The Impact of Jumps and Leverage in Forecasting the Co-Volatility of Oil and Gold Futures[*]

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Abstract

The paper investigates the impact of jumps in forecasting co-volatility in the presence of leverage effects. We modify the jump-robust covariance estimator of Koike (2016), such that the estimated matrix is positive definite. Using this approach, we can disentangle the estimates of the integrated co-volatility matrix and jump variations from the quadratic covariation matrix. Empirical results for daily crude oil and gold futures show that the co-jumps of the two futures have significant impacts on future co-volatility, but that the impact is negligible in forecasting weekly and monthly horizons.

1 Introduction

The severity and global nature of the recent financial crisis highlighted the risks associated with portfolios containing only conventional financial market assets (Muteba Mwamba et al., 2017). Such a realization triggered an interest in considering investment opportunities in the energy (specifically oil) market (Bahloul et al., 2018). In fact, the recent financialization of the commodity market (Tang and Xiong, 2012; Silvennoinen and Thorp, 2013) and, in particular, oil has resulted in an increased participation of hedge funds, pension funds, and insurance companies in the market, with investment in oil now being considered as a profitable alternative intrument in the portfolio decisions of financial institutions (Fattouh et al., 2013; Büyükşahin and Robe, 2014).

With gold traditionally considered as the most popular 'safe haven' (Baur and McDermott, 2010; Baur and Lucey, 2010; Reboredo, 2013a), recent studies have analyzed volatility spillovers across the gold and oil markets (Ewing and Malik, 2013; Mensi et al., 2013; Yaya et al., 2016),[1] where volatility spillovers are defined as the delayed effect of a returns shock in one asset on the subsequent volatility or co-volatility in another asset (Chang et al., 2018a).

In this regard, it must be realized that mode l ling and forecasting the co-volatility of gold and oil markets is of paramount importance to international investors and portfolio managers in devising optimal portfolio and dynamic hedging strategies (Chang et al., 2018b). By definition, (partial) co-volatility spillovers occur when the returns shock from financial asset k affects the co-volatility between two financial assets, i and j, one of which can be asset k (Chang et al., 2018a).

Against this backdrop, the objective of this paper is to forecast the daily co-volatility of gold and oil futures derived from 1-minute intraday data over the period September 27, 2009 to May 25, 2017.[2] In particular, realizing the importance of jumps, that is, discontinuities, in governing the volatility of asset prices (Andersen et al. (2007), Bollerslev et al. (2009), Corsi et al. (2010)), we investigate the impact of jumps by simultaneously accommodating leverage effects in forecasting the co-volatility of gold and oil markets, following the econometric approach of Asai and McAleer (2017) (applied to three stocks traded on the New York Stock Exchange (NYSE)).

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[1] For corresponding studies on comovements in gold and oil returns, see Reboredo (2013b), Bampinas and Panagiotidis (2015), Balcilar et al. (2018), and references cited therein. A literature review on return and volatility spillovers across asset classes can be found in Tiwari et al. (2018).

[2] Although the variability of daily gold and oil price returns have traditionally been forecasted based on Generalized Autoregressive Conditional Heteroskedasticity (GARCH)-type models of volatility, recent empirical evidence suggests that the rich information contained in intraday data can produce more accurate estimates and forecasts of daily volatility (see Degiannakis and Filis (2017) for a detailed discussion).

Although studies dealing with forecasting gold and oil market volatility has emphasized the role of jumps in forecasting realized volatility[3] (see, for example, Sévi (2014), Prokopczuk et al. (2015), and Demirer et al. (2019)), this paper would seen to be the first attempt to incorporate their role in predicting the future co-volatility path of these two important commodities.

The remainder of the paper is s follows. Section 2 lays out the theoretical details of the econometric framework, while Section 3 presents the data, empirical results, and analysis. Section 4 gives some concluding remarks.

2 Theoretical Framework

2.1 Model Specification

Let $p^*(s)$ denote a q-dimensional latent log-price vector at time s, and $W(s)$ and $Q(s)$ denote q-vectors of independent Brownian motions and counting processes, respectively. Let $K(s)$ be the $q \times q$ process controlling the magnitude and transmission of jumps, such that $K(s)dQ(s)$ is the contribution of the jump process to the price diffusion. Under the assumption of a Brownian semimartingale with finite-activity jumps (BSMFAJ), $p^*(s)$ follows:

$$dp^*(s) = \mu(s)ds + \sigma(s)dW(s) + K(s)dQ(s), \quad 0 \leq s \leq T \tag{1}$$

where $\mu(s)$ is a q-dimensional vector of continuous and locally-bounded variation processes, and $\sigma(s)$ is the $q \times q$ matrix, such that $\Sigma(s) = \sigma(s)\sigma'(s)$ is positive definite.

Assume that the observable log-price process is the sum of the latent log-price process in equation (1) and the microstructure noise process. For $q = 2$, define the log-price process as $p(s) = (X_s, Y_s)$. Consider non-synchronized trading times of the two assets, and let $\mathcal{T}$ and Θ be the set of transaction times of X and Y, respectively. Denote the counting process governing the number of observations traded in assets X and Y up to time T as n_T and m_T, respectively. By definition, the trades in X and Y occur at times $\mathcal{T} = \{\tau_1, \tau_2, \ldots, \tau_{n_T}\}$ and $\Theta = \{\theta_1, \theta_2, \ldots, \theta_{m_T}\}$, respectively. For convenience, the opening and closing times are set as $\tau_1 = \theta_1 = 0$ and $\tau_{n_T} = \theta_{m_T} = T$, respectively.

The observable log-price process is given by:

$$X_{\tau_i} = X^*_{\tau_i} + \varepsilon^X_{\tau_i} \quad \text{and} \quad Y_{\theta_j} = Y^*_{\theta_j} + \varepsilon^Y_{\theta_j}, \tag{2}$$

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[3]For a given fixed interval, realized volatility is defined as the sum of non-overlapping squared returns of high frequency within a day (Andersen and Bollerslev, 1998) which, in turn, presents volatility as an observed rather than a latent process.

where $\varepsilon^X \sim \text{iid}(0, \sigma_{\varepsilon X}^2)$, $\varepsilon^Y \sim \text{iid}(0, \sigma_{\varepsilon Y}^2)$, and $(\varepsilon^X, \varepsilon^Y)$ are independent of (X, Y).

Define the quadratic covariation (QCov) of the log-price process over $[0, T]$ as:

$$\text{QCov} = \operatorname*{plim}_{\Delta \to \infty} \sum_{i=1}^{\lfloor T/\Delta \rfloor} [p(i\Delta) - p((i-1)\Delta)] [p(i\Delta) - p((i-1)\Delta)]'. \tag{3}$$

Then we obtain:

$$\text{QCov} = \int_0^T \Sigma(s)ds + \sum_{0 < s \leq T} K(s)K'(s). \tag{4}$$

The first term on the right-hand side of (4) is the integrated co-volatility (ICov) matrix over $[0, T]$, while the second term is the matrix of jump variability. We are interested in disentangling these two components from the estimates of QCov for the purpose of forecasting QCov.

We explain below the consistent estimation method of the integrated co-volatility matrix suggested by Koike (2016), under non-synchronized trading times, jumps and microstructure noise for the bivariate process in (2)[4]. First, we consider the q-variate case, which consists of the estimators of integrated volatility and co-volatility, obtained using the approach of Koike (2016). Denote the estimators of QCov, ICov and jump component at day t as $\hat{\Omega}_t$, $\hat{C}_t$ and $\hat{J}_t$, respectively, where $\hat{J}_t = \hat{\Omega}_t - \hat{C}_t$. By the definitions in (1)-(4), the estimators should be positive (semi-) definite. One approach for guaranteeing its positive definiteness is to regularize the estimated covariance matrix by the use of thresholding.

Bickel and Levina (2008a,b) and Tao et al. (2011) showed consistency of the regularized estimator, assuming a sparsity structure. Define the thresholding operator for a $q \times q$ matrix A as:

$$\mathbb{T}_h(A) = [a_{ij}\mathbf{1}(|a_{ij}| \geq h)], \tag{5}$$

which can be regarded as A *thresholded at* h. Define the Frobenius norm by $||A||_F^2 = \text{tr}(AA')$. For the selection of h, we follow Bickel and Levina (2008b). In order to obtain $\tilde{A} = \mathbb{T}_h(\hat{A})$, we minimize the distance by the Frobenius norm $||\mathbb{T}_h(\hat{A}) - \hat{A}||_F^2$, with the restriction that $\tilde{A}$ is positive semi-definite. Using this approach, we obtain $\tilde{C}_t = \mathbb{T}_h(\hat{C}_t)$ and $\tilde{J}_t = \mathbb{T}_h(\hat{J}_t)$, which are consistent and positive semi-definite. Note that $\hat{\Omega}_t$ is positive semi-definite as it is the sample analogue of QCov.

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[4]The realized kernel (RK) estimator of Barndorff-Nielsen et al. (2011) is positive (semi-)definite and robust to microstructure noise under non-synchronized trading times. However, the robustness to jumps is still an open and unresolved issue for the multivariate RK estimator.

2.2 Estimator of Quadratic Covariation

We introduce below the pre-averaged Hayashi–Yoshida (PHY) estimator, suggested by Christensen, Kinnebrock and Podolskij (2010) for improving the estimator of Hayashi and Yoshida (2005) for non-synchronized trading times. For the PHY estimator, Koike (2016) shows that it is a consistent estimator of QCov under non-synchronized trading times, jumps and microstructure noise.

Consider a sequence, k_n, of integers and a number, $\psi_0 \in (0, \infty)$, satisfying $k_n = \psi_0 \sqrt{n} + o(n^{1/4})$, where n is the observation frequency. Assume that a continuous function $g : [0, 1] \to \Re$ is piecewise C^1 with piecewise Lipschitz derivative g' that satisfies $g(0) = g(1) = 0$ and $\psi_{HY} = \int_0^1 g(x)dx \neq 0$. We also consider pre-averaged observation data of X and Y based on the sampling designs $\mathcal{T}$ and Θ as:

$$\bar{X}^i = \sum_{p=1}^{k_n-1} g\left(\frac{p}{k_n}\right) \left(X_{\tau_{i+p}} - X_{\tau_{i+p-1}}\right)$$

$$\bar{Y}^j = \sum_{q=1}^{k_n-1} g\left(\frac{q}{k_n}\right) \left(Y_{\theta_{j+q}} - X_{\theta_{j+q-1}}\right), \quad i, j = 0, 1, \ldots.$$

Then the PHY estimator is defined by:

$$PHY(X, Y) = \frac{1}{(k_n \psi_{HY})^2} \sum_{\substack{i,j=0 \\ \max(\tau_{i+k_n}, \theta_{j+k_n}) \leq T}}^{\infty} \bar{X}^i \bar{Y}^j \bar{K}^{ij}, \tag{6}$$

where $\bar{K}^{ij} = \mathbf{1}([\tau_i, \tau_{i+k_n}) \cap [\theta_j, \theta_{j+k_n}) \neq \varnothing)$. Christensen, Kinnebrock and Podolskij (2010) derived the consistency and asymptotic mixed normality of the PHY estimator under non-synchronized trading times and microstructure noise without jumps. Koike (2016) shows that the PHY estimator is a consistent estimator of the quadratic covariation of (X, Y). Following Koike (2016), we use the specifications $g(x) = \min(x, 1 - x)$, $k_n = \lceil \psi_0 \sqrt{n} \rceil$ and $\psi_0 = 0.15$.

We can obtain the estimator of quadratic variation of X, by using $PHY(X, X)$. Hence, we can construct a consistent estimator of QCov for the q-variate case. As noted above, the estimator of QCov is generally positive definite, as it is the sample analogue of preaveraged data.

2.3 Estimator of Integrated Co-Volatility

This section explains the estimator of Koike (2016) for ICov under non-synchronized trading times, jumps and microstructure noise. Koike (2016) uses a truncation technique for removing the jump

components in order to obtain the pre-averaged truncated Hayashi–Yoshida (PTHY) estimator:

$$PTHY(X,Y) = \frac{1}{(k_n \psi_{HY})^2} \sum_{\substack{i,j=0 \\ \max(\tau_{i+k_n}, \theta_{j+k_n}) \leq T}}^{\infty} \bar{X}^i \bar{Y}^j \bar{K}^{ij} \bar{\Upsilon}^{ij}, \tag{7}$$

where $\bar{\Upsilon}^{ij} = \mathbf{1}(|\bar{X}^i|^2 \leq \varrho^X(\tau_i), |\bar{Y}^j|^2 \leq \varrho^Y(\theta_j))$, and $\varrho^X(t)$ and $\varrho^Y(t)$ are sequences of positive-valued stochastic process. Aït-Sahalia, Jacod, and Li (2012) suggested a similar idea for the truncation in the univariate case. Koike (2016) shows the consistency and asymptotic mixed normality of the PTHY estimator and, using the difference between the PHY and PTHY estimators, also shows the consistency of the quadratic co-variation of the jump component.

For the process of the threshold value of X, Koike (2016) uses:

$$\varrho^X(\tau_i) = 2 \log(N)^{1+\varepsilon} \hat{\sigma}_{\tau_i}^2,$$

with $\varepsilon = 0.2$, where:

$$\hat{\sigma}_{\tau_i}^2 = \frac{\mu_1^{-2}}{K - 2k_n + 1} \sum_{p=i-K}^{i-2k_n} |\bar{X}^p||\bar{X}^{p+k_n}|, \quad i = K, k+1, \ldots, N,$$

and $\hat{\sigma}_{\tau_i}^2 = \hat{\sigma}_{\tau_K}^2$ if $i < K$. Here, $\mu_1 = \sqrt{2/\pi}$, $K = \lceil N^{3/4} \rceil$, and N is the number of the available pre-averaged data $\bar{X}^i$. We can obtain $\varrho^Y(\theta_j)$ in the same manner.

We construct a consistent estimator of ICov for the q-variate case, $\hat{C}_t$, using the estimators based on $PTHY(X,X)$ and $PTHY(X,Y)$. In order to guarantee the positive semi-definiteness of the estimators of ICov and the jump component, we use the approach of Bickel and Levina (2008b), as explained above.

Using estimation techniques for $PTHY(X,X)$ and $PTHY(X,Y)$ (equation (7)), we can construct a consistent estimator of the $q \times q$ integrated co-volatility matrix at day t, $\hat{C}_t$, under jumps and microstructure noise. We can also obtain the estimator of QCov, which we denote as $\hat{\Omega}_t$, by using $PHY(X,X)$ and $PHY(X,Y)$ (equation (6)), which leads to the jump estimator, $\hat{J}_t = \hat{\Omega}_t - \hat{C}_t$. Applying the threshold operator defined by (5), we obtain the final estimates as $\tilde{\Omega}_t = \mathbb{T}_h(\hat{\Omega}_t)$, $\tilde{C}_t = \mathbb{T}_h(\hat{C}_t)$ and $\tilde{J}_t = \mathbb{T}_h(\hat{J}_t)$.

2.4 Estimation of Jump Component in Returns

Aït-Sahalia and Jacod (2012) suggest a simple methodology to decompose asset returns sampled at a high frequency into their base components (continuous and jumps). For the process of log-prices, X, the return is defined by $R^X = \sum_{i=2}^{n_T}(X_{\tau_i} - X_{\tau_{i-1}}) = X_m - X_1$, where $m = \max\{i : \tau_i \leq$

T}. Following the idea of Aït-Sahalia and Jacod (2012), we define continuous, jump, and noise components of the return by:

$$RC^X = \frac{1}{k_n \psi_{HY}} \sum_{i=0}^{m-k_n} \bar{X}^i \mathbf{1}(|\bar{X}^i|^2 \leq \varrho^X(\tau_i)),$$

$$RJ^X = \frac{1}{k_n \psi_{HY}} \sum_{i=0}^{m-k_n} \bar{X}^i \mathbf{1}(|\bar{X}^i|^2 \geq \varrho^X(\tau_i)),$$

$$RN^X = R^X - RC^X - RJ^X.$$

In the empirical analysis in the next section, we use the continuous component rather than the observed return itself, in order to examine leverage and co-leverage effects on volatility and co-volatility, respectively.

3   Empirical Analysis

We examine the effects on jumps and leverage in forecasting co-volatility, using the estimates of QCov, ICov and jump variation, for two futures contracts traded on the New York Mercantile Exchange (NYMEX), namely West Texas Intermediate (WTI) Crude Oil and Gold. With the CME Globex system, the trades at NYMESX cover 24 hours. Based on the vector of returns for the $q = 2$ futures for a 1-minute interval of trading day at t, we calculated the daily values of $\tilde{\Omega}_t$, $\tilde{C}_t$ and $\tilde{J}_t$, as explained in the previous section, and also the corresponding open-close returns and their continuous components, r_t and rc_t, respectively, for the two futures. The sample period starts on September 27, 2009, and ends on May 25, 2017, giving 1978 observations. The sample is divided into two periods: the first 1000 observations are used for in-sample estimation, while the last 978 observations are used for evaluating the out-of-sample forecasts.

Table 1 presents the descriptive statistics of the returns, r_t, and estimated QCov, $\tilde{\Omega}_t$. The empirical distribution of the returns is highly leptokurtic, and significant jumps occurred for 45% of the period. Regarding volatility, their distributions are skewed to the right, with evidence of heavy tails in the two series. More than 90% of the sample period contains significant jumps in volatility. For co-volatility, the empirical distribution is highly leptokurtic, and co-jump variations were found for 61% of the period.

Figures 1 and 2 show the estimates of quadratic variation, integrated volatility, and jump variability, namely the diagonal elements of $\tilde{\Omega}_t$, $\tilde{C}_t$, and $\tilde{J}_t$, respectively. It is known that spot and future prices of crude oil are effected by a variety of geopolitical and economic events. For

instance, the estimates of volatility in Figure 1 are relatively high for the period following the Arab Spring of 2011, and the extreme jump in 2015 is caused by the oversupply and the technological advancements of US shale oil production. On the other hand, the spot and futures prices of Gold reflect news and recessions, as investigated by Smales (2014). The estimates of volatility are high in 2011 during the European debt crisis, and the last one-third in Figure 2. For the latter, it corresponds to China's economy growing at its slowest pace for 24 years in 2014.

Figure 3 illustrates the estimates of quadratic covariation, integrated co-volatility, and jump co-variability, namely the $(2,1)$-element of $\tilde{\Omega}_t$, $\tilde{C}_t$, and $\tilde{J}_t$. Figure 3 indicates that crude oil and gold futures are negatively correlated for the first half in 2011, but the sign changes for the latter half. A large and positive co-jump variability is found in 2013, which reflects the political unrest in Egypt and the updates of the highest values of the Dow Jones Industrial Average.

Let $\tilde{\Omega}_{t-h+1:t}$ denote the h-horizon average, defined by:

$$\tilde{\Omega}_{t-h+1:t} = \frac{1}{h}\left(\tilde{\Omega}_t + \cdots + \tilde{\Omega}_{t-h+1}\right).$$

In order to examine the impact of jumps and leverage for forecasting volatility and co-volatility, we use three kinds of heterogeneous autoregressive (HAR) models for forecasting the (i,j)-elements of $\tilde{\Omega}_{t-h+1:t}$ $(h = 1, 5, 22)$, as follows:

$$\tilde{\Omega}_{ij,t-h+1:t} = \beta_0 + \beta_d \tilde{\Omega}_{ij,t-1} + \beta_w \tilde{\Omega}_{ij,t-5:t-1} + \beta_m \tilde{\Omega}_{ij,t-22:t-1} + u_{ij,t} \tag{8}$$

$$\tilde{\Omega}_{ij,t-h+1:t} = \beta_0 + \beta_d \tilde{C}_{ij,t-1} + \beta_w \tilde{C}_{ij,t-5:t-1} + \beta_m \tilde{C}_{ij,t-22:t-1} + \beta_j \tilde{J}_{ij,t-1} + u_{ij,t} \tag{9}$$

$$\tilde{\Omega}_{ij,t-h+1:t} = \beta_0 + \beta_d \tilde{C}_{ij,t-1} + \beta_w \tilde{C}_{ij,t-5:t-1} + \beta_m \tilde{C}_{ij,t-22:t-1} + \beta_j \tilde{J}_{ij,t-1} + \beta_a rc_{i,t-1}^- rc_{j,t-1}^- + u_{ij,t}, \tag{10}$$

where $rc_{i,t}^- = rc_{i,t} I(rc_{i,t} < 0)$, which is the negative part of the return of the i-th asset. In the second model, we use the previous values of the estimated continuous sample path component variation, $\tilde{C}_t$, rather than those of the estimated quadratic variation, $\tilde{\Omega}_t$, following the volatility forecasting models of Andersen et al. (2007) and Corsi et al. (2010). We exclude weekly and monthly effects of the jump component, $\tilde{J}_t$, in order to evaluate the impact of a single jump on future volatility and co-volatility. Note that $\tilde{C}_t$ and $\tilde{J}_t$ are positive (semi-) definite by the thresholding in (5). In addition to jump variability, the third model includes the asymmetric effect, as in the specification of the asymmetric BEKK model of Kroner and Ng (1998). For $i \neq j$, $\beta_a rc_{i,t-1}^- rc_{j,t-1}^-$ represents the 'co-leverage' effect, which is caused by simultaneous negative returns in two assets. Note that we use the continuous components of returns rather than the observed

returns, r_t. We refer to equations (8), (9) and (10) as the HAR, HAR-TCJ and HAR-TCJA models, respectively.

In the following empirical analysis, we:

(a) check the robustness of the positive effects of jump components under microstructure noise for the volatility equation $(i = j)$;

(b) test the robustness of the leverage effects under jump and microstructure noise for the volatility equation;

(c) examine the effects of co-jumps and co-leverage effects for the co-volatility equation;

(d) compare the out-of-sample forecasts of the above models.

For the above models for the volatility equation, the estimates of β_j are expected to be positive. However, the empirical results of Andersen et al. (2007) indicate that the estimates of β_j are generally insignificant. Corsi et al. (2010) noted that the puzzle is due to the small sample bias of the integrated volatility, and found that the estimates of β_j are positive and significant. Since the estimators of quadratic variation and integrated volatility used in Corsi et al. (2010) are biased in the presence of microstructure noise, we re-examine the robustness of the result, and this is the motivation of (a). With respect to (b), we also test $\beta_a > 0$ under microstructure noise in order to check the robustness of the result of Corsi and Renò (2012). Among (a)-(d), (c) is the main purpose of the current empirical analysis.

Although the estimates of β_j and β_a are expected to be positive and significant for the volatility equation $(i = j)$, their signs are not determined for the co-volatility equation $(i \neq j)$. As noted in (d), we compare the forecasting performances of the HAR, HAR-TCJ and HAR-TCJA models for daily, weekly and monthly horizons. For crude oil futures, Wen, Gong, and Cai (2016) found that HAR-TCJA is the best forecasting one-day-ahead volatility, while the HAR model is preferred for weekly and monthly forecasts, after removing the effects of structural breaks. In addition to crude oil, we examine the forecasting performances of the volatility of gold futures and the co-volatility of the two futures.

We estimate each model using the first 1000 observations, and obtain a forecast, $\tilde{\Omega}_{1001}^f$ We re-estimate each model fixing the sample size at 1000, and obtain new forecasts based on updated parameter estimates. For evaluating the forecasting performance of the different models, we report

R^2 of the Mincer-Zarnowitz (MZ) regression, namely:

$$\tilde{\Omega}_{ij,t} = \alpha_0 + \alpha_1 \tilde{\Omega}^f_{ij,t} + \text{error}_t, \quad t = 1001, \ldots, 1978.$$

We also use the heteroskedasticity-adjusted root mean square error suggested in Bollerslev and Ghysels (1996), namely:

$$\text{HRMSE} = \sqrt{\frac{1}{978} \sum_{t=1001}^{1978} \left(\frac{\tilde{\Omega}_{ij,t} - \tilde{\Omega}^f_{ij,t}}{\tilde{\Omega}_{ij,t}} \right)^2}.$$

For the latter, we examine equal forecast accuracy using the Diebold and Mariano (1995) test at the 5% significance level, and use the heteroskedasticity and autocorrelation consistent (HAC) covariance matrix estimator, with bandwidth 25. We also examine the forecasts of $\tilde{\Omega}_{ij,t-4:t}$ and $\tilde{\Omega}_{ij,t-21:t}$ in the same manner.

Table 2 shows the estimates of the daily regressions for the first 1000 observations. The estimates of the jump parameter, β_j, are positive and significant in all cases. The results for the volatilities support the empirical analysis of Corsi et al. (2010). The estimates of the coefficient of the asymmetric effect, β_a, are positive and significant in all cases, supporting the negative relationship between return and future volatility, as in Corsi and Renò (2012). For the co-volatility equation, the results indicate that a pair of negative returns and/or co-jumps of two assets increases future co-volatility. The HAR-TCJA model gives the highest $\bar{R}^2$ in all cases. Table 3 presents R^2 of the MZ regressions and HRMSE for the daily regressions. The HAR-TCJA has the highest R^2 for two-thirds of the cases, while the results of the Diebold and Mariano tests regarding the HRMSE indicate that there are no significant differences for the three models in all cases.

Table 4 reports the estimates of the weekly regressions. The estimates of the jump parameter, β_j, and the parameter of the asymmetric effect, β_a, are positive and significant. Unlike the daily regressions, the HAR model gives the highest $\bar{R}^2$ values in all cases. Table 5 gives the R^2 values of the MZ regressions, and HRMSE for the out-of-sample forecasts for the weekly regressions. Tables 4 and 5 indicate that the values of R^2 (and $\bar{R}^2$) are higher than those for the daily regressions in Tables 2 and 3, respectively. Table 5 shows that the HAR model is the best in forecasting volatility. For co-volatility, R^2 for MZ selects the HAR model, while HRMSE chooses the HAR-TCJA model. However, the Diebold and Mariano tests show that there are no significant differences for the three models for co-volatility.

Table 6 shows the in-sample estimates of the monthly regressions, while Table 7 reports the results of the corresponding out-of-sample forecasts. Unlike the daily regressions, the HAR model gives the highest $\bar{R}^2$ values in all cases. Tables 6 and 7 indicate that the values of R^2 are higher than those for the daily regressions in Tables 4 and 5, respectively. Table 7 shows that the HAR model is the best model in all cases and, moreover, there are significant differences between HAR and the other models in forecasting volatility.

The empirical results for the volatility models support the findings of Corsi et al. (2010), Corsi and Renò (2012), and Wen, Gong, and Cai (2016). Regarding co-volatility, the impacts of co-jumps of two assets are positive and significant for the daily, weekly, and monthly regressions. Although the HAR-TCJ model performs better than the HAR model for the daily regressions, the monthly regressions prefer the HAR model. We may improve the HAR-TCJ and HAR-TCJA models by accommodating the positive and negative jumps of Patton and Sheppard (2013), in addition to the weekly and monthly averages of jumps and leverage effects.

4   Concluding Remarks

The paper examined the impacts of co-jumps and leverage of crude oil and gold futures in forecasting co-volatility. We suggested disentangling the estimates of the integrated co-volatility matrix and jump variations so that they are positive (semi-) definite for coherence of the estimator. The empirical results showed that the co-jumps of any two assets have significant impacts on future co-volatility, but that the impacts are minor in forecasting weekly and monthly horizons.

The empirical results also showed that the impacts of the co-leverage effects caused by the negative returns of two assets are significant, but the impact decreases in forecasting longer horizons. The results of in-sample and out-of-sample forecasts showed that the datasets generally prefer the HAR-TCJA model for the daily horizon, but tend to choose the HAR model for weekly and monthly horizons.

Overall, the analysis given in the paper should be useful for investment analysis, and both public and private policy prescription, in terms of accommodating jumps and leverage, as well as choice of models and time frequency horizons, in forecasting the co-volatility of oil and gold futures. Extensions to other financial, precious and semi-precious metals, and alternative sources of renewable and non-renewable energy, are the subject of ongoing research.

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Table 1: Descriptive Statistics of Returns, Volatility and Co-Volatility

Stock	Mean	Std.Dev.	Skew.	Kurt.	Jump
Return					
Crude Oil	0.0256	0.6824	−0.4784	8.0378	0.4550
Gold	0.0185	1.4887	−0.0573	5.0225	0.4388
Volatility					
Crude Oil	0.4238	0.5070	7.2211	90.688	0.9267
Gold	1.5152	1.6510	3.3689	19.071	0.9459
Co-Volatility					
(Crude Oil, Gold)	0.1368	0.2844	−0.7851	37.359	0.6127

Note: The sample period is from September 27, 2009 to May 25, 2017.
'Jump' denotes the percentage of occurrence of significant jumps.

Table 2: In-sample Estimates for Daily Regressions

HAR $\quad\tilde{\Omega}_{ij,t} = \beta_0 + \beta_d\tilde{\Omega}_{ij,t-1} + \beta_w\tilde{\Omega}_{ij,t-5:t-1} + \beta_m\tilde{\Omega}_{ij,t-22:t-1} + u_{ij,t}$

HAR-TCJ $\quad\tilde{\Omega}_{ij,t} = \beta_0 + \beta_d\tilde{C}_{ij,t-1} + \beta_w\tilde{C}_{ij,t-5:t-1} + \beta_m\tilde{C}_{ij,t-22:t-1} + \beta_j\tilde{J}_{ij,t-1} + u_{ij,t}$

HAR-TCJA $\quad\tilde{\Omega}_{ij,t} = \beta_0 + \beta_d\tilde{C}_{ij,t-1} + \beta_w\tilde{C}_{ij,t-5:t-1} + \beta_m\tilde{C}_{ij,t-22:t-1} + \beta_j\tilde{J}_{ij,t-1} + \beta_a rc^-_{i,t-1}rc^-_{j,t-1} + u_{ij,t}$

Model	β_0	β_d	β_w	β_m	β_j	β_a	R^2	$\bar{R}^2$
Volatility: Crude Oil								
HAR	0.0958	0.4428	0.1047	0.2665			0.3514	0.3494
	(0.0021)	(0.0054)	(0.0050)	(0.0056)				
HAR-TCJ	0.1239	0.5182	0.1012	0.2879	0.2410		0.3546	0.3520
	(0.0017)	(0.0083)	(0.0064)	(0.0064)	(0.0112)			
HAR-TCJA	0.1301	0.2285	0.2431	0.3176	0.2575	0.1292	0.4140	0.4110†
	(0.0015)	(0.0067)	(0.0054)	(0.0064)	(0.0097)	(0.0032)		
Volatility: Gold								
HAR	0.2039	0.4730	0.2067	0.1462			0.4396	0.4378
	(0.0028)	(0.0060)	(0.0053)	(0.0043)				
HAR-TCJ	0.2520	0.5629	0.1512	0.1767	0.2913		0.4414	0.4391
	(0.0027)	(0.0082)	(0.0057)	(0.0048)	(0.0094)			
HAR-TCJA	0.2193	0.4612	0.2066	0.1621	0.2427	0.1189	0.5266	0.5242†
	(0.0024)	(0.0066)	(0.0051)	(0.0041)	(0.0069)	(0.0014)		
Co-Volatility: Crude Oil & Gold								
HAR	0.0431	0.3938	0.3199	0.0923			0.4034	0.4016
	(0.0009)	(0.0043)	(0.0049)	(0.0064)				
HAR-TCJ	0.0515	0.5028	0.3268	0.1447	0.2047		0.4116	0.4092
	(0.0008)	(0.0065)	(0.0060)	(0.0071)	(0.0057)			
HAR-TCJA	0.0444	0.4086	0.3486	0.1933	0.1664	0.0564	0.4391	0.4362†
	(0.0008)	(0.0069)	(0.0057)	(0.0068)	(0.0049)	(0.0010)		

Note: Standard errors are given in parentheses. '$\dagger$' denotes the model which has the highest $\bar{R}^2$ value of the three models.

Table 3: Out-of-Sample Forecast Evaluation for Daily Regressions

Model	R^2	HRMSE	J-R^2	J-HRMSE	C-R^2	C-HRMSE
Volatility: Crude Oil (900 Times Jump)						
HAR	0.0796	2.5591	0.0818	2.6478	0.0590	1.1057
HAR-TCJ	0.0928	2.0476	0.0944	2.1156	0.0775†	0.9635
HAR-TCJA	0.0995†	2.0205	0.1037†	2.0869	0.0641	0.9672
Volatility: Gold (950 Times Jump)						
HAR	0.6900	0.9009	0.6918	0.9066	0.6017	0.7857
HAR-TCJ	0.6907	0.9103	0.6929	0.9132	0.5891	0.8534
HAR-TCJA	0.6925†	0.8493	0.6938†	0.8521	0.6211†	0.7945
Co-Volatility: Crude Oil & Gold (616 Times Co-Jump)						
HAR	0.2199†	40.906	0.1897	45.317	0.3247†	32.033
HAR-TCJ	0.2103	36.572	0.1832	40.316	0.3127	29.114
HAR-TCJA	0.2119	37.543	0.1971†	41.018	0.2806	30.739

Note: The table reports Mincer-Zarnowitz R^2 and heteroskedasticity-adjusted root mean squared error (HRMSE). J-R^2 and J-HRMSE are R^2 and HRMSE conditionally on having a jump at time $t-1$, respectively, while C-R^2 and C-HRMSE are conditional on no jump at time $t-1$. '$\dagger$' denotes the model which has the highest R^2 value of the three models. For the Diebold-Mariano test of equal forecast accuracy, 'a', 'b' and 'c' denote significant improvements in forecasting performance with respect to the HAR, HAR-TCJ and HAR-TCJA models, respectively.

Table 4: In-sample Estimates for Weekly Regressions

HAR $\qquad \tilde{\Omega}_{ij,t-4:t} = \beta_0 + \beta_d \tilde{\Omega}_{ij,t-1} + \beta_w \tilde{\Omega}_{ij,t-5:t-1} + \beta_m \tilde{\Omega}_{ij,t-22:t-1} + u_{ij,t}$

HAR-TCJ $\qquad \tilde{\Omega}_{ij,t-4:t} = \beta_0 + \beta_d \tilde{C}_{ij,t-1} + \beta_w \tilde{C}_{ij,t-5:t-1} + \beta_m \tilde{C}_{ij,t-22:t-1} + \beta_j \tilde{J}_{ij,t-1} + u_{ij,t}$

HAR-TCJA $\qquad \tilde{\Omega}_{ij,t-4:t} = \beta_0 + \beta_d \tilde{C}_{ij,t-1} + \beta_w \tilde{C}_{ij,t-5:t-1} + \beta_m \tilde{C}_{ij,t-22:t-1} + \beta_j \tilde{J}_{ij,t-1} + \beta_a rc_{i,t-1}^- rc_{j,t-1}^- + u_{ij,t}$

Model	β_0	β_d	β_w	β_m	β_j	β_a	R^2	$\bar{R}^2$
Volatility: Crude Oil								
HAR	0.0203	0.1731	0.7803	0.0070			0.9208	0.9206†
	(0.0006)	(0.0015)	(0.0028)	(0.0017)				
HAR-TCJ	0.0562	0.1843	0.8853	0.0002	0.2655		0.9060	0.9057
	(0.0006)	(0.0024)	(0.0039)	(0.0022)	(0.0028)			
HAR-TCJA	0.0571	0.1441	0.9050	0.0043	0.2678	0.0179	0.9081	0.9076
	(0.0006)	(0.0026)	(0.0039)	(0.0023)	(0.0026)	(0.0008)		
Volatility: Gold								
HAR	0.0446	0.1793	0.7970	-0.0142			0.9403	0.9402†
	(0.0008)	(0.0016)	(0.0023)	(0.0012)				
HAR-TCJ	0.1086	0.2244	0.8310	-0.0058	0.2440		0.9240	0.9237
	(0.0009)	(0.0023)	(0.0033)	(0.0016)	(0.0019)			
HAR-TCJA	0.1027	0.2059	0.8411	-0.0084	0.2351	0.0217	0.9285	0.9281
	(0.0009)	(0.0021)	(0.0033)	(0.0016)	(0.0015)	(0.0003)		
Co-Volatility: Crude Oil & Gold								
HAR	0.0099	0.1441	0.8321	-0.0207			0.9346	0.9344†
	(0.0002)	(0.0012)	(0.0020)	(0.0017)				
HAR-TCJ	0.0237	0.1790	0.9493	0.0140	0.1996		0.9160	0.9157
	(0.0003)	(0.0021)	(0.0031)	(0.0024)	(0.0013)			
HAR-TCJA	0.0222	0.1594	0.9538	0.0241	0.1916	0.0117	0.9180	0.9176
	(0.0003)	(0.0022)	(0.0031)	(0.0023)	(0.0011)	(0.0003)		

Note: Standard errors are given in parentheses. '$\dagger$' denotes the model which has the highest $\bar{R}^2$ value of the three models.

Table 5: Out-of-Sample Forecast Evaluation for Weekly Regressions

Model	MZ R^2	HRMSE	J-R^2	J-HRMSE	C-R^2	C-HRMSE
Volatility: Crude Oil (900 Times Jump)						
HAR	$0.8060^{\dagger}$	$0.1980^{b,c}$	$0.7964^{\dagger}$	$0.2011^{b,c}$	$0.8792^{\dagger}$	$0.1578^{b,c}$
HAR-TCJ	0.7321	0.2444	0.7286	0.2469	0.8064	0.2141
HAR-TCJA	0.7381	0.2465	0.7340	0.2492	0.8120	0.2128
Volatility: Gold (929 Times Jump)						
HAR	$0.9748^{\dagger}$	$0.1258^{b,c}$	$0.9750^{\dagger}$	$0.1262^{b,c}$	0.9639	$0.1168^{b,c}$
HAR-TCJ	0.9692	0.1592	0.9693	0.1595	0.9601	0.1546
HAR-TCJA	0.9697	0.1557	0.9698	0.1559	$0.9647^{\dagger}$	0.1510
Co-Volatility: Crude Oil & Gold (616 Times Co-Jump)						
HAR	$0.8884^{\dagger}$	9.2664	$0.8695^{\dagger}$	11.509	$0.9138^{\dagger}$	2.5693
HAR-TCJ	0.8567	4.7787	0.8391	5.0081	0.8834	4.3606
HAR-TCJA	0.8564	4.0257	0.8409	3.9249	0.8799	4.1917

Note: The table reports Mincer-Zarnowitz R^2 and heteroskedasticity-adjusted root mean squared error (HRMSE). J-R^2 and J-HRMSE are R^2 and HRMSE conditionally on having a jump at time $t-1$, respectively, while C-R^2 and C-HRMSE are conditional on no jump at time $t-1$. '$\dagger$' denotes the model which has the highest R^2 value of the three models. For the Diebold-Mariano test of equal forecast accuracy, 'a', 'b' and 'c' denote significant improvements in forecasting performance with respect to the HAR, HAR-TCJ and HAR-TCJA models, respectively.

Table 6: In-sample Estimates for Monthly Regressions

HAR $\quad\quad \tilde{\Omega}_{ij,t-21:t} = \beta_0 + \beta_d\tilde{\Omega}_{ij,t-1} + \beta_w\tilde{\Omega}_{ij,t-5:t-1} + \beta_m\tilde{\Omega}_{ij,t-22:t-1} + u_{ij,t}$

HAR-TCJ $\quad \tilde{\Omega}_{ij,t-21:t} = \beta_0 + \beta_d\tilde{C}_{ij,t-1} + \beta_w\tilde{C}_{ij,t-5:t-1} + \beta_m\tilde{C}_{ij,t-22:t-1} + \beta_j\tilde{J}_{ij,t-1} + u_{ij,t}$

HAR-TCJA $\quad \tilde{\Omega}_{ij,t-21:t} = \beta_0 + \beta_d\tilde{C}_{ij,t-1} + \beta_w\tilde{C}_{ij,t-5:t-1} + \beta_m\tilde{C}_{ij,t-22:t-1} + \beta_j\tilde{J}_{ij,t-1} + \beta_a rc_{i,t-1}^- rc_{j,t-1}^- + u_{ij,t}$

| Model	β_0	β_d	β_w	β_m	β_j	β_a	R^2	$\bar{R}^2$
Volatility: Crude Oil								
HAR	0.0010	0.0148	0.0261	0.9573			0.9913	0.9913†
	(0.0001)	(0.0003)	(0.0003)	(0.0005)				
HAR-TCJ	0.0441	0.0117	0.0430	1.0977	0.0752		0.9741	0.9740
	(0.0003)	(0.0004)	(0.0008)	(0.0016)	(0.0009)			
HAR-TCJA	0.0444	-0.0031	0.0503	1.0992	0.0760	0.0066	0.9746	0.9745
	(0.0003)	(0.0003)	(0.0008)	(0.0016)	(0.0008)	(0.0002)		
Volatility: Gold								
HAR	-0.0017	0.0201	0.0288	0.9518			0.9931	0.9931†
	(0.0003)	(0.0003)	(0.0004)	(0.0005)				
HAR-TCJ	0.0819	0.0198	0.0335	1.0575	0.0850		0.9820	0.9819
	(0.0004)	(0.0005)	(0.0007)	(0.0008)	(0.0008)			
HAR-TCJA	0.0805	0.0154	0.0360	1.0569	0.0829	0.0052	0.9824	0.9823
	(0.0004)	(0.0004)	(0.0007)	(0.0008)	(0.0007)	(0.0001)		
Co-Volatility: Crude Oil & Gold								
HAR	-0.0004	0.0166	0.0363	0.9482			0.9922	0.9922†
	(0.0001)	(0.0002)	(0.0003)	(0.0005)				
HAR-TCJ	0.0184	0.0131	0.0279	1.1670	0.0551		0.9790	0.9790
	(0.0001)	(0.0004)	(0.0006)	(0.0011)	(0.0006)			
HAR-TCJA	0.0181	0.0093	0.0288	1.1690	0.0536	0.0023	0.9792	0.9791
	(0.0001)	(0.0004)	(0.0006)	(0.0011)	(0.0006)	(0.0001)		

Note: Standard errors are given in parentheses. '$\dagger$' denotes the model which has the highest $\bar{R}^2$ value of the three models.

Table 7: Out-of-Sample Forecast Evaluation for Monthly Regressions

| Model	MZ R^2	HRMSE	J-R^2	J-HRMSE	C-R^2	C-HRMSE
Volatility: Crude Oil (900 Times Jump)						
HAR	$0.9794^\dagger$	$0.0542^{b,c}$	$0.9786^\dagger$	$0.0552^{b,c}$	$0.9856^\dagger$	$0.0424^{b,c}$
HAR-TCJ	0.8563	0.1386	0.8571	0.1384	0.8625	0.1404
HAR-TCJA	0.8581	0.1381	0.8589	0.1379	0.8640	0.1403
Volatility: Gold (929 Times Jump)						
HAR	$0.9978^\dagger$	$0.0340^{b,c}$	$0.9978^\dagger$	$0.0342^{b,c}$	$0.9975^\dagger$	$0.0299^{b,c}$
HAR-TCJ	0.9944	0.0864	0.9945	0.0865	0.9933	0.0834
HAR-TCJA	0.9945	0.0856	0.9945	0.0858	0.9939	0.0827
Co-Volatility: Crude Oil & Gold (616 Times Co-Jump)						
HAR	$0.9888^\dagger$	3.0184	$0.9855^\dagger$	1.8945	$0.9941^\dagger$	4.3019
HAR-TCJ	0.9668	8.7480	0.9652	3.8293	0.9698	13.483
HAR-TCJA	0.9670	8.6250	0.9657	3.7655	0.9696	13.299

Note: The table reports Mincer-Zarnowitz R^2 and heteroskedasticity-adjusted root mean squared error (HRMSE). J-R^2 and J-HRMSE are R^2 and HRMSE conditionally on having a jump at time $t-1$, respectively, while C-R^2 and C-HRMSE are conditional on no jump at time $t-1$. '$\dagger$' denotes the model which has the highest R^2 value of the three models. For the Diebold-Mariano test of equal forecast accuracy, 'a', 'b' and 'c' denote significant improvements in forecasting performance with respect to the HAR, HAR-TCJ and HAR-TCJA models, respectively.

Figure 1: Estimates of Quadratic Variation, Integrated Volatility, and Jump Variability for Crude Oil Futures
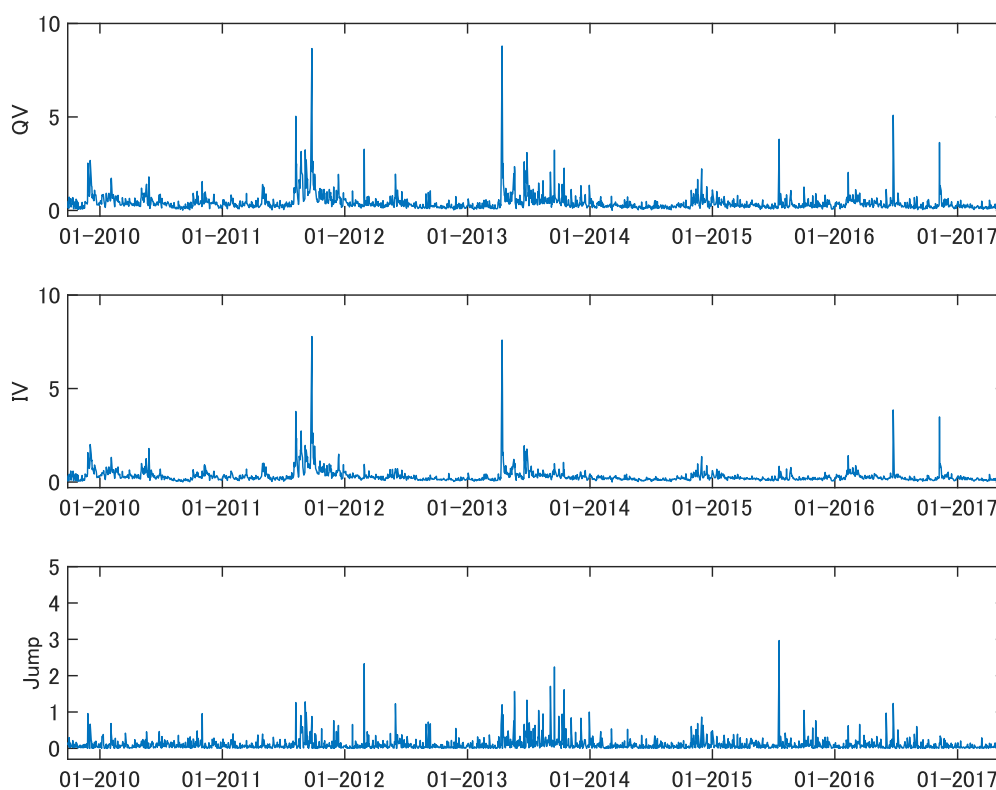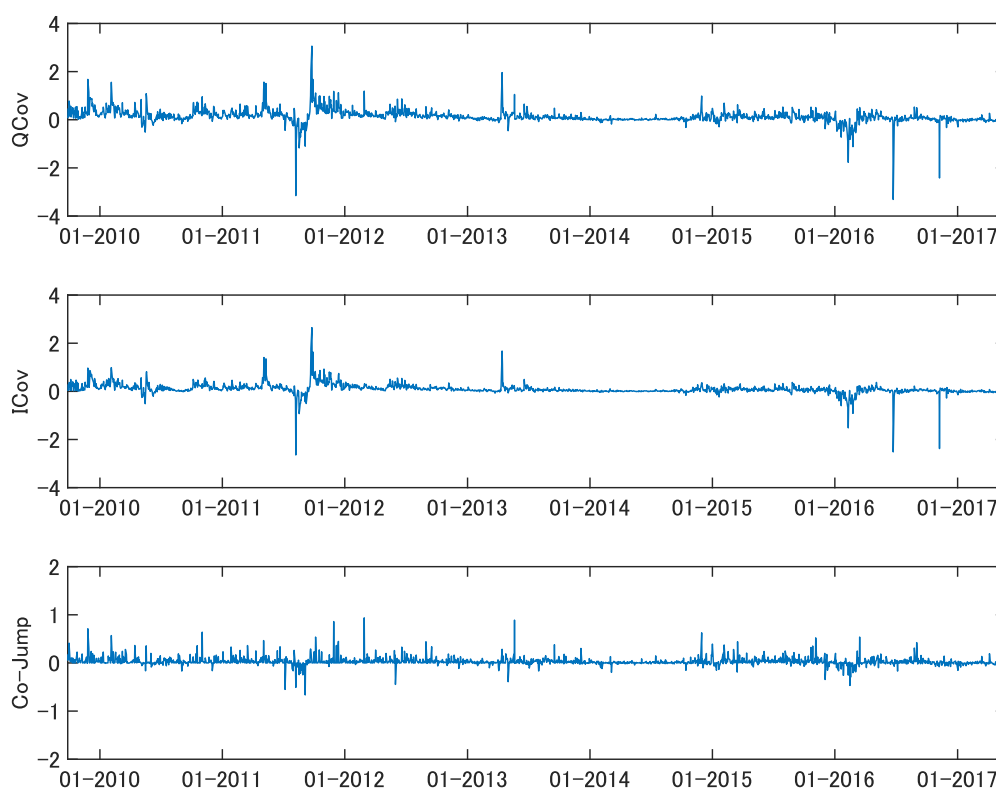

Note: Figure 1 shows the $(1,1)$-elements of $\tilde{\Omega}_t$, $\tilde{C}_t$, and $\tilde{J}_t$.

Figure 2: Estimates of Quadratic Variation, Integrated Volatility, and Jump Variability for Gold Futures
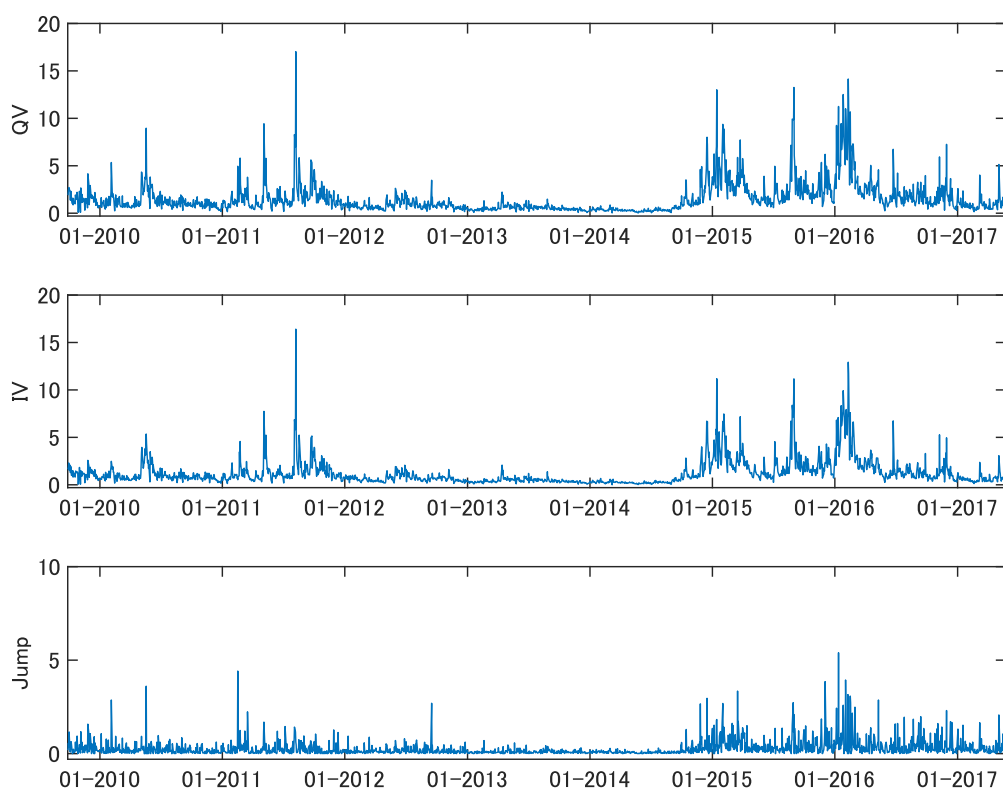
Note: Figure 2 shows the $(2,2)$-elements of $\tilde{\Omega}_t$, $\tilde{C}_t$, and $\tilde{J}_t$.

Figure 3: Estimates of Quadratic Covariation, Integrated Covolatility, and Jump Co-variability for Crude Oil and Gold Futures

Note: Figure 3 shows the $(2, 1)$-elements of $\tilde{\Omega}_t$, $\tilde{C}_t$, and $\tilde{J}_t$.