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Authors: Harwin de Vries, Jories van de Klundert, Albert P.M. Wagelmans

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The Roadside Healthcare Facility Location Problem

A Managerial Network Design Challenge

Harwin de Vries ∗1, Joris van de Klundert2, and Albert P.M. Wagelmans3

1Rotterdam School of Management, Erasmus University, Rotterdam, The Netherlands, harwin.devries@rsm.nl
2Prince Mohammad Bin Salman College for Business & Entrepreneurship, King Abdullah Economic City, Kingdom of Saudi Arabia, jklundert@mbsc.edu.sa
3Econometric Institute, Erasmus University Rotterdam, Rotterdam, The Netherlands, wagelmans@ese.eur.nl

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Abstract

The population of truck drivers plays a key role in the spread of HIV and other infectious diseases in sub-Saharan Africa. Truck drivers thereby affect the health and lives of many, but also suffer from poor health and significantly reduced life expectancy themselves. Due to professional circumstances, their health service needs are generally not well addressed. Therefore, the non-governmental organization North Star Alliance builds a network of healthcare facilities along the largest trucking routes in sub-Saharan Africa. This paper studies the problem where to place additional facilities, and which health service packages to offer at each facility. The objective combines the maximization of the patient volume at these facilities and the maximization of the effectiveness of the health service delivery to the population served. The latter criterion is modeled through three novel access measures which capture the needs for effective service provisioning. The resulting optimization problem is essentially different from previously studied healthcare facility location problems because of the specific mobile nature of health service demand of truck drivers. Applying our model to the network of major transport corridors in South-East Africa, we investigate several prominent questions managers and decision makers face. We show that the present network expansion strategy, which primarily focuses on patient volumes, may need to be reconsidered: substantial gains in effectiveness can be made when allowing a small reduction in patient volumes. We furthermore show that solutions are rather robust to data impreciseness and that long term network planning can bring substantial benefits, particularly in greenfield situations.

∗Corresponding author.
Keywords: roadside healthcare; truck drivers; facility location; continuous access; effectiveness

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1 Introduction

Sub-Saharan African truck drivers work under difficult conditions. Their trips last up to several weeks, during which time they are separated from their spouses and social-cultural norms and in which they are challenged by monotony, risk of hijacking, loneliness, and long waiting times at border posts. This working environment has been shown to be conducive to their engagement in behaviors that bring about high risks of communicable diseases such as tuberculosis (TB), malaria, HIV, and other sexually transmitted infections (STIs) (Botão et al. 2015, Delany-Moretlwe et al. 2014, Gomez et al. 2013, Apostolopoulos and Sönmez 2007). For instance, Ramjee and Gouws (2002) report a study on a population of 310 truck drivers in South Africa, 70% of whom had wives or girlfriends in rural areas, 13% used a condom during their last sexual encounter, and 56% were HIV positive. Similar behaviors and high prevalences of communicable diseases in general and STIs in particular are also reported for truck driver populations in India (Pandey et al. 2008, Roa et al. 1997), Brazil (Malta et al. 2006, Lacerda et al. 1997), China (Wong et al. 2007, Chen et al. 2006), the Baltic Region (Kulis et al. 2004), and the USA (Lichtenstein et al. 2008, Solomon et al. 2004, Stratford et al. 2000).

The high prevalence of communicable diseases among sub-Saharan truck drivers and their sex partners has a dramatic impact on their health and life expectancy (Delany-Moretlwe et al. 2014, Matovu and Ssebadduka 2012, Apostolopoulos and Sönmez 2007). For example, Wilson (2005) mentions a firm that “lost 39 out of 144 drivers in the last three years”. Moreover, because of their mobility and behaviors, truck drivers have substantially contributed to the spread of HIV and other communicable diseases among the sub-Saharan population at large (Ramjee and Gouws 2002, Gomez et al. 2013, Apostolopoulos and Sönmez 2007, Morris and Ferguson 2007, 2006, Laukamm-Josten et al. 2000, Caldwell et al. 1999, Hudson 1996). In addition, the disease burden has impoverishing effects on patients, decreases labor productivity of firms, and slows down economic growth (Sachs and Malaney 2002, Ahlburg 2000, Stover and Bollinger 1999).

Delivering prevention, diagnosis, and treatment services to sub-Saharan truck drivers is consid-
ered highly effective to reduce their burden of disease (Matovu and Ssebadduka 2012, ILO 2005, Ramjee and Gouws 2002). Unfortunately, traditional healthcare facilities struggle to provide these services as they are often not easily accessible by trucks, have insufficient parking space, require detours, have long waiting times, and are only open during the day whereas drivers usually stop during the evening (Delany-Moretlwe et al. 2014, Gatignon and Van Wassenhove 2008, Ferguson and Morris 2007, IOM 2003, Ramjee and Gouws 2002, Ferguson and Morris 2007, ILO 2005).

The non-governmental organization (NGO) North Star Alliance (North Star) aims to improve the health of the sub-Saharan truck driver population and roadside communities. It operates 55 Roadside Wellness Centers (RWCs) or “Blue Boxes”, which are easily accessible by truck and located at major truck stops and border crossings along sub-Saharan Africa’s main transport corridors. The RWCs provide primary care services, behavior change communication, condom distribution, HIV and STI voluntary counseling and testing, screening for TB and malaria, and treatment for HIV, malaria, TB, and STI. Part of these services are provided during outreach sessions at “hotspots” (e.g., parking lots, bars, and brothels).

This paper focuses on important managerial network design challenges/ questions that were identified during our collaboration and conversations with North Star and were not addressed in earlier work related to our topic (see Ares et al. 2016, and our discussion below). The first relates to the criteria considered when expanding the network – i.e., adding new RWCs. The NGO has so far based location decisions mostly on patient volume, positioning new RWCs where the expected number of patient visits is highest. Because of the long travel times and the relative sparsity of the network of roadside clinics, however, the effectiveness of health service provisioning – the degree to which it improves health – is often hampered by access discontinuities. In other words, travel time gaps between RWCs along a truck route are often too large. Hence, maximizing patient volume solely may not bring maximum gains in health effects for the population. To incorporate effectiveness in future network expansions, it is key to precisely define how it relates to access. Doing so is, however, far from trivial, partly since the relationship differs per health service. Accessibility requirements for “effective” HIV care may differ from those for primary care. Our first research objective is to develop models for health service effectiveness that capture this variety.

Second, North Star has been offering virtually identical health services at all of its RWCs. As indicated, accessibility requirements differ per health service and disease prevalence varies across the
network. In view of the scarce financial resources, this suggests that differentiation or centralization of certain service packages may be beneficial. Next to a standard primary care service package, North Star considers to introduce optional service packages. For example, North Star has piloted offering a package of HIV treatment services (see Gomez et al. 2013, Lalla-Edward et al. 2017). Evidently, potential locations of service packages are determined by locations of new facilities (and current facilities), which makes integrated decision making beneficial. Our second research objective is to model and solve the problem of optimizing integrated location and service differentiation choices.

Third, we aim to explore trade-offs between volume and effectiveness for this problem. How does the present network expansion strategy, which primarily focuses on patient volumes, perform in terms of effectiveness? How would an increased focus on effectiveness affect patient numbers? Answers to these questions form important inputs for strategic choices on objectives (cf. Ferdows and De Meyer 1990). They also fuel discussions with donors, which commonly evaluate impact through volume indicators rather than actual health impacts (Bolton et al. 2007).

Our fourth research objective is to assess the importance of long term network planning. The NGO makes expansion decisions sequentially and has not explicitly accounted for future expansions while doing so. The advantage of this method is its simplicity: advanced solution methods may not be required when choosing locations of few new facilities. On the other hand, the present myopic approach may lead to sub-optimality in the long run. At present, scientific and practical understanding of this matter is lacking.

Finally, North Star operates in a context where data on truck routes and volumes is scarce and imprecise. Quantification of objectives may also be biased. Impreciseness generally impedes trust in data-driven decision making (De Vries and Van Wassenhove 2019). Our fifth research objective is to inform decision makers on the magnitude of this risk by assessing robustness of “optimal” location decisions.

We pursue these five objectives by modelling and studying the roadside healthcare facility location problem (RHFLP): Given an existing transportation network and set of RWCs, and a limited expansion budget, where to place additional RWCs, and which service packages should each of the RWCs offer, so as to maximize a weighted sum of health service effectiveness and volume of truck drivers served? We present both analytical and numerical results, using the network of major
transport corridors in South-East Africa as a case study.

Our work is the result of the network design project we started in 2011, together with the NGO and ORTEC Consulting Group. The research presented by Ares et al. (2016) forms a first publication that spun off from this project. They analyze the problem of locating a fixed number of clinics based on the objectives to enhance equity of access to healthcare and to maximize patient volume and effectiveness. Our research advances on Ares et al. (2016) in the first place by distinguishing multiple health service packages and analyzing service differentiation, which motivate research objectives 1 and 2. As our aim is to address key managerial challenges posed to us by practitioners, which deal with volume and effectiveness measures, we especially focus our analyses on these two criteria, and present corresponding extensive trade-off analyses (see research objective 3). In section 5.8, we turn our attention to equity considerations along the lines of Ares et al. (2016). Our comparison between long-term and incremental planning approaches and sensitivity analyses on data impreciseness (see research objectives 4 and 5) are also novel contributions. To these purposes, we extend the mixed-integer linear programming (MILP) formation presented by Ares et al. (2016) and use a more elaborate case study: it incorporates more recent demand data and adds two major truck corridor networks that span eight East African countries.

Our results show that standard software solves real life RHFLP instances within reasonable computation times (less than 2.5 hours). This is an important finding, given that we prove that RHFLP is strongly $\mathcal{NP}$-hard. We also show that the current practice of selecting locations primarily based on patient volumes may need to be reconsidered. Substantial gains in effectiveness can be made when allowing a small reduction in patient volumes (e.g., $+13.3\%$ vs. $-1.7\%$ for our baseline case). A third key insight is that solutions are rather robust to various types of data impreciseness. We analyze hundreds of instances in which “true” parameter values differ substantially from assumed values (up to 100%), and show that the resulting optimality gap rarely exceeds 4%. Fourth, we show that long-term network planning can bring substantial benefits in greenfield situations (up to 7.3% in the instances considered). These benefits tend to disappear when the density of the network increases. Our results suggest the existence of a “tipping point” at which one can switch to consecutive decision making without risking substantial suboptimality. This suggests that there is no need to invest in expensive decision support software in such cases, which is again an important finding.

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The remainder of this paper is organized as follows. Section 2 presents a literature review. In Section 3 we present measures of access to health services for mobile patients. A MILP formulation of the location problem and some analytical results are presented in Section 4. The results of our numerical experiments are described in Section 5. Finally, in Section 6 we summarize our findings and present conclusions.

2 Literature Review

Our work extends the growing literature on operations research applications in global health/public health in low and middle income countries (see Yadav (2010) and White et al. (2011) for reviews). Papers from this field typically deal with the use or allocation of resources that improve health in heavily resource-constrained contexts. Kraiselburd and Yadav (2013) stress the importance of this field by describing the deficiencies in global health supply chains. The World Health Organization (WHO) also acknowledges the added value these disciplines can have for global health (Royston 2011).

Applications that use facility location models in this context have been reviewed by Daskin and Dean (2004), Rahman and Smith (2000). The authors show that existing models are variants of the set covering model, the maximal covering model, and the P-median model, which optimize or put constraints on some function of the patients’ distance or travel time to nearby health facilities. The applicability of these models in the context at hand is, however, rather limited. Truck drivers are highly mobile, so that their distance or travel time to a facility is constantly changing. In more recent work, Deo and Sohoni (2015) develop a model for allocating a health service to health clinics, Smith et al. (2009) consider the problem of locating primary healthcare workers, Griffin (2012) constructs strategies to allocate treatments to health centers and populations, and McCoy and Lee (2014) consider the allocation of motorcycle capacity to outreach sites. McCoy and Johnson (2014) investigate how to optimally allocate clinic capacity over time, and explicitly model the impact on HIV treatment adherence. None of these articles, however, considers optimizing access to mobile patients.

Our work is also related to mobile healthcare facility planning problems (see e.g. Doerner et al. 2007, Hachicha et al. 2000, Hodgson et al. 1998), as these problems also consider the time intervals
between moments of health service provisioning. These papers, however, consider the optimization of routing decisions whereas we consider location decisions.

Our problem seems to most closely resemble a facility location problem which balances the maximization of the total node demand covered by the facilities with the maximization of the total flow demand covered by the facilities. Such flow represents a collection of truck drivers traveling the same route and demanding a given service. Their “degree of coverage” represents the effectiveness of service provisioning, i.e. the resulting improvement in health, which we define as a function of the level of access to the service along the route. Our problem is therefore closely related to flow interception facility location problems (FIFLPs), which have the objective of intercepting demand units on their pre-planned paths (see Boccia et al. (2009) and Hodgson (1998) for reviews of FIFLPs). Most FIFLPs assume that a demand unit is covered if and only if there exists at least one “conveniently located facility”, such as a facility along its path or within a given distance from its path. The fact that it is beneficial to offer a service at multiple locations along a path makes our problem a multi-coverage FIFLP. Other than the aforementioned work of Ares et al. (2016), an extensive search yielded only two other multi-coverage FIFLPs: the billboard location problem (Averbakh and Berman 1996) and the flow refueling location problem (De Vries and Duijzer 2017, Kuby and Lim 2005). Though these problems show much similarity to the RHFLP, it is not possible to directly apply models for these problems to solve the RHFLP. As we will explain in the next sections, we measure the benefits of providing access to facilities by means of a continuous variable which depends on the travel time intervals between adjacent facilities along a route. In contrast, the billboard location problem measures these benefits based on the number of billboards along a route. The flow refueling location problem considers the driving times between adjacent facilities but differs from our problem in that it defines that a fixed amount of benefits are obtained if and only if specified travel time interval constraints are met. For a comprehensive literature review on related facility location problems, we refer to De Vries (2011).

Finally, the closely related paper by Ares et al. (2016) considers the problem of locating a fixed number of roadside clinics using a three-criteria objective function which considers volume, effectiveness and equity. They propose an advanced column generation algorithm to deal with the computational complexities induced by equity considerations. The model considered in their work differs from our model in that 1) it includes an equity criterion and 2) it does not consider service
differentiation decisions and the corresponding variety of effectiveness models (their work uses one of the access measures elaborated below, following expert opinion). See Section 1 for our discussion on how our work builds and advances upon theirs.

3 Continuous Access

We collected expert opinion by interviewing ten of North Star’s staff members from the headquarters and regional offices on model objectives and requirements. See Appendix A for interviewee characteristics. Interviewees were added till saturation was reached (Saunders et al. 2017). These interviews confirmed the importance of capturing the relationship between access and effectiveness, and revealed how this relationship varied among diseases.

For some health services, a critical time limit (CTL) exists which defines a time limit for access to healthcare. For instance, providing antimalarial drugs within 24 hours of onset of symptoms is essential for treatment effectiveness (Khatib et al. 2013, Johnson et al. 2013). The dichotomous variable of having access to a health service facility within 24 hours is therefore commonly used as performance indicator (see e.g. Chuma et al. 2009, WHO/UNICEF 2003). A critical time-limit may also apply to health services that need to be accessed at fixed time-intervals. One example is directly observed therapy for TB patients, in which a health provider frequently monitors a patient swallowing the required drugs (Volmink and Garner 2007).

For many services, however, responsiveness is less dichotomous. Instead, health impacts of postponing access gradually build up over time. In such cases, a recommended time limit may apply, in addition to a critical time-limit. Such a distance-decay relationship has been confirmed in a variety of studies (see e.g. McLaren et al. 2014, Cooke et al. 2010, Tanser et al. 2006), and are common for STIs (see e.g. Meyer-Weitz et al. 2000, Moses et al. 1994), for which a positive relation exists between treatment delay and disease progression and transmission (Hook III et al. 1997).

Thirdly, there are services for which specific time limits are lacking, but improved access is associated with higher utilization, less treatment delay, and improved treatment adherence, which in turn lead to lower mortality and morbidity, less drug resistance, and less disease transmission. Examples include HIV testing and treatment services (De Vries et al. 2014, Mills et al. 2006).
3.1 Types of Service Packages

On the basis of the three aforementioned relationships between access and effects, we now propose a classification of service packages in three service package types. The mapping of North Star’s service packages to the three service package types is presented in Section 5.1.

**CTL. Service packages with a critical time-limit of access.**

This package type corresponds to diseases for which access within a time-limit from the moment of (self) diagnosis is crucial and to health services that need to be accessed at fixed time-intervals.

**RCTL. Service packages with a recommended time-limit and a critical time-limit of access.**

This service package type refers to health service packages for diseases for which a recommended time-limit and a critical time-limit are well defined, and that are also characterized by a continuous relationship between access and effects.

**ASAP. Service packages that are to be accessed as soon as possible when needed.**

This last package type corresponds to health service packages for diseases for which there exists a distance-decay relationship without a clear time-limit of access.

3.2 Access Measures

We now propose access measures corresponding to the service package types introduced in the previous section. Following interview results we assume that truck drivers cyclically travel from origin \(O\) to destination \(D\) and back to origin \(O\) (cf. Ferguson and Morris 2007, Orubuloye et al. 1993). Figure 1 illustrates the access measures for part of such a round trip.

The boxes with the cross represent a moment at which some service package \(s\) is passed. Let us consider a given moment \(t\) during a trip. We define the truck driver’s access time to package \(s\) as the travel time to the next facility along his route that offers package \(s\). (Our interviews revealed that returning to a facility the driver already passed is generally not an option.) We refer to a facility that offers package \(s\) as a package \(s\) location from now on.

Suppose that service package \(s\) is of package type \(CTL\). At a given point in time, we say that a truck driver is “covered” if his access time to package \(s\) is at most the critical time-limit.
Obviously, the larger the part of the time-line the truck driver is covered, the more effective the service provisioning can be. Therefore, we define $a_s^{\text{CTL}}$, the access measure for a package $s$ that is of package type $\text{CTL}$, as the fraction of time the truck driver is covered (see the upper part of Figure 1: the truck driver is covered in the light parts of the time-line). More formally, letting $T$ denote the duration of the round trip, $\tau_s^{\text{CTL}}$ the critical time-limit for accessing package $s$, and $\Delta_s(t)$ the access time to package $s$ at time $t \in [0, T]$, measure $a_s^{\text{CTL}}$ is calculated as:

$$a_s^{\text{CTL}} = \frac{1}{T} \int_{t=0}^{T} 1(\Delta_s(t) \leq \tau_s^{\text{CTL}}) dt$$

(1)

Second, suppose that service package $s$ is of package type $\text{RCTL}$. Now, we say that a truck driver is “covered” if his access time is smaller than the recommended time-limit, that he is “not covered” if his access time exceeds the critical time-limit, and that he is “partially covered” otherwise. We assume that the degree of coverage with package $s$ decreases linearly from 1 (covered) to 0 (not covered) as the access time increases from the recommended time-limit to the critical time-limit (see the middle part of Figure 1: the lower the truck driver’s degree of coverage, the darker the shade of grey). Next, we define $a_s^{\text{RCTL}}$, the access measure for a package $s$ that is of package type $\text{RCTL}$, as the average degree of coverage with package $s$ during a trip. Formally, let the recommended time-limit and the critical time-limit of access be represented by $\tau_1^{\text{RCTL}}$ and $\tau_2^{\text{RCTL}}$, respectively. Then $a_s^{\text{RCTL}}$ is calculated as:
\[
\sigma_s^{RCTL} = \frac{1}{T} \int_{t=0}^{T} \left( 1_{(\Delta_s(t) < \tau_s^{RCTL})} + 1_{(\tau_s^{RCTL} \leq \Delta_s(t) \leq \tau_s^{RCTL})} \cdot \left( \frac{\tau_s^{RCTL} - \Delta_s(t)}{\tau_s^{RCTL} - \tau_s^{RCTL}} \right) \right) dt
\]  

(2)

For later use, we refer to the value of the integral \( \int \ldots dt \) in equation (2), taken from \( t_1 \) to \( t_2 \), as the “effective” time the truck driver is covered during the period \([t_1, t_2]\).

Finally, suppose that the service package is of package type \( ASAP \). Then, for each point of time during the truck driver’s trip, the following holds: the lower the access time, the more effective the health service accessed can be. Based on this observation, we define \( a_s^{ASAP} \), the access measure for a package \( s \) of package type \( ASAP \), as the average value of the truck driver’s access time during his trip (i.e., the average length of the arrow in the lower part of Figure 1):

\[
a_s^{ASAP} = \frac{1}{T} \int_{t=0}^{T} \Delta_s(t) dt
\]  

(3)

One may interpret \( a_s^{ASAP} \) as the expected travel time to the next package \( s \) location from the moment the truck driver needs this service package.

4 Model

In this section, we formally model the RHFLP. Some notations are introduced in Section 4.1. Section 4.2 models the value of a set of location decisions in terms of volume of truck drivers served and effectiveness of healthcare provision. Next, Section 4.3 introduces a MILP formulation of the problem. Finally, Section 4.4 provides a worst case analysis with respect to data inaccuracy.

4.1 Notation

We model a problem instance by means of a graph \( G(L, E) \), where \( L \) and \( E \) denote a set of vertices and a set of edges connecting these vertices, respectively. Table 1 introduces the notations we use to denote the graph, service packages, demand parameters, decision variables, and auxiliary variables.

Here, a truck flow \( q \) represents a collection of truck drivers who travel the same long distance route. A route defines a path in the graph, starting from origin \( O_q \), visiting the ordered set of
**General:**

- $p$: number of roadside health facilities to be newly located
- $p_s$: number of facilities service package $s$ can be newly allocated to
- $\Omega$: set of all feasible solutions $\omega = (x, y) = (\{x_k\}, \{y_{ks}\})$
- $r$: relative importance of the patient volume criterion
- $1 - r$: relative importance of the effectiveness criterion

**Network:**

- $KC$: set of current facility locations
- $KP$: set of potential facility locations
- $KP_s$: set of potential locations for package $s$
- $K$: set of facility locations: $K = KC \cup KP$
- $O$: set of truck route origins, $O \cap K = \emptyset$
- $D$: set of truck route destinations, $D \cap K = \emptyset$
- $L$: set of locations: $L = KC \cup KP \cup O \cup D$
- $Q$: set of long distance truck flows
  - $O_q$: origin of flow $q \in Q$, $O_q \in O$
  - $D_q$: destination of flow $q \in Q$, $D_q \in D$
- $K_q$: ordered set of facility locations along the route of flow $q \in Q$, $K_q \subseteq K$
- $\chi_{ks}$: parameter indicating whether a current facility at location $k$ offers package $s$ ($\chi_{ks} = 1$) or not ($\chi_{ks} = 0$)

**Service packages:**

- $S$: set of health service packages
- $J$: set of service package types $j = \{CTL, RCTL, ASAP\}$
- $S_j$: set of service packages that are of service package type $j$, $S_j \subseteq S$
- $\alpha_{qs}^j$, $\tau_{CTL}^j$, $\tau_{RCTL}^j$, $\tau_{2s}^j$: break points of piecewise linear function $e_{qs} = g^j(t(a_{qs}(y)))$
- $\tau_{RCTL}^1$, $\tau_{RCTL}^2$: recommended & critical time-limit for accessing package $s$, $s \in S_{RCTL}$
- $w_s$: maximal attainable effectiveness per truck driver demanding package $s$

**Demand parameters:**

- $d_k$: expected daily patient volume at facility location $k$
- $f_{qs}$: number of truck drivers in flow $q$ who need service package $s$
- $f_q$: number of truck drivers in flow $q$
- $\theta_{qs}$: fraction of truck drivers in flow $q$ who need service package $s$

**Decision variables:**

- $x_k$: \[
\begin{cases}
1 & \text{if a health facility is placed at location } k \\
0 & \text{otherwise}
\end{cases}
\]
- $y_{ks}$: \[
\begin{cases}
1 & \text{if service package } s \text{ is offered at location } k \\
0 & \text{otherwise}
\end{cases}
\]

**Auxiliary variables:**

- $Z_{PV}$: expected daily number of patient visits captured by the facilities in the network
- $Z_s$: effectiveness of package $s$ provisioning
- $Z_E$: effectiveness score
- $e_{qs}$: effectiveness of health service provisioning per truck driver in flow $q$ demanding service package $s$, given solution $\omega$, $e_{qs} \in [0, w_s]$
- $a_{qs}(y)$: level of access to package $s$ provided for truck drivers in flow $q$, given package allocation decisions $y$ and the access measure corresponding to the selected service package type $j$

**Table 1:** Table of Notations.
facility locations $K_q$, and ending at destination $D_q$. Note that, although $O_q$ (or $D_q$) may physically represent the same location as a facility location $k \in K_q$, we regard them as separate locations. $K_q$ may include potential locations from the set $KP$ as well as locations from the set of locations which are currently already in use, $KC$. The latter may include facilities operated by other organizations. For instance, if a facility is accessible, and contributes to continuity of access, it can be viewed as being part of the NGO’s network. Finally, we note that allocation of new service packages can be restricted to the newly located facilities only (by setting $KP_s = KP$) or to any existing or newly located facility (by setting $KP_s = \{k : y_{ks} = 0\}$).

### 4.2 Optimization Criteria

A solution $\omega = (x, y) \in \Omega$ specifies a network of roadside healthcare facilities, including all current and new facilities and their service packages. The network expansion budget allows for placing $p$ new facilities and assigning service package $s$ to $p_s$ facilities. For a given solution $\omega$, we define the patient volume $Z_{PV}$ as the expected daily number of patient visits captured by the roadside healthcare facilities in this network. Network expansion is commonly preceded by extensive assessments of potential locations in terms of truck volumes, layout of the truck stop, existing healthcare infrastructures, etc. This data can be used to estimate daily patient volumes. This can include visits by truck drivers, as well as sex workers and other roadside communities, which are also target groups for North Star. Hence, letting $d_k$ denote the daily patient volume at location $k$, we derive that (cf. Ares et al. 2016):

$$Z_{PV} = \sum_{k \in K} d_k x_k$$

(4)

This definition of $Z_{PV}$ implicitly assumes that the patient volume at a given location is independent of our location decisions.

Let us now quantify for a given solution $\omega \in \Omega$ the effectiveness of health service provisioning. We denote the number of truck drivers in flow $q$ demanding service package $s$ by $f_{qs}$. This can be estimated as a package-specific fraction of the volume of flow $q$, where the latter can be measured through vehicle tracking data, mobile phone data, and traffic counts. For each of these truck drivers,
\( e_{qs} \) represents the effectiveness of service package \( s \) provisioning. Then the total effectiveness, i.e., the total health improvement – of package \( s \) provisioning is defined as (cf. Ares et al. 2016):

\[
Z_s = \sum_{q \in Q} f_{qs} e_{qs}
\]  

(5)

We let functions \( g^j_s \) define the relationship between effectiveness and access. More precisely, for package \( s \), we firstly select the most fitting of the three proposed service package types and corresponding access measures. Let \( j \in \{CTL, RCTL, ASAP\} \) denote the selected service package type. Furthermore, given package allocation decisions \( y \), let \( a^j_{qs}(y) \) denote the level of access to package \( s \) for flow \( q \), as measured by the access measure corresponding to package type \( j \) (see Section 3.2). Then, the effectiveness of package \( s \) provisioning for truck drivers in flow \( q \) is measured as:

\[
e_{qs} = g^j_s (a^j_{qs}(y)) \quad s \in S_j
\]  

(6)

These functions \( g^j_s (\cdot) \) reflect which levels of access are ineffective, partially effective, and effective (cf. Ares et al. 2016). Lacking empirically or theoretically validated alternatives and for ease of modeling, we define \( g^j_s (\cdot) \) as piecewise linear functions. Figures 2a and 2b illustrate these functions.

In case that \( a^j_{qs}(y) \) is to be maximized (i.e., when \( j \in \{CTL, RCTL\} \)), we define that \( e_{qs} \) increases linearly from 0 to the maximal attainable effectiveness \( w_s \) when \( a^j_{qs}(y) \) increases from a given lower bound threshold \( \alpha^j_s \) (ineffective access) to a given upper bound threshold \( \bar{\alpha}^j_s \) (effective access). In case that this variable is to be minimized (i.e., when \( j = ASAP \)), \( e_{qs} \) decreases linearly from \( w_s \) to 0 when \( a^j_{qs}(y) \) increases from the lower bound threshold \( \underline{\alpha}^j_s \) (effective access) to the upper bound threshold \( \bar{\alpha}^j_s \) (ineffective access). Section 5.1 discusses the calibration of these parameters.

Parameters \( w_s \) can represent the expected health improvement per truck driver demanding service package \( s \) due to effective access to this package. This improvement can for instance be expressed in terms of the commonly accepted effectiveness measure of quality-adjusted life years (QALYs) gained. In practice, obtaining reliable QALY estimates may well be infeasible or too
Figure 2: Effectiveness of package $s$ provisioning as a function of $a_{qs}^j(y)$.

expensive. Reliable QALY estimates are, however, not necessary as long as decision makers can balance the two main optimization criteria. Multiplying $w_s$ by a given constant does not change optimal location decisions in that case. Consequently, it suffices if $w_s$ represents relative effectiveness, which is significantly easier to estimate. Moreover, $w_s$ can also be chosen so as to mimic the strategic objectives of the NGO’s donors, who may have preferences to address certain disease(s). In this case, effectiveness is to be interpreted as the extent to which the service provisioning increases weighted health improvements, where the weights represent donor preferences.

For later use, we represent the total effectiveness of health service provisioning by $Z_E$, and refer to this variable as the effectiveness score:

$$Z_E = \sum_{s \in S} Z_s$$  \hspace{1cm} (7)

4.3 MILP Formulation

The measures presented in Section 4.2 allow us to introduce the following problem formulation, which we refer to as the Roadside Healthcare Facility Location Problem (RHFLP). Here, parameters $r \in [0,1]$ and $(1 - r) \in [0,1]$ capture the relative importance of the patient volume criterion and the effectiveness criterion, respectively:
\[
\max \quad r \sum_{k \in K} d_k x_k + (1-r) \sum_{s \in S} \sum_{q \in Q} f_{qs} e_{qs} \\
\text{s.t.} \quad e_{qs} = g^j_s \left( a^j_s(y) \right) \quad q \in Q, j \in J, s \in S_j \quad (8)
\]

\[
\sum_{k \in KP} x_k = p \quad (10)
\]

\[
\sum_{k \in KP_s} y_{ks} = p_s \quad s \in S \quad (11)
\]

\[
x_k \geq y_{ks} \quad k \in K, s \in S \quad (12)
\]

\[
x_k = 1 \quad k \in KC \quad (13)
\]

\[
y_{ks} = \chi_{ks} \quad k \in KC, s \in S \quad (14)
\]

\[
x_k, y_{ks} \in \{0, 1\} \quad k \in K, s \in S \quad (15)
\]

The objective function (8) maximizes a weighted sum of \(Z_{PV}\) and \(Z_E\). Constraints (10) and (11) specify the number of new facilities and the number of service packages \(s\) to be allocated to potential facility locations. Note that these constraints could easily be replaced by a more general budget constraint, restricting the costs of network expansions. Constraints (12) define that a service package can only be allocated to facility location \(k\) if a health facility is located there. The current network of health facilities is described in constraints (13) and (14). Constraints (15) define our decision variables as binary variables. Effectiveness variables \(e_{qs}\) (see Section 4.2) are defined in equations (9).

The non-linearity of these equations makes (8) - (15) a mixed-integer \textit{nonlinear} programming (MINLP) model, which is generally very hard to solve. In Appendix B we show how this model can be non-trivially transformed into a mixed integer \textit{linear} programming (MILP) model. Specifically, it suffices to replace equations (9) by constraints (20) - (23) and (25) - (39). We also prove that the vast majority of the binary variables needed to perform this transformation can be relaxed to continuous variables, since the corresponding constraint matrix is totally unimodular (TU). This substantially decreases the model complexity.
In Online Appendix D, we prove the following by reduction from the problem CLIQUE:

**Proposition 1.** The RHFLP, as formulated by (8) - (15), is strongly NP-hard, even in case that $|S| = 1$.

Compared to the model presented by Ares et al. (2016), model (8) - (15) has a simplified objective function (8), as it disregards equity. Section 5.8 discusses how this criterion can be added to model (8) - (15). At the same time, constraint (9) now captures three instead of one effectiveness modalities, allowing for the introduction of service package differentiation, as expressed in constraints (11), (12), and (14). Constraints (20) - (39) in Appendix B form the non-trivial linearization of (9). See section 1 for our discussion on contributions of both papers.

### 4.4 Worst-case Effect of Imprecise Input Data

Parameters $f_{qs}$, the number of truck drivers in flow $q$ who need package $s$, and $d_k$, the expected daily patient volume at location $k$ tend to be imprecise in the data-scarce environment North Star operates in. Though this does not affect the feasible solution space, it may affect the value of a solution and thereby induce an optimality gap.

Proposition 2 provides an upper bound on this gap (see Online Appendix E for the proof). Here, $\omega^\#$ denotes the true optimal solution and $\omega^*$ the optimal solution based on the presently used parameter values. We denote the value function using the true parameters by $v^\#$, and the value function using the presently used parameters by $v^*$. For instance, $v^\#(\omega^*)$ denotes the value of the solution $\omega^*$ in case that the true parameter values were used.

**Proposition 2.** If the true values of $f_{qs}$ and $d_k$ deviate at most by a fraction $\delta$ from the presently used parameter values, then $\frac{v^\#(\omega^\#)}{v^\#(\omega^*)} \leq \frac{1+\delta}{1-\delta}$. This bound is tight.

To illustrate this bound, let $\delta$ be equal to 0.1. Then in the worst case, the true value of $\omega^*$ is about 18% lower than the true value of $\omega^\#$. As this bound is only attained with equality under three very specific circumstances (see Online Appendix E), Section 5.5 investigates realistic levels of sensitivity by means of a real life case study.

Let us now analyze the optimal solution’s sensitivity with respect to the parameters $w_s$, the maximal attainable effectiveness of package $s$ provisioning, and parameters $r$ and $1-r$, the relative
values of the patient volume criterion and the effectiveness criterion, respectively. Suppose that there exists an “optimal” value of these parameters. For example, optimal values could be those values yielding the largest health improvement. As it is very difficult to identify these values, it is likely that the chosen parameter values deviate from them. An upper bound on the resulting optimality gap is given in Proposition 3 (see Online Appendix F for the proof). Here, \( w_{ob} \in \{r\} \cup \{(1-r)w_s\} \) and \( \gamma \) denotes some instance-dependent constant between 0 and 1.

**Proposition 3.** If the “optimal” values of \( w_{ob} \) deviate at most a fraction \( \delta \) from the presently used parameter values, then

\[
\frac{v^*(w^*)}{v^*(w^*)} \leq \frac{(1+\delta)^2}{(1-\delta)(1+\delta+2\gamma)}.
\]

This bound is tight.

For example, if \( \gamma = 0.5 \) and \( \delta = 0.1 \), then in the worst case, the optimality gap is at most 11%.

It is easy to see that, in the worst case, the true value of a given parameter is either the fraction \( \delta \) larger or the fraction \( \delta \) smaller than the presently used values. Let an extreme realization of the parameters of interest be represented by \( \psi \in \Psi \), and let \( v^\psi(\omega) \) denote the resulting value function. An instance-specific bound can be found by solving the following problem, as we do in Section 5.5:

\[
\max_{\omega \in \Omega, \psi \in \Psi} v^\psi(\omega) - v^\psi(\omega^*)
\]

5 Numerical Analysis

This section illustrates how our model can be used for strategic planning and reveals practical insights for decision makers. We use CPLEX 12.6.2 to solve our MILP model, on a PC with a 3.4 AMD A4-5300 processor and 8 GB RAM.

5.1 South-East Africa Case

We base our baseline case on the network of major transport corridors in South-East Africa. This network of roads spans 14 sub-Saharan countries and connects the main ports in the region with the main inland cities and other areas of economic importance such as the copper belt in the DRC.

It consists of three subsets of corridors: (1) the North-South Corridor, which spans the region from Tanzania to South Africa, (2) the Central Corridor, which runs from the port of Dar es Salaam (Tanzania) into Burundi, Rwanda, and the DRC, and (3) the Northern Corridor, which connects...
the port of Mombasa (Kenya) with landlocked countries lying east and north of Kenya. Figure 3 depicts the corridor network as a graph consisting of the 29 RWCs North Star operated in this network at the time our case study was finalized (August 2016) and 85 potential RWC locations.

The case study data sources are summarized in Table 2 and a full description of the data is provided in Online Appendix I. In line with package allocation choices currently faced by North Star, we choose the primary care (PC) package to be a standard service package and the HIV care (HC) package to be the only optional service package. Hence, in the baseline case, $S = \{PC, HC\}$. The data sources identify 95 major truck flows, and their health service needs are calculated as $f_q s = f_q \theta_{qs}$. Here, $f_q$ and $\theta_{qs}$ denote the number of truck drivers in flow $q$ and the fraction of truck drivers in flow $q$ who need service package $s$, respectively. For truck flows making use of the Northern and the Central Corridor, estimates of $f_q$ were obtained from inter-country cargo volumes presented by Nathan Associates Inc. (2011) and Berger (2011), respectively. For the North-South Corridor, we use least squares optimization to fit $f_q$ to annual average daily truck volume (AADT) estimates ($R^2 = 0.93$), which Odoki and Anyala (2014) provide for most arcs in the network, and to a “target matrix” describing road-based inter-country cargo volumes ($R^2 = 0.81$), as presented by the SADC (2012). The use of such a target matrix is a common way to deal with the limitations of OD matrix estimation methods (Peterson 2007). We choose $\theta_{q, PC} = 1$, reflecting the assumption that every truck driver has a potential need for primary care, and estimate $\theta_{q, HC}$ as 1.41 times the average of the adult HIV prevalence in the flow’s origin country and the prevalence in the flow’s destination country. The multiplier 1.41 reflects the difference between the prevalence levels in the general population and the truck driver population, and is based on data for South Africa and Mozambique (see Online Appendix I). A truck flow is assumed to take the shortest route from origin to destination.

Finally, we use least squares optimization to fit the relationship between truck volume along current RWC locations (AADT$_k$) and daily patient volume at these locations ($d_k$) ($R^2 = 0.33$). This yields the following estimation method for patient volume at potential RWC locations: $d_k = 10.65 + 0.31 \cdot \sqrt{\text{AADT}_k}$.

Our calibration of the relationship between access and effectiveness is based on structured expert surveys. Specifically, we asked two experts from North Star to quantify the relative importance $w(s)$ of the two service packages $s$, to connect to each service package a corresponding package...
Figure 3: Map of the South-East African Transport Corridor Network.

type, yielding $S_j$, and to assign scores to 10 configurations of RWCs offering the PC package along an imaginary truck route and to 10 configurations of RWCs offering the HC package along an imaginary truck route. The functions $g^*_k(\cdot)$ providing the best fit to these scores are determined by least squares optimization ($R^2 = 0.77$ for PC, $R^2 = 0.69$ for HC). The resulting parameter values are given in Table 3, and the survey we used for the PC package can be found in Online Appendix J.

When asked to elaborate on their service type allocation – PC to the RCTL type and HC to the ASAP type – the experts made references to the criticality of corresponding health services, the aforementioned distance decay relationship, and the benefits of repeatedly passing a health
facility:

**Expert 1 about truck drivers accessing the PC package (translated):** Generally, the people do not go to the clinic for nothing. Certainly not the truck drivers. They have a problem that builds up till a certain critical level. Then they decide to come, and if they cannot access a clinic within two days, they have a very big problem.

**Expert 2 about the benefits of continuous access to the HC package (translated):**

One is biomedical, two is psychological – a sense of urgency among the patients – (...) and there is a third (...) the power of repetition is in that, if he thinks “should I do it or not” [go to a clinic] (...) the third time [he passes a clinic] he thinks “oh no, now I’m gonna do it” (...). So when it comes to HIV, you never have too many [clinics].

**Expert 1 about the time criticality of the HC package (translated):**

The problem evolves rather gradually. Problems arise when they run out of medication. Then you got a problem... then you need to come (...). When they get an opportunistic infection (...) it builds up rather slowly.

Experience suggests that decision makers will vary the parameter \( r \) in the software package we developed (see North Star Alliance 2011) until one or more satisfactory solutions are obtained. This
is essentially a basic way to calibrate parameter $r$. We initially assume that decision makers can identify the “optimal” value. Sensitivity towards this assumption is analyzed in Section 5.6. Under this assumption, it suffices that parameters $w_s$ represent the relative importance of the different service packages (i.e., the scaling of these parameters does not matter).

The service package type allocation obtained from the expert surveys – $PC$ to the $RCTL$ package type and $HC$ to the $ASAP$ package type – is largely influenced by the time criticality of the corresponding health services, the aforementioned distance decay relationship, and the benefits of repeatedly passing a health facility, as reflected by the quotes in Section 3.

<table>
<thead>
<tr>
<th>Package $s$</th>
<th>Service package type $j$</th>
<th>Service package type parameters (days)</th>
<th>$\alpha_j$</th>
<th>$\pi_j$</th>
<th>$w_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$PC$</td>
<td>$RCTL$</td>
<td>$\tau_{1PC}^{RCTL} = 1.29$ $\tau_{2PC}^{RCTL} = 3.00$</td>
<td>0.09</td>
<td>1.00</td>
<td>$1 \cdot \sigma$</td>
</tr>
<tr>
<td>$HC$</td>
<td>$ASAP$</td>
<td></td>
<td>0.40</td>
<td>5.48</td>
<td>$2 \cdot \sigma$</td>
</tr>
<tr>
<td>$MC$</td>
<td>$CTL$</td>
<td></td>
<td>0.50</td>
<td>0.90</td>
<td>$5 \cdot \sigma$</td>
</tr>
<tr>
<td>$TC$</td>
<td>$RCTL$</td>
<td>$\tau_{1TC}^{RCTL} = 1.00$ $\tau_{2TC}^{RCTL} = 2.00$</td>
<td>0.50</td>
<td>0.90</td>
<td>$2 \cdot \sigma$</td>
</tr>
<tr>
<td>$SC$</td>
<td>$RCTL$</td>
<td>$\tau_{1SC}^{RCTL} = 1.00$ $\tau_{2SC}^{RCTL} = 2.00$</td>
<td>0.50</td>
<td>0.90</td>
<td>$1 \cdot \sigma$</td>
</tr>
</tbody>
</table>

Note: the $TC$, $SC$, and $MC$ packages are only used in Section 5.7. W.l.o.g. we set scaling factor $\sigma = 1/25$

Table 3: Parameters describing the relationship between access and effectiveness of package $s$ provisioning.

We finally note that the parametrization for $HC$ is much more conservative than the requirement of facilitating monthly or bimonthly follow-up visits for drivers under HIV treatment.

### 5.2 Network Expansion Strategies

Based on the parameter values from the baseline case, we optimize locations of 4 new RWCs and the allocation of the $HC$ package among two established RWCs (i.e., $p = p_{PC} = 4$, $p_{HC} = 2$), which represents a realistic network expansion scenario. We analyze two network expansion strategies, characterized by the parameter $r$. The first strategy is the “balanced strategy”, which we model by setting $r = 0.5$. Many Pareto efficient solutions that balance both objectives are found around this value, as we show in Section 5.3. The second investment strategy reflects the strategy North Star had been employing before this research was implemented. The main focus of this strategy was put on maximizing the patient volume served, which we mimic by setting $r = 1.0$. We refer to this strategy as the “patient volume strategy”. Finally, we also consider the case in which we not
only locate new RWCs and service packages, but also allow current clinics to be relocated. Here, we again use \( r = 0.5 \).

![Diagram showing optimal location decisions for the baseline case.](image)

(a) Balanced solution.  
(b) Patient volume solution.

**Figure 4:** Optimal location decisions for the baseline case.

<table>
<thead>
<tr>
<th>Solution</th>
<th>( Z_{PV} )</th>
<th>( Z_{PC} )</th>
<th>( Z_{HC} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>562</td>
<td>19840</td>
<td>1646</td>
</tr>
<tr>
<td>Balanced</td>
<td>632 (12.3%)</td>
<td>22964 (15.7%)</td>
<td>2353 (43.0%)</td>
</tr>
<tr>
<td>Patient volume</td>
<td>662 (17.6%)</td>
<td>20070 (1.2%)</td>
<td>1652 (0.4%)</td>
</tr>
<tr>
<td>Redesigned</td>
<td>682 (21.2%)</td>
<td>24342 (22.7%)</td>
<td>2673 (62.4%)</td>
</tr>
</tbody>
</table>

Table 4: Daily patient volume (\( Z_{PV} \)), effectiveness of primary care provisioning (\( Z_{PC} \)), and effectiveness of HIV care provisioning (\( Z_{HC} \)) in the initial network, the networks obtained by the balanced strategy and the patient volume strategy, and in the redesigned network.

Figures 4a and 4b and Table 4 describe the optimal solutions yielded by the balanced strategy and the patient volume strategy. We see that the “balanced solution” results in a relatively lower
expected daily patient volume (5.3 percentage points lower) but also in a substantially higher effectiveness (14.5 and 42.6 percentage points higher for PC and HC, respectively). The latter is illustrated by the bubble plots in Figures 5a and 5b, which depict the solutions’ impact on the effectiveness of package PC provisioning for the 95 truck flows separately. The reason is that the patient volume solution places RWCs in the busiest parts of the network, which are already well-served by current RWCs (see Figure 4b). The balanced strategy, instead, yields locations that fill up gaps in the RWC network. For example, Figure 6 shows how the new RWCs improve the degrees of coverage with the PC package during trips along the routes connecting the ports of Durban, Beira, Nacala, and Dar es Salaam with the copper belt (Kolwesi, DRC).

![Balanced solution.](image1)

**Figure 5:** Effectiveness of package PC provisioning for the 95 truck flows before and after implementing the balanced solution and the patient volume solution. A bubble’s size reflects the number of truck drivers in the corresponding flow.

The optimal network obtained when relocation of clinics is allowed moves 17 of the current 29 clinics and all of the 6 current HC packages to a different location. Compared to the case when relocation is not allowed, this significantly increases patient volume (8.9 percentage point increase) and the effectiveness of PC provisioning (7.0 percentage point increase) and HC provisioning (19.4 percentage point increase). This provides a strong argument for critically assessing the fitness of the current network.

Finally, the optimal solutions illustrate an important property of the RHFLP: adding a given
Figure 6: Degree of coverage (see Section 3.2) with the PC package before and after locating the new RWCs at each point of time during a trip from a major regional ports (Durban, Beira, Nacala, and Dar es Salaam) to the copper belt (Kolwesi, DRC). The numbers correspond to the location numbers introduced in Figure 3.

package to multiple locations can bring about synergy effects. For instance, when adding the PC package to Kasumbalesa North or to Kasumbalesa South only, the effectiveness of package PC provisioning to truck flow Nacala Port - Kolwesi increases by 0.09, whereas it increases by 0.27 if this package is added to both locations. This is a direct implication of the assumption that a certain level of access is required before this translates into a strictly positive level of effectiveness. These effects suggest that long term network planning instead of making location decisions consecutively can be beneficial, as we further explore in Section 5.4.

5.3 Importance of Balancing Patient Volume and Effectiveness

Let us analyze the trade-off between the two main objectives in some more detail now. Figure 7 summarizes the results when solving the RHFLP for the baseline case with the number of facilities (with PC packages) to be placed $p \in \{2, 4, 8, 12, \ldots, 44, 48, 50\}$, the number of HC packages to be allocated $p_{HC} = \frac{p}{2}$, and relative importance parameter $r \in \{0.0, 0.1, \ldots, 1.0\}$. All other parameters are kept as before.

The results show that focusing solely on one criterion – i.e., choosing $r = 0.0$ or $r = 1.0$ – results
in significant sub-optimality in terms of the other criterion. When also assigning some weight to
the other criterion, large gains in effectiveness (patient volume) can be made at a marginal loss in
patient volume (effectiveness). The “irregular behavior” of the effectiveness score for $r = 1.0$ (i.e.,
the lowest point of each curve) is caused by the indifference on where to place the $HC$ package,
leading to more or less random package allocation decisions.

5.4 Importance of Network Planning
To assess the added value of long term network planning over consecutive decision making, we
now calculate for the baseline case the optimality gap when locating clinics one at a time for
$p \in \{2, 4, \ldots, 50\}$ and $p_{HC} = \frac{p}{2}$. Specifically, we allocate a clinic to the potential location for which
the objective value increase is largest till $p$ clinics are added. The first $p_{HC}$ clinics that are located
also offer package $HC$ and parameter $r$ again equals 0.5.

The results show that consecutive decision making yields near-optimal location decisions: the
optimality gap does not exceed 0.28%. This seems surprising, given the earlier observation that
network planning allows for a better exploitation of synergy effects. We note, however, that such
effects are only attainable for a given truck flow when its current level of access is lower than the
“access threshold”: the level of access required for a strictly positive level of effectiveness (e.g.
$\Omega_{PC}^{RCTL}$). In the baseline case, this applies to few truck flows.
We hence argue that network planning is mostly beneficial when the network of clinics is still small and/or when relatively many clinics are required to exceed the access threshold. To assess this, we calculate the optimality gap for consecutive decision making when no clinics are presently in the network and when \( p = p_{PC} = 20 \) and \( p_{HC} = 10 \). Two subcases are considered: one where all parameters are assigned their baseline value and one where access threshold \( \alpha_{PC}^{RTL} \) is increased to 0.50 (baseline value: 0.09). We refer to the corresponding solutions as “greenfield baseline” and “greenfield synergistic”.

<table>
<thead>
<tr>
<th>Solution</th>
<th>Opt. gap (%)</th>
<th>Gap ( Z_{PV} ) (%)</th>
<th>Gap ( Z_{PC} ) (%)</th>
<th>Gap ( Z_{HC} ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Greenfield baseline</td>
<td>1.3</td>
<td>-0.1</td>
<td>1.4</td>
<td>3.2</td>
</tr>
<tr>
<td>Greenfield synergistic</td>
<td>7.3</td>
<td>-15.2</td>
<td>17.6</td>
<td>10.0</td>
</tr>
</tbody>
</table>

Table 5: Optimality gap (%) for consecutive decision making in a greenfield situation; overall and for separate objectives.

Table 5 describes the results, which confirm our hypothesis. Consecutive decision making can substantially underperform in greenfield situations, in particular when the access threshold is large. The effectiveness increase attainable when locating a single clinic is comparatively small in such cases, so that location decisions are mostly determined by patient volumes. As a consequence, the solutions perform well in terms of patient volume, but weakly exploit possible gains in effectiveness.

These observations suggest the existence of a tipping point: a point at which most of the synergy effects have been obtained and from which one can switch to consecutive decision making without risking substantial suboptimality. To further investigate this, we analyze the optimality gap when switching to consecutive decision making, depending on the density of the network. Specifically, we again consider the greenfield synergistic situation and \( p = 20 \). We perform network planning in the startup phase, corresponding to \( p_{\text{switch}} \) clinics. Next, we switch to consecutive decision making for the remaining \( p - p_{\text{switch}} \) clinics. Figure 8 shows the resulting optimality gaps.

The results show that the optimality gap drops to almost 0% when \( p_{\text{switch}} \) exceeds 16. Switching to consecutive decision making before that can induce a serious optimality gap of up to 7.7%. The percentage of truck drivers for which the access threshold is not met indeed serves as an indicator of this tipping point. E.g., substantial optimality gaps are attained while this percentage exceeds 20% for the \( PC \) package, whereas (near-)optimality is attained as this percentage approaches zero.
5.5 Sensitivity Analysis on Demand Parameters

Proposition 2 proves worst-case bounds for the optimality gap induced by impreciseness in the demand parameters \(d_k\) and \(f_{qs}\). To gain insight into the actual sensitivity, we randomly generate the “true” values of the parameters in the baseline case, and determine the resulting optimality gap for the balanced solution (see Section 5.1) expressed as a percentage of the “true” optimal solution value. Specifically, we draw for each of these parameters from the uniform distribution on the interval \([-\delta, \delta]\), represented by \(U[-\delta, \delta]\), and determine the true values of \(f_{qs}\) and \(d_k\) as \(f_{qs}^\# = f_{qs} \cdot (1 + U[-\delta, \delta])\) and \(d_k^\# = d_k \cdot (1 + U[-\delta, \delta])\). We test different values of \(\delta\), and generate 50 demand scenarios for each of them. Table 6 shows for each value of \(\delta\) the resulting average optimality gap, the maximum optimality gap, the non-parametric worst case bound on this gap (which is not tight for our specific instance) and the tight parametric bound. The latter is obtained by solving a MILP formulation of the problem defined in (16).

<table>
<thead>
<tr>
<th>(\delta)</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. (%)</td>
<td>0.2</td>
<td>0.5</td>
<td>0.9</td>
<td>1.6</td>
<td>1.7</td>
</tr>
<tr>
<td>Max. (%)</td>
<td>0.5</td>
<td>1.2</td>
<td>2.7</td>
<td>2.9</td>
<td>3.8</td>
</tr>
<tr>
<td>Worst Case Par. (%)</td>
<td>2.5</td>
<td>4.2</td>
<td>10.7</td>
<td>26.6</td>
<td>36.9</td>
</tr>
<tr>
<td>Worst Case Nonpar. (%)</td>
<td>33.3</td>
<td>57.1</td>
<td>75.0</td>
<td>88.9</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Note: Worst Case Par.: tight bound found by solving (16).
Worst Case Nonpar.: non-tight bound provided by Proposition 2.

Table 6: Optimality gap (%) by imprecise demand data.
The results indicate that the optimal solution is highly insensitive to impreciseness. Even when
the true values of the demand parameters deviate by up to 100% from the presently used values, the
maximum optimality gap attained in the 50 demand scenarios is only 3.8%. This can be explained
by the fact that near-optimal solutions tend to locate facilities along the busiest truck routes and
at the busiest truck stops. The location decisions in these solutions therefore tend to overlap
considerably. As implied by the proof of Proposition 2 (see Online Appendix E)), the effects of
impreciseness are strongly correlated for such overlapping solutions, thus reducing the maximum
optimality gap.

5.6 Sensitivity Analysis on Model Parameters

As argued, setting “optimal” values for $r$ and $w_s$ is challenging, which begs to question the impact
of impreciseness. Figure 9a depicts the optimality gap when using $r = 0.5$ while the optimal
value $r^\#$ differs, expressed as a percentage of the “true” optimal solution value. We consider
$r^\# \in \{0.0, 0.1, \ldots, 1.0\}$. Similarly, Figure 9b depicts the optimality gap when the optimal ratio
$w_{PC}/w_{HC}$ is a factor $\rho \in \{2^{-5}, 2^{-4}, \ldots, 2^5\}$ times the presently used ratio 2. Specifically, we
analyze the impact of using $w_{HC} = 2$ instead of the optimal value $w_{HC}^\# = \rho \cdot w_{HC}$.

![Figure 9](https://via.placeholder.com/150)

(a) Gap (%) when using $r = 0.5$. (b) Gap (%) when using $w_{HC} = 2$.

**Figure 9:** Optimality gap (%) when using baseline parameter values for $r$ and the relative impor-
tance of service packages instead of the “real” values.

The results show that solution values are very robust with respect to impreciseness in $r$ and $w_s$.
Up to a substantial level of impreciseness in $r$, the optimality gap remains below 1.5%. For $w_s$, the
optimality gap does not exceed 1.0%.
Selecting the “true” package type to a given package \( s \) may be subject to errors as well, as understanding of access requirements is still in early stages of development. We therefore assess the potential impact of assigning the “wrong” package type to package \( PC \). For each package type \( j \in J \), we determine the parameters of function \( g^j_{PC}(\cdot) \) providing the best fit to the assigned network scores (see Section 5.1). Next, we assess the optimality gap when choosing package type \( j \) while the “true” package type is \( j^\# \). To assess the impact of building a substantial network with a “wrong” package type choice, we analyze this for the baseline case with \( p = p_{PC} = 20 \) and \( p_{HC} = 10 \).

<table>
<thead>
<tr>
<th>( j^# )</th>
<th>Selected package type ( j )</th>
<th>CTL</th>
<th>RCTL</th>
<th>ASAP</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTL</td>
<td>x</td>
<td>0.33</td>
<td>0.43</td>
<td></td>
</tr>
<tr>
<td>RCTL</td>
<td>0.25</td>
<td>x</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>ASAP</td>
<td>0.98</td>
<td>0.18</td>
<td>x</td>
<td></td>
</tr>
</tbody>
</table>

Table 7: Optimality gap (%) when choosing package type \( j \) while the “true” package type is \( j^\# \).

Table 7 summarizes results, which show that selecting a wrong package type can induce an optimality gap of up to approximately one percent for the case considered. This on the one hand makes it beneficial to adequately assess the package type, but on the other hand suggests that solution values are rather robust with respect to wrong package type choices. For example, applying the three functions to the 10 configurations of RWCs included in the survey yields rather similar function values (see Online Appendix G).

5.7 Model Statistics

Table 8 describes the solution times and model statistics of 12 problem instances. The problem instances are defined by the number of truck flows (55 or 95), the number of different service packages (2 or 5), and the structure of the network (baseline, dense, or sparse). The three additional service packages are: tuberculosis care (\( TC \)), STI care (\( SC \)), and malaria care (\( MC \)). Table 3 shows the definitions of the corresponding effectiveness variables. The fractions of truck drivers who need these packages were again estimated based on data from the global health data repository (WHO 2016) and are provided in Online Appendix I. The sparse network is generated by selecting the minimum spanning tree of our graph \( G(L, E) \), and the dense network is generated by adding for each vertex two imaginary edges that connect this vertex with the two closest neighbor vertices it was not connected to in the baseline network. We generate the time needed to traverse such
imaginary edge by dividing the Euclidian distance of the edge by the average speed of 40 km/hour. For both alternative network structures, we determine the shortest origin-destination routes. All other parameters are kept as before.

<table>
<thead>
<tr>
<th>Instance</th>
<th></th>
<th></th>
<th>Structure</th>
<th>Variables (Integer)</th>
<th>Constraints</th>
<th>LP gap (%)</th>
<th>CPU (sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D55</td>
<td>55</td>
<td>2</td>
<td>Dense</td>
<td>5679 (558)</td>
<td>2933</td>
<td>0.5</td>
<td>0.6</td>
</tr>
<tr>
<td>D95</td>
<td>95</td>
<td>2</td>
<td>Dense</td>
<td>9123 (798)</td>
<td>4825</td>
<td>0.5</td>
<td>0.7</td>
</tr>
<tr>
<td>B55</td>
<td>55</td>
<td>2</td>
<td>Baseline</td>
<td>9769 (558)</td>
<td>3625</td>
<td>2.2</td>
<td>7.8</td>
</tr>
<tr>
<td>B95</td>
<td>95</td>
<td>2</td>
<td>Baseline</td>
<td>15871 (798)</td>
<td>5977</td>
<td>1.8</td>
<td>4.2</td>
</tr>
<tr>
<td>S55</td>
<td>55</td>
<td>2</td>
<td>Sparse</td>
<td>13725 (558)</td>
<td>4109</td>
<td>3.0</td>
<td>19.5</td>
</tr>
<tr>
<td>S95</td>
<td>95</td>
<td>2</td>
<td>Sparse</td>
<td>22665 (798)</td>
<td>6837</td>
<td>3.3</td>
<td>73.7</td>
</tr>
<tr>
<td>D55e</td>
<td>55</td>
<td>5</td>
<td>Dense</td>
<td>14196 (1395)</td>
<td>7502</td>
<td>4.9</td>
<td>60.4</td>
</tr>
<tr>
<td>D95e</td>
<td>95</td>
<td>5</td>
<td>Dense</td>
<td>22806 (1995)</td>
<td>12232</td>
<td>5.2</td>
<td>91.3</td>
</tr>
<tr>
<td>B55e</td>
<td>55</td>
<td>5</td>
<td>Baseline</td>
<td>24421 (1395)</td>
<td>9232</td>
<td>8.2</td>
<td>274.4</td>
</tr>
<tr>
<td>B95e</td>
<td>95</td>
<td>5</td>
<td>Baseline</td>
<td>39676 (1995)</td>
<td>15112</td>
<td>8.6</td>
<td>364.5</td>
</tr>
<tr>
<td>S55e</td>
<td>55</td>
<td>5</td>
<td>Sparse</td>
<td>34311 (1395)</td>
<td>10442</td>
<td>11.2</td>
<td>3577.9</td>
</tr>
<tr>
<td>S95e</td>
<td>95</td>
<td>5</td>
<td>Sparse</td>
<td>56661 (1995)</td>
<td>17262</td>
<td>11.7</td>
<td>7868.0</td>
</tr>
</tbody>
</table>

Table 8: Model statistics for 12 problem instances.

We observe that the CPU time needed to solve an instance tends to grow rapidly when $|Q|$ increases, when $|S|$ increases, and when the network becomes sparser. This is well explained by the observations that, in each of these cases, the number of variables and constraints increase and the LP relaxation of the MILP becomes weaker, which tends to negatively impact the performance of a branch-and-bound algorithm. Note that sparser networks bring about many additional variables in (25) and additional constraints in (20)-(23), because the number of RWC locations along a route, $|K_q|$, tends to increase when the network gets sparser.

5.8 Incorporating Equity

The presented MILP model could be extended to include equity of access to healthcare as an additional objective along the lines of Ares et al. (2016). Their paper proposes to take a weighted sum of pairwise absolute differences in effectiveness between different truck flows as a measure of equity. As effectiveness is calculated for each service package in our context, we propose to additionally sum the equity terms over the packages. Specifically, letting $Z_{PV\&E}$ denote our initial objective function, as defined in (8), and $w_EQ \in [0, 1]$ the importance of the equity term, we propose the following objective function:

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\[(1 - w_{EQ})Z_{PV&E} + w_{EQ}Z_{EQ}, \quad (17)\]

where \(Z_{EQ} = \delta_{EQ} \sum_{s \in S} \sum_{q_1, q_2 \in Q|q_2 > q_1} f_{q_1} s f_{q_2} (\Lambda^+_{q_1q_2s} + \Lambda^-_{q_1q_2s}). \) Here, \(\delta_{EQ}\) denotes a scaling factor, which we set equal to \(-10^{-5}\) in our experiments. Furthermore, \(\Lambda^+_{q_1q_2s} = \max\{e_{q_1}s - e_{q_2}s, 0\}\) and \(\Lambda^-_{q_1q_2s} = \max\{-(e_{q_1}s - e_{q_2}s), 0\}\), so that \(\Lambda^+_{q_1q_2s} + \Lambda^-_{q_1q_2s}\) represents the absolute difference between \(e_{q_1}s\) and \(e_{q_2}s\). This relationship is enforced by adding the following constraints to the MILP:

\[
e_{q_1}s - e_{q_2}s = \Lambda^+_{q_1q_2s} - \Lambda^-_{q_1q_2s} \quad q_1, q_2 \in Q|q_2 > q_1, s \in S \quad (18)
\]

\[
\Lambda^+_{q_1q_2s}, \Lambda^-_{q_1q_2s} \geq 0 \quad q_1, q_2 \in Q|q_2 > q_1, s \in S \quad (19)
\]

To analyze sensitivity of optimal solutions towards equity, we assess the trade-off between \(Z_{PV&E}\) and \(Z_{EQ}\). Specifically, we repeat the numerical experiments from Section 5.3 with \(r = 0.5\) and \(w_{EQ} \in \{0.0, 0.1, \ldots, 1.0\}\). Figure 10 depicts the results. Note that \(Z_{EQ} = 0\) represents the situation in which no differences in effectiveness between truck routes exist. As a first observation, the equity criterion is quite aligned with the other objectives while \(p < 16\). For the short term, improvements in equity hence seem to be a matter of making the network grow rather than assigning importance to this criterion. For the longer term, substantial gains in equity can be made at a marginal loss in the other criteria, which is in line with the findings of Ares et al. (2016). On average, moving from \(w_{EQ} = 0.0\) to \(w_{EQ} = 0.2\) increases \(Z_{EQ}\) by 25.3% and decreases \(Z_{PV&E}\) by 0.3% only. This provides a strong argument for incorporating equity in future facility location and package allocation decisions.
To assess the impact of incorporating equity on computation times, we repeat the numerical experiments from Section 5.7, using $w_{EQ} = 0.1$. Results are provided in Online Appendix H. Interestingly, despite the fact that the number of constraints and linear variables approximately doubled and the LP gap in the root node is much larger, solution times remain relatively low (up to 1161 sec.). This is in line with findings by Ares et al. (2016), who show that solving their similar MILP only gets challenging for very large networks.

6 Discussion and Conclusions

This paper considers the design problems arising when developing an integrated network of roadside healthcare facilities to meet health service needs of truck drivers, as operated by NGO North Star Alliance. More specifically we consider the problem of choosing optimal locations for clinics that are added to the existing network, and allocating service packages to clinics. The optimization considers two criteria: the number of truck drivers served and the effectiveness of the service delivery to the population served. We model the latter criterion through three novel access measures.

The resulting optimization problem differs from previously studied healthcare facility location problems, which typically assume patients to be static instead of mobile. This paper develops a mixed-integer linear programming model which captures the three access measures.
model has been implemented in a software package called POLARIS and installed at headquarters (North Star Alliance 2011). We also present computational analyses and address prominent managerial questions and challenges, using the network of major transport corridors in South-East Africa as a case study.

One such question deals with the suitability of heuristic approaches to make network design decisions. North Star NGO has predominantly made such decisions based on the first objective: maximize patient volumes. While intuitively appealing, our experiments indicate that this approach may need to be reconsidered, as such decisions may be quite different from those maximizing health. We show that solutions that are close to optimal for both patient volume and effectiveness can be obtained by balancing the weights of the criteria in the objective function. In other words, considerable gains in effectiveness of the services are attainable when taking the second criterion into account, without having to reduce the number of truck drivers served much.

We also analyze the heuristic of making decisions consecutively, as is current practice, as opposed to planning ahead future expansion decisions. In a context where many clinics are presently operating, consecutive decision making is shown to yield near-optimal decisions. This is an important finding, as it suggests that advanced and possibly expensive optimization tools might not be needed in such contexts. Long term network planning, however, substantially outperforms such heuristic in greenfield situations and in cases when synergy effects of placing multiple locations are substantial.

Impact of impreciseness in data and model parameters on solution quality presents another prominent question. Availability and quality of data is essentially limited in sub-Saharan Africa. Moreover, understanding the relationship between access to services and effectiveness is still in early stages of development and parameter calibration may be weak. We present extensive theoretical and computational analyses on the relationship between data inaccuracy and solution values. These reveal that the solutions found are highly robust to impreciseness, suggesting that difficulties in acquiring precise input data do not necessarily hinder effective location choices.

Although, as we prove, the RHFLP is strongly \( NP \)-hard, our numerical experiments show that the real life test instances can be solved within reasonable computation times using standard software. Further research on approximation and exact solution methods will be valuable if future applications consider more complex and larger problem instances.
As access to health services is a human right laid down in the WHO constitution (United Nations 1946), we advocate that, once the high volume needs are addressed, equity considerations are taken into account in future network expansions. Along the lines of Ares et al. (2016), our work proposes how to extend our model to include this criterion and shows that substantial gains in equity can be attained at a marginal loss in the other criteria. Much additional work is yet to be done to better understand adequate models and assess trade-offs. One specific challenge is to develop methods that identify so-called equitably efficient solutions (see Ogryczak 2000). Our model also does not capture possible effects of location and service package decisions on patient volumes. These may include competition effects, marketing or repetition effects, and effects of offering a more comprehensive set of health services. For example, Morris and Ferguson (2007) show that demand is partly affected by the availability of public healthcare providers. Follow-up research could investigate the magnitude of such effects for the truck driver population, propose richer models for $Z_{PV}$, and analyze the benefits of incorporating them in network design decisions. Likewise, models which include stochastic parameters can be of value to solve practical instances, given the many uncertainties in the data.

The proposed model captures the effectiveness of services provided via three newly proposed measures of access. The validity of these measures and their parametrization should be strengthened. For example, there is much room for improvement in terms of understanding the mechanisms relating access to effectiveness and the role of the composition of a package in these relationships. For instance, integrating HIV and TB care is known to bring about significant synergy effects. One specific area is to investigate possibilities to calibrate model parameters through empirical research relating differences in health outcomes to differences in access. We believe that the models, and subsequently the solutions proposed and health services provided to the truck drivers can greatly benefit from advancing scientific understanding in this area.
A Interviewee Characteristics

<table>
<thead>
<tr>
<th>Role</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. CEO*</td>
</tr>
<tr>
<td>2. Partner relations manager</td>
</tr>
<tr>
<td>3. Public health and medical advisor*</td>
</tr>
<tr>
<td>4. International health expert</td>
</tr>
<tr>
<td>5. Finance and HR manager</td>
</tr>
<tr>
<td>6. Regional director East Africa</td>
</tr>
<tr>
<td>7. West Africa manager</td>
</tr>
<tr>
<td>8. Site coordinator Southern Africa region</td>
</tr>
<tr>
<td>9. IT &amp; information manager</td>
</tr>
<tr>
<td>10. Manager IT strategy</td>
</tr>
</tbody>
</table>

* Surveyed for calibration of model parameters.

**Table 9:** Interviewee characteristics.
B Linearization of Effectiveness Variables $e_{qs}$

In order to model $e_{qs}$ by means of a set of linear constraints, we now introduce additional notations and definitions related to the *trip of a truck driver* in flow $q$ (see Table 10 for an overview). For a given solution $\omega$, we represent this trip by the following vector of vertices: $\pi_{qs} = [\pi_{qs}(1), \pi_{qs}(2), \ldots, \pi_{qs}(n-1), \pi_{qs}(n)]$. Here, $\pi_{qs}(1)$ and $\pi_{qs}(n)$ are the start vertex $O_q$ and the end vertex $D_q$, respectively. The vector $[\pi_{qs}(2), \ldots, \pi_{qs}(n-1)]$ is the sequence of *package s locations* that are passed during a trip from $O_q$ to $D_q$. Recall that for a given solution $\omega$ a package $s$ location is a location $k$ with a facility that offers package $s$, and note that $\pi_{qs}(1)$ and $\pi_{qs}(n)$ are not a package $s$ location. As motivated in the main text, we make the following assumption:

**Assumption 1.** A truck driver cyclically travels from $O_q$ to $D_q$ to $O_q$ to $D_q$ et cetera, as follows:

$$[\pi_{qs}(1), \pi_{qs}(2), \ldots, \pi_{qs}(n-1), \pi_{qs}(n), \pi_{qs}(n), \pi_{qs}(n-1), \ldots, \pi_{qs}(2), \pi_{qs}(1), \pi_{qs}(2), \ldots]$$

<table>
<thead>
<tr>
<th>$t_{kl}$</th>
<th>travel time between two locations $k$ and $l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_q$</td>
<td>set of vertices in the path of flow $q$, $L_q = {O_q \cup K_q \cup D_q}$</td>
</tr>
<tr>
<td>$L_{kl}$</td>
<td>set of locations that are passed after passing location $k$ during a trip from $O_q$ to $D_q$</td>
</tr>
<tr>
<td>$\pi_{qs}$</td>
<td>set of vertices representing a trip from $O_q$ to $D_q$, $\pi_{qs} = [O_q, \text{package s locations along path}, D_q]$</td>
</tr>
<tr>
<td>$i_{klqs}$</td>
<td>$\begin{cases} 1 \text{ if location } l \text{ is the immediate successor of location } k \text{ in vector } \pi_{qs} \ 0 \text{ otherwise} \end{cases}$</td>
</tr>
<tr>
<td>$t_{C1Tq}^{KL}$</td>
<td>total time a truck driver in flow $q$ is covered in segment $k \rightarrow l, (k, l) \in K_q \times K_q$</td>
</tr>
<tr>
<td>$t_{C1Tq}^{DL}$</td>
<td>total time a truck driver in flow $q$ is covered in segment $k \rightarrow D_q \rightarrow k, k \in K_q$</td>
</tr>
<tr>
<td>$t_{C1Tq}^{DL}$</td>
<td>total time a truck driver in flow $q$ is covered in segment $l \rightarrow O_q \rightarrow l, l \in K_q$</td>
</tr>
<tr>
<td>$t_{C1Tq}^{KL}$</td>
<td>effective time a truck driver in flow $q$ is covered in segment $k \rightarrow l, (k, l) \in K_q \times K_q$</td>
</tr>
<tr>
<td>$t_{C1Tq}^{DL}$</td>
<td>effective time a truck driver in flow $q$ is covered in segment $k \rightarrow D_q \rightarrow k, k \in K_q$</td>
</tr>
<tr>
<td>$t_{C1Tq}^{DL}$</td>
<td>effective time a truck driver in flow $q$ is covered in segment $l \rightarrow O_q \rightarrow l, l \in K_q$</td>
</tr>
</tbody>
</table>

**Note:** see Section 3.2 for the definition of the effective time covered during period $[t_1, t_2]$

| Table 10: Table of Notations. |

Let us consider the case when $|\pi_{qs}| \geq 3$ (i.e., there is at least one package $s$ location along the path). Because of Assumption 1, we know that a truck driver in flow $q$ always returns to $\pi_{qs}(1)$ after reaching $\pi_{qs}(n)$, and vice versa. We refer to the cycle from $\pi_{qs}(1)$ via $\pi_{qs}(n)$ to $\pi_{qs}(1)$ as the *flow q cycle*. This cycle could be regarded as a set of paths between successively passed package $s$ locations. This set is given by: $\{\pi_{qs}(h) \rightarrow \pi_{qs}(h + 1)|2 \leq h \leq n - 2\} \cup \{\pi_{qs}(n - 1) \rightarrow \pi_{qs}(n) \rightarrow \pi_{qs}(n - 1)\} \cup \{\pi_{qs}(h + 1) \rightarrow \pi_{qs}(h)|2 \leq h \leq n - 2\} \cup \{\pi_{qs}(2) \rightarrow \pi_{qs}(1) \rightarrow \pi_{qs}(2)\}$. We refer to these paths as *package s cycle segments* from now on. The length of a cycle segment represents the travel time of the truck driver for that segment, and is obtained via the parameters $t_{kl}$. 

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Let variable $i_{klqs}$ be equal to 1 if location $l$ is the immediate successor of location $k$ in vector $\pi_{qs}$ and equal to 0 otherwise. In Online Appendices A, B, and C we prove the following results:

**Proposition 4.** Constraints (20) - (24) ensure that variable $i_{klqs}$ is equal to 1 if and only if location $l$ is the immediate successor of location $k$ in vector $\pi_{qs}$.

\[
\sum_{l \in L_q} i_{klqs} = y_{ks} \quad q \in Q, k \in K_q, s \in S \tag{20}
\]
\[
\sum_{l \in L_q} i_{klqs} = 1 \quad q \in Q, k \in O_q, s \in S \tag{21}
\]
\[
\sum_{k \in L_q} i_{klqs} = y_{ls} \quad q \in Q, l \in K_q, s \in S \tag{22}
\]
\[
\sum_{k \in L_q} i_{klqs} = 1 \quad q \in Q, l \in D_q, s \in S \tag{23}
\]
\[
i_{klqs} \in \{0, 1\} \quad q \in Q, k \in L_q, l \in L_{kq}, s \in S \tag{24}
\]

**Lemma 1.** For solution $\omega$, the polyhedron $P_B(\omega) = \{i|0 \leq i \leq 1, (20) - (23)\}$ is a singleton.

**Lemma 2.** The constraint matrix $B$ defined by the left-hand side of constraints (20)-(23) is totally unimodular (TU).

Note that, for a given a solution $\omega$, the right-hand side of constraints (20)-(23) is integral. It follows from the unimodularity of $B$ that the polyhedron $P_B(\omega)$ is integral as well (Schrijver 1998).

By Lemma 1 we know that this polyhedron consists of one feasible solution, which must hence be integral. This shows that the solution space remains the same when defining variables $i_{klqs}$ as continuous variables instead of binary variables:

\[
i_{klqs} \in [0, 1] \quad q \in Q, k \in L_q, l \in L_{kq}, s \in S \tag{25}
\]

Variables $i_{klqs}$ hence provide a complete description of the package $s$ cycle segments. Let us consider a given segment $k \to l$ for which $i_{klqs} = 1$. For this cycle segment the average access time is given by $\frac{1}{2}t_{kl}^2$, and the total time covered $t_{c_{kl}^{\text{CTL}}}$ and the effective time covered $t_{c_{kl}^{\text{RCTL}}}$ (see equation (3)) are calculated as:

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For segments $k \to D_q \to k$ and $l \to O_q \to l$ the values are obtained by replacing $t_{kl}$ in these definitions by their durations: $t_{kD_q} + t_{D_qk}$ and $t_{Oql} + t_{lO_q}$, respectively. Using these parameters and the variables $i_{klqs}$, the value of access measure $a_{qs}^j(y)$ is calculated by auxiliary (linear) constraints (26) - (28). For conciseness, we refer to this variable as $a_{qs}^j$ from now on. The idea behind these constraints is that they calculate the value of $a_{qs}^j$ per package $s$ cycle segment and aggregate these in the correct way (the aggregation for $a_{qs}^{CTL}$ and $a_{qs}^{RCTL}$ is obvious, the aggregation for $a_{qs}^{ASAP}$ is explained in De Vries (2011)):

\[
a_{qs}^{CTL} = \frac{1}{T_q} \left( \sum_{l \in K_q} i_{Oqlqs} t_{C_{O_ql}}^{CTL} + \sum_{k \in D_q} i_{kD_qqs} t_{C_{kD_q}}^{CTL} + \sum_{k \in K_q} \sum_{l \in K_q} i_{klqs} (t_{C_{kl}}^{CTL} + t_{C_{lk}}^{CTL}) \right) \tag{26}
\]

\[
a_{qs}^{RCTL} = \frac{1}{T_q} \left( \sum_{l \in K_q} i_{Oqlqs} t_{C_{O_ql}}^{RCTL} + \sum_{k \in D_q} i_{kD_qqs} t_{C_{kD_q}}^{RCTL} \right.
+ \left. \sum_{k \in K_q} \sum_{l \in K_q} i_{klqs} (t_{C_{kl}}^{RCTL} + t_{C_{lk}}^{RCTL}) \right) \tag{27}
\]

\[
a_{qs}^{ASAP} = \frac{1}{2T_q} \left( \sum_{l \in K_q} i_{Oqlqs} (t_{Oql} + t_{lO_q})^2 + \sum_{k \in D_q} i_{kD_qqs} (t_{kD_q} + t_{D_qk})^2 \right.
+ \left. \sum_{k \in K_q} \sum_{l \in K_q} i_{klqs} (t_{C_{kl}}^2 + t_{C_{lk}}^2) \right) + i_{O_qD_qqs}M \tag{28}
\]

Here, $M$ is a large number that sets $a_{qs}^{ASAP}$ to a large constant if service package $s$ is not offered along the path of flow $q$. Namely, constraints (21) and (23) ensure that $i_{O_qD_qqs} = 1$ in this case.
Let $M_{qs}$ represent an upper bound on $a^j_{qs}$. The set of auxiliary (linear) constraints that model $e_{qs}$ as a piecewise linear function of $a^j_{qs}$ can be formulated using the so-called Lambda Method (Lee and Wilson 2001):

$$e_{qs} = \lambda_{3qs} + \lambda_{4qs} \quad q \in Q, j \in \{CTL, RCTL\}, s \in S_j$$  \hspace{1cm} (29)

$$e_{qs} = \lambda_{1qs} + \lambda_{2qs} \quad q \in Q, j = ASAP, s \in S_j$$  \hspace{1cm} (30)

$$\lambda_{1qs} + \lambda_{2qs} + \lambda_{3qs} + \lambda_{4qs} = 1 \quad q \in Q, j \in J, s \in S_j$$  \hspace{1cm} (31)

$$\lambda_{1qs}0 + \lambda_{2qs}a^j_s + \lambda_{3qs}a^j_s + \lambda_{4qs}M_{qs} = a^j_{qs} \quad q \in Q, j \in J, s \in S_j$$  \hspace{1cm} (32)

$$\lambda_{1qs} \leq z_{1qs} \quad q \in Q, j \in J, s \in S_j$$  \hspace{1cm} (33)

$$\lambda_{2qs} \leq z_{1qs} + z_{2qs} \quad q \in Q, j \in J, s \in S_j$$  \hspace{1cm} (34)

$$\lambda_{3qs} \leq z_{2qs} + z_{3qs} \quad q \in Q, j \in J, s \in S_j$$  \hspace{1cm} (35)

$$\lambda_{4qs} \leq z_{3qs} \quad q \in Q, j \in J, s \in S_j$$  \hspace{1cm} (36)

$$z_{1qs} + z_{2qs} + z_{3qs} = 1 \quad q \in Q, j \in J, s \in S_j$$  \hspace{1cm} (37)

$$\lambda_{iqs} \geq 0 \quad i \in \{1, 2, 3, 4\}$$  \hspace{1cm} (38)

$$z_{iqs} \in \{0, 1\} \quad i \in \{1, 2, 3\}$$  \hspace{1cm} (39)

Summarizing, to transform model (8) - (15) into a mixed-integer linear programming model, it suffices to replace equations (9) by constraints (20) - (23) and (25) - (39).

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References


