A welfare economics foundation for health inequality measurement

Han Bleichrodt & Eddy van Doorslaer
Erasmus University Rotterdam

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Abstract

The empirical literature on the measurement of health inequalities is vast and rapidly expanding. To date, however, no foundation in welfare economics exists for the proposed measures of health inequality. This paper provides such a foundation for commonly used measures like the health concentration index and the Gini index and for Wagstaff's (2002) extended concentration index. Our results indicate that these measures require assumptions that appear restrictive. One way forward may be the development of multi-dimensional extensions.

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Address correspondence to:
Han Bleichrodt, Dept. of Economics, H13-27, Erasmus University, P.O. Box 1738, 3000 DR Rotterdam, The Netherlands. E-mail: bleichrodt@few.eur.nl

1. Introduction

Preference foundations give conditions that are necessary and sufficient for a particular model and, thereby, allow assessing the empirical content of a model. The conditions that are identified can serve to justify or to refute a given model. In this paper we will derive preference foundations for the models that underlie the most common ways to measure health inequality.

The literature on the measurement of health inequalities is vast and rapidly growing and has benefited from contributions from a number of disciplinary perspectives.[1] Economists have made substantial contributions to the empirical literature on this subject, thereby often drawing on the accumulated knowledge in the field of income inequality measurement. Le Grand (1989) and Le Grand and Rabin (1986) have proposed the use of the Gini coefficient for the measurement of *pure* inequality in mortality. Wagstaff, van Doorslaer, and Paci (1989) have proposed the use of the concentration index for the measurement of relative *socioeconomic* inequality in health and health care. More recently, van Doorslaer and Jones (2003) and Wagstaff and van Doorslaer (2004) have drawn attention to the simple relationship between both types of rank-dependent health inequality measures, while Koolman and van Doorslaer (2004) have illustrated the redistribution interpretation of the concentration index. Wagstaff, van Doorslaer, and Watanabe (2003) have shown how the concentration index can be decomposed by sources, and Clarke, Gerdtham, and Connelly (2003) have illustrated its decomposition by components. Clarke et al. (2002) have shown that inequality

[1] Early reviews include Wagstaff, Paci and Van Doorslaer (1991) and Mackenbach and Kunst (1997). A recent review is Regidor (2004).

comparisons based on absolute and relative inequality measures need not coincide. Finally, building on results obtained by Yitzhaki (1983) and Lerman and Yitzhaki (1984), Wagstaff (2002) has made the implicit weighting of individuals' health states in the concentration index more explicit, and has proposed the use of a so-called *achievement index* to simultaneously embody concerns about the mean and the degree of inequality of a health distribution.

While a welfare economics foundation for the measurement of income inequality and the comparison of income distributions has long been provided (Kolm, 1969, Atkinson, 1970, Lambert, 2001, Dutta, 2002), such a foundation has so far been lacking for the proposed measures of health inequality. Stecklov and Bommier (2002) have explored how the Atkinson and Bourguignon (1982) approach to measuring multi-dimensional inequality (e.g. in income and mortality) could be used to provide a welfare economics foundation for health inequality measurement and arrived at a negative conclusion. They restricted attention, however, to a specific notion of a just or equitable distribution of health, namely equality of access (see p.502) and their negative conclusions are a consequence of this notion. The commonly used measures of health inequality are based on a different notion of equity, namely equality of health.[2] Fleurbaey (2005a) has justified the use of the concentration curve for health by interpreting it as a component in the decomposition of the Lorenz curve for welfare.

This paper takes a different route than Stecklov and Bommier (2002) and Fleurbaey (2005a) by demonstrating how some conditions for preference relations can

[2] For a criticism of equality of access as a notion of equity see Culyer and Wagstaff (1993) and Fleurbaey (2005a).

usefully be applied to provide a welfare economics foundation for the commonly used measures of inequality in health. It draws on the work by Bleichrodt, Diecidue, and Quiggin (2004) to characterize the rank-dependent QALY model and on some results that have been derived in the theoretical literature on inequality measurement. By providing such a general preference foundation we hope that our paper will clarify what assumptions are implicit in the adoption of a particular measure.

In what follows, Section 2 describes the main measures of health inequality. Section 3 gives a preference foundation for these measures. We characterize the Gini index, the concentration index, and Wagstaff's (2002) achievement index. We also briefly discuss the extension of our techniques to characterize and design absolute measures of health inequality. Section 4, which concludes the paper, discusses the appeal of the conditions introduced throughout the paper and, hence, of the measures that they characterize, and considers possible extensions and generalizations. All proofs are in the appendix.

2. Measures of health inequality

The literature on the measurement of inequalities in health has drawn on the development of rank-dependent measures in the income inequality literature. A long-standing issue in the literature on health inequality is whether *all* inequalities ought to be measured or only those which show some systematic association with indicators of socioeconomic status (Gakidou, Murray, and Frenk, 2000, Wagstaff, 2001) Some of the earlier contributions by economists (e.g. LeGrand, 1989) used Lorenz curves and the Gini index to measure inequality in mortality rates. A Lorenz curve describes the cumulative distribution of health in a population ranked by health and the Gini index, henceforth denoted as *G*, measures the deviation from an equal distribution as (twice) the area between the Lorenz curve and the diagonal.

Wagstaff, Paci, and van Doorslaer (1991) proposed to use the related concepts of a concentration curve and index to measure the extent to which inequalities in health are related to indicators of socioeconomic status like income or education. They argued that the concentration index meets three minimal requirements of an inequality index: (i) it reflects the experience of the entire population studied, (ii) it reflects the socioeconomic dimension of health inequalities, and (iii) it is sensitive to changes in the composition of the underlying socioeconomic ranking variable. A concentration curve describes the cumulative distribution of health in a population ranked by socioeconomic status and the concentration index, henceforth denoted as *C*, measures the deviation from an equal distribution as (twice) the area between the concentration curve and the diagonal.

The Gini index *G* can only take positive values and becomes zero when the Lorenz curve coincides with the diagonal. A Lorenz curve can only lie below the diagonal. The concentration index *C* can be both negative and positive depending on whether the concentration curve lies above or below the diagonal. For individual-level data, both the Gini and the concentration coefficient can be written as:

$$1 - \frac{\sum_{i=1}^{n}(2R_i - 1)h_i}{n^2\mu(h)} . \tag{1}$$

where n is the sample size, h_i is the health indicator for person i that is larger the better is health, $\mu(h)$ is the mean level of health, and R_i is the relative rank of the *i*th person, where 1 means best rank or "best-off".[3] For the Gini index the ranking is in terms of health, for the concentration index the ranking is in terms of socioeconomic status.

Equation (1) shows that the weighting scheme and degree of inequality aversion embodied in the concentration index is arbitrary. Drawing on Yitzhaki's (1983) extended Gini coefficient, Wagstaff (2002) proposed an extended concentration index which, for the purposes of this paper, is most conveniently defined as

$$C(\nu) = 1 - \frac{\Sigma_{i=1}^{n} (R_i^{\nu} - (R_i - 1)^{\nu}) h_i}{n^{\nu} \mu(h)}, \tag{2}$$

where ranking is by socioeconomic status. The parameter ν reflects distributional sensitivity. When $\nu = 1$, everyone's health is weighted equally and inequality, as measured by (2), equals zero, irrespective of how unequal the distribution of health is across socioeconomic groups. For $\nu > 1$, a larger weight is attached to a worse-ranked person's health than to a better-ranked person's health and, thus, there is some aversion to inequality. If $\nu = 2$ then (2) reduces to (1) and becomes equal to the standard concentration index. As ν gets larger, the weighting becomes more pro-poor. If $\nu = \infty$ then (2) becomes $1 - \min_i \frac{h_i}{\mu(h)}$, i.e. all weight goes to the worst-ranked individual.

[3] In the empirical literature it is more common to let 1 mean worst rank. Obviously this is just a matter of convention. The notation in what follows is, we believe, easier if we let 1 denote best rank.

Wagstaff (2002) went on to propose what he labeled an *achievement index*, which in the notation of this paper is defined as

$$A(\nu) = \frac{\sum_{i=1}^{n} (R_i^{\nu} - (R_i - 1)^{\nu})h_i}{n^{\nu}}. \tag{3}$$

A comparison between (2) and (3) reveals immediately that $A(\nu) = \mu(h)\big(1 - C(\nu)\big)$.

The latter expression bears a striking resemblance to an abbreviated social welfare function that is quite common in the income inequality measurement literature. Lambert (2001) has given several welfare economics rationalizations for the use of an abbreviated social welfare function in the evaluation of income profiles. His approach is different from ours in that he did not give preference foundations for functional forms, but, instead, assumed particular functional forms.

3. A theoretical foundation for rank-dependent inequality measures

We will now examine the preference conditions that underlie the inequality measures introduced in Section 2. For ease of exposition, it is useful to start with comparisons of health distributions across populations with a fixed size before considering the case of variable population sizes.

3.1. Fixed population size

We consider a policy maker who has to make a choice between distributions of health for a population of fixed size $n \geq 2$. Let h_i denote the amount of health of individual i. A health profile $(h_1,\ldots,h_n)$ specifies the health of each individual in society. Amounts of health are nonnegative real numbers and, hence, health profiles are elements of $\mathbb{R}^n_+$.[4] The policy maker's preferences over health profiles are expressed by a preference relation $\succcurlyeq$, denoting "at least as preferred as". As usual, $\succ$ denotes strict preference and $\sim$ denotes indifference. The preference ordering over health profiles does not depend on other variables such as the level of consumption. This is a consequence of the fact that the purpose of the paper is to characterize common measures of health inequality and these measures depend only on health and not on other variables. The restrictiveness of the assumption that we can focus on health alone, which is implicit in these measures, will become apparent when we discuss the conditions that we introduce next.

We assume that the relation $\succcurlyeq$ is a weak order, i.e. it is *complete* (for all health profiles h,h', either $h \succcurlyeq h'$ or $h' \succcurlyeq h$ or both) and *transitive* (if $h \succcurlyeq h'$ and $h' \succcurlyeq h''$ then $h \succcurlyeq h''$). We also assume that $\succcurlyeq$ satisfies *monotonicity*: if health profiles h and h' are such that each individual in society has more health under h than under h', then h should be preferred to h'. Formally, if for all i in $\{1,\ldots,n\}$ $h_i > h'_i$ then $h \succ h'$. Further, we assume that for each health profile h there exists an *equally-distributed equivalent* e, that is, a constant level of health, which, if received by each individual, results in a distribution that is socially indifferent to h: $(e,\ldots,e) \sim (h_1,\ldots,h_n)$.

Under the assumptions made, there exists a social welfare function W that represents $\succcurlyeq$: for all health profiles h,h', $W(h) \geq W(h')$ if and only if $h \succcurlyeq h'$. For a proof of

[4] The set of health levels can also be a subset of the reals. In applications this subset is often $[0,1]^n$.

this assertion see the proof of Theorem 1. The equally distributed equivalent e corresponding to a given health profile h is then given by

$$W(e,\ldots,e) = W(h_1,\ldots,h_n). \tag{4}$$

Under the assumptions made, (4) can be solved uniquely for e and we can write

$$e = F(h), \tag{5}$$

where F denotes the functional relationship between e and h. By monotonicity, it follows that for all health profiles h,h′, W(h) ≥ W(h′) if and only if F(h) ≥ F(h′). Hence, W and F both represent ≽ and they must be related by a positive monotonic transformation.

The concern with inequality stems primarily from the feeling that reductions in inequality should lead to increases in social welfare, provided that mean health remains constant. Given any health profile h, we can define an index of inequality I(h) as the proportion of total health that can be discarded without affecting social welfare. This is the methodology associated with Atkinson (1970), Kolm (1969), and Sen (1973) (AKS). See Dutta (2002) for an overview. In our decision context, this means that

$$e = \mu(h)\bigl(1 - I(h)\bigr). \tag{6}$$

Equation (6) implies that social welfare is a positive monotonic transformation of $\mu(h)(1 - I(h))$. From (6) we obtain that an AKS index of inequality for the social welfare function W is given by

$$I(h) = 1 - \frac{e}{\mu(h)}, \tag{7}$$

Every AKS index of inequality is *normatively significant* in the sense that for every two health profiles h and h′ that have the same mean level of health, $I(h) \leq I(h')$ if and only if $W(h) \geq W(h')$.

We can use the general framework outlined above to analyze the value judgments implied by health inequality measures. We will do this first for the Gini index and then for the concentration index.

Gini index

A *rank-ordered health profile* $\tilde{h}$ is a permutation of h such that $\tilde{h}_1$ is the health of the individual with the best health status, $\tilde{h}_i$ the health of the individual who has rank i in the distribution of health status, and $\tilde{h}_n$ the health of the individual who has the worst health status. Using this notation, (1) shows that the Gini index is defined as

$$G = 1 - \frac{\tilde{h}_1 + \ldots + (2i-1)\tilde{h}_i + \ldots + (2n-1)\tilde{h}_n}{n^2\mu(h)}. \tag{8}$$

By (7) $G = 1 - \frac{e}{\mu(h)}$ and, thus, by (5) $G = 1 - \frac{F(h)}{\mu(h)}$. Because the sum of the first n odd numbers (1+3+…+(2i−1)+…+(2n−1)) equals n^2, (8) yields

$$G = 1 - \frac{\dfrac{\tilde{h}_1 + \ldots + (2i-1)\tilde{h}_i + \ldots + (2n-1)\tilde{h}_n}{1 + \ldots + (2i-1) + \ldots + (2n-1)}}{\mu(h)} \tag{9}$$

and, thus, we obtain that the social welfare function underlying the Gini index is equal to

$$F(h) = \frac{\tilde{h}_1 + \ldots + (2i-1)\tilde{h}_i + \ldots + (2n-1)\tilde{h}_n}{1 + \ldots + (2i-1) + \ldots + (2n-1)}. \tag{10}$$

As noted, the coefficients in (10) are arbitrary and we, therefore, start by characterizing a more general index, which we label the *generalized Gini index*:

$$F(h) = \frac{a_1\tilde{h}_1 + \ldots + a_n\tilde{h}_n}{a_1 + \ldots + a_n} = \frac{\Sigma_{i=1}^{n} a_i\tilde{h}_i}{\Sigma_{i=1}^{n} a_i}, \tag{11}$$

where, for monotonicity, $a_i > 0$, i = 1,…,n. Equation (11) is an adaptation of Weymark's (1981) generalized Gini index to our decision context.

Besides the requirement that they are positive, we have imposed no other restrictions on the weights a_i. Intuitively, we would like the weights to increase with the individual's ranking position to reflect that more weight is given to individuals who are worse-off. The following condition ensures this. The *principle of health transfers* holds if a transfer of health from someone who is in better health to someone who is in worse health does not lead to a reduction in social welfare provided the transfer does not change the ranking of the individuals in terms of health.[5] [6]

We will now formalize the principle of health transfers. It will be convenient in the subsequent discussion to consider the set $K = \{h: h_1 \geq \ldots \geq h_n\}$. That is, K contains all health profiles such that the individual with index number j also has rank j in terms of health. So if there are three (groups of) individuals in society (n = 3) then the health profile (0.8, 0.5, 0.2) belongs to K, but the profile (0.5, 0.8, 0.6) does not. Let $\alpha_i h$ denote the health profile h with h_i replaced by α: $\alpha_i h = (h_1, \ldots, h_{i-1}, \alpha, h_{i+1}, \ldots, h_n)$. That is, $\alpha_i h$ is the health profile that obtains when the health of individual i is changed from h_i to α, while leaving the health of all other individuals unaffected. Similarly, $\alpha_i \beta_j h$ is the health profile h with h_i replaced by α and h_j by β. Let 0 denote the vector of zero health for all individuals. The principle of health transfers holds if for all $h \in K$, for all $i < j$, and for all $\alpha > 0$, $h + (-\alpha)_i \alpha_j 0 \in K$ implies $h + (-\alpha)_i \alpha_j 0 \succcurlyeq h$. The requirement that $h + (-\alpha)_i \alpha_j 0 \in K$ reflects that the transfer should not change the ranking of the individuals in terms of health. A consequence of the principle of health transfers is that $a_1 \leq \ldots \leq a_n$ in (11). The principle of health transfers is similar to Yaari's (1988) condition of equality-mindedness applied to the health domain.

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[5] We discuss the acceptability of this principle in the final section.

[6] That health transfers are in practice generally not possible is not relevant here. What matters is that more equal health profiles are preferred to less equal health profiles. We used the term “transfers” because it is common in the literature on inequality measurement.

Anonymity holds if for all health profiles h, h′, if h is a permutation of h′ then h ~ h′. So if n = 3, anonymity implies, for example, that (0.8,0.6,0.3) ~ (0.3,0.8,0.6), because the latter profile is a permutation of the first. In words, anonymity says that it does not matter who gets which health state; the identity of the individuals does not matter. This also implies that individual characteristics like gender, age, occupation etc. should not influence social welfare judgments according to anonymity. The only characteristic that is relevant is the individual's health.

Finally, we assume that if h,h′, and h″ are all in K, then h ≽ h′ if and only if h+h″ ≽ h′+h″. We will refer to this condition as *additivity*. Additivity means, for instance, that if there are two (groups of) individuals in society and h = (0.5,0.5) ≻ (0.8,0.2) = h′ then also (0.7,0.5) ≻ (1.0,0.2). Adding h″ = (0.2,0) to both h and h′ does not affect preferences because h, h′, and h″ are all such that the first individual gets at least as much health as the second. Additivity does not claim that if, h = (0.5,0.5) ≻ (0.8,0.2) = h′ then also (0.5,0.7) ≻ (0.8,0.4), because now h″ = (0,0.2) is such that the second individual gets more health than the first, whereas in h and h′ the first individual gets at least as much as the second. Hence, h and h′ are in K, but h″ is not.

We can now state a first result.

Theorem 1: The following two statements are equivalent:

(i) $F(h)$ satisfies (11) with $0 < a_1 \leq \ldots \leq a_n$.

(ii) $\succcurlyeq$ is a weak order that satisfies monotonicity, the principle of health transfers, anonymity, and additivity and there exists for each health profile an equally distributed equivalent level of health.

□

To obtain (10), i.e. the social welfare function underlying the Gini index, we replace the principle of health transfers by the following condition: for all $h \in K$, for all j, and for all $\alpha > 0$, $h + \left(-(2j-1)\alpha\right)_1 \alpha_j 0 \in K$ implies $h + \left(-(2j-1)\alpha\right)_1 \alpha_j 0 \sim h$. We will refer to this condition as the *Gini condition*. In the definition of the Gini condition the subscript 1 refers to the individual who is best-off in terms of health. The Gini condition specifies the trade-off in units of health between the best-off individual and any other individual that leaves the policy maker indifferent. The requirement that $h + \left(-(2j-1)\alpha\right)_1 \alpha_j 0 \in K$ reflects that the health transfer should not change the rank-ordering of the individuals in terms of health. The Gini condition implies for example that $h = (0.8, 0.4) \sim (0.5, 0.5)$. In this example $\alpha = 0.1$.

Note that the Gini condition is stronger than the principle of health transfers. The Gini condition implies the principle of health transfers given the other conditions in Theorem 1. The principle of health transfers, however, does not imply the Gini condition: the principle of health transfers does not imply the indifference $h + \left(-(2j-1)\alpha\right)_1 \alpha_j 0 \sim h$. It only says that $h + (-\alpha)_1 \alpha_j 0 \succcurlyeq h$ whenever h and $h + (-\alpha)_1 \alpha_j 0$ both belong to K. Hence, the Gini condition can be interpreted as a stronger or more restrictive version of the principle of health transfers.

To summarize,

Theorem 2: The following two statements are equivalent:

(i) F(h) satisfies (10)

(ii) $\succcurlyeq$ is a weak order that satisfies monotonicity, the Gini condition, anonymity, and additivity and there exists for each health profile an equally distributed equivalent level of health.

□

Concentration Index

The characterization of the concentration index resembles the characterization of the Gini index. To avoid introducing extra notation, we will, with two exceptions, use the same names for the various conditions as we used above instead of distinguishing between the Gini and the concentration index version of the condition. The context in which a condition is used will clarify which version of a condition is meant.

To characterize the concentration index we should define $\succcurlyeq$ over profiles of vectors $h = \big((h_1,R_1),\ldots,(h_n,R_n)\big)$, where R_i denotes individual i's rank in terms of socioeconomic status. A *rank-ordered health profile* $\tilde{h}$ is now such that $\tilde{h}_1$ is the health of the individual with the highest socioeconomic status, $\tilde{h}_i$ the health of the individual who has rank i in the distribution of socioeconomic status, and $\tilde{h}_n$ the health of the individual who has the lowest socioeconomic status. Using this notation, (1) shows that the concentration-index is defined as

$$C = 1 - \frac{\tilde{h}_1 + \ldots + (2i-1)\tilde{h}_i + \ldots + (2n-1)\tilde{h}_n}{n^2\mu(h)}. \qquad (12)$$

And, thus, by a similar line of argument as in the derivation of (10), the social welfare function underlying the concentration index is equal to

$$F(h) = \frac{\tilde{h}_1 + \ldots + (2i-1)\tilde{h}_i + \ldots + (2n-1)\tilde{h}_n}{1 + \ldots + (2i-1) + \ldots + (2n-1)}. \qquad (13)$$

The *generalized concentration index* is defined as

$$F(h) = \frac{a_1\tilde{h}_1 + \ldots + a_n\tilde{h}_n}{a_1 + \ldots + a_n} = \frac{\sum_{i=1}^{n} a_i\tilde{h}_i}{\sum_{i=1}^{n} a_i}, \qquad (14)$$

where, for monotonicity, $a_i > 0$, $i = 1,\ldots,n$.

To ensure that the a_i decrease with the individual's ranking position, we impose the following *principle of income-related health transfers*: transferring health from someone who is better-off in terms of socioeconomic status to someone who is worse-off in terms of socioeconomic status does not lead to a reduction in social welfare provided the transfer does not change the ranking of the individuals in terms of socioeconomic status.

To formalize the principle of income-related health transfers, let K^{id} be the set of health profiles such that the numbering of the individuals corresponds to their rank in terms of socioeconomic status: h_1 is the health of the individual with the highest socioeconomic status, h_i is the health of the individual who has rank i in terms of socioeconomic status and h_n is the health of the individual with the worst socioeconomic status. That is, $K^{id} = \{h: R_1 <\ldots< R_n\}$. The principle of income-related health transfers holds if for all $h \in K^{id}$, for all $i < j$, and for all $\alpha > 0$, $h + (-\alpha)_i\alpha_j0 \in K^{id}$ implies $h + (-\alpha)_i\alpha_j0 \succcurlyeq h$. The requirement that $h + (-\alpha)_i\alpha_j0 \in K^{id}$ reflects that the health transfer should not affect the rank ordering of the individuals in terms of socioeconomic status.

Anonymity holds if for all profiles $h = \big((h_1,R_1),\ldots,(h_n,R_n)\big)$, $h' = \big((h'_1,R'_1),\ldots,(h'_n,R'_n)\big)$, if h is a permutation of h′, where health status and socioeconomic rank are permuted jointly, then $h \sim h'$. Anonymity implies, for instance, that $\big((0.8,1),(0.4,2),(0.6,3)\big) \sim \big((0.4,2),(0.6,3),(0.8,1)\big)$. According to anonymity, the only characteristics that are allowed to influence social welfare judgments are the individual's health and his rank in terms of socioeconomic status. Finally, we assume *additivity*: if h,h′, and h″ are all in K^{id}, then $h \succcurlyeq h'$ if and only if $h+h'' \succcurlyeq h'+h''$. We can now characterize the generalized concentration index.

Theorem 3: The following two statements are equivalent:

(i) F(h) satisfies (14) with $0 < a_1 \leq \ldots \leq a_n$.

(ii) $\succcurlyeq$ is a weak order that satisfies monotonicity, the principle of income-related health transfers, anonymity, and additivity and there exists for each health profile an equally distributed equivalent level of health.

□

To obtain (13), the social welfare function underlying the concentration index, we replace the principle of income-related health transfers by the following condition: for all $h \in K^{id}$, for all j, and for all $\alpha > 0$, $h + \left(-(2j-1)\alpha\right)_1 \alpha_j 0 \in K^{id}$ implies $h + \left(-(2j-1)\alpha\right)_1 \alpha_j 0 \sim h$. We will refer to this condition as the *CI condition*. The CI condition specifies a trade-off between the individual that is best-off in terms of socioeconomic status and any other individual in society that leaves the policy maker indifferent. The CI condition is a stronger condition than the principle of income-related health transfers: the CI condition implies the principle of income-related health transfers, but the principle of income-related health transfers does not imply the CI condition.

Theorem 4: The following two statements are equivalent:

(i) F(h) satisfies (13)

(ii) $\succcurlyeq$ is a weak order that satisfies monotonicity, the CI condition, anonymity, and additivity and there exists for each health profile an equally distributed equivalent level of health.

□

3.2. Variable population size

In Section 3.1. the population size was held fixed. We will now generalize the results of the previous subsection to the case where the population size is variable. A complication if we allow for variable population size is that the weights a_i in (11) and (14) depend on the population size n. A natural restriction is to assume that the a_i are independent of n and can be characterized by a single nondecreasing sequence $\{a_1, a_2, \ldots\}$. For example, for the Gini index or the concentration index this sequence would have to be $\{1,3,5,7,\ldots\ldots\}$. Using the terminology of Donaldson and Weymark (1980) we refer to (11) with the assumption that the a_i are independent of n as the *single-series Gini model* and to (14) with the assumption that the a_i are independent of n as the *single-series CI model*.

Let h be a given health profile. We call h^m an *m-fold replication of h* if $h^m = (h^{(1)},\ldots, h^{(m)})$ where each $h^{(i)} = h$. For example, if $h = (h_1,h_2)$ then its 2-fold replication $h^{(2)} = (h_1,h_2,h_1,h_2)$. In the case of the concentration index, replication also concerns socioeconomic status and $h^{(2)} = \left((h_1,R_1),(h_2,R_2),(h_1,R_1),(h_2,R_2)\right)$. The *principle of population*, introduced by Dalton (1920), requires that social welfare for a population of size n is the same as social welfare for a population of size mn if the larger population is an m-fold replication of the smaller population. Formally, for all m, if h^m is an m-fold replication of h then $h \sim h^m$. Intuitively, the principle of population claims that only per-capita quantities matter.

Theorem 5. Suppose that the single-series Gini model (single-series CI model) holds. Then the following two statements are equivalent:

(i) $F(h) = \dfrac{\Sigma_{i=1}^{n} (i^{\nu} - (i-1)^{\nu})\tilde{h}_i}{n^{\nu}}$ where the ranking is in terms of health (socioeconomic status).

(ii) $\succcurlyeq$ satisfies the principle of population

□

Theorem 5 shows that under the single-series CI model the principle of population implies Wagstaff's (2002) achievement index, which is, in fact, the abbreviated social welfare function underlying his extended concentration index. As noted in Section 2, the parameter ν captures distributional concerns; the higher ν, the more sensitive the policy maker is to the distribution of health. If we replace the principle of health transfers (the principle of income-related health transfers) in the single-series Gini model (single-series CI model) by the Gini condition (CI condition) then we obtain a preference foundation for the Gini index (concentration index) for variable population size.

In the preceding discussion, we focused on relative measures of health inequality. The above analysis can also be used to give a preference foundation for absolute measures of health inequality. Absolute measures of health inequality are defined by

$$I_A(h) = \mu(h) - F(h). \qquad (15)$$

Thus Theorem 5 can be used to define an absolute measure of health inequality that corresponds to Wagstaff's (2002) achievement index.

4. Discussion

In this paper we have discussed under what conditions a welfare economics foundation can be given to the largely empirical literature on the measurement of health inequality. The conditions of anonymity, the existence of an equally distributed equivalent level of health, weak ordering, and monotonicity generally seem acceptable. Sen (1973) has argued that completeness may be too restrictive given that many of our intuitions on inequality only support partial orderings. Monotonicity may in some cases be too restrictive, e.g. when health in society is concentrated among few people a policy maker may not consider it desirable to give these "happy few" a large gain in health when the health of the others in society, the "left-behinds", improves only marginally. Such decision situations will be rare, however, and empirical evidence does not seem to support such preferences (Bleichrodt, Doctor, and Stolk, 2005). Therefore, monotonicity does not seem restrictive as a general principle. Finally, anonymity may be violated if the policy maker cares about other attributes of an individual than only his health and his socioeconomic status.

Additivity is more controversial. It is sometimes observed that societies become more sensitive to inequalities when they grow richer and the same may be true for health. When the total level of health is low, inequalities in health may not be important. As societies become healthier, the concern for inequalities in health may increase. If this were to happen, violations of additivity may arise. Consider the following example where there are two (groups of) individuals and health is measured on the scale [0,1]. It may be that a policy maker prefers (0.5,0.1) to (0.3,0.3) because then at least one individual has a reasonable quality of life. However, the policy maker may also prefer (0.8,0.8) to (1.0,0.6), because the former is more equitable and all individuals have a reasonable quality of life. These two preferences violate additivity because (0.8,0.8) = (0.3,0.3) + (0.5,0.5) and (1.0,0.6) = (0.5,0.1) + (0.5,0.5). That said, we believe that in most real-world decision contexts additivity will not be violated and, hence, we tend to consider additivity a plausible condition.

Doubts may be raised about the validity of the principle of population. If only the health of people is replicated, but not other characteristics like consumption and income can we really say social welfare is the same in the original situation and in the m-fold replication? We are inclined to answer this question in the negative.

Let us finally turn to the principle of (income-related) health transfers. The principle of health transfers, used in the characterization of the generalized Gini index, says that transferring health from someone with higher health to someone with lower health does not lead to a reduction in social welfare provided the transfer does not change the health ranking of the individuals. One may object against this that it is not always desirable to transfer health from a healthier person to a less healthy person, e.g. when the healthier person is poor and the less healthy person is rich. The principle of income-related health transfers, which is the socioeconomic inequalities version of the principle of health transfers and which was used in the characterization of the generalized concentration index, requires that transferring health from someone who is better-off in terms of socioeconomic status to someone who is worse-off does not lead to a reduction in social welfare provided the transfer does not change the ranking of the individuals in terms of socioeconomic status. The plausibility of this principle is contestable. It does not seem desirable to transfer health from a person with high living standards to a person with lower living standards when the person with high living standards is in poor health and differences in living standards are small. In general, the principle of health transfers will, both in its pure inequalities version and in its socioeconomic inequalities version, be more acceptable the stronger the correlation between health and other attributes such as income. The Gini condition and the CI condition, the stronger versions of the principle of health transfers and the principle of income-related health transfers, respectively, seem arbitrary and the equity weights implied by these conditions will not meet with unanimous approval.

The above observations on the principle of population and the principle of (income-related) health transfers suggest that it may be desirable to study multivariate concepts of inequality: the policy maker generally cares not only about the distribution of health, but also about the distribution of other attributes, e.g. income, educational attainment, etc. To characterize health inequality measures in such a setting requires that additional assumptions be imposed. In particular, we should assume that the social decision maker's preferences over the various attributes are separable in the attributes. Also, the conditions imposed in Section 3 should be rephrased in a multidimensional setting, which raises complications for the formulation of the principle of (income-related) health transfers and additivity. The translation of the other conditions is straightforward. Theoretical treatments of multidimensional inequality measurement have been provided by Tsui (1995) and Gajdos and Weymark (2005) but we are not aware of any empirical applications other than the initial attempt of Atkinson and Bourguignon (1982) to illustrate their dominance conditions for the comparison of inequality in two dimensions (per capita incomes and life expectancy).

Fleurbaey (2005b) circumvented the problem of the existence of multiple attributes by reducing what is basically a two-dimensional problem (both health and income matter) to a one-dimensional problem. He introduced the concept of “healthy-equivalent consumption”. This is basically equal to the individual’s income minus his willingness to pay for perfect health. By converting the health dimension into an income dimension, Fleurbaey rephrased the two-dimensional problem into the income space and then the usual conditions for a social welfare function defined over distributions of income apply. In that sense, the results of Section 3 can also be applied to Fleurbaey’s rephrased problem.

Appendix: Proofs.

Proof of Theorem 1.

That (i) implies (ii) is easy to verify. Hence, we assume (ii) and derive (i). First we show that e(h), the equally distributed equivalent of health profile h, represents $\succcurlyeq$. Consider two health profiles h and h′ and let e and e′ be their corresponding equally distributed equivalents, which exist by assumption. If $h \succcurlyeq h'$ then $e\mathbf{1} \sim h \succcurlyeq h' \sim e'\mathbf{1}$, where **1** denotes the n-dimensional unit vector. By transitivity, we have $e \succcurlyeq e'$ and, hence, by monotonicity $e \geq e'$. Conversely, if $e \geq e'$, then by monotonicity $e \succcurlyeq e'$ and by transitivity $h \succcurlyeq h'$. Hence, e represents $\succcurlyeq$ and we can define a social welfare function W(h) = e(h).

We next show that e(h) satisfies Cauchy's functional equation: $e(h+h') = e(h) + e(h')$, whenever h and h′ are in $K = \{h_1 \geq \ldots \geq h_n\}$. Let $h, h' \in K$. Note that for all h, e(h) is in K. The indifference $h \sim e(h)$ implies by additivity that $h + h' \sim e(h) + h'$. Additivity and $h' \sim e(h')$ imply that $e(h) + h' \sim e(h) + e(h')$. Transitivity gives $h + h' \sim e(h) + e(h')$. Hence, $e(h + h') = e(h) + e(h')$. This and monotonicity imply by a similar line of proof as in Lemma 3 and Theorem 4 in Weymark (1981) that e(h) is linear: $e(h) = \Sigma_{i=1}^{n} \lambda_i h_i$. Weymark (1981) assumed continuity of e, but his proof also holds if monotonicity is assumed instead (see the argument in Aczel 1966, p.33). Because $e(\mathbf{1}) = 1$, it follows that the λ_i sum to one. By monotonicity the λ_i are all positive. By the principle of health transfers it follows that $\lambda_i \leq \lambda_j$ when $i<j$.

Now consider h not in K. Let $\tilde{h}$ be a permutation of h such that $\tilde{h} \in K$. By anonymity, $h \sim \tilde{h}$ and, hence, we can define $e(h) = \Sigma_{i=1}^{n} \lambda_i \tilde{h}_i$. Choosing a_i such that $\lambda_i = \frac{a_i}{\Sigma_{i=1}^{n} a_i}$ gives the desired result.

□

Proof of Theorem 2.

(i) ⇒ (ii) is immediate. Suppose (ii). By the proof of Theorem 1 we obtain that $h \succcurlyeq h' \Leftrightarrow \Sigma_{i=1}^{n} \lambda_i \tilde{h}_i \geq \Sigma_{i=1}^{n} \lambda_i \tilde{h}'_i$. Let $a_1 = 1$. By the Gini condition it follows immediately that $a_i = (2i-1)$. Hence, (10) represents $\succcurlyeq$.

□

Proof of Theorem 3.

(i) $\Rightarrow$ (ii) is immediate. Suppose (ii). For $h \in K^{id}$, the same line of proof as was used in the proof of Theorem 1 yields that $\Sigma_{i=1}^{n} \lambda_i h_i$ represents $\succcurlyeq$. If h is not in K and $\tilde{h}$ is a permutation of h such that $\tilde{h} \in K^{id}$, then by anonymity, $h \sim \tilde{h}$. The rest of the proof is identical to the proof of Theorem 1.

□

Proof of Theorem 4.

(i) $\Rightarrow$ (ii) is immediate. The proof that (ii) $\Rightarrow$ (i) is identical to the proof of Theorem 2.

□

Proof of Theorem 5.

That (i) implies (ii) is easy to verify. Assume that the principle of population holds. Assume that either the single-series Gini model or the single-series CI model holds. Define the function f by

$$f(0) = 0$$

$$f(m) = \Sigma_{i=1}^{m} a_i,\ m \in \mathbb{N}.$$

It follows that $a_i = f(i) - f(i-1)$. By the principle of health transfers or the principle of income-related health transfers, $f(n+1) - f(n) \geq f(n) - f(n-1)$ for all $n \in \mathbb{N}$. It now follows from the proof of Theorems 1 and 2 in Donaldson and Weymark (1980) that $f(n) = n^{\nu}$ for all $n \in \mathbb{N}$. Substitution gives (i).

□

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