutch pension fund for the hospital staff and experts: http://www.pggm.nl/TDSImages/2_98791.pdf.
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The general advice in practice is to avoid active portfolio management and stick to the passive strategy if one is uncertain about whether the manager has extraordinary skill or not (Cochrane (1999)). The roots of passive portfolio investment strategy can be traced back to the work of Tobin (1965), which has been extended by Samuelson (1969) and Merton (1969). The portfolio-selection decision is constant and independent of the consumption decision under the assumptions of i) iso-elastic marginal utility that implies constant rate of risk aversion, which means that the attitude of an individual towards financial risk is independent of his wealth level, and ii) independent increments assumption of the Wiener process. Consequently, price changes in the securities and the resulting level of total wealth have zero relevance for the portfolio selection decision. Samuelson (1969) has proved this point for the discrete time case and Merton (1969) for the continuous time case. Empirically, Malkiel (2003) has shown for the sample period between 1971 and 2001 that i) passively managed mutual funds in the U.S. have, on average, outperformed the active ones and ii) outperformance of actively managed mutual funds, if any, never persists through time in the long term. The fundamental belief behind benchmark replication strategy is the efficiency of the markets.

This belief of market efficiency is widely disputed in the literature. Treynor and Black (1973) have shown how to increase the Sharpe ratio (SR) of the benchmark portfolio by forming an additional portfolio containing all the stocks that had outperformed the benchmark in the past. Then, by combining the passive benchmark with the additional portfolio increases the SR of the benchmark. Daniel et al. (1997) have found that the aggressive-growth mutual funds seem to be able to select outperforming stocks while lacking market timing ability. From the empirical data, there seems to be time persistency in mutual fund performance. However, it strongly depends on the time period of study (Brown and Goetzmann (1995)) because mutual fund performance seems to be only persistent in the short-term (Bollen and Busse (2005), and Huij and Verbeek (2007)). Fama and French (1993) and the consequent series of publications have convincingly argued for the existence of risk premium in the stocks of smaller companies and companies with higher book-to-market ratio. If the market capitalization weighted index was used to select

2 See e.g. Amenc et al. (2004).
stocks, then the small-cap premium is automatically forfeited. The list of factors is augmented by Carhart (1997) to include the momentum factor, which prescribes to invest in recent winners while selling recent losers to obtain a premium. In general, the seemingly predictable stock returns\textsuperscript{3} invite a more active approach towards asset allocation as the standard passive index totally disregards this predictability. In the Bayesian environment, Baks et al. (2001) have shown that an investor should not avoid active managers totally based on the statistical results alone because the inability to reject a null hypothesis of “no active management skill” does not imply that all active managers have no skill in practice. In the practitioners’ literature, Grinold (1989, 1994) and Grinold and Kahn (1999) are fully dedicated to find and extract the abnormal return in excess of the market return (alpha) from the market.

The second type of investment strategy is the active investment strategy. The true virtue of a benchmark should be an objective reference point to measure the performance of managers instead of a “leash” to restrain the skillful managers. The managers with skill should be allowed to use it. Again, the active investment strategy can be sub divided into two distinct categories.

In the first category, a fixed IOS is defined from which the manager must build his POS. The manager is only allowed to construct a self-financing portfolio based on the elements in the fixed IOS.

**Definition 14:** in a self-financing portfolio, the overweight in the undervalued stocks is financed by a short position in the overvalued stocks. Hence, all the positions in a self-financing portfolio sum to zero.

**Definition 15:** an active portfolio is a self-financing portfolio constructed from an IOS containing a fixed number of investment opportunities, with long position in undervalued opportunities and short position in overvalued ones.

Definition 16: An actively managed portfolio contains a passive benchmark component and a self-financing active part.

In the second category, the IOS is not necessarily fixed. Although the actively managed portfolio still has a passive and active part, the active part does not necessarily contain the same investment opportunities as the passive part. A good example is the core-satellites investment strategy reported in Amenc et al. (2004). The passive core contains investment opportunities that are considered to be stable and market mimicking such that a “mediocre” manager can manage it. In contrast, the active satellite portfolio(s) are specialized portfolio(s) containing high yield and high-risk investment vehicles that demands specialized knowledge to maintain and exploit.

In this dissertation, the active portfolio refer to the first category of the active investment strategy unless stated explicitly otherwise. Also, the terms active investment strategy and active portfolio management are used interchangeably in this dissertation. The same convention holds for the passive investment strategy and passive portfolio management.

The institutional investor enforces his preference on the asset managers through his mandates to the managers. Typical mandates include weight constraints and risk exposure restrictions. In active portfolio management, a popular restriction is the tracking error volatility constraint.

Definition 17: Tracking Error Volatility (TEV) is the standard deviation of the return difference between the portfolio and the benchmark.

The TEV restriction limits the additional risk exposure of the overall portfolio, as it reins in any risk-seeking behavior of asset managers to maximize his remuneration scheme (Jorion (2003)). Hence, the active managers are allowed to deviate from the benchmark,

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4 In the practitioners' literature, tracking error (TE) is often used to denote the standard deviation of the return difference between the holding portfolio and the benchmark (see e.g. Grinold and Kahn (1999), Kahn (2000) and Blitz and Hottinga (2001)), while the academics use the TEV definition (see e.g. Roll (1992) and Sharpe (1994)). The TEV definition is used in this dissertation not only because of its academic roots, yet the name of TEV directly conveys the content of the definition.
yet not so much that the risk exposure of the overall portfolio becomes unacceptable for
the investor.

Another popular type of constraints is the bandwidth weight constraints.

**Definition 18: bandwidth constraint** is defined as a bandwidth around the benchmark weight within which the portfolio weight of the stock may fluctuate.

The bandwidth constraint limits heavily tilted portfolios towards a small number of securities, which is the main drawback of the Markowitz (1952) mean-variance (MV) optimization (Jagannathan and Ma (2003)). Disappointing ex-post performances in a tilted and hence concentrated portfolio may ruin the investor's solvency and consequently its social responsibility.

At performance assessment, each manager is assessed based on his risk-adjusted return measure of appraisal ratio (AR) or information ratio (IR). Treynor and Black (1973) first introduced the AR, which uses the residual return from the benchmark, $\alpha$, divided by the corresponding residual risk to appraise the added value of a security in the active portfolio. A number of authors affiliated with BARRA have introduced the IR (Sharpe (1994)). The IR is essentially the *Sharpe ratio* (SR) with a different benchmark. SR is the excess return (return above the risk free rate) divided by its standard deviation. IR is the portfolio return minus the benchmark return divided by the corresponding standard deviation: TEV.

Within the willingness of the institutional investors to allow for active portfolio management to acquire a bit of extra return, **two fundamental paradoxes** are hidden. *Firstly*, if the benchmark is taken as the efficient choice, then by definition of efficiency there exist no other choices that dominate the benchmark choice. Hence, why choose for active portfolio management that costs more and is not expected to earn any additional return? *On the other hand*, if the benchmark is perceived to be inefficient, then active portfolio management seems to be logical. But, why should one follow something that is dominated by other efficient choices at the first place? Replication of an inefficient
benchmark seems to be unwise and futile because the investor only gets the benchmark return adjusted with all the costs.

1.2 Research questions

Although HPM has been widespread in practice for decades, academia has been surprisingly indifferent towards the topic, as the number of publications in the past decades dealing with the subject is limited. Noticeable publications are the presidential address of Sharpe (1981) to the American Finance Association and Elton and Gruber (2004). More recently, van Binsbergen et al. (2007) have tackled the horizon problem in the HPM, as the investor's long-term investment horizon differs from the short-term remuneration scheme driven investment horizon of the managers and proposed an endogenous benchmark to align the management optimal portfolio choice with the investor's optimal portfolio choice.

The problems for the investor listed by the aforementioned publications include i) the economic loss in diversifications due to the hierarchical decision structure, ii) optimal asset allocation to the managers, iii) formulation of mandates to insure overall portfolio optimality at security selection in the lower level, iv) devising incentive plans to ensure optimal behavior of managers, v) evaluation and selection of managers, and vi) the problem of different investment horizons for the investor and manager.

This dissertation combines the knowledge gathered in Roll (1992), Jorion (2003) and Scherer (2004) about active portfolio management with the decentralized decision hierarchy in which the investor employs multiple active managers. Then, a general methodological framework is developed to account for the problems from ii) to v) conditional on the realistic setting to minimize the economic loss due to the hierarchy.

In essence, the HPM process has introduced sub optimal portfolio choices based on partial information in the lower level, as the managers are oblivious of each other's existence in managing the investor's wealth. Interactions between the sub portfolios and thus,

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1 See for example, Russ and Clasing (1982, pg. 513) and Grinold (1989, pg. 31).
diversification opportunities are lost. The overall portfolio is then the aggregated portfolio of a collection of sub optimal local portfolios. Had the investor invested directly without any hierarchy, then it is very likely that the “utopian” portfolio composition differs from the one based on the decentralized choice (Sharpe (1981), Elton and Gruber (2004), and van Binsbergen et al. (2007)).

**Definition 19:** the *utopian portfolio* is the overall portfolio obtained without any hierarchy. The goal function of the investor is optimized by directly choosing investment opportunities from the IOS instead of the POS.

The direct investment choice bears the name “utopian” because its execution in practice is constrained by knowledge, time and physical limitations. The investor may hire just one manager to take control of the whole process. However, the manager may lack the skill and knowledge to fully comprehend the complexities of all the products available in the markets. Also, the manager may be overwhelmed by the sheer amount of information available to make a sound judgment in the limited time span to exploit it: large optimization programs need days of running time on the computers before an answer is produced. Finally, the manager has physical limitations, as a day only holds 24 hours. Besides, the single manager setup is also undesirable from the investor's perspective due to risk diversification concerns: all the eggs are in the single basket that is the manager.

Thus, the principal concern in this dissertation is the economic loss in terms of investment return suffered as result of the HPM: the return difference between the utopian and the aggregated overall portfolio. Secondly, the current convention in HPM of using benchmark tracking with limited decision freedom certainly does not help the matter, as portfolio choices outside the feasible set defined by the benchmark and the decision freedoms are automatically ignored. This in turn also aggravates the information loss problem. Although already recognized by Sharpe (1981), Elton and Gruber (2004), Hallerbach et al. (2004) and van Binsbergen et al (2007), yet the size of the economic loss due to HPM with multiple self-financing active portfolios managed by different managers is still an open question. Next, I list the specific research questions of this dissertation.
Research question 1:
*Given the HPM process with multiple (non-) overlapping sub portfolios in a single period setting, what is the magnitude (i.e. significant or negligible) of the economic loss incurred by the hierarchy and the benchmark tracking strategy with limited decision freedom?*

Research question 2:
*How can we decrease this economic loss?*

Research question 3:
*If there is improvement in the performance of the overall portfolio with respect to its benchmark, is this improvement persistent through time or is it a lucky throw of the dice?*

Research question 4:
*In case a lower level actively managed portfolio has outperformed its lower level benchmark, does it automatically imply that the overall portfolio with the actively managed portfolio has outperformed the benchmark in the overall level?*

Research question 5:
*At performance assessment, how should an investor judge (active) portfolio manager if the sub portfolios are correlated?*

### 1.3 Two-step bottom-up portfolio selection

The logical solution to the information loss problem is to increase the information flow between the decision levels such that an increasing part of the available information is incorporated during the selection process. This dissertation proposes a *two-steps bottom-up* decision approach in which the managers supply the investor more information than the aggregated benchmark index combination. Hence, the IPOS contains more feasible sub portfolio choices for the investor than the benchmark choice alone. Also, the information set is increased with all the possible combinations of the portfolios in the IPOS, thus
mitigating the economic loss during aggregating the sub portfolios. In the proposed solution, the investor now actively utilizes his unique position of supervising the entire investment process and coordinates the activities of the managers by allocating selectively to them. If additional information presents no superior investment opportunities, then the investor can always fall back on the benchmark choice. Hence, it is expected that extra information cannot hurt portfolio selection.

At portfolio selection, the decision rule of the investor produces the optimal allocation to the managers that is expected to realize the investor’s goals based on the projections supplied by the managers. Thus, to get the account, the managers must enter a bid to compete for it. The managers may be oblivious of each other’s existence in managing (part of) the investor’s money. Even if they do know, it is highly unlikely that they will disclose their information to each other. Hence, the bidding process is a closed one. Together with the knowledge that the investor prefers higher performance than low ones, it is expected that each bidder will enter his best bid. Of course, managers that underperform their projection time after time must be aware of legal consequences and possible penalties. The bottom line is that the investor gets the best performance out of the managers through this first-bid closed auction of the available funds.

### 1.4 Decision rule, optimality and constraint

In the two-level hierarchy, both the investor as well as the managers take decisions and make choices based on certain rules.

In the economics literature, the prevailing decision rule is the expected utility maximization of Von Neumann and Morgenstein (1944). A well-behaved utility function is concave and has the non-satiation and risk aversion property. The non-satiation property states that the individual prefers more wealth to less. The risk aversion property captures the phenomenon of decreasing marginal utility: the utility gained by acquiring an additional Euro is different for a homeless person and a millionaire. The MV optimization
rule of Markowitz (1952) for the individual investor has a quadratic utility function that is well behaved. If the market is in an equilibrium state, then the Capital Asset Pricing Model (CAPM) of Treynor (1961), Sharpe (1964), and Lintner (1965) describes the expected return\(^6\) of a risky investment (Markowitz, 1991).

However, there is evidence very early on that managers like “long shots”, i.e. lottery tickets with high positive skewness of yields (Marschak (1938) pg. 320). This has been called the probability-dependent risk attitude in the literature. The expected utility model is unable to cope with this risk-seeking behavior that depends on the probability distribution of outcomes. Experimental literature on probability-dependent risk attitude is extensive\(^7\). The inclusion of human psychology into the decision process has "killed" the \textit{homo economicus}, whom the economists always have taken as the subject in the decision process. It seems that “man” is not a cold, calculating, self-interest driven subject who is rational. Rather, “man” is emotional who tends to overreact (De Bondt and Thaler (1985)) and systematically makes mistakes (the prospect theory of Kahneman and Tversky (1979)).

There are two principal hurdles preventing the rapid proliferation of these new models that clearly have superior properties over the traditional expected utility maximization model (Polkovnichenko (2005)). Firstly, there is a lack of empirical data to confirm the claim that the behavior prompting non-expected utilities is widespread outside of the laboratory environment. So, is “man” really that irrational in reality? Secondly, the additional complexity introduced by the alternative preferences other than the expected utility is often so significant, which does not seem to be an easily applicable alternative. This second problem is aggravated by the lack of evidences that support the usefulness of these alternative preferences in applications where the expected utility model has fallen short.

\(^6\) The security market line dictates that the return of a portfolio equals the risk exposure with respect to the market (\(\beta\)) times the market premium plus the risk free rate.

\(^7\) See for example, Starmer and Sugden (1989), Battalio, Kagel and Jiranyakul (1990), Tversky and Kahnemann (1992), Kachelmeier and Sheikh (1996) and the references in these papers. Shoemaker (1982), Camerer (1995) and Starmer (2000) provide excellent surveys of this literature.
The Markowitz (1952) model for an individual decision maker and the Capital Asset Pricing Model (CAPM) under market equilibrium remain to be widely used in practice precisely because of their easily understood characteristics and straightforward interpretation of the results. In this dissertation, the Markowitz (1952) MV optimization is chosen as the investment decision rule for both the investor as well as the managers. By doing so, I have chosen to stick with the current convention so that I avoid potential endless debate about the explanation of the results, as the results can be attributed to either the elimination of the information problem or a superior model that delivers better decisions. Hence, by sticking to the most elementary decision model, I can isolate and focus on the illustration of the extent of the main problem in this dissertation: the information loss problem in the multi-level hierarchical portfolio selection process. This information loss problem will not disappear if other models are used because the problem is a result of the hierarchical decision process.

Next, the Markowitz (1952) decision rule is illustrated in more details together with its shortcomings. By showing the irrelevance of these shortcomings on the research problem at hand increasingly motivates the choice of the MV decision rule. Then, the condition of decision optimality in this dissertation is defined. At the end, the no-short constraint that is always present throughout the dissertation is defined and explained in the last subsection.

1.4.1 The Markowitz decision rule

The Markowitz (1952) paper was the seminal work that has arguably started the modern portfolio theory, in which the portfolio choice in a single-period setting depends on two parameters: the expected return and risk of the portfolio choice. The expected return is measured by the average return of the investment choice over a period of time while the expected risk is measured by the variance of the return time series over the same period. The efficient minimum variance frontier (efficient frontier henceforth) is the collection of portfolio choices that have the highest return at a fixed risk level and simultaneously the lowest risk at a fixed return level. If there exists a risk free investment choice, then the
Tobin (1958) two-fund separation theory dictates that the single-period optimal choice of an investor is a linear weighted sum of the risk free investment and the efficient risky portfolio, defined by the tangent portfolio on the efficient frontier. The angle between the tangent and the horizontal line is the SR, which defines the excess return of the portfolio per unit of risk.

**Definition 20:** the excess return is defined as the portfolio return minus the risk free rate.

The bottom line in the Markowitz model is that the investor must aim for the efficient choice because only then does the investor maximize his excess return per unit of risk taken.

Unfortunately, neither the pension funds nor the insurers are MV believers, as they are more engaged in preventing default on their obligations. Then, why the Markowitz model? To an institutional investor, if some risk level has been chosen, then the efficient portfolio gives the highest obtainable return. In case the return level is inadequate to cover the future obligations, strategic decisions need to be adjusted to increase the risk to a level with an expected return that can cover the future obligations. The MV optimization procedure is flexible enough to cope with the adjustments and amendments. If additional constraints like social responsible investment criteria are applied to the optimization process, then the results obtained under the additional constraints are contained in a subset of the old results because some original results may no longer be feasible under the new additional constraints. However, the MV optimization procedure can still cope with these new constraints.

The shortcomings of the MV optimization rule have long been recognized in the portfolio theory and asset pricing literature. The fundamental concern with the MV setting is the implicit distribution of the security returns in the IOS. If only the mean and variance is used to describe a statistical distribution, then it is an elliptical, symmetric distribution around the mean with 95% of the masses within the confidence interval delimited by twice the standard deviation: it is a Gaussian distribution.
The return distribution of investment is skewed due to the existence of the limited liability property\(^8\) in all equity investments (see e.g. Black (1972), Christie (1982), Nelson (1991), Golec and Tamarkin (1998), and Harvey and Siddique (2000)). Also, in decentralized portfolio management the manager may prefer a positively skewed portfolio (Brennan, 1993). As such, the aggregated overall portfolio may be skewed as well. What if there is kurtosis in the distribution? So there are extreme losses\(^9\) and extreme gains. Dittmar (2002) uses nonlinear pricing kernels in which the risk factors are determined endogenously from the data to account for the extreme eventualities in the kurtosis. The results from the nonparametric model show that the nonlinear pricing kernel significantly improves the single- and multifactor pricing kernels. Furthermore, in line with Bansal et al. (1993), Bansal and Viswanathan (1993) and Chapman (1997), the nonlinear, nonparametric model explains the cross-sectional variation in expected return much better than the CAPM. Fama and French (1993, 1995, and 1996) also advocate the use of multi-factor pricing kernel instead of the CAPM. There are premiums associated with \(i\) the small firms and \(ii\) the value companies with a high book-to-market ratio. Carhart (1997) adds momentum to the list of factors, which captures the premium obtained by buying the recent winners while selling the recent losers.

The unrealistic property of the variance that classifies both the high positive returns and big negative returns as the same is also mentioned in Markowitz (1952). Extensive illustrations given in Markowitz (1952) using the semi-variance, which only accounts the negative returns as volatility suggest that better portfolio choice can be attained. The other methods to estimate the volatility include the stochastic dominance (Hanoch and Levy (1969), Post (2003), Post and Versijp (2007)) and the multi factor approaches (Chan et al. (1999)). However, at this moment the stochastic dominance (SD) methods for estimating volatility are still incapable of producing volatility estimates that can be used in portfolio selection. The flexibility of the SD methodology of capturing the "true" distribution of the

\(^8\) The limited liability property implies that the maximum loss incurred cannot exceed the total investment while the maximum attainable return is unbounded from above.

\(^9\) Examples of extreme losses are natural disasters like the Kobe earthquake in Japan in 1993 and hurricane Katrina in U.S. in 2005. These disasters not only put a lot of companies out of service, but it also triggered the migration of workers to other places and reallocation of companies that had a domino effect on the local and national economy.
return is also its Achilles heel because there is no single unequivocal choice. I refer to Versijp (2007) for further details.

Last but not least, the related problem to the skewness and kurtosis problem is the estimation error corresponding to the MV decision model. Britten-Jones (1999) used a 20-year database containing 11 country indexes and found that the sampling error\(^{10}\) in estimates of the weights of a global efficient portfolio is large.

Be as it may, Jagannathan and Ma (2003) reports that the sample covariance matrix performs as well as covariance matrix estimates based on factor models, shrinkage estimators and daily data when the no-short constraint is in place.

1.4.2 Conditional normative choice

In this dissertation, the conditional normative rule is used for decision making under uncertainty. Under the conditional normative rule, a portfolio choice is taken as ex-ante optimal if it suffices all the constraints and dominates the rest of the choice space. Naturally, this optimal choice is also scrutinized in its ex-post performance. If the MV decision rule is not dismissed as totally useless (Jagannathan and Ma (2003)), then the optimization results cannot be utterly worthless. At least, by regularly optimizing the portfolio composition the portfolio is regularly updated with new information available in the market unlike the fixed composition of a benchmark, which remains close to the benchmark composition disregarding changed market condition.

\(^{10}\) Sampling error occurs when the drawn samples differ from their population in the means and standard deviations.
1.4.3 The no-short constraint

Throughout this dissertation, the no-short constraint\footnote{The no-short constraint prevents negative position (short position) in the securities. An active position may be negative while not violating the no-short constraint because the original position minus the active position may still be positive.} is always imposed on the decision rules of both the managers as well as the investor. The reason for imposing such a constraint is both logical and practical.

In the decision setting defined in this dissertation, each manager competes for a piece of the overall available fund of the investor. If the proposed portfolio of a manager is classified by the investor's decision rule as inferior, then a short position in this portfolio seems to be justified. But how can an investor realize such a strategy? A manager gets part of the total fund if the investor takes a long position in this specific portfolio. However, if the investor's position in a specific portfolio is a short position, must a manager then pay the investor an amount of money? It is a very difficult position to realize. Hence, it has been excluded from the feasible set of POS.

Also, the well-know problem with the MV approach is the possible extreme holding weights (see e.g. Jagannathan and Ma (2003)). Hence, in its realization, the MV holdings can be extremely sensitive to estimation errors. However, as Jagannathan and Ma (2003) have shown, the sample covariance matrix performs as well as covariance matrix estimates based on factor models, shrinkage estimators and daily data when the no-short constraint is in place. The results in this dissertation substantiate this finding by illustrating that the MV portfolio performance obtained after imposing the no-short constraint can dramatically improve the passive benchmark tracking portfolio performance.
1.5 Academic contribution

The research conducted in this dissertation is on the crossroad where many different research directions in the field of finance intersect. The fundamental problem investigated in this dissertation is how to optimally allocate funds to the managers if the investor employs multiple managers.

If the investor is assumed to be ignorant of the possibilities and the managers are the experts, then this information asymmetry may induce moral hazard and the possible adverse selections in a manager’s behavior: the manager may be tempted to take excessive risk to increase his active return such that his remuneration is maximized. The research objective then becomes a quest for a reward scheme that minimizes the moral hazard and persuades the managers to behave in the interest of the investor. In van Binsbergen et al. (2007) a method is presented that produces an endogenous benchmark, which aligns the short-term objective of the managers with the long-term objective of the investor. Vayanos (2003) has concluded that to best process the information flow through hierarchy is to allow each level to have only one single subordinate. Stracca (2007) gives an excellent overview of the theoretical literature on the principal-agent problem.

In practice, the investor is not totally ignorant of the possibilities in the investment markets, especially the institutional investors with their in-house experts. Also, from the practical perspective the institutional investors cannot afford to be complacent. They must fulfill their long-term social and economic responsibilities like paying out the pensions or compensate the insurance policy takers for damages when accidents do occur. As such, the institutional investor cannot be passive and must actively seek for the optimal allocation scheme amongst the managers to extract the best out of the entire IOS.

This dissertation offers an alternative approach towards information exchange between the investor (principal) and manager (agent). Instead of the investor giving the

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12 Increased risk of immoral behavior on the part of the contract taker who may benefit from the immoral behavior.
13 A market process in which bad results occur due to information asymmetry between buyers and sellers: The “bad” products or customers are more likely to be selected. The most famous example is the “markets for lemons” of George Akerlof.
managers mandates and the managers filling in the blanks, the investor wants to know the choices available to him. Hence, the investor wants to know his IPOS. Then, the overall portfolio choice is an optimal portfolio of all the available choices in the IPOS. In this way, the investor is able to coordinate the different managers’ actions to maximally benefit from the interactions between the stand-alone portfolios. The main academic contribution of this dissertation is this new approach towards information exchange and integration.

Also, this research joins the ongoing debate about the choice between passive and active portfolio management. Given the theoretical results in Merton (1969) and Samuelson (1969), and empirical results in Malkiel (1995, 2003), it seems that passive management is the better choice in the single and multi-period portfolio context: the market efficiency is so effective that systematic outperformance over the market is impossible. A prevailing perception on active portfolio management is that it is expensive and it does not add any value after market frictions like transaction costs are properly taking into account. This dissertation provides tentative evidence supporting the claim that active portfolio management based on disaggregated information with periodic rebalancing can add value, even after transaction costs. Using the MV decision rule, monthly rebalancing of a portfolio with MSCI country and regional indexes significantly outperforms the fixed passive benchmark portfolio and the MSCI world index. More significantly, the bootstrapping results reveal that the net results of the MV optimization strategy only underperforms the passive benchmark portfolio in less than 1.3% of the 10,000 simulations over the period between Dec. 2002 and May 2006.

At performance assessment, the practitioners’ literature usually only focuses on active portfolio performance and assumes that the different sub portfolios are independent because the managers built the portfolios independently from each other (the independence assumption) when evaluating the overall portfolio (see e.g. Grinold and Kahn (1999) and Blitz and Hottinga (2001)). However, if the sub portfolios are correlated, then the risk adjusted return of the overall portfolio under the independence assumption is imprecise at best and exaggerated at worst. From the investor’s perspective, the overall portfolio performance should be the only relevant result.
Chapter 4 in this dissertation illustrates how severe the inaccuracy of performance assessment under the independence assumption at performance assessment can be. Additionally, a threshold is provided for assessing the active portfolio performance, which divides active portfolios that have added value to the overall portfolio from the rest.

1.6 Potential social impact

Private wealth of households managed by financial institutions has sharply increased over the past decade (Stracca (2007)). An increasing share of private personal wealth has been invested in the stock market via financial intermediaries for various reasons like pension needs, college funds and endowment funds. As such, the personal investment process has become one with more than one decision layer. It is the managers who translate the ideas and needs of the investor into concrete investment decisions from which the investor takes his decision. Overall, the sharp increase in private wealth of households managed by financial institutions is not isolated to the Anglo-Saxon countries alone, similar increase is also observed in other European countries and Japan (Davis and Steil (2001)).

Moreover, given the world wide population aging problem and the historically low interest rate, there is a steady growth\textsuperscript{14} in the size of the pension funds and their willingness to invest in the stock markets. In the period between 2000 and 2005, the annual investment of Dutch pension funds in risk bearing investments through financial intermediaries on average has exceeded 38\% of their total wealth\textsuperscript{15}. In absolute numbers, the total amount invested by the Dutch pension funds in the global stock markets averages € 200 billion per annum between 2000 and 2005. In 2002, however, there is a significant reduction in the amount invested as a result of the adverse market conditions. Thus, from both the social and economic perspective a thorough understanding of the HPM is of paramount importance.

\textsuperscript{14} Since 2000, the total wealth of pension funds in The Netherlands has grown by more than 32\% to 637 billion Euros under management in 2005 (www.statistics.dnb.nl/index.cgi?lang=nl&todo= PenFinGeg).

\textsuperscript{15} Calculated using data from the Dutch Central Bank: www.statistics.dnb.nl/index.cgi?lang=nl&todo= PenFinGeg.
For each person that saves for retirement, better investment results of the pension fund not only reduce the pension contribution by every participant in the pension scheme, but also help to guarantee the solvency and continuity of the pension fund. The domino effect on the economy as a whole is that people will have more money to spend on other things, which inevitably will increase economic activities.

1.7 The roadmap

This dissertation is organized according to the research questions. First we start in the single period setting and explore the cross section of the IOS. Then, Chapter 3 expands to the multi-period setting to examine the time persistency of the single period results. At the end, the assumptions at performance assessment are scrutinized with reference to the overall portfolio performance.

The first experiment in Chapter 2 is conducted in the static one-period setting conditional on the inescapable fact of hierarchy in HPM process. Proofs are provided to support the claim that benchmark index choice is not the efficient portfolio choice. Furthermore in Chapter 2, I present the Achilles heel of the TEV optimization procedure for any predetermined benchmark and show that an alternative strategy is a better choice. Given the IOS of each manager and the accompanying restrictions, the resulting POS contains more choices than the benchmark choice. By reporting more choices from the POS upwards to the overall level, the institutional investor can improve his overall portfolio choice. The Jobson and Korkie (1981) test is used to measure the significance of the portfolios.

In the second experiment that makes up Chapter 3, the investigation focuses on whether the improvement in overall portfolio performance in the single period is persistent through time. The transaction costs are taken into account in the experiment. The performance is measured by the cumulative absolute value of a hypothetical portfolio. The Jobson and Korkie (1981) test is also used to measure the significance of the improvements in
periodically rebalancing the holding portfolio. Using the bootstrap resampling methodology we rule out the possibility that the results are data specific.

Chapter 4 dissects the overall portfolio risk into different parts using a risk attribution methodology and explores the impact of each part of the portfolio risk on the overall portfolio risk. Also, Chapter 4 summarizes the performance assessment of the active portfolio and overall portfolio using AR, IR, and SR together with the thresholds for value added of the active portfolios.

Chapter 5 concludes and is followed by the customary summary in Dutch.
Chasing the global efficient frontier in two steps

The aim of this chapter is to provide answers to the first two research questions: what is the magnitude of the economic loss introduced by hierarchical portfolio management (HPM) and the current practice (research question 1), and how can we decrease the economic loss (research question 2)?

In this chapter, I use a hierarchy structure with three levels: an overall level; a sector level; and a stock level. The global efficient frontier is constructed without any decision hierarchy. It is a utopian solution because all the available information on the stocks has been utilized without any information aggregation. The investor in the overall level directly selects from the stocks. Hence, there is no information loss.

If the investor chooses the HPM strategy with benchmark tracking in the sector level, then each sector portfolio has a fixed composition of the stocks within the sector based on some chosen benchmark index. Assume there are 5 sectors, when the investor runs an optimization to select the optimal overall portfolio combinations, he is effectively selecting from a IIOS with 5 benchmark portfolios. Interactions between the different elements within each sector portfolio were never taken into account because the information in the stock level is aggregated into the sector portfolio based on the benchmark composition. This information loss may lead to loss in diversification opportunities (see e.g. Sharpe (1981), Vayanos (2003), Elton and Gruber (2004), Hallerbach et al. (2004) and Binsbergen et al. (2007)), which may induce economic losses. Figure 2.1 illustrates the hierarchy and information flows.
Hence, the magnitude of economic loss induced by HPM is defined by the distance between the global efficient frontier based on the full information set and the efficient frontier constructed based on the HPM strategy with benchmark tracking in the sector level.

The information loss problem induced by HPM is not solved by adding limited degree of decision freedom in the form of tracking error volatility (TEV) constraint to the decision process. As will be explained in more details later on, the TEV constraint is conceived to prohibit the volatility of the active portfolio to exceed a maximum: hence preventing excessive risk taking behavior of the managers. Therefore, the active strategy with TEV constraint only contemplates part of the entire investment choice space. All the opportunities outside of the predefined feasible space is automatically ignored and hence forfeited. Relevant information may have been lost.

In this chapter I propose a two-steps bottom-up optimization procedure to mitigate the impact of information loss problem. The first step is to allow the managers to build portfolios from the sub IOS under the investor’s preference to form the sub POS, thus creating more portfolio choices in the lower level than the benchmark choice alone.
In the second step, each manager reports (part of) the sub POS to the overall level to form the IIOS of the investor. Then, the investor optimizes his allocation over the portfolio choices in the (increased) IIOS based on his preference constraints, thus forming the IPOS of the investor. Finally, the investor makes a portfolio choice from his IPOS.

In practice, it is difficult, if not impossible, for a manager to “surrender” all his portfolio information. Therefore, the results in this chapter also serve as an indication of what kind of information is useful. For example, the results in this chapter show that the portfolio combinations close to the minimum risk portfolio are often used by the risk averse investor in the overall level while the portfolios near the maximum return portfolio are seldom used. Hence, the investor in the overall level should specifically ask for the portfolio compositions near the minimum risk portfolio and not waste energy and fees on the portfolios near the maximum return portfolio. Moreover, the results also show that the benchmark compositions like market capitalization weighted or equally weighted also adds value in the overall level. The reason for this lies with the diversified nature of these benchmark compositions.

The remainder of this chapter is organized as follows. Section 2.1 starts with the portfolio choices in the utopian full information case. Then, section 2.2 presents the current HPM practices of two-stage optimization procedure with benchmark tracking in the lower sector level. Section 2.3 provides the consequence of TEV optimization in the lower level on the performance of the overall portfolio. The example of bandwidth optimization and the impact of increased information exchange between the decision levels are illustrated in section 2.4. An empirical exposition is showcased in section 2.5 together with the main findings and discussions. Finally, section 2.6 concludes the chapter by summarizing the relevant findings and answers to the research questions.
2.1 The utopian choice

The utopian HPM choice is obtained under full information without decision hierarchy. This is the best result obtainable in absence of forecasting error.

In active portfolio management, the investor does not believe that the market portfolio is the best choice. Managers are allowed to deviate from the benchmark holding to realize an outperformance over the benchmark.

Under the standard Markowitz (1959) portfolio theory, the ideal HPM is the scenario without any decision hierarchy: the investor possesses the full knowledge over the entire opportunity set. If there are $m$ investment opportunities in the IOS, then $\mathbf{R}_U$ denotes the expected return vector in the top level under the ideal HPM scenario:

$$\mathbf{R}_U = [\tilde{r}_1, \ldots, \tilde{r}_m]^T,$$

(2.1)

with the subscript $U$ stands for "Utopia" and superscript $T$ for vector transpose. The bold letters indicate matrices and vectors. As such, $\mathbf{R}_U$ in eq. (2.1) is a vector whereas the expected return of the $i$-th stock, $\tilde{r}_i$ for $i = 1, \ldots, m$, is a scalar. Let $\mathbf{\Omega}_U$ denote the full covariance matrix of the IOS:

$$\mathbf{\Omega}_U = \begin{bmatrix}
\sigma_{1}^2 & \sigma_{1,2} & \ldots & \sigma_{1,m} \\
\sigma_{2,1} & \sigma_{2}^2 & \ldots & \sigma_{2,m} \\
\vdots & \vdots & \ddots & \vdots \\
\sigma_{m,1} & \sigma_{m,2} & \ldots & \sigma_{m}^2 
\end{bmatrix},$$
in which $\sigma_i^2$ denotes the variance of investment opportunity $i$ and $\sigma_{ij}$ denotes the covariance between investment opportunity $i$ and investment opportunity $j$ for all $i$ and $j$.

Let $W_U$ denote the weight vector with each element as the weight of a specific stock in the holding portfolio. Under these conditions, the investor allocates his wealth over the risky investment opportunities in the IOS and a risk free asset. After all, the investor may always choose to put his money in the bank. Given a risk free return $R_f$, the mean-variance (MV) efficient portfolio with expected return $\bar{R}$ is the solution to the constrained optimization

$$\min_{W_U} W_U^T \Omega_U W_U,$$

under the constraint of

$$W_U^T R_f + (1 - W_U^T i) R_f = \bar{R},$$

where $i$ is the unity vector $[1, 1, \ldots, 1]^T$. As described in section 5.2 (pg. 187) of Campbell, Lo and MacKinlay (1997) the solution to the constrained optimization problem in (2.2) can be written as

$$W_U^* = \frac{\bar{R} - R_f}{(\bar{R}_U - R_f)^T \Omega_U (\bar{R}_U - R_f)} \Omega_U^*(\bar{R}_U - R_f),$$

which can be decomposed into a scalar and a vector:

$$W_U^* = c_U \tilde{W}.$$  

In eq. (2.4), the scalar $c_U$ depends on the mean of the minimum variance portfolio return, $\bar{R}$.
Chapter 2 Chasing the global efficient frontier in two steps

\[ c_u = \frac{\bar{R} - R_f}{(\bar{R} - R_f)^\top \Omega_u^{-1} (\bar{R} - R_f)}. \]

Whereas the portfolio weight vector \( \bar{W} \) does not. Thus, all the minimum-variance portfolios are a combination of a given risky asset portfolio with weights proportional to \( \bar{W} \) and the risk free asset: two-fund separation of Tobin (1958). The portfolio containing all the risky assets is called the tangency portfolio and its portfolio weight vector is the standardized weight vector by dividing the elements of \( \bar{W} \) by their sum:

\[ W_{TP} = \frac{1}{\Omega_u^{-1} (\bar{R} - R_f)} \Omega_u^{-1} (\bar{R} - R_f). \]

The subscript \( TP \) identifies the tangency portfolio and it is the portfolio on the minimum variance frontier that optimally exploits the risk-return trade-off.

To summarize, the optimal utopian portfolio is a combination of a risk free and a risky investment with the weights of both components sum to one, as was predefined by the constraint in problem (2.2). The optimal weight of each element in the risky tangent portfolio is determined by the mean of the minimum variance portfolio return, \( \bar{R} \) and the expected excess return of each element, rescaled to sum to unity: the full investment constraint. Under the full investment constraint, the weight of each element can be interpreted as a percentage.

If an additional constraint of no-short is added to problem (2.2), then the closed form solution no longer holds. To the best of my knowledge, there is no closed form solution to problem (2.2) under the full investment and no-short constraints. Later on in the empirical part, the solution to the problem is also solved using computer optimization routines when the constraints are applied.
2.2 Passive benchmark tracking

This section illustrates the HPM in which all investment opportunities other than the benchmark choices are ignored. The inherent reduction in overall portfolio efficiency due to loss in investment opportunities is also illustrated. In this dissertation the replication strategy is referred to as the HPM strategy. The passive investors are skeptical about the skill of the managers and hold the benchmark portfolio to be efficient. Hence, the mandates given to the managers prohibit any deviation from the benchmark: each manager simply replicates his benchmark and the aggregated overall portfolio produces the benchmark return minus the transaction costs and fees. Thus, the benchmark replication portfolio is expected to always underperform its benchmark by definition.

In HPM, the investor can choose a benchmark replication portfolio in the overall level and mandate all the sub portfolios to be benchmark replicating as well. Consequently, the entire portfolio is passive without any active component. Given the general setup of an overall level with one investor and a lower level with multiple managers, the return vector of the investor who employs \( N \) managers, \( R_I \), contains \( N \) portfolio returns:

\[
R_I = \begin{bmatrix} \bar{p}_1, \ldots, \bar{p}_N \end{bmatrix}^T,
\]

where \( \bar{p}_i \) the expected portfolio return of manager \( i \) for \( i = 1, ..., N \) and the subscript \( I \) stands for "investor". The full POS of the investor contains \( N \) portfolios and a risk free investment with return \( R_f \). Each manager has an IOS that is not necessarily disjoint.
lower level, the bold lower case letters denote the return vector to distinguish it from the one in the overall level. The stochastic return vector of manager $i$, $\tilde{r}_{pi}$, contains $m_i$ stochastic stock returns.

$$\tilde{r}_{pi} = [\tilde{r}_{p1}, ..., \tilde{r}_{pm_i}]^T.$$ 

Over a single investment period, the $i$-th manager’s expected replication portfolio return is given by

$$\bar{r}_{pi} = \bar{r}_{bi} = (w_{bi})^T E(\tilde{r}_{pi}),$$

(2.5)

where $w_{bi}$ is the $m_i \times 1$ weight vector of the $i$-th manager’s benchmark. The total expected return for the investor in the overall level, $\bar{R}_r$, is the sum of all the portfolios’ expected return weighted according to the benchmark-weighting scheme $W_B$:

$$\bar{R}_r = W_B^T \bar{R}_r,$$

(2.6)

Clearly, the benchmark replication strategy suffers from both the selection bias as well as the aggregation bias. The expectation is that the benchmark replication strategy forms the minimum attainable performance.

### 2.2.1 Loss of investment opportunities

Assume that an investor is investing in 2 indexes. Index 1 contains the stocks A and B. Index 2 holds stock C only. Hence, the IOS is a list of three stocks and the full POS under the no-short and full investment constraints is defined by the surface area of the triangle ABC in the $\mathbb{R}^3$ space in Figure 2.2. If the composition of index 1 is 40% in stock A and
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the rest in stock B, then the choice space spanned by index 1 and index 2 is the line CD in Figure 2.2. Clearly, there are other choices than those represented by the line CD. If the optimal portfolio choice is not on the line CD, then the selection bias component of the information loss problem has caused an economic loss when the investor invests in a portfolio on the line CD.

The economic loss due to information loss problem aggravates when all the lower level sub optimal portfolios are aggregated to form the overall portfolio: the aggregation bias. In practice, the information loss problem is not confined to the financial world. Kouvelis and Lariviere (2000) have shown that it also exists in production companies with multiple agents.

Figure 2.2: The investment opportunity space defined by index 1 and index 2. Index 1 is composed from the stocks A and B. Index 2 only has stock C. With a fixed composition of 40% in A and 60% in B, the line CD spans the total investment opportunity space available by investing in index 1 and index 2.

The solution to the information loss problem is to increase the information flow between the decision levels such that an increasing part of the available information is incorporated during the selection process. In Figure 2.2, if the concentration of stock A in index 1 is allowed to fluctuate between 20% and 40%, then the POS of the investor is represented by
the surface of the triangle CDD’, which represents an increased choice space of the investor.

In the lower manager level, the loss of investment opportunity for the benchmark replication portfolio is zero if the benchmark weights happen to be efficient. Haugen and Baker (1991) have shown the inefficiency of a Mcap replication portfolio in the MV space. The EW index is a simple strategy that has proven to be difficult to beat (see e.g. DeMiguel et al. (2005)). The following two theorems and corollary illustrate i) when the EW replication portfolio is MV efficient if there are 2 investment opportunities in the IOS and ii) why the EW replication portfolio cannot be MV efficient if there are more than 2 investment opportunities in the IOS. Thus, there exist portfolio compositions that dominate the EW replication portfolio when there are more than 2 investment opportunities in the IOS.

**Theorem 2.1**: assume that there are only two investment opportunities in the entire investment opportunity space with the expected return vector \( \bar{r}_{x} = [\bar{r}_{a}, \bar{r}_{b}] \) and the covariance matrix \( \Omega_{x} = \begin{pmatrix} a & c \\ c & b \end{pmatrix} \)

with a, b, and c as different scalar values. The equally weighted index of the two investment opportunities is mean-variance efficient if and only if

\[
\bar{r}_{x} - r_{j} = \frac{(ab - c^2)(a + c)}{(ab - c^2) + (a + bc - ac - c)} \quad \text{and} \quad \bar{r}_{x} - r_{j} = \frac{(ab - c^2)(b + c)}{(ab - c^2) + (a + bc - ac - c)}
\]

hold.
Proof: let \( W^* \) denote the MV efficient weight vector and \( W_{EW} \) denote the equally weighted (EW) benchmark weight vectors. If the EW benchmark is MV efficient, then the necessary condition \( W_{EW} = W^* \), which is also sufficient, must hold. Given the necessary condition, from solution (2.3) we know that the following equation must hold for \( W_{EW} \) to be MV efficient:

\[
W_{EW} = \frac{\bar{f} - r_f}{(\bar{f} - r_f)^T \Omega^{-1}_p (\bar{f} - r_f)} \Omega^{-1}_p (\bar{f} - r_f).
\]

As we have already seen in eq. (2.4), \( W_{EW} \) is the product of a scalar and the vector \( \Omega^{-1}_p (\bar{f} - r_f) \). Work the vector out for the return vector \( \bar{f} \) and the covariance matrix \( \Omega_p \) we get

\[
\Omega^{-1}_p (\bar{f} - r_f) = \begin{bmatrix}
    b \bar{f}_{1f} - c \bar{f}_{2f} \\
    ab - c^2 \\
    a \bar{f}_{2f} - c \bar{f}_{1f} \\
    ab - c^2
\end{bmatrix}
\]

From the property of EW benchmark index we know that each element in the index vector is per definition identical. Hence the equation

\[
b \bar{f}_{1f} - c \bar{f}_{2f} = a \bar{f}_{2f} - c \bar{f}_{1f}
\]

must hold and from which follows the relationship between the excess return of the elements in the investment space:

\[
\bar{r}_{1f} = \bar{r}_{2f} \left( \frac{a + c}{b + c} \right)
\]  

(2.7)
Since the index holdings must always sum to unity, the full investment condition states that the sum of all the elements in the index vector must sum to one:

$$\frac{b\bar{r}_{1f} - c\bar{r}_{2f}}{ab - c^2} + \frac{a\bar{r}_{2f} - c\bar{r}_{1f}}{ab - c^2} = 1 .$$

(2.8)

After substituting $A_1$ into $A_2$ and solving $A_2$ for the returns we find the expressions:

$$\bar{r}_{1f} = \frac{(ab - c^2)(a + c)}{(ab - c^2) + (a + bc - ac - c)}$$

and

$$\bar{r}_{2f} = \frac{(ab - c^2)(b + c)}{(ab - c^2) + (a + bc - ac - c)} .$$

Q.E.D.

**Theorem 2.2:** in the case with $n>2$ investment opportunities, the equally weighted (EW) index is only MV efficient when the cofactors in each column of the adjoint of the covariance matrix are identical. Otherwise the EW index cannot be a MV efficient choice.

**Proof:** in case with $n$ investment opportunities, the expected return vector $\bar{r}_p \in \mathbb{R}^n$ and the covariance matrix $\Omega_p$ is an $n \times n$ symmetric matrix. The MV optimality equation in Theorem 2.1 still holds:

$$W_{EW} = \frac{\bar{r}_f - r_f}{(\bar{r}_p - r_f, t)^T \Omega_p^{-1} (\bar{r}_p - r_f, t)}$$

(2.9)
Since the first part with the fraction is a scalar, it is the second part, $\Omega_p^{-1} (\bar{r}_p - r_f)$, that determines the optimal weight vector. The inverse of the covariance matrix $\Omega_p$ follows from the equation

$$\Omega_p^{-1} = \frac{1}{\det(\Omega_p)} \text{adj}(\Omega_p),$$

where $\det(\Omega_p)$ denotes the determinant of the matrix $\Omega_p$ and $\text{adj}(\Omega_p)$ its adjoint. Just like the covariance matrix $\Omega_p$, its cofactor matrix and adjoint are also $n \times n$. Let $w_i^*$ denote the weight of element $i$ in the MV efficient EW index. The EW index property dictates that

$$w_i^* = w_j^* \quad \forall i, j \in [1, \ldots, n], i \neq j. \quad (2.10)$$

Let $c_{ij}$ denote the first cofactor of the $i$-th element and $\bar{r}_{ij}$ the excess return of the first element in the index. Using the cofactors, the optimal weight vector of (2.9) can be written as

$$W_{EW} = C \begin{pmatrix} c_{11} & c_{12} & \cdots & c_{1m} \\ c_{21} & c_{22} & \cdots & c_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nm} \end{pmatrix} \begin{pmatrix} r_{1f} \\ r_{2f} \\ \vdots \\ r_{nf} \end{pmatrix}$$

with

$$C = \frac{\bar{r} - r_f}{(\bar{r}_p - r_f)^T \Omega_p^{-1} (\bar{r}_p - r_f)} \frac{1}{\det(\Omega_p)}.\$$

The expression for the optimal weight of the $i$-th stock $w_i^*$ can be written as

$$w_i^* = C \left[ c_{i1} \bar{r}_{1f} + c_{i2} \bar{r}_{2f} + \ldots + c_{in} \bar{r}_{nf} \right].$$
and the expression for the optimal weight $w_j^*$ is

$$w_j^* = C \left[ c_{ij} r_{ij} + c_{ij} r_{ij} + \ldots + c_{ij} r_{ij} \right].$$

Since the excess return vector is given and consequently so is the covariance matrix, if condition (2.10) is to hold, then all the cofactors must be identical. More formally, the following relations must hold if condition (2.10) is to hold:

$$c_{ij} = c_{ij} \wedge c_{ij} = c_{ij} \wedge \ldots \wedge c_{ij} = c_{ij}.$$

Thus, the MV optimality of the EW portfolio only holds when the cofactors in each column of the adjoint of the covariance matrix are identical. The MV optimality of the EW portfolio is rejected in all the other cases.

Q.E.D.

The intuition here is that if there are $n$ investment opportunities, is every single investment opportunity also an efficient one? If not, then why follow a strategy that gives every investment opportunity an equal weight?

**Corollary 2.1:** with $n > 2$ investment opportunities, the EW index must invest equal amount in each investment opportunity, whereas the MV efficient portfolio only selects the efficient opportunities. When the EW index contains redundant investments from the MV perspective, then the equally weighted index cannot be MV efficient.

Just as the EW replication portfolio in the lower level, the Mcap replication portfolio is also MV inefficient (Haugen and Baker (1991)). Tracking the popular Mcap based EAFE index for international investor is not an achievement, as it can be easily beaten (Wilcox (1994)).
2.2.2 Aggregating benchmark tracking portfolios

In the utopian case, the investor can choose from the $M$ investment opportunities in the investment opportunity space (IOS). After employing $N < M$ managers the investor's IOS becomes POS with $N$ portfolios. As shown by Theorem 2.1 and 2.2, the benchmark replication portfolios in the lower manager level that tracks EW indexes are not MV efficient. The implication from the result is that there exist other portfolio compositions that dominate the benchmark choice in terms of return or risk, or both. This is the selection bias. The aggregated overall portfolio containing sub optimal portfolio is probably globally inefficient. This is the aggregation bias. Figure 2.3 illustrates the intuition for the case $N = 2$.

Figure 2.3: loss in overall portfolio performance by aggregating benchmark replication portfolios. Each index has 5 elements. The squares in the lower level are the equally weighted index portfolios. In the overall level, the vertical distance between the thick line and the global efficient frontier (thin line) is the loss in return and the horizontal difference is the loss in risk reduction.
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In the lower level of Figure 2.3, the bias in selection due to benchmark tracking is self-evident. Instead of a portfolio choice on the efficient frontier, the benchmark tracking strategy forces the EW benchmark choices (squares) to be reported to the overall level. If only the EW index replication portfolios are communicated to the overall level, then the interaction between the individual elements of the indexes are ignored and forfeited. After aggregating the EW benchmark choices in the overall level, it is straightforward to observe that the resulting combinations are inefficient.

To solve the problem of selection bias, the managers must be granted decision freedom to find better alternatives than the benchmark choice. To avoid problems at aggregation, the investor must better coordinate the lower level portfolios. Better coordination requires more information from the lower levels because local optimal portfolio is not unique: the efficient frontier is a collection of efficient portfolios and hence containing multiple optimal portfolios. If the investor only has the choice out of one local optimal portfolio, then other combinations of locally optimal portfolios are automatically lost. Once again, the proposition here is that more information from the lower level needs to be communicated to the overall level such that an increasing part of the available information is utilized in the overall portfolio selection.

2.3 Active hierarchical portfolio selections

Active tracking error volatility (TEV) optimization does not solve the selection bias in the information loss problem.

Aggregating TEV efficient sub portfolios to the overall level still leads to inefficient overall portfolio.

An active investor believes that higher return than the benchmark portfolio return can be realized and allows the managers to deviate from the benchmark as long as additional active risk defined by the tracking error volatility (TEV) remains within some stipulated boundaries. In practice, the prevailing assertion is also that the specialists do have a
better\textsuperscript{16} ability in interpreting market signals. Unfortunately, TEV optimization hardly ever yields the desired effect of efficient sub portfolios that dominate the benchmark choice with inherently dominating aggregated overall portfolio. As it turns out, additional decision freedom granted in the form of TEV always leads to additional active risk and sometimes without proportionally higher return.

This section explores the portfolio choice based on TEV optimization. Not only is the inadequacy of the TEV optimization in lower level portfolio selection demonstrated, but also its specific effect on the overall portfolio selection.

### 2.3.1 Two-step optimization

For ease of exposition, the multi-level hierarchical decision process for an MV investor is reduced to a two-steps bottom-up optimization. In the first step, the managers optimize their preferences and report the optimal portfolios on the efficient frontier to the investor. Then, the investor chooses the allocation of funds over the managers and the risk free investment under his own preferences. As the investor is the only person who is aware of all the sub portfolios in the overall portfolio, it is up to the investor to coordinate the actions of the managers to improve his overall asset allocation. To achieve that goal, the investor needs more investment choices than the locally optimal portfolios, as the number of combinations increases exponentially with each additional lower level portfolio choice.

The first step in the staged optimization is to find the local efficient frontiers. The minimum-variance portfolio with mean return $\overline{\text{p}}_p$ is the solution to the constrained minimize variance problem

$$
\min_{w_j \in \Lambda_i} \ w_j^T \Sigma_j w_j,
$$

\text{(2.11)}

\textsuperscript{16} See e.g. Black and Litterman (1992).
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with $A_i = \left\{ w_i \mid w_i^T \tilde{\mu}_i = \tilde{\mu}_i, w_i^T 1 = 1 \right\}$. The constraints are the return and full investment budget constraint. The problem of loss of diversification opportunities in HPM arises when the manager only partially communicates his opportunity space. Each rational manager only reports the portfolio that optimally exploits the risk-return trade-off in his local universe. However, the optimal manager’s choice is not necessarily optimal when the entire universe is contemplated. Local optimality does not automatically imply global optimality.

In the second step, the investor in the overall level first collects all the available portfolio returns in $\mathbf{R}'$, the IPOS return vector, and the corresponding volatilities in the covariance matrix $\Omega_p$. Then, the investor determines his optimal asset allocation over the risk free and the risky assets by solving the same problem as in (2.2) for different levels of $\mathbf{R}'$

$$\min_{\mathbf{w}_p} \mathbf{w}_p^T \Omega_p \mathbf{w}_p \quad (2.12)$$

subject to $\mathbf{w}_p^T \mathbf{R}_p + \left(1 - \mathbf{w}_p^T 1\right) \mathbf{1}_p = \tilde{\mathbf{R}}_p$. The obvious difference here with respect to the problem of the lower-level manager is that the investor does not have to invest all his wealth into the risky assets, as there exists the alternative of a risk free asset. Thus, the constraint above states that the minimum-variance portfolio invests $\left(1 - \mathbf{w}_p^T 1\right)$ in the risk free asset and the rest in the risky assets. As such, the budget constraint in problem (2.11) is no longer a binding restriction and can therefore be neglected.

Following Merton (1972) and section 5.3 of Campbell, Lo and MacKinlay (1997), the optimal portfolio holdings for manager $i$ given his $\Sigma_i$ and $\tilde{\mu}_i$ equals

$$\mathbf{w}_{p,i}^* = \mathbf{g} + h \tilde{\mu}_i \quad (2.13)$$
where \( g \) and \( h \) are \((m \times 1)\) vectors,

\[
g = \frac{1}{D} \left[ B \left( \Sigma_i^{-1} \mathbf{1} \right) - A \left( \Sigma_p^{-1} \bar{r}_p \right) \right],
\]

\[
h = \frac{1}{D} \left[ C \left( \Sigma_i^{-1} \bar{r}_p \right) - A \left( \Sigma_p^{-1} \mathbf{1} \right) \right],
\]

and \( A = \mathbf{1}^T \Sigma_i^{-1} \bar{r}_p \), \( B = \bar{r}_p^T \Sigma_i^{-1} \bar{r}_p \), \( C = \mathbf{1}^T \Sigma_i^{-1} \mathbf{1} \), and \( D = BC - A^2 \). Thus, the entry to

the optimized return vector \( R' \) by manager \( i \) is given by:

\[
R'_{i,j} = \left( w^*_p \right)^T \bar{r}_p.
\] (2.14)

The variance of the \( i \)-th optimized portfolio is given by:

\[
\sigma_{pi}^2 = \left( w^*_p \right)^T \bar{r}_p, \quad \text{and the covariance between portfolio } \ i \ \text{and portfolio } \ j \ \text{is given by:}
\]

\[
\text{cov}(p_i, p_j) = \left( w^*_p \right)^T \Sigma_{ij} w^*_p.
\] (2.16)

The return vector \( R' \) and the covariance matrix \( \Omega_j \) follow by repeating the steps from eq. (2.13) to (2.16) for all the optimized portfolios in the lower level. The solution to the optimization problem in (2.12) and the solution in eq. (2.3) only differ in the return vector and covariance matrix.
The loss in return as a result of the different portfolio weightings in eq. (2.3) and eq. (2.17) is the economic loss for the investor due to loss of diversification.

\[ \text{economic loss} = (W_p^*)^T R_i - (W_p^*)^T R_i' \]

This loss of diversification is due to lack of communication between the different managers, which is very common if the managers are each other's competitors for investment funds (Sharpe (1981)). On the other hand, the investor as a client is entitled to be kept posted by the managers about their investment plans for the invested funds. Thus, it is up to the investor himself to coordinate all the managers to achieve his optimal portfolio.

### 2.3.2 Problems with tracking error volatility (TEV) optimization

| Active return optimization gives constant active risk adjusted return. |
| More tracking error volatility demands more risk tolerance from the investor. |
| The overall efficiency of a portfolio depends on the efficiency of the benchmark portfolio. |

A common procedure to maximize active return is to treat the active portfolio as a self-financing portfolio in optimizing the portfolio choice. The active portfolio holds both long and short positions: long position in undervalued stocks and short position in overvalued ones. Continuing with the lower level manager \( i \) who is investing in \( m_i \) stocks, the objective of this, by now active, manager is
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\[
\max_{w_a} \quad w_a^T \tilde{\mathbf{r}}_i - \frac{1}{2\lambda} \mathbf{w}_a^T \Sigma_i \mathbf{w}_a, \tag{2.18}
\]

where \( \tilde{\mathbf{r}}_i \) is the expected return vector of \( i \)-th manager’s investment opportunity space and \( \lambda \) the risk tolerance of the investor. The objective in (2.18) states that while the extra active return is appreciated the additional active risk is discounted from the return via the risk tolerance measure of the investor, \( \lambda \). Using a \( m_i \times 1 \) unity vector \( \mathbf{i} \), the self-financing constraint dictates that the sum of all the active weights must be zero:

\[
\mathbf{i}^T \mathbf{w}_a = 0.
\]

Under this constraint, the solution to the maximization problem of (2.18) is an active weight vector \( \mathbf{w}_a \) that is linear in \( \lambda \), the risk tolerance of the investor and optimizes active return while minimizing the active risk.

\[
\mathbf{w}_a = \lambda \Sigma_i^{-1} \left( \tilde{\mathbf{r}}_i - \mathbf{i} \left( \mathbf{i}^T \Sigma_i^{-1} \mathbf{i} \right)^{-1} \left( \mathbf{i}^T \Sigma_i^{-1} \tilde{\mathbf{r}}_i \right) \right) \quad \tag{2.19}
\]

For the \( i \)-th manager, his active portfolio only depends on how risk tolerant the investor is, since the expected return vector \( \tilde{\mathbf{r}}_i \) and the covariance matrix \( \Sigma_i \) of the investment opportunity space for the manager are ex-ante fixed. If the investor is a passive investor with infinite risk aversion, then the almost zero \( \lambda \) forces the active weights towards zero. On the other hand, a risk tolerant investor with positive finite risk tolerance has fixed active weighting and it can be calculated using eq. (2.19).

After obtaining the active portfolio weights \( \mathbf{w}_a \) via eq. (2.19), the investor has fixed the ex-ante tracking error volatility (TEV)
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\[ TEV = \sqrt{w_a' \Sigma_a w_a} \]  \hspace{1cm} (2.20)

and the ex-ante active return \( r_a \):

\[ r_a = w_a' \bar{r}_a. \]  \hspace{1cm} (2.21)

From eq. (2.20) and (2.21) it follows that \( i \) the ex-ante active return adjusted with the active risk is constant and \( ii \) it is independent with respect to the investor's risk aversion \( \lambda \). Thus, the conditional normative ex-ante information ratio (IR) of the active portfolios is constant. By definition, the information ratio is the ratio between the active return and the active risk: \( IR = \frac{r_a}{TEV} \). Substituting eq. (2.19) into the IR definition we get

\[
IR = \frac{\left[ \Sigma_i^{-1} \left( \bar{r}_p - \bar{t} \left( \bar{r}_p \Sigma_i^{-1} \bar{t} \right) \right) \right]^T \bar{r}_p}{\sqrt{ \left[ \Sigma_i^{-1} \left( \bar{t} \Sigma_i^{-1} \bar{t} \right) \Sigma_i^{-1} \left( \bar{r}_p - \bar{t} \left( \bar{r}_p \Sigma_i^{-1} \bar{t} \right) \right) \right]^T}}. \]  \hspace{1cm} (2.22)

Clearly, active weight vector in eq. (2.22) is constant. If the expected return vector \( \bar{r}_p \) and the corresponding covariance matrix \( \Sigma_i \) are known, then eq. (2.22) yields a scalar. Thus, whether the manager gets 0.01% TEV constraint or infinite decision freedom over a fixed set of IOS, the resulting ex-ante IR of the portfolios in the active risk and return space is constant. Figure 2.4 illustrates.

The disturbing news for an investor who is willing to take active risk is that extra active risk will not provide a disproportionally higher risk adjusted active return, IR. Instead, the total active portfolio return can be scaled up or down by increasing or decreasing the total wealth invested rather than providing active managers with more decision freedom.
The active component reenters the mean-standard deviation (MS) space by adding it to the benchmark holding to construct the \(i\)-th manager's total portfolio return:

\[
\tilde{r}_{pi} = (w_{bi} + w_a)^T E(\tilde{r}_p),
\]

and the corresponding portfolio variance

\[
\sigma^2_{pi} = (w_{bi} + w_a)^T \Sigma_i (w_{bi} + w_a). \tag{2.24}
\]

The TEV efficient frontier in the MS space can be traced out by solving eq. (2.23) and (2.24) repeatedly for different levels of active risk by changing the risk tolerance in eq. (2.19). The benchmark portfolio lies on the TEV efficient frontier with zero active risk. Figure 2.5 illustrates.

**Figure 2.5** exposes the first fundamental problem of the TEV optimization procedure: it is a relative optimization procedure defined around the benchmark. If the benchmark is
efficient, then so is the TEV efficient frontier. Otherwise, the TEV efficient frontier is inefficient in the MS space. Consequently, there exist portfolio choices with lower risk at the same return level or higher return at identical risk level, or both.

**Figure 2.5:** lower level TEV efficient frontier. The TEV efficient frontier goes through benchmark $b$ and is anchored by the benchmark. If the benchmark is efficient, then the TEV efficient frontier collapses into the efficient frontier.

**In Figure 2.5 hides a second fundamental problem of TEV optimization:** the no-short condition may be violated during TEV optimization. The self-financing property of the active portfolio guarantees that the full investment property is not violated when an active portfolio is added to a benchmark. However, the short position of an asset in the active portfolio may be so large that the total holding of the asset becomes a short position. For example, the active weight of a stock in an index is calculated as -3% based on equation (2.19). The benchmark weight of the stock is 2%. If the active portfolio is added to the benchmark, then the total weight of the stock is -1%: violation of the no-short constraint. Hence, in Figure 2.5 there is a cut-off point on the TEV efficient frontier due to the no-short constraint that divides the portfolios with and those without short positions. As such, the TEV efficient frontier in the MS space is limited and the optimal tangent portfolio may be a corner solution.
The minimum risk portfolio of the TEV efficient frontier in Figure 2.5 is the solution to the constrained optimization problem of

$$\min_{w_a} \left( w_{bi} + w_a \right)^T \Sigma_i \left( w_{bi} + w_a \right)$$

subjected to the self-financing portfolio constraint. Under the independence assumption and the assumptions of the single index model, the cross multiplication terms in problem (2.25) are zero:

$$w_a^T \Sigma_i w_{bi} = w_{bi}^T \Sigma_i w_a = 0.$$ (2.26)

Due to the implications in eq. (2.26) and the fact that the benchmark variance is fixed, the problem in (2.25) simplifies to

$$\min_{w_a} \sigma_b^2 + w_a^T \Sigma_i w_a,$$ (2.27)

where the benchmark variance, $\sigma_b^2$, is constant and exogenous to the active weights. Given that the active weight squared is always positive, the active part of the total portfolio's variance in (2.27) is only negative if and only if the total weight on the negative elements in the covariance matrix $\Sigma_i$ is larger than the weights on the positive entries. If the negative covariance part is so large that it mitigates or even nullifies the positive variance part, then active portfolio creates no additional risk. If the negative covariance part is larger than the variance part, then we have a risk reducing effect through active portfolio management. This is exactly what we seek: allow for deviation from the benchmark holding to find either risk reduction or return generation, or both. So, the possibility of a total portfolio variance lower than the benchmark variance cannot be excluded a priori: all depends on the covariance structure.
2.3.3 Aggregating benchmark tracking portfolios with TEV constraint

The actively managed portfolio in the lower level consists of a benchmark and an active component. Both the benchmark and the active component are constructed from identical IOS assigned to the manager. The active component is constrained by the restrictions mandated by the investor. In aggregating the actively managed portfolios, the investor is effectively aggregating all the benchmarks each with an active component. The selection bias and the aggregation bias are still present here in the benchmark tracking portfolio with TEV constraint.

The efficiency of the TEV optimized portfolios depends on the efficiency of the benchmark. The TEV efficient portfolios are only globally efficient if and only if the benchmark is efficient (see e.g. Roll (1992), Jorion (2003) and Scherer (2004)). Otherwise, there exist other portfolio choices in the lower level that dominate the TEV efficient portfolio in terms of risk and returns. Aggregating inefficient portfolio choices with positive correlations rarely lead to efficient portfolio choices in the overall portfolio level.

Secondly, the choice of the lower level managers has always been a stand-alone choice. By only reporting one specific local optimal portfolio the managers have severely restricted the POS of the investor in the overall level, even if the stand-alone choices are locally TEV efficient. Figure 2.6 illustrates.

In Figure 2.6, the TEV efficient frontier (thick solid frontier) is added to the lower level diagrams. Since the TEV efficient frontier is a relative optimization with respect to the benchmark, it also runs through the benchmark when the active weights are zero. Due to the inefficiency of the chosen benchmark, the TEV efficient portfolios (diamond) are unfortunately sub optimal. The TEV efficient portfolio chosen by each manager is reported to the overall level. Clearly, the combinations between the TEV efficient portfolios in the overall level do not exhaust all the possibilities in the lower level, as there are other globally efficient choices.
2.4 A solution

The problems facing an investor who delegates his investment needs to multiple managers are i) the selection problem in the lower level portfolio choice and ii) the aggregation problem of aggregating the sub optimal choices to the lower level.

Depending on the efficiency of the chosen benchmark, tracking error volatility (TEV) optimization procedure generates portfolio composition that maximizes the portfolio return.
within the active risk mandate. If the chosen benchmark is efficient, then so is the TEV efficient portfolio. Otherwise, a sampling bias arises when only the inefficient lower level portfolio is reported back to the overall level. If there are $N$ managers with $M$ possible portfolio choices each, then the choice obtained by only contemplating the combinations of $N$ portfolios is clearly inferior to the choice based on $NM$ combinations, which exhausts all the possibilities. Hence, we first need to improve the lower level portfolio efficiency and then we need to improve the communication between the lower and overall level such that all the available possibilities are also exploited.

In the next subsection, the concept of bottom-up portfolio selection is explained in more details. Section 2.4.2 provides an example of how to improve lower level portfolio selection as TEV optimization is inadequate. In section 2.4.3 the bandwidth constrained optimization is explained in a more general setting than the hierarchical context. I would like to stress here that weight bandwidth is only one example of improving the lower level portfolio selection. By no means is the weight bandwidth optimization method the solution to the selection problem in the lower level.

### 2.4.1 Bottom-up portfolio selection

The solution to the efficiency problem in the lower level and aggregation problem is to increase the information flow between the decision levels such that better portfolio choices can be made during the selection process. This chapter proposes an active bottom-up decision approach in which the managers supply the investor more choices than the aggregated benchmark choice alone. In such a bottom-up portfolio selection process, the information flows from the lower level upwards to the overall level: each manager informs the investor about (part of) the available portfolio choices in his sub POS. By increasing the information flow, the investor increasingly obtains a more complete picture of the full POS. The advantage of extra information is at least twofold. Firstly, more and often better investment choices are discovered when the aggregated information contained in the benchmark is opened up. Based on the improved IPOS, the investor in the overall level can
make a better allocation of his investments. Secondly, the increase in information can also mitigate the economic losses during aggregation of the sub portfolios. The investor now actively utilizes his unique position of supervising the entire investment process and coordinates the management activities via selective allocation of funds to the managers.

The current chapter illustrates just how effective this simple bottom-up strategy with additional information can be in improving the overall portfolio performance of the investor. Not only does this chapter serve to illustrate the workings of additional information, but also what kind of information is needed to effectively improve the investor’s asset allocation choices. In practice, it is very difficult to ask an external expert to surrender all his knowledge and private information. Hence, the investor must aim to extract the relevant parts of the external expert’s information set. This chapter also serves to show what kind of information is relevant for the decision-making process in the overall level.

2.4.2 Improving lower level efficiency: weight bandwidth

Optimizing sub portfolio holdings directly in the mean-standard deviation space yields dominating sub portfolios.

Instead of only altering the active weights as in TEV minimization, the weight bands changes the entire portfolio weight.

The overall portfolio efficiency improves by increasing the amount of information to the overall level.

An alternative form of active portfolio management to TEV optimization is to allow for certain bandwidth around the benchmark weight. The holding weight vector \( w_i \) of manager \( i \) is the solution to the utility maximization problem of

\[
\max_{w_i \in \Lambda_i} w_i^T \bar{r}_i - \frac{1}{2\lambda} w_i^T \Sigma_i w_i, \tag{2.28}
\]
subject to the bandwidth constraint over the holding weights:

\[ w_{bi} - \Delta w \leq w_i \leq w_{bi} + \Delta w, \]

where \( \Delta w \) denotes the maximum allowed deviation from the benchmark weight and the set \( A_i \) defined as

\[ A_i = \left( w_i \mid t^T w_i = 1, w_i \geq 0 \right). \]

The difference here with respect to problem (2.18) is not only technical like changing the decision variable and the constraints. The entire objective has been changed. No longer are we optimizing the active weight vector \( w_a \) in the active return and active risk space irrespectively of the benchmark. In the problem of (2.28) we are optimizing the holding weights vector \( w_i \), which is now benchmark specific. If the benchmark is an equally weighted index with \( N \) elements, then the maximum allowed deviation from any of the \( N \) elements is \( 1/N \) due to the no-short constraint. For the market capitalization weighted index, it is the smallest company weight that defines the bottleneck formed by the no-short constraint. Also, instead of the active risk TEV constraint, the entire portfolio is now constrained according to the holding weights.

The main advantage of the bandwidth constraint over the TEV constraint is that the optimization procedure can improve both the portfolio return as well as risks if the benchmark is inefficient. The reason is that the efficient frontier from the bandwidth optimization is not anchored by the benchmark portfolio. The benchmark inefficiency prompts the procedure to find alternative holdings that dominate the benchmark. Since the efficient frontier is no longer anchored by the benchmark, the benchmark is then an interior point in the convex feasible set enveloped by the efficient frontier. Figure 2.7 summarizes the geometric interpretation of the active portfolio management strategies.

In Figure 2.7, Portfolio B is the benchmark portfolio. Portfolio \( P_1 \) is the tangent portfolio on the bandwidth efficient frontier traced out by repeatedly solving the minimum variance
problem for different return level between $r_{p_i}^{\min}$ and $r_{p_i}^{\max}$. The recursive heuristic is explained below. The TEV efficient frontier is constructed by varying active weight in eq. (2.19) for different values of $\lambda$ with $P_2$ as the tangent portfolio. $P_3$ is the portfolio with the same risk as $P_2$ by leveraging the benchmark B. Only the return difference between $P_2$ and $P_3$ can be attributed to the active manager.

![Figure 2.7: active portfolio management strategies in the mean-standard deviation space. Portfolio B is the benchmark portfolio. Portfolio $P_1$ is the tangent portfolio on the bandwidth efficient frontier traced out by repeatedly solving the minimum variance problem for different return level between $r_{p_i}^{\min}$ and $r_{p_i}^{\max}$. The TEV efficient frontier is constructed by varying active weight in eq. (2.19) for different values of $\lambda$ with $P_2$ as the tangent portfolio. $P_3$ is the portfolio with the same risk as $P_2$ by leveraging the benchmark B. Only the return difference between $P_2$ and $P_3$ can be attributed to the active manager.](image)

The bandwidth efficient frontier is obtained using a heuristic to solve the problem in (2.28). After sorting the return vector in the descending order with the highest expected return at the top, the highest attainable expected portfolio return $r_{p_i}^{\max}$ under the bandwidth constraint is obtained by overweighing the top performers with the maximum exposure financed by the underweight of the poorest performing stocks. By reversing this strategy, we get the lowest attainable expected portfolio return $r_{p_i}^{\min}$ under the bandwidth constraint. Given the optimal weight vectors, the corresponding volatilities follow immediately from
Chasing the global efficient frontier in two steps

the standard formulas. Then, the bandwidth efficient frontier can be traced out by repeatedly solving the standard portfolio risk minimization problem:

\[
\min_w \ w^T \Sigma w
\]

subject to full investment and the return constraint \( w^T \bar{r}_p = \mu \) for \( \mu \in (\bar{r}^{\min}_p, \bar{r}^{\max}_p) \). Note that the bandwidth constraint is no longer relevant because it nests in the return interval.

From sub section 2.2 we know that the TEV efficient frontier in Figure 2.7 can be traced out by changing the coefficient of risk aversion \( \lambda \) in eq. (2.19).

Just as before, Figure 2.7 illustrates that a significant part of the active return in portfolio \( p_1 \) can be obtained by simply leveraging the benchmark portfolio \( B \), i.e. the return difference between portfolio \( p_3 \) and the benchmark \( B \). Jorion (2003) has explicitly shown that this leverage return might account for substantial part of the active return. Implication here for the investor is that the active manager is not earning his fee since the investor can obtain part of the active return by simply selling the risk free portfolio short and leveraging the benchmark with the proceeds.

From sub section 2.3.1 we know that the TEV efficient frontier in Figure 2.7 can be traced out by changing the coefficient of risk aversion \( \lambda \) in eq. (2.19). If no TEV is granted, then the frontier collapses into the benchmark portfolio \( B \). If a TEV constraint of x% is granted, then the resulting TEV efficient portfolio in the MS space has both a higher return as well as a higher risk than the benchmark (portfolio \( p_2 \)). What is unsettling in Figure 2.7 for an investor is that a significant part of the TEV active return in portfolio \( p_3 \) can be obtained by simply leveraging the benchmark portfolio \( B \), i.e. the return difference between portfolio \( p_3 \) and the benchmark \( B \). Jorion (2003) has explicitly shown that this leverage return might account for substantial part of the active return. Implication here for the
An investor who seeks wealth creation should be most interested in the risk-adjusted return of his overall portfolio because the risk-adjusted return or Sharpe ratio precisely indicates how much return he has obtained from each unit of risk taken. Irrespective of the investor's risk behavior, the risk-adjusted return always provides a consistent measure for the obtained performance from different (active) managers (Sharpe (1994)). In the bandwidth efficient frontier case, the overall optimal portfolio performance may be improved upon from both the return as well as the risk dimension, which improves SR dramatically. Yet, the popular TEV efficient frontier seeks outperformance using risky active portfolio elements that usually increase the overall portfolio risk as we have seen in section 2.3.1: for every bit of extra active return there is positive active risk. To control for this possible excessive risk taking behavior in the TEV constrained portfolio optimizations, Jorion (2003) has proposed to add an extra risk constraint to the standard minimum risk problem, which forces the overall portfolio risk to match that of the benchmark. The result is a paradox: To improve the risk-return trade-off, the manager needs substantial decision freedom in terms of tracking error. Yet, ample decision freedom just makes the original objective of controlling the overall portfolio risk at the benchmark level a virtually impossible task. However, this paradox does not exist if the whole portfolio is optimized instead of the active portfolio alone, as we have seen in this section.

### 2.4.3 Bandwidth constrained optimization

Benchmark tracking with bandwidth constraint is a special case of the bandwidth constrained optimization. Instead of allowing the portfolio weight to deviate from the benchmark weight with a fixed bandwidth, the bandwidth constrained optimization allows the portfolio weights to fluctuate within a predetermined bandwidth. For example, if the benchmark weight of a stock is 10%, then a bandwidth of 3% in benchmark tracking with bandwidth constraint allows the weight of the specific stock to fluctuate between 7% and
13%. In bandwidth constrained optimization, a bandwidth of 10% with no-short constraint implies that the stockholding may fluctuate between 0 and 10%. Hence, in benchmark tracking the bandwidth constraint is a relative constraint while it is an absolute constraint irrespective of any benchmark in bandwidth constrained optimization. More formally, the problem in eq. (2.28) remains the same. But the set $A_i'$ is now defined as

$$A_i' = \left\{ w_i | t^T w_i = 1, w_i \geq 0, w_i \leq w_{\text{max}} \right\},$$

with $w_{\text{max}}$ denoting the positive scalar that represents the maximum exposure.

Disconnecting from a benchmark presents a number of advantages for portfolio management. Firstly, the bandwidth constraint presents an explicit demarcation of the feasible set for the portfolio holdings. No longer is the final portfolio weight dependent on the choice of the benchmark. Also, Jagannathan and Ma (2003) has shown that the no-short constraint and an upper limit on the portfolio holdings greatly improves the Markowitz (1952) portfolio selection procedure such that the MV optimization procedure does not underperform the other more sophisticated methodologies. Thirdly and finally, the bandwidth constraint forces the portfolio to be diversified, as there is an equal maximum exposure to each choice in the POS.
2.5 Empirical exposition

This section illustrates the improvement in lower level portfolio performance and the impact of increased communication flow between the different levels on the overall portfolio performance.

2.5.1 Enlarge the investor POS with superior portfolio choices

In absence of estimation errors or any other biases, the utopian case represents the maximum attainable portfolio performance for the investor in the conditional normative context, as all the available information in the choice space has been utilized. Therefore, the utopian solution is the global efficient frontier with no hierarchical decision process. The investor directly solves the allocation problem (2.2) over the securities under the no-short and full investment constraints utilizing all the available information. At the other extreme, the investor can always fall back on the benchmark choice. In the benchmark case the lower-level portfolio weights in eq. (2.5) are either Mcap or EW weighted and thus fixed. The passive portfolio return in the overall level follows from eq.(2.6). Hence the result space of the investor is defined by the utopian and benchmark cases. The effect of decision freedom in terms of tracking error volatility (TEV) minimization and bandwidth optimization on the overall portfolio choice lies within these boundaries. Moreover, by increasing the amount of information communicated from the lower level to the overall level, the distance to the utopian efficient frontier becomes smaller and smaller.

In chasing the global efficient frontier, a sequential bottom-up method is used to construct the overall portfolio using additional information from the lower level. Similar to Black and Litterman (1992), the implicit assumption here is that a lower-level manager possesses superior knowledge and expertise to improve the level specific efficient frontier, and consequently improving the overall level performance. The improvement in the lower level
can be directly attributed to the decision freedom granted to the manager with skill and expertise.

In illustrating the improvement in overall portfolio performance as a result of more communication to the overall level I distinguish between two specific cases in the lower level.

As argued by Jorion (1985, 1986), the global minimum portfolio has the best out-of-sample performance, superior even to the classical tangent portfolio, the tangent portfolio constructed using the Bayes-Stein estimator for the vector of mean returns, and the value-weighted and equally weighted portfolios. Chan et al. (1999) also document that the constrained global minimum variance portfolios outperform the equally weighted portfolios. Thus, in the first specific case the lower level managers communicate the minimum risk portfolio, the classical tangent portfolio and the two benchmarks to the top-level. This choice of the benchmark is partly motivated by the portfolio spanning theory in Kandel and Hubermann (1987) and Cochrane (2001), as the possibility for locally inefficient portfolio choices to produce an efficient choice in the overall level is not zero, a priori. Repeating the procedure for $N$ sub portfolios, we obtain $N$ times 4 portfolios for the overall level choice space. Hence, this is the “$Nx4$ portfolios” case.

In the second specific case, the lower level management must report all the portfolios that span the sub portfolio’s efficient frontier. As such, I strive to utilize all the available information in the lower level. Here I use 20 equal distance portfolios for each sub portfolio starting in the minimum risk portfolio and goes to the maximum return portfolio, and this is called the “$Nx20$ portfolios” case. The primary differences between the two specific scenarios are i) the benchmarks cannot enter the top-level opportunity set in the $Nx20$ case and ii) the $Nx4$ case does not communicate any information around the maximum return portfolio in the lower to the overall level.

2.5.2 The Jobson-Korkie test

The test used to determine whether the maximize Sharpe ratio (SR) tangent portfolios of the specific cases are significantly different to those of the passive benchmark is the
Jobson and Korkie (1981) test. The same test is repeated to test whether the difference between the utopian and the specific cases’ tangent portfolio are significant or not.

The null hypotheses of the Jobson-Korkie (JK henceforth) test states that the Sharpe ratio (SR) of portfolio A is the same as the SR of portfolio B:

\[ H_0 : \text{SR}_A = \text{SR}_B. \]

After transformation, the null hypothesis can be reformulated in terms of the difference between the SR of portfolio A and portfolio B.

\[ H_0 : \text{SR}_A - \text{SR}_B = 0 \]

The test statistics are the sample differences \( \hat{\text{SR}}_A - \hat{\text{SR}}_B = 0 \). Transforming the sample difference using the sample standard deviation \( s \) and sample mean return \( \bar{r} \), the transformed difference for the Sharpe measure becomes

\[ \hat{\text{SR}}_{ab} = s_A \bar{r}_B - s_B \bar{r}_A, \]

with an asymptotically normal distribution. The expectation \( E(\hat{\text{SR}}_{ab}) \) is given by

\[ E(\hat{\text{SR}}_{ab}) \approx (\sigma_A \mu_B - \sigma_B \mu_A) \left(1 - \frac{1}{4T} + \frac{1}{32T^2}\right), \tag{2.29} \]

with the following variance \( \theta \)

\[ \theta = \frac{1}{T} \left[ 2\sigma_A^2 \sigma_B^2 - 2\sigma_A \sigma_B \sigma_{ab} + \frac{1}{2} \mu_A^2 \sigma_B^2 + \frac{1}{2} \mu_B^2 \sigma_A^2 - \frac{\mu_A \mu_B}{2\sigma_A \sigma_B} \left( \sigma_{ab}^2 + \sigma_A^2 \sigma_B^2 \right) \right]. \tag{2.30} \]
\( \sigma_{AB} \) in eq. (2.30) is the covariance between the portfolios and \( T \) is the number of observations in the sample. The test statistic for the null hypotheses is a z-score by dividing the expected SR difference in (2.29) with the standard deviation obtained from eq. (2.30):

\[
 z_{SR} = \frac{E(\hat{SR}_{AB})}{\sqrt{\hat{\theta}}}.
\]

For the resulting pair-wise test statistic, \( z_{SR} \), the null hypotheses of normality can only be rejected at large Type I error level of around 40\% (Jobson and Korkie (1981)).

### 2.5.3 Sample data

From the DataStream database, the monthly total return of 125 actively traded U.S. stocks evenly divided in 5 sectors over the period of December 1990 until May 2002 are extracted. Also, the 3-month T-Bill rate is collected and used as a proxy for the risk free rate over the sample period. The 5 sectors are Aerospace (AEROS), food and drugs retailers (FDRET), life assurances (LIFEA), oil (OIL), and real estate development (RLDEV). A specific manager is assigned to each sector portfolio. These sector portfolios form the lower level and the aggregated portfolio of these 5 sector portfolios is the overall portfolio. The level within each sector is called the sub sector level. To calculate the market capitalization (Mcap) weight of each stock in the sector Mcap benchmark, I have also extracted the market capitalization of each stock over the sample period. Table 2.1 summarizes the descriptive statistics of the excess returns over the risk free rate.

The first striking observation in Table 2.1 is that the difference in Mcap within each sector is vast between the top and bottom company. Secondly, the difference in size between sectors can be large as well. For example, the value of the largest OIL company is more than 14 times the value of the largest real estate company. Thirdly, the ranking order of the average return does not obey the ranking by the company size. The smaller companies
Table 2.1: descriptive statistics of monthly returns for the period of Jan. 1991 - May 2002 of 125 stocks evenly divided over 5 sectors. The market capitalization is the average market capitalization of the stock over the sample period. The stocks in each sector are ranked based on the market capitalization of the stock in a descending order. The sum of the normalized weights of the biggest 5 companies equals 94.03% for “Aerospace”, 73.40% for “Food & Drugs retail”, 75.23% for “Life insurers”, 56.32% for “Oil”, and 63.78% for “Real estate developers”.

<table>
<thead>
<tr>
<th>Sector</th>
<th>Average Market cap (in mln US$)</th>
<th>Mean (%)</th>
<th>Median (%)</th>
<th>Maximum (%)</th>
<th>Minimum (%)</th>
<th>Std. Dev. (%)</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>P-value JB</th>
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For the period of Jan. 1991 - May 2002 (Number of observations (N) = 137)
## Table 2.1 continued.

For the period of Jan. 1991 - May 2002 (Number of observations (N) = 137)

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<th></th>
<th>Average Market cap (in mln US$)</th>
<th>Mean (%)</th>
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<td>9 ALFA</td>
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<td>401.31</td>
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<td>19 CNG.INDS.</td>
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</table>

-68-
Table 2.1 continued.

For the period of Jan. 1991 - May 2002 (Number of observations (N) = 137)

<table>
<thead>
<tr>
<th>Real Estate Developers</th>
<th>Mean Market cap (in mln US$)</th>
<th>Mean (%)</th>
<th>Median (%)</th>
<th>Maximum (%)</th>
<th>Minimum (%)</th>
<th>Std. Dev. (%)</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>P-value JB</th>
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<td>Archstone Smith TST.</td>
<td>1699.25</td>
<td>21.40</td>
<td>17.06</td>
<td>277.76</td>
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<td>Duke Realty</td>
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<td>285.45</td>
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<td>21.84</td>
<td>0.36</td>
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<td>16.75</td>
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<td>20.32</td>
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</table>

belong to the best performers in their sector. Moreover, the sample period has been a very prosperous period for most of the companies. However, there are two stocks ([14] CHIQUITA BRANDS INTL. and [21] DRUG EMPORIUM) in the food and drugs retailers sector with negative average return over the sample period. Furthermore, the sample period has also been very volatile given the large standard deviations of the total return.

At 1% significance level, the Jarque-Bera statistic is insignificant for only 32 stocks. At 5% significance level that number shrinks to 23. Although these observations raise concerns about using the Markowitz mean-variance framework, but our purpose is to illustrate the economic loss in portfolio selection due to the selection bias in the lower level and the aggregation bias at aggregating the stand-alone optimal lower-level portfolios to the overall level: information loss in the decision hierarchy.
Chapter 2               Chasing the global efficient frontier in two steps

2.5.4 Results and discussions

The discussion in this section starts with the impact of the TEV minimization and bandwidth optimization on the lower level portfolio choice efficiency. Then, the discussion expands to the impact of improved communication between the different levels on the overall portfolio choice efficiency.

2.5.4.1 Improvement in lower level portfolio efficiency

Figure 2.8 presents the scatter diagram of the monthly performance of the companies and sector benchmarks over the sample period between January 1991 and May 2002 in the MS space.

Figure 2.8: scatter diagram of the average performance of the 125 companies and 5 sector benchmarks (Market capitalization and Equally weighted) over the period Jan. 1991 - May 2002.
In Figure 2.8, not only is the observation of the two negative average return in the FDRET sector striking, but also the outliers with extreme level of risk from the sector LIFE A and extreme level of returns from the sector AEROS. From Figure 2.8 it seems that top-level decisions based on the aggregated benchmark information is not a bad one, as the benchmark portfolios seem to be globally efficient. Figure 2.9 is a magnified picture of Figure 2.8 around the origin with the sector benchmarks, either equally or Mcap weighted.

**Figure 2.9:** Lower level myopic passive portfolio choices. It is a magnification of Figure 2.8 around the origin with the benchmark index portfolios. The choice space for the investor in the overall level is restricted to the benchmarks: either market capitalization (Mcap) or equally weighted (EW).

As we have seen in eq. (2.19), the active weights are linear in the risk tolerance of the investor. As such, the IR should be constant and the TEV efficient frontiers in the active risk and return space should be linear. In absence of liquidity risk, the investor is much
better served by the manager if the investor just scaled up his investment in the actively portfolio instead of granting more active risk, as it will not yield a higher ex-ante IR. All these expectations are confirmed by the observations from Figure 2.10. From Figure 2.10 it seems that the common TEV of 4% demands massive risk tolerance from the investor as standard risk tolerance is fixed somewhere between 2 and 3 (Sharpe (1994)). The 4% active risk intersection with the TEV efficient frontier is well beyond the lambda value of 8 in Figure 2.10.

Figure 2.10: sector TEV efficient frontier in the active risk and active return space for various level of risk tolerance of the investor.

In the active risk and return space, it is difficult to judge the conventional efficiency of the active portfolios because all active portfolios are optimized regardless of the benchmark. Only after reintroducing the TEV efficient frontier back into the MS space by adding the
active portfolio to a benchmark, we obtain the insight of the active portfolio's efficiency. In Figure 2.10, although the OIL sector has a lower IR than the RLDEV sector, but the difference is not that large to produce a conclusive judgment over the sector active performance. However, after reintroducing the TEV efficient frontier back into the MS space, it becomes painfully obvious that the sector OIL is dominated by the RLDEV sector in terms of both return as well as risk. Figure 2.11 illustrates.

Figure 2.11: TEV efficient frontiers in the MS space based on the market capitalization (Mcap) and equally weighted (EW) benchmarks.

Compared to the OIL TEV efficient frontiers in the MS space, the RLDEV TEV efficient frontiers clearly have a lower level of risk at the same level of return: the risk level of a TEV efficient portfolio in the RLDEV sector is barely half of the risk bore by a TEV efficient portfolio in the OIL sector with identical return. Hence, an investment in the OIL sector...
sector is clearly inefficient for the investor when compared to an investment in the RLDEV sector. Therefore at overall portfolio selection, the investor must seek the efficient investment opportunities, as a lower level manager is powerless to alter the overall level choice. The manager of the OIL sector may have realized the maximum of what is possible within the OIL sector, yet in this case the overall portfolio that overweighs the OIL sector will underperform an overall portfolio that allocates the RLDEV sector an overweight.

Another interesting observation from the results in Figure 2.11 is that the benchmark choice is not a straightforward affair. The reason is that the equally weighted (EW) index does not always dominate the market capitalization (Mcap) weighted benchmark index or vice versa. In the specific OIL sector, the size effect reported by Banz (1981) and the size premium associated with small companies (Fama and French (1993)) do not guarantee a dominating equally weighted index. A possible explanation may lie in the economic condition that an oil company must be big enough to extract oil cost effectively and thus capable to produce a positive return.

Figure 2.11 also verifies the claim in section 2.3.1 that the benchmark with zero active risk does not have to be the global minimum risk portfolio\(^{17}\). Due to the covariance structure of the investment opportunities in each sector, a reduction in return sometimes also reduces the level of risk. The additional observation here is that when the benchmark is not the global minimum risk portfolio, they are located very close to the global minimum risk portfolios.

If there is a no-short constraint on the holding portfolio in the MS space, which contains a benchmark with an active portfolio, then the constraint may be violated if too much decision freedom in terms of TEV is granted. This is because an active portfolio contains short positions, which may be so massive due to too much decision freedom such that it drags the benchmark holding into negativity. On the other hand, the full investment constraint is never violated for the holding portfolio as the active portfolio was, and will
always remain self-financing by definition. The TEV efficient frontier under the no short constraint is plotted in Figure 2.12 together with the bandwidth efficient frontiers and the dashed unconstrained utopian efficient frontier for the Mcap case. Figure 2.13 contains the frontiers for the EW case.

Figure 2.12: for the market capitalization (Mcap) benchmarks, the solid frontiers are the bandwidth efficient frontiers with 8% bandwidth decision freedom. The dashed frontier is the unconstrained utopian efficient frontier. Unobservable, the benchmark squares also contain the TEV efficient frontier under the no-short constraint. For each sector, the TEV efficient frontier collapses into the benchmark under the no-short constraint.

Under the no-short constraint the TEV efficient frontier collapses into the benchmark in the Mcap case. In the EW case it shrinks so much that it almost collapses into the

Each portfolio on the efficient frontier is the minimum risk portfolio at that particular return level. Global minimum risk portfolio is the portfolio with the lowest absolute value of risk amongst the minimum risk
benchmark. Compared to the bandwidth efficient frontier, the inefficiency of the TEV optimization due to Mcap benchmark choice follows directly from Figure 2.12. Clearly visible in Figure 2.12 is also the fact that the bandwidth efficient frontier is not anchored by the benchmark. The TEV efficient frontier in this case under the no-short constraint can be regarded as the benchmark choice. Clearly, the bandwidth efficient frontier is capable to reach the unconstrained utopia efficient frontier if the allowed bandwidth is large enough.

Figure 2.13: For the equally weighted (EW) benchmarks, the solid frontiers are the bandwidth efficient frontiers with 8% bandwidth decision freedom. The dashed frontier is the unconstrained utopian efficient frontier of the sector. For each sector, the TEV efficient frontier lies very close to the benchmark under the no-short constraint.
Clearly observable in Figure 2.12 and 2.13 is that the bandwidth efficient frontier of some sectors obtained under the bandwidth constraint of 8% approaches the utopia frontier near the global minimum risk portfolio. If a risk-averse investor grants full decision freedom to his managers and the managers only report the portfolios with the highest return and consequently highest risk, then the investor has been forced to choose between bad and worse. Thus, to any risk-averse investor, it is never advisable to run an unconstrained optimization especially in the HPM where the investor in the overall level only gets to see part of the POS via the managers.

Next, the impact of an expanded POS because of more communication between the lower and overall level on the overall portfolio selection is presented.

2.5.4.2 Impact of increased communication between the decision levels

Figure 2.9 contains the full POS for a passive investor who is only investing in the benchmark portfolios. After adding the minimum risk and maximum Sharpe ratio tangent portfolios to the picture, Figure 2.14 presents the new POS for an investor who allows for active portfolio management. The improvement in efficiency in the overall portfolio lies in the discovery of the portfolios to the left of the benchmarks. If the investor had chosen the passive benchmarks, then these portfolios would never have been selected in the overall portfolio. The improved overall portfolio is now a convex combination of all the portfolios in the POS illustrated in Figure 2.14. Now, what if we increase the amount of information even more by communicating the entire bandwidth efficient frontier in the lower level to the overall level?
Figure 2.14: lower level myopic active portfolio choices. Besides the benchmark portfolios in Figure 2.9 there are also the maximum Sharpe ratio tangent portfolio and global minimum risk portfolio of the bandwidth efficient frontier with a bandwidth of 8%.

What are missing in Figure 2.14 are the more extreme choices in the lower level. The portfolio choices with a return higher than the maximum Sharpe ratio portfolio are absent in Figure 2.14. Depending on the curvature of the efficient frontier in the lower level, the additional information carried to the overall level also include the area under the efficient frontier that exceeds the convex combination of the minimum risk and the maximum Sharpe ratio portfolio. Figure 2.15 illustrates the POS of the investor who also contemplates the entire lower level bandwidth efficient frontiers.
Figure 2.15: expansion of Figure 2.14. Lower level myopic active portfolio choices including the benchmarks and the bandwidth efficient frontier with 8% bandwidth decision freedom. The dashed lines are the bandwidth efficient frontiers based on the equally weighted benchmark while the solid frontiers are based on the market capitalization weighted benchmark when using the bandwidth optimization procedure.

The impact of increased information exchange to the overall portfolio is quite dramatic as illustrated in Figure 2.16. Different to Figure 2.15, the efficient frontier in Figure 2.16 represent portfolio choices in the overall level. At one extreme of the choice spectrum are the benchmark choices while the utopian unrestricted efficient frontier represents the maximum attainable efficient portfolios.
Figure 2.16: single-period portfolio choice in the overall level. The space between the global efficient frontier tangent portfolio with the highest Sharpe ratio (SR) and market capitalization weighted benchmark tangent portfolio is the space for improvement in portfolio performance. The tangent portfolios of the specific cases (5x4 and 5x20) approach the maximum attainable SR tangent portfolio, which shows the importance of i) the use of disaggregated information in the lower level and ii) more extensive communication between the different levels. Clearly, the monthly improvement in terms of portfolio risk and return is large enough to merit the efforts.

Clearly observable in Figure 2.16, the overall efficient frontier based on 20 portfolios with a bandwidth of 8% approaches the utopian frontier and it converges to the utopian case for cases at higher level of risk. If we take the tangent portfolio of the Mcap benchmark efficient frontier as the new origin, then the distance between the intersections of the axes with the utopian frontier and the origin represent the space for improvement. In terms of return, the monthly improvement equals 1.15% or almost 14% annually. In terms of risk, the monthly reduction is about 2.74% or approximately 9.49% per year. The solid line tangent to the global efficient frontier defines the maximum Sharpe ratio portfolio while
the other solid line tangent to the Mcap benchmark efficient frontier defines the minimum. The dashed tangent lines of the 5x4 and 5x20 portfolios have greatly improved the efficiency of the overall portfolio. Moreover, the ranking of the dashed lines is consistent with our view that more information and thus more diversification opportunities inevitably improve the portfolio efficiency.

The last relevant observation from Figure 2.16 is that HPM still demands a price in terms of risk and return, as there is a gap between the global efficient frontier and the specific cases. In the 5x4 case, the price paid in terms of risk at the global minimum risk portfolio is 0.87% per month or about 3% per annum. The price in terms of monthly return equals 0.60% or 7.23% on the annual basis. In the 5x20 case, the price at the global minimum risk portfolio is considerably lower: 0.65% in risk on a monthly basis or 2.25% per annum and 0.20% in return on the monthly basis or 2.36% per annum. These observations are not surprising since the staged optimization procedures still utilize aggregated information, whereas the global utopian efficient frontier uses all the available information.

2.5.4.3 Jobson-Korkie test results

The z-test statistic in the Jobson-Korkie (JK) test examines whether two tangent portfolios that maximizes the Sharpe ratio generates significant different return for a specified sample period. In Jobson and Korkie (1981) the normality of the z-test statistic for the Sharpe measure cannot be rejected for any level of significance. If the test statistic is significant at a significance level, then the null hypotheses of zero difference between the Sharpe ratios is rejected for the corresponding significance level. The Table 2.2 summarizes the JK test statistic for 6 pair-wise comparisons of the four Sharpe tangent portfolios in Figure 2.16.

At the 5% significance level, the null hypothesis of identical SR portfolios is rejected for all the pair-wise comparisons. At 1% significance level, the null hypothesis cannot be rejected for the comparison between the 5x4 and 5x20 SR portfolios. Thus, the SR portfolios generate significant different return pattern for the sample period. This is not a
Chasing the global efficient frontier in two steps

strange observation, as all the portfolios are constructed differently that are only optimal at certain point in time.

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**Table 2.2:** the Jobson Korkie (1981) \( z \)-statistics for the four tangent portfolios maximizing the Sharpe ratio (SR) over the sample period of Jan.-1991 until May-2002. The test statistic shows whether the two tangent portfolios are significantly different from each other over a sample period. A positive sign indicates there is outperformance. Take for example the number 6.24 in the Utopia row and Mcap column, the number is significant at the 1% level and it indicates that the Utopia SR have on average outperformed the Mcap SR over the sample period of Jan.-1991 until May-2002.

*: Significant at 5% level. **: Significant at 1% level.

Naturally, the Utopia SR with its full information set has outperformed all the other SR choices while the benchmark has underperformed. The interesting observation in Table 2.2 is the 5x4 portfolio outperforming the 5x20 choice. It seems that over the sample period between January 1991 and May 2002, the choice of abandoning the Mcap and EW benchmarks was not a good idea. In the cross section of the decision period, it was self-evident from Figure 2.16 that the passive benchmark choices are inferior choices.

However, over the entire sample period the MV optimal choices in the decision period are not always the better choice when compared to the passive benchmarks.

Hence, the natural extension is to test the time persistency of the current approach. This is precisely the topic of Chapter 3: the multi-period portfolio choice.


2.6 Concluding remarks

The magnitude of economic loss due to HPM can be substantial. On the annual basis, the difference in terms of portfolio return and risks can amount to a double digit percentage, as illustrated in the empirical exposition.

This is not surprising because, in HPM, the managers are mandated to track the benchmark. The POS of the investor only holds benchmarks and all the rest of the opportunities are ignored. Theorem 2.2 states that the EW benchmark can never be MV optimal when there are more than two investment opportunities in a sub IOS, which implies that there must be at least one portfolio that dominates the EW benchmark in that sub IOS. Hence, together with the results presented in Haugen and Baker (1991) and Wilcox (1994), the investor should allow for active portfolio management in the lower level to find the portfolio choice that dominates the benchmark choice.

In active HPM, the lower-level TEV minimization strategy first mandates the manager to find an optimal self-financing active portfolio of the sub IOS irrespective of the benchmark. Then the active portfolio is added to some existing benchmark. As it turns out, the ex-ante information ratio (IR) of the optimal active portfolio is constant given a sub IOS. It seems to be futile to allow for more decision freedom in terms of TEV, as the ex-ante IR of a sub IOS will remain constant by construction. In the MV space, the efficient frontier based on TEV minimization is only MV efficient if and only if the chosen benchmark is also MV efficient (see e.g. Roll (1992), Jorion (2003) and Scherer (2004)). Unfortunately, the TEV efficient frontiers based on EW and Mcap benchmarks are not MV efficient given the results of Theorem 2.2 and Haugen and Baker (1991).

Another severe problem with TEV minimization is the possible violation of the no-short constraint when the active portfolio reenters the MV space. Although the active portfolio is self-financing by construction, but the short positions in the active portfolio may be so large such that the entire holding weight becomes negative. In the empirical exposition, the TEV efficient frontiers collapse into the benchmark without any exception after imposing the no-short constraint in the MV space. Hence, these observations raise
doubt about the contribution of the TEV minimization approach to wealth creation for the investor.

However, the lower-level portfolio efficiency improves substantially when the bandwidth optimization procedure is used. The portfolio weight deviation from the benchmark weight is constrained by a pre-selected bandwidth and the bandwidth optimization is directly conducted in the MV space. Hence, the active portfolio is benchmark specific and always obeys the no-short constraint. The bandwidth efficient frontier can shift to the left upper corner in the mean-standard deviation space, which implies increase in portfolio return and reduction in the portfolio risk. The reason is that the bandwidth efficient frontier is no longer anchored by the benchmark as in the TEV minimization case.

The overall portfolio efficiency improves with respect to the passive benchmark tracking case after communicating more and better portfolio choices to the overall level. The improvement in the overall portfolio performance of the 5x4 and 5x20 special cases as a direct consequence of bandwidth optimization is so large that it is difficult to ignore. However, when compared to the utopian case, there is still room for improvement around the global minimum risk portfolio.

It seems that to mitigate the economic loss due to HPM, the managers must be granted decision freedom to deviate from the benchmark holding restricted by a bandwidth constraint. The managers must report as much as possible the available portfolios in his sub IOS to enlarge the POS of the investor. The IPOS not only holds more portfolio choices, but also more superior ones. Hence, the overall portfolio based on the enlarged IPOS should also yield better performances for obvious reasons.

Despite the inefficiency of the benchmarks, I remain to be an advocate of using them in performance assessment because it is i) an reference point for measuring portfolio performance and ii) an objective measure to compare the managers’ performance. This chapter illustrated how the benchmark performance can be improved upon in the single period setting using simple optimization procedures. Next, the logical extension of the research is to explore whether the improvement is also persistent through time.
3

Improving HPM in the multi-period context

In portfolio management practice, nothing is more expedient than generating returns while keeping the risk exposures within the strategic boundaries. In the case with benchmark tracking portfolios with limited decision freedom the focus is on how to fully utilize the given decision freedom to generate extra returns. One possible motivation is the cost aspect, which is intimately linked with implementing the benchmark tracking strategy. Chapter 2 has illustrated how extra return can be added to the overall portfolio while mitigating its risk level in a single-period context. Chapter 3 extends the results of Chapter 2 to a multi-period context such that insights into the workings of the two-step optimization procedure through time can be obtained. Hence, this chapter focuses on research question 3.

The main purpose of this chapter is to present an example that illustrates how an additional return premium can be realized if the portfolio is periodically rebalanced, even after transaction cost within the standard mean-variance (MV) framework. The reason against active portfolio management in practice is that periodically rebalancing of the portfolio can be costly. The additional return, if any, can be nullified by the additional costs. In the worst case scenario, the cost may reduce the additional return premium to a negative premium. Hence, passive benchmark tracking seems to be the better choice. However, there is evidence in the literature that shows the return and risks can be very different in different states of the economy, or business cycle regimes, and its impact on asset allocations can be substantial (see e.g. Ang and Bekaert (2002, 2004), and Guidolin and Timmermann (2005, 2006, 2007)). This changing volatility structure through time is also substantiated by the volatility clustering effect observed in the longitudinal return data (see e.g. Engle (1982),
Chapter 3 Improving HPM in the multi-period context

Bollerslev (1986), and Granger and Machin (2002)). In a recent publication, van Binsbergen et al. (2007) illustrates that portfolio selection may change and become quite complex when the prices of risk vary over time.

Following Sharpe (1981), Elton and Gruber (2004) and van Binsbergen et al. (2007), this chapter gives an illustrative example of the impact on portfolio return by periodically rebalancing the portfolio composition. In light of the HPM, this chapter keeps the hierarchy component to its minimum. There are only two levels in this chapter. The overall level portfolio is the investor portfolio and this portfolio is divided into three regional indexes and one country index due to home bias. The benchmark tracking strategy is the strategy that invests in the four indexes. The active strategy dissects the Europe regional index and invests in the country indexes within the Europe index. By doing so, additional information is communicated to the overall level such that more combination than the Europe index can be realized within the investment hierarchy. The additional advantage is that the volatility structure is also increased such that more risk reduction opportunities can be utilized.

This chapter presents a portfolio management example that illustrates the additional net premium that can be gained after accounting for the changing volatility structure, even within the simple MV framework. As such, this chapter serves as an illustration of whether the economic loss due to the changing economic states is a serious problem or not for the investors.

From the results it seems that periodically rebalancing the portfolio not only adds value in good times when the stock markets are generally performing well, but also during the bad times. Using the *bootstrap* method (see e.g. Efron (1979), Efron and Gong (1983), and Davison and Hinkley (1997)), the average outperformance over the benchmark is approximately 21% over a sample period of three and half years with a maximum outperformance of around 54% while only 127 cases out of the 10,000 scenarios have underperformed the benchmark. The average net underperformance is approximately 3.85% with a maximum of around 24%. The implication from these results is that passive
portfolio management within HPM costs the investor money. That is, given the set of data used for this study and the associated bootstrapping exercise.

The remainder of this chapter proceeds as follows. Section 3.1 describes the details of the multi-period models and decision framework. The models for sensitivity testing are listed in section 3.2. Section 3.3 presents the sample data of country indexes and transaction costs used in the multi-period models. The relevant findings of each model are in section 3.4 together with its discussion and interpretation. Section 3.5 contains the discussion and interpretation of the sensitivity analyses results. At the end, section 3.6 concludes and gives a concise summary of the main findings.

3.1 Towards the multi-period context

This section presents a simple three-period model to create intuition and sets the stage for the multi-period rolling window model of section 3.2. The main objective of this section is to show the properties of the benchmark portfolio and the (efficient) alternatives during different periods with different economic states: recession, revival and the entire period. The implications of the observations are i) the IOS is time-varying and ii) the fixed benchmark holdings based on aggregated information yield suboptimal portfolio performance.

To avoid problems like survivorship bias (see e.g. Brown et al. (1995)) and selection bias in the stocks selection, this chapter abandons the randomly chosen database of 125 U.S. companies of Chapter 2 and uses the MSCI country indexes instead. Another motivation for using the country indexes is that it allows the investor to invest internationally.18

Sub section 3.1.1 starts with the detailed description of the three-period model. Then, sub section 3.1.2 illustrates the bottom-up portfolio selection process. At the end, the size

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of the discrepancy in portfolio performance between the benchmark choice and the bottom-up choice is summarized in sub section 3.1.3.

3.1.1 Splitting the sample period

The first model divides the entire period into two sub periods: a recession and a revival period because for the first part of the sample period a downward slope can be detected in all the country indexes while a upward slope is clearly visible for the rest of the sample period as Figure 3.1 illustrates.

![MSCI country and regional indexes](image)

**Figure 3.1:** MSCI country and regional indexes for the period between November 2000 and October 2005

Although most of the country indexes have reached their bottom in March 2003, but for simplicity of exposition I have taken August 2003 as the cut-off point that divides the recession period from the revival period. At least in August 2003 we can be certain that the
most indexes have entered the revival period with an upward trend. Data details can be provided upon request.

For each period, the expected return vector and covariance matrix obtained from the historical data is used to construct the benchmark portfolio. The MV efficient frontier of each period is the set of solutions of the minimization problem of (2.2) for different levels of expected return.

The same procedure is repeated for the entire period, which forms the third state of economy. The reason that the sum of the two sub periods is taken as the “third” state of the economy is because the average risk and return characteristics may result in a totally different portfolio allocation, which underscores our main message: the investment opportunity space changes through time and fixed benchmark weighting schemes may not be the best choice.

The three single-period models demonstrate the merits of portfolio optimization in asset allocation when compared to tracking a fixed mixture of the elements in the IPOS: a benchmark. These performance differences are indicative for the different strategies’ effectiveness in dealing with the changing market environment. Later, an extra restriction is added to the optimization procedure in the spirit of bandwidth constrained optimization of Chapter 2 to ensure portfolio diversification: each country index’s holding within the optimal portfolio must remain below the 10% threshold. Thus, the optimal portfolio holds at least 10 country indexes. This is a common procedure employed in practice to ensure portfolio diversification and thus restricting impact of estimation error.

3.1.2 Illustrating the bottom-up portfolio selection process

This section gives the 19 MSCI country and regional indexes used in the benchmark and provides an illustration of the two-step bottom-up portfolio selection method explained in section 2.3.1 in the multi-period context.
Chapter 3       Improving HPM in the multi-period context

The benchmark in our illustration invests in 19 MSCI country and regional indexes. The composition of the benchmark is 20% in the U.S., 33% in Europe excluding The Netherlands, 12% in the Far East with the rest of the portfolio, 35%, invested in The Netherlands. Hence, the lower level is composed of 3 regional and one country index. The MSCI Europe index contains 16 countries: all the Euro countries except Luxemburg plus Switzerland, U.K. and the Nordic countries of Denmark, Norway and Sweden.

The MSCI Europe index is only one possible combination of the 16 country indexes. As explained before, it makes sense to break the Europe regional index up to explore other alternative combinations of the country indexes. Thus, we add the 16 country indexes to the 3 regional indexes.

The total return index of all the relevant indexes is collected for the 67 months of the total sample period (Nov. 2000 - May 2006). The total sample period is divided into two non-overlapping periods of equal length. The recession period is the first 33 months (Dec. 2000 – Aug. 2003) and the economic revival period is the following 33 months (Sep. 2003 - May 2006). Thus, there are 66 return observations in the entire sample period (December 2000 - May 2006). Figure 3.2 shows the expected return and volatility of each index in the MS space for three periods.

In Figure 3.2, the first striking observation is that the aggregated regional indexes used in the benchmark, on average, have performed relatively poorly over the three periods. Also, Figure 3.2 illustrates the fundamental problem associated with using aggregated information: the top performing country indexes like Austria or Norway would never have been discovered if one only considers the regional indexes. In turn, this selection bias may induce a significant opportunity cost. Thirdly, the average performance of the country indexes fluctuates over different periods and under different market conditions: all ships rise when the tide comes in. Indeed, the indexes have far better performance during economic revival period relative to the recession period. Finally in Figure 3.2, the average performance of individual country index changes over time. For example, during the recession period investing in Austria has the lowest risk. But that no longer holds in the revival period.
Figure 3.2: Mean-Variance (MV) performance of the indexes over the sample period (Dec/2000 – May/2006).
Chapter 3       Improving HPM in the multi-period context

From Figure 3.2 it seems that i) better choices than the regional indexes are hidden in the regional indexes, ii) given the observed performances, it is expected that the benchmark choice based on the regional indexes is an inferior choice, and iii) following a fixed composition like a benchmark throughout implies forfeiting opportunities to reduce portfolio risk or enhance portfolio return. Therefore, the expectation is that by first enlarging the IIOS of the investor and then optimizes the overall portfolio choice, a far better portfolio than the benchmark can be obtained for the investor.

Table 3.1 contains the correlation matrix between the different country and regional index returns. Panel A holds the correlations between the index returns during the economic recession period (Dec. 2000 - Mar. 2003). Panel B captures the correlations in the revival period (Apr. 2003 - May 2006), and panel C contains the correlations over the entire period (Dec. 2000-May 2006).

From Table 3.1, the correlations between the indexes are clearly not constant during different periods under different market conditions. A fix-mix buy and hold strategy can never be optimal from a risk minimization perspective as the correlations between the countries indexes are heterogeneous. Additionally, the only negative correlation observed in Table 3.1 is the correlation between the country index of Austria and Finland. If only the aggregated MSCI Europe index was used in constructing the optimal portfolio, then this risk reduction opportunity would have been ignored and consequently lost. Finally, although the correlations are heterogeneous over time, the numbers in Table 3.1 suggest that the correlations between the MSCI equity indexes are very high during both the recession and revival periods. The high correlation numbers over the entire period substantiate this observation.

A preliminary worrying conclusion is that portfolio risk reduction is a difficult task due to high and positive correlations. Risk reduction must come from somewhere else.

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3.1.3 The portfolio performance in the single-period models

The three single-period models are conceived to explore the performance of the benchmark and the possible opportunities within the IOS within different period. Hence, the efficient frontier in each sub period is compared with the performance of the benchmark choice. Figure 3.3 summarizes the efficient frontiers and the benchmark portfolios of different period in the mean-standard deviation (MS) space.

![Efficient frontiers & fix-mix benchmark portfolios during different states of the economy](image)

**Figure 3.3:** the benchmark and the efficient frontier during recession, revival and the whole period.

From Figure 3.3 it seems that portfolio optimization can offer superior asset allocation than the benchmark in any state of the economy. The general observation in Figure 3.3 is that portfolio optimization can improve the portfolio performance over the cross-section of the stock risk and returns. More specifically, portfolio optimization could have kept the portfolio in the profit zone, even during recession. During the other periods, even the minimum risk portfolio (MRP) on the efficient frontier has outperformed the benchmark portfolio of the same period in terms of both risk and return. Figure 3.4 illustrates the differences on a monthly basis.
On an annual basis, the numbers show the true extent of the impact of portfolio optimization. As shown in Table 3.2, the actively managed portfolio outperforms the fix-mix benchmark in terms of standard deviation and expected return.

Table 3.2: monthly and annualized difference in risk and return between the fix-mix benchmark portfolio and the minimum risk portfolio on the efficient frontier, as illustrated in Figure 3.3.

The results in Table 3.2 suggest that the portfolio performance can be improved upon by using disaggregated information and simple MV optimization procedures to optimize the portfolio in different states of the economy. Of course, the procedure is only interesting when it produces ex-ante portfolio composition that yields ex-post abnormal portfolio
return superior to the benchmark return. Another fundamental difference between the MRP on the efficient frontier and the fixed mixture of the benchmark portfolio is the portfolio composition: as Figure 3.5 shows, the composition of the MRP during different periods is far from constant and fixed.

**Figure 3.5:** The portfolio composition of the minimum risk portfolio on the efficient frontier during different state of the economy.

The efficient minimum risk portfolio in Figure 3.5 only invests in 4 country and 1 regional index during the different periods. The bad news for the benchmark holder is that the efficient portfolio choice only holds 1 regional index and The Netherlands country index never enters the efficient holding portfolio. However, the MRP in Figure 3.5 does have a concentrated holding, which may be viewed as undesirable by some investors. Thus, an extra constraint is needed to control the maximum country or regional allocation. Here an arbitrary ceiling of 10% is chosen. By doing so we can mitigate some of the concentration problem and guarantee portfolio diversification as the portfolio holds at least 10 countries and regions. Figure 3.6 shows the efficient frontiers and the benchmark in the original setting and the new efficient frontiers under the 10% constraint. Note that the benchmark portfolio has violated the 10% constraint as it invests 35% of the total portfolio in The Netherlands index.
Hierarchical portfolio management

Figure 3.6: the benchmark and the efficient frontier under the 10% country and regional constraint during the three periods: Recession, revival and the whole period. The solid thick black line is the efficient frontier without the 10% constraint during revival with the black square representing the performance of its benchmark. The efficient frontier restricted by the 10% constraint is the thin solid black line in between. Similar convention holds for the other two periods.

The 10% upper limit to the country and regional holding restricts the full exploitation of the information contained in the dataset. Thus, certain investment choices, which were available before, are no longer part of the opportunity space. Consequently, we observe both a change in the shape of the efficient frontiers and a shift of the efficient frontiers in the MS space. Also, the loss of investment opportunities has led to less abnormal return above the benchmark return in Figure 3.6. In particular, the 10% holding restriction is so stringent for the recession and the entire period that the maximum attainable return under the 10% constraint is lower than the minimum efficient return in the unconstrained case. Moreover, the 10% efficient frontiers are less sensitive to risk alterations. Finally, the reduction of portfolio risk in Table 3.3 has suffered due to reduction in the opportunity set and is much less significant as in Table 3.2.
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Table 3.3: monthly and annualized difference in risk and return between the benchmark portfolio and the minimum risk portfolio on the efficient frontier with the 10% holding constraint, as illustrated in Figure 3.6.

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Table 3.4: risk and return intervals for the different cases.

-102-
In Table 3.4, the left column summarizes the risk and return intervals for the case without the 10% holding constraint. The upper part presents the monthly risk and return intervals for the different periods and the bottom part the intervals on an annual basis. The right column presents the numbers under the 10% weight constraint.

The numbers in Table 3.4 show the fundamental problem with index tracking under constraints. If the constraints are taken without the full knowledge of their effects, then the constraints may be more restrictive than one has bargained for. In the current case, the extra 10% constraint has severely reduced the investment opportunity set of the manager and hence the expected outcome space containing all the ex-ante expected portfolio performance. For example, during the recession period, no positive return can be realized due to the 10% holding constraint. However, the portfolio composition has become more diversified as was initially intended. During each period there are at least 11 indexes in the portfolio as shown in Figure 3.7.

![Composition of the minimum risk portfolio on the efficient frontier with 10% upper limit for each country and regional index](image)

**Figure 3.7:** The portfolio composition of the minimum risk portfolio on the efficient frontier during different state of the economy with 10% upper limit for each country and regional index.

In Figure 3.7, the regional indexes are forced into the holding portfolio through the 10% holding constraint. As the optimal positions are exhausted, the lesser choices are also
Chapter 3       Improving HPM in the multi-period context

considered and bought into the portfolio. There are two drawbacks here. Firstly, the inferior performance as a result of the constraint is obvious. Secondly, the extra transaction costs due to buying more country indexes is another less appreciated feature of such a strategy.

3.2 The multi-period context

In this section, the cross-sectional results of Chapter 2 using the bottom-up portfolio selection process and disaggregated information are validated in the multi-period context. The multi-period model of choice is the rolling window model. Sub section 3.2.1 presents the details of this model. Then, periodically rebalancing in the multi-period context introduces additional transaction cost. Sub section 3.2.2 provides the detailed description of the transaction cost scheme used in the rolling window model. Finally, sub section 3.2.3 gives the results of the rolling window model of the observed return sample. The sensitivity analysis of the rolling window results are presented in Section 3.3.

3.2.1 Rolling window

The multi-period model chosen in this chapter is the rolling window method and it illustrates the impact of monthly rebalancing on the investor’s total wealth. Figure 3.7 gives a sketch of the rolling window model.

In Figure 3.8, the historical average return vector and covariance matrix calculated based on an estimation period of length \( t \) is used to construct the minimum risk portfolio\(^{19} \left( P^*_t \right) \) under the 10% holding, no-short sales and the budget constraint. The rest of the total sample period, \( T - t \), is the testing period. The ex-post performance of the minimum risk

\(^{19}\) The minimum risk portfolio is taken as the optimal portfolio here. Other choices on the efficient frontier were not excluded from the choice space.
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portfolio is calculated based on the realized return and realized risk in month $t$. The passive benchmark holding at time $t$ ($B_t$) is held constant over time. The fix-mix benchmark performance follows from the realized return and covariance matrix in month $t$. Thus, any deviation in the fixed mixture of benchmark holding as a result of return realizations in period $t$ automatically induces rebalancing in the benchmark holdings. The goal is to keep the fixed mixture of the benchmark holdings constant. The entire process is repeated for each month in the sample period with a fixed estimation period of length $t$.

![Figure 3.8: the rolling window method.](image)

In the empirical exposition, a 24 months estimation period has been chosen. Given the country and region indexes, at least 20 observations are needed to construct the invertible covariance matrix for the MV-optimization. To include more data points in the estimation period is a waste of valuable information points. Also, by taking bigger periods we will have averaged out the extreme events in the data, which is undesirable. Hence, the choice has fallen upon a 24 month estimation period.

In the rolling window multi-period model, the cumulative return after transaction cost of the different strategies over the entire sample period is used as the measure to assess portfolio performance. The MSCI world index cumulative return is also added as an
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The expectation is that the actively rebalanced portfolio should have a lower risk, and preferably a higher return, when compared to the passive fix-mix portfolio in the sample period.

The cumulative return of the actively managed portfolio should not only outperform the fix-mix benchmark cumulative return in a booming market, it should also incur lower losses when the market sentiment turns. The reason is clear-cut: the actively managed portfolio is constructed based on the minimum risk principle. As it turns out, the MSCI index also has an inferior cumulative return compared to the actively managed portfolio. This is significant because the MSCI index portfolio only incurred a cost when the investor initially bought the index portfolio. Thereafter, no costs were subtracted from the index return, as no information was available concerning index updates. Hence, the MSCI cumulative index return is presumably overvalued. To be able to outperform an overvalued index benchmark just shows the potential of the proposed active strategy.

Defined as the difference between the expected and the realized value, the forecast error remains a point of concern. In this chapter, the expected risk and return of an investment opportunity are calculated based on its historical information. Clearly, much can be improved in the naive forecasting model. However, forecasting error is not the central point and message in this dissertation. Although intertwined, the objective of this dissertation is to show the effects of using disaggregated information set in portfolio selection, ceteris paribus. If both disaggregated data and better forecasting model is applied, then attribution of the superior results to the improvements of the model is less clear-cut.
3.2.2 Transaction cost

The transaction cost data come from a Dutch Internet brokerage bank: Binck bank\textsuperscript{20}. Table 3.5 summarizes the fee charged per transaction in the different markets of the world. In Table 3.5, the fee in each market used at rebalancing is the highest, which implies that the resulting active return after transaction costs is lower than the "real" value. Even though, the return above benchmark from active management remains significantly positive. Obviously, the institutional investor can negotiate for even lower tailor-made fee schemes using volume leverage and economies of scale.

<table>
<thead>
<tr>
<th>Stock markets</th>
<th>Transaction fees</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euronext and Eurex</td>
<td>8 Euros + 0.1%</td>
</tr>
<tr>
<td>Amsterdam, Brussels, Paris and Lisbon</td>
<td></td>
</tr>
<tr>
<td>Euro markets</td>
<td>15 Euros + 0.15%</td>
</tr>
<tr>
<td>Dublin, Frankfurt, Madrid, Milan and Vienna</td>
<td></td>
</tr>
<tr>
<td>U.S. Markets</td>
<td>US$15 + 0.15%</td>
</tr>
<tr>
<td>NYSE, Nasdaq and AMEX</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.5: single trip fee per transaction for the major stock markets in the world.

Also in Table 3.5, the transaction costs on the U.S. markets are in U.S. dollars. As such, there are currency exchange concerns. During the period between Nov. 2002 and May 2006 the exchange rate of Dollars per Euro has fluctuated between 0.9871 (02 Dec. 2002) and 1.3633 (28 Dec. 2004).\textsuperscript{21} As such, the fixed cost of U.S.$15 per transaction has fluctuated between € 11 and € 15.20. Using the maximum of € 15.20, the cumulative return at the end of the sample period only decreases by a few hundred euros, less than a third of a percentage point. The transaction cost scheme applied here is the highest fee scheme. Therefore, the after costs risk adjusted return reported here is also the most conservative estimate. When the volume aspect is taken into account, the resulting

\textsuperscript{20} It is an internet broker under the supervision of the Dutch central bank and the Dutch Authority of Financial Markets (Autoriteit Financiële Markten) with over 45,000 clients with entrusted funds of well over € 2.1 billion (http://www.binck.com/nl/welkom/over_binckbank/).

\textsuperscript{21} European central bank: http://sdw.ecb.int/browse.do
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transaction cost should be lower than the costs already taken into account here in this chapter.

At inception of the MSCI world index tracking portfolio, the passive investor pays a front load fee of 0.18%.\textsuperscript{22} The additional annual fee to cover the costs of holding the MSCI world index-tracking fund, the expense ratio, is neglected. The reason is that the other portfolios also have to pay such a holding fee. By excluding it altogether does not change the relative performances between the different portfolios. Comparison between the different portfolios still holds and remains consistent. Also, any changes in the MSCI index composition with the resulting rebalancing costs were neglected due to lack of reliable information. Hence, the net cumulative wealth of the MSCI world index tracking fund is probably higher than the real wealth had the costs been properly taken into account.

\subsection{3.2.3 The rolling window results}

In assessing the portfolio performance I use 5 criteria: 1) the absolute cumulative return of each strategy; 2) The return differences between the strategies, both absolute as well as relative difference; 3) Transaction costs; 4) Discrepancy between realized and expected return, which is the forecast error; 5) The hit rate, which is defined as the number of month of outperformance over the benchmark.

The first three criteria are straightforward. The forecast error criterion is used to establish the forecast power of each strategy, which may explain the performance of each strategy. The hit rate shows the number of months that the active strategy yields an outperformance over the benchmark. Hence, it also serves to explain the performance of the active strategy.

Figure 3.9 gives the net, after transaction costs, cumulative growth path of a starting capital of €1 million for the period between Dec. 2002 (month 1) and May 2006 (Month 42). The active portfolio is the MRP on the efficient frontier, just as before. The

\textsuperscript{22} The common fee charged by passive index tracking funds offered by Vanguard.
benchmark is still the portfolio with fixed mixture of the three MSCI regional indexes and The Netherlands index. Also given in the figure is the growth path of the investor’s wealth if he had bought the MSCI world index over the same period.

In active portfolio management, the flexibility in portfolio composition introduced by regular rebalancing is the greatest strength of the active strategy. In May 2006, the starting capital of € 1 million has grown to € 1,765,939 in the active case whereas in the fix-mix benchmark case the end capital lingers at € 1,390,731: an absolute difference of € 375,208 or relative difference of 96%. The abnormal return realized by the actively managed portfolio in our example comes from higher profits and relatively low losses during the entire testing period. After accounting for the transaction costs, the active portfolio management based on disaggregated information significantly outperforms the benchmark in terms of net total wealth.

![Figure 3.9: cumulative value of the MRP, the benchmark, and the MSCI world index in the rolling window model. The period spans 42 months starting in Dec. 2002 (month 1) and ends in May 2006 (Month 42). The initial value of each portfolio is € 1 million.](image)

Also interesting from Figure 3.9 is that the benchmark has had similar performance as the MSCI world index. This is not surprising as the benchmark holds three major regional
indexes of the MSCI world index. The difference in the cumulative portfolio value between the MSCI world index portfolio and the benchmark at the end of the testing period is caused by an extraordinary series of high returns on the Dutch market. Combined with the disproportional large weight of the Dutch index in the benchmark due to home bias, the result is a transient higher cumulative return than the MSCI world index. However, this outperformance is negligible when compared to the alternative active strategy.

Thirdly, the three lines of cumulative returns exhibit high correlating behavior. When one line increases, the other two usually follow. The only difference lies in the magnitude of the reaction. Often, it is the active portfolio line that has the strongest reaction. The reason for the highly correlated behavior lies in the construction of the benchmark and the actively managed portfolio: both use the country indexes as their basic building blocks. The only difference is that by using the regional indexes the benchmark keeps a fixed composition of the country indexes, whereas the actively managed portfolio chooses a more flexible combination of the country indexes based on the MV optimization rule. Given this similarity between the portfolios, it is not surprising that the two growth paths exhibit high correlating behavior. However, the composition difference between the two portfolios causes the disparity in the size of reaction to return realization of different country indexes.

Figure 3.10 summarizes the difference in cumulative portfolio values between the benchmark and the actively managed portfolio.

The general observation from Figure 3.10 is the ascending trend in the cumulative return difference between the benchmark and actively managed portfolio. More specifically, there are both increases in difference as well as decreases. When the difference decreases, the gap narrows. Fortunately, there are not only more instances of increases than decreases, the magnitude of increases is more than capable to offset subsequent decreases. The implication here is the widening of the gap between the cumulative benchmark return and
the cumulative return of the actively managed portfolio throughout the entire testing period.

![Graph of After cost cumulative difference MRP & benchmark](image)

**Figure 3.10:** difference in after cost cumulative return of the MRP and the benchmark.

As for the transaction costs at rebalancing, the benchmark is a much cheaper strategy to follow than the active strategy. However, the benchmark transaction cost is by no means zero due to the relatively small adjustment needed to return the benchmark composition to its original fixed mixture, as Figure 3.11 shows. The relatively small adjustments in the benchmark only induce the fixed part of the transaction cost. Since its size is much smaller than in the active case, the variable part of the transaction cost of these relatively small adjustments is much lower than in the active case. Thus, the total transaction cost of the benchmark is much lower.

In Figure 3.11, the peak in month 1 is the inception cost associated with buying the entire portfolio, which means that the positions go from zero to the optimal holding or the fixed benchmark holding. Then, costs are only incurred during rebalancing. At the end, there is no transaction cost as no portfolio rebalancing is carried out in the last month in the testing period for the next month. Clearly, rebalancing portfolios induces much less radical transaction costs when compared to acquisition of the portfolio.
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Turning to the fourth criteria forecasting error, Figure 3.12 contains the realized return and the expected optimal return of the strategies. The realized return is the observed return in month $t$ times the portfolio holdings and the expected optimal return is the expected return for month $t$ times the portfolio holdings, for all $t \in [1, 42]$. As was expected, the realized return shows much more volatility than the expected return in both cases.

In Figure 3.12, the dashed line of the expected portfolio return in each month is smooth and follows the economic cycle as we leave the recession period behind us and enter the revival period. Due to an estimation window of 24 months, the negative expected portfolio return drags on for almost 20 months into the testing period. In the first 5 months of the testing period, the realized return has underperformed due to the market conditions. Once entered the revival period, the realized return has been consistently above expectation with occasional underperformance. In the second half of the testing period when the lagged negative expectation is rolled out of the estimation period, the occurrence of realized return underperforming the expectation increases. Note that a return of 0.05 stands for a return of 5%.
However, the magnitude and frequency of the underperformance pales compared to the cases in which the realization outperforms the expectation, as shown by Figure 3.13.

Figure 3.12: the expected and realized return of the MRP and benchmark.

Figure 3.13: the forecast error of the MRP and benchmark.
In Figure 3.13, the forecast error is defined as expectation minus realization, a negative number in the diagram represents a realization exceeding expectation. Hence, it seems that the number of months with higher than expected return surpasses the number of months with lower than expected return for both the benchmark and the MRP. The general observation from Figure 3.13 is that the actively managed portfolio MRP and the benchmark have very similar exposure to forecast error. The visual evidence in Figure 3.13 suggests that the actively managed portfolio does not possess superior forecast ability than the benchmark portfolio, which is not surprising since the inadequacy of the MV method's forecast power is well documented. Then, what is the reason and origin of the superior outperformance of the MRP portfolio over the benchmark?

The reason lies with the fifth performance criteria: the hit rate, which is defined as the number of month of outperformance over the benchmark. Figure 3.14 summarizes the monthly performance of the MRP and the benchmark in the MS space for each of the 42 months in the testing period. Comparing the two strategies in the illustration, the MRP has outperformed the benchmark portfolio in 27 out of the 42 months with an average excess return of 1.09%. Although the MRP has underperformed the benchmark in the other 15 months, but the average underperformance per month is only -0.93%. Furthermore, the MRP always had a lower risk than the benchmark. As such, the MRP seems to be a far better choice for a risk adverse investor than the benchmark.

To summarize, Table 3.6 lists the relevant results of the rolling window model.

<table>
<thead>
<tr>
<th></th>
<th>Starting wealth</th>
<th>End wealth</th>
<th>Transaction costs</th>
<th>Gross return</th>
<th>Net return</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRP</td>
<td>1,000,000.00</td>
<td>1,765,939.31</td>
<td>20,977.00</td>
<td>78.69%</td>
<td>76.59%</td>
</tr>
<tr>
<td>Benchmark</td>
<td>1,000,000.00</td>
<td>1,390,731.00</td>
<td>12,995.00</td>
<td>40.37%</td>
<td>39.07%</td>
</tr>
</tbody>
</table>

Absolute difference: € 375,208.31
Relative difference: 96.03%
Correlation: 0.9859
Jobson-Korkie: -4.46***

Table 3.6: summary of the relevant numbers of the rolling window model.

---

Figure 3.14: Portfolio risk and return in the mean-standard deviation space with the asterisk representing the MRP portfolio and the cross the benchmark. 0.05 stands for 5% and 0.1 for 10%.
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Looking at the numbers in Table 3.6 reveals that the investor can almost doubles his net profit by actively rebalancing the portfolio based on disaggregated information on a monthly basis instead of holding on fixed benchmark constructed using aggregated information. Also, the Jobson-Korkie (JK) statistic is significant at the 1% level, which implies that the portfolio Sharpe ratio (SR) is significant different from the benchmark. Since the JK statistic is defined as benchmark SR minus portfolio SR, the negative sign indicates that the portfolio SR is superior to the benchmark SR. Moreover, the cumulative returns exhibit highly correlated behavior in the testing period. As explained earlier, the reason for this correlating behavior is due to the fact that both portfolios are built with identical country and regional indexes. Although the actively managed portfolio MRP is a much more costly strategy to pursue, yet the additional costs are more than compensated for by the obtained additional profit. The sensitivity analysis results in the next sub section addresses the question whether the rolling windows results is just a lucky throw of the dice or something more persistent.

3.3 Sensitivity analyses of the rolling window results

In this section, 3 types of sensitivity analysis have been performed to test whether the rolling window results are data path dependent. The natural solution is to repeat the procedures for different dataset to explore whether the results change due to the alterations in the dataset. Ceteris paribus, the dataset is first “rotated” at the point that divides the recession and revival period. Then, the impact of extreme momentum in the Dutch index is explored, as it accounts for large portion of the equity holding due to home bias. At the end, the bootstrap resampling method is used to obtain statistical inference about the bottom-up portfolio selection method using disaggregated data.
3.3.1 Rotating the sample dataset

In the rolling window method, the return vector and covariance matrix in the testing period depend on the estimation period. As mentioned before, the total sample period can be roughly divided into a recession period at the beginning and a revival period later on. To show that the actively managed portfolio always outperforms the benchmark irrespective of any state of nature, the sample dataset is "rotated" around the month April in 2003, whereby the revival period is shifted to the beginning of the sample period and the recession period pushed back towards the end. Figure 3.15 illustrates how the data ranges change.

![Figure 3.15: estimation and testing periods before and after "rotating" the data.](image)

There are two major changes in the dataset. In the original situation the portfolio is constructed during a recession period. Hence, the active holdings are conservative and more pessimistic in nature. As the window rolls forward, the realization in returns has consistently exceeded expectation. However, in the "rotated" scenario, the estimation period is no longer seeded in the recession period, as it now starts in the revival period. The second implicit change here is that the length of the revival period has been reduced by 24 months. Instead of the original revival period of 38 months, the new revival period is reduced to 14 months only. The direct consequence is that the testing period is
3.3.2 Extreme momentums

Due to home bias, a large portion of the benchmark is invested in the MSCI Netherlands country index. Needless to say, the benchmark portfolio is highly sensitive to shocks in The Netherlands index. I use two extreme momentum scenarios to stress test the actively managed portfolio. Ceteris paribus, The Netherlands index is first allowed to continuously grow at a fixed rate for six months. Then, around the same time, The Netherlands index is forced to endure a six-month continuous losing streak. The value of the two-steps bottom-up strategy is determined in comparison to the benchmark strategy.

3.3.3 Resampling by bootstrapping

The bootstrap method, first introduced by Efron (1979) and Efron and Gong (1983), is a computational intensive, general resampling method used to analyze the sensitivity of empirical estimators to sampling variation. It is used to test the path dependency of the results. There are many forms of the bootstrapping method (Davison and Hinkley (1997)); here the case resampling aspect of the bootstrapping method is used to generate new datasets that is considered to be statistically different with respect to the original dataset. Each new return time series is created by drawing return with replacement randomly from the original dataset until the new return time series has been filled. The procedure is repeated if other return time series is needed.

Clearly, the choice of the number of bootstrap samples is subject to arbitrariness and computational power of the computer, which may be a point of contention. Here we follow the conventional choice of 10,000 bootstrap samples.
Hierarchical portfolio management

Both the benchmark and the rolling windows strategy are repeated for the different bootstrap samples. From the 10,000 possible scenarios we obtain the non-parametric distribution of the cumulative return net of transaction cost for each of the strategies. Of course, if the net cumulative return of the monthly rebalancing active strategy always dominates the net cumulative return the benchmark strategy, then the superiority of the active strategy over the benchmark strategy is self-evident.

3.3.4 The results of sensitivity analyses

This section presents the results of test scenarios designed to explore the sensitivity of the cumulative return to the selected data. In the first scenario the data sample is rotated to assess how the results change when the dataset start in the revival period and enters into the recession period. Then, the impact of extreme momentum on The Netherlands country index was explored in two different scenarios. Finally, the bootstrap results provide insights into the statistical properties of the rolling window results and reveal how data sensitive and path dependent the proposed two-steps bottom-up method is. If the active rebalancing strategy based on disaggregated data dominates the passive benchmark strategy, then the active strategy should always yield a higher cumulative return than the passive strategy.

The performance assessment of the two strategies is still based on the 5 criteria of 1) cumulative portfolio value, 2) absolute and relative difference in portfolio net value at the end of the sample period, 3) the transaction cost, 4) forecast error, and finally 5) the hit rate of the actively managed MRP.

3.3.4.1 Results of rotating the sample data

The principal difference with the original setting is that the data period in the “rotated” case starts in the economic revival period and moves into the recession period. The
implications for the mean-variance optimization procedure are i) a sense of overconfidence, ii) ignorance about the upcoming adverse market conditions, and iii) the economic revival period has been shortened, as 24 months of it disappeared into the estimation period. In contrast, the recession period has become longer and the exposure time of the portfolio to the adverse market conditions has been increased, as Figure 3.16 shows.

![Graph showing the after cost cumulative value of the MRP and the benchmark in the rolling window model when the data have been rotated.](image_url)

**Figure 3.16:** after cost cumulative value of the MRP and the benchmark in the rolling window model when the data have been rotated.

Clearly, the adverse market conditions and over-exuberant manager sentiment have helped little to protect the cumulative wealth of the investor. There are two interesting observations in Figure 3.16. Firstly, the actively managed portfolio of MRP has added very little to the total value of the portfolio after transaction cost. As can be observed in Figure 3.16, the cumulative values of the two portfolios in the first 14 months are almost identical. Indeed, all ships rise at high tide. In contrast, the second interesting observation is that the active manager shows his value during adverse market conditions. At the end of the period, the gap between the benchmark and portfolio value amounts to more than €100,000, as can be observed in Figure 3.17.
Figure 3.17: the cumulative portfolio after cost return difference between the minimum risk portfolio (MRP) and the benchmark. Clearly, monthly rebalancing does add value to the investor who otherwise would have lost additional 100,000 Euros.

In Figure 3.17, the first 9 months of the testing period together with the 24 months of the estimation period forms the economic revival period in the sample. The difference between the cumulative return of the MRP and the benchmark during the first 9 months is hardly distinguishable from zero and it zigzags along the x-axis. As matter of fact, instances whereby the fix-mix benchmark outperforms the active portfolio occur more often than otherwise and this has led to more negative cumulative difference for a longer period. However, once we hit the recession period, active portfolio management starts to show its worth. Active rebalancing of the portfolio kept loses in check and realized a smaller cumulative loss at the end of the testing period when compared to the benchmark case.

The comparison between the transaction costs in Figure 3.18 and those in Figure 3.12 is not straightforward because the testing period in Figure 3.18 is not the same as in Figure 3.12. The cutoff month is April 2003, which implies that the majority of months in the period with positive returns have disappeared into the estimation period. As explained before in Figure 3.4, the first 24 of the 33 months in the original revival period disappear in the estimation period after the dataset is tilted over. In the remaining 9 overlapping months, the transaction cost of the actively managed portfolio in Figure 3.18 shows similar
volatility as its transaction cost during the same period in Figure 3.12. However, the size of the transaction cost seems to be smaller than in the original situation, as the peak in some period no longer exceeds the €1000 level.

The forecast error in the current scenario is expected to be much more severe due to the longer exposure to the recession period. Figure 3.19 gives the expected and realized return of the MRP and the benchmark.

In Figure 3.19, the expected return during the first 30 months of the testing period is positive while it becomes negative towards the end of the period. Also striking is the poor performance of both the actively managed MRP and the passive benchmark after the first 9 months. Although the frequency of months in which the realization exceeds expectation has not significantly diminished when compared to Figure 3.13, but the magnitude of underperformance is more extreme. The biggest monthly losses in Figure 3.13 have never fall below the 10% level whereas the biggest monthly loss in Figure 3.19 is 15%. Thus, the forecast error of the portfolios should be higher with more realizations fall short of
expectation, as Figure 3.20 illustrates. Hence, the poor performance of the portfolios in the flipped data scenario is not an enigma.

**Figure 3.19:** the expected and realized return of the MRP and benchmark based on the tilted data.

**Figure 3.20:** the forecast error of the portfolios based on the tilted data.
Figure 3.21: portfolio risk and return in the mean-standard deviation space for the case with tilted data. The asterisk indicates the MRP portfolio and the cross the benchmark. 0.05 stands for 5% and 0.1 for 10%.
With the forecast error still defined as expectation minus realization, a redistribution of positive and negative values can be clearly observed in Figure 3.20. Compared to Figure 3.13, not only did the average exhibit an upward level shift, but the outliers on the positive side are also more extreme than before. Again, the explanation lies with the rotated data. However, the monthly ex-post performance in Figure 3.21 still supports the original finding of better performing actively managed MRP than the benchmark. Just as before, the actively managed MRP always has a lower risk than the passive benchmark. In terms of hit rate, the MRP has outperformed the benchmark in 23 months out of the 42 with an average monthly outperformance of 1.65%. The monthly underperformance averages 1.13%. Table 3.7 summarizes all the relevant numbers in this scenario.

<table>
<thead>
<tr>
<th></th>
<th>Starting wealth</th>
<th>End wealth</th>
<th>Transaction costs</th>
<th>Gross return</th>
<th>Net return</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRP</td>
<td>€ 1,000,000.00</td>
<td>€ 718,265.67</td>
<td>€ 19,780.00</td>
<td>-26.20%</td>
<td>-28.17%</td>
</tr>
<tr>
<td>Benchmark</td>
<td>€ 1,000,000.00</td>
<td>€ 608,113.76</td>
<td>€ 12,966.00</td>
<td>-37.89%</td>
<td>-39.19%</td>
</tr>
<tr>
<td>Absolute difference</td>
<td>€ 110,151.91</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative difference</td>
<td>28.11%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correlation</td>
<td>0.9830</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jobson-Korkie</td>
<td>-0.88</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 3.7:** summary of the relevant numbers in the rolling window model based on the rotated data.

From the numbers in Table 3.7, the tentative conclusion is that active rebalancing should be particularly attractive to the investors because it is capable to reduce losses during adverse market conditions. Although the Jobson-Korkie statistic is no longer significant statistically, but there is still outperformance. Hence, the flexibility of the proposed strategy to incorporate the relevant information allows it to outperform the benchmark: Higher return during better times and lower losses during times of adversity. However, the portfolio returns are highly correlated, as can be observed in Table 3.6 and 3.7.
3.3.4.2 Results of positive momentum in The Netherlands index

In month 8 (July 2003) of the testing period, the MSCI The Netherlands index had realized a positive return of 6.60%. In this extreme positive scenario, the Dutch index is assumed to have continued this positive return for 6 months. From month 8 to 13 (July till Dec. 2003), the MSCI The Netherlands index has a monthly return of 6.60%. The consequence of this extreme positive momentum of 6 months is the narrowing of the gap in cumulative value between the portfolios, as Figure 3.22 illustrates.

![Figure 3.22: net cumulative value of the MRP and the benchmark in the rolling window model whereby The Netherlands country index had a six-month positive return of 6.60% in each month.](image)

The 6 months positive return has narrowed the gap between the cumulative return of the benchmark and the actively managed portfolio. After the six months, the original return series reenters and everything proceeds as before in the original scenario: the gap widens again and the actively managed MRP outperforms the benchmark. But the extreme positive momentum has left its mark, as the actively managed portfolio does not outperform the benchmark by the same margin as in the original case, as shown in Figure.
3.23. The outperformance of the actively managed portfolio is just above the €200,000 mark in Figure 3.23 in contrast to the margin of over €370,000 in the original case.

![Graph showing the difference in after cost cumulative value of the MRP and benchmark](image)

**Figure 3.23:** difference in after cost cumulative after cost value of the MRP and the benchmark in the case of 6 months positive momentum for The Netherlands country index.

The extreme positive momentum has minimum effect on the transaction cost because the continuous positive returns did not prompt a major rebalancing of the benchmark. Also, since the actively managed portfolio drains information from an estimation period of 24 months, it seems that extreme positive momentum has drawn little attention of the MV optimization procedure. Figure 3.24 illustrates.
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Figure 3.24: monthly transaction costs of the MRP and the benchmark in the case with 6 months positive momentum for the Dutch country index.

Figure 3.25: the expected and realized return of the MRP and benchmark in the case of 6 months positive momentum for The Netherlands country index.

In Figure 3.25, given the nature of historical average, the expected return of both the minimum risk portfolio (MRP) as well as the benchmark is smoother and less volatile than its realization. When comparing the time series of benchmark return in Figure 3.25 with the original observed time series in Figure 3.13, then it is striking that in the period
between month 8 and month 13 (the encircled part in Figure 3.25) shows an upward level shift in the benchmark value.

The outperforming Dutch index in the overall benchmark has turned the total benchmark loss in month 11 in the original situation into a profit in the positive momentum scenario as Figure 3.25 shows. Above all, the underperformance of the benchmark with respect to the expectation has turned into an outperformance. This outperformance is also observable in Figure 3.26 that contains the forecast errors (the encircled part).

During the momentum period, with the forecast error defined as expectation minus realization, we observe a lower forecast error for the benchmark portfolio. In Figure 3.26, the positive momentum has induced a downward parallel shift in the forecast error of the benchmark. Once outside of the momentum period, the forecast errors in Figure 3.26 should be the same as those in the original situation displayed in Figure 3.14. However, when putting the two figures besides each other, there is a small, yet noticeable increase in the benchmark forecast error in Figure 3.26. The reason for this can be found in the definition of the forecast error: defined as expectation minus realization, the benchmark
The influence of the extreme positive momentum in the Dutch market on the benchmark return is transitory by construction. This is also observable when we look at the ex-post portfolio performances in Figure 3.27.

In terms of risk, the extreme positive momentum has had little influence, as the actively managed MRP still holds lower risk than the benchmark in every month. As for the return, the benchmark has outperformed the actively managed MRP in 17 months compared to the original 15 months in Figure 3.15. In two extra months during the extreme momentum period: month 11 and 13. The benchmark had a higher return of 1.20%, on average over the 17 months: an increase of 0.27% on a monthly basis compared to the original case. The actively managed MRP used to outperform the benchmark by 2.57% and 1.64%, respectively, in month 11 and 13. However, this outperformance becomes an underperformance of respectively 3.27% and 1.03% in the extreme positive momentum case. Also, the underperformance of the actively managed MRP in other months during the positive momentum period has been aggravated, which in part has led to the extra 0.27% average outperformance of the benchmark. Table 3.8 summarizes all the relevant numbers in this scenario.

<table>
<thead>
<tr>
<th></th>
<th>Starting wealth</th>
<th>End wealth</th>
<th>Transaction costs</th>
<th>Gross return</th>
<th>Net return</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRP</td>
<td>€ 1,000,000.00</td>
<td>€ 1,748,873.88</td>
<td>€ 20,909.00</td>
<td>76.98%</td>
<td>74.89%</td>
</tr>
<tr>
<td>Benchmark</td>
<td>€ 1,000,000.00</td>
<td>€ 1,548,311.69</td>
<td>€ 13,134.00</td>
<td>56.14%</td>
<td>54.83%</td>
</tr>
<tr>
<td>Absolute difference</td>
<td>€ 200,562.19</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative difference</td>
<td>36.58%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correlation</td>
<td>0.99319</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jobson-Korkie</td>
<td>-1.49</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 3.8:** summary of the relevant numbers in the rolling window model in which The Netherlands country index has a six-month positive surge in return of 6.60% per month.
Figure 3.27: portfolio risk and return in the mean-standard deviation space for the case of positive momentum in the Dutch index. The asterisk indicates the MRP and the cross the benchmark. 0.05 stands for 5% and 0.1 for 10%.
Chapter 3       Improving HPM in the multi-period context

Given the extreme positive momentum on the Dutch market, there seems to be no effect on
the active portfolio composition, as the total transaction cost of both the benchmark and
the actively managed MRP has remained virtually the same as in the original case. As
mentioned before, the extreme momentum has slashed the original active cumulative
return premium by more than 45%, from € 375.208 to € 200.562. This is not surprising, as
by construction a significant part of the benchmark has been allowed to grow continuously
for 6 months with a return of 6.60% per month. While the Dutch index is absent from the
MRP, the constructed situation implicitly states that the benchmark has made an excellent
bet in the Dutch index. However, at the end the cumulative return of the benchmark is still
inferior to the cumulative return of the actively managed MRP. Although the Jobson-
Korkie statistic is not significant, yet the size of average outperformance of the MRP over
the benchmark is still considerable. This result shows the robustness of the active
rebalancing methodology. If the optimization procedure had detected the momentum and
consequently exploited the situation, then the premium reduction is would have been much
smaller.

3.3.4.3 Results of negative momentum in The Netherlands index

The other extreme of momentum is the negative momentum. The Netherlands index had
realized a negative return of 6.97% in month 10 (September 2003) of the testing period.
Just as before, the Dutch index is assumed to have continued this negative return for 6
months. From month 10 to 15 (Sep. 2003 till Feb. 2004), The Netherlands index has
realized a negative monthly return of 6.97%. The consequence of this extreme negative
momentum of 6 months can be clearly observed in Figure 3.28.

The cascade in the benchmark cumulative return started in month 10 reaches its bottom in
month 23 (October 2004). The benchmark cumulative return recovers to its initial value in
month 33 (August 2005). The benchmark gains are realized in the last 9 months of the
testing period. The actively managed portfolio remains virtually unaffected by this turn of
events and features a very similar growth pattern as in Figure 3.10.
The consequence of the negative momentum is intuitively clear: Since the benchmark has an overweight on the Dutch index due to the home-bias, the fixed nature of the benchmark can only result in a loss in portfolio cumulative value. Looking at Figure 3.29, the jump in cumulative return difference supports the intuition.

Figure 3.28: after cost cumulative value of the MRP and the benchmark in the rolling window model where The Netherlands country index has a six-months losing streak of 6.90% per month.

Figure 3.29: difference in after cost cumulative value of the MRP and the benchmark in the case of six months negative momentum for The Netherlands country index.
In Figure 3.29, it seems that there has been a level shift in the cumulative return difference after the period with extreme negative momentum, which was not observed in the case with positive momentum. Furthermore, due to poor investment choice and the fixed benchmark composition, the premium associated with the actively managed MRP has now well exceeded the € 600,000 level.

The benchmark transaction costs during the momentum period exhibit some small peaks unobserved elsewhere. Although the benchmark has been more active during the momentum period to restore the old weightings, the overall increase in the benchmark transaction costs is negligible. The actively managed portfolio is not entirely unaffected during the momentum period, as the transaction cost has increased slightly.

![Figure 3.30: monthly transaction costs of the MRP and the benchmark in the case with 6 months negative momentum for The Netherlands country index.](image)

After the momentum period has fully entered the estimation period, there is in general more transaction costs observed in Figure 3.30. This is because that The Netherlands index enters the MRP with significant increasing weight (>1% of the total portfolio holding) starting in month 22 (Sep. 2004) just as in the positive momentum case. Due to the effect of the negative momentum on the realized benchmark return, the benchmark realized return in Figure 3.31 is inferior to what it used to be in Figure 3.13.
Again, the expected return of both the minimum risk portfolio, MRP, as well as the benchmark in Figure 3.31 is smoother and less volatile than its realization. Clearly, there is an observable downward level shift for the benchmark-realized return between Sep. 2003 (month 10) and Feb. 2004 (month 15). With the forecast error defined as before, expectation minus realization, we observe a worsening performance of the benchmark portfolio in the period between month 10 and 15. Compared to Figure 3.14, the negative momentum induced an upward parallel shift in the forecast error of the benchmark in Figure 3.32.

From the ex-post performances documented in Figure 3.33, the outperformance of the MRP portfolio in terms of risk and returns is clearly observable for the negative momentum period. Besides the observation that the actively managed MRP has always realized a lower risk than the passive benchmark, the actively managed MRP has also realized a higher return than the benchmark in 29 out of the 42 months. The average monthly active return in addition to the benchmark return is 1.69% or 20.33% on annual basis. In the remaining 13 months the MRP has underperformed with a monthly average of 1.02% or 12.18% annually. Thus, the origin of the abnormal above benchmark return summarized in Table 3.9 is clear.
Chapter 3       Improving HPM in the multi-period context

Figure 3.32: the forecast error of the portfolios in the negative momentum case.

<table>
<thead>
<tr>
<th></th>
<th>Starting wealth</th>
<th>End wealth</th>
<th>Transaction costs</th>
<th>Gross return</th>
<th>Net return</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRP</td>
<td>€ 1,000,000.00</td>
<td>€ 1,758,436.18</td>
<td>€ 21,085.00</td>
<td>77.95%</td>
<td>75.84%</td>
</tr>
<tr>
<td>Benchmark</td>
<td>€ 1,000,000.00</td>
<td>€ 1,131,205.08</td>
<td>€ 13,104.00</td>
<td>14.43%</td>
<td>13.12%</td>
</tr>
</tbody>
</table>

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolute difference</td>
<td>€ 627,231.10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative difference</td>
<td>478.05%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correlation</td>
<td>0.81448</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jobson-Korkie</td>
<td>-3.31**</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.9: summary of the relevant numbers in the rolling window model in which The Netherlands country index is assumed to have had a six-month continuous monthly negative momentum in return of 6.90%.

At the 1% significance level, the Jobson-Korkie statistic of –3.31 indicates that the Sharpe ratio of the MRP is significantly higher than the benchmark Sharpe ratio. Also, besides the relative difference between the MRP and benchmark cumulative return of more than 470%, the relatively low correlation of 0.81 is also striking. It seems that the cascade started in month 10 has significantly reduced the similarity in the growth patterns, as the correlation is almost perfect positive in the original scenario and the positive momentum case.
Figure 3.33: portfolio risk and return in the mean-standard deviation space for the case of negative momentum in the Dutch index. The asterisk indicates the MRP and the cross the benchmark. 0.05 stands for 5% and 0.10 for 10%.
3.3.4.4 The bootstrapping results

The bootstrapping results provide statistical inferences concerning the different strategies. If the active rebalancing strategy perfectly dominates the passive fix-mix benchmark strategy, then the active strategy must always yield a higher cumulative return than the passive strategy. Additionally, the bootstrap results reveal how sensitive and path dependent the proposed method is.

Figure 3.34 gives the distribution of the cumulative return of the benchmark (diagram A) and MRP (diagram B) for the 10,000 bootstrap scenarios. Panel C presents the difference in cumulative return between the MRP and the benchmark. Note that the x-axis of the diagram A and B are in millions of Euros while the x-axis of the difference in diagram C is in hundreds of thousand Euros.

Diagram A and B in Figure 3.34 present the distribution of the cumulative return of the benchmark and MRP for the 10,000 samples in the bootstrap method. Although indicative of the superior realized return of the actively managed MRP, they do not convey any information about the magnitude in return difference of scenario specific benchmark and MRP: the return difference between the MRP and benchmark in a specific scenario cannot be obtained by subtracting the returns in diagram A from the returns in diagram B. Diagram C fills that gap. The average net return difference is €205,290. Maximum gain equals €543,290. In 127 scenarios out of the 10,000, the active rebalancing strategy has produced a net cumulative return lower than the benchmark. Average underperformance is €38,467 with a maximum net underperformance of €242,230. Table 3.10 presents the percentile critical values of the return difference in Diagram C of Figure 3.34.
Figure 3.34: the distribution of the cumulative return of the benchmark (diagram A) and MRP (diagram B) for the 10,000 bootstrap scenarios. Diagram C presents the difference in cumulative return between the MRP and the benchmark.

<table>
<thead>
<tr>
<th>Percentile</th>
<th>0%</th>
<th>5%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>95%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return difference</td>
<td>-€ 242,232.52</td>
<td>-€ 69,380.16</td>
<td>-€ 156,333.03</td>
<td>-€ 204,450.01</td>
<td>-€ 256,395.00</td>
<td>-€ 340,784.33</td>
<td>-€ 543,287.68</td>
</tr>
<tr>
<td>Variance difference</td>
<td>-0.21%</td>
<td>-0.14%</td>
<td>-0.11%</td>
<td>-0.10%</td>
<td>-0.08%</td>
<td>-0.06%</td>
<td>-0.03%</td>
</tr>
</tbody>
</table>

Table 3.10: the percentile critical value of the bootstrap empirical distributions in Figure 3.34.

From Table 3.10, with a net outperformance of € 69,380.16 at the 5th percentile, the investor has a 95% probability of obtaining a net return of at least 6.94% in excess of the benchmark return given his initial investment of € 1 million, 75% probability of obtaining a net return of at least 15.63% in excess of the benchmark and so forth. If the empirical distribution is representative to the true distribution, then the investor should always use our approach based on decision measures like the value-at-risk at the 5% significance level. From the risk perspective, the probability for the portfolio variance to be lower than that of the benchmark is 100%.
Figure 3.35 presents the distribution of the average variance of the benchmark (diagram A) and the MRP (diagram B) for the 10,000 scenarios in the bootstrapping. Diagram C gives the difference in average variance between the MRP and the benchmark.

Over the 42 months in the rolling window model, each portfolio has a variance in each month calculated based on its performances over the prior 24 months. The average variance over the 42 months is taken as indication of the portfolio variance. The distribution is obtained from the 10,000 data samples in the bootstrap method. The actively rebalancing strategy using MRP has always produced a portfolio with lower risk than the benchmark portfolio, which is precisely the objective when the global MRP was chosen.

Figure 3.36 presents the distribution of the transaction cost. As expected, the active rebalancing strategy is a more costly strategy to pursue than the benchmark. However, the average difference in transaction cost is merely €6,000, which is only 0.6% of the initial
wealth. In absolute numbers, the benchmark strategy costs on average €13,019 to follow and the active rebalancing strategy on average about €19,161. Minimum cost for the benchmark strategy is €12,686 and €15,224 for the active strategy. The maximum cost is €13,904 for the benchmark strategy and €26,838 for the active strategy.

![Graph showing transaction costs](image)

**Figure 3.36:** transaction costs of the actively managed MRP and the passive benchmark.

Clearly, the rigid benchmark tracking strategy with limited decision freedom has worked and confined the transaction costs of the passive strategy to a relatively narrow interval. Also, the observation that MRP always has a higher cost than the passive strategy underscores the concerns in practice about following an active strategy: the active strategy has a higher threshold to overcome before it produces outperformance. However, as the critical values in Table 3.10 indicate, pursuing an active strategy does not necessarily spell gloom and doom. Far from it, a well designed methodology may be quite profitable indeed for all parties involved.

Figure 3.37 gives the distribution of the hit rate: the number of months in which the active rebalancing strategy has realized a higher return than the benchmark.
On average, the active rebalancing portfolio got a higher realized return than the benchmark in 25 months out of 42 months. At best, the active strategy got a higher realized return in 37 months and 13 in the worst case scenario. It seems that higher return is possible even using the most naive return forecast model. Therefore, the consistent higher cumulative return produced by the active rebalancing strategy is not an enigma.
3.4 Concluding remarks

The purpose of this paper is to explore the time persistency of the actively managed portfolio based on disaggregated information. The proposed two-steps bottom-up approach based on disaggregated information has led to consistent improvement in portfolio performance in terms of increased return and decreased risk. The sensitivity analyses have shown that the result is not a lucky throw of the dice. The core idea is that the allocation decision at the overall level of the investment hierarchy uses too little information on the possibilities at the lower levels. Consequently, valuable lower level investment opportunities (including their interactions which are crucial for overall portfolio risk) are ignored. The proposed approach transfers more information about the opportunities in the lower levels to the overall level, which can then be used for the overall level allocation decision and thus can yield better overall results.

The paper describes a real life case in which the benchmark is a fixed combination of three MSCI regional and one MSCI country index. After plotting the risk and return performance of the regional indexes and the country indexes in the MV space, it is clearly observable that the regional indexes used by the benchmark are inferior choices when compared to the top-performing country indexes in terms of average return and risk over the sample period and two non-overlapping sub periods of the sample period. It seems that the benchmark choice based on aggregated regional information is dominated by some of the country index performance. Hence, above benchmark return can be achieved if the top-performing country indexes are chosen instead of the regional indexes: portfolio choice based on disaggregated data dominates portfolio choice based on aggregated data. Also, the performances of the regional and country indexes fluctuate during different periods, which suggest periodical rebalancing instead of holding on a fixed benchmark.

The ex-post results of the rolling window model confirm that the benchmark choice based on aggregated data is sub optimal in terms of realized risk and returns. For an initial portfolio value of € 1 million, the benchmark strategy has yielded a net absolute return of € 390,731 over the sample period between Dec. 2002 and May 2006. The minimum risk
portfolio (MRP) has yielded a net absolute return of €765,939: an absolute difference of €375,208 and a relative difference of 96.03%. The concern that transaction costs will kill the alpha does not seem to hold in this case. The tantalizing conclusion is that active portfolio management using disaggregated information adds value for the investor in a hierarchical portfolio selection context.

After resampling the dataset for 10,000 times using the bootstrap method, the realized results reveal that the outperformance of the MRP is not a lucky throw of the dice. Far from it, the MRP portfolio has outperformed the benchmark in 9873 scenarios out of the 10,000 by more than €200,000 on average with a maximum outperformance of about €543,000.

The general conclusion of this paper is that a passive fixed composition of the available investment choices is a sub optimal choice in a multi-period decision framework. Also, portfolio construction using aggregated investment opportunity information is less than ideal. Far better results can be obtained using disaggregated information that better describes the whole IPOS for the investor. Finally, the holding portfolio should be rebalanced periodically to incorporate new information when it becomes available.
How should investors assess active managers?

The performance of the investor’s overall portfolio is the aggregated performance of the lower level sub portfolios managed by different managers. In the practitioners’ literature, the managed sub-portfolios are assumed to be uncorrelated (the independence assumption) because different managers construct their portfolios independently from each other (see e.g. Grinold and Kahn (1999) and Blitz and Hottinga (2001)). In reality, the independence assumption does not hold as managers draw their investment choices from the same pool of opportunities: the global markets. For example, if manager A draws from the equity asset class while manager B draws from the bond asset class, then it is rarely the case that the two portfolios are uncorrelated with each other. Since the total risk depends on the risk of each sub portfolio and the correlations between the sub portfolios, the overall portfolio’s risk-adjusted return may be overstated because the overall portfolio risk may be understated by ignoring the correlations at performance assessment.

This chapter provides answers to the last two research questions. Does positive active performance automatically imply improvement of the overall portfolio’s performance (research question 4)? How should an investor judge portfolio manager if the sub portfolios are correlated (research question 5)?

Hence, this chapter focuses on the effects of possible correlations in the lower level on the overall portfolio risk adjusted return. In the lower level I distinguish between 4 types of correlations between the returns. Assume for simplicity of exposition that there are only two benchmarks with an active portfolio attached to each benchmark Figure 4.1 summarizes all the possible types of return correlations.
Chapter 4 How should investors assess active managers?

Figure 4.1: the correlations between two benchmarks with return $r_{bI}$ and $r_{bII}$. Each benchmark has one self-financing active portfolio with return $r_{aI}$ and $r_{aII}$. The two-headed arrows indicate the correlations. There are 4 types of correlations: (i) Correlations between the benchmark returns; (ii) The correlation between the benchmark and its self-financing active portfolio; (iii) The active portfolios may be correlated; (iv) The cross correlation between the active portfolio with the other benchmark.

In Figure 4.1, the first type of correlation is the correlation between the benchmarks due to the macro economic factors that govern the global markets. For example, a rise in the interest rate in the U.S. will have its repercussions on all the asset class indexes in the markets from Japan to Europe. Secondly, the active portfolio and its benchmark are correlated due to the definition of the self-financing active portfolio\(^\text{24}\). The active portfolio contains identical investment elements that also constitute the benchmark index. Hence, each active portfolio is very likely to be correlated with its benchmark. Moreover, the active portfolios must be correlated due to the definition of the active portfolio and the fact that the benchmarks are correlated. Finally, as benchmarks are correlated while the active portfolios are also correlated with their own benchmark, it is likely that the active portfolios and other benchmarks are also correlated. Formal derivation of these correlations is postponed to section 4.1.

\(^{24}\) As defined before, the active portfolio is self-financing because the active long positions in the undervalued stocks are financed by the short positions in the active portfolio. In total, the positions in the active portfolio cancel each other out and hence the active portfolio is self-financing.
Hierarchical portfolio management

H. Ning

The significance of the correlated sub portfolios in the lower level lies with its direct impact on the overall portfolio’s level of risk. Implicitly, the aggregated risk-adjusted return also depends on the correlations in the lower level. The bias introduced by the independence assumption in calculating the risk-adjusted return of the overall portfolio can be massive. The empirical results in this chapter show that the benchmark with independent active portfolios only accounts for 67% of the overall volatility.

A related point is that the active performance measures in practice only give an account of the performance of the active manager relative to a benchmark index. Positive appraisal ratio (AR) or information ratios (IR) do not automatically imply a higher level of risk adjusted overall return, Sharpe ratio (SR), for the investor. Occasionally, the correlation between the benchmark and the active portfolio may lead to a higher level of overall risk, which in turn reduces the overall portfolio efficiency.

The remainder of this chapter is divided into two parts. In the first theoretical part I illustrate the precise workings of aggregating return and risk from the lower level to the overall level. The theoretical basis of the approach for aggregating risk is illustrated by decomposing the overall portfolio covariance matrix into smaller blocks attributable to the benchmarks and the active portfolios. The impact of the lower level correlations on the overall portfolio risk becomes apparent after adding each correlation stepwise to the augmented covariance matrix. The overall portfolio risk increases as more positive correlations enter the augmented covariance matrix. Moreover, the theoretical background of the active performance measures in current use is given. Not only is there a difference between the definitions of the measures, but also the magnitude of the difference can be massive at the asymptotes. Then in the second empirical part, the effect of correlated portfolios is illustrated using a hierarchical investment example for a U.S. investor who is investing domestically as well as internationally.
4.1 Aggregated performance attributes

The model considered here is the standard myopic single period model with a hierarchy consisting of two levels: an overall and a lower level. At the start of the period, the Mean-Variance (MV) investor in the overall level allocates funds over the MV optimized portfolios of managers in the lower level. At the end of the period the managers are not only appraised based on their active performances, but also on their contribution to the overall portfolio performance. Given a benchmark, the corresponding active portfolio is a self-financing portfolio. The benchmark plus the active portfolio is the sub portfolio managed by a manager. The overall portfolio is the aggregate of all the sub portfolios.

In this chapter, the overall portfolio is being appraised according to its risk adjusted return: Sharpe Ratio (SR). Hence, the illustration in this section starts with return aggregation from the lower level to the overall level. Then, subsection 4.1.2 illustrates the size of the error in risk aggregation due to the independence assumption by means of an augmented covariance matrix. Using such a matrix allows us to decompose the overall portfolio risk and attribute part of the risk to each specific part of the sub portfolios in the lower level. By stepwise adding extra correlation component to the augmented covariance matrix, changes in the overall portfolio’s level of risk and consequently in the overall risk adjusted return become clear.

4.1.1 Return aggregation

Return aggregation of all the portfolios under management is straightforward. It is the weighted sum of all the portfolios’ realized return. If the sub portfolios are benchmark replication portfolios, then the realized returns are the benchmark returns net of costs. Unfortunately as shown in Section 2.1, such a strategy will always lead to aggregate portfolio performance that underperforms those of the benchmark due to market frictions like transaction cost, loading fee and management fees.
If active portfolio management is allowed, then things start to change in both the overall as well as in the lower level. What makes active portfolio management interesting is that the changes may be unforeseen and thus unexpected, which may lead to undesired results. In the lower level, the change is obvious as the managers place bets on individual investment opportunities in the benchmark space according to their own interpretation of the signals. Hence, each sub portfolio return may deviate from the benchmark return such that the entries in the expected return vector in the overall level change. Then in the overall level, the investor may change his view on individual manager due to his expectation of the manager’s skill on market timing and/or stock picking. Thus, the weighting vector in the overall level may also change. After having adjusted for all these changes, the aggregate portfolio return is still the linear weighted sum of all the sub portfolios’ return in the lower level.

4.1.2 Correlations and risk aggregation

Risk aggregation is a quadratic weighted sum of individual risk components in the portfolio plus the correlating components. If these existing correlations between the different components in the overall portfolio are ignored, then the overall performance cannot be assessed accurately. In the following 4 sub sections we first identify the specific type of correlation given in Figure 4.1. Then, the non-trivial impact of the correlation on the overall level of risk is presented by using the augmented covariance matrix that decomposes the overall risk into smaller blocks attributable to the benchmarks, active self-financing portfolios, and the correlated parts.

4.1.2.1 Correlating benchmarks

The first type of correlation in Figure 4.1 is the correlation between the benchmarks. Typical benchmarks are market capitalization or equally weighted indexes divided by geographical regions, countries, asset classes or sectors. The empirical evidences suggest
that the major indexes (Dow, NASDAQ, S&P, FTSE, Nikkei, etc.) are correlated. Given the nature of the current information based economy, the markets over the globe are governed by similar macroeconomic concerns: a sell-off in equities in Japan prompt by a local interest rate raise will lead to speculations of similar move from the European Central Bank, which in turn will probably trigger a day of sell-offs in the European equities. On the other hand, a positive session on the U.S. market is likely to increase investor confidence elsewhere and induce positive trading sessions in the other markets. Also, uncorrelated bond and equity markets seems to be farfetched as movements in macroeconomic factors like interest rate, inflation, and unemployment inevitably influence the attractiveness of both asset classes.

Given a situation with two benchmarks, each benchmark has two different securities and there is no overlap between the benchmarks. Let \( \tilde{r}_I \) and \( \tilde{r}_II \) be the return vectors that contain the securities' stochastic return in excess of the risk free rate for benchmark I and II, respectively. Let \( w'_I = \begin{bmatrix} w_1, w_2 \end{bmatrix} \) and \( w'_II = \begin{bmatrix} w_3, w_4 \end{bmatrix} \) define the benchmark weights. Assume that the budget constraint holds, and then the sum of the weights in each benchmark equals unity. Hence the benchmark expected excess returns \( \bar{r}_I \) and \( \bar{r}_II \) can be written as

\[
\begin{align*}
\bar{r}_I &= \tilde{r}_I + w_I \left( \tilde{r}_I - \tilde{r}_II \right) \quad \text{and} \quad \bar{r}_II &= \tilde{r}_II + w_II \left( \tilde{r}_II - \tilde{r}_I \right) \\
\end{align*}
\]

where \( \tilde{r}_i = E(\tilde{r}_i) \) for \( i = 1, \ldots, 4 \). The augmented matrix divides the overall covariance matrix into four parts: a passive part with the benchmarks, an active part containing the active portfolios, and two symmetric correlating parts. If the investor is a passive investor who requires his managers to replicate the benchmark, then both the active parts as well as the correlating parts of the augmented matrix are zero.
Given the situation in augmented matrix (i), the overall portfolio covariance matrix, $\Omega$, is equal to the benchmark covariance matrix $\sigma_{bI}^2$, $\sigma_{bII}^2$, $\sigma_{bI,bII}$, $\sigma_{bII,bI}$, $\sigma_{bII,bII}$, with the subscript $i$ denoting the case number in Figure 4.1 and $\sigma_{bI,bII}$ denoting the covariance between the benchmark returns. The correlation between the different parts of the overall portfolio has direct consequences in the overall portfolio covariance matrix.

4.1.2.2 Correlation between benchmark and the active portfolio

The correlation between the benchmark portfolio and the corresponding active portfolio is non-zero because of the definition of the active portfolio. Since the active portfolio is self-financing, it consists of identical securities as in the benchmark portfolio. The only difference with the benchmark is that the active portfolio has a weighting of the securities that sums to zero. As such, by adding the active portfolio to the benchmark portfolio, we obtain a tilted portfolio that is expected to outperform the benchmark.

25 The matrix $\Omega_i$ is an augmented covariance matrix, which implies that the entire covariance matrix has been dissected into a benchmark part, active part and the correlating parts. On page 34 we were talking about the “plain” covariance matrix and the covariance matrix contains variance and covariances between stocks, whereas the covariance matrix $\Omega_i$ here contains covariances between portfolios.
Expanding the illustration in the last sub section with an active portfolio for each benchmark, the self-financing active portfolio weight vectors are $w_{aI}^I = \left[ \Delta_a, -\Delta_a \right]$ and $w_{aII}^I = \left[ \Delta_a, -\Delta_a \right]$ for benchmark I and II, respectively. The active portfolio returns can be written as

$$\bar{r}_a = \Delta_a (\bar{r} - \bar{r}_I) \quad \text{and} \quad \bar{r}_{aII} = \Delta_{aII} (\bar{r} - \bar{r}_I).$$ \quad (4.2)$$

Given (4.1) and (4.2) we can write the covariance between the benchmarks and its active portfolio's return as

$$\text{cov} (\bar{r}_a, \bar{r}_I) = \Delta_a \left( w_1 \sigma_1^2 - (1-w_1) \sigma^2 + (1-2w_1) \text{cov}(\bar{r}_a, \bar{r}_I) \right)$$ \quad (4.3)$$

for benchmark I and

$$\text{cov} (\bar{r}_{aII}, \bar{r}_I) = \Delta_{aII} \left( w_3 \sigma_3^2 - (1-w_3) \sigma^2 + (1-2w_3) \text{cov}(\bar{r}_{aII}, \bar{r}_I) \right)$$ \quad (4.4)$$

for the other benchmark. In eq. (4.3) and (4.4), $\text{cov}(\cdot, \cdot)$ denotes the covariance operator and $\sigma_i^2$, for $i = 1, 2, 3$ and 4, denotes the variance of security $i$. From eq.(4.3) and (4.4) the covariance between the benchmark and its active portfolio is zero if

$$\Delta_a = \Delta_{aII} = 0$$ \quad (4.5)$$

or

$$w_1 = \frac{\sigma_1^2 - \text{cov}(\bar{r}_a, \bar{r}_I)}{\sigma_1^2 + \sigma_2^2 - 2 \text{cov}(\bar{r}_a, \bar{r}_I)} \quad \text{and} \quad w_3 = \frac{\sigma_3^2 - \text{cov}(\bar{r}_{aII}, \bar{r}_I)}{\sigma_3^2 + \sigma_4^2 - 2 \text{cov}(\bar{r}_{aII}, \bar{r}_I)}$$ \quad (4.6)$$

hold. Intuitively, eq. (4.5) states the obvious: there is no correlation between the benchmark and the active portfolio if there is zero deviation from the benchmark. The
benchmark weight choices in eq. (4.6) show that the active portfolio can be made uncorrelated with respect to the benchmark. However, the choice lies with the benchmark composition instead of the active portfolio weight choices. This may prove to be a difficult strategy to pursue if the benchmark is a given fixed reference point.

In current investment practice the popular Single Index Model (SIM) is used to disentangle the residual return of a portfolio \( \alpha_p \) from the market index return. By construction, the covariance between the index return and the residual risk in the SIM for a portfolio \( p \) is zero. By assumption, the residual return of different portfolios in the SIM is uncorrelated with each other. The covariance structure between portfolio \( i \) and portfolio \( j \) under SIM reduces to

\[
\text{cov}(\hat{r}_i, \hat{r}_j) = \beta_{im} \beta_{jm} \sigma_{\alpha_i}^2, \quad (4.7)
\]

where \( \beta_{im} \) denotes the beta of portfolio \( i \) with the market index \( m \), and \( \sigma_{\alpha_i}^2 \) is the variance of the market return in excess of the risk free rate. Under SIM, it seems that the different portfolios are only correlated via their common component described by the market portfolio. As we will see later in this chapter, the active performance measures AR and IR only give consistent performance assessment under beta neutrality \( (\beta = 1) \) and hence the covariance between the different portfolios is just the market portfolio variance.

The problem here is that the uncorrelated residual returns across different portfolios in SIM only holds true by assumption. In practice, it is highly unlikely that the residual returns of different portfolios are uncorrelated since the active portfolio has the same elements as the benchmark. If the investor allows for active management while assuming general orthogonal conditions like those of the SIM, then the augmented matrix \( (i) \) becomes
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By assumption, the additional risk introduced due to active portfolio management is limited to the variance of each active portfolio and only the diagonal elements in the overall covariance matrix $\Omega$ increase with the active variances.

$$\Omega_{ii} = \begin{pmatrix} \sigma_{bl}^2 + \sigma_{al}^2 & \sigma_{bl,al} \\ \sigma_{bl,al} & \sigma_{al}^2 + \sigma_{all}^2 \end{pmatrix}$$

Case (ii) is the independent active portfolio case.

However, the correlation between the active portfolio and its benchmark is not zero since the active portfolio contains the same elements as the benchmark. The augmented matrix (ii) should be

$$\Omega_{ii'} = \begin{pmatrix} \sigma_{bl}^2 & \sigma_{bl,bl} & 0 \\ \sigma_{bl,bl} & \sigma_{all}^2 & 0 \\ 0 & 0 & \sigma_{all}^2 \end{pmatrix}$$

where the correlations between the active portfolios and the benchmarks are added to the correlating components in the augmented portfolio. Consequently, the diagonal elements in the overall covariance matrix $\Omega$ have been increased again, this time by the correlation between the benchmark and its own active portfolio.
But different to the variance elements, the additional covariance elements can be negative. Hence, accounting for the covariance elements may actually reduce the overall risk when the covariances are negative. Case \( \text{ii}' \) is the active portfolios correlated with own benchmark case.

### 4.1.2.3 Correlating active portfolios

The third type of correlation is the correlation between the active portfolios. The active portfolios contain identical stocks and securities as their benchmark, and since the benchmarks are correlated, it is then highly likely that the active portfolios are also correlated with each other. Continuing with the example of two benchmarks with an active portfolio each, we can write the covariance between the benchmarks as

\[
\text{cov} \left( \hat{r}_a, \hat{r}_w \right) = w_i w_j \text{cov} \left( \hat{r}_i, \hat{r}_i \right) + w_i \left( 1 - w_i \right) \text{cov} \left( \hat{r}_i, \hat{r}_i \right) + \left( 1 - w_i \right) w_j \text{cov} \left( \hat{r}_j, \hat{r}_j \right) + \left( 1 - w_i \right) \left( 1 - w_j \right) \text{cov} \left( \hat{r}_i, \hat{r}_j \right)
\]

\[(4.8)\]

We know that \( w_i \in [0,1] \) and \( w_j \in [0,1] \) hold under the budget and no-short selling constraints. Hence, if the covariance between the benchmarks is non-zero, then there must be at least one covariance term on the right hand side in eq. (4.8) that is also non-zero. Therefore, the covariance between the active portfolios is also non-zero since we can write it as follows:

\[
\text{cov} \left( \hat{r}_a, \hat{r}_w \right) = \Delta_a \Delta_w \left( \text{cov} \left( \hat{r}_i, \hat{r}_i \right) + \text{cov} \left( \hat{r}_j, \hat{r}_j \right) - \text{cov} \left( \hat{r}_i, \hat{r}_j \right) - \text{cov} \left( \hat{r}_j, \hat{r}_i \right) \right)
\]

\[(4.9)\]

In eq. (4.9), if any of the active weights is zero, then the covariance between the active portfolios is zero because one active portfolio just ceased to exist. Also, if the covariances
of the individual stocks and securities sum to zero, then the covariance between the active portfolios goes to zero.

When the active portfolios do covariate with each other given their definition, then we get

\[
\begin{pmatrix}
\sigma_{bl}^2 & \sigma_{bl,bl} & 0 \\
\sigma_{bl,bl} & \sigma_{bl}^2 & 0 \\
0 & 0 & \sigma_{bl,bl}^2 \\
\end{pmatrix}
\]

as the augmented covariance matrix. In case (iii), the overall portfolio variance is increased by the covariance between the active returns.

\[
\Omega_{iii} = \begin{pmatrix}
\sigma_{bl}^2 + \sigma_{al}^2 + 2\sigma_{bl,al} & \sigma_{bl,bl} + \sigma_{al,all} \\
\sigma_{bl,bl} + \sigma_{al,all} & \sigma_{al}^2 + \sigma_{all}^2 + 2\sigma_{bl,all} \\
\end{pmatrix}
\]

Again, as in case (ii'), the additional elements do not have to increase the overall portfolio risk as long as the covariance elements are non-positive. This third case is called the correlated active portfolio case.

### 4.1.2.4 Cross correlations

Under the same analogy of the correlated active portfolio case, the cross correlation between different benchmark and active portfolio cannot be zero either. The cross covariance between the first benchmark and the second active portfolio can be written as
The cross covariance between the second benchmark and the first active portfolio can be written as

\[ \text{cov}(\tilde{r}_w, \tilde{r}_o) = w_1 \Delta_w \left( \text{cov}(\tilde{r}_1, \tilde{r}_1) + \text{cov}(\tilde{r}_2, \tilde{r}_1) - \text{cov}(\tilde{r}_1, \tilde{r}_2) - \text{cov}(\tilde{r}_2, \tilde{r}_2) \right) + \Delta_w \left( \text{cov}(\tilde{r}_2, \tilde{r}_2) - \text{cov}(\tilde{r}_2, \tilde{r}_2) \right). \] (4.10)

Since there must be at least one term on the right hand side of eq. (4.8) for the covariance between the benchmarks to be non-zero, there must be at least one non-zero term on the right hand side of eq. (4.10) and (4.11). Eq. (4.10) can be zero if the following equalities hold:

\[ \text{cov}(\tilde{r}_1, \tilde{r}_1) + \text{cov}(\tilde{r}_2, \tilde{r}_1) = \text{cov}(\tilde{r}_1, \tilde{r}_2) + \text{cov}(\tilde{r}_2, \tilde{r}_2), \] (4.12)

For eq. (4.11) the necessary conditions for zero covariance are

\[ \text{cov}(\tilde{r}_1, \tilde{r}_1) + \text{cov}(\tilde{r}_2, \tilde{r}_2) = \text{cov}(\tilde{r}_1, \tilde{r}_2) + \text{cov}(\tilde{r}_2, \tilde{r}_1), \] (4.13)

From eq. (4.12) and (4.13), \textit{zero cross covariance} between a benchmark and the active portfolio of the other benchmark implies that the equation

\[ \text{cov}(\tilde{r}_1, \tilde{r}_1) = \text{cov}(\tilde{r}_2, \tilde{r}_2) \] (4.14)

also holds. The implication from the equations (4.12) through to (4.14) is that each element in the benchmarks has identical covariance with the elements in the other benchmark. In
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In our illustration the covariances all have the value \( \text{cov}(\tilde{r}_i, \tilde{r}_j) \). Due to triangular property, the elements in each benchmark must also have the same covariance. As such, zero cross covariance implies that all the investment opportunities' movements synchronize in the same direction and amplitude. Such property may be hard to find and construct in practice. Hence, the cross covariance between a benchmark and the active portfolio of another benchmark is highly unlikely to be zero. The full augmented covariance matrix becomes:

\[
\begin{pmatrix}
\sigma_{bl}^2 & \sigma_{bl,bl} & \sigma_{bl,al} & \sigma_{bl,all} \\
\sigma_{bl,bl} & \sigma_{bl}^2 & \sigma_{bl,al} & \sigma_{bl,all} \\
\sigma_{al,bl} & \sigma_{al,bl} & \sigma_{al}^2 & \sigma_{al,all} \\
\sigma_{all,bl} & \sigma_{all,bl} & \sigma_{all,al} & \sigma_{all}^2 \\
\end{pmatrix}
\]

Each cross-correlation enters the overall covariance matrix twice due to the symmetric property of the variance-covariance matrix.

\[
\Omega_p = \begin{pmatrix}
\sigma_{bl}^2 + \sigma_{al}^2 + 2\sigma_{bl,al} & \sigma_{bl,bl} + \sigma_{al,al} + \sigma_{bl,all} + \sigma_{bl,al} \\
\sigma_{bl,bl} + \sigma_{all,al} + \sigma_{al,bl} + \sigma_{al,bl} & \sigma_{al}^2 + \sigma_{all}^2 + 2\sigma_{all,all} \\
\end{pmatrix}
\]

In the overall level, the total risk faced by the investor's portfolio \( P \) that is investing \( W \) in benchmark \( I \) and \( (1-W) \) in the other with the corresponding active portfolios equals

\[
\sigma_{P,v}^2 = \begin{bmatrix} W & 1-W \end{bmatrix} \Omega_p \begin{bmatrix} W \\ 1-W \end{bmatrix},
\]

\[
= W^2 \left( \sigma_{st}^2 + \sigma_{st}^2 + 2\sigma_{st,al} \right) + (1-W)^2 \left( \sigma_{st}^2 + \sigma_{st}^2 + 2\sigma_{st,all} \right) + W(1-W) \left( \sigma_{st,al} + \sigma_{st,all} + \sigma_{st,al} + \sigma_{st,all} \right). \tag{4.15}
\]
Capital $W$ is used to denote the weight in the overall portfolio. Under the independence assumption, the overall variance-covariance matrix is $\Omega$ and the overall risk of the portfolio in (4.15) becomes

\[
\sigma^2_{p,\text{tot}} = \begin{bmatrix} W & 1-W \end{bmatrix} \Omega \begin{bmatrix} W \\ 1-W \end{bmatrix},
\]

\[
= W^2 \sigma^2_{bd} + \sigma^2_{al} + (1-W)^2 \left( \sigma^2_{bl} + \sigma^2_{al} \right) + 2W(1-W)\sigma_{bl,al}.
\]

The discrepancy in the overall portfolio risk amounts to

\[
\sigma^2_{p,\text{tot}} - \sigma^2_{p,\text{act}} = 2W^2\sigma_{bl,al} + 2(1-W)^2\sigma_{bl,al} + 2W(1-W)(\sigma_{bl,al} + \sigma_{al,al}).
\]

(4.16)

In the worst case, if the covariance between the benchmark and the active return is positive and non-zero, and if the cross-covariances and covariance amongst the active portfolios are also positive, then the discrepancy in overall portfolio risk in eq. (4.16) is positive. As covariance is not scaled like correlation, it is therefore unbounded. As such, there is no upper limit to the discrepancy defined in eq. (4.16). The inherent implication is that the risk-adjusted overall portfolio performance may be grossly overstated under the independence assumption.

### 4.2 Active and overall performance assessment

The multi-level hierarchical portfolio management (HPM) introduces at least two types of performance assessment. In the lower level, there is the active performance of each manager. Then aggregated to the overall level, the appraisal of the total performance informs the investor of the bottom line of all his investments.
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Clearly, the active performance assessment is more than adequate from the managers’ perspective. Since each manager is assigned part of the total portfolio, he can only be held responsible for his own actions and of no one else's. As the remuneration scheme is also commonly linked to the manager's active performance to improve agent behavior, an accurate measure of the active performance assessment is often the pursuit of practitioners' literature. However from the investor's perspective, the only important assessment is the assessment of his overall portfolio. For the investor, it is important to know whether his managers have realized wealth creation or not. As we have seen, the aggregated risk is partly driven by the correlations between the sub-portfolios assigned to different managers. If the benchmarks are efficient, then it is possible that the Sharpe ratio of the overall portfolio actually decreases as a result of bigger rise in overall risk than in overall return. Coordination from the investor is needed, as he is the only person at the top of the decision chain who has an overview of the sub portfolios. If the benchmarks are inefficient, then there are other investment opportunities that dominate the benchmark choice and await discovery. Better coordination and improved information exchange between the different levels can improve overall portfolio risk and return level, as Chapter 2 has shown for the single-period case and Chapter 3 for the multi-period context.

This section starts with the definitions of different active performance measures in the literature and the difference between the two measures. As the AR and IR are ratios by definition, it is important to understand their behaviors at the asymptotes. The reason is that the difference in assessment of the same investment opportunity at the asymptotes can be massive. Then, we show the insufficiency of the active assessment from an investor’s perspective, as overall performance assessment may differ dramatically from the active assessment. In particular, a threshold value of the active performance measure is provided that marks the divide between value-added by active management in the overall portfolio's context and otherwise. Hence, the investor no longer assesses the active manager based on the active performance alone, yet the manager is also appraised based on his contribution to the overall portfolio.
4.2.1 The active performance measures

The appraisal ratio (AR) and information ratio (IR) use different definition of active return.
Different active return has different active risk.
AR and IR only produce identical performance assessment at beta neutrality ($\beta = 1$).

In the literature, there are two definitions commonly used for the abnormal return above the benchmark return. The active return is the difference between the holding portfolio return and the benchmark return. The residual return is the alpha obtained by running a regression of the portfolio return on the benchmark return. The appraisal ratio (AR) in Treynor and Black (1973) divides the residual return by the residual risk of the regression. Others (see e.g. Rudd and Clasing (1982), Grinold (1989), Grinold and Kahn (1999)) divide the active return by its own risk and named it the information ratio (IR). This section illustrates the difference between the two measures.

4.2.1.1 The appraisal ratio (AR)

By definition, the purpose of the appraisal ratio (AR) is to add performance to an existing optimal portfolio. The active portfolio $a$ is constructed using securities that have a positive residual return, $\alpha$, which is determined by the Sharpe (1963) Single Index Model (SIM)

$$\bar{r}_{if} = \alpha_i + \bar{R}_{im} \bar{r}_{mf} + \tilde{e}_i$$  \hspace{1cm} (4.17)

where $\bar{r}_{if}$ denotes the stochastic return of security $i$ in excess of the risk free rate $r_f$ and $\bar{r}_{mf}$ is the market portfolio return in excess of the risk free rate. The active portfolio $a$ improves the standard Sharpe ratio measure of the market portfolio $M$ as the extra risk-adjusted performance can be simply added to the market portfolio’s SR due to the zero
correlation between the market portfolio return and the residual return $\alpha$ in the SIM, as Figure 4.2 illustrates.

Compare to the market portfolio $M$, the combination of the active portfolio $a$ and the market portfolio $M$ in Figure 4.2 produces a new tangent portfolio $P$ with a higher risk adjusted return or SR. As such, the Sharpe ratio (SR) of the new optimal portfolio $P$ can be written as the quadratic sum of the market portfolio's SR and the appraisal ratio.

$$SR_P^2 = SR_a^2 + \left( \frac{\alpha}{\sigma_a} \right)^2$$

(4.18)

Since the decomposition of the Sharpe measure is only valid for the optimal portfolio, each portfolio manager must seek to construct an active portfolio $a$, such that the appraisal ratio in eq. (4.18) is maximized (Treynor and Black (1973)). Sharpe (1994) has pointed out the weakness of eq. (4.18) because a negative alpha also adds value to the overall portfolio in eq. (4.18). Hence, performance assessment using the squared SR does not always yield the correct performance judgment.

**Figure 4.2:** adding extra return to the market portfolio $M$ while reducing the portfolio risk. Portfolio $a$ contains all the investments that have a positive $\alpha$ with respect to the market. By combining the active portfolio $a$ and the market portfolio $M$, a new tangent portfolio $P$ can be generated with a higher risk adjusted return or Sharpe ratio.
4.2.1.2 The information ratio (IR)

Following Sharpe (1966), a number of authors associated with BARRA introduced a new return-to-volatility measure to assess the active portfolio performance: the information ratio (IR). The IR is simply the SR where portfolio return is measured against the benchmark return instead of a risk free rate.\textsuperscript{16} Let $\tilde{r}_{pf}$ denote the portfolio’s return in excess of the risk free rate $r_f$ and $\tilde{r}_{bf}$ the benchmark portfolio’s excess return. The Single Index Model in terms of the return difference of excess returns becomes

$$
\tilde{r}_{pf} - \tilde{r}_{bf} = \alpha_p + (\beta_{pf} - 1)\tilde{r}_{bf} + \tilde{\varepsilon}_p,
$$

where $\alpha_p$ is the mean return difference and $\tilde{\varepsilon}_p$ is the zero-mean stochastic part of return difference. Let $\sigma_{bf}^2$ denote the benchmark variance. From eq. (4.19) the covariance between the differential excess return and the benchmark excess return is

$$
\text{cov}(\tilde{r}_{pf} - \tilde{r}_{bf}, \tilde{r}_{bf}) = (\beta_{pf} - 1)\sigma_{bf}^2.
$$

Unlike the original Single Index Model, the covariance between the differential excess return and the benchmark excess return is zero if and only if beta neutrality ($\beta_{pf} = 1$) holds. Let $\tilde{r}_{bf}$ denote the expected excess return on the benchmark. The IR of the active portfolio in the overall portfolio $p$ is defined as:

$$
IR_p = \frac{E(\tilde{r}_{pf} - \tilde{r}_{bf})}{\sigma(\tilde{r}_{pf} - \tilde{r}_{bf})} = \frac{\alpha_p + (\beta_{pf} - 1)\tilde{r}_{bf}}{\sqrt{(\beta_{pf} - 1)^2 \sigma_{bf}^2 + \sigma_{ep}^2}}
$$

(4.20)
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The IR in eq. (4.20) provides identical information as AR when beta neutrality holds. The benchmark return part in the numerator and the benchmark risk part in the denominator nullify and disappear. The preliminary results hint that active performance assessment based on the IR and AR is only identical at beta neutrality. If beta deviates from neutrality, then the active performance measures provide different performance assessment of the same fund. Next, we explore how big the discrepancy between the two measures can become.

4.2.1.3 Differences between AR and IR

From eq. (4.20) we know that IR only produces the same active performance assessment as AR at beta neutrality. Then, the assessment by the different active performance measures of a portfolio with beta other than unity will be different. The question now is how big the difference is. Additionally, since both measures are by definition ratios, the behavior of the measures near the asymptotes cannot be ignored. In this subsection, we first reformulate the definitions of AR and IR in terms of beta. Then, by examining whether the two functions are identical over the feasible domain of beta and its behavior around the asymptotes, we explore the difference between the two measures.

The details of the results are in Appendix A1. Here I only summarize the most important findings. First of all, if beta neutrality holds, then the active performance measures, AR and IR, produce identical performance assessment of the same active portfolio. However, if beta neutrality does not hold, then different active performance measures yield different active assessments as the active performance assessments diverge from each other for other values of beta other than unity. The third finding is that the difference between \( AR(\beta_{pf}) \) and \( IR(\beta_{pf}) \) may become massive for portfolios with beta close to the upper limit of \( AR(\beta_{pf}) \) as the AR value goes to infinity in case there is an outperformance with respect to the benchmark. The practical implication here is that the active portfolio's

\( ^{26} \) See Sharpe (1994) pg. 51 for more detailed discussion.
performance assessments may be meaningless unless the portfolio beta is well behaved and kept at or very close to unity.

### 4.2.2 Insufficiency of the active performance measures

Positive information ratio (IR) does not automatically imply a portfolio Sharpe ratio (SR) higher than the benchmark SR.

By construction, neither AR nor IR conveys more information than the active portfolio performance. More specifically, the AR was created with the sole purpose of adding extra risk-adjusted return to an existing optimal market portfolio. Therefore, to try to construct an overall optimal portfolio in the mean-variance (MV) space based on AR or IR alone may prove to be the Labor of Sisyphus: more information is needed than what the IR or AR holds if one wants to construct the overall optimal portfolio in the MV space. This subsection explores the insufficiency of the active performance measures for constructing overall optimal portfolio in the MV space.

Consider an actively managed investment portfolio $p$ consisting of a benchmark and a self-financing active portfolio, just as in Chapter 2. Similar to Hallerbach (2006), I write the excess return of this portfolio $p$ as

$$\tilde{r}_{pf} = \tilde{r}_{bf} + (1 + w)(\alpha_p + \tilde{e}_p)\Big|_{w=0},$$

with $w$ denoting the additional weight of the active portfolio and $w$ conditioned to be zero because the optimized active portfolio is fixed given the constraints. If portfolio $p$ is the

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27 In Greek mythology, Sisyphus was a devilish mischief who tried to get his way by playing tricks on the gods. He even chained Thanatos who personified death in order to cheat death. Hades, the god of the underworld, had to intervene and Sisyphus was condemned to eternal hard labor by the gods. Sisyphus has to push a bolder up a hill, only to have it roll back again down to the foot of the hill each time he finally gets it to the top. Labor of Sisyphus is a metaphor for all difficult and repetitive labor that is frustrating, unrewarding, and above all futile.
tangent portfolio, then the Sharpe ratio (SR) of portfolio $p$, denoted by $SR_{p^*}$, is at its maximum. The SR of the optimal portfolio $p$ is

$$SR_{p^*} = \frac{\bar{r}_{bf} + (1 + w)\alpha_p}{\sqrt{\sigma_{bf}^2 + (1 + w)^2 \sigma_{ep}^2 + 2(1 + w)\sigma_{ep,bf}}}$$

where $\sigma_{ep,bf}$ is the covariance between the active portfolio return and benchmark return.

The Euler equation for the overall portfolio at MV-optimality must hold:

$$\frac{d SR_{p^*}}{d w} = \frac{\alpha_p}{\sigma_{pf}} - \frac{\bar{r}_{pf}}{\sigma_{pf}^2} \left[ \frac{\sigma_{ep}^2 + \sigma_{ep,bf}}{\sigma_{pf}^2} \right] = 0 ,$$

which can be written as

$$AR_{p^*} = SR_b \left[ \frac{\rho_{ep,bf} + \frac{\sigma_{ep}}{\sigma_{bf}}}{1 + \rho_{ep,bf} \frac{\sigma_{ep}}{\sigma_{bf}}} \right] \leq SR_b . \quad (4.21)$$

The detailed derivation of equation (4.21) can be found in Appendix A2. The inequality in (4.21) is satisfied for the restrictions of $i$) limited active portfolio volatility ($\sigma_{ep} < \sigma_{bf}$) and $ii$) positive benchmark Sharpe ratio ($SR_b > 0$). The correlation between the active portfolio return and the benchmark return ($\rho_{ep,bf}$) is restricted between positive and negative unity. In case of the SIM, $\rho_{ep,bf}$ is by construction zero and the inequality of (4.21) still holds under the restrictions. Thus, portfolio $p^*$ in the SIM is still the optimal choice. If the correlation between the active return and the benchmark return are perfect
positive ($\rho_{e_p,b_f} = 1$), then the appraisal ratio of the optimal portfolio is just the benchmark’s Sharpe ratio ($AR_{p^*} = SR_b$). In the other extreme case of perfect negative correlation ($\rho_{e_p,b_f} = -1$), the AR is negative because of limited active portfolio volatility ($\sigma_{e_p} < \sigma_{b_f}$) restriction. The first-order condition is only violated when the AR of the holding portfolio is greater than the benchmark's Sharpe ratio ($AR_p > SR_b$).

Figure 4.4 illustrates.

The minimum risk "bullet" frontiers in Figure 4.4 can be either tracking error efficient or globally efficient: if benchmark $b$ is inefficient, then the minimum risk frontiers in Figure 4.4 are tracking error efficient; Otherwise, the bullet frontiers in Figure 4.4 are globally efficient, as there exist no other choices in the investment space that would produce a portfolio with a higher return at the same level of risk or lower risk at equal level of return.

In Figure 4.4, the diagram on the left illustrates the case in which positive AR has actually reduced the performance of the overall portfolio. Clearly the optimal portfolio $p^*$ in the left diagram of Figure 4.4 is the benchmark. No deviations from the benchmark portfolio should be allowed, as the first order condition in (4.21) clearly holds. Had portfolio $p$ been
chosen, then the SR of the overall portfolio diminishes because we are moving away from the optimal portfolio. The diagram on the right hand side in Figure 4.4 shows the case when the first order condition in (4.21) is violated: \( \text{AR}_p > \text{SR}_b \). Consequently, the overall portfolio performance can be improved upon, as the benchmark does not represent the optimal portfolio with the highest return per unit of risk.

The first order condition in (4.21) clearly indicates when an overall portfolio is optimal or not. Although portfolio \( p' \) has realized a positive AR in Figure 4.4, but the overall portfolio's SR is identical to the benchmark SR. After accounting for all the costs, it is very likely that portfolio \( p' \) has done more evil than good. So, what is the minimum value of AR or IR to realize \( \text{SR}_p > \text{SR}_b \) ?

### 4.2.3 Minimum active performance required

A performance metric should be able to specify whether active management has added value to the overall portfolio or not. Per definition, active management has added value if and only if the SR of the overall portfolio is higher than the SR of the benchmark.

\[
\text{SR}_p = \frac{\bar{r}_{pf}}{\sigma_{pf}} > \frac{\bar{r}_{bf}}{\sigma_{bf}} = \text{SR}_b
\]  

(4.22)

For the residual return-based performance measure AR, active management has added value for any portfolio \( p \) if condition (4.22) holds:

\[
\text{SR}_p = \frac{\alpha_p + \beta_p \bar{r}_{bf}}{\sigma_{pf}} > \frac{\bar{r}_{bf}}{\sigma_{bf}} = \text{SR}_b.
\]
Using the definitions of AR, and after some simplification, condition (4.22) can be expressed in terms of the benchmark SR and the portfolio's correlation with the benchmark:

\[ AR_p > SR_b \left[ \frac{1 - \rho_{pf,bf}}{\sqrt{1 - \rho_{pf,bf}^2}} \right] \quad \text{for} \quad \left| \rho_{pf,bf} \right| < 1. \quad (4.23) \]

The details of derivation are given in Appendix A2. Here the minimum value of AR is determined by the correlation between the overall portfolio and the benchmark \( \rho_{pf,bf} \). As this correlation goes to positive unity, the term between square brackets becomes smaller and smaller. The implication is that the minimum value of AR also decreases. In the limit case in which the correlation becomes perfect positive \( \rho_{pf,bf} = 1 \), a positive AR always indicates value-added by the active management. This behavior of AR at the limit is due to the definition of the SIM. For portfolio \( p \) it can be written as:

\[ \tilde{r}_p = \alpha_p + \beta_p \tilde{r}_b + (1 - \beta_p) \tilde{r}_f + \tilde{\epsilon}_p \quad (4.24) \]

When the quadratic term of the correlation between portfolio \( p \) and benchmark \( b \) (\( R^2 \) of regression (4.24)) approaches unity, \( \tilde{\epsilon}_p \) disintegrates and its volatility goes to zero. If \( \alpha_p \) is positive, then \( AR_p \) goes to positive infinity. As such, a positive alpha implies that the managed portfolio \( p \) with probability one outperforms a portfolio that consists of a fraction \( \beta_p \) invested in the benchmark and the remainder invested in the risk free rate: active management has added value by investing in an arbitrage opportunity!

For the differential return-based TEV and IR, condition (4.22) can be rewritten as
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\[ IR_p > SR_b \left[ \frac{\sigma_{pf} - \sigma_{bf}}{\sigma_{xp}} \right] = SR_b \left[ 1 + \frac{\sigma_{bf}}{\sigma_{xp}} \right] \left( \frac{\sigma_{bf}}{\sigma_{xp}^2} + 2 \rho_{xp,bf} \right) - \frac{\sigma_{bf}}{\sigma_{xp}} \] \tag{4.25} \]

The details of derivation can be found in Appendix A2. In the extreme case with perfect positive correlation between the active portfolio return and the excess benchmark return, \( \rho_{xp,bf} = 1 \), condition (4.25) for value-added reduces to \( IR_p > SR_b \), which also indicates sub optimality of the chosen portfolio according to eq. (4.21). Of course, if the active portfolio adds value to the overall portfolio, then the weight of the active portfolio needs to be scaled up. Unfortunately, the no-short sale constraint and tracking error restrictions will limit the weight of the active portfolio. Also, the number of mispriced securities may be limited given the (weak) market efficiency. For the other extreme of \( \rho_{xp,bf} = -1 \), the value-added condition (4.25) boils down to \( IR_p > -SR_b \), which is difficult to interpret because negative SR is meaningless. At zero correlation between the active portfolio return and the excess benchmark return, \( \rho_{xp,bf} = 0 \), the minimum IR value for value-added equals

\[ IR_p > SR_b \left[ 1 + \frac{\sigma_{bf}}{\sigma_{xp}^2} - \frac{\sigma_{bf}}{\sigma_{xp}} \right] \]

The value between the brackets is positive, which makes the minimum active portfolio performance requirement a feasible mandate.

To use eq. (4.25) in practice, precise knowledge about i) the correlation between the active portfolio return and the excess benchmark return and ii) the magnitude of the tracking error volatility is needed to determine the minimum active performance required. A correct ex-ante critical value for IR is very difficult to compute due to the need of all the information. Firstly, it is very difficult to know the precise correlation between each active portfolio and
the corresponding benchmark. Secondly, it is not certain that the portfolio managers would fully use their risk budgets. Hence, we do not know how much the tracking error volatility ($\sigma_{cp}$) would be, a priori. When the overall portfolio consists of multiple mandated portfolios, each with a corresponding benchmark, the situation becomes even more complex. Then, not only is the pair-wise correlation between each active portfolio and the corresponding benchmark needed, but also i) all the cross correlations between the active portfolios and the other benchmarks, and ii) the correlations amongst all active portfolios.

### 4.3 Empirical illustration

This section shows the empirical impact of the independence assumption in practice. Theoretically the impact can be massive, as section 4.1.2 has shown. Although the augmented covariance matrix (ii) is assumed to be true, yet in reality the augmented matrix (iv) probably holds due to the existing correlations. The results in this section give an indication of i) the size of overall risk increment due to correlations, ii) the impact on overall portfolio Sharpe ratio, and iii) the minimum active performance required in terms of IR.

The illustrative example mimics the investment behavior of a U.S. investor who entrusts Fidelity Investments with all his investment needs. The investor invests in both the U.S. domestic market and in the international emerging markets. As such, the investor's overall holding portfolio contains four assets: U.S. equity, U.S. bonds, emerging markets equity and emerging markets bonds. All asset class funds are product of Fidelity Investments. The holding weight of the four assets in the overall portfolio is identical: the overall portfolio is an equally weighted portfolio of the four asset funds. Besides its simplicity, this asset allocation setup does not tilt the aggregated volatility or return because the investor's overall holding portfolio exposure to each asset is equal. Ceteris paribus, the change in the

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28 The largest provider of mutual fund products in U.S.A. Established in 1930, Fidelity Investments serves more than 20 million individuals and institutions, and holds more than US$ 1100 billion in managed assets as of June 30 2005.
overall portfolio’s volatility can only be attributed to the change in the correlation assumptions in the funds level.

The active portfolio performance for the U.S. investor is measured in terms of both IR and AR. As it turns out, there is discrepancy between the assessments by different active performance measure. Also, the correlation structure has significant influence on the overall portfolio’s SR. Furthermore, an after cost IR of 0.40 is not enough to produce an overall SR higher than the benchmark SR. Given the transaction costs, a pre-cost IR of 0.50 may not be sufficient to add value to the overall portfolio. Last but not least, the influence of the correlated portfolios can be mitigated through minimizing the active portfolio volatility.

4.3.1 The Fidelity mutual funds

The data contains the monthly total return index of 4 benchmarks and 4 mutual funds for the period between September 2001 and September 2006. All data come from the CRSP dataset Via WRDS29. The total number of in-sample observations equals 60. A fund must i) be a fund that has been traded during the sample period and ii) have an explicitly stated benchmark for it to be considered. The mutual funds are arbitrarily chosen from a set of Fidelity mutual funds that fit the specification of the investor. The first mutual fund Fidelity Advisor Equity Income has S&P 500 as its benchmark. Fidelity Emerging Markets has the MSCI EAFE index as its reference point. The Lehman Brothers U.S. Aggregate Bond Index Fidelity serves as the benchmark of U.S. Bond Index Fund and finally, Fidelity Advisor Emerging Markets Inc T. uses Lehman Brothers Global Aggregate Bond Index. Table 4.1 summarizes the descriptive statistics of the mutual funds and their benchmark.

In Table 4.1, it is remarkable that despite the Internet bubble at the turn of the second millennium and the ensuing volatile years, a rational investor should be satisfied with the

29 Wharton Research Data Service
annual average performance of the equity funds. Given that the 3-months U.S. treasury bills has grossed an average return of 2.22% per annum for the sample period, the mutual funds not only have realized a positive equity premium, they also have realized a positive active premium (defined as the return differential between the portfolio and its benchmark). Although less impressive in size, the fixed income mutual funds have also obtained a higher return than the risk free rate and its benchmark. Thus, from the return perspective the first observations suggest that all the mutual fund managers deserved every penny of their fee.

<table>
<thead>
<tr>
<th>Mutual fund</th>
<th>Mean</th>
<th>Median</th>
<th>Maximum</th>
<th>Minimum</th>
<th>Std. Dev</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Jarque-Bera</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fidelity Advisor Equity Income instl</td>
<td>10.02%</td>
<td>12.57%</td>
<td>103.32%</td>
<td>-139.06%</td>
<td>43.76%</td>
<td>-0.62</td>
<td>4.29</td>
<td>7.94</td>
<td>0.019</td>
</tr>
<tr>
<td>Fidelity Emerging Markets</td>
<td>27.26%</td>
<td>31.45%</td>
<td>158.13%</td>
<td>-134.33%</td>
<td>67.24%</td>
<td>-0.48</td>
<td>2.71</td>
<td>2.52</td>
<td>0.283</td>
</tr>
<tr>
<td>Fidelity US Bond</td>
<td>4.89%</td>
<td>7.57%</td>
<td>33.71%</td>
<td>-41.48%</td>
<td>13.74%</td>
<td>-0.83</td>
<td>4.32</td>
<td>11.29</td>
<td>0.004</td>
</tr>
<tr>
<td>Fidelity Advisor Emerg Mkts Inc T</td>
<td>14.88%</td>
<td>19.13%</td>
<td>81.33%</td>
<td>-66.45%</td>
<td>31.96%</td>
<td>-0.62</td>
<td>3.54</td>
<td>4.69</td>
<td>0.100</td>
</tr>
<tr>
<td>Benchmark index</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>7.56%</td>
<td>11.11%</td>
<td>105.62%</td>
<td>-130.42%</td>
<td>44.14%</td>
<td>-0.48</td>
<td>4.14</td>
<td>5.62</td>
<td>0.060</td>
</tr>
<tr>
<td>MSCI EAFE</td>
<td>14.70%</td>
<td>21.92%</td>
<td>118.99%</td>
<td>-128.57%</td>
<td>46.64%</td>
<td>-0.68</td>
<td>4.02</td>
<td>7.25</td>
<td>0.027</td>
</tr>
<tr>
<td>Lehman US Aggregate Bond Index</td>
<td>4.78%</td>
<td>7.75%</td>
<td>31.76%</td>
<td>-40.34%</td>
<td>13.72%</td>
<td>-0.90</td>
<td>4.03</td>
<td>9.09</td>
<td>0.011</td>
</tr>
<tr>
<td>Lehman Global Aggregate Bond Index</td>
<td>6.85%</td>
<td>7.12%</td>
<td>57.67%</td>
<td>-43.88%</td>
<td>19.88%</td>
<td>0.12</td>
<td>3.18</td>
<td>0.23</td>
<td>0.892</td>
</tr>
</tbody>
</table>

Table 4.1: descriptive statistics of the 4 Fidelity mutual funds and their benchmark for the period of Oct./2001 - Sept./2006. Number of observations equals 60.

However, closer inspection of the return volatilities reveals that the conclusion in the last paragraph may have been drawn too hastily. First of all, most mutual funds have a bigger return spread\(^{30}\) when compared to the benchmarks with only one exception: Fidelity Advisor Equity Income. Also, the standard deviation of most active mutual funds is generally higher than that of the passive benchmarks. Thus, the risk adjusted return of the mutual funds may not be higher than that of the benchmarks. Moreover, the mutual funds' return distribution has a significant negative skew with significant fat tails. It seems that there are more extreme negative realizations in the sample period, which is partly confirmed by the fact that the mean is smaller than the median. In Table 3.1, the null

\(^{30}\) The spread is defined by the difference between the sample minimum and sample maximum return.
Chapter 4  How should investors assess active managers?

hypothesis of a normal distribution\(^{31}\) is rejected at the 5\% level for the mutual funds Fidelity Advisor Equity Income and Fidelity U.S. Bond Index Fund, and the benchmark indexes MSCI EAFE and Lehman Brothers U.S. Aggregate Bond Index. At the 1\% level, the null hypothesis is only rejected for the Fidelity U.S. Bond Index Fund.

The correlations between the mutual funds active premiums and the benchmark index returns are presented in Table 4.2.

<table>
<thead>
<tr>
<th></th>
<th>S&amp;P 500</th>
<th>MSCI EAFE</th>
<th>F. Advisor Equity Income Instl</th>
<th>F. Emerging Markets</th>
<th>F. US Bond</th>
<th>F. Advisor Emerg Mkts Inc T</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500</td>
<td>100%</td>
<td>84%</td>
<td>-31%</td>
<td>19%</td>
<td>47%</td>
<td>60%</td>
</tr>
<tr>
<td>MSCI EAFE</td>
<td>100%</td>
<td>100%</td>
<td>14%</td>
<td>19%</td>
<td>51%</td>
<td>41%</td>
</tr>
<tr>
<td>F. US Bond Index</td>
<td>100%</td>
<td>71%</td>
<td>7%</td>
<td>6%</td>
<td>3%</td>
<td>5%</td>
</tr>
<tr>
<td>F. Global Aggregate Bond Index</td>
<td>100%</td>
<td>14%</td>
<td>14%</td>
<td>13%</td>
<td>32%</td>
<td></td>
</tr>
<tr>
<td>F. Advisor Equity Income Instl</td>
<td>100%</td>
<td>8%</td>
<td>15%</td>
<td>2%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F. Emerging Markets</td>
<td>100%</td>
<td>4%</td>
<td>42%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F. US Bond</td>
<td>100%</td>
<td>53%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F. Advisor Emerg Mkts Inc T</td>
<td>100%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.2: correlations between the 4 F(idelity) mutual funds returns and their benchmark for the period of Oct./2001 - Sept./2006. The augmented matrix is divided into four quadrants. The left upper quadrant is the benchmark correlation matrix, which contains the correlations between the four benchmark indexes (type (i) correlation). The right lower quadrant is the active correlation matrix that documents the correlation between the active portfolios of the mutual funds (type (iii) correlation). The other two symmetric components contain the correlations between the benchmarks and the active portfolios. The diagonal elements are the correlation between a benchmark with its own active portfolio (type (ii) correlation) and the off diagonal elements are the cross correlation elements (type (iv) correlation). “L.” is the abbreviation of “Lehman”.

In total, there are 18 negative correlations and 38 positive correlations in Table 4.2. The cells highlighted with light gray are cells containing high positive correlations. For example, the MSCI EAFE index has a correlation of 84\% with respect to the S&P 500 index. Also remarkable are the high positive cross correlations between the bond mutual funds and the equity benchmarks in the right upper corner of the matrix. Last but not least, the equity and bond active portfolios in the emerging markets are positively correlated with each other. The cells highlighted with dark gray are the cells containing the highest

\(^{31}\) The p-values in the last column are the probability that a Jarque-Bera statistic exceeds in absolute value the observed value under the null hypothesis. The Jarque-Bera statistic is distributed as \(\chi^2\) with 2 degrees of freedom under the null hypothesis of a normal distribution.
negative correlations. The bond index and equity index in the U.S. are negatively correlated with each other. This observation makes the positive correlation between the bond and equity index portfolios in the emerging markets remarkable. Also, the positive correlation between the bond and equity active portfolios in the same markets stands out. Due to the highly correlated benchmarks, the cross correlation between the mutual funds and other benchmarks than its own is not zero. Consequently, it is highly unlikely that the risk adjusted portfolio return under the independence assumption would be the same as the one obtained after all the correlations have been taken into account.

4.3.2 Portfolio performance assessment

An active manager who has realized an information ratio (IR) of 0.50 is considered as an exceptionally good performing active manager (see e.g. Grinold and Kahn, 1999).

In the sample dataset, an IR of 0.40 is not enough to guarantee a higher portfolio SR than the benchmark SR after having accounted for all the correlations.

A pre-cost IR of 0.50 may be insufficient to assure value-added to the overall portfolio performance.

Our results of the appraisal ratio (AR) as well as the information ratio (IR) reveal that the mutual funds have added value to the overall portfolio, as their AR and IR are relatively high. Unfortunately, the active performance measures give different assessment of the mutual funds. If the investor wants to know how much value the managers have added to the overall portfolio, then the active performance measures do not give a clear-cut picture. Therefore, in the second part, the focus shifts to the aggregated total portfolio return. The Sharpe ratio (SR) defines in one number the overall portfolio's return per unit of risk taken: a higher Sharpe ratio implies a superior overall performance as the investor obtains more return per unit of risk. In Chapter 3 we have seen that the aggregated overall portfolio risk changes when the independence assumption in the correlation structure is lifted. Here, in the sample data, the aggregated total portfolio risk more than doubles when all the
correlations are accounted for. With constant portfolio excess return, the implication for the SR is clearly significant.

At performance assessment, the fundamental problem for the investor is that a positive AR or IR does not automatically lead to an improved overall holding portfolio SR, which is higher than that of the benchmark. Given the four mutual funds, a pre-cost IR of 0.50 may not be a guarantee that the portfolio SR is higher than the benchmark SR. After taking all the costs into account, the holding portfolio’s SR may plunge due to either lower after cost active return or higher overall portfolio risk, or both. The direct consequence is of course that the holding portfolio underperforms the benchmark. Thus, the empirical finding in this chapter underscores the need of assessing the overall portfolio risk instead of the active risk alone.

At the end of this section, a solution is given to mitigate the correlation problem. The basic idea is intuitively straightforward: although correlation between two portfolios can be substantial, but the correlation cannot exceed unity by definition. As covariance is defined as the product of correlation and the standard deviations, the solution lies naturally in minimizing the standard deviations. If the passive benchmark risk is taken as given, the only instrument candidate is then the active portfolio's risk. By minimizing the active portfolio's standard deviation, the additional value to the portfolio variance due to correlation can be nullified. Thus, the bottom line is minimizing active risk while maximizing the active return.

### 4.3.2.1 Active portfolio performance assessment

As a reminder, the information ratio (IR) takes the return difference between the portfolio and the benchmark as the active return (differential return) with the corresponding standard deviation as the active risk. The appraisal ratio (AR) takes the alpha or the

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32 In Grinold and Kahn (2000), an active manager who has realized an IR of 0.50 is hailed as an exceptionally good performing active manager.
33 Transaction costs, load costs (front and back) and expenses defined by the expense ratio.
residual return in the single index model (SIM) with the benchmark as the market portfolio as the active return. The residual risk of the regression is the corresponding active risk. The fundamental problem with AR and IR is twofold. Firstly, there is no clear-cut unambiguous definition of the active return and consequently the active risk. Secondly, there is the correlation problem. Treynor and Black (1973) have tried to elude this problem by dividing the alpha in the single index model of Sharpe (1964) by the residual risk of the regression. However in such a setup, the portfolio return is only uncorrelated with the benchmark index while none of the other 3 correlation types have been addressed.

The aggregated active portfolio return is straightforward. Using equal weighting, it is simply the average of the fund returns. Table 4.3 contains the annualized active returns of each fund and the overall portfolio with the corresponding volatilities. The AR and IR follow from these inputs.

<table>
<thead>
<tr>
<th>Fund</th>
<th>( \alpha_r )</th>
<th>( \beta_r )</th>
<th>( \varepsilon_r )</th>
<th>AR</th>
<th>( r_r - r_{bf} )</th>
<th>( \sigma(r_r - r_{bf}) )</th>
<th>IR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fidelity Advisor Equity Income Instl</td>
<td>2.70%</td>
<td>0.9673</td>
<td>2.78%</td>
<td>0.97</td>
<td>2.45%</td>
<td>2.79%</td>
<td>0.88</td>
</tr>
<tr>
<td>Fidelity Emerging Markets</td>
<td>10.17%</td>
<td>1.1630</td>
<td>11.27%</td>
<td>0.88</td>
<td>12.56%</td>
<td>11.68%</td>
<td>1.08</td>
</tr>
<tr>
<td>Fidelity US Bond</td>
<td>0.12%</td>
<td>0.9971</td>
<td>0.36%</td>
<td>0.34</td>
<td>0.11%</td>
<td>0.36%</td>
<td>0.31</td>
</tr>
<tr>
<td>Fidelity Advisor Emerg Mkts Inc T</td>
<td>11.64%</td>
<td>0.4724</td>
<td>8.90%</td>
<td>1.31</td>
<td>8.02%</td>
<td>9.32%</td>
<td>0.86</td>
</tr>
<tr>
<td>Overall portfolio</td>
<td>3.66%</td>
<td>1.2508</td>
<td>4.31%</td>
<td>0.85</td>
<td>5.79%</td>
<td>4.56%</td>
<td>1.27</td>
</tr>
</tbody>
</table>

Table 4.3: performance assessment of the mutual funds’ active portfolio. The subscript F stands for fund and bF stands for benchmark of the fund. The appraisal ratio (AR) is the ratio of the intercept (\( \alpha_r \)) and the residual error (\( \varepsilon_r \)) of the regression in which the mutual fund return is regressed against its benchmark return. \( \beta_r \) is the coefficient of the benchmark return in the regression. It represents the factor loading of the benchmark return on the mutual fund return. The information ratio (IR) is the ratio of the differential return and the corresponding volatility measured by the standard deviation of the differential return.

The first striking observation in Table 4.3 is the different values of AR and IR for each fund and the equally weighted overall portfolio. It seems that the active return performance appraisal for the same mutual fund does not match. The reason for this lies in the beta value of AR. The IR definition implicitly puts the beta at unity. However, the beta values in Table 4.3 are not exactly unity. As we already know, the AR and IR values of an active return only match under beta neutrality (\( \beta_{pf} = 1 \)). Figure 4.5 illustrates the values of AR (gray line) and IR (black line) for different level of \( \beta_{pf} \). In Figure 4.5, the x-axis
is $\beta_{af}$ and the y-axis gives the value of AR and IR of the mutual fund. The difference in active performance assessment between the AR and the IR are illustrated for each mutual fund in the right column of Figure 4.5.

At beta neutrality the difference between the active performance measures is clearly zero. The fund "Fidelity Advisor Equity Income Instl." seems to be the only exception, as its difference between the active performance measures never hits the zero mark. The reason is that both the AR as well as the IR has an upper bound in $\beta$ that is smaller than unity.

Table 4.4 summarizes the limits of the active performance measures.

<table>
<thead>
<tr>
<th>Fund</th>
<th>IR Lower bound</th>
<th>IR Upper bound</th>
<th>AR Lower bound</th>
<th>AR Upper bound</th>
<th>AR - IR Lower bound</th>
<th>AR - IR Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fidelity Advisor Equity Income Instl</td>
<td>--</td>
<td>0.99127</td>
<td>--</td>
<td>0.99123</td>
<td>-0.99123</td>
<td>-0.00004</td>
</tr>
<tr>
<td>Fidelity Emerging Markets</td>
<td>--</td>
<td>1.53930</td>
<td>--</td>
<td>1.44173</td>
<td>-1.44173</td>
<td>-0.09756</td>
</tr>
<tr>
<td>Fidelity US Bond</td>
<td>--</td>
<td>1.00126</td>
<td>--</td>
<td>1.00126</td>
<td>-1.00126</td>
<td>0.00000</td>
</tr>
<tr>
<td>Fidelity Advisor Emerg Mkts Inc. T</td>
<td>--</td>
<td>1.79234</td>
<td>--</td>
<td>1.60769</td>
<td>-1.60769</td>
<td>-0.18465</td>
</tr>
</tbody>
</table>

Table 4.4: upper and lower limits of the active performance measures for the sample of Fidelity mutual funds. For each mutual fund the appraisal ratio (AR) and information ratio (IR) are given in column 2 and 3. The last column contains the interval limits of the discrepancy between the active performance measures.

Clearly, the active performance measures do not yield a straightforward consistent appraisal of the mutual funds. Then, which measure represents the true or correct measure of the active performance? From the investor's perspective the answer is *neither* because the active performance measures do not convey any information about the overall portfolio performance. What complicates things is that the portfolio's overall risk changes due to different assumptions on the covariance structure. Next we explore the effects of the different assumptions on the covariance structure and its consequences on the overall portfolio's level of risk.
Figure 4.5: discrepancy in active performance assessment for the mutual funds. The left column of diagrams contains both the appraisal ratio (AR) and the information ratio (IR) for different values of $\beta_{pf}$ on the x-axis for each mutual fund. In the right column the discrepancy in active performance assessment for each mutual fund is presented in a diagram for different value of $\beta_{pf}$. The value of the 
asymptotic bounds for $\beta_{pf}$ is given in Table 4.4.
### 4.3.2.2 Overall portfolio risk and the Sharpe ratio

The overall aggregated portfolio risk heavily depends on the assumption made on the covariance structure. When all correlations have been taken into account, the aggregated portfolio risk containing the sample mutual funds almost doubles. Table 4.5 summarizes the augmented covariance matrices in section 4.1.2.

<table>
<thead>
<tr>
<th>Case Description</th>
<th>Augmented Covariance Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Benchmark</td>
<td>0.40% 100.00%</td>
</tr>
<tr>
<td>2) Independent active portfolios</td>
<td>0.40% 73.68%</td>
</tr>
<tr>
<td>3) Independent active portfolio correlated with own benchmark</td>
<td>0.40% 0.00%</td>
</tr>
<tr>
<td>4) Correlated active portfolios</td>
<td>0.40% 0.00%</td>
</tr>
<tr>
<td>5) Full correlation</td>
<td>0.40% 0.00%</td>
</tr>
</tbody>
</table>

Table 4.5: five augmented covariance matrices of the overall portfolio. The five cases are the cases discussed in section 4.1.2.

Table 4.5 documents the augmented covariance matrices of the aforementioned 5 cases starting with the "benchmark" and ends with the "full correlation" case. Each augmented matrix is divided into 4 quadrants: the left upper quadrant is the benchmark matrix; the right lower quadrant is the active portfolio covariance matrix with the remaining quadrants.
as the symmetric correlating parts. The first number in each quadrant represents the variance attributable to that quadrant and the second number represents the weight of the quadrant risk in the aggregated overall portfolio risk. For example, in the "benchmark" case the total portfolio variance is simply the aggregated variance of the benchmarks, which is 0.40% and it is the total portfolio risk. The double percentage sign indicates that the number is a variance instead of a standard deviation. Last, the highlighted cells indicate which part of the quadrant covariance matrix has been accounted for. For instance, only the diagonal values in the active covariance matrix have been taken into account in the "independent active portfolio" case. In the "correlated active portfolio" case, all the correlating cells are also taken into the calculation.

As the numbers in Table 4.5 show, the overall portfolio risk changes dramatically as more and more correlations were taken into account. In the "benchmark" case, the benchmark risk represents the total portfolio risk. Yet in the "full correlation" case, that same level of benchmark risk only accounts for 49.61% of the total portfolio risk. Thus, more than half of the total portfolio risk is attributed to the active portfolios and the correlations among the portfolios. Consequently, the overall portfolio performance cannot escape the negative influence at a constant level of total portfolio excess return.

The benchmark SR equals 0.99. If the active portfolios are assumed to be totally uncorrelated, then the overall portfolio SR equals 1.63, as illustrated in Figure 4.6. That number plummets to 1.34 when all the correlations were properly taken into account.

Although the aggregated portfolio SR is still higher than the benchmark SR, but the difference in portfolio SR of almost 0.30 cannot be ignored. According to Grinold and Kahn (1999) a manager who has realized an IR of 0.50 is considered to be extremely good, a portfolio manager that produces an IR of 1.27 must be a super star among his colleagues. Hence, the fact that the portfolio SR is higher than the benchmark SR is not an enigma. Then, the question becomes whether it holds true that a positive IR or AR always lead to a higher portfolio SR than the benchmark SR. In plain English, is it enough for the investor
to only focus on the active performance? Unfortunately, the answer is no! The next section explains why.

![Figure 4.6: aggregated overall portfolio performance for different correlation structure. In the expected return and portfolio risk space, the annualized overall portfolio's performance in terms of Sharpe ratio (SR) changes significantly for different correlation structure assumptions between the sub portfolios.](image)

### 4.3.2.3 When is active portfolio performance sufficient?

As we have seen in the last section, positive AR and IR realized by the mutual fund managers have improved the overall portfolio performance. Even after taking all the correlation into account, the portfolio SR is still significantly higher than the benchmark SR. The explanation lies in the fact that the active portfolio managers have performed extremely well over the sample period. This has led us to ponder whether it is enough to just contemplate the active performance alone at performance assessment. Unfortunately, a positive IR does not always lead to a higher portfolio SR as empirical evidences in Figure 4.7 will show. The active risk added to the overall portfolio is constrained by the tracking
error volatility restrictions, the only remaining instrument influencing the active performance measure is the active return. In Figure 4.7 I hold the risk structure constant and only change the level of active return. Then, by calculating the SR that corresponds to the overall portfolio containing both the benchmark and the active portfolios we gain insight of whether the active portfolio has added value or not. What is more interesting is that the intersection point of the overall portfolio SR and the benchmark SR gives the minimum IR needed to add value. Figure 4.7 illustrates.

Figure 4.7: active performance (IR) versus overall performance (SR). The x-axis is the overall portfolio return.

In Figure 4.7: the x-axis in is the aggregated return of the holding portfolio; the information ratio (IR) is the ratio of the differential return and the corresponding standard deviation of the differential return; the Sharpe ratio (SR) is the ratio of the overall portfolio excess return and the standard deviation of the excess return; the horizontal solid line is the SR of the benchmark portfolio at different level of overall portfolio return; the dashed lines represent the IR and SR of the case in which the active portfolios are assumed to be independent. The other solid lines represent the IR and SR of the case whereby full
correlation has been taken into account; the vertical solid lines cross the SR lines where the SR of the actively managed portfolios intercepts that of the benchmark.

The overall benchmark SR in Figure 4.7 is constant for the entire domain because the aggregated benchmark composition is fixed. The solid vertical lines give the intersection points of the overall portfolio SR and the benchmark SR. Only the extreme cases, the "independent active portfolios" and "full correlation" cases, are illustrated in Figure 4.7. Clearly, any IR lower than 0.30 will not produce a portfolio SR higher than the benchmark SR in the "independent active portfolios" case. In the "full correlation" case, it is even more dramatic: any IR lower than 0.47 will not yield an overall portfolio SR that is higher than the benchmark SR.

The problem aggravates when we take transaction and other costs into account. Typically, the maximum front-end load\(^{34}\) that mutual funds may charge is 5\(^{\%}\)\(^{35}\) and an average annual expense ratio of 1.54\(^{\%}\)\(^{36}\) in 2005. The passive index funds have an expense ratio as low as 0.25\(^{\%}\). For example, the Vanguard 500 index that tracks the S&P 500 index has an expense ratio of merely 0.18\(^{\%}\). Needless to say, the impact of the costs cannot be neglected.

### 4.3.2.4 A remedy to the correlation problem

The overall portfolio risk depends on i) the benchmark volatilities, ii) the active portfolios' risk, and iii) the correlating components. Besides the benchmark correlations, the remaining correlating components can be categorized into a) correlations among active portfolios, b) correlation between the active portfolio and its benchmark, and c) cross

---

\(^{34}\) Load fees are one-time sales fees paid by the investor to buy the fund. *Front load fees* are paid when the shares in the mutual fund are bought and *back load fees* at time when the shares are redeemed.

\(^{35}\) Investment Company Institute (ICI): [http://www.icifactbook.org/06_fb_sec5.html#trends](http://www.icifactbook.org/06_fb_sec5.html#trends)

\(^{36}\) ICI is the national association of U.S. investment companies. Founded in 1940, its membership as of April 1, 2007 included 8,821 mutual funds, 664 close-end funds, 385 exchange-traded funds, and 4 sponsors of unit investment trust. Its mutual fund members serve 93.9 million individual shareholders and manage $10.481 trillion in investor assets. [http://www.ici.org/about_ici.html](http://www.ici.org/about_ici.html)

\(^{37}\) Investment Company Institute (ICI): [http://www.icifactbook.org/06_fb_sec5.html#trends](http://www.icifactbook.org/06_fb_sec5.html#trends)
correlation between the active portfolio and other benchmarks. If the benchmark is chosen, then the benchmark composition is fixed and exogenous to the model. Therefore, only the active part of the portfolio can be used to control the overall portfolio risk. Since the benchmark volatility is conditionally fixed and the absolute value of correlation is limited by positive unity, minimizing the impact of the correlating component is equivalent to minimizing the active risk. The statement may sound like a cliché for it recommends the asset managers to pursue the arbitrage opportunities, as active portfolios are per definition self-financing.

In the pair-wise illustration, the overall portfolio risk in eq. (4.15) can be written as

$$
\sigma_{\text{p},\text{b}}^2 = (W \sigma_{\text{b}} + (1-W) \sigma_{\text{a}})^2 + (W \sigma_{\text{a}} + (1-W) \sigma_{\text{a}})^2 + 2W^2 \sigma_{\text{b},\text{a}} + 2(1-W)^2 \sigma_{\text{a},\text{a}} + 2W(1-W) (\sigma_{\text{a},\text{a}} + \sigma_{\text{b},\text{a}}).
$$

(4.26)

Obviously, when the active risks $\sigma_{\text{a}}$ and $\sigma_{\text{a}}$ go to zero, the overall portfolio risk is simply the benchmark risk. Theoretical explanation is that the covariance terms will also go to zero since the covariance terms are per definition the correlation multiplied by the standard deviations:

$$
\sigma_{\text{b},\text{i},\text{j}} = \rho_{\text{b},\text{i},\text{j}} \sigma_{\text{b}} \sigma_{\text{i},\text{j}} \quad \text{for} \quad i = I, II \text{ and } j = I, II.
$$

(4.27)

Even if the correlation equals unity, eq. (4.27) cannot produce high additional covariance terms when the active risk term is close to zero. Consequently, the additional covariance effect on the overall portfolio risk is also negligibly small or disappears altogether. In case the active risk is zero while the active return is positive, then together with the definition that the active portfolio is always self-financing we obtain an arbitrage portfolio.

As long as the active return is positive and active portfolio risk infinitely small, the IR or AR may be infinite large and the portfolio SR is always higher than that of the benchmark. In essence, the recommendation here is to find a self-financing risk less
investment that enhances the overall portfolio return while refrains from increasing the overall portfolio risk. Of course, one can always resort to choose other benchmarks like cash to safeguard one's IR, AR and SR.

4.4 Concluding remarks

In theory, the influence on the overall portfolio efficiency coming from the correlations between the actively managed portfolios can be substantial. As illustrated by the pair-wise example, besides the correlations among the benchmarks there are also i) the correlation amongst active portfolios, ii) correlations between the benchmark and the active portfolio, and iii) the cross-correlation between an active portfolio and other benchmarks. The direct consequence is that the overall portfolio's efficiency in the MV space can be significantly reduced due to those correlations irrespective of positive active portfolio performance. Extending this finding to a broader context, it seems that an IR of 0.50 before transaction costs does not always guarantee a superior overall SR than the benchmark SR. Thus, the issue of high management fee for the active managed portfolio remains a point of contention. The warning to any private investor or sponsor of a pension fund is that performance assessment using the active performance measures alone is far from adequate!

In line with the findings in Jorion (2003), the overall portfolio performance cannot be accurately determined when parts of the correlation structure are ignored. Ex-ante it is indeed difficult, if not impossible, to form an accurate judgment of the correlation structure. Fortunately, all performance assessments are per definition an ex-post exercise. In this chapter I have illustrated the need to account for all the correlations and have provided tools to do it.
Appendix A1

The results of sub section 4.2.1.3: difference between AR and IR

From eq. (4.17), the portfolio alpha ($\alpha_p$) can be rewritten as $\alpha_p = \bar{r}_p - \beta_{pf} \bar{r}_{mf}$, and the residual risk ($\sigma_{ep}$) as $\sigma_{ep} = \sqrt{\sigma_{pf}^2 - \beta_{pf}^2 \sigma_{bf}^2}$. Substitute both equations into the definition of AR we get

$$AR(\beta_{pf}) = \frac{\bar{r}_p - \beta_{pf} \bar{r}_{mf}}{\sqrt{\sigma_{pf}^2 - \beta_{pf}^2 \sigma_{bf}^2}}, \quad (4.28)$$

where $\bar{r}_p$ denote the expected portfolio return in excess of the risk free rate and $\sigma_{pf}^2$ denote the corresponding portfolio variance. The feasible domain of $\beta_{pf}$ in eq. (4.28) is defined by the interval $\left(-\frac{\sigma_{pf}}{\sigma_{bf}}, \frac{\sigma_{pf}}{\sigma_{bf}}\right)$.\footnote{$\beta$ is defined as $\beta_{pf} = \rho_{bf} \frac{\sigma_{pf}}{\sigma_{bf}}$. As the correlation ($\rho_{bf}$) can neither exceed positive unity nor go below negative unity by definition, the domain of $\beta$ is bounded by the ratio of the portfolio risk with respect to the benchmark’s risk. As expected, the domain increases when more volatility is added to the portfolio.} Using eq. (4.19) and applying the same analogy as in eq. (4.28) to IR we get

$$IR(\beta_{pf}) = \frac{\bar{r}_p - \frac{\sigma_{bf}}{\sigma_{pf}} \bar{r}_{mf}}{\sqrt{\sigma_{pf}^2 + (1 - 2\beta_{pf}) \sigma_{bf}^2}}. \quad (4.29)$$

In eq.(4.29), the denominator limits the input of $\beta_{pf}$ to a ceiling of $\frac{\sigma_{bf}^2 + \sigma_{pf}^2}{2\sigma_{bf}^2}$ while no lower bound exists.

The slopes of both functions must be the same if the functions are identical. The partial derivative of $AR(\beta_{pf})$ with respect to $\beta_{pf}$ is
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\[
\frac{\partial AR}{\partial \beta_{pf}} = \frac{\beta_{pf} \sigma_{bf}^2 \overline{r}_{pf} - \sigma_{pf}^2 \overline{r}_{bf}}{(\sigma_{pf}^2 - \beta_{pf} \sigma_{bf}^2)^3}.
\]

The partial derivative of \( IR(\beta_{bf}) \) to \( \beta_{pf} \) is

\[
\frac{\partial IR}{\partial \beta_{pf}} = \frac{(\overline{r}_{pf} - \overline{r}_{bf}) \sigma_{bf}^2}{(\sigma_{pf}^2 + (1 - 2 \beta_{pf})\sigma_{bf}^2)^2}\sqrt{\sigma_{pf}^2 + (1 - 2 \beta_{pf})\sigma_{bf}^2}.
\]

At beta neutrality we would have

\[
AR(1) = \frac{\overline{r}_{pf} - \overline{r}_{bf}}{\sqrt{\sigma_{pf}^2 - \sigma_{bf}^2}}, \quad IR(1) = \frac{\overline{r}_{pf} - \overline{r}_{bf}}{\sqrt{\sigma_{pf}^2 - \sigma_{bf}^2}}.
\]

\[
\frac{\partial AR(1)}{\partial \beta_{pf}} = \frac{\overline{r}_{pf} \sigma_{bf}^2 - \overline{r}_{bf} \sigma_{pf}^2}{(\sigma_{pf}^2 - \sigma_{bf}^2)^2} \sqrt{\sigma_{pf}^2 - \sigma_{bf}^2}, \quad \frac{\partial IR(1)}{\partial \beta_{pf}} = \frac{(\overline{r}_{pf} - \overline{r}_{bf}) \sigma_{bf}^2}{(\sigma_{pf}^2 - \sigma_{bf}^2)^2} \sqrt{\sigma_{pf}^2 - \sigma_{bf}^2}.
\]

At beta neutrality, \( IR(\beta_{bf}) \) and \( AR(\beta_{bf}) \) have the same level value, but different slopes.

Although a small difference in the portfolio risk in the numerator, the implication is that the discrepancy between \( IR(\beta_{bf}) \) and \( AR(\beta_{bf}) \) increases as portfolio beta moves away from unity in either direction. Under tracking error optimization, the inequality \( \sigma_{pf}^2 \geq \sigma_{bf}^2 \) holds if \( r_{pf} \geq r_{bf} \) is required, then the inequality

\[
(\overline{r}_{pf} - \overline{r}_{bf}) \sigma_{bf}^2 \geq \overline{r}_{pf} \sigma_{bf}^2 - \overline{r}_{bf} \sigma_{pf}^2 - \overline{r}_{bf} \sigma_{bf}^2 + \overline{r}_{bf} \sigma_{bf}^2 = \overline{r}_{bf} \sigma_{bf}^2 - \overline{r}_{bf} \sigma_{bf}^2
\]

also holds. The implication from inequality (4.31) is that \( IR(\beta_{bf}) \) has a steeper slope than \( AR(\beta_{bf}) \) at beta neutrality. Thus, under tracking error optimization the IR is more sensitive to changes in beta and could potentially exaggerate either active profits or losses.

When tactical asset allocation weight bands are imposed on the portfolio optimization like the one in eq. (2.30), the resulting optimized portfolio has a lower volatility than the benchmark: the inequality \( \sigma_{pf}^2 \leq \sigma_{bf}^2 \) holds and the opposite of inequality (4.31) holds true. Under weight bands optimization it is the AR that is more sensitive to changes in the portfolio beta.
Next, we explore the behavior of both $AR(\beta_{pf})$ and $IR(\beta_{bf})$ at the domain asymptotes. Specifically, the question here is whether it is $IR(\beta_{pf})$ that dominates $AR(\beta_{pf})$ or is the contrary true at the domain limits?

Assume that the expected residual return in the numerator is positive.\(^{38}\) If we approach the lower limit of $AR(\beta_{pf})$ arbitrarily close from above we get

$$\lim_{\beta_{pf} \downarrow \sigma_{pf}} AR(\beta_{pf}) = \lim_{\beta_{pf} \downarrow \sigma_{pf}} \frac{\bar{r}_{pf} - \beta_{pf} \bar{r}_{bf}}{\sqrt{\sigma_{pf}^2 - \beta_{pf}^2 \sigma_{bf}^2}} = \infty,$$

and if we approach its upper limit arbitrarily close from below we get

$$\lim_{\beta_{pf} \uparrow \sigma_{pf}} AR(\beta_{pf}) = \lim_{\beta_{pf} \uparrow \sigma_{pf}} \frac{\bar{r}_{pf} - \beta_{pf} \bar{r}_{bf}}{\sqrt{\sigma_{pf}^2 - \beta_{pf}^2 \sigma_{bf}^2}} = \infty.$$

In the risk-seeking case with negative expected residual return, the sign of positive infinity flips over when we approach the domain limits arbitrarily close from either side.

When the beta goes to minus infinity, $IR(\beta_{pf})$ will converge to zero from above or below depending on the sign of the active portfolio return in the numerator:

$$\lim_{\beta_{pf} \downarrow -\infty} IR(\beta_{pf}) = \lim_{\beta_{pf} \downarrow -\infty} \frac{\bar{r}_{pf} - \bar{r}_{bf}}{\sqrt{\sigma_{pf}^2 + (1 - 2\beta_{pf})\sigma_{bf}^2}} = 0.$$

When beta approaches the positive limit arbitrarily close from below, $IR(\beta_{pf})$ will go to positive infinity if the holding portfolio outperforms the benchmark:

$$\lim_{\beta_{pf} \uparrow 0} IR(\beta_{pf}) = \lim_{\beta_{pf} \uparrow 0} \frac{\bar{r}_{pf} - \bar{r}_{bf}}{\sqrt{\sigma_{pf}^2 + (1 - 2\beta_{pf})\sigma_{bf}^2}} = \infty.$$

\(^{38}\) A negative expected residual return indicates risk-seeking behavior, which we do not consider here.
If underperformance is realized, then $IR(\beta_{pf})$ goes to negative infinity at the domain limits. Figure 4.3 illustrates the case with outperformance.

\[ IR(\beta_{pf}) = \frac{\sigma_{pf}}{\sigma_{\beta_{pf}}} \]
\[ AR(\beta_{pf}) = \frac{\sigma_{pf}^2 + \sigma_{\beta_{pf}}^2}{2\sigma_{\beta_{pf}}^2} \]

**Figure 4.3:** relative behavior of IR with respect to AR at the asymptotes. Here, the AR value first goes to infinity in the positive portfolio beta region and therefore dominates the IR value in that region. For negative values of $\beta_{pf}$, it is the IR value that dominates.

From eq. (4.32), eq. (4.33), and Figure 4.3 we observe that $AR(\beta_{pf})$ dominates $IR(\beta_{pf})$ at the lower limit. The difference between the two measures will go either to positive or to negative infinity depending on the holding portfolio's performance.

However, around the upper limit the matter is not straightforward. As can be observed in Figure 4.3, the upper limit of $AR(\beta_{pf})$ is $\frac{\sigma_{pf}}{\sigma_{\beta_{pf}}}$ and the domain ceiling of $IR(\beta_{pf})$ is...
defined by $\frac{\sigma^2_e + \sigma^2_f}{2\sigma^2_b}$. Clearly, the function that first approaches its upper limit will first
converge to infinity and thus dominate. The difference between the two upper limits can be
written as:

$$\frac{\sigma_{pf}}{\sigma_b} - \frac{\sigma_{bf}^2 + \sigma_{pf}^2}{2\sigma_{bf}^2} = \frac{\sigma_{pf}^2}{\sigma_b^2} - \frac{\sigma_{pf}^2}{2\sigma_{bf}^2} - \frac{1}{2},$$

(4.34)

The difference between the two upper limits in (4.34) is zero if and only if $\sigma_{pf}/\sigma_{bf}$
equals one, in which case the portfolio has the same risk as the benchmark (beta
neutrality). For any other value of $\sigma_{pf}/\sigma_{bf}$ the upper limit of $AR(\beta_{pf})$ is always lower
than that of $IR(\beta_{pf})$. This result implies that $AR(\beta_{pf})$ will always go to infinity before
$IR(\beta_{pf})$ does if beta neutrality does not hold.

**Appendix A2**

Derivation details of eq. (4.21), ineq. (4.23) and (4.25).

**Equation (4.21):**

Given the Euler equation of the Sharpe ratio (SR)

$$\frac{d}{dw} SR_{p^*} = \frac{\alpha_p}{\sigma_{pf}} - \frac{\bar{r}_{pf}}{\sigma_{pf}^2} \left[ \frac{\sigma_{e,p}^2 + \sigma_{e,p,bf}}{\sigma_{pf}^2} \right] = 0$$

we can write it as

$$\frac{\alpha_p}{\sigma_{pf}} = \frac{\bar{r}_{pf}}{\sigma_{pf}^2} \left[ \frac{\sigma_{e,p}^2 + \sigma_{e,p,bf}}{\sigma_{pf}^2} \right].$$

A2.1
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After inserting the definition of the portfolio excess return into eq. A2.1 and multiplying it by \( \frac{\sigma_{pf}}{\sigma_{ep}} \) we get

\[
\frac{\alpha_p}{\sigma_{ep}} = \frac{\alpha_p + \bar{r}_{bf}}{\sigma_{pf}} \left[ \frac{\sigma_{ep}^2 + \sigma_{e,bf}^2}{\sigma_{pf}^2} \right].
\]

Clearly, the left hand side of the equation is the appraisal ratio (AR) and rearranging the right hand side to get all the AR part to the left hand side of the equation we get

\[
AR_p = \left[ 1 - \frac{\sigma_{ep}^2}{\sigma_{pf}^2} \right] = \frac{\sigma_{e,bf}^2}{\sigma_{pf}^2} \left[ \frac{\sigma_{ep}^2 + \sigma_{e,bf}^2}{\sigma_{pf}^2} \right].
\]  A2.2

The portfolio variance is defined as

\[
\sigma_{pf}^2 = \sigma_{bf}^2 + \sigma_{ep}^2 + 2\sigma_{e,p,bf}.
\]

After substituting the portfolio variance definition into eq. A2.2 and some rearranging the AR of the optimal portfolio can be written in terms of the benchmark's SR:

\[
AR_p^* = \left[ 1 + \frac{\sigma_{e,bf}^2}{\sigma_{bf}^2} \right] = SR_b \left[ \frac{\sigma_{ep}^2 + \sigma_{e,bf}^2}{\sigma_{bf}^2} \right].
\]  A2.3

Given the definition of the correlation we know that the following equations hold:

\[
\frac{\sigma_{e,bf}}{\sigma_{bf}} = \rho_{e,bf} \frac{\sigma_{ep}}{\sigma_{bf}} \quad \text{and} \quad \frac{\sigma_{e,bf}}{\sigma_{ep} \sigma_{bf}} = \rho_{e,bf}.
\]

After inserting these equations back into eq. A2.3 we obtain the expression in eq. (4.21).

Inequality (4.23):

If the portfolio Sharpe ratio (SR) is higher than the benchmark SR, then inequality (4.22) must hold. For the active performance measure information ratio, the portfolio excess return is defined by the portfolio alpha \( \alpha_p \), beta \( \beta_p \) and expected benchmark excess return \( \bar{r}_{bf} \). After inserting the definition into inequality (4.22), it becomes
The portfolio beta is defined as:

$$\beta_p \equiv \rho_{pf, bf} \frac{\sigma_{pf}}{\sigma_{bf}}$$

and the portfolio standard deviation is defined by the squared root of the benchmark and residual variance:

$$\sigma_{pf} = \sqrt{\beta_p^2 \sigma_{bf}^2 + \sigma_{ep}^2}.$$ 

Using the definition of the portfolio beta, the portfolio standard deviation can be written as

$$\sigma_{pf} = \frac{\sigma_{ep}}{\sqrt{1 - \rho_{pf, bf}^2}}.$$ 

Substituting everything back into inequality A2.4 we get

$$\frac{\alpha_p}{\sigma_{ep}} \left[ \sqrt{1 - \rho_{pf, bf}^2} \right] + \rho_{pf, bf} \frac{\bar{r}_{bf}}{\sigma_{bf}} > \frac{\bar{r}_{bf}}{\sigma_{bf}},$$

which simplifies to

$$AR_p > SR_b \left[ \frac{1 - \rho_{pf, bf}}{\sqrt{1 - \rho_{pf, bf}^2}} \right].$$

**Inequality (4.25):**

Just as before, if the portfolio Sharpe ratio (SR) is higher than the benchmark SR, then the inequality in (4.22) must hold. After subtracting the benchmark excess return from both side of the inequality (4.22) and some rearranging we get

$$\frac{\bar{r}_{pf} - \bar{r}_{bf}}{\bar{r}_{bf}} > \frac{\sigma_{pf} - \sigma_{bf}}{\sigma_{bf}}.$$
Rearrange everything according to the SR definition we can write the last inequality as

\[
\frac{\bar{r}_{pf} - \bar{r}_{bf}}{\sigma_{xp}} \left[ \frac{\sigma_{xp}}{\sigma_{pf} - \sigma_{bf}} \right] \geq \frac{\bar{r}_{bf}}{\sigma_{bf}} = SR_b.
\]

As the active return in the information ratio (IR) is defined by the return difference between the portfolio excess return and benchmark excess return, the coefficient on the left hand side of the last inequality is simply the IR. Divide \(SR_b\) by the ratio within the square brackets we obtain the inequality in (4.25).
Conclusions

The main purpose of this dissertation has been to explore whether the current hierarchical portfolio management (HPM) process can be improved by increasing the interaction and information exchange between the different decision levels. Five research questions have been formulated and explored in this dissertation. Here in this chapter I present the obtained answer to each of the research questions as conclusions of this dissertation.

Research question 1:
Given the HPM process with multiple (non-) overlapping sub portfolios in a single period setting, what is the magnitude (i.e. significant or negligible) of the economic loss incurred by the hierarchy and the benchmark tracking strategy with limited decision freedom?

The hierarchical decision structure divides the overall portfolio decision into a finite number of sub portfolio decisions. There is no interaction between the competing managers in their portfolio choices. The sub portfolio choices are based on stand-alone local optimizations. The results in chapter 2 show that the current convention of benchmark tracking with tracking error volatility (TEV) limit certainly does not help: it depends heavily on the efficiency of the chosen benchmark. Unless the benchmark is mean-variance (MV) efficient, the selected portfolio under TEV optimization is rarely MV efficient otherwise. As most of the benchmarks are either market capitalization (Mcap) weighted or equally weighted (EW), it is often the rule than exception that the benchmark is MV inefficient. Inherently, the current procedure of TEV optimization is sub optimal: the TEV efficient frontier always goes through the benchmark, as we get the benchmark portfolio when the TEV is zero. By putting a limit on the TEV, the investor prevents the
active portfolio from deviating too much from the benchmark too often. Hence by only focusing on the active weights, the optimized portfolio is always the benchmark portfolio with some adjustment to it.

After implementing the bottom-up portfolio optimization using enlarged investor portfolio opportunity space (IPOS) of a sample containing 10 year data of 125 U.S. companies equally divided into 5 sectors, the gain in terms of risk and return in the exposition is considerable. On the annual basis, the gap in portfolio return is 14.15% at the risk level of 11.05% between the benchmark and the MV efficient portfolio. At the fixed return level of 18%, the annual risk reduction is 9.45%. Still, there is room for improvement between the MV efficient frontier and the utopian efficient frontier.

Hence, the magnitude of the economic loss in the single-period setting is economically significant.

Research question 2:
How can we decrease this economic loss?

To improve overall portfolio performance in the cross section of the equity data, chapter 2 uses a two-stage optimization procedure to reduce the gap in efficiency between the MV efficient frontier and the benchmark portfolio in the MV space. The bottom line is that current practice of active portfolio management can improve the performance of the overall portfolio if 

i) more sub-POS are sent to the overall level allowing for more portfolio combinations and

ii) the benchmark tracking strategy with TEV constraints in the lower level does not yield the desired results.

In the first stage, Chapter 2 has illustrated the limitation of TEV optimization in the MV space for portfolio optimization in the lower level. The ex-ante information ratio (IR) is constant if only the active weights are optimized. Thus, taking on extra active risk will not yield higher expected risk adjusted active return. Consequently, by allowing for a higher
TEV will not provide any disproportional higher expected return. Rather, the standard advice then becomes to scale the investment up by investing more or less in the active investments instead of giving more decision freedom to the active managers.

As an alternative, the lower level portfolio efficiency can be increased through direct portfolio optimization: instead of focusing on the active weights, the portfolio composition is directly optimized according to the available information set. The optimization procedure fully exploits the available opportunities in the manager’s sub IOS such that the resulting MV efficient frontier envelops the benchmark.

In the second stage of optimization in the overall level, the managers have reported the minimum risk portfolio, maximum Sharpe ratio (SR) tangent portfolio and both the Mcap as well as the EW benchmarks to the overall level: the POS of the investor has just quadrupled in size. Besides these additional portfolio choices, the IPOS now also contains the combinations of the additional portfolio choices. During portfolio optimization in the overall level, the additional information helps the investor to make a better portfolio selection, as illustrated by the numerical double digit gains in portfolio return in the exposition.

Hence, the economic loss due to HPM can be reduced based on the enlarged IPOS.

Research question 3:
If there is improvement in the performance of the overall portfolio with respect to its benchmark, is this improvement persistent through time or is it a lucky throw of the dice?

Chapter 3 has explored the persistency of the cross sectional improvement in portfolio performance in a multi-period setting. Using an experiment, the results in Chapter 3 confirm that the cross sectional improvement documented in Chapter 2 is also persistent through time. For the observed performance of the MSCI regional and country indexes in the period between December 2002 and May 2006, the comparative static model based on the bottom-up framework has produced an overall return of almost 77% after transaction
cost in contrast to the 39% realized by the benchmark. At the same time, the monthly actively managed portfolio always has a lower risk than the benchmark. The bootstrap results net of transaction cost reveal that the actively managed portfolio using the bottom-up framework outperforms the passive benchmark in 98.73% of the 10,000 scenarios. The outperformance over the benchmark return fluctuates between 6.94% and 54.33%.

The empirical findings in chapter 2 and 3 reaffirm a simple intuition: given the dynamic nature of the modern economic environment, it seems strange that a fixed portfolio composition always suffices in capturing the optimal portfolio.

A byproduct of the results found in Chapter 3 is that the current internet based transaction cost no longer presents a hurdle preventing active portfolio management. The total transaction cost for the actively managed portfolio has fluctuated between 1.5% and 2.7% of the initially invested wealth in the 10,000 scenarios of the bootstrap method. Although much lower in the benchmark replication case, the transaction cost here is anything but zero. It fluctuates between 1.2% and 1.4% of the initially invested wealth. Clearly, given the size of the outperformance, the additional transaction cost incurred by the actively managed portfolio is more than compensated for, at least in the illustration in Chapter 3.

**Research question 4:**

*In case a lower level actively managed portfolio has outperformed its lower level benchmark, does it automatically imply that the overall portfolio with the actively managed portfolio has outperformed the benchmark in the overall level?*

The results in Chapter 4 show that a positive IR in the lower level by no means automatically implies a higher SR of the overall portfolio. The reason is that the correlation between the sub portfolios may be so high that the risk element in the denominator of the SR is inflated such that the aggregated SR actually decreases. In other words, the extra return of the positive IR is inadequate to compensate for the additional
risk taken on. There is a threshold that the IR must overcome to add value to the benchmark SR.

The risk attribution results in Chapter 4 reveal that the impact of the correlations between the sub portfolios on the overall risk-adjusted return, SR, can be massive. In our numerical example using Fidelity mutual funds, a pre cost IR of 0.50, a generally accepted indicator of good active portfolio management in practice (see e.g. Grinold and Kahn (1999)), is no guarantee that the overall portfolio's SR is higher than that of the benchmark. This insight is crucial: a positive active performance is not always a guarantee for value added to the overall portfolio in HPM.

**Research question 5:**

*At performance assessment, how should an investor judge active portfolio manager if the sub portfolios are correlated?*

The current practice treats each managed portfolio as an independent entity that is uncorrelated with each other. Although the managers may not know each other's views, ideas and strategy, but the chosen portfolios are correlated either through common benchmark or consensus choices. Given there is only one global market to choose from, everyone is drawing from the same pool of investment opportunities or IOS. Also, the asset classes are rarely uncorrelated: the equity markets are correlated and the bond market is a resort for the equity investor during bearish equity market conditions. Therefore, the assumption of uncorrelated sub portfolios is unrealistic.

Per definition, the overall portfolio’s risk increases when the portfolios are positively correlated and decreases when the correlation is negative. Thus, at performance assessment, the risk component in the risk-adjusted return of the aggregated portfolio, SR, can be substantially different than the perceived amount. Consequently, the risk-adjusted overall portfolio return under the independence assumption in case of correlated sub portfolios is inaccurate.
Chapter 5                  Conclusions

By rewarding the individual manager according to the active performance, the investor seems to have forgotten his main objective. Instead of only assessing part of the total portfolio, the investor should evaluate the overall portfolio performance by looking at the SR of the overall portfolio: a manager’s contribution should be measured in terms of attribution to total returns and total risk. After all, an investor is only willing to postpone consumption if he can consume more in the future through investments.

Chapter 4 illustrates the problem of correlated actively managed portfolios. The correlation can be split up into three parts: firstly, the active portfolios are correlated; secondly, the active portfolio is correlated with its own benchmark; and finally, the active portfolio can be correlated with other benchmarks due to correlations between the benchmarks. As it turns out, correlation in itself is not dangerous because it is by definition limited in size. Correlated active portfolios are only hazardous when the active risk is so high that it inflates the covariance part dramatically. The influence of high correlation disappears after reformulation in terms of covariance. This finding is in line with the findings in chapter 2. The active performance assessment measures, the information ratio and appraisal ratio, only produce consistent assessment of the same portfolio if the portfolio beta equals unity (beta neutrality). If the portfolio beta is unity or close to unity, then it implies that the active portfolio adds no or contributes slight risk to the portfolio. As such, the overall portfolio beta should stay close to unity to avoid any aftershocks of correlated active portfolios.

So, given the finding above, it seems that everything boils down to the traditional pursuit of maximizing (active) return while minimizing the (active) risk.

The general view of this dissertation is that the investor in current practice plays a too passive role in fund allocation. To the best of my knowledge, allocation of funds to, for example, country managers is based on the MSCI world index at best or totally random at worst. The role given to the investor is a passive one and the investor should leave the
Hierarchical portfolio management

matter to the professional managers. Although valid to some extent, it remains a question whether such view still holds when the investor is a pension fund or large insurer. As the only person who has the overview, the investor is in a unique position of coordinating the managers who choose their portfolios based on stand-alone portfolio optimizations. By increasing the portfolio opportunities reported to the investor by the managers, the POS of the investor increases not only with the additional portfolio choices, but also with the combinations of these new choices. Hence, the investor has more and often better alternatives to choose from. The quality of the additional information is partly guaranteed by the competitiveness between the managers and their knowledge of the investor’s preferences of non-satiation and risk aversion. In such a setting, the investor can optimize manager allocation based on the bids and track record of the managers.
Samenvatting

Hiërarchisch Portefeuille Beheer (HPB) ontstaat wanneer de eigenaar van het te investeren bedrag, de investeerder, meerdere vermogensmanagers inhurkt. De investeerder kan verschillende redenen hebben om zijn geld te spreiden. Ten eerste reduceert het spreiden onder meerdere vermogensmanagers de kans op misallocatie; één manager kan een slecht jaar hebben, maar de kans dat alle managers een slecht jaar hebben is aanzienlijk kleiner. Een tweede reden is specialisatie. Gegeven de groeiende complexiteit van de investeringsproducten wordt het steeds moeilijker voor één vermogensmanager om alle investeringsmogelijkheden te doorgronden. Als er niet genoeg kennis in huis is, moeten er andere specialisten aangetrokken worden. De laatste reden heeft te maken met fysieke capaciteit. Het is moeilijk en vaak haast onmogelijk voor één vermogensmanager om alles in de gaten te houden. Na de sluiting van de Aziatische beursen is het onmogelijk voor een manager om nog een productieve dag te realiseren op de Europese markten: meerdere managers moeten worden ingehuurd en verdeeld in regio teams.

In de praktijk krijgt iedere vermogensmanager doorgaans een referentieportefeuille, een benchmark, toegewezen die door iedereen waarneembaar is, samen met de opdracht om deze te volgen of te verslaan. De keuze van een benchmark tracking strategie kent tenminste twee voordeLEN voor de investeerder. Niet alleen wordt de investeringsruimte gedefinieerd, maar ook een minimale prestatie. De investeerder weet precies waarin zijn geld geïnvesteerd wordt en hij weet wat de “markt”, vertegenwoordigd door de benchmark, gepresteerd heeft.

Om een benchmark te kunnen verslaan moet er beslissingsvrijheid gegeven worden aan de managers. De managers mogen beslissen hoe en hoeveel zij gaan afwijken van de benchmark, maar alles moet binnen de perken blijven. De randvoorwaarden zijn meestal gedefinieerd door een tracking error volatility (TEV) restrictie of een gewicht bandwidth voorwaarde. Een TEV restrictie geeft aan hoeveel extra actief risico bovenop het benchmark risico mag komen. Hiermee beperkt de investeerder het mogelijke risicozoekende gedrag van de manager wiens beloning vaak prestatie afhankelijk is. Een gewicht bandwidth voorwaarde heeft dezelfde doelstelling als een TEV restrictie, maar beperkt de keuze van de manager door middel van een bandwijdte om de benchmark gewichten.

Een extra motivatie voor een benchmark tracking strategy met beperkte beslissingsvrijheid is het kostenaspect. Bij het repliceren van de benchmark rendementen zijn er kosten gemaakt. Het behaalde replicatie rendement is een bruto rendement. Het netto rendement na aftrek van de kosten zal altijd iets lager zijn dan het benchmark rendement bij de replicatie-strategie. Daarom mogen managers beperkt afwijken om een klein beetje extra rendement te behalen om de kosten te dekken. Als puur rendements-maximalisatie de doelstelling zou zijn geweest, dan is het volgen van een benchmark niet de beste keuze omdat de benchmark vaak geconstrueerd is zonder de rendements-maximalisatie doelstelling. Vaak zijn de benchmarks gewogen naar prijs (DOW), gelijkgewogen (NASDAQ-100 equally weighted index), of gewogen naar markt kapitalisatie (S&P500, NASDAQ, AEX, etc.).
Samenvatting

Het fundamentele probleem van de hiërarchische structuur is het informatiefiltereffect dat optreedt binnen de hiërarchie en het gebrek aan communicatie tussen zowel de investeerder en de managers als tussen de managers onderling. Neem als voorbeeld een elementaire beslissingsstructuur met twee beslissingslagen. De investeerder beheert de toplaag met de managers in de onderlaag. Twee problemen kunnen zich voordoen. Het eerste probleem is de efficiëntie van de portefeuillekeuze van de managers in de onderlaag. Hoogstens is de keuze van ieder manager slechts lokaal optimaal omdat iedere gekozen subportefeuille het resultaat is van een optimalisatie onafhankelijk van de andere subportefeuilles. Dit wordt het communicatieprobleem genoemd. Het tweede probleem is het aggregeren van alle subportefeuilles. Na het aggregeren van alle suboptimale portefeuilles in de onderlaag hoeft de topportefeuille ook niet optimaal te zijn. Als iedere manager slechts één keuze doorgeeft zonder te weten of dat past in de topportefeuille, dan heeft de investeerder geen andere keuze behalve de doorgegeven subportefeuilles. Dit wordt het filtereffect genoemd. Een mogelijke oplossing is dat de investeerder een keuze uit meerdere portefeuilles van iedere manager vraagt. Dan kan de investeerder een afweging maken zodat gegeven de hiërarchisch structuur de beste combinatie van de subportefeuilles gemaakt kan worden.

Deze dissertatie concentreert zich op een vijftal onderzoeksvragen die betrekking hebben op de bovengenoemde twee problemen. Deze onderzoeksvragen zijn de volgende:

1. Gegeven HPB met meerdere (niet-) overlappende subportefeuilles in één enkele periode, wat is de grootte (economisch significant of verwaarloosbaar) van het economische verlies veroorzaakt door de combinatie van hiërarchie en de “benchmark tracking” strategieën met beperkte beslissingsvrijheid?
2. Hoe kan dit verlies verminderd worden?
3. Als er verbetering is in de prestatie van de topportefeuille met betrekking tot zijn benchmark in de één periode context, is deze verbetering dan vervolgens ook haalbaar door de tijd heen of is het slechts een gelukkige worp van de dobbelsteen?
4. Als een actief beheerde portefeuille in het lagere niveau een betere prestatie heeft geleverd dan zijn benchmark, betekent dit dan automatisch dat de topportefeuille waar de actief beheerde portefeuille deel van uitmaakt ook zijn benchmark verslaat?
5. Hoe moet een investeerder bij prestatie evaluatie een oordeel vellen over de (actieve) portefeuille manager als de subportefeuilles gecorreleerd zijn?

In hoofdstuk 2 worden de onderzoeksresultaten gepresenteerd die een antwoord geven op de eerste twee vragen. In de praktijk is de “tracking error volatility” (TEV) minimalisatie-procedure zeer populair in het bepalen van de “optimale” actieve portefeuille gegeven een keuzeruimte van investeringsmogelijkheden. Helaas kampt de TEV minimalisatie-procedure met een tweetal fundamentele problemen. Ten eerste is de hele procedure een relatieve procedure. Dat wil zeggen, om de definitieve portefeuille te kunnen bepalen moet de “optimale” actieve portefeuille toegevoegd worden aan een benchmark. Dus de benchmark verankert de hele keuzeruimte en de bijbehorende efficiënte grenslijn. Als die benchmark inefficiënt is, dan is de TEV optimale grenslijn ook inefficiënt. Met andere woorden, iedere “optimale” portefeuillekeuze is inefficiënt. Het tweede probleem met TEV minimalisatie is de mogelijkheid dat de “no-short” restrictie overtreden wordt bij het
Samenvatting


Als een beter alternatief bepleit hoofdstuk 2 het gebruik van de “bandwidth” optimalisatie-procedure. Zoals de naam suggereert laat de hele procedure het gewicht van iedere investeringsmogelijkheid schommelen binnen een bandwijdte die van tevoren door de investeerder is bepaald. Hoewel deze procedure nog steeds gebruik maakt van een benchmark, staat de bandwijdte nu toe dat de gehele efficiënte grenslijn verschoven kan worden. De efficiënte grenslijn is niet langer verankerd door de benchmark. Als in dit geval de benchmark inefficiënt is, dan kan deze optimalisatie procedure ervoor zorgen dat de grenslijn naar links boven in de rendement-risico ruimte gaat verschuiven en vervolgens de benchmark domineert. Naar verwachting zal deze methode, ex-ante, altijd een betere keuze opleveren dan de benchmarkkeuze.

Uit de empirische resultaten in hoofdstuk 2 blijkt dat het economische verlies op de topportefeuille, veroorzaakt door de combinatie van hiërarchie- en TEV-optimalisatie, economisch zeer significant is. Vergeleken met de benchmark keuze in de steekproefperiode van januari 1991 tot en met mei 2002 zien wij een sprong van 12% op jaarbasis in gerealiseerde rendementen van de topportefeuille door het toepassen van de “bandwidth” optimalisatieprocedure en het vergroten van het aantal subportefeuilles gerapporteerd aan de investeerder. Als men risicoreductie zoekt, dan levert de procedure in hoofdstuk 2 bijna 9% reductie op per jaar in de standaarddeviatie van de topportefeuille. Uiteraard, de logische vervolgvraag is of deze verbetering in portefeuilleprestatie ook haalbaar is door de tijd heen. Hoofdstuk 3 gaat nader in op deze onderzoeksvraag.

In hoofdstuk 3 wordt de “rolling window” methode gebruikt om een dynamisch portefeuilleselectieproces te simuleren. Voor iedere periode wordt een portefeuille geconstrueerd en een periode aangehouden. Vervolgens wordt alles herhaald in de volgende periode. De portefeuillecompositie aan het begin van iedere periode wordt vastgesteld op basis van de informatie over een schattingperiode van 24 maanden voorafgaand aan de periode in kwestie. De waarde van de portefeuille wordt vastgesteld aan het einde van iedere periode met behulp van de realisatie van de investeringsmogelijkheden. Aan het einde van de totale steekproefperiode wordt de absolute waarde van de portefeuille vastgesteld en vergeleken met zijn beginwaarde om het rendement over de hele steekproefperiode uit te rekenen. De superieooriteit of inferioriteit van de strategieën wordt vanzelf duidelijk als wij die rendementen naast elkaar zetten.

In de berekening van de waarde van een portefeuille in iedere periode is ook rekening gehouden met de transactiekosten. Tijdens de simulatie wordt iedere transactie belast met de kosten die een Nederlandse internetbank vraagt aan haar klanten. Op jaarbasis zijn de transactiekosten van de passieve “benchmark tracking” portefeuille ongeveer 1.2% van de iniële portefeuillewaarde en 1.8% voor de procedure uiteengezet in hoofdstuk 2. In de waargenomen steekproef periode tussen november 2000 en mei 2006 heeft de voorgestelde procedure een bovenprestatie van 96% behaald, wat bijna een verdubbeling
van het benchmark rendement impliceert. In absolute termen heeft de initiële portefeuille
van € 1 miljoen een eindwaarde van € 1.765.939,31 als hij beheerd zou zijn geweest door
middel van de voorgestelde procedure. De portefeuille heeft een eindwaarde van
€1.390.731 als de “benchmark tracking” methode was gebruikt, wat leidt tot een
bovenprestatie van ruim €375.000. Bovendien is het risico van de portefeuille beheerd
door middel van een “bandwidth” optimalisatie procedure altijd lager dan die van de
benchmark in iedere periode afzonderlijk.

Aan het einde van hoofdstuk 3 wordt de “bootstrapping” methode gebruikt om de
statistische stabiliteit van de procedure voorgesteld in hoofdstuk 2 te testen. Uit de 10.000
steekproeven is er slechts in 127 gevallen door de procedure onderprestatie gerealiseerd
met betrekking tot de benchmark. De kans op onderprestatie in de steekproef door het
volgen van de procedure is dus kleiner dan 1.3%. De kritieke waarde van het 5° percentiel
is €69.380 en de kritieke waarde van het 95° percentiel is €340.784. Met andere woorden,
in deze steekproef is de kans dat de bovenprestatie onder €69.380 valt 5% en in 90% van
de gevallen valt de bovenprestatie tussen €69.380 en €340.784.

De gevolgtrekking uit deze dataset is dat de prestatieverbetering op basis van de
voorgestelde procedure in hoofdstuk 2 haalbaar is door de tijd heen. Het ziet ernaar uit dat
de voorgestelde procedure in staat is om consistent een bovenprestatie te leveren boven
op de benchmarkprestatie in deze dataset. Meer onderzoek is nodig om de algemene validiteit
van de procedure vast te leggen.

Hoofdstuk 4 richt zich op evaluatie van de prestaties van de managers door de investeerder
en concentreert zich daarmee op de laatste twee onderzoeksvragen. De huidige prestatie-
evaluatie wordt meestal gedaan vanuit het perspectief van de manager. Een populair
argument voor zulke praktijk is dat niemand verantwoordelijk gesteld moet worden voor
de activiteiten van anderen. Zolang een manager een positief actief rendement realiseert
behoort die manager beloond te worden. Dit mag wel redelijk klinken, maar de manager
hoefst niet een positieve bijdrage te hebben geleverd aan de topportefeuille. Vanuit het
perspectief van de investeerder heeft de manager mogelijk over een enorme stuk (actief) risico
toegevoegd aan de topportefeuille, waardoor de Sharpe Ratio (SR) van de
topportefeuille eigenlijk daalt en zelfs onder de benchmark SR kan vallen.

Een tweede probleem voor de investeerder is dat de subportefeuilles gecorreleerd
kunnen zijn, waardoor het risico van de geaggregeerde portefeuille in de toplaag veel
hoger kan liggen dan men zou verwachten. Als de investeerder iedere manager
afzonderlijk zou evalueren, dan wordt dat stuk extra onzekerheid verwaarloosd, resulterend
in een verkeerde evaluatie van prestaties.

In hoofdstuk 4 wordt het totale risico van de topportefeuille ontleed en de totale
covariantiematrix opgesplitst in een benchmarkgedeelte, een actief portefeuille gedeelte en
de correlatiegedeeltes. Door steeds elementen in de verschillende onderdelen te variëren
vergaren wij inzicht in de werking en invloed van ieder onderdeel van de covariantiematrix
op het totale risico van de topportefeuille.

De resultaten zijn opmerkelijk. De populaire aanname dat de portefeuilles van
verschillende managers niet gecorreleerd met elkaar zijn omdat de managers
onafhankelijke beslissingen hebben genomen wordt niet ondersteund door empirische
resultaten. De gerealiseerde rendementen van verschillende portefeuilles zijn vaak wel
Samenvatting
degelijk met elkaar gecorreleerd. Nog opmerkelijker is dat het actieve rendement vaak ook gecorreleerd is met het benchmarkrendement. Als de verschillende benchmarks gecorreleerd zijn, dan kan de actieve portefeuille ook gecorreleerd zijn met de andere benchmarks omdat de actieve portefeuille identieke elementen bezit als zijn benchmark. Het gevolg van al deze correlaties en de symmetrische eigenschap van de correlatiematrix is dat alle correlaties twee keer terugkomen in de covariantiematrix waardoor het risico van de topportefeuille aanzienlijk groeit. Het directe gevolg is dat de SR van de topportefeuille enorm kan dalen als alle correlaties correct in rekening worden gebracht. In de empirische illustratie van hoofdstuk 4 daalt de topportefeuille SR van 1.63 naar 1.34 wanneer alle correlaties correct zijn verrekend.

In het empirische gedeelte van hoofdstuk 4 wordt ook de link tussen SR en Informatie Ratio (IR) gelegd. Met name wordt gekeken hoe hoog de IR moet zijn, gegeven de steekproef, om de benchmark SR te kunnen evenaren. Uit de resultaten blijkt dat een IR onder 0.47 een lagere SR oplevert dan de benchmark. Als kosten zoals instap (front load) en uitstap (back load) in rekening gebracht worden, dan hoeft een IR van 0.50 (indicatie van een goede actieve performance in de praktijk (Grinold and Kahn, 1999)) geen garantie te zijn voor betere prestaties dan de benchmark. Vanuit het perspectief van de investeerder rest de vraag of de actieve manager al dan niet beloond moet worden.

Samenvattend levert deze dissertatie drie aanbevelingen op. Ten eerste, als de investeerder beslissingsvrijheid toelaat in het subportefeuille beheer, dan moet hij goed beseffen wat voor implicaties die vrijheden met zich meebrengen. Ten tweede zal de toegenomen communicatie tussen de onderlaag met alle managers en de strategische toplaag leiden tot verbetering in allocatie van de beschikbare middelen van de investeerder. Als laatste dient de investeerder heel voorzichtig te zijn bij de prestatieevaluatie van de managers. Een positieve IR staat niet altijd garant voor een hogere SR dan de benchmark SR.
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Hierarchical Portfolio Management
Theory and applications

Under his own preference, how should an investor coordinate the asset managers such that his aggregated portfolio is optimized? The efficiency of each managed sub portfolio and the aggregation of all the sub portfolios are the two main underlying problems considered in this dissertation.

Contrary to popular believes, the tracking error volatility (TEV) optimization, commonly used to find the optimal active portfolio, often yields inferior portfolio choices. The results in this dissertation together with those in Jagannathan and Ma (2003) underscore how effective simple portfolio optimization techniques can be.

In aggregating all the sub portfolios, the investor’s choice is limited if the managers only report the local optimal portfolio. Since the reported portfolios are the result of a stand-alone optimization within the sub portfolio while disregarding all the rest, each reported portfolio can only be optimal locally. A rational investor should and must demand for more choices than the locally optimal choice alone.

Using simple examples in the single and multi period setting, this dissertation illustrates how significant the improvement in aggregated portfolio performance can be, both in terms of expectation as well as realization.

Given the insufficiency of the TEV optimization, the inherent question is whether the active performance measures like the information ratio still suffice in judging a manager’s performance. As it turns out, the investor should be very careful when applying the active performance measures. Preferably, the Sharpe ratio should be used to judge the added value of a manager to the aggregated portfolio.

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