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INTERNATIONAL GROWTH WITH FREE TRADE IN EQUITIES AND GOODS: A COMMENT*

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1. INTRODUCTION

Within a trade model for a small, two-sector barter economy, Fischer and Frenkel [1974] discussed the behavior of the trade balance as the economy approaches the steady state. Hori and Stein [1977] extended the analysis to a two-country, two-commodity model with complete specialization. In the last section of their paper, Hori and Stein analyze the different stages of the creditor-debtor relationship when moving from the steady state equilibrium without trade in equities toward the steady state with free trade in equities. In this note, we show that Hori and Stein's dynamic analysis is erroneous. Consequently, in important cases where Hori and Stein perceive a switch from a net debtor to a net creditor position or vice versa, such a switch is either impossible or occurs under an artificial economic assumption. Nevertheless, there is a case not considered by Hori and Stein, where a switch in the creditor-debtor position is generated; this case then offers a description of the stages of the balance of payments as discussed in Fischer and Frenkel. Some other studies have dealt with the question raised by Fischer and Frenkel; see for example Buiter [1981], Ramanathan and Roberts [1981], Ruffin [1979], Van Bochove [1982], and Woodland [1982]. These studies all quote the work by Hori and Stein, but the shortcomings of the analysis appear to have gone unnoticed in the literature.

Hori and Stein conclude their analysis by noting the absence of a relationship between the richness of a country and its financial position. "Our analysis is compatible with Halevi's findings that 'there is no discernibly orderly progression through balance of payments stages connected with rising income per capita level'." [1977, p. 98]. "There is no relation between the flow of capital and the level of output per capita measured in consumer goods." [1977, p. 99].

We demonstrate that these results follow trivially from the assumptions underlying the model. These assumptions are simply too rigid to allow for a variation in the richness of one country relative to the other.

We use equations derived by Hori and Stein as we need them, referring to them by double parentheses $((\cdot))$; single parentheses $(\cdot)$ refer to our own equations.

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2. SOME FEATURES OF HORI AND STEIN'S MODEL

Two curves in the (k_1, k_2) space determine the steady state positions of the capital-labor ratios. In any steady state, the capital-labor ratios are constant; their values are related by the constant intensity curve (CIC)

$$k_2 = \frac{1}{n}\frac{N_1}{N_2}[f_1(k_1) - nk_1]. \tag{1}$$

In the steady state without trade in equities, k_1 and k_2 are given by the intersection of the CIC curve with

$$k_2 = \frac{N_1}{N_2}\frac{1-s}{s}k_1.$$

With trade in equities, the steady state values of k_1 and k_2 are given by the intersection of the CIC curve with the curve where values of a unit of equity are equal in both countries. The latter curve is named the iso capital value curve (ICV), and reads as follows:

$$k_2 = \frac{N_1}{N_2}\frac{1-s}{s}\frac{a_2}{a_1}k_1. \tag{2}$$

Denote by P^e and P^* the points displaying the steady state values of the k_i's ($i=1, 2$) with (k_i^e) and without (k_i^*) trade in equities, respectively. It readily follows that if the share of capital in country II exceeds the share of capital in country I, $a_2 > a_1$, then P^* lies *below* the ICV curve and vice versa:

$$k_1^* \gtreqless k_1^e \quad \text{insofar as} \quad a_2 \gtreqless a_1. \tag{3}$$

The graph depicting the intersection of the CIC and ICV curves is drawn by Hori and Stein [Figure 1, p. 91] for the case $a_2 < a_1$. However, in their analysis of the balance of payments (Section 3) they deal with the case $a_2 > a_1$, but still employ the same figure. Because the positions of the CIC and ICV curves differ essentially depending on whether $a_2 < a_1$ or $a_2 > a_1$, we drew two relevant graphs for the case $a_2 > a_1$. The shapes and locations of the two loci can be inferred from the assumed neoclassical properties of the production functions.

Whether P^* lies above or below the ICV curve is not immaterial for the following reason. Anywhere *below* the ICV curve equity in country II is priced higher and therefore all investment takes place in country II; the reverse is true above the ICV curve.

An important part of the paper by Hori and Stein deals with the dynamics of the creditor-debtor relationship. For an analysis thereof we need the following concepts and equations. Define per capita wealth w in country I as the sum of the stock of per capita capital k_1 existing in the country, plus the *net* per capita ownership of foreign capital. Denote the latter by x. Note that the net

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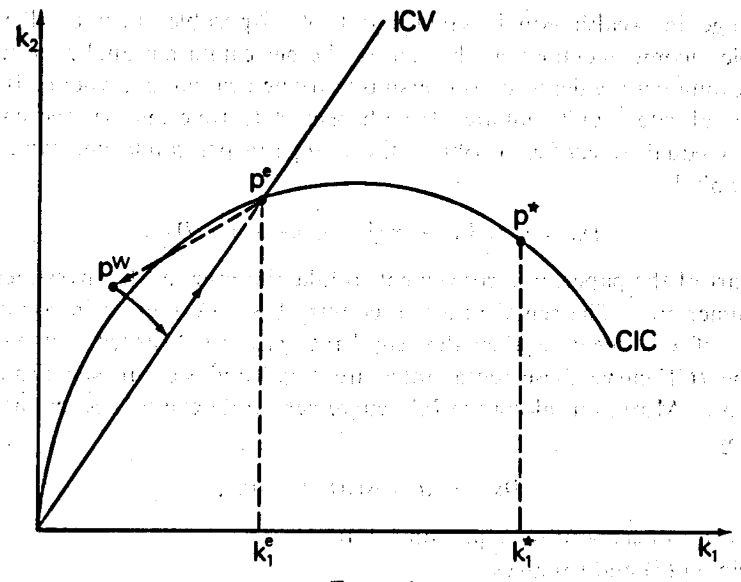

FIGURE 1

THE CASE OF $a_2 > a_1$ AND $a > s$.

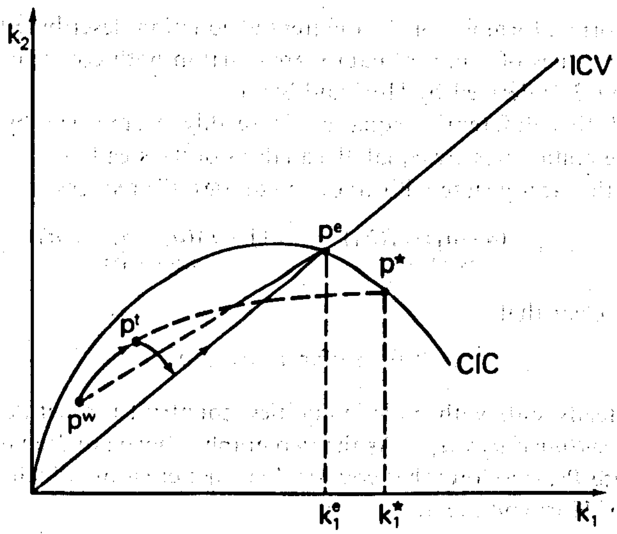

FIGURE 2

THE CASE OF $a_2 > a_1$ AND $a < s$.

ownership of foreign capital x is positive if country I is a creditor, and it is negative if country I is a debtor. Symbolically

(4) $$x = w - k_1.$$

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The change in wealth equals savings out of disposable income. Per capita disposable income is defined as the sum of the per capita national product $f_1(k_1)$ and per capita net dividends on ownership of foreign capital zx, where z is defined as the rental rate. Differentiate (4) with respect to time and set the change in wealth Dw equal to savings to obtain the change in per capita net ownership of foreign capital

$$Dx = s[f_1(k_1) + xz] - n(x+k_1) - Dk_1. \tag{5}$$

In this part of the paper, it is convenient to take the price of the investment good as the numeraire. The rental rate z in country I is $f'_1(k_1)$ and z in country II is $f'_2(k_2)/p$. If x is negative, then the rental rate $f'_1(k_1)$ is "earned" in country I. Along the *ICV* curve these rental rates are equal and we can substitute $f'_1(k_1)$ for z in (5). Moreover, along the *ICV* curve per capita capital accumulation Dk_1 is given by

$$Dk_1 = (a_1/a)sf_1(k_1) - nk_1, \tag{6}$$

where $a = a_1 s + a_2(1-s)$; see equation ((23)).

Combining (5) and (6) gives

$$Dx = \left(\frac{a-a_1}{a}\right)sf_1(k_1) + x[sf'_1(k_1) - n], \tag{7}$$

which is the correct formula for the differential equation describing the movement of x when the values of a unit of equity are equal in both countries; compare (7) with equation ((38)) derived by Hori and Stein.

Stability of this differential equation is readily established by using ((24)). Thus, once the rental rates are equal, the analysis of the steady state is a meaningful exercise. In the steady state with trade in equities, (7) reduces to

$$x^e = \frac{(a-a_1)sf_1(k_1)}{na(1-a)} = \frac{s(1-s)(a_2-a_1)f_1(k_1)}{na(1-a)}. \tag{8}$$

From (8) it is clear that

$$x^e \gtreqless 0 \text{ insofar as } a_2 \gtreqless a_1. \tag{9}$$

Thus in the steady state with trade in equities, country I is a net *debtor* if $a_2 < a_1$ and is a net *creditor* if $a_2 > a_1$. As the two graphs above are drawn for $a_2 > a_1$ at the steady state P^e, it follows that country I is a net creditor, which coincides with the analysis of Hori and Stein.

3. THE STAGES OF THE BALANCE OF PAYMENTS

With the above preparations in mind, we can discuss the development of the creditor-debtor relationship when moving from one steady state without trade in equities P^*, towards the steady state with trade in equities P^e. In particular,

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we will deal with Hori and Stein's case $a_2 > a_1$ [1977, pp. 97, 98]. Initially in the steady state without trade in equities at P^*, net ownership of foreign securities must be zero, i.e. $x=0$: Since $a_2 > a_1$, P^* lies below the *ICV* curve and the value of a unit of equity in country II is higher than its value in country I. This implies that after the opening of trade in securities all capital formation occurs in country II, and thus country I becomes a net *creditor*. Hori and Stein, however, argue the opposite on page 97. It is beneficial to support our conclusion with a more formal analysis.

To this end, we first describe how equation (5) looks when the values of a unit of equity are unequal and trade in equities is allowed for. Since $a_2 > a_1$, P^* lies below the *ICV* curve and we need the differential equations which describe the capital accumulation there:

$$\left.\begin{aligned} &(10) \qquad Dk_1 = -nk_1 \\ &(11) \qquad Dk_2 = f_1(k_1)\frac{N_1}{N_2} - nk_2 \end{aligned}\right\} \text{ if } Q_2 > Q_1.$$

See equations ((18b)) and ((19b)) in Hori and Stein. We also need to account for the fact that the rental rates in the two countries differ. The change in per capita net ownership of foreign capital below the *ICV* curve is

$$(12) \qquad Dx = \begin{cases} sf_1(k_1) + x[sf_1'(k_1) - n], & \text{if } x < 0 \\ sf_1(k_1) + x\left[\dfrac{s}{p}f_2'(k_2) - n\right], & \text{if } x > 0, \end{cases}$$

where the terms of trade p are known from ((3)) as

$$(13) \qquad p = \frac{s}{1-s}\frac{N_2}{N_1}\frac{f_2(k_2)}{f_1(k_1)}.$$

This differential equation is conveniently written as

$$(14) \qquad Dx(t) = A(t)x(t) + B(t),$$

where $B(t) = sf_1(k_1)$, and $A(t) = [sf_1'(k_1) - n]$ if $x<0$, or $A(t) = \left[\frac{s}{p}f_2'(k_2) - n\right]$ if $x>0$. The general solution for this equation is known as

$$(15) \qquad x(t) = \exp\left[\int_{t_0}^{t} A(s)ds\right]\left\{c + \int_{t_0}^{t} B(s)\exp\left[-\int_{t_0}^{s} A(u)du\right]ds\right\},$$

where c is a constant determined by the initial condition.

In the discussion on pages 97 and 98 in Hori and Stein, the economies are initially in the steady state without trade in equities; hence, $x(t_0) = 0$. This boundary value implies that $c=0$ in the above integral (15). When $c=0$, the sign of $x(t)$ is determined by the sign of $B(t)$. Because $B(t) = sf_1(k_1)$ below the *ICV* curve, $x(t)$ is unambiguously positive. Country I immediately becomes a net *creditor* when trade in equities is opened, and the economies were initially in the

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steady state without trade in equities. The net creditor position is kept by country I when moving from one steady state to the other. This can be seen by integrating the differential equation (7), and using, as the boundary value, the positive value of $x(t)$ provided by (15) at the time the *ICV* curve is reached. It was shown above that country I is a net creditor in the new steady state. *We conclude that no switch can occur in the creditor-debtor relationship between the countries on the path from the one steady state to the other.* Because Hori and Stein use the figure on page 91 in their analysis for the case $a_2 > a_1$, they reach the conclusion that country I is initially a debtor but becomes a creditor in the steady state; thus erroneously observe a switch in the creditor-debtor relationship.

Let us now discuss the general case, where the initial position of the economies is not necessarily the steady state without trade in equities. Because the case for $a_2 < a_1$ is very similar to the one for $a_2 > a_1$, we will confine ourselves to the latter. On page 89 in Hori and Stein, it is assumed that once on the *ICV* curve: "...newly produced investment is allocated in a way that will maintain this state." The *ICV* curve can never be crossed. We first describe the evolution where the initial position is below the *ICV* curve and then that where it is above the *ICV* curve.

Anywhere below the *ICV* curve the integrals (15), or the differential equations (12), describe the state of $x(t)$; once on the *ICV* curve, equation (7), or its corresponding integral which is very similar to (15), describes the state of $x(t)$. The only way in which country I can ever be a debtor is when initially $x(t_0) < 0$; this implies $c < 0$ in (15). We regard the boundary value $x(t_0) < 0$ as somewhat artificial because given $a_2 > a_1$, the differential equations (12) and (7) imply that $x(t)$ must have had a larger negative value for $t < t_0$. Presumably, $x(t)$ has been zero at some date in the past. *Within the model it cannot be explained how, below the ICV curve, an initial net debtor position for country I could arise.* A switch in the financial regimes between the countries relies on an artificial economic assumption.

The differential equations (12) have to be replaced by another pair if we start somewhere above the *ICV* curve. The state of $x(t)$ once on the *ICV* curve is still correctly described by equation (7). Capital accumulation above the *ICV* curve is given by the equations

$$\left.\begin{aligned} &(16) & Dk_1 &= f_1(k_1) - nk_1 \\ &(17) & Dk_2 &= -nk_2 \end{aligned}\right\} \quad \text{if} \quad Q_2 < Q_1.$$

Using (5), the change in per capita net ownership of foreign capital above the *ICV* curve is

$$(18) \qquad Dx = \begin{cases} -(1-s)f_1(k_1) + x[sf_1'(k_1) - n], & \text{if } x < 0, \\ -(1-s)f_1(k_1) + x\left[\dfrac{s}{p}f_2'(k_2) - n\right], & \text{if } x > 0. \end{cases}$$

If we choose $B(t) = -(1-s)f_1(k_1)$, and $A(t) = [sf_1'(k_1) - n]$ if $x < 0$, or $A(t) =$

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$\left[\frac{s}{p}f_2'(k_2)-n\right]$ if $x>0$, then the appropriate integral of (18) is again given in (15).

In this case, a nonpathological switch in the creditor-debtor relationship can be generated. Assume, for example, that the economies are moving toward the steady state without trade in equities and are above the *ICV* curve when the trade in equities is opened. Hence, the boundary value implies $c=0$, and because $B(t)=-(1-s)f_1(k_1)$ is negative, $x(t)$ will be negative for some finite time period. *We conclude that if the initial position of the economies lies above the ICV curve, it is conceivable that a switch in the creditor-debtor relationship occurs.* The sign of $x(t)$ along the growth path towards the *ICV* curve is determined by the expression

$$c+\int_{t_0}^{t} B(s)\exp\left[-\int_{t_0}^{s} A(u)du\right]ds.$$

Hence, once $x(t)$ is negative, it cannot become positive again before touching the *ICV* curve. This follows from the fact that $B(t)$ is negative above the *ICV* curve and that c is necessarily negative if $x(t_0)$ is negative. It is not hard to see that $x(t)$ becomes positive in some finite timespan once the economies are moving along the *ICV* curve. To summarize, while the economies are growing towards the steady state, *at most two switches in the creditor-debtor relationship can occur.*

We are now in a position to describe a case in which the countries experience distinct balance of payments stages. For the sake of economic clarity we repeat the conditions for a switch in the financial position to occur: $a_2>a_1$, the share of capital in country II exceeds the share of capital in country I; $Q_2<Q_1$, initially the value of a unit of equity is higher in country I, and, therefore, all capital goods are installed in country I; $x(t_0)$ has at most a small positive value, the initial net ownership of foreign capital in country I is small or negative. Given these conditions, country I evolves from a young and growing debtor to a mature debtor, then to a young creditor, and finally, into a major creditor.

How could a process like the one above arise? Suppose[2] the economies are initially in the steady state with trade in equities, when a war between the countries destroys some parts of the capital stocks k_1 and k_2. Assume that this puts the economies somewhere to the left of the *ICV* curve, as shown by points P^w in Figures 1 and 2. In Figure 1 we have illustrated the trajectory the economies would follow if trade in all goods were restored after the war. At first, capital accumulation in country I brings the economies back to the *ICV* curve; see equations (16) and (17). Country I was a net creditor before the war but now has the potential to become a net debtor. Once on the *ICV* curve, the economies grow towards P^e and country I becomes a net creditor again.

Another possible scenario is sketched in Figure 2. Suppose that because of the hostilities claims on foreign capital are liquidated, hence at P^w: $x=0$. After the war, trade in consumption and investment goods is reopened, but there are still

[2] In private correspondence, Professor Stein suggested this example to point out what Hori and Stein intended to envisage. The elaboration is mine.

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impediments to trade in securities. The economies grow toward P^*; the differential equations ((4)) and ((5)) are now the relevant ones. As time elapses, international relations become normalized and trade in equities is liberalized. Suppose this happens at point P^t. Then, country I definitely becomes a net debtor until the *ICV* curve is reached. A switch in the creditor-debtor relationship occurs somewhere along the *ICV* curve while the economies are growing towards the steady state, P^e.

This description of the development of the balance of payments is similar to the one offered by Fischer and Frenkel [1974, p. 513] for a poor country. Fischer and Frenkel define the initial richness of a country relative to the steady state; a country is poor if it has low initial levels of wealth and per capita capital. We can identify country I as poor in a similar fashion, but this does not rule out the possibility that *both* countries are poor. The above conditions contain a subset of boundary values such that $x(t_0)$ is smaller than its steady state value x^e, and that $k_1(t_0)$ is smaller than the steady state level, k_1^e. These boundary values are sufficient to identify country I as poor, as can be inferred from equation (4). *Thus the model admits patterns of the balance of payments related (in a definite way) to the richness of a country as discussed by Fischer and Frenkel.* Fischer and Frenkel have to define the richness relative to the steady state because they analyze the small country case. Hori and Stein adopt a different definition of richness; we now turn to the implications thereof.

In Hori and Stein's view, the richness of a country can be defined as the level of per capita output measured in consumer goods. (The price of the consumption good is the numeraire.) For country I this amounts to $pf_1(k_1)$ and for country II this is just $f_2(k_2)$. From definition (13) of the terms of trade we immediately find that the richness of country I relative to country II is given by

$$\frac{pf_1(k_1)}{f_2(k_2)} = \frac{s}{1-s}\,\frac{N_2}{N_1}. \tag{19}$$

Independently of any endogenous variable like the net credit or debit position, and hence of the initial positions, the terms of trade adjust in such a way that the richness of one country relative to the other is completely determined by the parameters s, N_1 and N_2 of the model. The explanation for this fact can be found on page 85 of Hori and Stein's paper. There they argue that because consumption demand is chosen proportional to disposable income and with identical savings ratios: "Transfer payments between countries will not affect the world consumption demand..." As a result of these assumptions, the richness of one country relative to the other is not affected by the opening of trade in equities. Hence, no wonder that Hori and Stein could not detect any "...orderly progression through balance of payments stages connected with rising income per capita level." *Because the richness of a country vis à vis the other is predetermined, it cannot change during the growth process.*

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