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Theory and Methodology

Performance approximation of pick-to-belt orderpicking systems

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Abstract: In this paper, an approximation method is discussed for the analysis of pick-to-belt orderpicking systems. The aim of the approximation method is to provide an instrument for obtaining rapid insight in the performance of designs of pick-to-belt orderpicking systems. It can be used to evaluate the effects of changing the layout of the system, the number of picking stations, the number of pickers, the conveyor speed, the number of bins to be processed per day, the number of orderlines per bin, etc. Especially in the design phase, modeling and analysis speed is more important than accuracy. The method presented in this paper is based on Jackson network modeling and analysis. The method is fast and sufficiently accurate. The method is used by Ingenieursbureau Groenewout B.V., for early-stage evaluation of design alternatives of pick-to-belt orderpicking systems and general transportation systems.

Keywords: Performance; Orderpicking systems; Heuristics; Queues; Practice

1. Introduction

In orderpicking from shelf storage systems, use is often made of so-called pick-to-belt orderpicking systems. Such a system usually consists of a central conveyor, which connects all picking stations in a shelf storage area and leads to a sorting and packing area. Orderlines are manually picked from the shelf storage area. An orderline is a number of pieces of one article, belonging to one customer order. A customer order may contain several orderlines. After picking an orderline, each picker receives information which orderline is to be picked next.

Two different types of pick-to-belt orderpicking systems can be distinguished.

In the first type, the picked items are directly deposited on the conveyor. A barcode, containing the ordernumber, may be attached to each piece, for sorting purposes. The picker needs preliminary information on the orderlines that have to be picked (for example, by computer visual display unit) and furthermore a printing device for barcodes is needed. This method is mostly used when articles are prepacked in cartons: barcodes are easily attached to cartons and can easily be read automatically.

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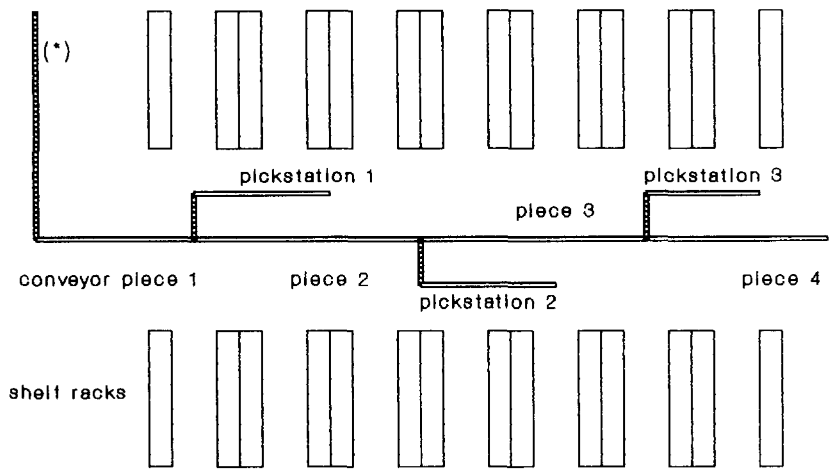

Figure 1. Central conveyor, consisting of 4 pieces, connecting 3 picking stations. Bins are placed on the conveyor at position (*). At the end of conveyor piece 4 the bins are removed from the system

Another possibility is, that at the beginning of the conveyor, bins are placed on the conveyor. Each bin is assigned to one or more customer orders that have to be picked within the bin. The simplest way to achieve this is having an orderlist printed, containing all orderlines that have to be picked from the shelf storage area into the bin. This list is deposited in the bin, after which the transportation system automatically transports the bin sequentially to all stations where an orderline has to be picked. In Figure 1, a top view is sketched of such an orderpicking system, with shelf racks on both sides of the conveyor and three picking stations.

Each picking station is a separate part of the conveyor. If an orderline has to be picked at a certain station, the transportation system automatically pushes out the bin to the station, so that the main flow of bins cannot become blocked by bins waiting for picking. At the station, the picker reads the orderlist in the bin, walks to the aisles and shelves indicated on the orderlist, picks the articles and deposits them into the bin. Having finished the orderlist, the bin is pushed back onto the main conveyor. The conveyor transports the bin to the next picking station, etc.

This type of pick-to-belt orderpicking systems can be used for items which differ in shape. On the other hand, the items are restricted in weight and size, since they have to be so small as to fit in the bins (and in the shelves, of course). A common size for the bins is $60 \times 40 \times 35$ cm ($l \times w \times h$).

The way the bins are identified by the transportation system is either by fixed barcodes, transponders or similar devices on the bins, or by a dynamic identification device that has to be adjusted by the pickers each time the bin has been pushed out at a picking station.

In this paper, the focus is on the second type of pick-to-belt orderpicking systems.

The advantage of pick-to-belt orderpicking systems is, that they are highly flexible in storage capacity as well as in pick capacity. By dividing the shelves into more slots, more (different) products can be stored. By simply bringing in more orderpickers, more orders can be handled. By reducing the number of orderpickers and letting each orderpicker handle several picking stations, the capacity can be reduced.

In a well-organised operation, the orderpickers are not in each other's way.

Because of these advantages and a fair price-performance ratio, pick-to-belt orderpicking systems of both types become more and more common in the Netherlands.

Although pick-to-belt orderpicking systems are flexible in capacity in principle, they are of course restricted by the length and the speed of the transportation system. The length of the transportation system is mainly determined by the number of shelf-storage aisles that have to be reached. Nevertheless, also the conveyor length is a parameter, since the number of aisles can be influenced by changing the length and height of the aisles. The length and the number of the aisles are determined so as to create efficient average walking distances for the pickers.

A common bin conveyor speed is about 0.5 m/s. Higher speeds (of up to about 2.3 m/s for fast sorting systems) can be reached, depending on the application. Higher speeds often involve substantial additional costs, partly due to problems with barcode or bin scanning response times.

A specific application area of pick-to-belt orderpicking systems is in high-volume picking operations. In such operations, the capacity of the system (length and speed), the number and location of picking stations and the number of pickers needed is of special importance.

In designing pick-to-belt orderpicking systems (of the second type discussed above), it is therefore important to have available a fast modeling and analysis tool to investigate the possible alternatives.

Pick-to-belt orderpicking systems, or systems with transportation systems involved in general, are hard to analyse. In an exact analysis, all possible positions of bins on the transportation system have to be included in the state space. Since the transportation system can contain a considerable number of bins, the state space becomes very large and hence unfit for exact analysis. In the literature, some attempts have been made to model and analyse transportation systems. See, for example, Van Ryzin (1987) or Van Sleeuwen (1987). Most commonly, the models dealt with are either simple, or the analysis is approximate.

In view of the difficulties in analysing pick-to-belt orderpicking systems, usually simulation is applied. A major drawback of simulation, especially in the design phase of a system is, however, that evaluating different alternatives requires different models, each model consuming a considerable amount of computer time for evaluation.

In this paper, a modeling and analysis method is proposed for pick-to-belt orderpicking systems. The aim of the method is, for a particular design, to provide fast information on throughput times of bins, picker utilization and the average number of bins in the system. Since the method is fast, it is possible to investigate the effects of changing the speed or the length of the transportation system, the number of picking stations or the number of pickers per station.

Since the modeling and analysis methods are fast, it is also possible to use them in a reversed way. They can be used, for a given approximate layout and given desired throughput and throughput times, to estimate such quantities as the necessary conveyor speed, the number of picking stations and pickers required, etc. In fact, this is the main purpose for which the model is used at Ingenieursbureau Groenewout B.V., since these quantities are directly related to investments and operational costs.

Both the modeling and the analysis method are approximate. In the model, it is assumed that picking times can be represented by exponential distributions, independent of the number of orderlines to be picked at the picking station into the bin. Furthermore, the probability that a particular bin has to be pushed out at a particular picking station is assumed to be representable by a Markov process, and hence independent of the particular bin.

The approximate analysis of the model is based on a Jackson network analysis (see for example Gelenbe and Pujolle, 1987). It is shown in this paper, that the approximate analysis is accurate. Furthermore, it is demonstrated by means of some examples that the approximate way of modeling pick-to-belt orderpicking systems leads to acceptable results.

The organisation of the paper is as follows. In Section 2, both the approximate model and the approximate analysis of this model are presented. In Section 3, numeric approximation results are compared with simulation of the approximate model. Section 4 compares the approximate model with more realistic models, for a number of example situations. Conclusions are drawn in Section 5.

2. The approximate model

A pick-to-belt orderpicking system is modeled as a network of connected conveyor pieces (often a roller or belt conveyor) and picking stations. Each conveyor piece can contain a finite number of bins. At the end of a conveyor piece, a transition is made by the bin to another conveyor piece, or to be pushed out to some picking station. These transitions are determined by Markov probabilities. At the picking stations, a number of order pickers pick orderlines into the bins arriving at the particular station. The service time per bin depends only on the picking station and is assumed to be exponentially distributed for all picking stations. After the bin has received service it is pushed onto another conveyor piece or it transfers to another picking station, with a Markov transition probability.

It is assumed that each picking station has infinite storage capacity for bins. This appears to be not a serious assumption in reality, because a picker responsible for a picking station will make sure that the system does not become blocked by his picking station becoming full. If a picking station tends to become full, the order picker can be instructed to temporarily put the bins on the floor.

Bins that have to be processed from outside arrive, with exponentially distributed interarrival times at the conveyor pieces and the picking stations. The speed of the conveyor pieces is assumed to be constant. It is also assumed that the conveyor pieces are preceded by an infinite buffer. This means that arrivals are not lost and picking stations and conveyors pieces cannot become blocked because of lack of output possibility. Although this assumption may seem to have drawbacks, the implications will be limited, since in reality conveyors are sufficiently long. In Section 4, it will be shown by some examples that the influence on performance is slight.

The notations used are as follows:

N = Number of conveyor pieces.

M = Number of picking stations.

v_i = Constant speed (expressed in bins per time unit) of conveyor piece i, $i = 1,\dots, N$.

K_i = Capacity (expressed in bins) of conveyor piece i, $i = 1,\dots, N$.

l_r = External arrival rate (expressed in bins per time unit) at conveyor piece or picking station r, $r = 1,\dots, N, N+1,\dots, N+M$. Interarrival times are assumed to be exponentially distributed.

c_j = Number of order pickers at station j (a picking station), $j = N+1,\dots, N+M$.

m_j = Service rate (expressed in bins per time unit) at station j (a picking station), $j = N+1,\dots, N+M$. Service times are assumed to be exponentially distributed.

The total system can be represented as a network of queues consisting of $N+M$ stations preceded by uncapacitated queues. Stations $1,\dots, N$ represent conveyor pieces and stations $N+1,\dots, N+M$ represent picking stations. At each station a bin receives service, after possible waiting for this service in a queue, which means either transportation, or a picking activity. After the service at a station, the bin jumps to another station to undergo a next service. The transition between these stations are determined by Markov transition probabilities:

p_{ij} = Probability of a transition after the service at station i, from station i to station j. $i,j = 1,\dots, N, N+1,\dots, N+M$; $p_{ij} \geqslant 0$, $i,j = 1,\dots, N+M$. Furthermore, $\Sigma_j p_{ij} \leqslant 1$, $i = 1,\dots, N+M$. $\Sigma_j p_{ij} < 1$ means that with probability $1 - \Sigma_j p_{ij}$ the bin is finished after service at station i and leaves the system.

The matrix of probabilities is indicated by $\boldsymbol{P}$. $\boldsymbol{P}$ is substochastic, since in the networks studied, there always is an i with $\Sigma_j p_{ij} < 1$, as the bins have to be able to leave the system. Such a station i is called an absorbing station. In the system studied, it also holds that $p_{ii} = 0$, for all i.

As an example, consider the network sketched in Figure 1, assuming that at the end of each conveyor piece (except piece 4), a bin has a probability of 0.5 to be pushed out to the next picking station or to continue its way to the next conveyor piece. Bins are placed on the conveyor at position (*) and they leave the system at the end of conveyor piece 4. In this case, $N = 4$, $M = 3$, and $l_r > 0$ for $r = 1$, and 0 otherwise.

$$\boldsymbol{P} = \begin{pmatrix} 0 & 0.5 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}.$$

The transition probabilities depend on the number of orderlines to be picked in the bin and the probability that one or more of these orderlines has to be picked at a particular picking station. This last mentioned probability depends on the number and type of articles stored in the shelf racks served by the picking station. If a picking station serves only a limited number of stored, slow moving products, the probability that an orderline has to be picked at that station will be small. In Section 4, some attention will be paid as to how these transition probabilities and also the picking times per bin have to be estimated, by means of some example cases.

Although the model may seem simple, it is still difficult to analyse. This is because in the analysis the positions of bins on the conveyor pieces have still to be incorporated and because of the fact that the speeds of the conveyor pieces are constant. Therefore the model is approximated.

One way of doing this could be the replacement of a conveyor piece of length K by K tightly coupled deterministic servers in series and applying an approximation method for this system. See Van Ryzin (1987) for a related approach. Here, this approach is not attractive, since in pick-to-belt orderpicking systems many long conveyor pieces can be involved.

The approach chosen in this paper is an approximation by a Jackson network, as follows. Conveyor piece i, for $i = 1, \dots, N$, is approximated by K_i exponential servers in parallel with rates equal to v_i/K_i. The storage capacity of the queue before each station i ($i = 1, \dots, N$) is infinite.

Note that if all servers at station i (a conveyor piece) are busy (meaning that conveyor piece i is totally full with bins), then the output rate of station i is precisely v_i. In the approximation, the output rate is proportional to the number of occupied places on the corresponding conveyor piece.

By this approximation, a Jackson network of queues is obtained which can easily be analysed in a short time. See Gelenbe and Pujolle (1987). All queues in a Jackson network can be analysed as if they were stand alone (the network can be decomposed).

The total arrival rate at each station (both internal and external) is given by

$$L_r = l_r + \sum_s L_s p_{sr}, \quad r = 1, \dots, N+M. \tag{1}$$

This can be rewritten in matrix form as $\mathbf{L} = \boldsymbol{I} + \mathbf{L}\boldsymbol{P}$. This equation can be solved for the vector $\mathbf{L}$, since the square matrix $\boldsymbol{I} - \boldsymbol{P}$ is non singular.

In this paper, system (1) is solved using the Gauss–Seidel iterative method (see for example Burden and Faires, 1985).

The utilization of station i, r_i, is now given by

$$r_i = \begin{cases} L_i/v_i & \text{for } i = 1, \dots, N, \\ L_i/(c_i^* m_i) & \text{for } i = N+1, \dots, M, \end{cases} \tag{2}$$

provided that both $v_i > L_i$, for $i = 1, \dots, N$ and $c_i > L_i/m_i$, for $i = N+1, \dots, M$. If this is not the case, then the system is not stable.

All queues in the network can now be analysed as stand alone $M/M/c$ queues, by standard methods. See for example Kleinrock (1975). The arrival rate at queue i is given by L_i; the service rate by v_i/K_i, for $i = 1, \dots, N$, and by m_i, for $i = N+1, \dots, N+M$. The number of servers is given by K_i, for $i = 1, \dots, N$, and by c_i, for $i = N+1, \dots, N+M$.

In the next section, simulation results of the approximate model are compared with approximate analysis through Jackson networks. This will be done for 3 example cases. The performance criteria compared are the average throughput time per bin, the average number of bins per station and in the system as a whole, and the utilization of the picking stations.

3. Numerical results

In simulating conveyors, it is important how the simulation is carried out. There is a difference between accumulating and non-accumulating conveyors. Belt conveyors are non-accumulating, whereas roller conveyors can be accumulating. For accumulating conveyors, if a bin cannot be pushed out to a station, the upstream loads accumulate and therefore may use less space, thus creating the possibility to store more loads on the conveyor. In the model of Section 2, the conveyor type is not important, since it is assumed that all queues preceding the stations have infinite capacity.

Also the way bins are transferred to a conveyor piece may be of importance. The conveyor sections may have fixed interval spacing windows. In such a case, the distance between loads is always an integer (actually an integer multiple of the size of the space a bin uses). Fixed interval spacings can be realised in two ways. First, if a bin transfers to a conveyor piece, it must wait until the leading edge of an empty window appears at the transfer point. A bin that is ready to transfer when the window's leading edge is just beyond the transfer point, must wait for the next opportunity to transfer. The second way fixed interval spacings can be realised is by immediately transferring if the bin is ready and the window is empty and by assuming that such a bin uses less space on the conveyor (or equivalently: it takes less time to convey this bin to the next position). The time needed to convey this bin to the next position is proportional to the window size that has not yet passed the transfer point at the instant of transfer.

The simulation model used in this section uses fixed interval spacings in combination with the second way of transferring bins described above. Note that in the approximation, a bin both immediately transfers if a server is idle and furthermore that it uses the full service time K_i/v_i. The impact of this approximation error becomes smaller if the conveyor length becomes longer. Since in reality the capacity of the transportation pieces is not too small (about 10–20 positions is common), the approximation may be reasonable.

The fact that the approximation becomes better indeed if the conveyor length increases, as argued above, is demonstrated by the following example.

Example. Assume that the system consists of only 1 conveyor piece of length K. Hence, $N = 1$, $M = 0$. Further data are: $\boldsymbol{I} = 10$; $\boldsymbol{v} = 15$.

In Table 1, simulation results are compared with the approximation, for varying values of K. The simulations were carried out by 1 run of 10 000 time units, preceded by an initialisation run of 50 time units, for all cases. In this table, L denotes the average number of bins in the system, S denotes the average throughput time per bin and p_0 denotes the probability that the system is empty.

For $K = 1$, the situation is equivalent to an M/D/1 queueing system, where the service time of the first customer in a busy period of the server differs from that of other customers. The mean service time of 'first' customers is $\frac{1}{30}$. The service time of other customers is $\frac{1}{15}$ time units (constant). For the system

Table 1
Comparison of simulation versus numeric results as a function of the conveyor size for a system with a single conveyor

K	Performance measure	Simulation	Numeric	Rel.error (%)
1	L	1.0 (1.08)	2.0	−100.0
	S	0.1 (0.108)	0.2	−100.0
	$1-p_0$	0.525 (0.5)	0.667	−27.1
2	L	1.65	2.40	−45.5
	S	0.166	0.240	−45.5
	$1-p_0$	0.755	0.800	−6.0
3	L	2.32	2.89	−24.6
	S	0.232	0.289	−24.6
	$1-p_0$	0.874	0.889	−1.7
5	L	3.66	3.99	−9.0
	S	0.366	0.399	−9.0
	$1-p_0$	0.967	0.968	−0.1
10	L	6.97	7.02	−0.7
	S	0.699	0.702	−0.4
	$1-p_0$	0.999	0.999	0.0

Table 2
Comparison of simulation versus numeric results for case 1

Station	Average number of bins in station		
	Simulation	Numeric	Rel.error (%)
1	35.3	35.6	−0.9
2	4.44	4.46	−0.5
3	4.44	4.46	−0.5
4	1.09	1.12	−2.8
5	1.05	1.12	−6.7
6	1.07	1.12	−4.7
Total in the system	47.3	47.8	−1.1
	Utilization of station		
4	0.522	0.528	−1.2
5	0.524	0.528	−0.8
6	0.518	0.528	−1.9
Average throughput time per bin	0.596	0.598	−0.3

Data for case 1: $N = M = 3$; $\boldsymbol{I} = (80, 0, 0, 0, 0, 0)$; $\boldsymbol{v} = (180, 180, 180)$; $\boldsymbol{m} = (30, 30, 30)$; $\boldsymbol{K} = (80, 10, 10)$; $\boldsymbol{c} = (1, 1, 1)$;

$$\boldsymbol{P} = \begin{pmatrix} 0 & 0.8 & 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0.8 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.2 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

with $K = 1$, it is possible to obtain exact results for L, S and p_0 using mean value analysis. In Table 1, exact results for the case $K = 1$ are indicated between brackets.

In the next simulations, also use is made of a single simulation run, preceded by an initialisation run. The length of the initialisation run is chosen such that several hundreds of bins have been processed by the system and that the system has reached a steady-state situation. The runlength is chosen such that several thousands of bins have been processed. Hence, the lengths of both the run and the initialisation run depend mainly on the arrival rates (assuming that the system is stable).

Case 1. The first example has a layout as sketched in Figure 1, except that conveyor piece 4 is not present. At the end of conveyor piece 3, the bins either leave the system, or they receive service at station 6 (i.e. picking station 3), after which they also leave the system. At the end of conveyor piece i there is a probability of 0.2 of a transfer to station $i + 3$ (i.e. picking station i), for $i = 1, \ldots, 3$. The data for this example can be found in Table 2.

The system was simulated for 500 time units, after an initialisation run of 30 time units. This leads to approximately $500 \times 80 = 40\,000$ bins to be processed. Since the system is stable, the throughput of the system is known (80 bins per time unit). The simulation results for case 1 are compared with numeric results in Table 2 as well. From the results in Table 2, it appears that the approximation by a Jackson network is accurate.

Case 2. Case 2 has the same configuration as case 1. This case was simulated for 2000 time units, with an initialisation run of 20 time units. The data and results can be found in Table 3. The approximation results for case 2 are also accurate.

Case 3. The layout of this case is based on a recently realised pick-to-belt system in the Netherlands. It consists of 14 roller conveyor pieces, connecting 13 picking stations, the last three of which act as checking, packing and sorting station. Once a bin arrives at such a packing station, it is removed from the conveyor after checking and packing.

The system consists of three parts: Part 1 is situated on the ground floor. It consists of conveyor pieces $1, \ldots, 9$ with picking stations, $1, \ldots, 8$. It has layout similar to the system sketched in Figure 1. At the end of conveyor piece 9, a bin can either go to Part 2 or Part 3. Part 2 is situated on a mezzanine and consists

Table 3
Comparison of simulation versus numeric results for case 2

Station	Average number of bins in station		
	Simulation	Numeric	Rel.error (%)
1	4.49	4.52	−0.7
2	4.56	4.52	0.9
3	4.56	4.52	0.9
4	1.78	1.70	4.5
5	0.74	0.76	−2.7
6	1.66	1.70	−2.4
Total in the system	17.8	17.7	0.6
	Utilization of station		
4	0.635	0.630	0.8
5	0.431	0.432	−0.2
6	0.642	0.630	1.9
Average throughput time per bin	0.982	0.984	−0.2

Data for case 2: $N = M = 3$; $\boldsymbol{I} = (18, 0, 0, 0, 0, 0)$; $\boldsymbol{v} = (40, 40, 40)$; $\boldsymbol{m} = (14.3, 16.7, 20)$; $\boldsymbol{K} = (10, 10, 10)$; $\boldsymbol{c} = (1, 1, 1)$;

$$\boldsymbol{P} = \begin{pmatrix} 0 & 0.5 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.6 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.7 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

of two further picking stations (picking stations 9 and 10). 70% of all bins go to the mezzanine, 30% of the bins go immediately from Part 1 to Part 3. Part 3 consists of the three checking and packing stations.

A schematic (and slightly simplified) layout is sketched in Figure 2. The detailed data of case 3 (time units are measured in hours) can be found in Table 4.

Table 4
Comparison of simulation versus numeric results for case 3

Station	Utilization of station		
	Simulation	Numeric	Rel.error (%)
15	0.137	0.132	3.6
16	0.134	0.132	1.5
17	0.124	0.132	−6.5
18	0.141	0.132	6.4
19	0.135	0.132	2.2
20	0.138	0.132	4.4
21	0.134	0.132	1.5
22	0.131	0.132	−0.8
23	0.130	0.132	−1.5
24	0.109	0.092	15.6
25	0.864	0.880	−1.9
26	0.909	0.880	3.2
27	0.892	0.880	1.4
Total average number of bins in the system	131.4	129.8	1.2
Average throughput time per bin	1.642	1.622	1.2

Data for case 3: $N = 14$, $M = 13$;
$\boldsymbol{I} = (80, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$;
$\boldsymbol{v} = (180, 180, 180, 180, 180, 180, 180, 180, 180, 180, 180, 180, 180, 180)$;
$\boldsymbol{m} = (30, 30, 30, 30, 30, 30\ 30, 30, 30, 30, 30, 30, 30)$;
$\boldsymbol{K} = (80, 10, 10, 10, 15, 10, 10, 10, 20, 40, 30, 30, 10, 10)$;
$\boldsymbol{c} = (1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$;

Notes to table 4 (continued):

$P =$

0	0.95	0	0	0	0	0	0	0	0	0	0	0	0	0.05	0	0	0	0	0	0	0	0	0	0	0	0	conv.1
0	0	0.95	0	0	0	0	0	0	0	0	0	0	0	0	0.05	0	0	0	0	0	0	0	0	0	0	0	conv.2
0	0	0	0.95	0	0	0	0	0	0	0	0	0	0	0	0	0.05	0	0	0	0	0	0	0	0	0	0	conv.3
0	0	0	0	0.95	0	0	0	0	0	0	0	0	0	0	0	0	0.05	0	0	0	0	0	0	0	0	0	conv.4
0	0	0	0	0	0.95	0	0	0	0	0	0	0	0	0	0	0	0	0.05	0	0	0	0	0	0	0	0	conv.5
0	0	0	0	0	0	0.95	0	0	0	0	0	0	0	0	0	0	0	0	0.05	0	0	0	0	0	0	0	conv.6
0	0	0	0	0	0	0	0.95	0	0	0	0	0	0	0	0	0	0	0	0	0.05	0	0	0	0	0	0	conv.7
0	0	0	0	0	0	0	0	0.95	0	0	0	0	0	0	0	0	0	0	0	0	0.05	0	0	0	0	0	conv.8
0	0	0	0	0	0	0	0	0	0.3	0.65	0	0	0	0	0	0	0	0	0	0	0	0.05	0	0	0	0	conv.9
0	0	0	0	0	0	0	0	0	0	0	0	0.667	0	0	0	0	0	0	0	0	0	0	0	0.333	0	0	conv.10
0	0	0	0	0	0	0	0	0	0	0	0.95	0	0	0	0	0	0	0	0	0	0	0	0.05	0	0	0	conv.11
0	0	0	0	0	0	0	0	0	0	0	0	0.667	0	0	0	0	0	0	0	0	0	0	0	0.333	0	0	conv.12
0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	0	0	0	0	0	0	0	0	0	0	0	0.5	0	conv.13
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	conv.14
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	pick st.1
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	pick st.2
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	pick st.3
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	pick st.4
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	pick st.5
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	pick st.6
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	pick st.7
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	pick st.8
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	pick st.9
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	pick st.10
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	pack st.11
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	pack st.12
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	pack st.13

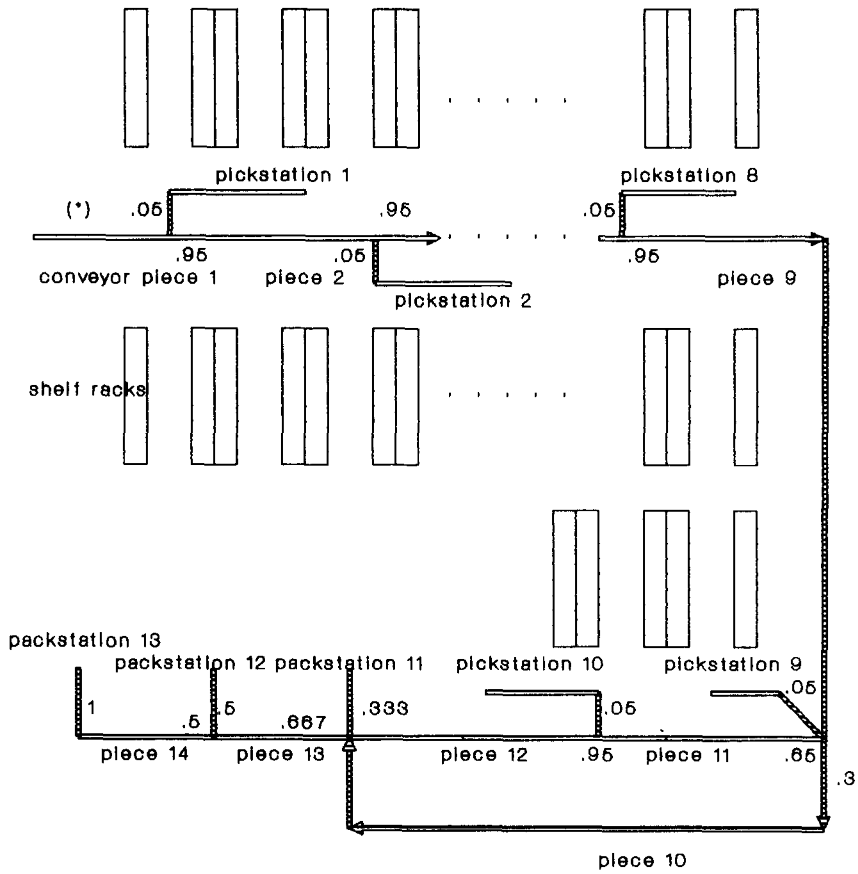

Figure 2. Central conveyor, consisting of 14 pieces, connecting 13 picking stations. Bins are placed on the conveyor at position (*). At packing stations 11, 12 and 13, the bins are removed from the system

From the transition matrix $\boldsymbol{P}$ in Table 4, it can be calculated that stations 25, 26 and 27 (the checking/packing stations 11, 12 and 13) are absorbing (the probability of a transition to all other stations is 0). The probability of absorption in each of these stations, starting in station 1, is $\frac{1}{3}$, which means that the average workload of these stations is the same and furthermore that each bin has a probability of $\frac{1}{3}$ of being dealt with in either station 25, 26 or 27.

In general, the probability of a visit to station i, starting in station j, q_j, can be calculated by solving the system of equations

$$(\boldsymbol{I}-\boldsymbol{P})\boldsymbol{q}=\boldsymbol{p}(i), \tag{3}$$

where $\boldsymbol{p}(i)$ is the vector $(p_{1i},\dots,p_{N+M,i})^{t}$.

Case 3 was simulated for 200 time units (hours), with an initialisation run of 10 time units. The results can be found in Table 4. The approximation results for case 3 are sufficiently accurate.

4. Modeling pick-to-belt orderpicking systems

In the model presented in Section 2, some questionable assumptions are being made. The most important ones are:

1. It is assumed that the probability that a certain bin has to be pushed out from a certain part of the conveyor system to a certain picking station is independent of the bin.
2. It is assumed that the service time of a bin at a picking station is independent of the actual number of orderlines that have to be picked within the bin.

Comment to point 1. The influence of this modeling error on the approximation results will be restricted, since the number of bins that have to be processed per day on pick-to-belt orderpicking systems is, in general, considerable. The probability that a bin has to be pushed out to a certain picking station from a certain part of the conveyor system in the model can be determined by making sure that the total probability of a visit to the station is approximately equal to the average fraction of bins that have to be processed at the station per day. The probability of a visit to a station, starting at station i, can be obtained by solving the system of linear equations (3).

For example, in the system of Figure 2 (case 3 of the previous section), the probability that a bin has to be pushed out at the end of conveyor piece i to be processed at station $i+14$ ($i=1,\ldots,8$) is 0.05. This is also the probability of a visit to station $i+14$, starting at station 1, as can be calculated easily from the matrix $\boldsymbol{P}$, by solving system (3). The probability of a visit to stations 23 and 24 (picking stations 9 and 10), starting at station 1, is $0.7\times 0.05=0.035$, for both stations. Hence, for the model to be accurate, these visit probabilities have to equal the fraction of bins to be processed at these stations.

Comment to point 2. The influence of the number of orderlines will increase if the probability that at a certain picking station more than one orderline has to be picked increases. The processing time at a picking station per bin can be approximated by an exponential distribution, with an average equal to the average picking time of the orderlines in the bin at the station, taking into account that more than one orderline may have to be picked at the station.

Suppose that the system contains M picking stations and has a layout similar to the one sketched in Figure 1. Assume that in each bin exactly n orderlines have to be picked. If the size of the picking areas is equal for all picking stations and if the distribution of product types (fast, medium and slow movers) over the storage slots is equal for all picking stations, then the probability that an orderline has to be picked at a particular picking station is $1/M$, where M is the number of picking stations. In such a case, the probability that a bin is pushed out from conveyor piece i to picking station i can be approximated by

$$\mathrm{P}[\text{bin is pushed out to picking station } i] = 1-((M-1)/M)^n. \tag{4}$$

The right-hand-side of (4) is the probability that at least one orderline has to be picked at picking station i.

The average service time per bin at station i given that the bin is pushed out at picking station i, $1/m_i$, can be approximated by

$$\begin{aligned}
&E[\text{service time per bin at picking station } i \,|\, \text{at least 1 orderline has to be picked at picking station } i]\\
&\quad=\sum_{j=1,\ldots,n} E[\text{picking time of } j \text{ orderlines at station } i \,|\, j-1 \text{ other}\\
&\qquad\qquad \text{orderlines have to be picked at picking station } i,\ |\, n-j\\
&\qquad\qquad \text{orderlines have to be picked at other stations}]\\
&\qquad\quad \times \mathrm{P}[j-1 \text{ other orderlines have to be picked at picking station } i,\\
&\qquad\qquad\qquad n-j \text{ orderlines have to be picked at other stations}].
\end{aligned}\tag{5}$$

In formula (5), P[$j-1$ other orderlines have to be picked at picking station i, $n-j$ orderlines have to be picked at other stations] is approximated by the prior probability for this event. That is, by $\binom{n-1}{j-1}(1/M)^{j-1}((M-1)/M)^{n-j}$, if all picking stations have equal probability for picking an orderline.

All examples studied in this section have a layout similar to Figure 1, except for a different number of picking stations (M) and conveyor pieces ($M+1$). It is assumed that the prior probability that an orderline has to be picked at station i, $i=1,\ldots,M$, is $1/M$.

In the following four examples, the effects of modeling the system as in Section 2, using formulas (4) and (5), are studied. This is done by comparing detailed simulation results of the exact model with

Table 5
Comparison of simulation versus numeric results for case 4

Station	Utilization of station		
	Simulation	Numeric	Rel.error (%)
8	0.488	0.50	−2.5
9	0.494	0.50	−1.2
10	0.489	0.50	−2.3
11	0.501	0.50	0.2
12	0.504	0.50	0.8
13	0.518	0.50	3.5
Average number of bins in the system	14.2	14.0	1.4
Average throughput time per bin	283.8	280.0	1.3

Data for case 4: $N = 7$, $M = 6$; $\boldsymbol{I} = (0.05, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$;
$\boldsymbol{v} = (0.5, 0.5, 0.5, 0.5, 0{,}5, 0.5, 0.5)$;
$\boldsymbol{K} = (20, 10, 10, 10, 10, 10, 10)$; $\boldsymbol{c} = (1, 1, 1, 1, 1, 1)$.
Formulas (4) and (5) lead to
$\boldsymbol{m} = (0.0167, 0.0167, 0.0167, 0.0167, 0.0167, 0.0167)$ and

$$\boldsymbol{P} = \begin{pmatrix} 0 & 0.833 & 0 & 0 & 0 & 0 & 0 & 0.167 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.833 & 0 & 0 & 0 & 0 & 0 & 0.167 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.833 & 0 & 0 & 0 & 0 & 0 & 0.167 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.833 & 0 & 0 & 0 & 0 & 0 & 0.167 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.833 & 0 & 0 & 0 & 0 & 0 & 0.167 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.833 & 0 & 0 & 0 & 0 & 0 & 0.167 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

approximate analysis of the approximate model. The simulation results were obtained using the Automod simulation package. For each bin, the number of orderlines that have to be picked into the bin is drawn according to a probability distribution. Then, for each orderline in the bin, the picking station where the orderline has to be picked is randomly drawn, according to a probability distribution. The transportation system sequentially transports each bin to all picking stations where it has to be processed. If i orderlines have to be picked at a picking station, then the total picking time per bin is obtained by i sequential draws from the probability distribution of the picking time per orderline at the picking station concerned. The simulation program assumes accumulating conveyor pieces (roller conveyors).

In contrast to the approximate model, in the exact model there is no buffer before the conveyor pieces (except for pieces where external arrivals occur, so that arrivals are not lost). Hence, blocking of conveyor pieces may occur (though this will be rare, since picking stations are preceded by infinite buffers). A bin, coming from a conveyor piece or picking station, can only enter a transportation piece if sufficient physical space is available.

Case 4. The layout of this system is similar to the layout sketched in Figure 1, but with $N = 7$, $M = 6$. Each bin contains exactly 1 orderline. It is assumed, that the probability than an orderline has to be picked at a picking station is $\frac{1}{6}$. The average service time per orderline is 60 time units at each station, exponentially distributed. At each picking station there is 1 orderpicker. The capacity of the first conveyor piece is 20 bins. The capacity of the other 6 conveyor pieces is 10 bins. The arrival rate of bins that have to be processed at conveyor piece 1 is 0.05. The speed of the conveyor is 0.5 bins per time unit. The data for the approximate model are given in Table 5.

The results of the approximation by the method of Section 2 and detailed simulation of the exact model can also be found in Table 5. The simulation results were obtained by 1 run of 288 000 time units, preceded by an initialisation run of 3600 time units. The results are good, which was to be expected, since exactly one orderline per bin has to be picked. This makes that the picking time per bin is exponentially distributed at each picking station. Therefore, in the next example, it is assumed that 1 or 2 orderlines have to be picked per bin. Hence, the processing time per bin is no longer exponentially distributed.

Case 5. This case is the same as case 4, except that P[1 orderline has to be picked within the bin] = P[2 orderlines have to be picked within the bin] = 0.5. The probability that a bin has to be pushed out from conveyor piece $i-1$ to picking station i is approximated by

$$\begin{aligned}&\mathrm{P}[\text{bin is pushed out to picking station } i]\\&\quad=\mathrm{P}[\text{bin is pushed out to picking station } i \mid 1 \text{ orderline has to be picked within the bin}]\\&\qquad\times \mathrm{P}[1 \text{ orderline has to be picked within the bin}]\\&\qquad+\mathrm{P}[\text{bin is pushed out to picking station } i \mid 2 \text{ orderlines have to be picked within the bin}]\\&\qquad\quad\times \mathrm{P}[2 \text{ orderlines have to be picked within the bin}]\\&\quad=\tfrac{1}{6}\times\tfrac{1}{2}+\tfrac{11}{36}\times\tfrac{1}{2}=0.2361.\end{aligned}$$

The average service time per bin at picking station i, given that the bin is pushed out at picking station i, $1/m_i$, is approximated by

$$\begin{aligned}1/m_i&=\mathrm{E}[\text{picking time of 1 orderline} \mid 1 \text{ orderline has to be picked into the bin}]\\&\quad\times\mathrm{P}[1 \text{ orderline has to be picked into the bin}]\\&\quad+\mathrm{E}[\text{picking time of 1 orderline} \mid 2 \text{ orderlines have to be picked into the}\\&\qquad\qquad\text{bin, 2nd orderline does not have to be picked at station } i]\\&\quad\times\mathrm{P}[2 \text{ orderlines have to be picked into the bin, 2nd orderline does}\\&\qquad\qquad\text{not have to be picked at station } i]\\&\quad+\mathrm{E}[\text{picking time of 2 orderlines} \mid 2 \text{ orderlines have to be picked into the}\\&\qquad\qquad\text{bin, 2nd orderline also has to be picked at station } i]\\&\quad\times\mathrm{P}[2 \text{ orderlines have to be picked into the bin, 2nd orderline also}\\&\qquad\qquad\text{has to be picked at station } i]\\&=60\times\tfrac{1}{2}+60\times\tfrac{5}{6}\times\tfrac{1}{2}+2\times 60\times\tfrac{1}{6}\times\tfrac{1}{2}=65\quad\text{for } i=8,\ldots,13.\end{aligned}$$

The resulting data for the approximate model can be found in Table 6. The simulation and approximation results are also given in Table 6. The simulation results were obtained by 1 run of 288000 time units, preceded by an initialisation run of 3600 time units. The results are again good.

In the last two cases, the bins all contain 3 orderlines. In case 6, the service time distribution per orderline is exponentially distributed. In case 7, the service time distribution per orderline is hyperexponentially distributed.

Case 6. This case is the same as case 4, except that P[3 orderlines have to be picked within the bin] = 1, $K=(40, 20, 20, 20, 20, 20, 20)$. Furthermore, the picking time per orderline is exponentially distributed with an average of 24 time units at each station.

Formula (5) leads to

$$1/m_i=24\times\left(\tfrac{5}{6}\right)^2+2\times 24\times\left[\tfrac{1}{6}\times\tfrac{5}{6}+\tfrac{5}{6}\times\tfrac{1}{6}\right]+3\times 24\times\left(\tfrac{1}{6}\right)^2=32\quad\text{for } i=8,\ldots,13.$$

Formula (4) leads to

$$\mathrm{P}[\text{bin is pushed out to picking station } i]=1-\left(\tfrac{5}{6}\right)^3=0.4213\quad\text{for } i=8,\ldots,13.$$

Additional data and simulation and approximation results can be found in Table 7. The simulation results were obtained by 1 run of 288000 time units, preceded by an initialisation run of 3600 time units.

Table 6
Comparison of simulation versus numeric results for case 5

Station	Utilization of station		
	Simulation	Numeric	Rel.error (%)
8	0.751	0.767	−2.1
9	0.744	0.767	−3.1
10	0.743	0.767	−3.2
11	0.784	0.767	2.2
12	0.718	0.767	−6.8
13	0.767	0.767	0.0
Average number of bins in the system	29.2	27.8	4.8
Average throughput time per bin	584.6	555.7	4.9

Data for case 5: $N=7$, $M=6$; $\boldsymbol{I}=(0.05, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$;
$\boldsymbol{v}=(0.5, 0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5, 0.5)$; $\boldsymbol{K}=(20, 10, 10, 10, 10, 10, 10)$;
$\boldsymbol{c}=(1, 1, 1, 1, 1, 1)$; $\boldsymbol{m}=(0.0154, 0.0154, 0.0154, 0.0154, 0.0154, 0.0154)$;

$$\boldsymbol{P}=\begin{pmatrix}
0 & 0.764 & 0 & 0 & 0 & 0 & 0 & 0.236 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0.764 & 0 & 0 & 0 & 0 & 0 & 0.236 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0.764 & 0 & 0 & 0 & 0 & 0 & 0.236 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0.764 & 0 & 0 & 0 & 0 & 0 & 0.236 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0.764 & 0 & 0 & 0 & 0 & 0 & 0.236 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0.764 & 0 & 0 & 0 & 0 & 0 & 0.236\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}.$$

Table 7
Comparison of simulation versus numeric results for case 6

Station	Utilization of station		
	Simulation	Numeric	Rel.error (%)
8	0.606	0.674	−11.2
9	0.623	0.674	−8.2
10	0.599	0.674	−12.5
11	0.612	0.674	−10.1
12	0.620	0.674	−8.7
13	0.623	0.674	−8.2
Average number of bins in the system	28.0	28.4	−1.4
Average throughput time per bin	545.0	568.2	−4.3

Data for case 6: $N=7$, $M=6$; $\boldsymbol{I}=(0.05, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$;
$\boldsymbol{v}=(0.5, 0.5, 0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5)$; $\boldsymbol{K}=(40, 20, 20, 20, 20, 20, 20)$;
$\boldsymbol{c}=(1, 1, 1, 1, 1, 1)$; $\boldsymbol{m}=(0.0313, 0.0313, 0.0313, 0.0313, 0.0313, 0.0313)$;

$$\boldsymbol{P}=\begin{pmatrix}
0 & 0.579 & 0 & 0 & 0 & 0 & 0 & 0.421 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0.579 & 0 & 0 & 0 & 0 & 0 & 0.421 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0.579 & 0 & 0 & 0 & 0 & 0 & 0.421 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0.579 & 0 & 0 & 0 & 0 & 0 & 0.421 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0.579 & 0 & 0 & 0 & 0 & 0 & 0.421 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0.579 & 0 & 0 & 0 & 0 & 0 & 0.421\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Table 8
Comparison of simulation versus numeric results for case 7

Station	Utilization of station		
	Simulation	Numeric	Rel.error (%)
8	0.608	0.674	−10.9
9	0.597	0.674	−12.9
10	0.586	0.674	−15.0
11	0.598	0.674	−12.7
12	0.607	0.674	−11.0
13	0.593	0.674	−13.7
Average number of bins in the system	26.1	28.4	−8.8
Average throughput time per bin	527.8	568.2	−7.7

The simulation results lead to greater relative errors in the utilization of the servers than in the previous two cases. Nevertheless, the results are still reasonable.

Case 7. This case is the same as case 6, except that the picking time per orderline is hyperexponentially distributed at each station. With probability 0.5, the picking time per orderline is exponentially distributed with an average of 12 time units and with probability 0.5, the picking time per orderline is exponentially distributed with an average of 36 time units. Hence, the average picking time per orderline is 24 time units at each station. This implies that the approximate model for this case is the same as that of case 6.

The simulation and approximation results are given in Table 8. The simulation results were obtained by 1 run of 432000 time units, preceded by an initialisation run of 7200 time units. The simulation results leads to even greater relative errors in the utilization of the servers and also in the average number of bins in the system and the average throughput time per bin. Nevertheless, the results are still reasonable. Absolute errors are small and furthermore, the numeric approximation leads to an overestimation of server utilization, average work-in-process and throughput times, which is for practical purposes more acceptable than underestimates.

5. Conclusion

In this paper, a modeling and approximate analysis method is proposed for pick-to-belt orderpicking systems. The method can be used in the design phase of such systems, to quickly evaluate alternatives. These alternatives may differ in the length and layout of the system, the number of picking stations, the number of pickers per station, the conveyor speed, the number of bins to be processed per day, the number of orderlines per bin, etc.

In the early stage of design, comparing the performance of different alternatives is often more important than accurately estimating the performance of one particular model. Especially for comparing different alternatives, the model and analysis method can be useful.

The quality of the approximation method is acceptable for practical purposes. The approximation mostly leads to an overestimation of the average number of bins in the system, the average bin throughput times and picker utilization. This means that the real design will most likely not perform worse than the approximation. Both the modeling method and the approximation method are fast, since Jackson networks are used.

At Ingenieursbureau Groenewout B.V., the method described in this paper is used for the fast evaluation of designs of pick-to-belt orderpicking systems. It is also used for sorting systems and, in general, for the evaluation of systems where conveyors are involved. It is used as a design tool, which can be followed by detailed simulation analysis of a chosen design, if so desired. The method is also used in a reversed way, for determining the number of picking stations, the number of pickers per station and the necessary conveyor speed, given a certain conveyor length and performance requirements.

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