Spare Parts Inventory Control based on Maintenance Planning

Sha Zhu[a,*], Willem van Jaarsveld[b], Rommert Dekker[c]

[a]*Research Centre for Operations Management, Katholieke Universiteit Leuven*
Naamsestraat 69 - box 3555, 3000 Leuven, Belgium
[b]*Industrial Engineering & Innovation Sciences, Technische Universiteit Eindhoven, P.O. Box 513, 5600 MB Eindhoven, The Netherlands*
[c]*Erasmus Research Institute of Management, P.O. Box 1738, 3000 DR, Rotterdam, The Netherlands*

EI2019-02

Abstract

For many maintenance organizations, on-condition maintenance tasks are the most important source of spare part demand. An uneven distribution of maintenance tasks over time is an important cause for intermittency in spare parts demand, and this intermittency complicates spare parts inventory control severely. In an attempt to partially overcome these complications, we propose to use the maintenance plan, i.e. the planned maintenance tasks, as a source of advance demand information. We propose a simple forecasting mechanism to estimate the spare part demand distribution based on the maintenance plan, and develop a dynamic inventory control method based on these forecasts. The value of this approach is benchmarked against state-of-the art time series forecast methods, using data from two large maintenance organizations. We find that the proposed method can yield cost savings of 23 to 51% compared to the traditional methods.

Keywords: Spare parts forecasting; spare parts inventory control; maintenance planning;

1. Introduction

Spare parts demand forecasting is essential to controlling spare parts inventories and avoiding high spare part shortage and holding costs. Time series methods estimate demand based on history (see e.g. Syntetos and Boylan (2005)), and as such they may work well when the historical situation is comparable with the future. They respond reactively to unprecedented factors and cannot predict

*Corresponding author.
Email addresses: sha.zhu@kuleuven.be (Sha Zhu), W.L.v.Jaarsveld@tue.nl (Willem van Jaarsveld), rdekker@ese.eur.nl (Rommert Dekker)

the timing of sudden peaks in demand. This is especially problematic for spare parts demand because of its intermittency and lumpiness (Petropoulos and Kourentzes (2015)).

Advance demand information (ADI) is information on demand, either perfect or imperfect, that is available ahead of the actual demand occurrence (cf. Tan et al., 2007). This concept has been widely used in various industrial settings outside of the spare parts context, e.g. the demand forecasting of e-commerce (Ozer (2003)), customized products (Johnson and Whang (2002), Gallego and Ozer (2001)), construction industry (van Donselaar et al. (2001)). To overcome the limitations of time series methods in dealing with spare part demand intermittency and lumpiness, in this paper we propose to use planned maintenance tasks as a source of ADI for spare parts inventory control.

We focus on maintenance tasks that prescribe to inspect a part of the asset, and depending on the condition of the part, it is then either immediately replaced by a spare part, or it may remain in the asset. Such *on-condition* maintenance tasks are a cost-effective tool for ensuring that parts continue to meet their functional and safety requirements, and they therefore constitute an important part of modern maintenance policies for aircraft, trains, and other capital assets.

The resources that enable maintenance, e.g. mechanics and a maintenance hangar, need to be planned ahead of the actual maintenance. To enable this maintenance logistics planning, companies specify which on-condition maintenance tasks will be performed some time periods into the future (see Section 3 for a detailed discussion). It is this maintenance logistical plan that we propose to use as a source of ADI in this paper. We need to overcome two complications when using this form of ADI to control spare parts inventories. First, on-condition maintenance tasks are a form of *imperfect* demand information: Only upon inspection does it become clear whether an on-condition maintenance task constitutes a spare part demand. Second, while the need for logistics planning forces companies to plan on-condition maintenance tasks ahead of time, this plan is only available and reliable a few months into the future.

We contribute to literature by proposing a new approach for joint forecasting and inventory control based on the maintenance plan. Moreover, we assess the value of this approach using company data. Our hybrid forecasting method combines information on historical part replacement rates with the maintenance plan to arrive at a forecasted demand distribution a few months into the future. This hybrid approach allows us to capture sudden changes in spare parts demand caused by on-condition maintenance tasks. For months outside the planning horizon the method reverts to a state-of-the-art time-series based forecast. As in different months there are different numbers of planned maintenance tasks, the demand forecast is not the same for all months. To deal with this nonstationarity in the forecast, we also contribute a dynamic, forward looking inventory policy.

An important advantage of our approach is that ADI in our approach can be obtained without incurring substantial additional operational costs. In contrast, some other approaches improve forecast accuracy at the expense of increasing the difficulty, the effort and the operational cost to predict demand, e.g. investing in a condition monitoring system (Topan et al., 2018) (see also Driessen et al., 2010). As we are only indirectly concerned with component degradation, we do not need any data or information on the degradation process. In particular, while our method needs more data than typical time-series methods, we need considerably less data and information than approaches based on the degradation process (e.g. Deshpande et al., 2006; Wang and Syntetos, 2011) or on system monitoring systems Topan et al. (2018); Lin et al. (2017); Poppe et al. (2017). Arguably, this makes our approach more suitable than these latter reliability based approaches for application in practical settings for managing thousands of spare parts. We use company data to assess the potential value of implementing our approach for inventory control of thousands of parts in two companies, and compare it to a state-of-the-art time series forecasting approach (viz. Syntetos and Boylan, 2001). We find that optimizing inventory using the maintenance plan yields a very substantial cost reduction of 23-51%, compared to the benchmarks.

The remainder of this paper is organized as follows. The next section gives an overview of the relevant literature. In Section 3 we discuss the availability of planned on-condition maintenance tasks in practice. In Section 4 we discuss our approach. Section 5 gives the setup and results of our experiments using two sets of real company data. The final conclusions are presented in Section 6.

2. Literature Review

Spare parts demand can either be forecasted based on historical data, advance demand information or a combination of both (Driessen et al. (2010)). Since our ADI-based method represents an alternative to time-series forecasting methods for intermittent spare parts demand, we first briefly review literature those latter methods (for a more detailed review see van Wingerden et al., 2014). Time series forecasting methods for intermittent demand include parametric and nonparametric methods. Croston's method (1972) is an early example of a parametric method. Recent works analyze and improve this method (Teunter et al. (2011); Syntetos (2001); Syntetos and Boylan (2005); Syntetos and Boylan (2001); Shale et al. (2006)) and propose alternative parametric methods (Willemain et al. (1994); Ghobbar and Friend (2003); Eaves and Kingsman (2004); Romeijnders et al. (2012)). For insightful discussions regarding parametric methods refer to Boylan and Syntetos (2010) and Prak and Teunter (2019). *Non-parametric* methods construct empirical distributions of demand, see e.g. Willemain et al. (2004), Porras and Dekker (2008), van Wingerden et al. (2014), and Zhu et al. (2017).

We next review contributions that apply ADI in demand forecasting and inventory control. ADI may take different forms (e.g. service contract, sensor data from machines, part age, etc) and topics in literature include how to derive demand from different forms of ADI and how to respond to ADI. ADI literature can be divided into two streams: *perfect ADI* refers to situations where the quantity and timing of demand is known in advance, and *imperfect ADI* refers to cases where some information regarding demand is known, but not the exact quantity and timing.

Hariharan and Zipkin (1995) study where customers place their orders in advance (i.e. a *demand leadtime*), and show that this form of perfect ADI is mathematically equivalent to a reduction in the supply leadtime. Gallego and Ozer (2001) also study perfect ADI and show that state-dependent (s, S) policies are optimal in the periodic review model for positive set-up cost. The study gives a lower bound to its extension to a distribution system, which is studied in Ozer (2003). Gallego and Ozer (2003) consider perfect ADI in a multi-echelon system. They find that the value of ADI on each echelon is influenced by the lead time of that echelon, and prove the optimality of state-dependent, echelon base-stock policies. The key finding for perfect ADI is that the inventory position should include outstanding demand.

We now discuss papers with imperfect ADI. Tan et al. (2007) study an inventory problem with ADI that might either be realized as demand, wait in the system one more period or leave the system without demand realization with given probabilities. They show that the optimal policy is of order-up-to type, with the order-up-to level depending on the ADI information. Tan et al.

(2009) consider imperfect ADI in an ordering and rationing problem with two demand classes. ADI is used to make a better rejection decision to lower class demand.

In addition to the above papers utilizing ADI to obtain (optimal) inventory policies, Abuizam and Thomopoulos (2005) and Tan (2008) apply ADI in demand forecasting. Abuizam and Thomopoulos (2005) propose a Bayesian method to update the expected amount of orders. However, Bayesian updates might fail in some problems as they rely on the distribution assumption, give one sided updates and fail to consider customer patterns (cf. Tan, 2008). Therefore, Tan (2008) combines expert judgmental prediction and demand estimation from ADI. ADI are subject to change in time and orders are partially materialized. Historic record is used to model the order changing behavior.

Using the imperfect ADI concept for improving spare parts inventory control could potentially aid practitioners to overcome the difficulties posed by spare part demand intermittency and lumpiness, and a few researchers have developed approaches in this general direction. Deshpande et al. (2006) track the age of parts in aircraft and use this information as ADI to improve spare parts inventory control. Romeijnders et al. (2012) develop a two-step method that makes use of the component repairs in spare parts demand forecasting. They focus on comparing various time series forecasting methods without ADI, but mention that ADI in this setting can considerably increase forecast accuracy. Pince et al. (2015) consider a manufacturer with contractual obligations to provide parts to its customers, and study the drop in demand rate resulting from contract expiration. Their proposed policy reduces the base stock level ahead of actual contract expiry. Basten and Ryan (2015) consider a single stocking point that satisfies demands resulting from corrective and preventive maintenance, and assume that perfect ADI is available for preventive maintenance. They propose heuristics for order and inventory allocation decisions, and find that the joint inventory requirement will be reduced due to the effect of risk pooling.

Wang and Syntetos (2011), Topan et al. (2018), Poppe et al. (2017) and Lin et al. (2017) pioneer new approaches towards spare parts inventory control driven by planned/foreseen maintenance, and as such they are arguably most closely related to our present work. Wang and Syntetos (2011) consider a block-based inspection policy. They model component degradation using a delay model and assume that distributions for both the initial and the delay time are known. They focus solely on forecasting, and develop a forecasting model by computing the conditional probabilities of parts needing replacement during inspection and between the inspection intervals. They find that this model can reduce the forecast error substantially. Topan et al. (2018) consider an asset monitored by a real-time condition monitoring system that generates imperfect warnings that may indicate that a part is failing. They develop effective spare parts inventory control policies in this situation. Poppe et al. (2017) consider an asset monitored by a real-time condition monitoring system, and investigate the impact of adopting a condition based maintenance policy on inventory control, where they use corrective maintenance and periodic maintenance as benchmarks. In contrast, Lin et al. (2017) investigate the value of condition monitoring systems for spare parts inventory control without changing the maintenance policy, and find that it may be substantial.

All these approaches rely heavily on component degradation information, in the form of real-time condition monitoring and/or complete distributional information on the degradation process. In contrast, our approach needs no such information. Instead, we propose to directly estimate the probability that a part needs replacement in an on-condition maintenance task from data. This greatly simplifies applying our approach in practice. In addition, we are the first to use real data to test the value of ADI approaches for spare parts inventory control. Note that our tests involve assessing the value of the model for inventory control, and as such they contribute to an understanding of the potential value of our approach in practice.

3. On-condition maintenance tasks as advance demand information

We first discuss the on-condition maintenance concept versus other maintenance concepts, and explain how planned on-condition maintenance tasks constitute ADI. We then discuss in more detail how information regarding planned on-condition maintenance tasks arises in practice. The discussion is based on the experience of the authors working closely with two maintenance organizations (cf. Section 5). The second author has been working with these two companies for over 10 years. The first author conducted site visits to these and other maintenance organizations to verify the validity of the idea using maintenance plan for spare parts demand forecasting. The third author has worked with many industrial partners and accumulated deep insights to the maintenance industry. Finally, we note that Driessen et al. (2010) bring up a similar discussion based on in-depth interviews with a wide range of maintenance organizations. (Unlike us, they do not develop forecasting methods.)

The simplest form of maintenance is perhaps break-down maintenance, i.e. run-to-failure. In contrast, preventive maintenance encompasses a wide range of maintenance strategies aimed at preventing failures, and we discuss such strategies next. In *time-based* maintenance a part or component is replaced periodically, e.g. after a fixed amount of time (e.g. every 6 months) or usage (e.g. every 20000 landings of an aircraft). Time-based maintenance can be planned ahead easily, and no condition information is needed to apply it, but it has the disadvantage that the useful life of the replaced parts may be poorly used. Therefore, when it is economically feasible to do so, companies *inspect* parts of the asset before deciding upon replacement; the part is then only replaced if degradation is above some threshold, hence the term *on-condition* maintenance task. This approach is typically motivated using a delay model for part degradation (Wang, 2011, 2012). Arguably, on-condition maintenance tasks are an example of *condition-based maintenance*, but most scholars reserve this latter term for situations where the condition is *real-time* monitored. In practice, the prevalence of real-time condition monitoring systems is still low because of their high associated cost (Topan et al., 2018).

In this paper, we propose to use *planned* on-condition maintenance tasks as spare parts ADI. This generic idea is broadly applicable across a wide range of maintenance organizations. The key and only requirements of the proposed approach are that on-condition maintenance tasks are known beforehand (e.g. 1 month into the future), and that information on past on-condition maintenance tasks and resulting spare parts usage is stored. In typical high-tech asset maintenance settings, the first requirement is satisfied in the sense that a broad range of on-condition maintenance tasks can be accurately predicted beforehand, and data collection is often compulsory because of traceability requirements. Indeed, asset maintenance must be planned ahead of time to organize availability of the asset, qualified mechanics, tools, maintenance hangar, etc. The scope of asset maintenance is typically also known beforehand. Therefore, for those assets which have a big ratio of planned maintenance, using information on the on-condition maintenance task is very beneficial from cost perspective. Moreover, increasing adoption of maintenance management software has made data on the maintenance plan available in formats useful for automatic decision making, which has increased the potential of and need for the approach we propose. In the following, we explain in more detail the applicability of the approach for modularly designed assets.

Modularly designed assets contain many line-replaceable units (LRUs) that can be removed during asset maintenance. Examples include aircraft, trains, trams, and many other high-tech machines. LRUs are typically removed periodically in order to inspect them. In particular, each LRU has an associated inspection interval and *inspection scope*. This scope consists in the on-condition maintenance tasks on *parts* of the LRU that together constitute the inspection. Both inspection intervals and degradation limits for on-condition tasks are typically prescribed by the manufacturer of the LRU, which bases its prescriptions on quantitative analysis in so-called reliability-centered/risk-based maintenance studies (see e.g. Moubray, 1997; Khan and Haddara, 2003). The inspection of LRUs after removal from the asset is typically carried out in specialized repair shops. Therefore, parts of the LRU that are replaced depending on their condition are typically referred to as shop-replaceable units (SRUs). **Example:** *the manufacturer may specify that the rear servomotor (=LRU) of a certain type of aircraft must be removed for inspection every 8000 flight hours. Moreover, the manufacturer specifies that if, during inspection, it is found that the coil (=SRU) of the servomotor shows any signs of corrosion, it must be replaced.*

Asset maintenance is *clustered* in order to efficiently satisfy the component safety requirements prescribed by manufacturers. E.g. in aviation it is common practice to define maintenance with several depth levels, e.g. A, B, C and D level maintenance. In A-level maintenance the scope is small, while D-level maintenance encompasses the removal and inspection of a wide range of LRUs. The various levels of asset maintenance and their frequency are designed such that inspection intervals of individual LRUs are guaranteed. As a consequence of this careful design of the various checks, the *work scope* of such checks is typically specified beforehand, i.e. it is known which LRUs will be removed for inspection in which check. Moreover, maintenance organizations make detailed plannings of the maintenance of their fleet, in order to align availability of bottleneck resources such as maintenance hangars, mechanics and tooling, and moreover to ensure that the operational capabilities of the fleet remain at a sufficiently high level. **Example (continued):** *The fleet maintenance plan of an operator specifies that in the upcoming four weeks, each week two aircrafts of a specific type will undergo C-level maintenance, which includes removal and inspection of the* *rear servomotor installed in those aircraft.*

The main idea of this paper is then to use the maintenance plan, and in particular the on-condition maintenance tasks that can be derived from the plan, as input to forecast spare parts demand. Maintenance plans may be made years in advance, but the plan is not reliable far into the future. This may for example be a consequence of cumulative forecasted usage (e.g. flight hours/ KMs) deviating from actual cumulative usage, or changes in the plan, etc. However, on the short term (e.g. a few months into the future instead of years) cumulative usage can be more accurately forecasted, making such deviations rare. Another reason for the plan to be more reliable on the short term is that deviations on the short term cause operational disruptions as well as unavailability and/or idle time of bottleneck resources. We propose to base ADI on this reliable time horizon, and we develop a method that reverts to time-series forecasting for periods beyond this horizon. **Example (continued):** *The coil in the rear servomotor has historically been replaced in one out of three repairs. Based on two planned removals of the rear servomotor per week, an expected demand of* $2/3 = 0.67$ *coils per week is forecasted for the next four weeks.*

We emphasize that while maintenance plans and tasks are nowadays increasingly available in maintenance management systems, such systems are not ubiquitous, and even if an investment in such systems is made, it is not necessarily trivial to extract information from such systems in a format usable for spare parts decision making. Apart from developing methods to use on-condition maintenance plans for spare parts inventory control, one of the goals of this paper is testing their potential value in practice using data from companies in aerospace and train maintenance that were able to extract the necessary data from their system. We believe this assessment may help in driving business cases for the proposed approach.

4. Methods

Our approach is applicable in general for maintenance organizations that perform on-condition maintenance tasks. For concreteness, in what follows we adopt terminology of a *repair shop*. We focus on one specific LRU/component that is regularly inspected by the repair shop, which is an establishment specialized in repairs of line-replaceable units (cf. Section 3). Inspection consists of determining the condition of parts of the component: if a part is degraded beyond some acceptable level, then it must be removed and discarded, and replaced by a spare part. So for the repair shop, each component repair corresponds to a number of on-condition maintenance tasks: one for each part in the component.

To complete repairs quickly, the repair shop keeps a local inventory of spare parts, and our focus is on inventory control of one specific spare part that may be used in the component. In case the part is needed but out of stock, an emergency order is placed, and after the emergency leadtime the repair continues. Emergency orders are a common way to avoid very long and costly delays of maintenance, and the emergency order leadtime is understood to be much shorter than the regular leadtime. Note that placing an emergency order is (mathematically) equivalent to a lost sale, and we take this latter perspective. We consider the penalty costs for the lost sale to be c_e, where c_e includes emergency order cost and costs of delaying the repair. Note that delaying a component repair may be costly as it requires the mechanic to store the inspected component, and to later retrieve it, which is time-consuming. As is customary in industry (cf. Romeijnders et al., 2012), inventory is reviewed periodically, resulting in a periodic-review, single-item, lost-sales inventory system. We denote periods by $t \in \{1, \ldots, T\}$. Here, T is the last period before the end of the horizon.

We consider a constant lead time L for a regular order. Parts ordered at the start of period t arrive at the start of period $t + L$. We let $L = 1$, which corresponds to a situation where parts ordered one period are available in the next period. This is reasonable because repair shop inventory is replenished from a central warehouse every period. Moreover, since many SRUs are relatively inexpensive, it is affordable to avoid stock-outs in this central warehouse. More importantly, focusing on this assumption avoids very technical inventory models and allows us to focus on the exposition of the key ideas regarding the integration of ADI and inventory control. For the same reasons, we assume no economies of scale in ordering. Inventory has holding cost h per part per time unit, leftover inventory at the end of period T is scrapped with cost s per part.

As described in Section 3, we focus on cases where the repair shop knows the number of on-condition maintenance tasks (component repairs) that will be carried out some periods in advance. In particular, at the start of period t, the repair shop knows the number of on-condition maintenance tasks for periods $t, t+1, \ldots, t+T_m$, where T_m corresponds to the number of periods that tasks are known in advance. Note that the spare part demand for period t is only revealed during period t: only upon inspection does it become clear whether a part needs replacement. Also, we assume that the repair shop keeps track of past on-condition maintenance tasks and the resulting spare parts demand. (This is often required for quality assurance reasons anyhow.)

4.1. Forecasting

The goal of this section is to arrive at a demand forecast for upcoming periods that can serve as a basis for inventory control. Note that for this latter purpose, a demand *distribution* forecast rather than a point estimate is needed. Let d_t denote the actual spare part demand in period t, and let A_t denote the number of maintenance tasks in period t. At the start of period t, we know the values d_{t+i} and A_{t+i} for $i < 0$, because those periods are in the past. We also know A_{t+i} for $0 \leq i \leq T_m$: this is the ADI.

Conceptually, an on-condition maintenance task results in a spare part demand with some (failure) probability p. In practice, such a probability needs not not be stationary; it may be subject to change as the components age, and as their usage pattern changes, etc. Moreover, the precise value of this probability is unknown. We therefore suggest to estimate the value of this unknown probability from data, by updating the forecasted failure probability $\widehat{p}_t$ in every period t as follows:

$$\widehat{p}_t = \begin{cases} (1-\alpha)\widehat{p}_{t-1} + \alpha \dfrac{d_{t-1}}{A_{t-1}} & \text{if } A_{t-1} > 0 \qquad (1) \\ \widehat{p}_{t-1} & \text{if } A_{t-1} = 0 \qquad (2) \end{cases}$$

Here, α is a smoothing factor. $\widehat{p}$ could be initiated as 0, or using the first few months of the demand history.

To forecast demand more than T_m periods in advance (that is, d_{t+i} where $i > T_m$) in our proposed method, we revert to standard time-series methods to forecast the average demand per period, which will be denoted by λ_t. We opt for the well-studied Syntetos-Boylan approximation (SBA) (Syntetos and Boylan, 2001), which is an improvement to Croston's method (1972):

$$\widehat{\lambda}_t = (1 - \frac{\alpha'}{2}) \frac{\widehat{S}_t}{\widehat{k}_t} \tag{3}$$

Here, α' is a smoothing parameter. The estimated demand size $\widehat{S}_t$ is updated by

$$\widehat{S}_t = \begin{cases} (1-\alpha')\widehat{S}_{t-1} + \alpha' d_{t-1} & \text{for } d_{t-1} > 0 \quad (4) \\ \widehat{S}_{t-1} & \text{for } d_{t-1} = 0 \quad (5) \end{cases}$$

and the estimated demand interval $\widehat{k}_t$ is updated by

$$\widehat{k}_t = \begin{cases} (1-\alpha')\widehat{k}_{t-1} + \alpha' k_{t-1} & \text{for } d_{t-1} > 0 \quad (6) \\ \widehat{k}_{t-1} & \text{for } d_{t-1} = 0 \quad (7) \end{cases}$$

At the start of period t, to arrive at a forecasted demand *distribution* $\widehat{D}_{t,t+i}$ for some upcoming period $t+i$, we distinguish between the cases $i > T_m$ and $i \leq T_m$. For $i \leq T_m$, the spare parts demand is binomially distributed with A_{t+i} trials and success probability p. Since p is unknown, we substitute the estimated value $\widehat{p}_t$ to obtain for $0 < i \leq T_m : \widehat{D}_{t,t+i} \sim B(A_{t+i}, \widehat{p}_t)$. For $i > T_m$, A_{t+i} is not available. Because the Poisson distribution is a good fit on spare part demand in general, for $i > T_m$ we forecast the demand distribution as $\widehat{D}_{t,t+i} \sim \text{poisson}(\widehat{\lambda}_t)$.

We note that there are substantial difference between using $B(A_{t+i}, \widehat{p}_t)$ for forecasting, versus using $\text{poisson}(\widehat{\lambda}_t)$. Most importantly, $B(A_{t+i}, \widehat{p}_t)$ reacts immediately to a large number of planned maintenance tasks, while $\text{poisson}(\widehat{\lambda}_t)$ only changes after the maintenance tasks have been executed. Secondly, using the binomial distribution $B(A_{t+i}, \widehat{p}_t)$ has the advantage that we explicitly know an upper bound on the number of replacements, which may help reduce the stock in certain situations.

4.2. Inventory optimization

We develop an approach for determining the amount x_t of spare parts to order in some arbitrary period t. Since demand is non-stationary, x_t should not myopically depend on the forecasted demand during leadtime alone, but it should be forward looking. This is easily seen based on an example: *Let $T_m = 1$ and consider two situations at time t: 1) $\widehat{p}_t = 0.2$, $\lambda_t = 0.01$, $A_t = 10$, $A_{t+1} = 10$; 2) $\widehat{p}_t = 0.2$, $\lambda_t = 2$, $A_t = 10$, $A_{t+1} = 10$. In both cases, demand on the short term is likely around 2 since $\hat{p}_t A_{t+1} = 2$, but in the first case, demand is expected to go down to 0.01 in subsequent periods, while in the second case, demand is expected to remain around 2. That means that any items remaining at the end of period $t+1$ will likely stay in stock longer in the first case than in* *the second case, which should be reflected in the order decision.*

To arrive at a forward-looking policy, in each period t we solve a stochastic dynamic program (SDP) over periods $t+i \in \{t, \dots, T\}$. This SDP uses the demand distributions over said periods, but the exact demand distribution is unknown. Instead, it is natural to use the forecasts constructed in period t: $\widehat{D}_{t,t+i}$. In the following, we briefly summarize the steps that occur in each period, and we subsequently give the SDP used to determine the order quantity x_t in each period t. For summarizing the steps, we introduce y_t and y'_t to denote the on hand inventory at the beginning and the end of period t, respectively. Here, y_t is understood to include the items that arrive in period t.

In each period, first the order placed in the previous period arrives. Thus $y_t = y'_{t-1} + x_{t-1}$. Then the order amount x_t is decided. Next, spare part demand D_t happens. Demand is satisfied by on hand inventory y_t. Thus at the end of period t we have on hand inventory $y'_t = (y_t - D_t)^+ = (y'_{t-1} + x_{t-1} - D_t)^+$. The holding cost $h \cdot y'_t$ and emergency ordering costs $c_e \cdot (D_t - y_t)^+$ are incurred. Subsequently, the next period starts.

To arrive at an SDP equation for deciding x_t, let $f_{t,t+i}(y_{t+i})$ denote the optimal total discounted cost from period $t+i$ until the end of the time horizon T, when the starting inventory in that period is y_{t+i}, and based on the forecasts obtained in period t. Then $f_{t,t+i}$ satisfies the recursive equation:

$$
\begin{aligned}
f_{t,t+i}(y_{t+i}) = \min_{x_{t+i} \in \{0,1,\dots\}} & \, h \sum_{d=0}^{\infty} P(\hat{D}_{t,t+i} = d)(y_{t+i} - d)^+ + c_e \sum_{d=0}^{\infty} P(\hat{D}_{t,t+i} = d)(d - y_{t+i})^+ \\
& + \sum_{d=0}^{\infty} P(\hat{D}_{t,t+i} = d) f_{t,t+i+1}(y_{t+i+1}) \qquad (8) \\
& = \min_{x_{t+i} \in \{0,1,\dots\}} g_{t,t+i}(x_{t+i}) \qquad (9)
\end{aligned}
$$

where $y_{t+i+1} = (y_{t+i} - d)^+ + x_t$, and with the boundary condition $f_{t,T+1} = -s \cdot y_{T+1}$ reflecting that inventory at the end of the final period T must be scrapped. Note that (9) implicitly defines $g_{t,t+i}(x_{t+i})$. We then obtain the amount to order in period t as $x_t \in \arg\min_x g_{t,t}(x)$.
Note that $f_{t,t+i}(y_{t+i})$ corresponds to *estimated* costs based on the forecast constructed at the start of period t. Hence, after updating the forecast, at the start of each period a new SDP is constructed to arrive at x_t.

5. Assessment of potential value of the method

In this section we use data from two maintenance companies in order to quantify the potential value of the ideas and methods developed in this paper. In Section 5.1 we discuss the data and the setup of the experiments. Section 5.2 gives the results.

5.1. Data and experimental setup

We first briefly describe the two companies and their data, and then discuss the experimental setup. The first company is Fokker Services (FS). FS provides comprehensive in-house component maintenance, repair and overhaul support to aircraft operators in dedicated repair shops. Components are typically delivered to FS according to the aircraft operator maintenance plan. FS subsequently determines the condition of parts during initial inspection. Failed parts generate demand for spare parts, which are delivered from a warehouse next to the repair shop. (The warehouse is replenished from a central warehouse in the Netherlands, but this replenishment is left out of the scope of this study.)

Repair data over a period of 134 months are available. Data are cleaned by removing some parts which are not applicable in our model, e.g. if the bill of material coefficients are larger than one for the part/if the part is used in a quantity larger than one in a single repair. Each type of component and each type of spare part has a unique serial number. In our analysis, we use the following information that is gathered about component repairs: Period in which the component arrived at the repair shop, component serial number, which spare parts (serial numbers) were used in the repair operation. As a component might generate demand of different spare parts types, we call the component and each corresponding type of spare part as one spare part-component pair. The data set includes 24,455 different spare part-component pairs.

We designate the first 84 months as the training set. Within this training set, we use the average of the first 48 months to arrive at an initial estimate for the model parameters. We then use the approach discussed in Section 4.1 to update the parameters (e.g. parts failure probability) for the remaining 36 months in the training set. The test set contains the last 50 months of data, i.e. months 85-134. In the test set, we keep updating the parameters, and we record performance statistics such as holding costs and emergency ordering costs.

Real SRU prices are not available. In the experiment, we consider the following 4 parameters as the experimental factors: (i) holding cost per item per time unit h, (ii) emergency shipping cost c_e, (iii) scrapping cost at the end of horizon s, (iv) maintenance plan lead time T_m.

For the *base case*, we use the following parameters. At 24% holding costs per year, a typical low part price of 5 euros amounts to $h = 0.1$ euros/month. Costs for scrapping parts at the end of the horizon are set at $s = 5$ euros, because the costs of scrapping are dominated by the lost investment. Regarding the emergency costs, we found in discussions at various repair shops that delaying repairs is inconvenient because it typically requires the mechanic to temporarily store the component, and to later retrieve it. Additionally, repair delays may harm customer satisfaction. As a consequence, we set the penalty costs for an emergency shipment as $c_e = 20$ euro. Finally, we set $T_m = 3$. We design our experiments around these base case parameters, and a sensitivity analysis is conducted to explore the effect of changes to the base case.

Since each component constitutes an on-condition maintenance task that may result in usage of the part, we can directly apply our methods for each spare part-component combination. Our method is used to determine the replenishment quantity in each period. Subsequently, we simulate the dynamics of the system using the real demand and maintenance data (cf. Section 4.2), and obtain holding, emergency shipping, and scrapping costs for all parts. To assess the value of ADI, we will use as a benchmark a method that does not use ADI. Like the method proposed in this paper, the benchmark uses the recursive approach (8-9) to set spare parts orders. However, the benchmark uses the time series forecast $\widehat{D}_{t,t+i} \sim \text{pois}(\widehat{\lambda}_t)$ for *all* future periods, including those with $0 < i \leq T_M$. Note that the value of A_t is not needed for the benchmark, and note that the Syntetos-Boylan approximation is used to determine λ_t. So the benchmark represents the state-of-the-art time-series method.

We also test our approach at another company: the Netherlands Railways (NS). NS is by far the largest operator of passenger railway transport in the Netherlands. The maintenance department of NS tracks the repair actions of main components of trains over 35 months, and we obtained that data. The history covers information over 138,347 repair actions on main components. At NS components may either be replaced as part of the maintenance plan, or upon unplanned failure. The former covers 2,727 types of components and 749 types of parts, and the latter covers 3,935 types of components and 1,485 types of spare parts. Ideas in this paper are applicable to the former case, and we only use that data. We designate the first 25 months of demand as training data, and the last 10 months as test data. Out of the training set the first 20 periods are used for initialization of forecast parameters, and the last 5 for updating those parameters. The other settings are the same as in the FS case.

5.2. Results

We compare the total cost of all spare parts of our proposed approach to the costs of the benchmark that only uses time-series forecasts. Table 1 shows the relative cost reduction of all the spare parts at Fokker Services, in the Total Costs (TC), Holding Costs (HC), Emergency Costs (EC), and Scrapping Costs (SC). Table 2 does the same for NS. Each row in Tables 1 and 2 represents a single setting of parameters: For each case the base values of parameters are $h = 0.1$, $p = 0$, $c_e = 20$, $s = 5$, $T_m = 3$.

As the FS case has a relatively long demand history, we can make a relatively accurate categorization of spare parts based on the number of months with positive demand during the training period (84 months) to explore the value of the maintenance plan on each category. The three categories are very-slow moving (1-5 months with positive demand), slow-moving (6-20 months), and fast moving (21-84 months). We have 24,455 types of part - component combinations in total. Very-slow moving includes 21,011 combinations, slow moving covers 2,846 combinations and relatively fast moving has 598 combinations. Table 3 shows the cost reduction in each category. We have the following observations.

- We observe that our approach reduces the total cost compared to the benchmark by 48% and 23% in average for Fokker Services and NS, respectively. This illustrates that the value of the maintenance plan is very high in inventory control. In eight out of ten instances in the FS case and in nine out of ten instances in the NS case, our approach outperforms the benchmark with regard to all three cost components. Cost reductions are mainly driven by reductions in emergency shipping cost, followed by the holding cost and scrapping cost. The emergency shipping cost contributes 89% to the total saving in the Fokker case and 68% in the NS case. Note that since many spare parts are very reliable, and since components have

Table 1: The effect of parameters on the value of maintenance planning (Fokker Service). All absolute cost figures are in thousands.

para-	Total Cost			Holding Cost			Emergency Shipping			Scrapping Cost		
meter	ADI	B[1]	Red.	ADI	B	Red.	ADI	B	Red.	ADI	B	Red.
h=**0.1**	212	431	51 %	47	73	36 %	90	270	67 %	76	88	14 %
h=0.5	338	576	41 %	138	152	9 %	151	388	61 %	49	36	-36 %
h=1	432	657	34 %	192	170	-13 %	204	466	56 %	36	20	-78 %
c_e=15	188	373	50 %	42	69	38 %	77	222	65 %	68	82	17 %
c_e=**20**	212	431	51 %	47	73	36 %	90	270	67 %	76	88	14 %
c_e=30	249	536	54 %	53	76	30 %	109	367	70 %	87	93	7 %
T_m=1	232	431	46 %	43	73	41 %	119	270	56 %	70	88	21 %
T_m=**3**	212	431	51 %	47	73	36 %	90	270	67 %	76	88	14 %
T_m=5	208	431	52 %	50	73	32 %	78	270	71 %	80	88	9 %
T_m=38	202	431	53 %	55	73	24 %	62	270	77 %	85	88	4 %

[1] Benchmark

Table 2: The effect of parameters on the value of maintenance planning (NS). All absolute cost figures are in thousands.

para-	Total Cost			Holding Cost			Emergency Shipping			Scrapping Cost		
meter	ADI	B	Red.	ADI	B	Red.	ADI	B	Red.	ADI	B	Red.
h=**0.1**	45	58	23 %	2	4	36 %	28	37	24 %	14	17	16 %
h=0.5	53	72	26 %	9	15	40 %	33	43	23 %	11	14	18 %
h=1	58	74	21 %	12	17	26 %	38	50	24 %	8	7	-11 %
c_e=15	38	48	22 %	2	4	38 %	23	29	22 %	12	15	19 %
c_e=**20**	45	58	23 %	2	4	36 %	28	37	24 %	14	17	16 %
c_e=30	59	76	23 %	3	4	31 %	39	54	28 %	17	18	8 %
T_m=1	47	58	20 %	3	4	32 %	29	37	24 %	15	17	10 %
T_m=**3**	45	58	23 %	2	4	36 %	28	37	24 %	14	17	16 %
T_m=5	45	58	23 %	2	4	37 %	29	37	23 %	14	17	19 %
T_m=10	45	58	24 %	2	4	42 %	30	37	21 %	13	17	25 %

Table 3: The effect of parameters on the value of maintenance planning-category (Fokker Service). All absolute cost figures are in thousands.

para-	VSM			SM			FM		
meter	ADI	B	Red.	ADI	B	Red.	ADI	B	Red.
h=**0.1**	146	273	46 %	48	109	56 %	18	48	63 %
h=0.5	208	309	33 %	93	181	49 %	38	86	56 %
h=1	240	300	20 %	135	237	43 %	57	120	52 %
c_e=15	127	236	46 %	44	95	54 %	17	42	60 %
c_e=**20**	146	273	46 %	48	109	56 %	18	48	63 %
c_e=30	175	343	49 %	54	135	60 %	19	58	66 %
T_m=1	160	273	41 %	53	109	52 %	19	48	60 %
T_m=**3**	146	273	46 %	48	109	56 %	18	48	63 %
T_m=5	143	273	48 %	47	109	57 %	18	48	63 %
T_m=38	139	273	49 %	46	109	58 %	17	48	64 %

a life cycle of 5-20 years, scrapping costs may be a substantial part of total costs because even with low stocks there is always a risk of leftovers (cf. van Jaarsveld and Dekker, 2011). This explains the substantial costs of scrapping for the cases. Finally, we note that the cost reduction for Fokker Services is higher than the cost reduction for NS. This is mainly due to the fact that 1) the maintenance plan is more stable for NS, which reduces the value of ADI, cf. Section 4.1; 2) The time horizon in FS is 28 time periods longer than that in NS, leading to more cost savings.

- The holding cost rate h has more effect on our method than the benchmark while the emergency shipping cost c_e has larger impact on the benchmark. When h is increased from 0.1 to 0.5, the cost of our approach is increased by 59% while 34% for the benchmark in Fokker case. For h from 0.5 to 1, it's 28% for our approach and 14% for the benchmark. The effect of h is monotonic in general. For c_m from 15 to 20 and to 30, the cost of our approach is increased by 13% and 17% respectively, while for the benchmark it's 16% and 24%. Therefore, we can conclude that our approach on average orders more than the benchmark as to have less penalty and holding cost. The method apparently orders at the right moment. When h/c_m is large enough, the value of the maintenance plan vanishes since the optimal policy under both methods does not place any order. When c_m/h is large, both of the methods have larger stocks. However, using the maintenance plan yields an upper bound for the spare parts demand while the benchmark does not have access to such an upper bound. In addition, the benchmark might place an order in the period when there is no component arrivals as it only uses the history demand in forecasting. Again, the ADI method places timely orders. This is verified by the observation of the increase in the holding cost reduction with the increase of c_m in both Table 1 and 2.

- The value of maintenance plan is not sensitive to T_m. In the Fokker case, obtaining the maintenance plan one lead time ahead achieves 46% in cost reduction, while obtaining it 5 lead times ahead achieves a 52% cost reduction. In the NS case, the cost reduction increases from 20% to 24% by increasing T_m from 1 to 10. The marginal benefit decreases dramatically with the increase of T_m. Therefore, we conclude that it is necessary to obtain the maintenance plan in advance as it brings substantial cost saving. However, obtaining the maintenance plan one lead time ahead contributes the most to the inventory cost. It is not very cost effective to invest more in order to obtain the maintenance plan much earlier.

- Table 3 shows that costs reductions are substantial over all categories, and relative cost reductions increase only slightly for faster moving categories. The biggest *absolute* contribution to the total cost savings is made by the very slow moving parts: 53% (averaged over the various cost parameter settings). This is because the very-slow moving category accounts for a large proportion of the parts. It is interesting that our approach performs well for the very-slow moving category, because that category is very hard to forecast using traditional techniques. Note furthermore that the very-slow moving category is less sensitive to the holding cost rate h than the other two categories in both the ADI method and the benchmark. This is because holding cost accounts for a smaller proportion of the total cost for very-slow moving items.

- The results are consistent with an intuitive interpretation. The proposed method can better take into account the time interval between positive demands. Therefore, it prevents the system from keeping redundant stocks. This leads to less inventory holding cost, as we observe in both cases. The proposed approach differentiates spare part demand forecasts in different time periods by building the dependence between spare part demand and its origin, component arrivals. By responding to the maintenance plan, our approach makes the spare part demand forecasting more accurate and the inventory decisions more appropriate. As a result, we have less penalty cost for emergency shipment and less scrapping for leftover stocks at the end of time horizon. In this way, our approach can better achieve the goal of having the right amount of stocks at the right moment. We expect the value of our approach in practice to be highest for the very slow moving items, because especially such items are notoriously difficult to control for human decision makers.

6. Conclusions and future research

Spare parts demand forecasting is essential to spare parts inventory control but difficult as the demand has the feature of irregularity and lumpiness. We proposed and tested ideas to apply ADI, in the form of planned on-condition maintenance tasks, to improve spare parts inventory control under these circumstances. Incorporating this form of ADI into forecasting makes the demand forecast nonhomogeneous over the forecast horizon. Accordingly, as argued in this paper, the inventory control must become forward looking, and we propose an inventory optimization approach to reflect this. We determined the potential value of the combined forecasting and inventory optimization approach using industry data, and found that potential savings are very substantial: 51% for the aerospace maintenance case, and 23% for the train maintenance case.

Some comments are needed to put these figures into perspective. First of all, while aircraft component maintenance typically results from checks planned by the operator, this information is currently only shared on an ad hoc basis, e.g. maintenance organizations inform the repair shop in advance when they expect a substantial number of removals of a specific component in a short time interval. This is mainly in their own interest: If the repair shop is prepared, then spare part shortages are rare. The present research solidifies these findings and underlines the economical value of such information sharing in the supply chain. Moreover, the study provides compelling evidence that investing in a more structured sharing of information, e.g. in the form of a data platform, can simultaneously reduce inventory and increase part availability. Train maintenance organizations likewise inform the repair shop typically on an ad hoc basis. The present study shows that it would be better to more structurally organize this, as structural sharing of information would allow for a substantial reduction in the mismatch between spare parts demand and supply. Finally, it is interesting to note the rather marked difference in cost reduction for Fokker Services and NS, though in both cases cost reductions are substantial. We believe that this is caused by a more uneven maintenance pattern in the aircraft industry compared to train maintenance.

Our approach focuses on the most common case: If parts are used in an on-condition maintenance task, they are used in quantity 1. While this holds for the vast majority of parts, one could generalize it to situations where multiple parts of the same type may be replaced in a single maintenance task (see e.g. van Jaarsveld et al., 2015). Another direction for future research is related to our forecast method: Suppose many on-condition maintenance tasks are upcoming, i.e. A_{t+i} is high. This will likely increase future demand, so future demand is likely to be higher than $\hat{\lambda}_t$. It would perhaps be interesting to adapt the forecasting method for $\hat{\lambda}_t$ such that it already responds to expected changes in demand. Finally, any experience on a broad implementation of the ideas pursued in this paper would likely teach us valuable lessons that cannot be learned from this preliminary study alone.

References

R. Abuizam and N.T. Thomopoulos. Adjusting an existing forecasting model when some future demands are known in advance; a bayesian technique. *working paper*, 2005.

R.J.I. Basten and J.K. Ryan. Inventory management with two demand streams: A maintenance application. *BETA working paper*, 2015.

J.E. Boylan and A.A. Syntetos. Spare parts management: a review of forecasting research and extensions. *Journal of Management Mathematics*, 21:227–237, 2010.

J.D. Croston. Forecasting and stock control for intermittent demands. *Operational Research Quarterly*, 23:289–303, 1972.

V. Deshpande, A.V. Iyer, and R. Cho. Efficient supply chain management at the u.s. coast guard using part-age dependent supply replenishment policies. *Operations Research*, 54(6):1028–1040, 2006.

M.A. Driessen, J.J. Arts, G.J.J.A.N. van Houtum, W.D. Rustenburg, and B. Huisman. Maintenance spare parts planning and control : a framework for control and agenda for future research. *BETA publicatie : working papers*, 325:1028–1040, 2010.

A.H.C. Eaves and B.G. Kingsman. Forecasting for the ordering and stock-holding of spare parts. *The Journal of the Operational Research Society*, 55(4):431–437, 2004.

G. Gallego and O. Ozer. Integrating replenishment decisions with advance demand information. *Management Science*, 47:1344–1360, 2001.

G. Gallego and O. Ozer. Optimal replenishment policies for multiechelon inventory problems under advance demand information. *Manufacturing Service Oper. Management*, 5(2):157–175, 2003.

A.A. Ghobbar and C.H. Friend. Evaluation of forecasting methods for intermittent parts demand in the field of aviation: a predictive model. *Computers & Operations Research*, 30:2097–2114, 2003.

R. Hariharan and P. Zipkin. Customer-order information, leadtimes, and inventories. *Management Science*, 41:1599–1607, 1995.

M.E. Johnson and S. Whang. E-business and supply chain management: an overview and framework. *Production and Operations Management*, 11:413–423, 2002.

F.I. Khan and M.M. Haddara. Risk-based maintenance (rbm): a quantitative approach for maintenance/inspection scheduling and planning. *Journal of loss prevention in the process industries*, 16(6):561–573, 2003.

X. Lin, Rob J.I. Basten, A.A. Kranenburg, and G.J.J.A.N. Van Houtum. Condition based spare parts supply. *Reliability Engineering & System Safety*, 168:240–248, 2017.

J. Moubray. *Reliability-centered maintenance*. Industrial Press Inc., 1997.

O. Ozer. Replenishment strategies for distribution systems under advance demand information. *Management Science*, 49(3):255–350, 2003.

F. Petropoulos and N. Kourentzes. Forecast combinations for intermittent demand. *Journal of the Operational Research Society*, 66:914–924, 2015.

C. Pince, J.B.G. Frenk, and R Dekker. The role of contract expirations in service parts management. *Production and Operations Management*, 24:1580–1597, 2015.

J. Poppe, R.J.I Basten, R.N Boute, and M.R. Lambrecht. Numerical study of inventory management under various maintenance policies. *Reliability Engineering & System Safety*, 168:262–273, 2017.

S. Porras and R. Dekker. An inventory control sytem for spare parts at a refinery: An empirical conparison of different re-order point methods. *European Journal of Operational Research*, 184: 101–132, 2008.

D. Prak and R. Teunter. A general method for addressing forecasting uncertainty in inventory models. *International Journal of Forecasting*, 35:224–238, 2019.

W. Romeijnders, R. Teunter, and W. van Jaarsveld. A two-step method for forecasting spare parts demand using information on component repairs. *European Journal of Operational Research*, 220 (2):386–393, 2012.

E.A. Shale, J.E. Boylan, and F.R Johnston. Forecasting for intermittent demand: The estimation of an unbiased average. *The Journal of the Operational Research Society*, 57(5):588–592, 2006.

A.A. Syntetos. *Forecasting of intermittent demand*. PhD thesis, Brunel University, 2001.

A.A. Syntetos and J.E. Boylan. On the bias of intermittent demand estimates. *International Journal of Production Economics*, 71:457–466, 2001.

A.A. Syntetos and J.E. Boylan. The accuracy of intermittent demand estimates. *International Journal of Forecasting*, 21:303–314, 2005.

T. Tan. Using imperfect advance demand information in forecasting. *Journal of Management Mathematics*, 19:163–173, 2008.

T. Tan, R. Gullu, and N. Erkip. Modelling imperfect advance demand information and analysis of optimal inventory policies. *European Journal of Operational Research*, 177:897–923, 2007.

T. Tan, R. Gullu, and N. Erkip. Using imperfect advance demand information in ordering and rationing decisions. *International Journal of Production Economics*, 121:665–677, 2009.

R. Teunter, A.A. Syntetos, and M.Z. Babai. Intermittent demand: Linking forecasting to inventory obsolescence. *European Journal of Operational Research*, 214:606–615, 2011.

E. Topan, T. Tan, G.J.J.A.N. van Houtum, and R. Dekker. Using imperfect advance demand information in lost-sales inventory systems with the option of returning inventory. *IISE Transactions*, 50(3):246–264, 2018.

K. van Donselaar, L.R. Kopczak, and M. Wouters. The use of advance demand information in a project-based supply chain. *European Journal of Operational Research*, 130:519–538, 2001.

W. van Jaarsveld and R. Dekker. Estimating obsolescence risk from demand data to enhance inventory control - a case study. *International Journal of Production Economics*, 133(1):423–431, 2011.

W. van Jaarsveld, T. Dollevoet, and R. Dekker. Improving spare parts inventory control at a repair shop. *Omega*, 57:217–229, 2015.

E. van Wingerden, R.J.I. Basten, R. Dekker, and W.D Rustenburg. More grip on inventory control through improved forecasting: A comparative study at three companies. *International Journal of Production Economics*, 157:220–237, 2014.

W. Wang. A joint spare part and maintenance inspection optimisation model using the delay-time concept. *Reliability Engineering & System Safety*, 96(11):1535–1541, 2011.

W. Wang. An overview of the recent advances in delay-time-based maintenance modelling. *Reliability Engineering and System Safety*, 106:165–178, 2012.

W. Wang and A.A. Syntetos. Spare parts demand: Linking forecasting to equipment maintenance. *Transportation Research Part E*, 47:1194–1209, 2011.

T.R. Willemain, C.N. Smart, J.H. Shockor, and P.A DeSautels. Forecasting intermittent demand in manufacturing: a comparative evaluation of croston's method. *International Journal of Forecasting*, 10:529–538, 1994.

T.R. Willemain, C.N. Smart, and H.F Schwarz. A new approach to forecasting intermittent demand for service parts inventories. *International Journal of Forecasting*, 20:375–387, 2004.

S. Zhu, R. Dekker, W. van Jaarsveld, R.W. Renjie, and A.J. Koning. An improved method for forecasting spare parts demand using extreme value theory. *European Journal of Operational Research*, 261(1):169–181, 2017.