

Forecasting the international diffusion of innovations: An adaptive estimation approach

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ERIM REPORT SERIES <i>RESEARCH IN MANAGEMENT</i>	
ERIM Report Series reference number	ERS-2003-073-MKT
Publication	2003
Number of pages	45
Email address corresponding author	yeverdingen@fbk.eur.nl
Address	Erasmus Research Institute of Management (ERIM) Rotterdam School of Management / Faculteit Bedrijfskunde Rotterdam School of Economics / Faculteit Economische Wetenschappen Erasmus Universiteit Rotterdam P.O. Box 1738 3000 DR Rotterdam, The Netherlands Phone: +31 10 408 1182 Fax: +31 10 408 9640 Email: info@erim.eur.nl Internet: www.erim.eur.nl

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BIBLIOGRAPHIC DATA AND CLASSIFICATIONS		
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Library of Congress Classification (LCC)	5001-6182	Business
	5410-5417.5	Marketing
	HD 45+	Technological Innovation
Journal of Economic Literature (JEL)	M	Business Administration and Business Economics
	M 31	Marketing
	C 44	Statistical Decision Theory
	O 31	Innovation and Invention
European Business Schools Library Group (EBSLG)	O 57	Comparative studies of countries
	85 A	Business General
	280 G	Managing the marketing function
	255 A	Decision theory (general)
	15 A	Technology, technological innovations
Gemeenschappelijke Onderwerpsontsluiting (GOO)		
Classification GOO	85.00	Bedrijfskunde, Organiseatiekunde: algemeen
	85.40	Marketing
	85.03	Methoden en technieken, operations research
	83.62	Economie van de Technologie
Keywords GOO	Bedrijfskunde / Bedrijfseconomie	
	Marketing / Besliskunde	
	Innovatiediffusie, Forecasting, Methode van Bayes, Internationale marketing	
Free keywords	Cross-country Diffusion, Forecasting, Bayesian estimation, International marketing	

**Forecasting the international diffusion of innovations:
An adaptive estimation approach**

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Forecasting the international diffusion of innovations: An adaptive estimation approach

Abstract

We introduce an international, adaptive diffusion model that can be used to forecast the cross-national diffusion of an innovation at early stages of the diffusion curve. We model the mutual influence between the diffusion processes in the different social systems (countries) by mixing behaviour. Furthermore, we apply the matching procedure as proposed by Dekimpe, Parker and Sarvary (1998). This international diffusion model is adaptively estimated using an augmented Kalman Filter with Continuous States and Discrete observations, developed by Xie, Song, Sirbu and Wang (1997). This is the first application of this procedure in an international context. We empirically applied this method to the diffusion of Internet access at home, and mobile telephony among households in the 15 countries of the European Union. The results show that our international, adaptive model performs well and is by far superior when compared to the classical method of estimating diffusion models for each country separately.

Key search words

Cross-country Diffusion, Forecasting, Bayesian estimation, International marketing

Introduction

Ever since the introduction of the Bass diffusion model (BDM) in 1969, a lot of attention has been paid by marketing researchers to the diffusion of new products in a particular market. Mahajan, Muller and Bass (1990) provide an extensive overview of the extensions of the standard Bass model. These studies mainly focused on one market or country, and so far limited attention has been paid to the international diffusion of new products. However, nowadays marketing managers launch new products not only into the domestic market, but sooner or later new products will often be introduced in foreign markets as well. Consequently, insight into the diffusion processes across countries is becoming more and more important, which has recently led to a growing interest of marketers to model these cross-national diffusion patterns (see e.g. Takada and Jain, 1991; Helsen, Jedidi and Desarbo, 1993; Putsis, Balasubramaniam, Kaplan and Sen, 1997; Dekimpe, Parker and Sarvary, 2000; Talukdar, Sudhir and Ainslie, 2002; Kumar and Krishnan, 2002; Tellis, Stremersch and Yin, 2003).

A significant gap in the global diffusion literature, however, is that to date little research has been conducted on forecasting the multi-country diffusion of an innovation, and especially on estimation approaches when little or no data are available. According to Putsis and Srinivasan (2000, p. 283), “procedures that combine Bayesian approaches with adaptive processes may be a promising avenue for future research addressing what to do when little or no data are available.” They suggest two approaches in particular, worth exploring when little or no data are available, i.e. the Hierarchical Bayes (HB) procedure used by Lenk and Rao (1990), and the adaptive procedure suggested by Xie, Song, Sirbu and Wang (1997), i.e. the Augmented Kalman Filter with Continuous State and Discrete Observations (AKF(C-D)).

Studies by Neelamegham and Chintagunta (1999) and Talukdar, Sudhir and Ainslie (2002) extended the HB method proposed by Lenk and Rao (1990) to international markets by pooling the data not only across multiple products, as Lenk and Rao (1990) did, but also across countries. However, these studies still have some limitations. First of all, the proposed HB method requires an analytical solution, i.e. the diffusion model is required to be solvable. However, diffusion models often are expressed by differential equations that do not have analytical solutions (Xie et al., 1997). Secondly, Talukdar et al (2002) have assumed that, consistent with most diffusion research, the parameters of the diffusion model are time invariant and use the time-invariant NLS estimation approach developed by Srinivasan and Mason (1986). A disadvantage of the time-invariant approach is that it often requires data to include the peak sales, which makes it less appropriate for early forecasting. Furthermore, it is unlikely that the parameters in a diffusion model will be constant over time. Van den Bulte and Lilien (1997, p. 338) have shown that “NLS estimates of the Bass model coefficients are biased and that they change systematically as one extends the number of observations used in the estimation.”

Our study overcomes these shortcomings. It contributes to the new product forecasting literature by developing an international, adaptive (IA) diffusion model, which can be used for early forecasts if only a few data points are available or even prior to launch, and by estimating this model with a time-varying Bayesian estimation procedure. Our model combines the approaches by Putsis et al. (1997) and Dekimpe, Parker and Sarvary (1998). Putsis and Srinivasan (2000) suggested this combination as a potential area for future study, but so far, none of the earlier global diffusion studies has combined these approaches. Following Putsis et al. (1997), we model the diffusion in multiple countries simultaneously, taking into account the cross-country interaction between the individuals of the different countries. Since it can be expected that some interaction of individuals takes place across countries, it is most likely that the adoption of an innovation in country A influences the innovation adoption in country B to some extent, dependent on the level of interaction. When studying international diffusion this cross-country interaction should be addressed, and is therefore explicitly incorporated in our model. We also apply a sample-matching procedure as proposed by Dekimpe et al. (1998). When pooling data across countries to estimate the diffusion parameters, Dekimpe et al. (1998) suggest that country samples should be matched on external criteria, such as country size, before valid cross-national comparisons can be made.

To estimate our IA diffusion model, we use the adaptive, Bayesian estimation procedure proposed by Xie et al. (1997), i.e. the Augmented Kalman Filter with Continuous State and Discrete Observations (AKF(C-D)). Xie et al. (1997) have demonstrated the superior prediction performance of the AKF(C-D) method over five other methods, amongst which the NLS estimation method by Srinivasan and Mason (1986) and the HB method of Lenk and Rao (1990). Nevertheless, this method has not yet been applied to an international diffusion model. Therefore, we extend the AKF(C-D) method to an international context, in order to be able to forecast the diffusion of an innovation across multiple countries.

To estimate and validate the proposed IA diffusion model, we use a novel data set on the penetration of the Internet access and mobile telephony among households in 15 European Union countries. The data sets we use have two major features. First of all, it concerns more recently introduced innovations as opposed to the products generally included in global diffusion studies, such as VCR's, color TVs, microwave ovens, etc (see Kumar, Ganesh and Echambadi, 1998). Consequently, the results of our study are not only useful for scientific purposes but also for the suppliers of these relative new products. Secondly, the yearly penetration rates included in these data sets are based on real adoption data at the household level, instead of annual sales data that are generally used in international diffusion studies.

The advantage of real adoption data is that no repeat purchases are included, which is the case with sales data.

The remainder of this article is organized as follows. The next section presents our forecasting model, and subsequently we discuss the adaptive estimation technique we use to estimate our IA diffusion model. Then, we discuss the data we use for the empirical validation of our forecasting approach, followed by a description of the forecasting results. These results are compared to the results obtained using the classical approach in order to evaluate the performance of our model. We conclude with a discussion of the limitations and suggestions for future research.

The international adaptive diffusion model

Cross-country interaction

Takada and Jain (1991) found faster innovation diffusion in the so-called lag countries, i.e. countries where the innovation was launched at a later point in time than in the lead country, i.e. the first country where the innovation is launched. This might be due to the communication between people from different countries. This cross-country interaction may influence the cross-country diffusion of a new product. For example, if a product is accepted fast in Belgium, does this automatically lead to a fast adoption in neighbor countries France and The Netherlands? So far, only a few studies modeled these cross-country influences (Mahajan and Muller, 1994; Putsis et al., 1997; Ganesh, Kumar and Subramaniam, 1997; Kumar and Krishnan, 2002). Table 1 provides an overview of these studies by summarizing the specific characteristics of each of these models. This table also includes the Staged estimation procedure by Dekimpe et al (1998), which will be discussed in the next section / paragraph.

- Table 1 about here -

Ganesh et al (1997) investigated the influence between the lead country and the lag countries. The learning effect they modelled, works only in one direction, from the lead country to the lag country. The diffusion in a lag country is supposed to have no influence on the diffusion in both other lag countries and the lead country. However, following communication, the influence between countries may take place between all the countries where the innovation is introduced (Mahajan and Muller, 1994; Putsis et al., 1997; Kumar and Krishnan, 2002). The mutual influence model proposed by Mahajan and Muller (1994) assumes, however, that the influence of adopters on potential adopters in another country is equal to the influence on potential adopters within their own country. This assumption is very hard to defend and there is no empirical evidence to support it. Since very little is known about the influence between countries, the model should allow for an intensity of the influence that is based on the observations of the diffusion process, which is done in the models of Putsis et al. (1997) and Kumar and Krishnan (2002). They both allow for a varying influence between all countries investigated, and found significant cross-country interactions. When studying international diffusion the cross-country interaction should be addressed.

To model this cross-country interaction, we use the model of Putsis et al. (1997) as the starting point. They model the mutual influence (called *mixing behavior*) as variable, ranging from no mixing at all (pure segregation, countries are isolated) to complete mixing (random mixing, mixing occurs freely). The empirical results obtained, show that the mean absolute percentage error is lowest when the intermediate form of Bernoulli mixing is used. We improve the model of Putsis et al. (1997) in two ways. First of all, Putsis et al. (1997) uses a

nonlinear least squares algorithm to estimate the model, which is non-adaptive, i.e. all the data is used in one pass. We reformulate the model to make it possible to adaptively estimate the parameters, i.e. the parameters estimations will be updated as additional data becomes available, a recommended estimation technique by for example Brettschneider and Mahajan (1980), Sultan and Farley (1990) and Xie et al. (1997). A second adjustment to the model of Putsis et al. (1997) is that we combine it with the matching procedure of the staged estimation approach of Dekimpe et al. (1998).

Sample Matching

When investigating the cross-national diffusion of an innovation, it is important to match the countries based on objective criteria, in order to be able to make a valid comparison of the diffusion patterns across those countries. Dekimpe et al. (1998) propose to match on three dimensions, i.e. the size of the countries, the penetration ceiling, and finally the time of origin. They apply this procedure to the diffusion of cellular phones across 184 countries, and show that the critical factor in explaining diffusion patterns across countries is the matched definition of (1) the social system size, (2) the adoption ceiling and (3) the time of introduction. In line with these results, we also apply the matching procedure on these three dimensions.

Ad 1. Matching on country size S_i

Of course one should reckon with the size of countries (e.g. in terms of inhabitants) before comparing the absolute number of adopters. A straightforward solution is expressing penetration as penetration per population.

Following Dekimpe et al. (1998) we define the market potential of country i , $m_i(t)$ as:

$$m_i(t) = C_i(t) * S_i(t) \quad \text{Equation 1}$$

Where:

- $S_i(t)$ is the exogenously estimated country size of country i expressed in the number of units of adoption (e.g. households or individuals),
- and $C_i(t)$ is the estimate at time t of the long-term ceiling of penetration in country i , varying between 0 and 100%.

The social system size, $S_i(t)$, will be determined using exogenous statistical data on the number of units per adoption, and is modeled as an exogenous variable:

$$S(t) = \text{exogenous variable} \quad \text{Equation 2}$$

To match on social system size when comparing the progress of diffusion in the different countries, we will report the diffusion using the penetration per unit:

$$PENETRATION_i = \frac{N_i(t)}{S_i(t)} \quad \text{Equation 3}$$

Where:

- $PENETRATION_i$ is the penetration per unit of adoption of the innovation in country i ,
- and $N_i(t)$ is the total number of units that have adopted the innovation at time t in country i .

Ad 2. Matching on penetration ceiling C_i

A certain percentage of individuals within a given country may never have sufficient intrinsic utility for the innovation in question. Therefore an exogenous ceiling, which is independent of the size of the country, should be introduced for each of the social systems studied.

Determining the long term ceiling, $C_i(t)$, is difficult. Dekimpe et al. (1998) estimate this ceiling completely exogenous, and don't adjust it as additional data points become available. Making a good assessment of the long-term utility of an innovation is, however, a complex and sometimes impossible task. In our opinion, it is necessary to use data available at later points in time to adjust a first assessment. Consequently, the penetration ceiling will be included in our model as a vector function of exogenous variables (e.g. income distributions) of which the parameters are allowed to vary²:

$$C(t) = G_C(\beta_C(t), \text{exogenous variables}) \quad \text{Equation 4}$$

Where:

- $C(t)$ is the $K \times 1$ vector of the long term ceiling in each of the K countries,
- $\beta_C(t)$ is a vector of time varying parameters,
- and G_C is a vector function of the parameter vector $\beta_C(t)$ and exogenous variables.

Ad 3. Matching on time of introduction t_{0i}

The time origin must be matched to correct for the fact that innovation introduction timing may vary widely across countries. Before the innovation is introduced in country i there will be no diffusion in this country. To match on the time of origin, the following constraint is imposed:

$$n_i(t) = 0 \quad \text{if } t < t_{0,i} \quad \text{Equation 5}$$

Where:

- $n_i(t)$ is the speed of diffusion in country i ,
- and $t_{0,i}$ is the time of introduction of the innovation in country i .

Model specification

We model the cross-country interaction by a Bernoulli noise mixing parameter Φ_i , as was found to be the best way to model it (see Putsis et al., 1997). Further, we extend the mixing behavior model of Putsis et al. in two ways. First of all, we replace the constant parameters by time varying parameters (see Equation 6) in order to be able to use an adaptive estimation technique, which allows us to update the estimates as additional data points become available. Secondly, we apply matching on country size, penetration ceiling and time of introduction (see equations 1 to 5).

For each of the K countries we model the diffusion as follows:

² This is allowed by the adaptive estimation technique used to adapt the initial estimates of the parameters based on the observations of the diffusion process.

$$n_i(t) = \frac{dN_i(t)}{dt} = \begin{cases} 0 & , \text{if } t < t_{0,i} \\ \left(m_i(t) - N_i(t) \right) * \left(p_i(t) + \sum_{j=1}^K q_i(t) * \rho_{ij}(t) * \frac{N_j(t)}{m_j(t)} \right) & , \text{if } t \geq t_{0,i} \end{cases}$$

with :

$$\rho_{ij}(t) = \begin{cases} \Phi_i(t) + (1 - \Phi_i(t)) * \left[\frac{q_j(t) * m_j(t) * (1 - \Phi_j(t))}{\sum_{k=1}^K q_j(t) * m_j(t) * (1 - \Phi_j(t))} \right] & , \text{if } i = j \\ 0 + (1 - \Phi_i(t)) * \left[\frac{q_j(t) * m_j(t) * (1 - \Phi_j(t))}{\sum_{k=1}^K q_j(t) * m_j(t) * (1 - \Phi_j(t))} \right] & , \text{if } i \neq j \end{cases}$$

Equation 6

Where:

- $N_i(t)$ is total number of units (e.g. households) that have adopted the innovation in country i ,
- $n_i(t)$ is the speed of adoption in country i (the derivative of $N_i(t)$),
- $m_i(t)$ is the market potential of country i ,
- $p_i(t)$ is the coefficient of external influence of country i ,
- $q_i(t)$ is the effective contact rate for country i (how gregarious are individuals in country i , and how susceptible are they to word-of-mouth influence),
- ρ_{ij} is the mixing probability for individuals in country i with individuals in country j ,
- $t_{0,i}$ is the time of introduction of the innovation in country i ,
- and Φ_i is the Bernoulli noise mixing parameter for country i , varying from $\Phi_i=0$ (random mixing; country borders do not exist) to $\Phi_i=1$ (no mixing at all; complete segregation).

Use of exogenous covariates

The possibility to use exogenous covariates to explain the cross-country diffusion differences in p and q is secured by the following equations:

$$\begin{aligned} p(t) &= G_p(\beta_p(t), \text{exogenous variables}) \\ q(t) &= G_q(\beta_q(t), \text{exogenous variables}) \end{aligned}$$

Equation 7

Where:

- $p(t)$ is the $K \times 1$ vector of the coefficient of external influence in each of the K countries,
- $q(t)$ is the $K \times 1$ vector of the effective contact rate in each of the K countries,
- $\beta_p(t)$ and $\beta_q(t)$ are vectors of time varying parameters,
- and G_p and G_q are vector functions.

Combining equations 1, 2, 6, and 7, we obtain the following differential equation with time varying parameters that describes the diffusion in the K countries:

$$n(t) = \frac{dN(t)}{dt} = f(N(t), \beta_C(t), \beta_p(t), \beta_q(t), \Phi(t), t_0, \text{exogenous variables})$$

Equation 8

Where:

- $n(t)$ is the $K \times 1$ vector of the speed of adoption in each of the K countries,
- $N(t)$ is the $K \times 1$ vector of the total number of units (e.g. households) that have adopted the innovation in each of the K countries,
- and $\beta_C(t)$, $\beta_p(t)$, $\beta_q(t)$, $\Phi(t)$, and t_0 are parameter vectors.

Model estimation

The Augmented Kalman Filter with Continuous State and Discrete Observations

We estimate the IA diffusion model (equation 8) using the Augmented Kalman Filter with Continuous State and Discrete Observations (AKF(C-D)) approach (Xie et al. 1997). This estimation procedure is an adaptive estimation procedure, providing a systematic way of incorporating prior information about the likely values of parameters, and a Bayesian updating mechanism to upgrade the estimates as additional data becomes available. Such an adaptive procedure facilitates forecasting early in the innovation life cycle.

Figure 1 shows how the AKF(C-D) estimation procedure works. The procedure is initiated by giving initial estimates of the parameters and the state (e.g. cumulative sales or the cumulative number of adopters) based on a-priori knowledge. The AKF(C-D) procedure updates the parameter estimations as soon as an additional data point becomes available. Two mechanisms are used for this updating procedure, a time-updating and a measurement-updating mechanism.

- Figure 1 about here -

At a certain point in time ($t=t_k$), the time-updating mechanism provides a-priori estimates of the cumulative number of adopters and the parameters for the period t_{k+1} . As soon as observations become available, this a-priori estimate is compared with the observation, and the forecasting error (i.e. the difference between the observed and forecasted number of adopters) is calculated. This forecasting error is subsequently used by the measurement-updating mechanism to update the estimation of the parameters, which leads to the a-posteriori estimates. This time- and measurement-updating mechanism can be repeated for a next period.

Xie et al.(1997) have developed their approach for diffusion models that have only one state. International diffusion models that describe the diffusion in multiple countries at a time have multiple states (e.g. the penetration in each country). Consequently, AKF(C-D) estimation approach is not directly applicable to international diffusion models, but using the underlying control engineering theory (Lewis, 1986; Stengel, 1986), it is possible to generalize the AKF(C-D) procedure to a procedure that can handle international diffusion models. We present this generalization in Appendix A.

The AKF(C-D) method compared to other adaptive estimation techniques

Table 2 compares four time-varying estimation approaches using adaptive estimation techniques.

- Table 2 about here -

Xie et al. (1997) have shown that the AKF(C-D) procedure provides better 1-year-ahead forecasts than the other three methods, due to three concrete advantages of this method. First of all, it can be applied directly to differential diffusion models, since it does not require a discrete analogue or an analytical solution as in case of the methods by Brettschneider and Mahajan (1980), Sultan, Farley and Lehmann (1990), and Lenk and Rao (1990). Given the fact that diffusion models are often represented as continuous differential equations, this is an important advantage. Secondly, the method of Xie et al. (1997) is better able to follow the changes in parameters over time. And finally, the AKF(C-D) procedure “explicitly

incorporates observation error in the estimation process, which is ignored in other procedures (Xie et al., 1997, p. 380).” The importance of this is twofold. It allows researchers to make better use of market data based on its’ reliability. An increase in the variance of the measurement noise, implying a less reliable measure, leads to a decreasing importance of the measure for the a-posteriori estimation. And the other way around, if the measure is characterized by small errors, then the measure becomes more important for the updating of the parameters. Additionally, taking into account the measurement noise, improves the estimation of the cumulative number of adopters. A procedure that not take into account the measurement noise, such as the Adaptive Filtering approach of Bretschneider and Mahajan (1980), provides at time t_k as the best estimation of the cumulative sales simply the measure of the sales at time t_k . The AKF(C-D) method estimates the sales by a weighted sum of the measure and the previous forecast.

Given the clear advantages of the AKF(C-D) method as compared to other three adaptive estimation procedures in Table 2, and given the fact that this method has not yet been applied in an international context, we decided to use this method to adaptively estimate our IA diffusion model. In order to be able to apply the AKF(C-D) method we had to reformulate the model as expressed in equation 8. Appendix B shows our IA diffusion model written in a form that allows the application of the AKF(C-D) estimation technique.

Empirical application

Data

Our data includes yearly information, covering the period 1990-1999, on the diffusion of Internet access and mobile telephony among the households in the fifteen countries in the European Union (Gallup Europe, 2000). The penetration data we use, has been calculated based on the responses to the following questions (Gallup Europe, 2000):

- “About your Internet access at home, in which year did your household first get Internet access at home?”
- “In which year did your household get your first mobile telephone?”

The Figures 2 and 3 show the penetration rates for Internet access and mobile telephony respectively. The data is based on over 44000 household interviews executed in the second half of 1999³ in 130 regions of the 15 Member States. It is by far the largest survey at a European level that has been undertaken in the sector. All the European regions have been covered and the sub-samples were of sufficient size as to obtain statistically reliable findings. A carefully set up sampling plan has been followed closely. The number of respondents interviewed per country varies from 1009 for Luxembourg to 5301 for France.

- Figures 2 and 3 about here -

Prior estimates: initiating the forecasting model

To initiate the filter in the AKF(C-D) method, we need a number of prior estimates. To be able to apply the sample matching procedure, we have to use estimates of the size of each country, the time of introduction of both products, and the estimated penetration ceiling as input in the model. Table 3 shows these input variables. The number of households is used as a measure for country size, because we use penetration data at the household level.

³ According to Gallup, the surveys for 1999 have been taken between June and October 1999, an exact date is not given. We have approximated this date by July 1, 1999 (1999.5).

- Table 3 about here -

In addition to the data in Table 3, we need an a-priori estimate of the cumulative adoption in each country at $t=t_0$ (=initial penetration), ($N(0) \sim (\bar{N}_0, P_{N0})$). We have fixed the expected values of the initial penetration for Internet access and mobile telephony at the values observed in 1994 (Internet access) and 1990 (mobile telephony), the years of introduction of these innovations. The variance is fixed at the square root of 0.5%, since we estimate that the standard error of the observations is 0.5%.

Finally, we need initial estimates of the distribution of the parameters p , q , and Φ at $t=t_0$ ($\beta(0) \sim (\bar{\beta}_0, P_{\beta0})$), and the covariance of the process noise Q and the measurement noise R .

The initial values of the parameters are assumed to be uncorrelated. Thus the initial distribution of the parameters, $\beta(0) \sim (\bar{\beta}_0, P_{\beta0})$, consists of the expected value and variance of each of the parameters at $t=t_0$. The variances will form the diagonal of the covariance matrix $P_{\beta0}$. All other elements of this matrix will be zero. We also assume that the process noises and the observation noises are uncorrelated. Consequently, in order to estimate the covariance matrices, we need to estimate the variances of both noises.

Determining the starting values for the parameters can be based either on expert judgments or on the experience with previous products. We do the latter, and construct the priors mainly based on the results reported in Sultan, Farley and Lehmann (1990), Putsis et al. (1997) and Xie et al. (1997), combined with the specific nature of the two types of innovation under study. The initial expected values and variances for the model parameters p , q , and Φ as well as for the noise statistics, used to initiate our model, are shown and elucidated in Appendix C.

Forecasting results

One-step ahead forecasts

Our IA diffusion model and the AKF(C-D) approach are formulated and estimated in a Matlab environment (Mathworks, 2001). First we present the one-step-ahead predictions and compare them with the actual observations. The step size is determined by the time between the observations, which is one year in our data set. In this application one-step-ahead is thus one-year-ahead for all but the last prediction, which is made for half-a-year.

Each time an observation becomes available, the IA method uses it to generate a new forecast (see Figure 1). This forecast consists of distributions that are assumed to be normal and are characterized by their expected value and variance. The variance and expected value can be used to calculate an interval estimate. We have chosen to show the 68% confidence interval estimate ($E(x) \pm 1SD(x)$), and the point estimate given by the expected value.

Figure 4 shows the results for the one-step-ahead predictions of the penetration of Internet access in Sweden (see Appendix D for the one-step-ahead forecast for mobile telephony in Sweden). Internet access was introduced in Sweden in 1994, and therefore the first forecast takes place for the year 1995. We have chosen to show the results for Sweden because the diffusion process has advanced the most in this country, which makes it also possible to compare the long-term forecasts with the observed penetration in the next section. Although we show the results for only one country here, we remind the reader that the IA method uses an international diffusion model and therefore provides a forecast for all countries simultaneously. Appendix E shows the one-step-ahead forecasts for Internet access for all countries.

- Figure 4 about here -

Figure 4 shows that the observed values always fall within the 68% confidence interval estimation. In 161 out of all 172 one-step-ahead forecasts made (15 countries, 2 innovations and different years), the observed values fall within the forecasted 68% confidence interval (i.e. in 94% of the cases). As such the interval estimation gives a reasonable estimation of the uncertainty. Furthermore, we can remark that the point estimate provided by the expected value is also close to the observed values. The average absolute one-step-ahead percentage prediction error (MAPE) is 12% for the Internet case and 10% for the mobile telephony case. Though these errors are substantial, these still are good results since a very simple model is used to forecast a process that is characterized by a high level of uncertainty.

Long-term forecasts

The forecasting method not only produces one-step-ahead forecasts. In fact, forecasts are obtained for the whole diffusion process. Since we only have observations up to July 1999, the long-term forecasts are shown only up to this date. The first available forecast is made using the initial estimates that are reported in Table 3 and Appendix C. Then, each time a new observation becomes available it is used to adapt the estimates of the parameters and a new forecast is made. Figure 5 shows the resulting long-term forecasts for Internet access in Sweden (the long-term forecast for mobile telephony in Sweden is shown in Appendix F, while Appendix G shows the long-term forecasts for Internet access for all countries).

- Figure 5 about here -

The initial forecast is shown in the upper left corner. We see that in the case of Internet access in Sweden our initial estimates have resulted in a forecast of the diffusion that is much too slow. In the following graphs, we see the adaptive forecasts provided by our IA approach, each time an observation becomes available. We see that the adaptive estimation results in forecasts that become better and better as more observations become available. Again, all observed values fall within the 68% confidence interval for all the forecasts. For the whole study (15 countries, 2 innovations and different years), this is true for 155 out of 172 long-term forecasts made (i.e. in 90% of the cases).

Like the expected values, the variances are estimated adaptively by the IA method. In the case of Sweden we see that as more observations become available the variance becomes smaller which causes the confidence interval to become narrower. This reflects the decreasing uncertainty of the predictions as more and more observations have been made, and past predictions have proved to be close to the observed values. This trend can also be seen in the long-term forecasts for mobile telephony in Sweden (see Appendix F) and the forecasts for Internet access for the other countries (see Appendix G).

The effect of the international dimension on the forecasting performance

In our IA diffusion model, the extent to which mixing takes place, or the extent to which the diffusion processes in the different countries influence each other, is regulated by the Bernoulli mixing parameter, Φ . This parameter can vary between zero, which implies ideal mixing, and one, which implies total segregation. When Φ equals zero, the European Union is modeled as one country without borders that influence the diffusion process. When Φ equals one, the diffusion is supposed to take place entirely within each country without any influence

between the countries. A value between zero and one implies that borders play a role (the closer Φ is to one the more), but the diffusion processes in the different countries do influence each other (the closer Φ is to zero the more).

In order to quantify the contribution of the international dimension of our IA diffusion model on the forecasting performance, we have fixed Φ at different values varying from zero to one. The mean absolute percentage errors (MAPE's) for Internet access in the one-, two- and three-step-ahead predictions are used as the performance indicators, and are shown in Figure 6. Figure 6 clearly shows the contribution of the international dimension. When the countries are considered to be isolated ($\Phi=1$), the forecasting errors are much higher than when a moderate influence between the countries is supposed to exist ($\Phi=0.7$). The results also clearly show that country-borders in the European Union still play an important role in the diffusion of innovations. When the borders are considered to be non-existent ($\Phi=0$) the forecasting error increases dramatically.

- Figure 6 about here -

The MAPE's are lowest for $\Phi=0.7$, a value which is close to the value for Φ found by Putsis et al. (1997) for the diffusion of personal computers (PCs), namely $\Phi=0.72$. This doesn't surprise, since these innovations are closely related (the device most widely used to obtain access to the Internet is the PC). When constructing our initial estimate of Φ (see Appendix C), we already indicated that we expected that Φ is likely to be similar across innovations with similar word-of-mouth networks like PCs and Internet access.

Based on these results, we can conclude that suppliers, who want to launch a new product across Europe, should take the country borders into account. The European Union cannot be considered to be one market, as the diffusion processes differ substantially across the countries. At the same time, however, the countries should not be treated as totally independent of each other, since our results clearly indicate that an important influence between the diffusion processes in the separate countries does exist.

The estimates of parameters p , q , and C

The IA forecasting method not only adaptively estimates the distribution of the diffusion or penetration itself, it also adaptively estimates the distributions of the parameters. Like the distributions of the penetration in the different countries, the parameter distributions are assumed to be normal and are characterized by an expected value and a variance.

The results show stable estimates of both the expected value of the penetration ceiling C and the coefficient of innovation p . For both parameters, no large adjustments with an important impact on the forecasts are made by the updating-mechanism. With respect to the effective contact rate q_i , however, substantial adjustments have been made to the a-priori estimate. Table 4 provides an overview of the initial estimates of q_i and the estimate after the final measurement update (halfway 1999) for all countries.

- Table 4 about here -

Comparing the estimates of the expected values of the q_i , we see that substantial adjustments, ranging from -27% for Luxembourg to +52% for Sweden, have been made for most countries. In Luxembourg, the penetration of Internet access happened to be slower than expected, while we found the opposite for Sweden. The adjustments to the expected values of

q_i made by the IA diffusion model can result from two sources. The initial estimate of the expected value may be different from the real value, or the diffusion process has changed in terms of q_i during its evolution. For Sweden, for example, the initial estimate appears to be a rather imprecise estimate. Figure 7 shows the evolution of the estimates for q for Sweden and Luxembourg. The initial estimate for Sweden was 0.81, and we see that the IA algorithm had adjusted this upward every time an observation became available. This upward adjustment was necessary because our initial estimate of q resulted in a forecast of the diffusion that was much too slow. However, not all adjustments of q are caused by a bad initial estimate. A good example of an adjustment that is caused by a changing diffusion process is the diffusion in Luxembourg. Figure 7 shows that the estimate of the expected value of q remains almost unchanged for the first four years. After the observation of the diffusion in 1998, however, the parameter is firmly adjusted downwards, due to a sudden change in the real diffusion process (see Appendix G – Long-term forecast for Luxembourg). This slowdown could be caused by all kind of effects. One possible explanation is the existence of Moore's chasm: the dissociation between the early adopters and the early majority (see Moore 1991, 1995).

- Figure 7 about here -

In addition to the parameter adjustments over time, Table 4 shows that the differences between the different countries in the final estimate are larger than our initial estimation. The expected values of the q_i after the final measurement update vary from 0.45 (Greece) to 1.25 (Sweden). This implies that the differences between the speeds of diffusion in the different countries are very large. Consumers from Portugal, Spain and Greece are amongst the slowest in adopting Internet, while consumers from Sweden, Denmark, and The Netherlands appear to embrace the Internet very rapidly. These differences between countries should be taken into account by managers when assessing the diffusion of innovations in the consumer markets in the European Union. When compared to the values of q found in other studies of the diffusion of innovations, the values of q shown in Table 4 are very high (see for instance Putsis et al. 1997, Dekimpe et al. 1998 and Sultan et al. 1990). The take up of Internet access in the EU has been very fast compared to other innovations like CD-players, VCRs and PCs. Finally, the last column of Table 4 shows a strong reduction of the standard deviations of the estimated parameters ($SD(q_i)$), ranging from -29% for Portugal to even -83% for Luxembourg, which reflects the reduced uncertainty. This implies that the estimates for parameter q_i have become more precise.

Evaluation of the forecasting performance of the IA diffusion model

A comparison with the classic approach

To evaluate the forecasting performance of our IA diffusion model, we compare the forecasting results of our model with the results obtained from the classical approach, which uses observations of the diffusion process in different countries to estimate the parameters p and q in a Bass model for each country separately. Consequently, it doesn't take into account the mutual influence of the diffusion processes across the countries, as our approach does. We estimate the parameters of this classical approach with the toolbox made available by Mahajan, Muller and Wind (2000). The classical diffusion model is estimated with a nonlinear least squares (NLS) estimation technique. Since this is a non-adaptive estimation technique, each period the whole diffusion curve is re-estimated, and the parameters p and q

are considered to be constant over time. To initiate the NLS estimation procedure, the market potential has to be determined exogenously. We have fixed it at the same value as we used to initiate our adaptive approach (see Table 3).

To evaluate the forecasting performance of the IA diffusion model, we focus on the forecasting errors. Table 5 shows the mean absolute percentage errors (MAPE's) for both methods, while we also include MAPE's for the IA approach with the international dimension switched off (i.e. IA with $\Phi=1$). These averages are calculated over the cases where both methods provide a forecast. Since the classic method needs at least two observations, it does not provide a forecast for the first two years following introduction. The results in Table 5 clearly indicate that the IA diffusion model provides better forecasts as compared to the classical approach. For both innovations, we found much smaller MAPE's for the IA diffusion model than for the classical diffusion model. This is even the case for the IA diffusion model with the international dimension switched off ($\Phi=1$). Thus both, modeling the mutual influence between the countries and the adaptive estimation technique contribute substantially to the better performance of our IA diffusion model. The biggest contribution to the better performance is, however, made by the adaptive estimation technique. We see only minor improvements in the forecasting errors for the IA approach with the international dimension switched on as compared to the IA method with $\Phi=1$. The better forecasting results suggest that the IA diffusion model should be preferred over a non-adaptive non-international forecasting approach when forecasting the international diffusion of innovations.

- Table 5 about here -

Sensitivity analysis

The IA diffusion model needs initial estimates to start the algorithm, which were discussed in the section on the empirical application. We have fixed very general initial estimates, which may not be the best initial estimates possible. Better initial estimates will of course give a better forecasting performance, especially for the first few years after introduction⁴. Therefore, another method to check the forecasting performance of our IA diffusion model is to perform a sensitivity analysis of the influence of changing the initial estimates on the overall performance indicators. Table 6 shows the results of this sensitivity analysis for the Internet case, and indicates that our findings are robust with respect to variations in the initial estimates. The results are the most sensitive for the initial estimate of parameter q .

- Table 6 about here -

Conclusions and implications

We have developed an international, adaptive diffusion model, which can be used to forecast the cross-national diffusion curves of a new product, just before, during, or immediately after a new product launch, if only a small number of data points are available. As soon as additional data points become available, the forecasts are re-estimated and adjusted. The model contributes to the literature by combining the approaches of Putsis et al. (1997) and Dekimpe et al. (1998). We model the mutual influence between the diffusion processes in the

⁴ As more observations have been used to update the estimates the influence of the initial estimates on the forecasts reduces.

different countries. Moreover, we apply a sample-matching procedure by matching the samples from the different countries on three dimensions, i.e. (1) the size of the country, (2) the penetration ceiling, and (3) the time of introduction of the innovation. To estimate the model we extend the AKF(C-D) estimation approach of Xie et al. (1997), which has not yet been used for estimating cross-national diffusion patterns, to an international context.

The data we use involve the diffusion of Internet access and mobile telephony among households in the 15 countries of the EU. The empirical application shows good forecasts of these two diffusion processes. The IA diffusion model result in much lower mean absolute percentage errors (MAPE's) for the 1-, 2- and 3-year-ahead forecasts as compared to the classical approach, i.e. estimating a Bass diffusion model for each country separately. Both the international dimension and the adaptive nature of the estimation approach contribute substantially to the better forecasting performance of the IA diffusion model. The biggest contribution, however, is made by the adaptive estimation, which is caused by the following advantages of this estimation technique: (1) the model parameters are allowed to change over time, which better reflects the reality than parameters that are not allowed to change, (2) the model uses a-priori knowledge through the usage of initial estimates, (3) the observation noise is modeled and used as an input for the adaptation of the estimates, and finally (4) it does not require the continuous diffusion process to be approximated by a discrete-time model.

The IA diffusion model leads to a better performance because it more accurately models reality. The assumption that diffusion processes in the different countries of the EU do not influence each other has little face value. Our results show that this assumption is indeed flawed. It appears that the diffusion processes in the different EU countries influence each other, but at the same time it is not possible to consider the EU as one big market, since the diffusion processes vary enormously across the countries. Especially the pace of the diffusion differs a lot.

When assessing the diffusion of high-tech or disruptive consumer innovations, marketing managers should take the mutual influence between the diffusion processes in different countries into account. Furthermore, they cannot take it for granted that the diffusion processes in the different EU countries show the same pattern. Thus our results indicate that when forecasting the diffusion in EU member countries, an international diffusion model, explicitly modeling the mutual influence, should be used.

Limitations and future research

To validate the IA diffusion model we used data involving only two innovations, Internet access and mobile telephony, among households in the 15 EU member states. Moreover, the diffusion processes of these two innovations are not yet finished, and therefore our forecasting method could only be tested on part of the diffusion process. Data on other innovations and / or from countries outside the EU should be used for further validation of our IA diffusion model. This will also provide the opportunity to make generalizations, e.g. new products diffuse always slower in Greece than in the other EU countries, or the parameter Φ is always about 0.7 for Europe.

To assess the forecasting performance of our IA diffusion model, we compared the results of our model with the forecasting results obtained from the classical approach, i.e. estimating a Bass model for each country separately. Future research could compare the forecasting results of our model with the results of other international diffusion models, such as the model of Putsis et al. (1997).

A final limitation concerns the fact that we did not use covariates (exogenous variables) in the empirical test of our model. For individual companies or managers, however, it could be important to see how the diffusion would look like if, for example, a certain amount of the budget were spent on advertising for the product. Although we didn't include covariates in our empirical test of the model, our model allows for including such covariates (see Equations 7 and 8). Including covariates will not only reduce the forecasting errors, but the model will also become more useful for the strategic planning of new product introductions, and for evaluating the effects of the marketing mix on the sales.

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Figure 1: The AKF(C-D) estimation procedure (Xie et al., 1997, p. 382).

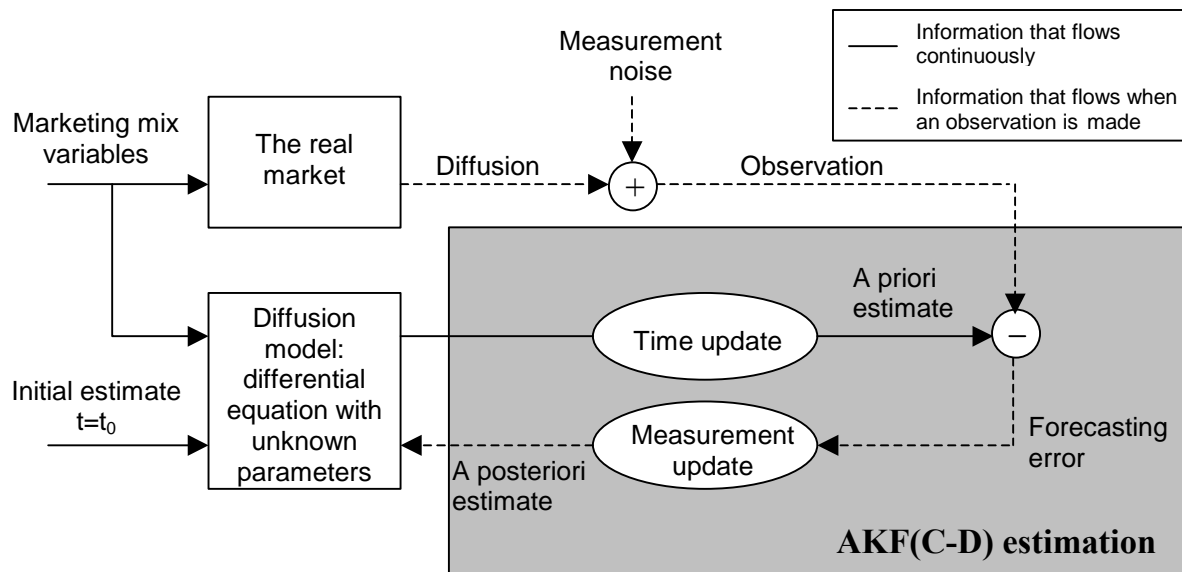


Figure 2: Penetration of Internet access in the households of the EU (Source: Gallup Europe, 2000).

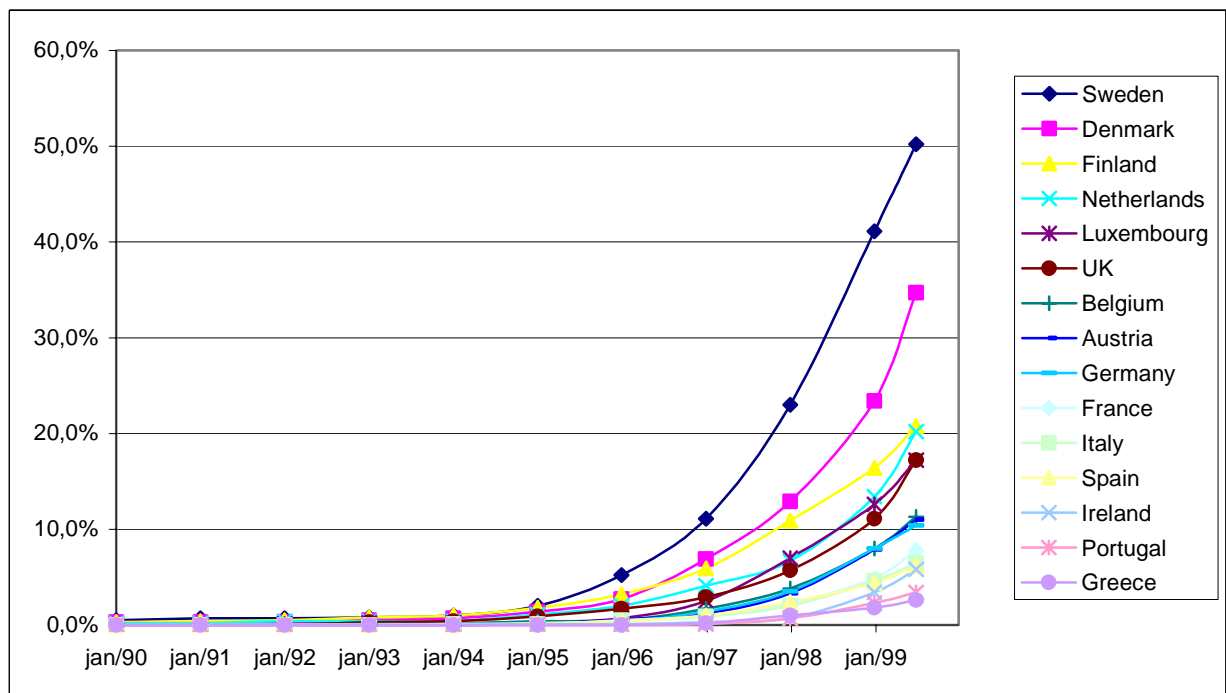


Figure 3: Penetration of mobile telephony in the households of the EU (Source: Gallup Europe, 2000).

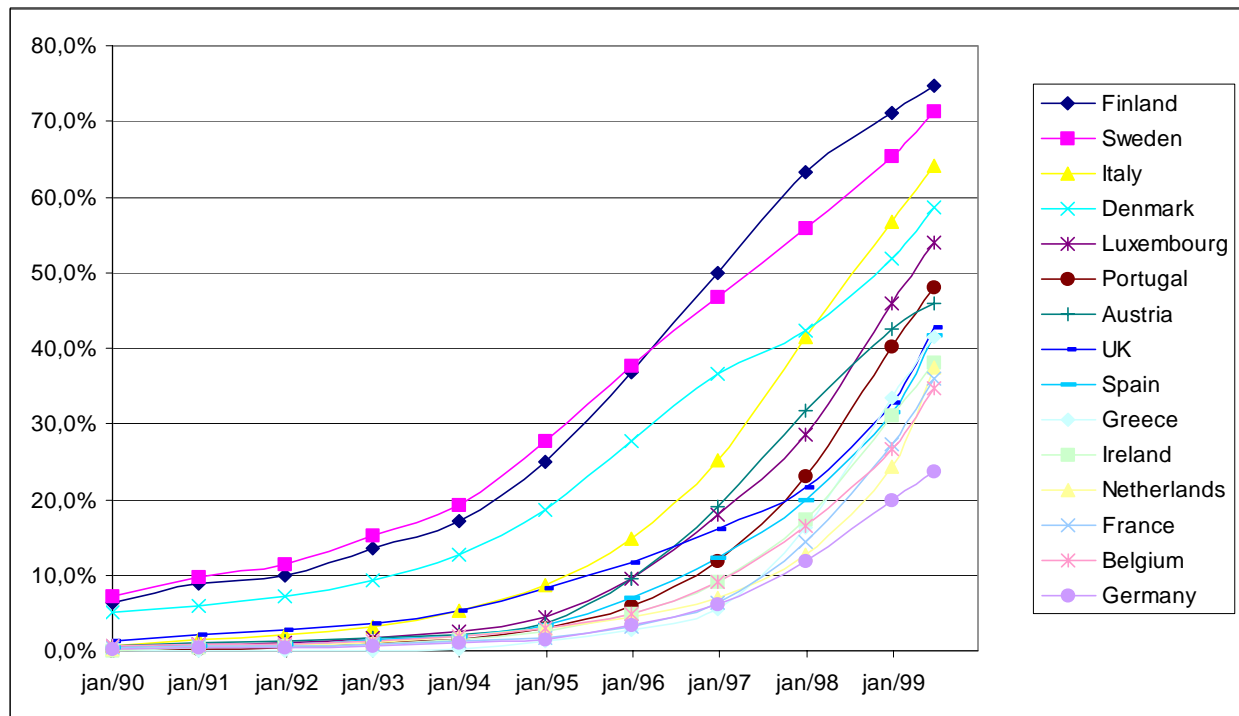


Figure 4: One-step-ahead prediction for Internet access in Sweden

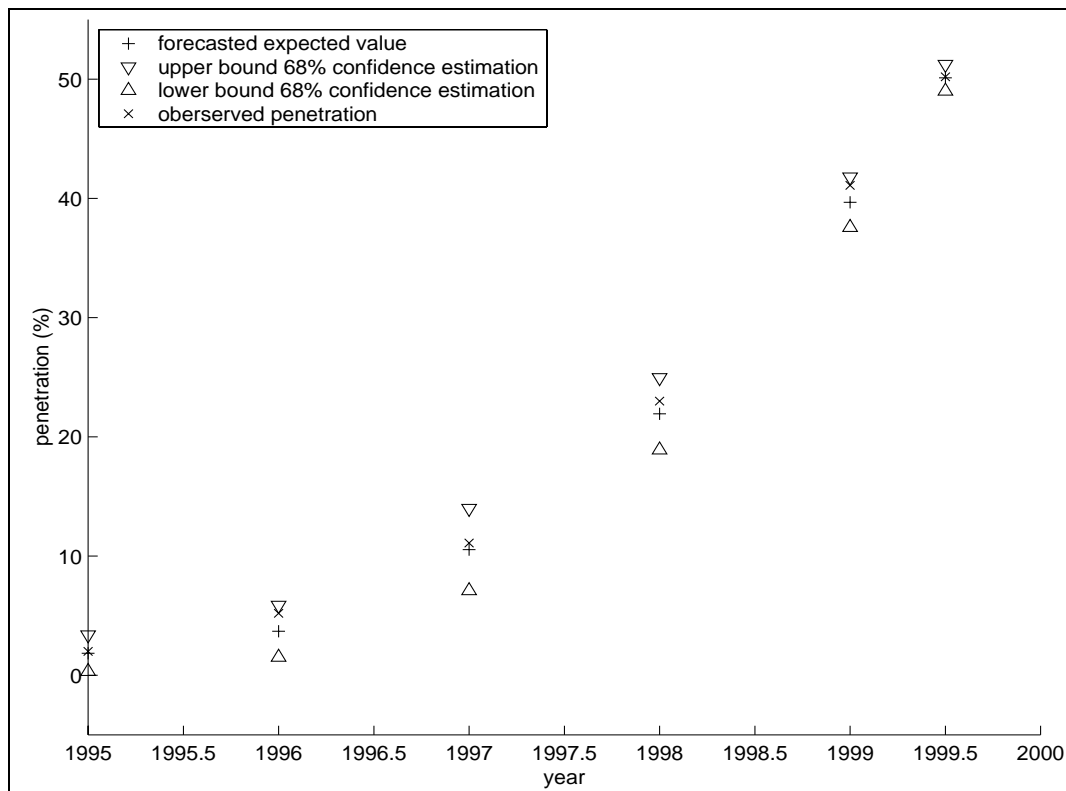


Figure 5: The long-term forecasts for Internet access in Sweden

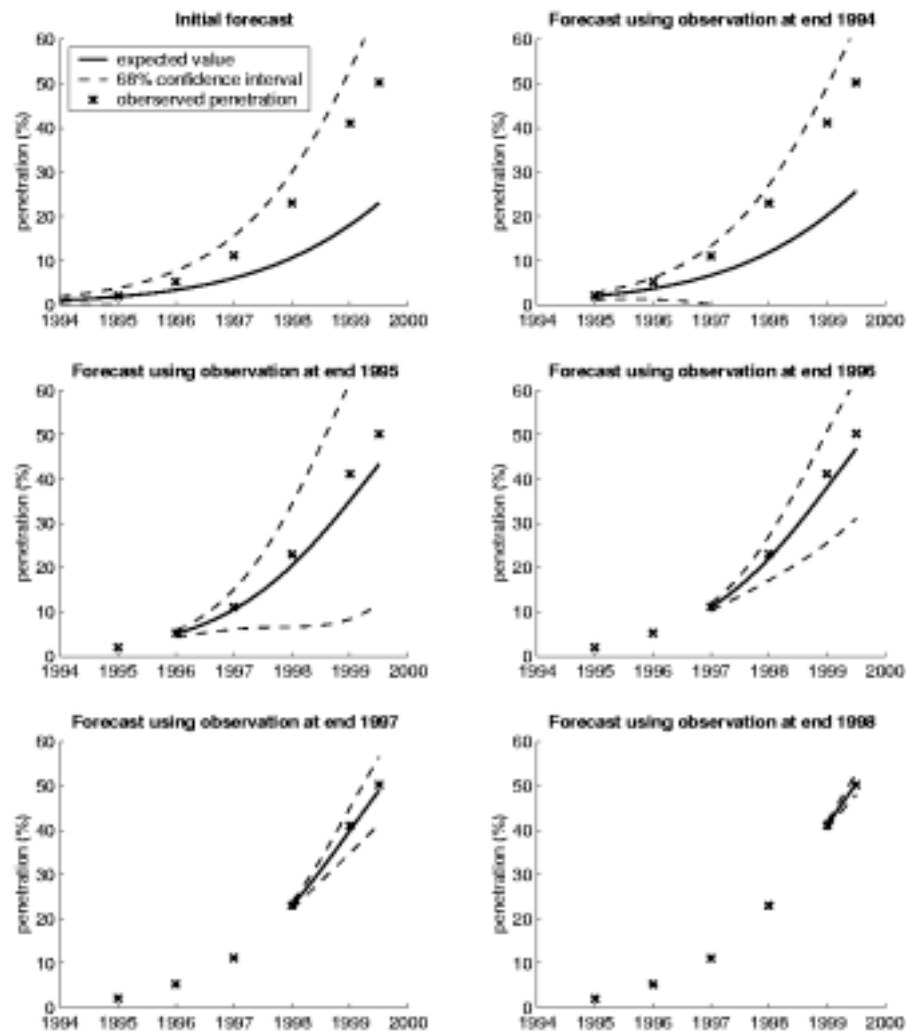


Figure 6: Forecasting errors for different values of Φ (Internet access case)

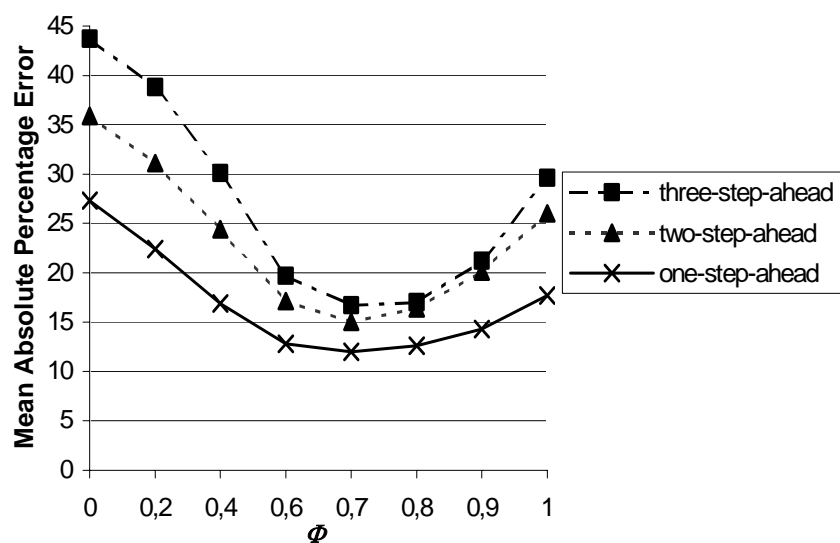


Figure 7: The evolvement of the estimates of q for Sweden and Luxembourg

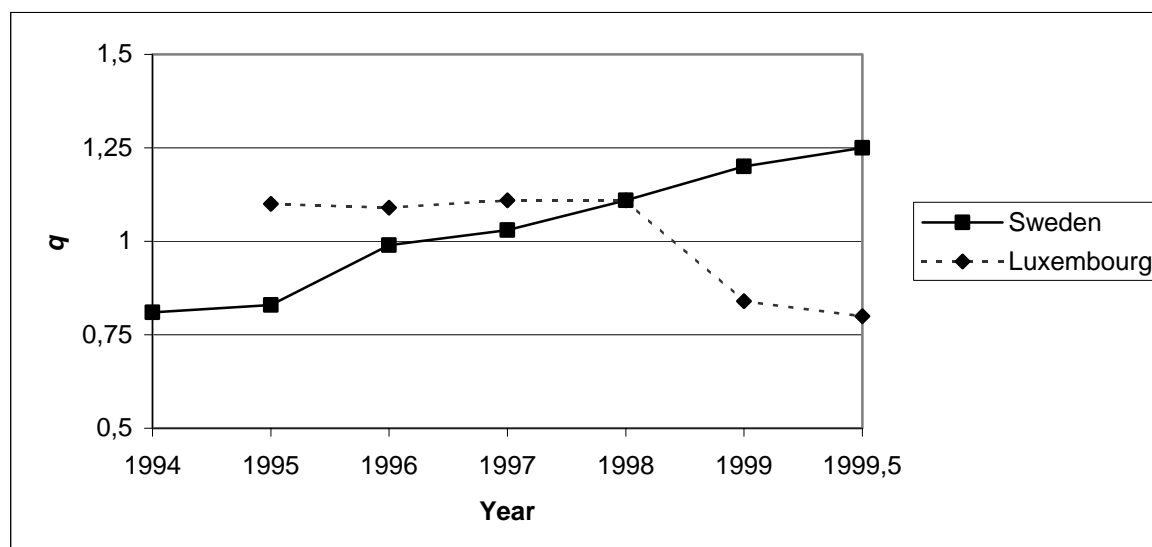


Table 1: Four approaches to international diffusion compared

	Mutual influence, Mahajan and Muller 1994	Mixing behavior, Putsis et al. 1997	Learning effect, Ganesh et al. 1997	Mutual influence, Kumar and Krishnan, 2002	Staged estimation, Dekimpe et al. 1998
Basic principle	Adopters from each country influence the potential adopters in other countries	Bernouilli mixing describes the way countries interact	Consumers in lag countries learn from adopters in lead country	Adopters from each country influence the potential adopters in other countries	Before estimating diffusion parameters a matching procedure has to be followed
Which influence of adoption in one country on adoption in other countries is included in the model?	Mutual influence between all countries	Mutual influence between all the countries	Lead country influences lag countries	Lead-lag, lag-lead and simultaneous influence between all countries	None
Does the model assume a fixed or variable intensity of the influence between the countries?	Fixed, assumed equal to influence within country (q)	Variable, mixing can be varied on the continuum from no mixing at all to complete mixing	Variable, learning coefficient may be varied	Variable, Interaction effects may vary.	Fixed, no influence
Does the approach include sample matching?	No	No	Yes, matching on social system size by using penetration per population	No	Yes, matching on country size, ceiling and time of introduction

Table 2: Four adaptive parameter estimation methods compared

	Adaptive filtering, Bretschneider and Mahajan 1980	Hierarchical Bayes, Lenk and Rao (1990)	Bayesian updating, Sultan and Farley 1990	AKF(C-D), Xie et al. 1997
Basic principle	Feedback filter is used to update time varying coefficients in regression model	Hierarchical Bayes procedure to update the a-priori parameter estimates	Bayesian updates of prior estimates, using data based estimates obtained by NLS	Augmented extended Kalman feedback filter applied directly to the diffusion model
Does the method allow for parameters that also vary according to prescribed dynamics (flexible diffusion models)?	No, the parameters are considered to vary randomly only	No, the parameters are considered to vary randomly only	No	Yes, they can simultaneously vary according to prescribed dynamics and a random process
Does method require model to have explicit solution or be discrete?	Yes, the method is based on a discrete diffusion model	Yes, the method requires analytical solution of the diffusion model	Yes, NLS requires the analytical solution of the diffusion model	No, the method is applied directly to the differential equation
Does the method already update the prior estimate when first data point becomes available?	Yes	Yes	No, three data points are needed for the first update.	Yes

Table 3: Initial estimates of country size, time of introduction, and expected penetration ceiling

Country	Size (number of households)¹	Time of introduction Internet access²	Time of introduction mobile phones³	Expected penetration ceiling Internet access⁴	Expected penetration ceiling mobile phones⁵
Austria	3.013.000	1995	1992	71%	98%
Belgium	3.953.000	1996	1992	73%	98%
Denmark	2.274.000	1994	1990	75%	99%
Finland	2.037.000	1994	1990	69%	100%
France	21.542.000	1996	1993	71%	99%
Germany	35.256.000	1996	1993	72%	99%
Greece	3.204.000	1997	1994	49%	96%
Ireland	1.029.000	1997	1993	60%	89%
Italy	19.909.000	1996	1990	56%	97%
Luxembourg	145.000	1995	1992	88%	100%
Netherlands	6.162.000	1994	1993	69%	100%
Portugal	3.146.000	1997	1992	44%	85%
Spain	11.836.000	1996	1992	54%	93%
Sweden	3.830.000	1994	1990	79%	100%
United Kingdom	22.422.000	1994	1990	66%	99%

1. Source: Eurostat (1996)

2. Operationalized as: “the first year in which 0,4% or more of the households acquired Internet access at home” (source: Gallup Europe, 2000)

3. Operationalized as: “the first year in which 0,4% or more of the households acquired a mobile phone” (source: Gallup Europe, 2000)

4. The percentage of households in the EU-member countries that have an income that is higher then the European poverty line (Dekimpe et al. 1998; source: European Communities, 2000)

5. The percentage of households that has a sufficient income to afford basic telephone service (source: European Communities, 2000)

Table 4: The initial and final estimates of the effective contact rate q (Internet access)

Country	Initial estimate ($t=t_0$)		Estimate after final measurement update (1999.5)		Difference between Initial and 1999.5 estimate	
	$E(q_i)$	$SD(q_i)$	$E(q_i)$	$SD(q_i)$	$E(q_i)$	$SD(q_i)$
Belgium	0.82	0.45	0.75	0.13	-8%	-72%
Denmark	0.89	0.47	1.11	0.12	+25%	-75%
Germany	0.83	0.46	0.72	0.14	-13%	-70%
Finland	0.79	0.44	0.73	0.10	-7%	-78%
France	0.81	0.45	0.69	0.18	-15%	-59%
Greece	0.57	0.38	0.45	0.26	-20%	-32%
Ireland	0.68	0.41	0.76	0.25	+13%	-40%
Italy	0.69	0.42	0.66	0.19	-5%	-55%
Luxembourg	1.10	0.52	0.80	0.09	-27%	-83%
Netherlands	0.81	0.45	0.93	0.09	+14%	-79%
Austria	0.85	0.46	0.78	0.14	-8%	-70%
Portugal	0.57	0.38	0.59	0.27	+5%	-29%
Spain	0.63	0.40	0.60	0.18	-4%	-55%
United Kingdom	0.69	0.42	0.91	0.10	+31%	-76%
Sweden	0.82	0.45	1.25	0.16	+52%	-64%

Table 5: Mean absolute percentage errors of the IA and the classical forecasting approach

Forecasting period	Internet access			Mobile telephony		
	<i>IA</i>	<i>IA</i> $\Phi=1$	<i>Classic</i> <i>approach</i>	<i>IA</i>	<i>IA</i> $\Phi=1$	<i>Classic</i> <i>approach</i>
One-year-ahead	8%	9%	14%	7%	8%	13%
Two-year-ahead	14%	16%	27%	15%	17%	24%
Three-year-ahead	12%	16%	39%	23%	25%	32%

Table 6: Sensitivity of the forecasting performance to changes in initial estimates (Internet access)

Change of initial estimate	Mean average absolute percentage error		
	one-step-ahead	two-step-ahead	three-step-ahead
With used values	12.2	16.0	17.6
C+ 25%	12.2	15.9	17.4
C- 25%	12.3	16.1	18.0
q+25%	16.5	28.4	40.2
q-25%	20.3	32.8	43.3
p+25%	12.2	16.0	17.6
p-25%	12.2	16.0	17.6
Φ +25%	13.1	18.1	18.5
Φ -25%	13.0	17.1	20.0
P+25%	12.3	15.8	17.1
P-25%	12.3	16.3	18.2

Appendix A: The AKF(C-D) estimation procedure.

In this appendix a generalized version of the AKF(C-D) estimation procedure used by Xie et al. (1997) is presented. It is based on the books of Lewis (1986, p.260-265) and Stengel (1986, p.386-396). The model used by Xie et al. describes the diffusion in one social system. They use one state to describe the evolvement in time of the diffusion process. Our generalization consists of augmenting the state from one state variable to a (Kx1) vector of state variables that describes the evolvement of the diffusion in K social systems (or countries) at a time.

When studying the diffusion of innovations in K countries at a time, we study a continuous process with K states. The observations of the diffusion process are made only at a finite number of discrete times, $z = t_z$. Following control engineering theory, such a dynamic system and its discrete observation can be described by two equations. The system equation describes the evolution of the state variable in time. The measurement equation describes how the observations are related to the state of the system.

$$\begin{aligned}\frac{dN(t)}{dt} &= f(N(t), u(t), t) + G(t) * w \\ y(z) &= H(z) * N(t) + v \\ N(0) &\sim (\bar{N}_0, P_0), w \sim (0, Q), v \sim (0, R)\end{aligned}\tag{Equation 9}$$

Where:

- $N(t)$ is the state variable vector with dimension (Kx1), and $N(0)$ is assumed to have an initial distribution with mean vector \bar{N}_0 and covariance matrix P_0 ,
- f is a vector function of the state, $N(t)$, the control vector $u(t)$, and time t ,
- $u(t)$ is the control variable vector (for instance the marketing mix variables),
- $y(z)$ is the measurement vector at time z ,
- $H(z)$ is the measurement matrix,
- w and v are respectively the process noise and the measurement noise vectors that both are assumed to be stationary white noise processes, that are uncorrelated with each other and with $N(0)$,
- and Q and R are covariance matrices of the process noise and the measurement noise.

In international diffusion research the observations are made directly of the diffusion itself. Therefore the measurement matrix will only have non-zero elements in the diagonal. At each observation moment $t=z$, observations of the diffusion process are made in one or more of the countries. At instant z , the diagonal elements of the measurement matrix $H(z)$ will be zero when the diffusion in the associated country is not observed, and one when the diffusion in the associated country is observed.

In this formulation the model parameters are considered to be constant. Following Stengel (1986, p.392-6), we can incorporate model parameters that vary with time by writing:

$$\begin{aligned}
\begin{bmatrix} \frac{dN(t)}{dt} \\ \frac{d\beta(t)}{dt} \end{bmatrix} &= \begin{bmatrix} f_n(N(t), \beta(t), u(t), t) \\ f_\beta(N(t), \beta(t), t) \end{bmatrix} + \begin{bmatrix} w_n \\ w_\beta \end{bmatrix} \\
y(z) &= H(z) * N(t) + v \\
N(0) &\sim (\bar{N}_0, P_{N0}), \beta(0) \sim (\bar{\beta}_0, P_{\beta0}), w \sim (0, Q), v \sim (0, R)
\end{aligned}
\tag{Equation 10}$$

Where:

- $\beta(t)$ is the parameter vector,
- f_β is a vector function of the state $N(t)$, the parameters $\beta(t)$ and the time t ,
- w_β is the process noise associated with the modeling of the dynamics of the parameters.

Combining the state and the parameter vector we define the augmented state, $x(t)$ as:

$$x(t) = \begin{bmatrix} N(t) \\ \beta(t) \end{bmatrix}
\tag{Equation 11}$$

Where:

- $x(t)$ is the augmented state vector of length $K+M$, where M denotes the number of model parameters that are incorporated.

Then we can rewrite the system equation as:

$$\frac{dx(t)}{dt} = \begin{bmatrix} f_n(N(t), \beta(t), u(t), t) \\ f_\beta(N(t), \beta(t), t) \end{bmatrix} + \begin{bmatrix} w_n \\ w_\beta \end{bmatrix} = f_x(x(t), u(t), t) + w_x
\tag{Equation 12}$$

The observations that are made are only a function of the non-augmented state, $N(t)$. The parameters of course are not observed at all. Then the observation function becomes:

$$y(z) = H_A(t) * x(t) + v = [H(t) \ 0] * \begin{bmatrix} N(t) \\ \beta(t) \end{bmatrix} + v
\tag{Equation 13}$$

Where:

- $H_A(t)$ is the augmented measurement matrix, that is augmented with zeros. This is done to give it the right dimension to multiply it with the augmented state $x(t)$, instead of the original state $N(t)$.

Following Lewis (1986), Stengel (1986) and Xie et al. (1997) we now define the AKF(C-D) estimation procedure that consists of three main steps (see Figure 1). The Filter is initialized with prior estimates of the state and the error covariance matrix of the state. Between measurements, estimates of the state and the error covariance are obtained by the “time update” step. When a measurement becomes available, the state and parameter estimates are updated in the “measurement update” step.

Step 1: Initialization

At $t=t_0$, based on prior information, the best prior estimate of the distributions of the augmented state (\hat{x}_0, \hat{P}_0) and the noise statistics, (Q, R) are developed and the filter is initialized:

$$\hat{\mathbf{P}}(0) = \hat{\mathbf{P}}_0, \quad \hat{\mathbf{x}}(0) = \hat{\mathbf{x}}_0 \quad \text{Equation 14}$$

Step 2: Time update

At a given time t , the filter predicts the future state at $t+\Delta t$, or future diffusion and parameter values, through the time updating process. Time updating is accomplished by the following equations over the time interval $(t, t+\Delta t)$.

$$\begin{aligned} \frac{d\mathbf{x}(t)}{dt} &= \mathbf{f}_x(\mathbf{x}(t), \mathbf{u}(t), t) \\ \frac{d\mathbf{P}(t)}{dt} &= \mathbf{F}(x, t) * \mathbf{P}(t) + \mathbf{P}(t) * \mathbf{F}^T(x, t) + \mathbf{Q} \end{aligned} \quad \text{Equation 15}$$

with :

$$\mathbf{F}(x, t) = \frac{\partial \mathbf{f}_x(\mathbf{x}(t), \mathbf{u}(t), t)}{\partial \mathbf{x}}$$

Where:

- $\mathbf{F}(x, t)$ is the Jacobian of \mathbf{f}_x .

The output of the time update step is an a priori estimate of the state at $t+\Delta t$, $\hat{\mathbf{y}}_{\text{apriori}}(t + \Delta t)$, and of the error covariance at $t+\Delta t$, $\hat{\mathbf{P}}_{\text{apriori}}(t + \Delta t)$.

Step 3: Measurement update

When a new observation becomes available at time z , the estimate is modified using the forecasting error (the difference between the observed diffusion and the predicted diffusion).

$$\begin{aligned} \hat{\mathbf{x}}(z) &= \hat{\mathbf{x}}_{\text{apriori}}(z) + \mathbf{K}_c * (\mathbf{y}(z) - \mathbf{H}_A(z) * \hat{\mathbf{x}}_{\text{apriori}}(z)) \\ \hat{\mathbf{P}}(z) &= [\mathbf{I} - \mathbf{K}_c * \mathbf{H}_A(z)] * \hat{\mathbf{P}}_{\text{apriori}}(z) \end{aligned} \quad \text{Equation 16}$$

with :

$$\mathbf{K}_C = \hat{\mathbf{P}}_{\text{apriori}}(z) * \mathbf{H}_A^T(z) [\mathbf{H}_A(z) * \hat{\mathbf{P}}_{\text{apriori}}(z) * \mathbf{H}_A^T(z) + \mathbf{R}]^{-1}$$

Where:

- \mathbf{K}_C is the Kalman gain,
- $(\mathbf{y}(z) - \mathbf{H}_A(z) * \hat{\mathbf{x}}_{\text{apriori}}(z))$ is the forecasting error,
- $\hat{\mathbf{x}}_{\text{apriori}}(z)$ and $\hat{\mathbf{P}}_{\text{apriori}}(z)$ denote respectively the a priori estimate of the state and the error covariance matrix obtained by the time updating step,
- and $\hat{\mathbf{x}}(z)$ and $\hat{\mathbf{P}}(z)$ are the posterior estimates of the state and the error covariance matrix that result from the measurement update step

The posterior estimates obtained by the measurement update step will now be used as the starting point for the estimates made by the time updating mechanism, until a new observation becomes available (see Figure 1).

Appendix B: Formalizing the IA diffusion model to apply the Kalman Filter.

Following Xie et al. (1997) and Putsis (1998), we will model parameter variation as purely stochastic. Because there are only very weak priors for the nature of parameter variation in diffusion models (Putsis 1998, p.235), we do not impose a systematic variation. This gives:

$$\frac{d\beta(t)}{dt} = 0 + w_\beta \quad \text{Equation 17}$$

Where:

- $\beta(t)$ is the vector of unknown time varying parameters of the model, $\beta(t) = [\beta_c(t), \beta_p(t), \beta_q(t), \Phi(t), t_0]^T$, it is assumed that $\beta(0) \sim (\bar{\beta}_0, P_{\beta 0})$ is a white noise
- w_β is vector of process noise in the parameters that is assumed to be white noise: $w_\beta \sim (0, Q_\beta)$.

Adding a noise term, w_N , to equation 8, combining it with equation 17 by augmenting the state and writing the measurement equation⁵, the AKF(C-D) model is:

$$\begin{aligned} \begin{bmatrix} \frac{dN(t)}{dt} \\ \frac{d\beta(t)}{dt} \end{bmatrix} &= \begin{bmatrix} f_N(N(t), \beta(t), u(t), t) \\ 0 \end{bmatrix} + \begin{bmatrix} w_N \\ w_\beta \end{bmatrix} \\ y(z) &= H(z) * N(t) + v \end{aligned} \quad \text{Equation 18}$$

Where:

- $N(t)$ is the vector of the of the total number of units that have adopted the innovation in each country, it is assumed that $N(0) \sim (\bar{N}_0, P_{N0})$ is a white noise,
- $\beta(t)$ is the vector of unknown time varying parameters of the model, $\beta(t) = [\beta_c(t), \beta_p(t), \beta_q(t), \Phi(t), t_0]^T$, it is assumed that $\beta(0) \sim (\bar{\beta}_0, P_{\beta 0})$ is a white noise,
- $\begin{bmatrix} w_N, w_\beta \end{bmatrix}$ is the vector of process noises that is assumed to be white noise: $\begin{bmatrix} w_N, w_\beta \end{bmatrix} \sim (0, Q)$,
- $y(z)$ is the vector of observations at time $t=t_z$,
- $H(z)$ is the observation or measurement matrix,
- v is the vector of measurement noises, that is assumed to be white noise, $v \sim (0, R)$.

It is assumed that all the white noise processes are not correlated to one another.

As is shown in the measurement equation, the observations $y(z)$ of the diffusion process are made only at a finite number of discrete times, $z = t_z$. The observations are made of the diffusion itself. Therefore the measurement matrix will only have non-zero elements in the diagonal. At each observation moment $t=z$, observations of the diffusion process are made in one or more of the countries. At instant z , the diagonal elements of the measurement matrix $H(z)$ will be zero when the diffusion in the associated country is not observed, and one when the diffusion in the associated country is observed.

⁵ This equation describes the relation between the observations and the dynamic states of the system

Appendix C: Prior estimates

The prior estimates of the parameter distributions

Following Xie et al. (1997), we define the variance of the parameters as follows: $\text{Var}_0(\beta) = 0.25 * E_0(\beta)$. The expected initial value for the parameter C has already been given in Table 3. The expected initial values we use for the other parameters are summarized in Table C1 and elucidated below.

Table C1 : Expected values of the parameters.

Parameter	Internet access	Mobile telephony
Coefficient of innovation (p)	$E_0(p_i)=1e-5$	$E_0(p_i)=1e-3$
Effective contact rate (q)	$E_0(q_i) = 0.5 + \frac{GDPPC_i - 5000}{25000}$	$E_0(q_i)=0.5$
Penetration ceiling (C)	% of households with income above European poverty line	% of households that can afford basic telephone service
Bernoulli mixing coefficient (Φ)	$E_0(\Phi)=0.7$	$E_0(\Phi)=0.7$

p

For Internet access we have initiated p with a low expected value: $E_0(p_i)=1e-5$. It is a new interactive medium that needs to build an entirely new network and that requires substantial learning to use. Moreover, network effects are at work, and therefore adopting it independently of others seems unlikely. For mobile telephony we have fixed the initial expected value of p higher: $E_0(p_i)=1e-3$. We expect a higher p for mobile phones than Internet because the mobile phone seamlessly fits into the existing telephone network and does not require learning to use it.

q

The effective contact rate q is closely related to the coefficient of imitation in the Bass model that is also labeled q . In their meta-analysis of applications of diffusion models, Sultan et al. (1990) show that the average value of q found in past research, mostly of diffusion processes before 1980, was about 0.4. We expect that the take up of the Internet will be quicker than the take up of the average innovation in the past, and also expect that the speed of diffusion will be strongly related to the GDP per capita (GDPPC). The Internet is complex to use, requiring computer skills, and a substantial investment in a PC. Furthermore, network externalities play an important role. The Internet is a new interactive medium that needs to build a new network to become really valuable. One of the most important applications of the Internet is e-mail that only has value when the people you want to communicate with have access. We expect word-of-mouth communication to play an important role in the adoption decision process. Therefore we have initiated q using the following equation that gives rather high values for q varying between 0.6 and 1.1⁶ (see Table C2 for the exact values for each country):

$$E_0(q_i) = 0.5 + \frac{GDPPC_i - 5000}{25000}$$

⁶ The GDP per capita of the richest country (Luxembourg) is about 30000 Euro and of the poorest country (Greece) it is 5000 Euro. We have used this equation to roughly fix the q_i at values between 0.5 for the poorest countries and 1 for the richest.

Table C2: Initial estimates of the expected value of the effective contact rate q

Country	E (q_i) Internet
Austria	0.85
Belgium	0.82
Denmark	0.89
Finland	0.79
France	0.81
Germany	0.83
Greece	0.57
Ireland	0.68
Italy	0.69
Luxembourg	1.10
Netherlands	0.81
Portugal	0.56
Spain	0.63
Sweden	0.81
United Kingdom	0.69

For mobile telephony we expect that the speed of diffusion will be slightly higher than the average innovation. We have initiated q on the same value for all countries ($E_0(q_i)=0.5$). Measures like GDP per capita will not be useful to predict the speed of mobile phone diffusion, because the innovation is easy to use and does not require a large upfront investment by the adopters.

Φ

Finally, as an initial estimate of the expected value of the Bernoulli mixing parameter we have used the value found by Putsis et al (1997) for Home Computers: $E_0(\Phi)=0.7$, both for the Internet access and the mobile telephony. The Φ may be considered to be similar across innovations with comparable word-of-mouth networks like PCs and Internet access. It is very likely that people, who talk to each other about PCs, also talk about Internet access at home, and other technological innovations like mobile telephones.

The noise statistics

Process noise Q

Xie et al. (1997) fix the process noise at a value of the order of magnitude of 1% of the penetration. We assume, however, that the process noise depends on the level of penetration, and we set Q by the following two rules:

- the minimum value of Q is 0.5%;
- Q equals 5% of the predicted penetration $N(t)$.

Thus we obtain:

$$Q(t) = \max(0.5, 0.05 * N(t))$$

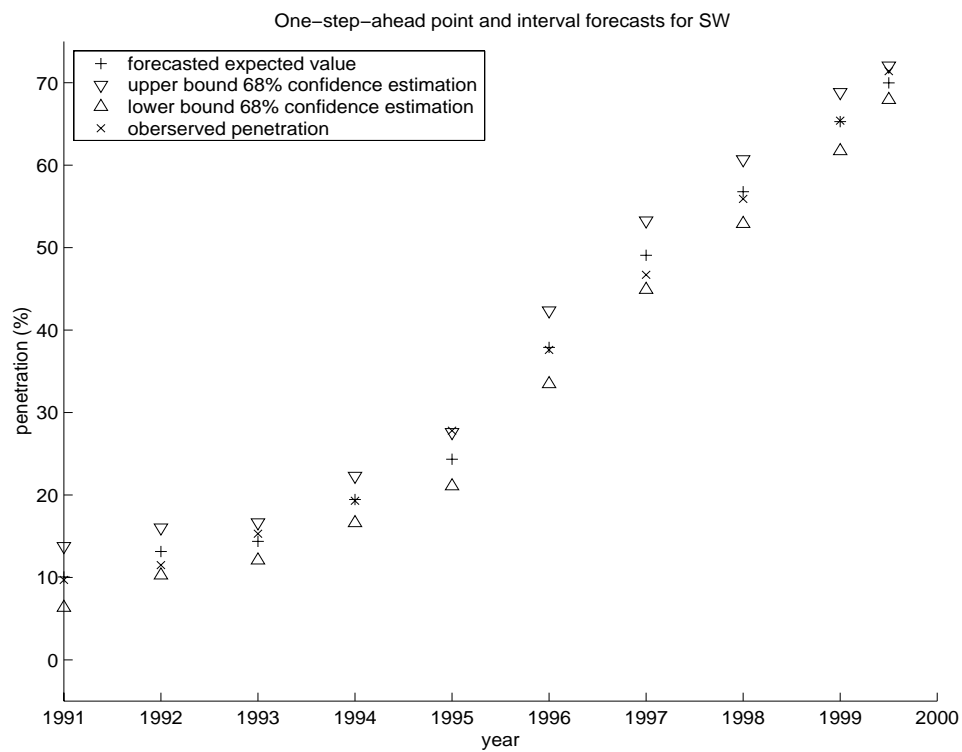
If the penetration $N(t)$ is below 10%, then we use a process noise of 0,5%; as soon as the penetration level is above 10%, the process noise is calculated as 5% of $N(t)$.

Measurement noise R

Based on the information Gallup (2000) gives about the accuracy of the results we have estimated the observation error (measurement noise) to be 0.5% for all countries. This resulted in the following variance: $R = \sqrt{0.5} * I^{K \times K}$.

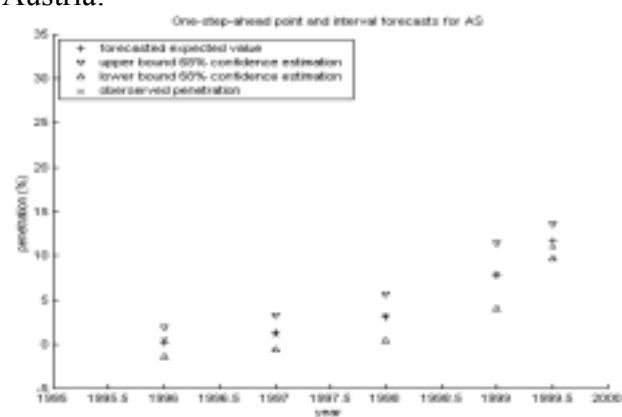
A very attractive feature of the AKF(C-D) estimation procedure that is used in the IA forecasting method is that the observation noise can be specified for each observation separately. This is especially useful when multiple sources of different reliability are used, as is often the case in the practice of forecasting. Since we have only one source, we do not use this possibility.

Appendix D: The one-step-ahead forecast for mobile telephony in Sweden

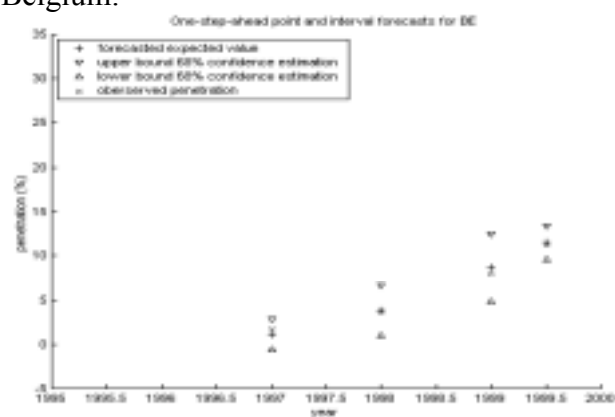


Appendix E: The one-step-ahead forecasts for Internet access for all countries

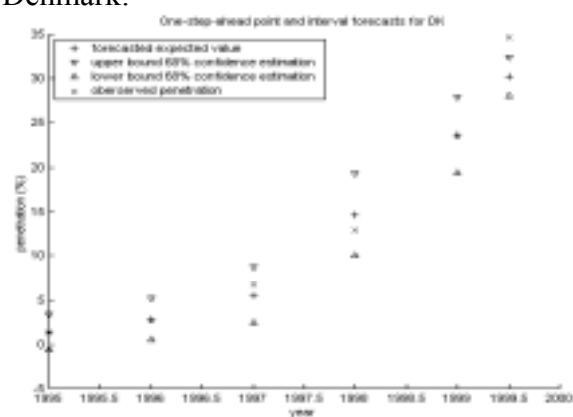
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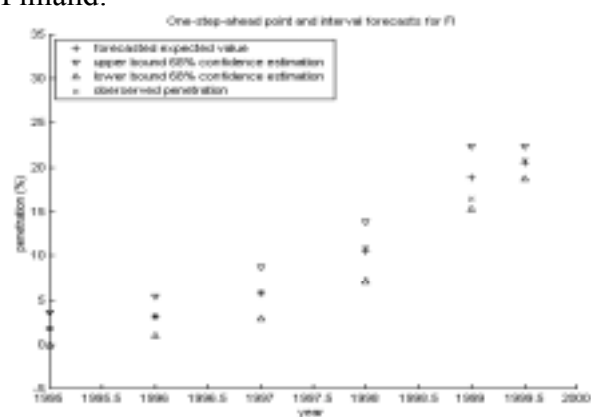
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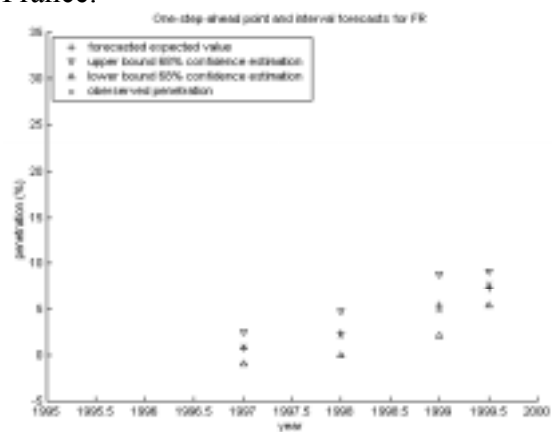
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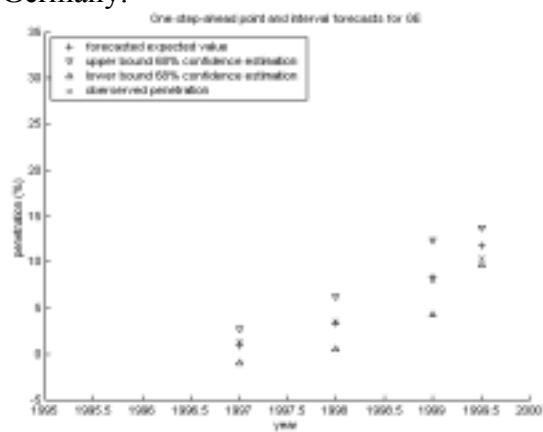
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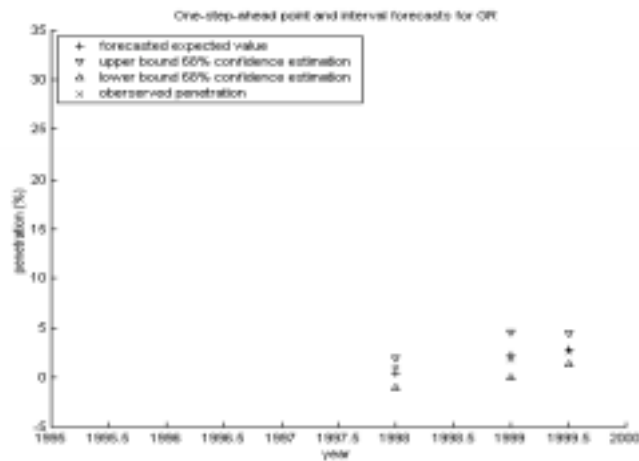
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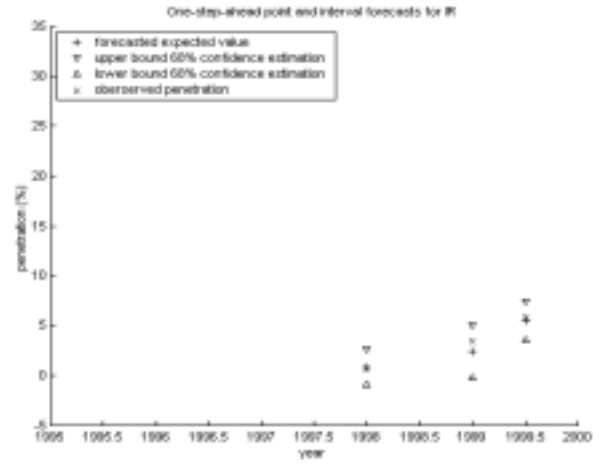
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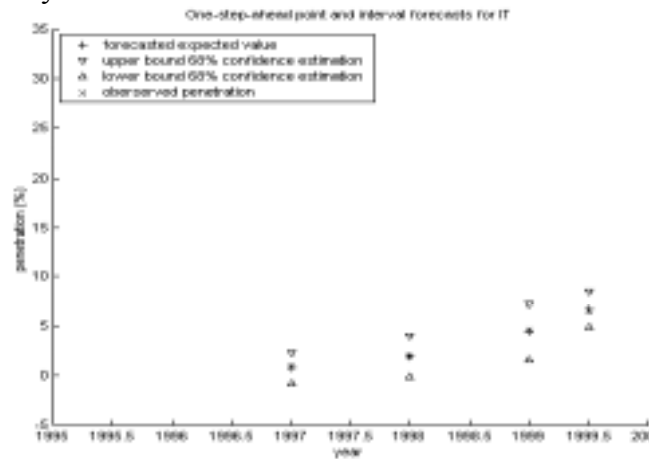
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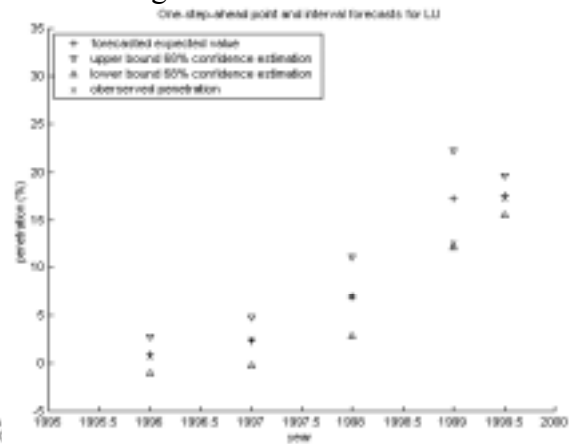
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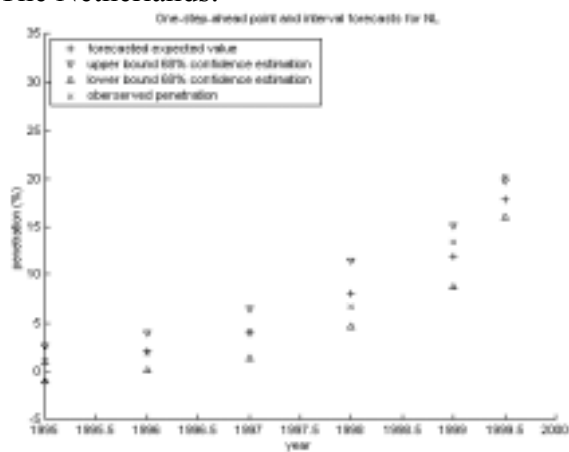
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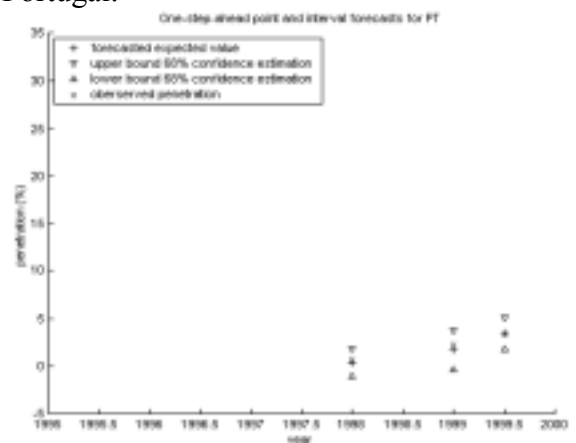
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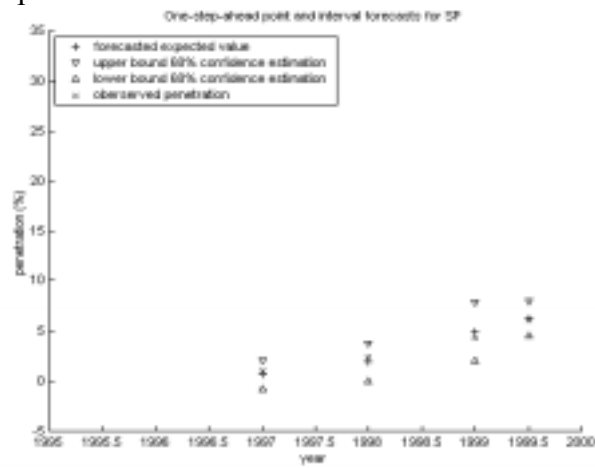
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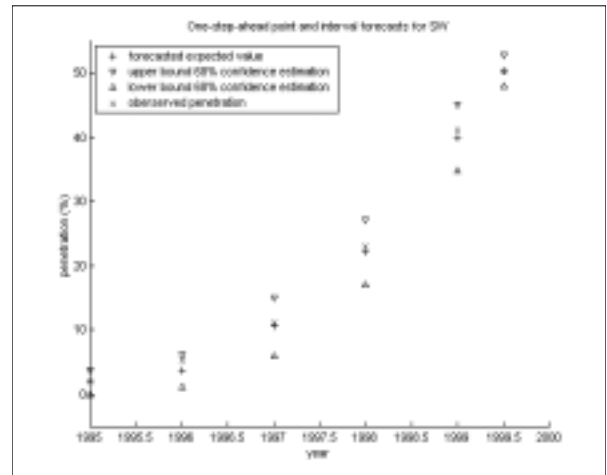
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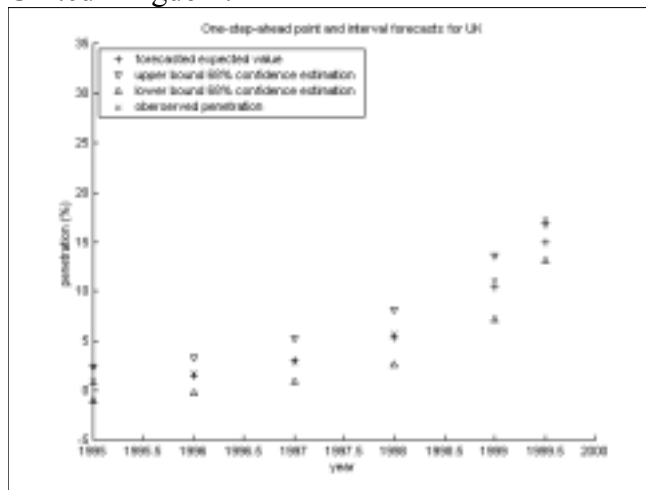
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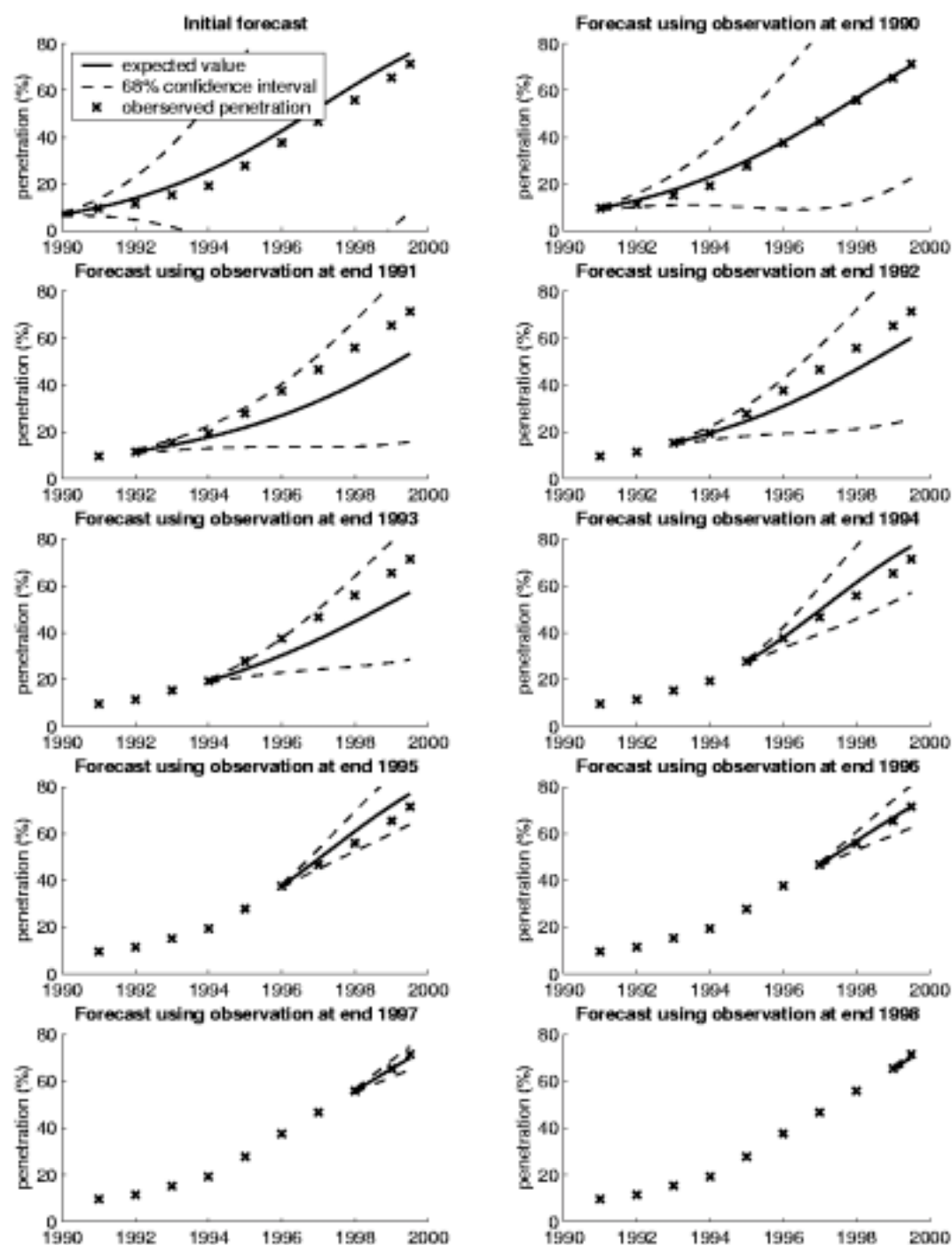
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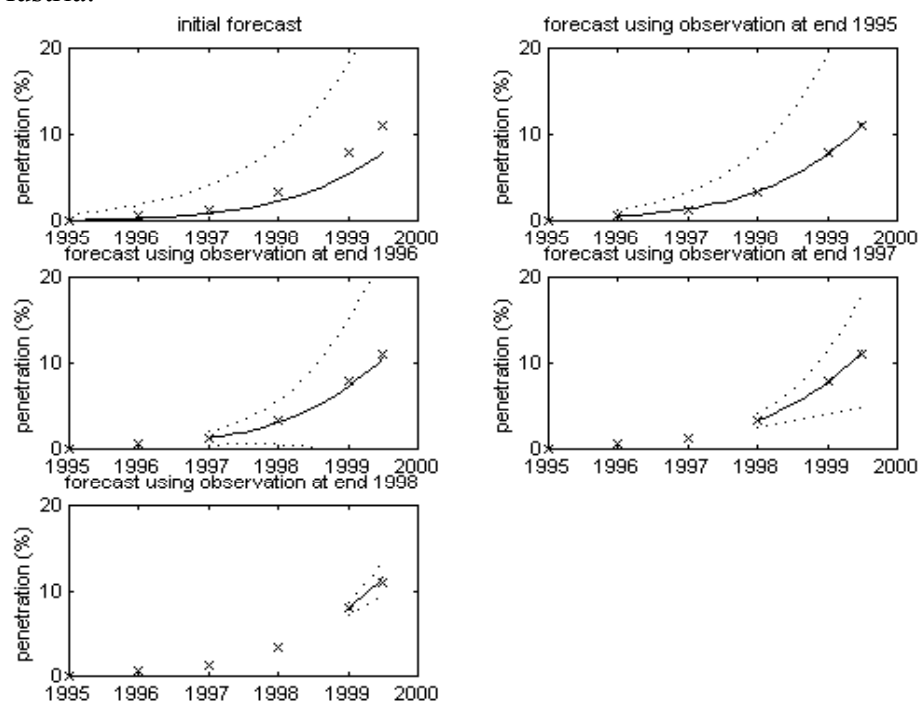


Appendix F: The long-term forecast for mobile telephony in Sweden

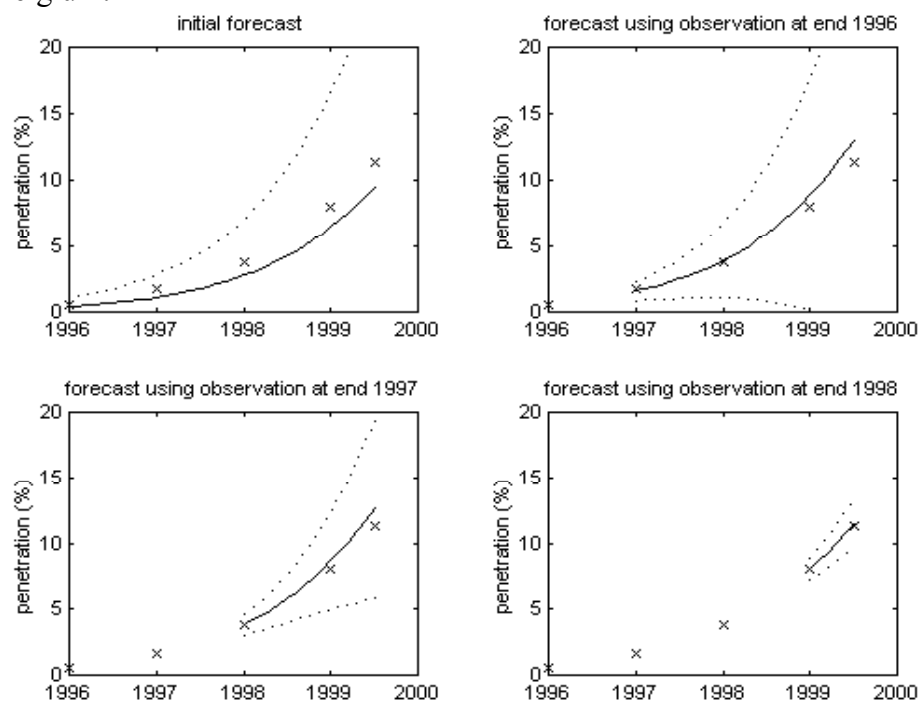


Appendix G: The long-term forecasts for Internet access for all countries

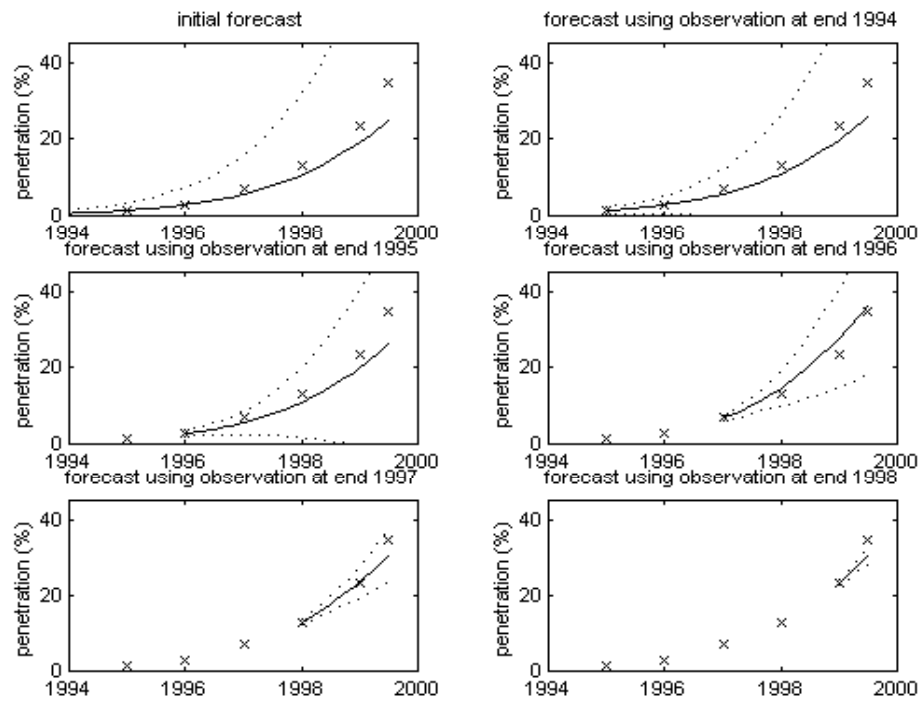
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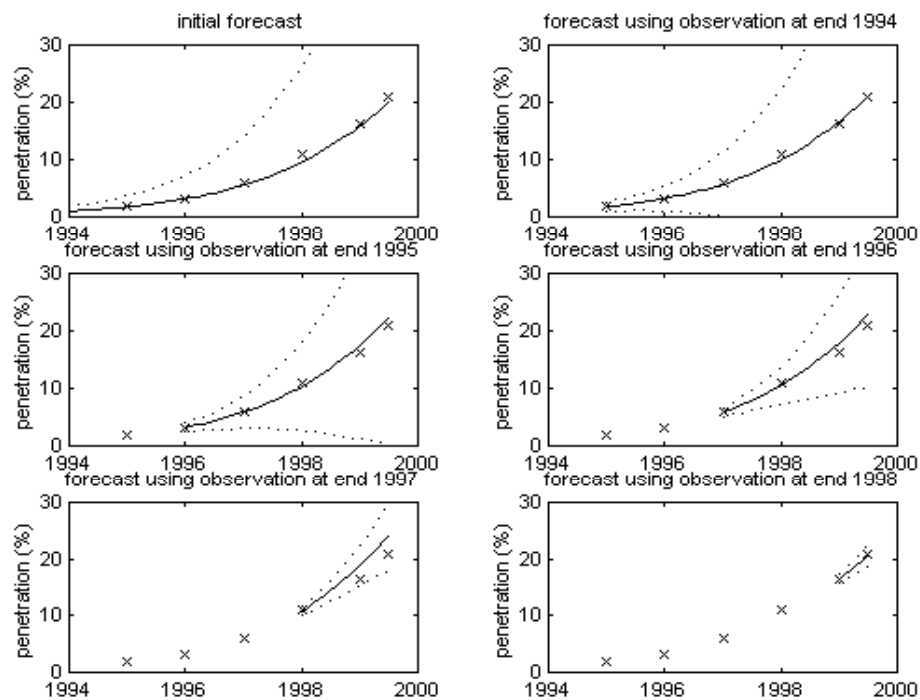
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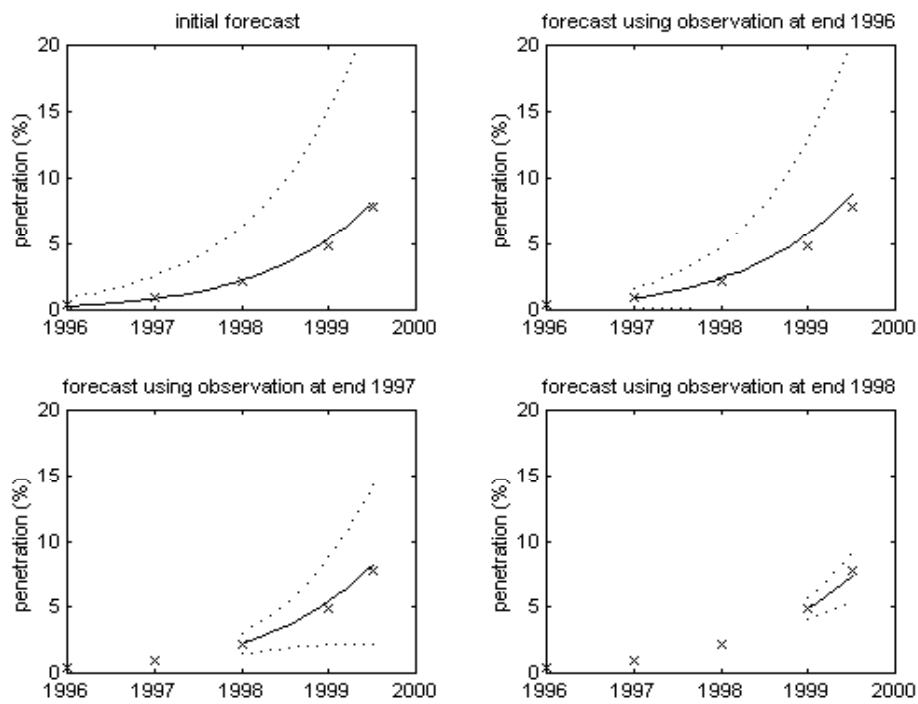
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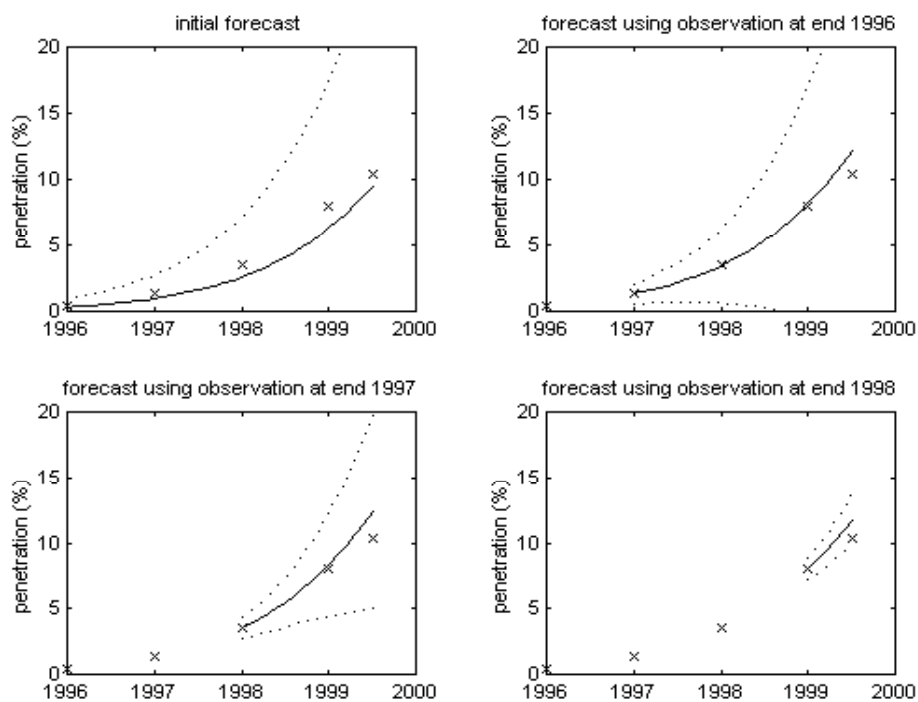
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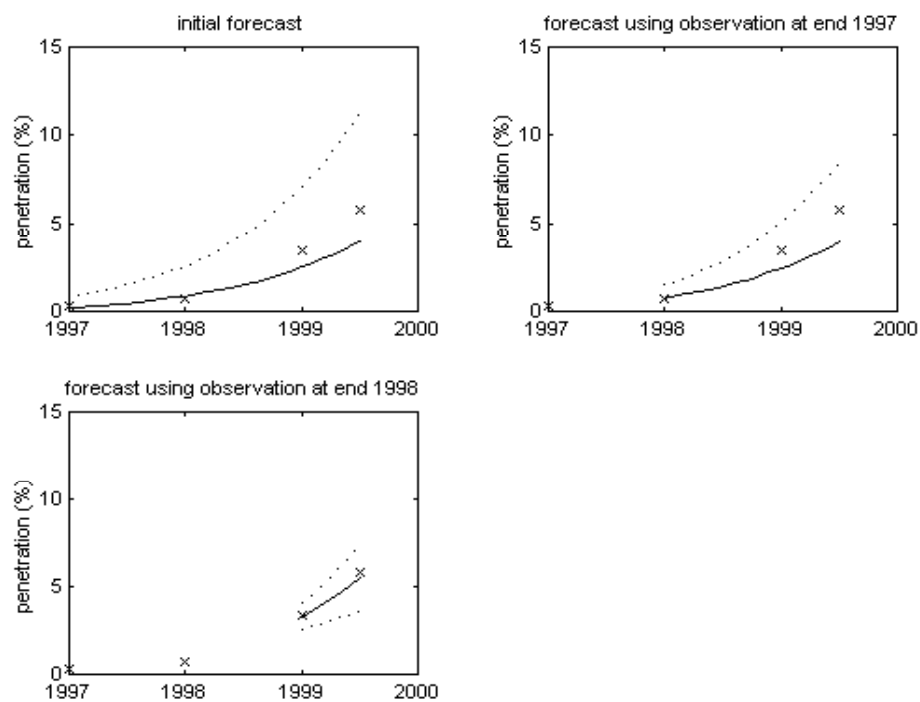
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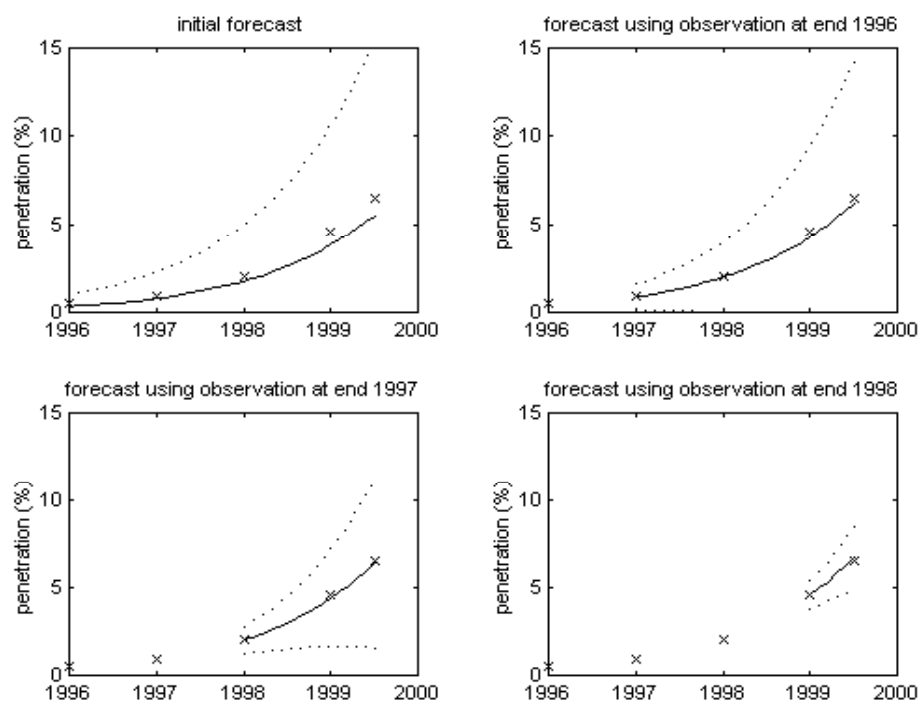
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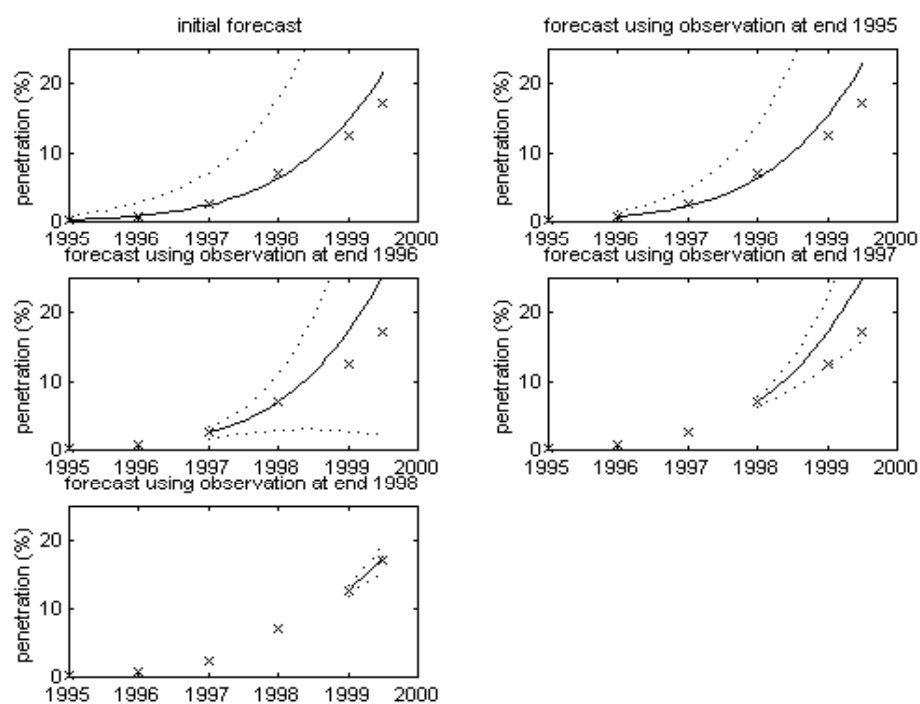
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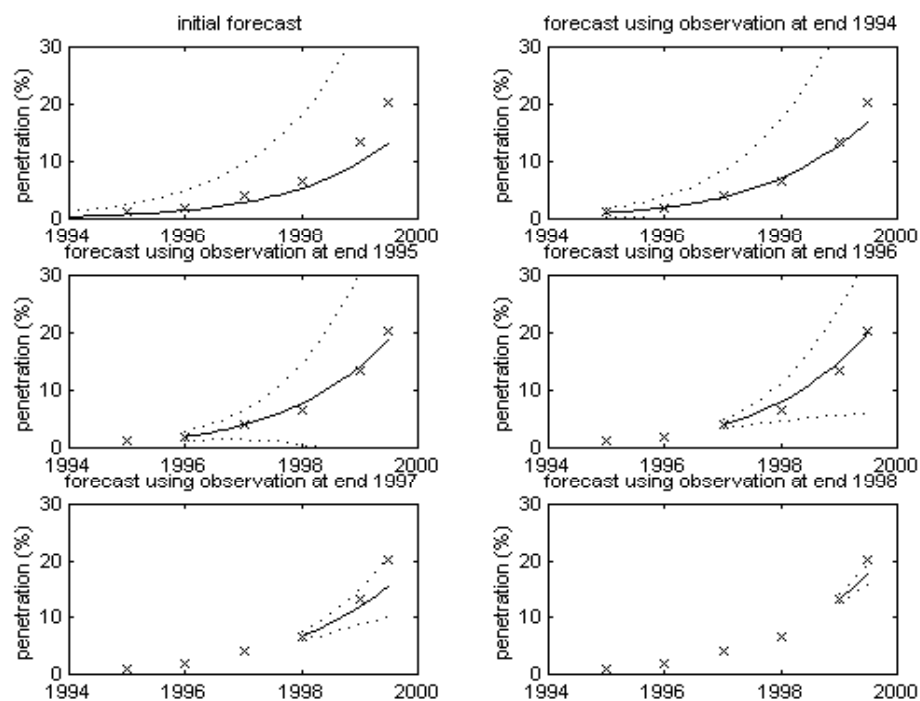
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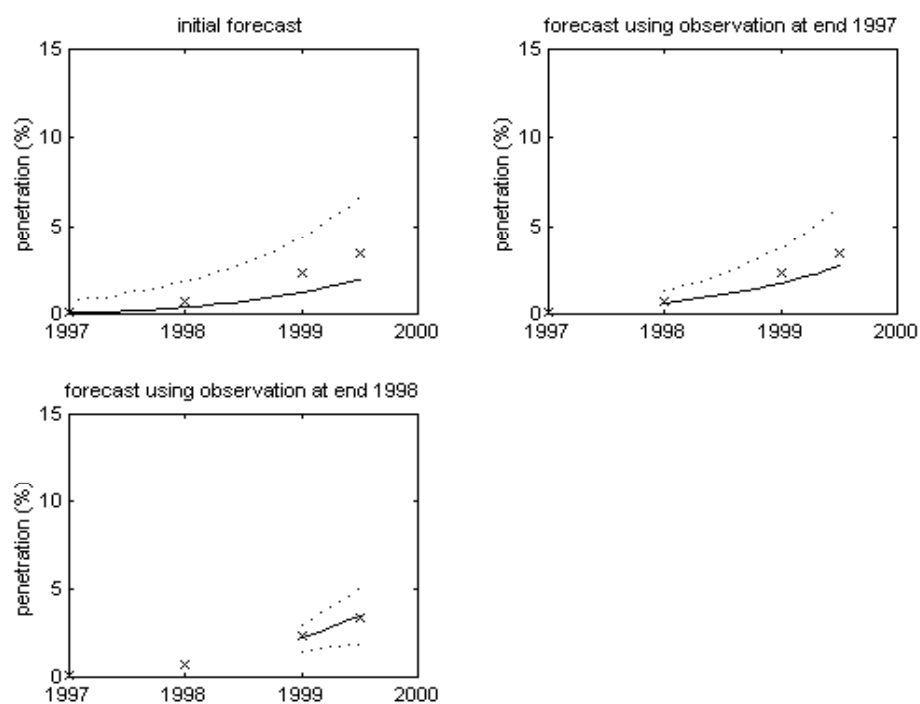
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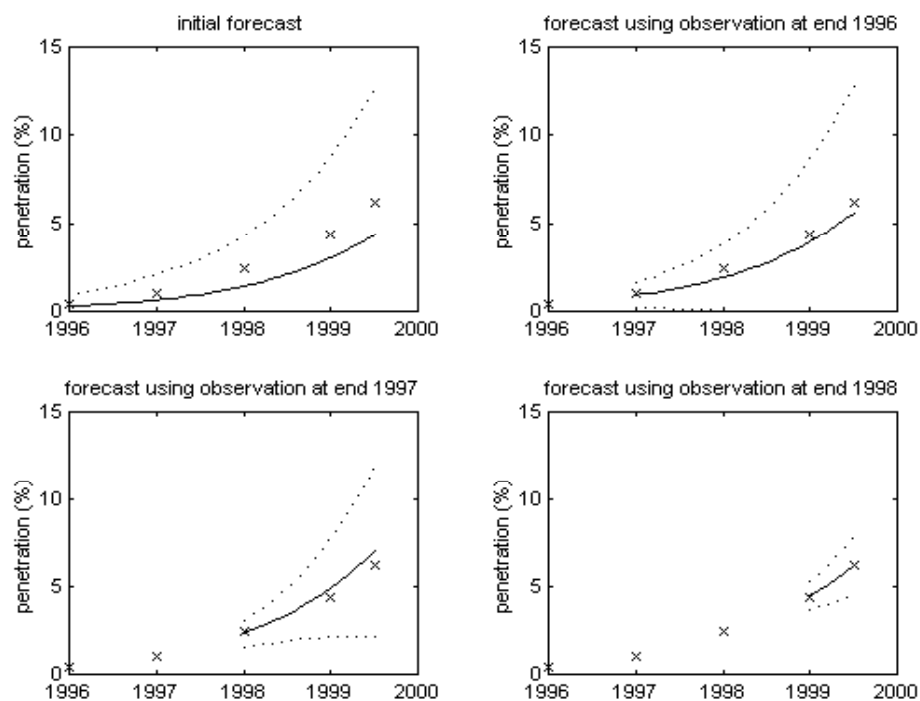
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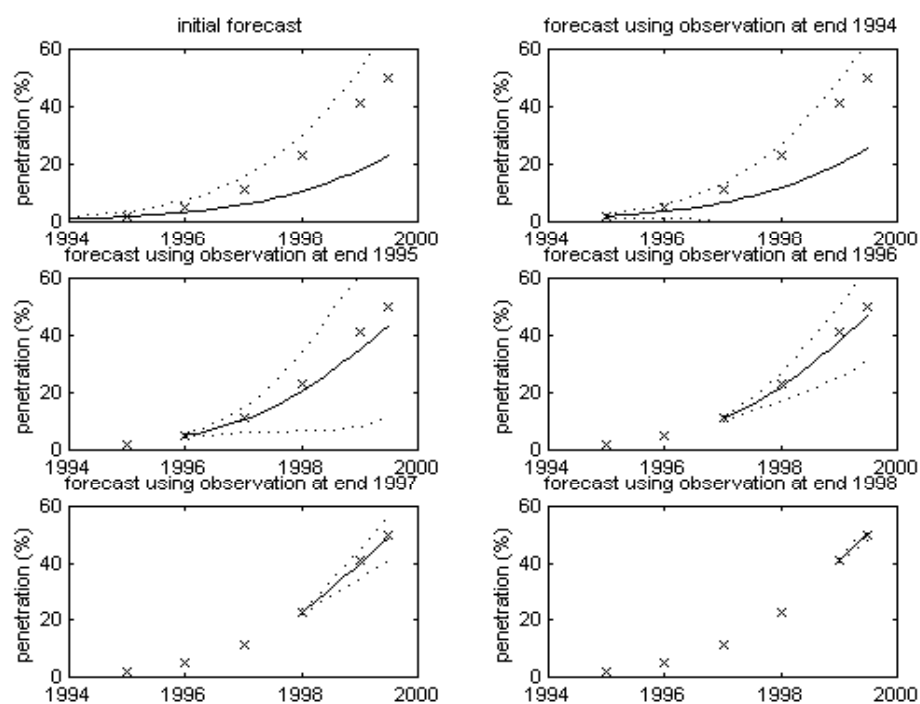
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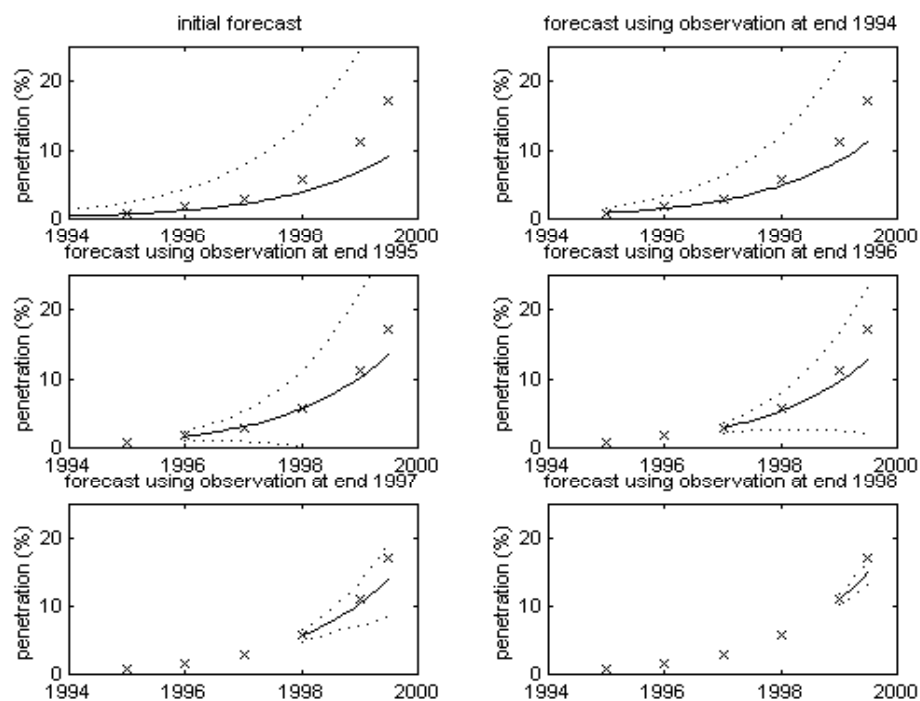
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