Mathematical Modeling of Floating Stock Policy in FMCG Supply Chains

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Abstract

The floating stock distribution concept exploits inter-modal transport to deploy inventories in a supply chain in advance of retailer demand. It is appropriate in case of batch production and containerized transport of standard product mixes. In this way response times are reduced and storage costs can be reduced as well by having products in the transport-pipeline. This concept was earlier analysed using a simulation approach and showed to be efficient under simplifying assumptions for the demand distribution.

In this paper we present two mathematical models to analyse this policy while backlogging is allowed. The first one tries to optimize the advanced shipping time of containers to inter-modal terminal, and the second one optimizes the total number of containers in pipeline and terminal. In fact, in both policies containers are shipped to a terminal before the demand is realized in order to benefit from less storage cost at the terminal by utilizing the shipping time and also free storage cost period at inter-modal terminals.

A comparison is made with the simulation outcomes of applying previously developed strategies which shows that this concept has advantages in inventories over other strategies.

Keywords: Supply chain, Floating Stock, Inter-modal Transport, Virtual Warehousing, Inventories.

1. Introduction

Fast delivery is used in many retail supply chains. The advantages are enjoyed mainly by the retailers as they can operate in a just-in-time mode: they need fewer inventories on-site which reduces operational costs (both holding and storage costs) and investment costs

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(through less amount of warehouse space required). When they call orders, they can rely on rapid fulfilment. This works well if the order lead times and production time allow the manufacturer to operate on a make-to-order basis. If this option is not available and substantial batches are made, the burden of keeping inventory is shifted from the retailer to the supplier. In this case the supplier has to either store it close to the retailer or use fast transport in order to meet the required order lead time (Ochtman et. al 2004). This leads to many transport movements with few opportunities for loaded return trips. The considered distribution concept in this paper, inter-modal floating stocks, supports just-in-time delivery with shorter order lead times for manufacturers that follow a make-to-stock strategy.

The floating stock concept exploits the opportunities inter-modal transport offers to deploy inventories in the supply chain. The idea is that by advanced deployment and carefully tuning demand with transport modes we can reduce non-moving inventories, shorten lead times and improve the order fill rate. This strategy benefits from floating of stocks and the existence of inter-modal terminals to postpone the selection of the destination so that a pooling effect can be obtained in comparison to direct road transport. We use inter-modal transport with deferred final transport instead of transshipments to achieve postponement of positioning in a sense built on Herer et al. (2002). By this setting, we create a kind of virtual warehouse at the inter-modal terminals, yet one different than commonly referred to in literature (see e.g. Landers et al. 2000, as they stress real-time global visibility of logistic assets).

Although the term floating stock is relatively new, the concept is applied already for a long time. It is used when shippers send their containers in advance of demand from Asia to Europe or to US and the final destination is determined only in the final port. Another example is North American lumber, where lumber producers would ship loads to north central and eastern customers before demand had finalized (Sampson et al. 1985). The flatcars or boxcars were held at transit yards in the mid-west until a customer order was received. This practice enabled western US producers to compete in the eastern markets with their southern competitors in terms of lead times. Yet, almost no literature is available on floating stocks, as the terminology is not yet standardized. The exceptions are Teulings & van der Vlist (2001) which do not deal with inventories and a companion paper by Ochtman et al. (2004) which applies a simulation approach for studying the floating stock concept, but do not propose a mathematical optimization model to deal with this policy.

To position our contribution in literature, we further relate it to three streams, viz. intermodal studies, inventory management and outbound dispatch policies. Inter-modal transport can be defined (ECMT 1993) as the movement of goods in one and the same loading unit or vehicle by successive modes of transport without handling of the goods themselves during transfers between modes, e.g. container transport via rail and road. The transfer points offer short term storage to decouple the successive steps in the transport chain and they often feature a limited amount of free time for which no storage costs are charged. Nowadays this transport method is strongly advocated by many European governments in order to reduce road congestion and pollution. Inter-modal transport is however, on short distances more costly than road transport since it requires more handling. Furthermore, its transit time is often longer than that of direct road transport and its reliability is not always high (Konings 1996). Transport studies such as Bookbinder & Fox (1998) and Rutten (1995) typically make such comparisons between road transport and inter-modal transport, but in these studies inventories are left out of consideration.

Inventory management is another important topic in supply chains (Chopra & Meindl 2004). The main emphasis here is on determining how much inventory should be kept at which stocking locations, while typically only one lead time (and hence transportation mode) is considered. A well-known result is that centralization or pooling can reduce inventories if demands are uncorrelated, at the expense of higher transportation costs and a longer response time. This has led to the creation of European Distribution Centres, from which goods are trucked to clients throughout Europe directly upon client's calls. Different transport modes are considered primarily in the case of emergency shipments to take care of stock-outs (Moinzadeh and Schmidt, Moinzadeh & Nahmias 1988). Some studies also consider lateral transshipments in multi-echelon chains, but mostly just in case of stock-outs (Minner 2003 and Diks et al. 1996). Herer et al. (2002) is an exception as they consider lateral transshipments to enhance postponement and hence leagility (i.e. a combination of lean and agility) in supply chains. There are a few studies that integrate transportation and inventory control (see e.g. Tyworth & Zeng (1998)), but they focus on the relation between either transport frequency or transit time reliability and inventory control. A negative effect of the floating stock concept is that few possibilities exist for pooling, as products are shipped already towards their destination. Evers (1996), (1997) and (1999) study risk pooling of demand and lead times in relation to transhipments. However, these studies do not consider transport costs. No studies seem to exist on integration of inter-modal transport and inventory control, according to recent reviews on inter-modal research, such as Bontekoning et al. (2004) and Macharis & Bontekoning (2004).

The ideas in this paper can also be related to outbound dispatch policies for integrated stock replenishment and transportation decisions. The logistic literature reports that two different types of such policies are popular in current practice. These are *time-based* and *quantity-based* dispatch policies. Considering the case of stochastic demand, it has been shown that the quantity-based policy has substantial saving over time based policy. Under a time-based policy each order is dispatched by a pre-specified shipment release date even though the dispatch quantity does not necessarily realize transportation scale economies. On the other hand, under a quantity-based policy, the dispatch quantity assures transportation scale economies, but a specific dispatch time cannot be guaranteed. An alternative to these two policies is a hybrid routine aimed at balancing the trade-off between the timely delivery advantages of time-based policies and the transportation cost savings associated with quantity-based policies. Under a hybrid policy, the objective is to consolidate an economical dispatch quantity, denoted by q_H. However, if this quantity does not accumulate within a reasonable time window, denoted by T_H, then a shipment of smaller size may be released. A dispatch decision is made either when the size of a consolidated load exceeds q_H, or when the time since the last dispatch exceeds T_H (Cetinkaya et al. 2006).

The main difference between the problem discussed in this paper and the previously discussed outbound dispatch logistic is that in those models first demand is realized and then a shipping is done by either time-based or quantity-based policies. But in the problem discussed in this paper, it is intended to ship before demand realization. It is worth nothing that when intermodal transport is used, the shipment time increases considerably and the order lead time will usually be exceeded if the order is shipped after it is received. Increasing the order lead time forms a great problem, especially in the retail industry in which even the short delays are not acceptable. To avoid this problem the shipment should be sent in advance, before the order is placed, and then in this case the system can be benefited from a fast delivery time to customer.

In this study we consider a Fast Moving Consumer Goods supply chain and we will present mathematical models for floating stock policy in that supply chain. These models address the question of how to schedule shipment of containers through intermodal channel. Next, we will compare the results of our models with other distribution strategies.

This paper is organized as follows. In section 2 the problem environment is explained and based on that a quick review on possible distribution strategies for this problem is presented. This review is done, to be able to compare the result of developed policies to the previously developed distribution strategies. Then in section 3 the floating stock strategy is formulated and two different policies to deal with this problem are developed. Section 4 focuses on the calculation of safety stock in DS/CSS strategy. In section 5 some numerical results of applying the developed policies for a real world case are shown. Finally, section 6 ends up with some conclusions.

2. Problem definition

In this paper we consider a Fast Moving Consumer Goods (FMCG) supply chain with two echelons (the manufacturer's warehouse and the intermodal terminals) and one type of product (or aggregated mix). The products can be stored in a storage location near the factory (which we call the factory storage) or can be transported to an intermodal terminal where they can wait before being sent to the final destination (see Figure 1). The advantage of factory storage is that it is cheaper than intermodal terminal, but the traveling time from that terminal to customer is shorter than in case of factory storage.

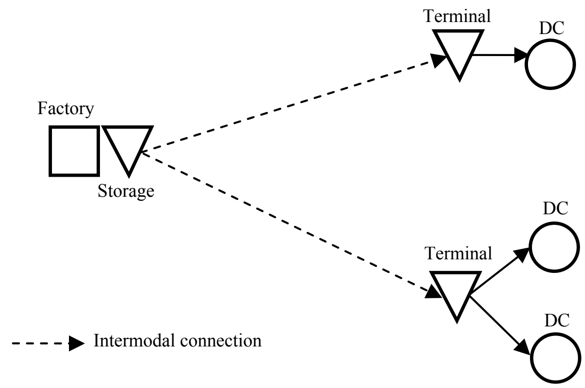

Figure 1. Conceptual representation of an FMCG supply chain with intermediary stocking points.

Generally Fast Moving Consumer Goods (FMCG), also known as Consumer Packaged Goods (CPG), is a business term with different interpretations. The main characteristic of these products is having a high turnover and relatively low cost. Though the absolute profit margin made on FMCG products is relatively small, the large numbers they sell in can yield a substantial cumulative profit. Examples of FMCG include a wide range of frequently purchased consumer products such as cosmetics, batteries, paper products and plastic goods. FMCG may also include pharmaceuticals, consumer electronics, packaged food products and drinks. What is essential in our analysis is that a batch production is applied and full containers are used for transportation. If turnover is low and product cost is high then typically smaller shipments are sent. This is the reason we focus on FMCG.

We assume that in each production run a batch of containerized products is produced and they are packaged in the factory. Determining the set of products that are packed into one container is left out of consideration. Moreover, we assume that the finishing of the production batch can be adjusted to the shipment of the last container from the factory.

In practice, intermodal terminals are not meant for long-time storage, since they have higher storage costs, but they benefit from a short delivery time to customers. That is, delivering the whole production batch directly to the terminal will cause higher storage costs but will also guarantee very fast delivery to the customers and lower backorder costs. In addition, once they have arrived, it is common in practice that intermodal terminals offer a period of free of holding costs for each delivery (e.g. due to discounts given by the terminal authorities), this feature is included in the proposed model. Moreover, we assume that when a customer arrives while there is no container available in the terminal, the demand is backlogged and will be satisfied as soon as a container becomes available. Therefore system endures shortage cost in this case.

Total costs are made up of transportation, shortage and storage costs. Transportation costs differ per transportation route. They contain all costs that result from using the specific transportation route. Therefore, transportation costs can cause differences in the total costs of each strategy, but these are independent of the inventory levels during a production cycle. The storage costs are the direct costs for storing a certain number of products for a certain period. These costs depend on the storage tariff at the specific point, the storage time, and volume of the products (or load units) stored. Shortage cost is stock-out cost, and is calculated base on the waiting time of customer for a container to arrive.

The main question in this paper is how to schedule the shipments of products from the factory such that the total cost is minimized. In the next section all the possible strategies for this problem are briefly reviewed.

2.1 Distribution strategies

For an FMCG supply chain with the above mentioned characteristics, we consider four distribution strategies. These strategies are based on two decisions. First, if every container will be stored in a centralized or a decentralized location, and second, if road or inter-modal is used for transportation.

Among these strategies, the first strategy is based on the just-in-time concept and applies direct road transport only. This is frequently used in FMCG-supply chains. The second strategy is completely based on distributed storage: all transports are inter-modal. This strategy is especially popular in supply chains where an inter-modal connection has lower transport costs than a road connection. The third and fourth strategies aim to take as much advantage of floating stock as possible. Below we will explain these strategies in detail.

Strategy CS: *Centralized storage and unimodal transport*: Using this just-in-time based strategy means that the whole production batch and a possible safety stock are stored on-site at the factory storage. When an order arrives, it is always fulfilled using road transport from the on-site inventory. In this strategy the emphasis is on fast transportations and easy coordination.

Strategy DS: *Decentralized storage and inter-modal transport*: The complete production batch is shipped to regional terminals using inter-modal transport. The safety stock is also stored in these regional terminals. This batch cannot be used to fulfill orders until it has arrived at the regional terminal. Any order which comes in during this transit time is assumed to be backlogged. This increases storage time and costs. A demand prediction is used to determine the split of the production batch over the regional terminals. Orders are delivered by truck from these terminals to the DCs. The emphasis is on using inter-modal transportation and short order lead times (because the orders lead time from the terminal will be shorter than the factory). If the safety stocks are depleted at a terminal, lateral transshipments from other terminals are made.

Strategy DS/CSS: *Decentralized storage, inter-modal transport, and centralized safety stock*: In this case the safety stock is stored at the factory storage, whereas the production batch is shipped to the terminals using inter-modal transport and stored there. The safety stock takes care of demands during shipment of the batch to the intermodal terminal. As soon as batches reach to terminals regular deliveries to the retailers are fulfilled from the terminals. The emergency deliveries from factory are done by road, because the inter-modal transit time is much longer.

The safety stock storage costs will probably be lower in the DS/CSS strategy when compared to the DS strategy. This is because long storage on-site is in general cheaper than long storage in an intermodal terminal. Furthermore, demand fill rate increases if the safety stock is stored in a central location.

Strategy FS: Floating Stock with staged arrivals: The Floating Stock strategy stores part of the production batch in the factory storage (centralized) and part of the production batch is stored in decentralized terminals. The shipment to the terminals is done by inter-modal transport. Once the products have arrived at the terminal, they are shipped to retailer from that point upon demand occurrence with a shorter order lead time. This strategy is designed to benefit from costs advantages of floating stock storage without having to increase the total inventory level in the supply chain. The FS strategy we consider here has staged arrivals, viz. for each container a shipment time is determined. This differs from the FS strategies in Ochtman et al. (2004) where the containers are sent together.

Note that if we assume the batching decision to be fixed beforehand and that the timing of the finishing of the batch coincides with the shipment of the last container, then the CS strategy needs no safety stock and is completely determined. For the DS/CSS strategy we still need to determine the amount of safety stock needed. This can be done with standard approaches such as marginal cost analysis and the approach will be described in section four. For the Floating Stock strategy however, we need to determine when to ship each container, which is the problem this paper focuses at. We will do that in the next section.

3. Formulation of the container shipping scheduling problem.

The shipment process can be explained as follows: when the last container has been sent out of the factory storage, say at time t, a batch of size m is produced (the production time is neglected) and therefore we have m containers ready for shipment at the production facility. Customers call off containers according to a stochastic demand processes. Production planning is outside the scope of this paper, thus we assume the value of m is given. To fulfill the demand, containers are shipped according to FS strategy from the manufacturer site to the terminal before demand actually occurs. It takes T_{fi} days for delivery from factory to the intermodal terminal and during these days and the initial period T_{nh} at the terminal no storage costs occur. T_{nh} is the number of days without storage cost at intermodal terminal. If a customer requests his container after termination of the free charge period, the system incurs storage cost h_i at the intermodal terminal per container per time unit. On the other hand, if a customer requests his container before end of the free of charge period, the container had spent long time at the factory and factory storage costs could be reduced by sending it earlier to the terminal. If a customer arrives while there is no container at the terminal the demand will be backlogged until the container arrives. This shortage costs c_b per container per time unit.

We assume an Erlang(k, λ) renewal process for demand, since the Erlang(k, λ) distribution can be interpreted as the batching of k exponentially distributed demands, which is simply treatable and it is straightforward to calculate the convolution of the same Erlang random variables. Other processes may need more computational effort to calculate the convolution.

In the rest of this section, we present two approaches to deal with the FS strategy. The first one is a time based policy, that tries to find the optimal shipping time and the second one is a quantity based policy that ships a new container whenever the total number of containers in pipeline and terminal drops down to a certain level. It is worth noting that, without loosing generality, the supply network is decomposed into serial chains including an intermodal terminal and a warehouse, and each chain is treated independent of other chains.

3.1 Time based policy

As we have already mentioned, we aiming at the optimization of shipping moments such that the average expected costs are minimal. As we assume a fixed batch of size m which is repeated all the time, the long-term average costs equal the expected costs per manufacturing batch. That is, for each batch we will optimize the shipment moments $r_1, \ldots, r_m$ of the containers in the batch such that the average cost per container is minimal:

$$\min_{r_1,\ldots,r_n \geq 0} \bar{C}\left(r_1, r_2, \ \ldots, \ r_m\right) = \frac{E\left[\text{batch cost}\left(r_1, r_2, \ \ldots, r_m\right)\right]}{\text{batch size}}$$

Let us construct now this cost function.

3.1.1 Cost function

The assumed cost structure incorporates factory, and inter-modal terminal storage and shortage cost. It is worth noting that during shipment of a container system does not endure holding cost. To formulate the total cost, assume that *k-1* demands have happened so far and A_k as the arrival time of the k^{th} container arriving to inter-modal after replenishment time and it is planned to satisfy k^{th} demand. Considering this situation three possible circumstances are likely to happen in backlogging case:

$D_k \leq A_k$

In this case, demand occurs before the arrival of the container, i.e. the demand is backlogged and the customer has to wait for the container to arrive. Total cost in this case consists of backlogging costs for the length of delay and holding costs at the factory until the shipment moment.

$A_k < D_k \leq A_k + T_{nh}$

Here, demand happens after arrival of the container and before end of the inter-modal terminal free holding cost period. Therefore, no holding cost is incured at inter-modal terminal, and total cost in this case includes only the holding costs at the factory until the shipment moment.

$A_k + T_{nh} < D_k$

In this circumstance, container arrives to inter-modal, spends the entire free holding charge period at the terminal and then the demand is realized. This case leads to holding cost at inter-modal terminal plus the holding costs at the factory until the shipment moment. Due to deterministic transportation time, the arrival moments A_k can be immediately computed from the shipment moment r_k as $A_k = r_k + T_{fi}$.

Note now that, due to the backlogging assumption, each shipped container will be in fact "assigned" to a certain demand moment D_t. Therefore, optimizing the shipment moments r_1, …, r_n of the containers in a certain batch, we can ignore the demand moment that were "assigned" to other containers. This allows us to separate the function of the total expected cost of a batch and rewrite it as:

$$E\left[C\left(r_1, r_2, ..., r_m\right)\right] = \sum_{k=1}^{m} E\left[C_k\left(r_k\right)\right] \tag{1}$$

Taking into account the demand model, the cost functions $C_k\left(r_k\right)$ for each container will be computed as:

$$C_k\left(r_k\right) = \begin{cases} h_f r_k + c_b\left(r_k + T_{fi} - D_k\right), & P\left(D_k \le r_k + T_{fi}\right) \\ h_f r_k, & P\left(r_k + T_{fi} < D_k \le r_k + T_{fi} + T_{nh}\right) \\ h_f r_k + h_i\left(D_k - r_k - T_{fi} - T_{nh}\right), & P\left(D_k > r_k + T_{fi} + T_{nh}\right) \end{cases} \tag{2}$$

Then, expected cost for each container is the following:

$$\begin{aligned} E[C_k(r_k)] = h_f r_k + c_b \int_0^{r_k + T_{fi}} \left(r_k + T_{fi} - D_k\right) f_{D_k}\left(\tau\right) d\tau \\ + h_i \int_{r_k + T_{fi} + T_{nh}}^{\infty} \left(D_k - r_k - T_{nh} - T_{nh}\right) f_{D_k}\left(\tau\right) d\tau \end{aligned} \tag{3}$$

Since total cost function is separable, to minimize the total expected cost of the batch, we need to minimize the expected cost of each container $E[C_k(r_k)]$.

In this case the following lemma is valid.

Lemma 1.

Given continuous distribution of the demand time D_t, optimal shipment moments r_k, that minimize the expected cost for each container $E[C_k(r_k)]$ either solve equation:

$$P\left(D_k \le r_k + T_{fi}\right) + \frac{h_i}{\left(c_b + h_i\right)} P\left(r_k + T_{fi} \le D_k \le r_k + T_{fi} + T_{nh}\right) = \frac{h_i - h_f}{c_b + h_i} \tag{4}$$

or should be set equal to the production moment, when there is no optimal r_k after the production moment.

Proof.

Assuming continuous distribution of the demand D_t, the expected cost function is continuous in r_k. Therefore, we can find the optimal shipment moment r_k by analyzing the first derivative of the cost function:

$$\begin{aligned}\frac{dE[C_k(r_k)]}{dr_k} &= h_f + c_b P\left(D_k \le r_k + T_{fi}\right) - h_i P\left(D_k \ge r_k + T_{fi} + T_{nh}\right) \\ &= h_f + c_b P\left(D_k \le r_k + T_{fi}\right) - h_i\left(1 - P\left(D_k \le r_k + T_{fi} + T_{nh}\right)\right) \\ &= h_f - h_i + \left(c_b + h_i\right) P\left(D_k \le r_k + T_{fi}\right) \\ &\qquad + h_i P\left(r_k + T_{fi} \le D_k \le r_k + T_{fi} + T_{nh}\right)\end{aligned} \tag{5}$$

From this expression it is easy to see that the first derivative is always greater than or equal to *0* in cases with $h_i \le h_f$:

$$\begin{aligned}&h_f + c_b P\left(D_k \le r_k + T_{fi}\right) - h_i P\left(D_k \ge r_k + T_{fi} + T_{nh}\right) \\ &\qquad \ge h_f - h_i P\left(D_k \ge r_k + T_{fi} + T_{nh}\right) \ge h_f - h_i \ge 0\end{aligned} \tag{6}$$

That is, the cost function $E[C_k(r_k)]$ is continuously increasing in r_k. Then, in order to minimize the costs, we have to set r_k as low as possible. In other words, we get a very natural conclusion that in the case with the holding cost at the factory higher than the holding cost at the terminal, it will be cheaper to send everything to the terminal as soon as it is produced. To find optimal shipping time r_k for the other case ($h_i > h_f$), we apply the standard method of optimization of convex functions and set the first derivative of the cost function $\frac{dE[C(r_k)]}{dr_k}$ to *0*, i.e. we have to solve the following equation:

$$h_f + c_b P\left(D_k \le r_k + T_{fi}\right) - h_i P\left(D_k \ge r_k + T_{fi} + T_{nh}\right) = 0 \tag{7}$$

It is easy to see that the first derivative is continuous non-decreasing function, since the second derivative

$$\frac{d^2 E[C_k(r_k)]}{dr_k^{\,2}} = c_b P\left(D_k = r_k + T_{fi}\right) + h_i P\left(D_k = r_k + T_{fi} + T_{nh}\right) \ge 0 \tag{8}$$

is always greater than or equal to *0*.

This means that the solution of the equation (4) minimizes the expected cost for each container $E[C_k(r_k)]$, if r_k is unbounded. However, in real life situations it is not possible to ship an order that is not produced yet. Therefore, optimal shipment moments r_k that minimize the expected cost for each container $E[C_k(r_k)]$ either solve equations $\frac{dE[C(r_k)]}{dr_k} = 0$ or should be set equal to the production moment (in cases when there is no optimal r_k after the production moment). ■

Note here, that for certain probability distributions, there is a possibility of multiple solution for the equation $\frac{dE[C(r_k)]}{dr_k} = 0$. However, since the first derivative is non-decreasing, all solutions of $\frac{dE[C(r_k)]}{dr_k} = 0$ will belong to one compact set and all of them will in fact produce the same value of $E[C(r_k)]$. The optimal solution can be easily found by a bisection search or an enumeration method.

3.2 Quantity based policy

To deal with this approach we make the extra assumption that the demand moments D_t are modeled as $D_t = D_{t-1} + \eta_t$, where D_{t-1} is the arrival moment of previous demand and η_t is stochastic interdemand time. As the second proposed policy, we will look for a shipment schedule defined by the system state. Namely, the schedule in which the shipment moments are defined by the total number of containers in the delivery "pipeline" and intermodal terminal, i.e. shipped but not picked-up yet by customers.

The cost function in this case can be defined similarly to the cost function (1) in the previous section. However, estimation and minimization of the expectation of the total cost for each container C_k requires different approach. First of all, we assume here that the container k will be shipped after the number of containers in the "pipeline" has changed from S_k to $S_k - 1$. S_k is the optimal total number of containers in the pipeline and intermodal terminal before realization of demand k. Since backorders are allowed, customers $k - S_k + 1, \ldots, k - 1$ will pick up their containers from the terminal before the customer k will arrive there. That is, we know exactly that $S_k - 1$ customers will come to the terminal before customer D_k. Then, the time from the moment of the system change from S_k to $S_k - 1$ until the moment of arrival of customer k will be distributed as $\sum_{i=0}^{S_k-1} \eta_{k-i}$.

Taking into account these assumptions about shipment policy, the total cost for container k can be defined as:

$$C(r_k, S_k) = \begin{cases} h_f r_k + c_b\left(r_k + T_{fi} - \sum_{i=0}^{S_k-1} \eta_{k-i}\right), & P\left(\sum_{i=0}^{S_k-1} \eta_{k-i} \le r_k + T_{fi}\right) \\ h_f r_k, & P\left(r_k + T_{fi} < \sum_{i=0}^{S_k-1} \eta_{k-i} \le r_k + T_{fi} + T_{nh}\right) \\ h_f r_k + h_i\left(\sum_{i=0}^{S_k-1} \eta_{k-i} - r_k - T_{fi} - T_{nh}\right), & P\left(\sum_{i=0}^{S_k-1} \eta_{k-i} > r_k + T_{fi} + T_{nh}\right) \end{cases} \tag{9}$$

where r_k has a different definition from that in the time based policy. It is defined as the time from the moment that the number of container in the system changes from S_k to $S_k - 1$ (moment D_{k-S_k}) until the container is shipped to customer k (see Figure 2).

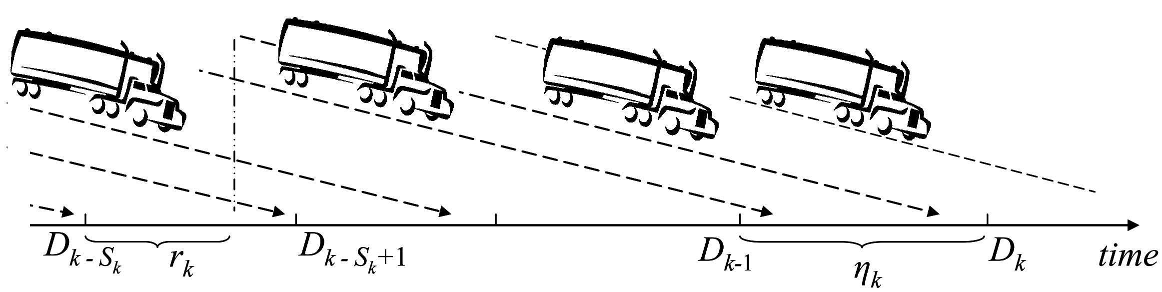

Figure 2. Arrivals of the customers (moments Dt) and of the containers to the terminal.

The expectation of the total cost for container k is shown in expression (10) and minimization of the system cost is done by the parameters S_k and r_k.

$$E\left[C_k\left(r_k,S_k\right)\right]=h_f r_k+c_b\int_0^{r_k+T_{fi}}\left(r_k+T_{fi}-\tau\right)f_{\sum_{i=0}^{S_k}\eta_{k-i}}\left(\tau\right)d\tau \\ +h_i\int_{r_k+T_{fi}+T_{nh}}^{\infty}\left(\tau-r_k-T_{nh}-T_{nh}\right)f_{\sum_{i=0}^{S_k}\eta_{k-i}}\left(\tau\right)d\tau \tag{10}$$

Lemma 2.

Given continuous distribution of the interdemand times ηt, optimal shipment moments $r_k(S_k)$, that minimize the expected cost for each container $E[C_k(r_k,S_k)]$ *either solve equation:*

$$P\left(\sum_{i=0}^{S_k}\eta_{k-i}\le r_k+T_{fi}-d_k\right)+\frac{h_i}{\left(c_b+h_i\right)}P\left(r_k+T_{fi}-d_k\le\sum_{i=0}^{S_k}\eta_{k-i}\le r_k+T_{fi}+T_{nh}-d_k\right)=\frac{h_i-h_f}{c_b+h_i}$$

or equal to 0.

Proof. Proof of this lemma is very similar to the proof of lemma 1. ■

This lemma, however, requires knowing convolutions of the interdemand times $\sum_{i=0}^{S_k}\eta_{k-i}$, which are not trivial in many cases. Still, for some distributions (such as normal or exponential), this convolutions are relatively easy. Let us find now optimal S_k and r_k for an Erlang demand process with identically distributed interdemand times.

3.2.1 Erlang distributed interdemand times

Let us assume that the interdemand times are Erlang distributed i.i.d's with parameters λ and k, say $\eta_k\sim$ Erlang(k, λ) Since the interdemand times are identically distributed, it is easy to conclude that for each arrival D_t the optimal parameters S_t and r_t will be identical. The convolutions $\sum_{i=0}^{S-1}\eta_i$ used in expression (10) will be then distributed according to Erlang(kS, λ) distribution. This means that we can write expression (10) as:

$$E\left[C\left(r,S\right)\right]=h_f r+c_b\int_0^{r+T_{fi}}\left(r+T_{fi}-\tau\right)\tfrac{\lambda(\lambda\tau)^{kS-1}}{(kS-1)!}e^{-\lambda\tau}d\tau \\ +h_i\int_{r+T_{fi}+T_{nh}}^{\infty}\left(\tau-r-T_{nh}-T_{nh}\right)\tfrac{\lambda(\lambda\tau)^{kS-1}}{(kS-1)!}e^{-\lambda\tau}d\tau \tag{11}$$

Further integration gives us the following close-form expression:

$$E\left[C\left(r,S\right)\right]=h_f r+c_b\left(r+T_{fi}\right)\left(1-e^{-\lambda\left(r+T_{fi}\right)}\sum_{n=0}^{kS-1}\frac{\left(\lambda\left(r+T_{fi}\right)\right)^n}{n!}\right)-c_b\frac{kS}{\lambda}\left(1-e^{-\lambda\left(r+T_{fi}\right)}\sum_{n=0}^{kS}\frac{\left(\lambda\left(r+T_{fi}\right)\right)^n}{n!}\right)$$
$$-h_i\left(r+T_{nh}+T_{nh}\right)e^{-\lambda\left(r+T_{fi}+T_{fi}\right)}\sum_{n=0}^{kS-1}\frac{\left(\lambda\left(r+T_{fi}+T_{nh}\right)\right)^n}{n!}+h_i\frac{kS}{\lambda}e^{-\lambda\left(r+T_{fi}+T_{fi}\right)}\sum_{n=0}^{kS}\frac{\left(\lambda\left(r+T_{fi}+T_{fi}\right)\right)^n}{n!} \quad (12)$$

It is easy to see, that for a fixed number S the optimal time $r(S)$ has to satisfy the following equation:

$$\frac{\partial E[C(r,S)]}{\partial r}=h_f+c_b\int_0^{r+T_{fi}}\frac{\lambda(\lambda\tau)^{kS-1}}{(kS-1)!}e^{-\lambda\tau}d\tau-h_i\int_{r+T_{fi}+T_{nh}}^{\infty}\frac{\lambda(\lambda\tau)^{kS-1}}{(kS-1)!}e^{-\lambda\tau}d\tau$$
$$=h_f-h_i+c_b\left(1-e^{-\lambda\left(r+T_{fi}\right)}\sum_{n=0}^{kS-1}\frac{\left(\lambda\left(r+T_{fi}\right)\right)^n}{n!}\right)+h_i\left(1-e^{-\lambda\left(r+T_{fi}+T_{fi}\right)}\sum_{n=0}^{kS-1}\frac{\left(\lambda\left(r+T_{fi}+T_{nh}\right)\right)^{n-1}}{n!}\right)=0$$

(13)

Simplifying the last equation we obtain an equation for the time $r(S)$ that is quite similar to the equation in the previous section:

$$\left(1-e^{-\lambda\left(r+T_{fi}\right)}\sum_{n=0}^{kS-1}\frac{\left(\lambda\left(r+T_{fi}\right)\right)^n}{n!}\right)+\frac{h_i}{\left(c_b+h_i\right)}e^{-\lambda\left(r+T_{fi}\right)}\left(e^{-\lambda T_{nh}}\sum_{n=0}^{kS-1}\frac{\left(\lambda\left(r+T_{fi}+T_{nh}\right)\right)^n}{n!}-\sum_{n=0}^{kS-1}\frac{\left(\lambda\left(r+T_{fi}\right)\right)^n}{n!}\right)=\frac{h_i-h_f}{c_b+h_i} \quad (14)$$

and the optimal time $r(S)$ is either equal to 0 or solves this equation. Substituting the optimal $r(S)$ into equation 8 for the expected cost per container we can simplify it further as:

$$E\left[C\left(r\left(S\right),S\right)\right]=c_b\left(T_{fi}-\frac{kS}{\lambda}\right)\left(1-e^{-\lambda\left(r(S)+T_{fi}\right)}\sum_{n=0}^{kS-1}\frac{\left(\lambda\left(r(S)+T_{fi}\right)\right)^n}{n!}\right)$$
$$+c_b\left(r\left(S\right)+T_{fi}\right)e^{-\lambda\left(r(S)+T_{fi}\right)}\frac{\left(\lambda\left(r(S)+T_{fi}\right)\right)^{kS-1}}{(kS-1)!}$$
$$-h_i\left(T_{nh}+T_{if}-\frac{kS}{\lambda}\right)e^{-\lambda\left(r(S)+T_{nh}+T_{fi}\right)}\sum_{n=0}^{kS-1}\frac{\left(\lambda\left(r(S)+T_{nh}+T_{fi}\right)\right)^n}{n!} \quad (15)$$
$$+h_i\left(\lambda\left(r\left(S\right)+T_{nh}+T_{fi}\right)\right)e^{-\lambda\left(r(S)+T_{nh}+T_{fi}\right)}\frac{\left(\lambda\left(r(S)+T_{nh}+T_{fi}\right)\right)^{kS-1}}{(kS-1)!}$$

Although, the function $E\left[C\left(\overline{r}\left(S\right),S\right)\right]$ and equation $\frac{\partial E[C(r,S)]}{\partial r}=0$ are not too complex for numerical computation, analytical derivation of the optimality conditions for S is not trivial. It is also not clear whether the function $E\left[C\left(r\left(S\right),S\right)\right]$ is convex. On other hand, S

is a one-dimensional discrete variable and in the most of real life situations it can not be extremely high (e.g. it can be limited by the number of available containers). Therefore, in many situations, optimization of S by enumeration will be still easy.

4. Calculation of safety stock for DS/CSS strategy

In DS/CSS strategy safety stock is hold in the factory and takes care of demand while the batch is being shipped to the intermodal terminal. The amount of containers which hold as the safety stock is calculated based on a marginal cost analysis. We define $C(ss)$ as the cost of stocking ss containers in factory and dispatching the rest of production batch to intermodal terminals. If this value is increased by one unit then extra holding cost should be paid at factory while it reduces the risk of shortage during shipment and decreases the holding cost at intermodal terminal since less containers are shipped to that terminal.

$$C(ss+1)-C(ss)=h_f E(\frac{D_{ss+1}-D_{ss)}}{2})-c_b E(T_{fi}-D_{ss+1})^+ - h_i E(D_{m-(ss+1)}-T_{fi}-T_{nh})^+ \geq 0$$

$$C(ss)-C(ss-1)=h_f E(\frac{D_{ss}-D_{ss-1}}{2})-c_b E(T_{fi}-D_{ss})^+ - h_i E(D_{m-ss}-T_{fi}+T_{nh})^+ \leq 0 \quad (16)$$

5. Numerical Results (Case Study)

Below we present a real case which has been made together with the logistic service provider Vos Logistics in the Netherlands to illustrate our model. Vos Logistics is primarily a European trucking company, but provides inter-modal transport as well. It faced more and more urgent transports with few opportunities for loaded return trips. Motivated by increasing road taxes, like the MAUD in Germany, she was considering expanding her inter-modal capabilities. The case is also described in Ochtman et al. (2004), but has been slightly adapted to allow the application of our models. The case is as follows. An FMCG-manufacturer runs a factory in Poznan (Poland) and distributes its products to two retail DCs in Germany, viz. one serving Dortmund and Köln and the other one covering demands from Rüsselsheim (near Frankfurt), and Appenweier (near Strassbourg). At this moment all orders are transported FTL by truck. The load unit is a 40 ft. container. An alternative inter-modal route is a rail connection from a station in Gadki (15 km from Poznan) to two train terminals in Duisburg and Mannheim. The conceptual network representation for this case is depicted in Figure 3.

The transit time for all two direct truck routes is two days including handling time for in- and outbound in the on-site DC. The inter-modal connection makes use of the rail connection. Due to the long time needed for shunting, the transit time of the train transport

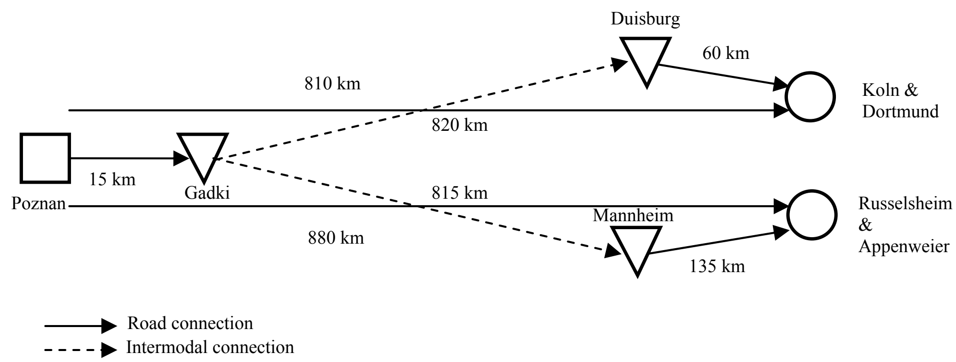

Figure 3. Conceptual network representation of the case

to both terminals is 4 days. The shipping time from an intermodal terminal to a customer is one day using road transport. If a stock-out happens in one of the terminal, then the customer has to wait until the container arrives to the terminal. The cost components which are used to estimate the costs are linear per FTL container delivery and are detailed in Table 1. It is worth noting that direct truck transport is slightly cheaper than inter-modal transport. The model presented in section three is now applied for each inter-modal terminal.

5.1 Simulation results

In order to be able to compare the developed policies with the other distribution strategies we implemented a simulation program using MATLAB 7.0. In this simulation the following criteria are considered: the expected total costs and the order fill rate. The order fill rate is the percentage of the orders that can be fulfilled in less than two days. This two day period is the direct shipping time from factory to the customer. In calculating total cost a fixed transportation cost component has been excluded since it has no effect on the optimal value. Therefore we have no cost for direct transport and 20 units per container for intermodal transport. Assuming that the size of the production batch is 80 containers and demand at each terminal is exponentially distributed with rate $\lambda = 1.5$ per day, we reach to the results shown in table 2. Floating stock policies outperform the other strategies in total cost. Comparing the fill rates shows that CS strategy and time based policies have the highest fill rate. The fill rate of CS strategy is always 100% since in case of direct shipment all containers are delivered to customer in less than two days and moreover we assume a zero production time of the batch. The DS strategy has the lowest fill rate, since for the first four days all demands that happen during this period have to wait for the whole production

Table 1: Cost Components

Components:	Costs (Euro per container):
Storage and Holding	
Centralized holding cost at factory storage h_f	8 / day
Decentralized holding cost in terminal (first 3 days are free) h_i	18 / day
Backlogging cost at inter-modal c_b	20 / day
Transportation	
For the direct road connection from factory to DC	880
For the intermodal connection from factory to DC	900

batch to arrive at intermodal terminal. Extra analysis showed that if we exclude the constant transportation cost from total cost more than 95% of the total cost is due to storage cost at factory and intermodal terminal, 3% transportation cost and just 2% of the total cost results from backlogging. In table 3, the optimal shipping times which are the resultant of the time based policy are shown (rounded off to an integer number of days). It shows for each day, how many containers should be shipped from factory to the each intermodal terminal. Since the arrival rates are assumed to be identical, number of containers that should be shipped to each terminal in each day is identical as well.

Table 2: Performance criterion for all policies

	CS	DS	CS/DSS	Floating Stock	
				Time based	Quantity based
Total Cost	79054.2	81958.9	82551.5	59337	73025
Fill Rate (%)	100	84	98	99.7	87

Table 3: Optimal shipping time of containers to each terminal

Shipping Day	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of containers	*4*	*4*	*3*	*4*	*3*	*3*	*4*	*3*	*2*	*2*	*2*	*3*	*3*

5.1.1 Sensitivity analysis

In this section, some sensitivity analysis is done on cost parameters. It is done for a case in which demand follows an Erlang distribution process. The cost parameters are changed and the results of different policies are shown in the tables 4 and 5. As it is obvious in table 5, the floating stock and particularly the time based policy outperform the other strategies in minimizing total cost. But as it is shown in table 4 the floating stock strategy is not the best one when we are dealing with slow moving items. We did further analysis on the basic problem with cost parameters shown in table 1 to find out under which circumstances it is feasible to use floating stock regarding total arrival rate to the system. Since the time based approach outperforms the other one for both criteria, we excluded the quantity based policy in figure 4. The results are depicted in figure 4 and it is obvious that for a total arrival rate of more than 0.6 per day, it is profitable to implement a floating stock policy.

Table 4: Performance criterion for all policies λ_D=0.11 and λ_M=0.13 and k=3

h_i	h_f	c_b	Performance	CS	DS	DS/CSS	Floating Stock			
							Time based	Quantity based	S_D	S_M
16	8	20	Total Cost	178720	278580	279350	240210	183040	1	1
			Fill Rate(%)	100	99.9	100	94.5	73.4		
24	8	20	Total Cost	178540	384190	382450	294490	190160	1	1
			Fill Rate(%)	100	99.9	100	96.6	70.8		
8	8	20	Total Cost	178120	176020	175600	185510	172210	1	1
			Fill Rate(%)	100	99.1	100	96.7	93.4		
16	2	20	Total Cost	97483	280800	278830	196410	139110	1	1
			Fill Rate(%)	100	99.1	100	90.1	85.0		
16	14	20	Total Cost	259010	280100	278770	281950	256380	1	1
			Fill Rate(%)	100	99.9	100	95.6	99.5		
16	8	50	Total Cost	178170	278430	279500	241560	179680	1	1
			Fill Rate(%)	100	99.1	100	95.7	95.7		

16	8	18	Total Cost	178580	279140	278850	239310	185420	1	1
			Fill Rate(%)	100	99.9	100	99.3	82.6		

D stands for Duisburg and M for Mannheim

Table 5: Performance criterion for all policies λ_D=2.5, λ_M=2 and k=3

h_i	h_f	c_b	Performance	CS	DS	DS/CSS	Floating Stock			
							Time based	Quantity based	S_D	S_M
16	8	20	Total Cost	22130	37597	29767	12453	19781	3	2
			Fill Rate(%)	100	95.6	100	92.5	84.8		
24	8	20	Total Cost	22090	55773	40057	14698	19484	3	2
			Fill Rate(%)	100	96	100	92.3	86.5		
8	8	20	Total Cost	22250	19684	20326	14563	18847	3	2
			Fill Rate(%)	100	96	100	100	81.2		
16	2	20	Total Cost	5543.7	37511	22637	5202	5484	3	2
			Fill Rate(%)	100	96	100	90.8	79.2		
16	14	20	Total Cost	38933	37550	36966	14133	25345	3	2
			Fill Rate(%)	100	96	100	99.4	80.3		
16	8	50	Total Cost	22210	37958	29614	10242	18334	3	2
			Fill Rate(%)	100	96.1	100	98.9	81.7		
16	8	18	Total Cost	22140	37647	29855	11481	19758	3	2
			Fill Rate(%)	100	95.9	100	96.6	81.4		

It is worth noting that, CS strategy depends only on holding cost at factory but we can see in both tables 4 and 5 that by changing other cost parameters the resultant total costs become slightly different. To analyze the accuracy of the simulation, 0.95% confidence intervals are derived. In table 4, for a case in which holding cost at factory is 8 the confidence interval is computed to be [178300, 178850] for 100 simulation runs. Therefore the difference among total costs in this case is not significant. This interval for arrival rates considered in table 5 is [22050, 22340], and again we can conclude that the differences are not significant.

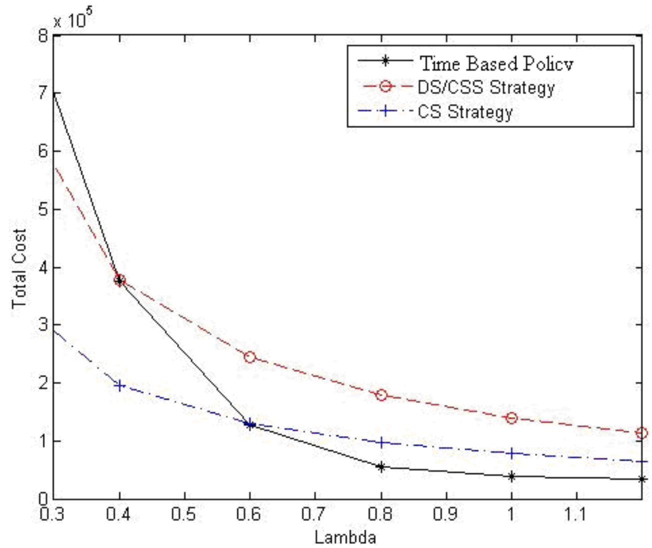

Figure 4. Total cost of different policies vs. λ_t

6. Discussion and conclusion

Floating stock is a concept where a new production batch is (partly) pushed into the supply chain, without determining the exact destination for each product beforehand. Use of this concept may lead to lower storage costs and a shorter order lead time, without a demand fill rate decrease. This is possible if the production batch is split up into some parts and being shipped to intermodal terminal in advance of demand realization at the right time. In this paper we developed mathematical approaches for floating stock strategy to determine the optimal shipping time of containers through intermodal route. In the developed policies, the production batch is split up into some parts. In the time based policy the shipping moments are optimized while in the quantity based policy the optimal total number of containers in the pipeline and intermodal terminal is determined. The simulation results show that the floating stock strategy offers the best opportunities to benefit from low storage costs without affecting fill rate level. The popular just-in-time strategy often uses centralized storage and road transport. The computational results show that the floating stock strategy can reduce costs and lead times, in spite of possibly higher transportation costs of an inter-modal connection. The main conclusion that can be drawn is that when an intermodal transport is used even though it can be slower and more expensive, if we integrate it with inventory control then the results show that it leads to cost saving without affecting the service level. So when considering a move from factory to DC, storage and holding costs as well as transportation costs should be taken into account.

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