Aversion to health inequalities and priority setting in health care

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Abstract

Traditionally aversion to health inequality is modelled through a concave utility function over health outcomes. Bleichrodt et al. (2004) have suggested a "dual" approach based on the introduction of explicit equity weights. The purpose of this paper is to analyze how priorities in health care are determined in the framework of these two models. It turns out that policy implications are highly sensitive to the choice of the model that will represent aversion to health inequality.

Key Words: Equity, health, social welfare**.**

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1. Introduction

The most common approach to aggregate health benefits in economic evaluations of health care is by unweighted summation, also referred to as QALY-utilitarianism. In this approach each individual in society gets the same weight and the aim is to maximize the benefits accruing from health care. Many authors have criticized QALY-utilitarianism for being blind to equity considerations (Nord, 1995, Williams, 1997, Williams and Cookson, 2000) and, indeed, empirical studies have typically observed that people care about equity in the distribution of health (for an overview see Dolan and Tsuchiya, 2006).

Two approaches have been put forward in the literature to incorporate equity considerations into economic evaluations of health care. The first approach defines a social utility function over health and uses health utilities rather than health itself as input in the medical decision process. The parameters in the social utility function then reflect the weight that different individuals should receive. The utility approach to equity was pioneered by Wagstaff (1991), who argued in favour of the iso-elastic utility function suggested by Atkinson (1970), and was also advocated by Dolan (1998), who suggested a Cobb-Douglas utility function, which is a special case of the iso-elastic utility function.

Bleichrodt, Diecidue, and Quiggin (2004) proposed a different approach to introduce equity weighting into economic evaluation. Instead of using a utility function over health they applied equity weights in the decision process. An advantage of modelling equity through explicit equity weights is that attitudes towards outcomes are separated from attitudes towards equity. Bleichrodt et al. (2004) gave an axiomatic foundation for their model and argued that the conditions on which the model depends are plausible. An important advantage of their model is that the empirical estimation of the equity weights is very simple, which fosters the practical applicability of the model. The approach of Bleichrodt et al. (2004) can be considered dual to the utility-based approach of Wagstaff (1991) and Dolan (1998) in the following sense. In the utility-based approach, the health outcomes are transformed but the proportion of people in each category of health outcome is not transformed, while in the approach of Bleichrodt et al. (2004) the proportion of people in each category of health outcome is transformed but the health outcomes themselves are left untransformed.

It may appear at first sight that the two approaches, the utility model and the dual model, are essentially equivalent ways of incorporating equity considerations into economic evaluation. This paper will argue that this is not true. As we will show, the two models have rather different implications for priority setting in health and the choice between the two models is not trivial. Hence, there is really something to choose between the two approaches and our analysis will clarify the differences between the two approaches.

Alternative models are often compared in terms of the appeal of their underlying axioms. However, they may also be evaluated in terms of their implications for choices and decisions. The purpose of this paper is to compare the utility approach to model health inequality with the dual approach in terms of their implications for priority setting in health. Intuitively, the following requirements make sense for an inequality-averse social planner. First, it seems plausible that the more inequality averse the social planner is, the more priority she will give to the worse-off. This requirement catches the basic idea behind inequality aversion and is a basic requirement for any model that seeks to capture inequality aversion. Our second requirement is related to Nord's (1993) idea that the priority given to the worse-off should increase with the severity of their illness. We will also consider a third requirement that the equity weights should depend on the prevalence of serious illnesses. Arguments why the prevalence of serious illnesses should matter come from the recent debate on the cost-effectiveness of orphan drugs, drugs for managing serious and rare diseases, (Gericke, Riesberg, and Busse, 2005, Hughes, 2006, McCabe, Claxton, and Tsuchiya, 2005). There it has been pointed out that society places a greater value on health gains in individuals if the number of cases is small and the condition is severe.

We will evaluate the utility model and the dual model in terms of the above three requirements. It turns out that both approaches satisfy the first requirement, that more inequality aversion entails more priority for the worse-off. The second requirement is only satisfied by the utility model, while the third requirement is only satisfied by the dual model, but to a limited degree.

In what follows, Section 2 describes the decision problem considered in this paper, which is very close to the one adopted by Hoel (2003) and subsequently by Bui, Crainich, and Eeckhoudt (2005) to explore the impact of treatment risks on health care priorities. Section 3 analyzes the benchmark case where the social planner is inequality neutral. Sections 4 and 5 analyze the utility and the dual model of inequality measurement, respectively. In the conclusion, we contrast the results obtained in each model and some potential extensions are suggested. All formal derivations of results presented throughout the paper are in the appendices.

2. The decision problem

Consider a social planner who has to allocate health care resources across a population. We assume that health can be quantified, for instance through quality-adjusted life-years (QALYs). The population consists of two types of individuals. A proportion α suffers from a severe disease (disease 1), which - if it remains untreated - produces a health level h_1. The remainder of the population $(1-\alpha)$ is affected by a minor disease yielding a health level h_2.

For each disease there exists a treatment in which the social planner can invest. The investment per patient is denoted c_1 for disease 1 and c_2 for disease 2. Expanding c_1 or c_2 improves the corresponding health levels in a way characterized by the continuous and everywhere differentiable functions $h_1(c_1)$ and $h_2(c_2)$. We assume that higher investments in the treatments improve the health of the patients, but that the marginal benefits from additional investments decrease. Hence, the marginal capacity to benefit from treatment is always positive but may converge to zero. Formally, the functions h_i are strictly increasing and concave: $h_j' > 0$ and $h_j'' < 0$ for j=1, 2.

We assume for the sake of simplicity that, regardless how much is invested in the treatment of disease 1, a patient with disease 1 will never end up in a better health state than a patient with disease 2 even if the latter were left untreated. That is, even if disease 1 were treated to the fullest extent (c_{1m}), i.e. up to the point where $h_1'(c_{1m}) = 0$, the resulting health level $h_1(c_{1m})$ would still be below h_2. This assumption is not crucial for our results. In fact, our conclusions also hold true when much less stringent restrictions are imposed on the medical technologies. For instance, if there are levels of c_1 and c_2 such that $h_1(c_1) = h_2(c_2)$ then it suffices to assume that $h_1'(c_1) \leq h_2'(c_2)$ to obtain the results.

The social planner has a fixed budget B that she has to allocate between c_1 and c_2. Hence, she faces the budget constraint:

$$\alpha c_1 + (1-\alpha)c_2 \leq B, \tag{1}$$

where we normalized so that the whole population is set equal to 1.

3. Inequality neutrality

Suppose that the social planner is inequality neutral, i.e. she does not care about inequality in the distribution of health and behaves according to QALY utilitarianism. Then she will try to maximize the total health level defined by:

$$\max_{c_1,c_2} R = \alpha h_1(c_1) + (1-\alpha)h_2(c_2) \tag{2}$$

subject to the budget constraint (1). This optimization gives the following result.

Result 1. Suppose the decision problem of Section 2 holds. If the social planner is inequality neutral then the marginal benefits of health care are equalized, i.e. at the optimum we have $h_1'(c_1^*) = h_2'(c_2^*)$, where c_j^* denotes the optimal level of investment in disease j, j=1,2.

Result 1 is, of course, not surprising. It merely asserts that neutrality towards inequality implies that investments should be pursued up to the point where the patients have equal capacity to benefit from additional treatment. Any resulting inequalities in health play no role in the resource allocation decisions of a social planner who is inequality neutral.

3.1. Consequences of inequality neutrality

Because the decision rule for the inequality neutral social planner will be used as a benchmark for the other two models, we will briefly specify some of its properties. First we consider the effect of a deterioration in the worst disease, disease 1. We will consider two ways in which the first disease may deteriorate: an *additive* deterioration, so that $h_1(c_1)$ becomes $h_1(c_1) - r$ with $r > 0$, and a *multiplicative* deterioration, so that $h_1(c_1)$ becomes $\theta h_1(c_1)$ with $0 < \theta < 1$. We obtain.

Result 2. An additive deterioration in the worst disease (disease 1) does not affect the priorities of an inequality-neutral social planner ($\frac{dc_1^*}{dr} = 0$). A multiplicative deterioration in disease 1 reduces the optimal investment in the treatment of disease 1 ($\frac{dc_1^*}{d\theta} > 0$).

The intuition behind result 2 is as follows. An additive deterioration does not affect the marginal efficiency of treating disease 1, additional investments still lead to the same increase in health as before, and, hence, the allocation of health care resources between the two diseases is not affected by the additive deterioration of the worst disease. When there is a multiplicative deterioration in disease 1, however, the return to additional investments in disease 1 will be reduced, because the marginal benefits of treatment 1 are also reduced by the factor θ. Consequently, the social planner will allocate fewer resources to the worst disease. Because the social planner is inequality neutral she only considers the effect of deteriorations in health on the marginal return of the two treatments and this leads her to invest less in disease 1. Consequently, a multiplicative deterioration in health leads to more inequality in health.

Consider next the impact of an increase in the health care budget.

Result 3. If the social planner is inequality-neutral and the health care budget is increased then both c_1^* and c_2^* increase.

Result 3 shows that both treatments are normal goods: when the budget expands, each disease receives a positive share of the additional funds. This result also holds in the utility model and in the dual model and we will not state it separately there.

Let us finally consider what happens when α, the prevalence of disease 1 in the population increases. The following result shows that the effect on the optimal investment in treating disease 1 depends on the sign of $(c_2^* - c_1^*)$.

Result 4. If the social planner is inequality-neutral and α, the prevalence of the worst disease, increases then

(i) c_1^* increases if $c_1^* < c_2^*$

(ii) c_1^* decreases if $c_1^* > c_2^*$

(iii) c_1^* remains constant if $c_1^* = c_2^*$.

The intuition behind Result 4 is as follows. Suppose that $c_1^* < c_2^*$, i.e., the social planner spends more on a patient with disease 2 than on a patient with disease 1. For an inequality neutral social planner the parameter α affects the allocation of resources only through a budget effect.[1] An increase in α induces money savings, because fewer people will need the expensive treatment for disease 2 and these savings will, in part, be redistributed to disease 1. In other words, what Result 4 says is that for an inequality neutral social planner changes in α affect her choices only through the impact on the budget and there is no consideration of the fact that changes in α may change the extent of health inequality.

4. The utility model

In the utility model, which we consider in this Section, the social planner expresses her inequality aversion by transforming the health outcomes through an increasing and concave utility function U(h). Her objective function then becomes:

$$\max_{c_1, c_2} R = \alpha U\big(h_1(c_1)\big) + (1-\alpha)\, U\big(h_2(c_2)\big) \tag{3}$$

under the same budget constraint as before (Eq. 1). The models considered by Wagstaff (1991) and Dolan (1998) are special cases of (3). Assuming (3) we obtain:

Result 5. Under the utility model, the optimal investment in the worst disease 1, $\hat{c}_1$, exceeds c_1^*, the optimal investment in this disease of an inequality-neutral social planner.

Of course, the counterpart of Result 5 is that the optimal investment in the less severe disease 2, $\hat{c}_2$, will be less than c_2^*. Result 5 shows that the utility model satisfies our first requirement that inequality aversion leads to more investment in the disease of the worse-off. The social planner's inequality aversion comes from the concavity of the utility function U joint with Jensen's inequality. Together these imply that the social planner has a preference for a more equal distribution of health.

Because of her inequality aversion the social planner reallocates funds to the benefit of the more severe disease. Besides, as we show in appendix A, when the social planner becomes more inequality averse this tendency is reinforced. These results are in line with common sense: the more inequality averse social planner is willing to accept a larger efficiency loss, as measured by the size of the difference between the marginal productivities of c_1 and c_2, in return for an increase in the equality of the distribution of health.

[1] This can be seen by inspecting the relevant formulas in Appendix A.

Let us now turn to the second requirement, Nord's (1993) conception that the priority given to a patient should increase with the severity of her disease. Result 6 shows that for an additive deterioration in health, the utility model satisfies this requirement.

Result 6. Under the utility model, an additive deterioration in the worst disease leads to an increase in the funds to treat this disease ($\frac{d\hat{c}_1}{dr} > 0$).

As disease 1 deteriorates relative to disease 2, the social planner reallocates funds towards the deteriorated disease in order to, at least partially, reduce the difference between the (final) health levels.

The analysis of a multiplicative deterioration is more complicated as Result 7 reveals.

Result 7. Under the utility model, a multiplicative deterioration in the worst disease leads to an increase in the funds to treat this disease if $1 + \theta h_1(\hat{c}_1)\frac{U''(\theta h_1(\hat{c}_1))}{U'(\theta h_1(\hat{c}_1))}$ is negative and to a decrease if $1 + \theta h_1(\hat{c}_1)\frac{U''(\theta h_1(\hat{c}_1))}{U'(\theta h_1(\hat{c}_1))}$ is positive.

Hence a multiplicative deterioration of disease 1 does not necessarily lead to an increase in $\hat{c}_1$. The intuition behind this result is simple: in the additive case the deterioration has no impact on the marginal productivity of the treatment so that only the concern for health inequality matters. Then the social planner's aversion to inequality leads to an increase in the investment in the worst disease. For a multiplicative shift there is not only an equity effect, an incentive to increase c_1 to reduce inequality, but also an efficiency effect. The reduction in θ implies a decrease in the marginal benefit of treating disease 1 and this gives an incentive to reduce c_1 for efficiency reasons.

As Result 7 shows, the effect of a multiplicative deterioration in health depends on $\theta h_1(\hat{c}_1)\frac{U''(\theta h_1(\hat{c}_1))}{U'(\theta h_1(\hat{c}_1))}$. Lambert (2001) refers to the expression $-x\frac{U''(x)}{U'(x)}$, with x any type of outcome, as an index of inequality aversion and, hence, we can interpret the term $-\theta h_1(\hat{c}_1)\frac{U''(\theta h_1(\hat{c}_1))}{U'(\theta h_1(\hat{c}_1))}$ as reflecting the degree of inequality aversion. Then Result 7 states that inequality aversion should be sufficiently high for a fall in θ to induce an increase in $\hat{c}_1$. The social planner will only invest more in treatment 1 when the degree of inequality aversion is strong enough to compensate for the reduced efficiency of treatment 1.[2]

While the results presented so far illustrate the relevance of the utility approach for policy choices, we now turn to its potential weakness: its relative insensitivity to changes in α. In the utility model c_1 is sensitive to α only through a budget effect just as we derived for an inequality-neutral social planner. The utility model does not take account of the fact that changes in α can change the attitude towards inequality, which an inequality averse social planner would like to compensate by increasing the optimal investment in treating disease 1.

Result 8. Under the utility model, the effect of a change in α, the prevalence of the worst disease, is the same as for an inequality-neutral social planner.

5. The dual model

As suggested by Bleichrodt et al. (2004), aversion to health inequality can alternatively be expressed by the introduction of an equity weighting function. In their model, the proportions of patients involved are transformed rather than the health outcomes. For that reason we will refer to the model of Bleichrodt et al. (2004) as the *dual approach*. A motivation for the dual model is that it seems more transparent to model equity weighting by explicitly introducing equity weights rather than by transforming the health outcomes. As mentioned before, a practical advantage of the model is that the estimation of the equity weights is simple, as Bleichrodt et al. (2004) explained.

In the dual model the ranking of the individuals is important as higher weights are assigned to individuals who are ranked lower in terms of health. Recall that we have assumed that h_1 is always less attractive than h_2 regardless of the level of investment in treating the two diseases. Then the dual model becomes

$$\max_{c_1,c_2} R = W(\alpha)h_1(c_1) + \big(1-W(\alpha)\big)h_2(c_2), \qquad (5)$$

where W can be interpreted as an equity weighting function. W is strictly increasing, the more

[2] In a different context, Sen (1973) already noted that utility must be sufficiently concave for a model like Eq. (3) to be consistent with his weak equity axiom.

people in the worst disease the more weight they get, and satisfies $W(0) = 0$ and $W(1) = 1$. The distinguishing property of W is that it is concave. This implies that $W(\alpha) > \alpha$ for all α in (0,1) and, thus, that the weight given to the people who are worse-off is higher than their proportion in the population. Note that for an inequality-neutral social planner the weight given to the worse-off is equal to their proportion in the population. Hence, the dual model gives more weight to the worse-off than an inequality-neutral planner does and this shows how the function W captures inequality aversion. Because $W(\alpha) > \alpha$ for all α and h_1 is always less than h_2, the value of R in (5) is always less than the value of $\alpha h_1(c_1) + (1-\alpha)h_2(c_2)$, the mean level of health in the population. Hence, the social planner is inequality-averse.

It can already be inferred from the above discussion that the dual model satisfies our first requirement that more is invested in treating the worse-off. The next result summarizes.

Result 9. Under the dual model, the optimal investment in the worst disease, $\check{c}_1$, exceeds c_1^*, the optimal investment in this disease of an inequality-neutral social planner.

Hence, whether it is expressed by a concave utility function as in the previous section or by a concave transformation of α, aversion to health inequality leads to more investment in the worst disease. We can also show that the investment in the worst disease increases with the degree of inequality aversion, which is captured by the degree of concavity of W. See Appendix A for a derivation of this result.

Let us next consider the effect of a deterioration in the worst disease. First we consider an additive deterioration. As it turns out, an additive deterioration in the worst disease does not affect the optimal level of investment in treating this disease.

Result 10. Under the dual model, an additive deterioration in the worst disease does not affect the priorities of the social planner ($\frac{d\check{c}_1}{dr} = 0$).

The intuition behind Result 10 is that an additive deterioration in the health of the worse-off does not change the rank ordering of the individuals. The individuals who were worse-off remain the worse-off after an additive deterioration in their health and, hence, their equity weight $W(\alpha)$ is not affected. The equity weight $W(\alpha)$ is only affected when either the rank of the individuals changes or when their proportion in the population changes. In other words, the dual model is only sensitive to changes in health to the extent that they change the rank ordering of the individuals. This is clearly a drawback of this model.

The next result shows that for a multiplicative deterioration the result is even less appealing: a multiplicative deterioration leads to less investment in treating the worse-off.

Result 11. Under the dual model, a multiplicative deterioration in the worst disease reduces the optimal investment in the worst disease ($\frac{d\check{c}_1}{d\theta} > 0$).

To understand Result 11, first note that, as in the case of an additive deterioration in the worst disease, the equity weight under the dual model is not affected because the ranking of the individuals in the population is not affected by the deterioration. Once this is understood, Result 11 should not be surprising in light of the discussion of Result 7. There we noted that the effect of a multiplicative deterioration in health depends on the value of the coefficient of inequality aversion. In the dual model the objective function is linear in terms of the health outcomes. Consequently, the index of inequality aversion must be equal to zero. We noted in Section 4 that a multiplicative deterioration would lead to a decrease in the investment in disease 1 if the coefficient of inequality aversion was less than 1. Since in the dual approach the value of the coefficient of inequality aversion is below 1, Result 11 becomes obvious.
The relative advantage of the dual model lies in fact in its ability to focus attention on the initial value of α and its importance for priority setting. We saw in Section 4 that the utility model treats changes in α as an income effect only, just as an inequality-neutral social planner would do. Result 12 shows that the dual theory is richer than the utility model when analyzing the effect of the prevalence of the worst disease.

Result 12. Under the dual model, changes in α, the prevalence of the worst disease, affect the optimal level of investment in treating the worst disease both through an income effect and through a substitution effect.

The income effect is under the dual model the same as the effect under inequality neutrality and under the utility model. However, in the dual theory the income effect is accompanied by a substitution effect, the sign of which is ambiguous. The precise expression of the substitution effect can be found in Appendix A.

6. Discussion

When a social planner is inequality averse, we often implicitly think that

(a) she should spend relatively more on the worst disease(s) and the more inequality averse she is, the more she should spend on the worst disease(s).

(b) she should spend more on the worst disease(s) when they deteriorate relative to more favourable diseases.

In addition, we have considered the requirement that the social planner should be sensitive to the prevalence of the worst disease. Quite interestingly neither of the two models used to express inequality aversion - the utility model and the dual model - satisfies these 3 conditions simultaneously. They both satisfy the first condition, but while the utility model satisfies condition (b) but cannot capture concerns about the prevalence of diseases, the dual model can capture such concerns but does not satisfy condition (b). Hence, the strengths and weaknesses of the utility model and the dual model are complementary: when one model performs well, the other performs worse and vice versa. Of course, the relative merits of the different models depend on how appealing the three requirements that we considered are.

If one aims for a model that satisfies all the above three requirements then a plausible candidate is a model that combines the utility model and the dual model. One example is the model used in Bleichrodt, Doctor, and Stolk (2005), which is similar to the dual model except that a utility function is defined over health. They did not explore the implications of this model but used it for empirical testing. They showed that this model can easily be measured and, hence, that it is tractable. Perhaps surprisingly, Bleichrodt et al. (2005) observed in a representative sample of the Dutch population that utility was close to linear, suggesting that equity weighting is more powerful than utility curvature in capturing people's aversion towards inequality.

A caveat that should be made in interpreting the results is that we have assumed that the effect of each treatment is known with certainty. Making the effect of treatment random should not be too difficult in the utility model. However, in the dual model matters would become much more difficult because the randomness of the outcomes can change the ranking of the diseases. For instance a patient suffering from disease 1 who is lucky with its treatment may end up better than another patient suffering from disease 2 who experiences adverse effects of treatment.[3]

Finally, we only assumed two diseases. The extension to an arbitrary number *n* of diseases seems not too difficult, but a warning from risk theory should be kept in mind: in risk theory properties that hold true in the binary case do not always automatically extend to a situation with more than two states of the world.

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Appendix A: Derivations.

Derivation of Result 1

The Lagrangean is equal to

$$\max_{c_1,c_2} L = \alpha h_1(c_1) + (1-\alpha)\, h_2(c_2) + \lambda(B - \alpha c_1 - (1-\alpha)c_2) \tag{A1}$$

and the first order conditions are

$$\frac{\partial L}{\partial c_1} = \alpha h_1'(c_1) - \lambda\alpha = 0 \tag{A2a}$$

[3]The same fact was observed by Doherty and Eeckhoudt, 1995) when they introduced background risk into the analysis of insurance decisions with Yaari's (1987) dual model of choice under risk.

$$\frac{\partial L}{\partial c_2} = (1-\alpha)h_2'(c_2) - \lambda(1-\alpha) = 0. \tag{A2b}$$

It is easily checked that the second order condition for a maximum holds. Rearranging (A2a) and (A2b) gives the equilibrium condition $h_1'(c_1^*) = h_2'(c_2^*)$.

Derivation of Result 2

Substituting $h_1(c_1) - r$ for $h_1(c_1)$ in (A1) does not change the first order conditions and, consequently, does not affect c_1^* and c_2^*. If we substitute $\theta h_1(c_1)$ for $h_1(c_1)$ in (A1) then (A2a) becomes

$$\frac{\partial L}{\partial c_1} = \alpha\theta h_1'(c_1) - \lambda\alpha = 0. \tag{A3a}$$

Because $\theta \in (0,1)$, the term $\alpha\theta h_1'(c_1^*)$ is smaller than $\alpha h_1'(c_1^*)$. To restore equality $h_1'(c_1^*)$ needs to increase, which, by the concavity of h_1, implies that c_1^* has to fall.

Derivation of Result 3

Because $c_2^* = \frac{B-\alpha c_1^*}{1-\alpha}$ by the budget constraint (1), we can write the equilibrium condition as

$$h_1'(c_1^*) - h_2'\left(\frac{B-\alpha c_1^*}{1-\alpha}\right) = 0 \tag{A4}$$

Totally differentiating (A4) with respect to c_1^* and B gives

$$\left(h_1''(c_1^*) + \frac{\alpha}{1-\alpha} h_2''\left(\frac{B-\alpha c_1^*}{1-\alpha}\right)\right)dc_1^* - \frac{1}{1-\alpha} h_2''\left(\frac{B-\alpha c_1^*}{1-\alpha}\right)dB = 0. \tag{A5}$$

Rearranging gives

$$\frac{dc_1^*}{dB} = \frac{h_2''}{(1-\alpha)h_1'' + \alpha h_2''}, \tag{A6}$$

where, for notational convenience, we have suppressed the arguments of h_1'' and h_2''. In a similar vein (now use $c_1^* = \dfrac{B - (1-\alpha)c_2^*}{\alpha}$) we can show that

$$\frac{dc_2^*}{dB} = \frac{h_1''}{(1-\alpha)\, h_1'' + \alpha h_2''}, \tag{A7}$$

The concavity of h_1'' and h_2'' implies that (A6) and (A7) are positive.

Derivation of Result 4

Totally differentiating (A4) with respect to c_1^* and α gives

$$\left(h_1''(c_1^*) + \frac{\alpha}{1-\alpha} h_2'' \left(\frac{B-\alpha c_1^*}{1-\alpha}\right)\right)dc_1^* + \left(\frac{B-\alpha c_1^*}{(1-\alpha)^2} - \frac{c_1^*}{1-\alpha}\right)h_2''\left(\frac{B-\alpha c_1^*}{1-\alpha}\right)d\alpha = 0. \tag{A8}$$

Rearranging and suppressing the arguments of of h_1'' and h_2'' gives

$$\frac{dc_1^*}{d\alpha} = \frac{(c_2^*-c_1^*)h_2''}{(1-\alpha)h_1'' + \alpha h_2''} = (c_2^*-c_1^*)\frac{dc_1^*}{dB}, \tag{A9}$$

By a similar line of argument we can show that

$$\frac{dc_2^*}{d\alpha} = \frac{(c_2^*-c_1^*)h_1''}{(1-\alpha)h_1'' + \alpha h_2''} = (c_2^*-c_1^*)\frac{dc_2^*}{dB}, \tag{A10}$$

The concavity of h_1'' and h_2'' implies that (A9) and (A10) are positive.

Derivation of Result 5.

The Lagrangean is equal to

$$\max_{c_1,c_2} L = \alpha U\big(h_1(c_1)\big) + (1-\alpha)U\big(h_2(c_2)\big) + \lambda(B - \alpha c_1 - (1-\alpha)c_2) \tag{A11}$$

and the first order conditions are

$$\frac{\partial L}{\partial c_1} = \alpha h_1'(c_1)U'\big(h_1(c_1)\big) - \lambda\alpha = 0 \tag{A12a}$$

$$\frac{\partial L}{\partial c_2} = (1-\alpha)h_2'(c_2)U'\big(h_2(c_2)\big) - \lambda(1-\alpha) = 0. \tag{A12b}$$

It is easily checked that the second order condition for a maximum holds. Rearranging (A12a) and (A12b) gives the equilibrium condition

$$h_1'(\hat{c}_1)U'\big(h_1(\hat{c}_1)\big) = h_2'(\hat{c}_2)U'\big(h_2(\hat{c}_2)\big). \tag{A13}$$

The concavity of U and h in combination with the fact that $h_1(c_1) < h_2(c_2)$ for every pair (c_1,c_2) implies that $\hat{c}_1 > c_1^*$ and, hence, that $\hat{c}_2 < c_2^*$.

Next we show that increases in the concavity of U imply more inequality aversion. Let V be a concave transformation of U, i.e. $V = k(U)$ with $k' > 0$ and $k'' < 0$. Then equilibrium condition (A13) becomes

$$h_1'(\hat{c}_{1,V})U'\big(h_1(\hat{c}_{1,V})\big)k'\big[U\big(h_1(\hat{c}_{1,V})\big)\big] = h_2'(\hat{c}_{2,V})U'\big(h_2(\hat{c}_{2,V})\big)k'\big[U\big(h_2(\hat{c}_{2,V})\big)\big], \tag{A14}$$

where $\hat{c}_{j,V}$, j = 1,2, denotes the optimum of c_j when V is used. Because $h_1(c_1) < h_2(c_2)$ for all c_1,c_2, and k is concave, $k'\big[U\big(h_1(c_1)\big)\big] > k'\big[U\big(h_2(c_2)\big)\big]$ for all c_1,c_2. It follows that

$$h_1'(\hat{c}_1)U'\big(h_1(\hat{c}_1)\big)k'\big[U\big(h_1(\hat{c}_1)\big)\big] > h_2'(\hat{c}_2)U'\big(h_2(\hat{c}_2)\big)k'\big[U\big(h_2(\hat{c}_2)\big)\big]. \tag{A15}$$

Hence, $\hat{c}_1$ and $\hat{c}_2$ cannot be optimal when V is used. To restore equality c_1 must increase (and consequently c_2 must decrease) by the concavity of h_j, j = 1,2, U, and k. Thus $\hat{c}_{1,V} > \hat{c}_1$ and $\hat{c}_{2,V} < \hat{c}_2$.

Derivation of Result 6

Substituting $h_1(c_1) - r$ for $h_1(c_1)$ gives as the new equilibrium condition

$$h_1'(\hat{c}_{1,a})U'\big(h_1(\hat{c}_{1,a}) - r\big) = h_2'(\hat{c}_{2,a})U'\big(h_2(\hat{c}_{2,a})\big), \tag{A16}$$

where $\hat{c}_{j,a}$, j = 1,2, denotes the optimum of c_j after the additive deterioration. Since U is concave, we obtain that

$$h_1'(\hat{c}_1)U'\big(h_1(\hat{c}_1) - r\big) > h_2'(\hat{c}_2)U'\big(h_2(\hat{c}_2)\big), \tag{A17}$$

so that c_1 must be increased to restore equality by the concavity of U and h_j, j=1,2.

Derivation of Result 7

Consider next the effect of a multiplicative deterioration in the worst disease. The new equilibrium condition is

$$\theta h_1'(\hat{c}_{1,m})U'\big(\theta h_1(\hat{c}_{1,m})\big) = h_2'(\hat{c}_{2,m})U'\big(h_2(\hat{c}_{2,m})\big), \tag{A18}$$

where $\hat{c}_{j,m}$, j = 1,2, denotes the optimum of c_j after the multiplicative deterioration. The effect of a multiplicative deterioration in the worst disease depends on whether $\theta U'\big(\theta h_1(\hat{c}_1)\big) \gtreqless U'\big(h_1(\hat{c}_1)\big)$, i.e. on the sign of $\frac{d}{d\theta}\theta U'\big(\theta h_1(\hat{c}_1)\big) = U'\big(\theta h_1(\hat{c}_1)\big) + \theta h_1(\hat{c}_1)U''\big(\theta h_1(\hat{c}_1)\big)$, which is positive (negative) if $1 + \theta h_1(\hat{c}_1)\frac{U''\big(\theta h_1(\hat{c}_1)\big)}{U'\big(\theta h_1(\hat{c}_1)\big)}$ is positive (negative). If $\theta U'\big(\theta h_1(\hat{c}_1)\big) > U'\big(h_1(\hat{c}_1)\big)$ then $\hat{c}_{1,m} > \hat{c}_1$ by the concavity of U. Hence, $1 + \theta h_1(\hat{c}_1)\frac{U''\big(\theta h_1(\hat{c}_1)\big)}{U'\big(\theta h_1(\hat{c}_1)\big)} < 0$ corresponds to an increase in c_1 (and consequently a decrease in c_2). Similarly $1 + \theta h_1(\hat{c}_1)\frac{U''\big(\theta h_1(\hat{c}_1)\big)}{U'\big(\theta h_1(\hat{c}_1)\big)} > 0$ corresponds to a decrease in c_1.

Derivation of Result 8

Because $c_2^* = \frac{B-\alpha c_1^*}{1-\alpha}$ by the budget constraint, we can write the equilibrium condition as

$$h_1'(\hat{c}_1)U'\big(h_1(\hat{c}_1)\big) = h_2'\left(\frac{B-\alpha\hat{c}_1}{1-\alpha}\right)U'\Big(h_2\Big(\frac{B-\alpha\hat{c}_1}{1-\alpha}\Big)\Big). \tag{A19}$$

Totally differentiating (A19) with respect to $\hat{c}_1$ and B gives after some rearranging

$$\frac{d\hat{c}_1}{dB} = \frac{R_2}{(1-\alpha)R_1 + \alpha R_2}, \tag{A20}$$

where $R_j = U''(\cdot)h_j'(\cdot)^2 + U'(\cdot)h_j''(\cdot)$, j = 1,2, which is negative by the concavity of U and h_j, j = 1,2. Hence, $\frac{d\hat{c}_1}{dB} > 0$ and c_1 is a normal good. Similarly we can show that c_2 is a normal good.

Totally differentiating (A19) with respect to $\hat{c}_1$ and α gives after some rearranging

$$\frac{d\hat{c}_1}{d\alpha} = (\hat{c}_2 - \hat{c}_1)\frac{R_2}{(1-\alpha)R_1 + \alpha R_2} = (\hat{c}_2 - \hat{c}_1)\frac{d\hat{c}_1}{dB}, \tag{A21}$$

an expression similar to (A9). Similarly we can show that $\frac{d\hat{c}_2}{d\alpha} = (\hat{c}_2 - \hat{c}_1)\frac{d\hat{c}_2}{dB}$.

Derivation of Result 9

The Lagrangean is equal to

$$\max_{c_1,c_2} L = W(\alpha)h_1(c_1) + \big(1-W(\alpha)\big)\, h_2(c_2) + \lambda(B - \alpha c_1 - (1-\alpha)c_2) \tag{A22}$$

and the first order conditions are

$$\frac{\partial L}{\partial c_1} = W(\alpha)h_1'(c_1) - \lambda\alpha = 0 \tag{A23a}$$

$$\frac{\partial L}{\partial c_2} = \big(1-W(\alpha)\big)h_2'(c_2) - \lambda(1-\alpha) = 0, \tag{A23b}$$

and hence the equilibrium condition becomes

$$\frac{W(\alpha)}{\alpha}\ h_1'(\check{c}_1) = \frac{1-W(\alpha)}{1-\alpha}\, h_2'(\check{c}_2). \tag{A24}$$

By the concavity of W, $\frac{1-W(\alpha)}{1-\alpha} < 1 < \frac{W(\alpha)}{\alpha}$. Hence, $\frac{W(\alpha)}{\alpha}\ h_1'(c_1^*) > \frac{1-W(\alpha)}{1-\alpha}\, h_2'(c_2^*)$ and, by the concavity of h_j, j = 1,2, $\check{c}_1 > c_1^*$.

We next show that the investment in the worst disease increases further when W becomes more concave. Let T = k[W] with k concave. Because T(0) = 0 and T(1) = 1, T(α) > W(α) for any α in (0,1). Then for any α in (0,1), $\frac{1-T(\alpha)}{1-\alpha} < \frac{1-W(\alpha)}{1-\alpha}$ and $\frac{W(\alpha)}{\alpha} < \frac{T(\alpha)}{\alpha}$.

Hence,

$$\frac{T(\alpha)}{\alpha}\ h_1'(\check{c}_1) > \frac{1-T(\alpha)}{1-\alpha}\, h_2'(\check{c}_2). \tag{A25}$$

To restore equality, $\check{c}_1$ has to increase and $\check{c}_2$ has to decrease by the concavity of h_1 and h_2.

Derivation of Result 10

Substituting $h_1(c_1) - r$ for $h_1(c_1)$ in (A22) does not change the first order conditions and, consequently, does not affect $\check{c}_1$ and $\check{c}_2$.

Derivation of Result 11

If we substitute $\theta h_1(c_1)$ for $h_1(c_1)$ in (A22) then the equilibrium condition becomes

$$\theta\frac{W(\alpha)}{\alpha}\ h_1'(\check{c}_{1,m}) = \frac{1-W(\alpha)}{1-\alpha}\, h_2'(\check{c}_{2,m}), \tag{A26}$$

where $\check{c}_{j,m}$, j = 1,2, is the optimal amount of investment in condition j after a multiplicative deterioration in the worst disease. Because $\theta \in (0,1)$, $\theta\frac{W(\alpha)}{\alpha}\ h_1'(\check{c}_1) < \frac{1-W(\alpha)}{1-\alpha}\, h_2'(\check{c}_2)$. To restore equality, $h_1'(\cdot)$ must increase and because h_1 is concave this can only be achieved by a decrease in c_1. Hence, $\check{c}_{1,m} < \check{c}_1$ and $\check{c}_{2,m} > \check{c}_2$.

This result was already obtained for an inequality neutral planner because the fall in θ makes treatment 1 less efficient at the margin. Intuition suggests that this effect should be mitigated when inequality aversion prevails. Unfortunately this does not happen when inequality aversion is expressed through the dual model. Quite to the contrary, because in the dual model θ is multiplied by a number $\frac{W(\alpha)}{\alpha}$ that is larger than unity, the effect of the fall in θ is amplified.

Derivation of Result 12

Because $\check{c}_2 = \frac{B-\alpha\check{c}_1}{1-\alpha}$ by the budget constraint, we can write the equilibrium condition (A24) as

$$\frac{W(\alpha)}{\alpha}\; h_1'(\check{c}_1) = \frac{1-W(\alpha)}{1-\alpha}\, h_2'\left(\frac{B-\alpha\check{c}_1}{1-\alpha}\right). \tag{A27}$$

Totally differentiating (A27) with respect to $\check{c}_1$ and B gives after some rearranging

$$\frac{d\check{c}_1}{dB} = \frac{\alpha(1-W(\alpha))h_2''}{(1-\alpha)^2 W(\alpha)\, h_1'' + \alpha^2(1-W(\alpha))h_2''}, \tag{A28}$$

which is positive by the concavity of h_j, j = 1,2. Hence, c_1 is a normal good. Similarly we can show that c_2 is a normal good.

Totally differentiating (A27) with respect to $\check{c}_1$ and α gives after some rearranging

$$\frac{d\check{c}_1}{d\alpha} = \frac{-(1-\alpha)\left[(1-\alpha)\left(\frac{W(\alpha)}{\alpha} - W'(\alpha)\right)h_1' + \alpha\left(\frac{1-W(\alpha)}{1-\alpha} - W'(\alpha)\right) h_2'\right]}{(1-\alpha)^2 W(\alpha)h_1'' + \alpha^2(1-W(\alpha))h_2''} + (\check{c}_2 - \check{c}_1)\,\frac{d\check{c}_1}{dB}. \tag{A29}$$

The second term is the same as in the utility model and the case of the inequality neutral social planner. The first term, the substitution effect, shows that the dual model is richer than the utility model for analyzing changes in the prevalence of the worst disease. By concavity of W, $\frac{W(\alpha)}{\alpha} - W'(\alpha)$ is positive and $\frac{1-W(\alpha)}{1-\alpha} - W'(\alpha)$ is negative. Hence, the substitution effect is sign-ambiguous.

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