



Forward-reserve storage strategies with order picking: When do they pay off?

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ABSTRACT

Customer order response time and system throughput capacity are key performance measures in warehouses. They depend strongly on the storage strategies deployed. One popular strategy is to split inventory into a bulk storage and a pick stock, or Forward-Reserve (FR) storage. Managers often use a rule of thumb: when the ratio m of average picks per replenishment is larger than a certain factor, it is beneficial to split inventory. However, research that systematically quantifies the benefits is lacking. We quantify the benefits analytically by developing response travel time models for FR storage in an Automated Storage/Retrieval system combined with order picking. We compare performance of FR storage with turnover class-based storage, and find when it pays off. Our findings illustrate that, in FR storage systems where forward and reserve stocks are stored in the same rack, FR storage usually pays off, as long as m is sufficiently larger than 1. The response time savings can go up to 50% when m is larger than 10. We validate these results using real data from a wholesale distributor.

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1. Introduction

Warehouses apply a multitude of storage strategies. A suitable storage strategy has a major impact on customer response time and system throughput capacity. One popular strategy, which can be combined with other storage rules, is to split up inventory into a bulk storage and a pick stock, also called forward-reserve storage (De Koster *et al.*, 2007). This saves effort when products are replenished from vendors in large quantities, as the inventory can be moved to stock locations in pallet quantities. It also may help to compact the forward pick zone, as only small quantities of product are stored there, which reduces travel distance in the order picking process. However, the downside of dividing inventory over two storage areas is that extra internal replenishments are required to move inventory from the bulk area to the pick area once the pick area stock falls below the internal reorder level. It also may cost additional space which, in turn, can increase travel times, warehouse size and land cost. Additionally, not every product has to be in the forward zone. Splitting up inventory may not benefit a particular product that is only stored in the bulk area.

In order to address this problem in practice, managers often fall back on the “consultants rule”, which states that as long as the average number of order picks per product exceeds the replenishments with a certain factor m , it is beneficial to split inventory over two systems. According to Figure 5 (Guidelines for designing a manual case-picking warehouse) in Thomas and Meller (2015), $m = 2$ is a good

value. In their numerical results, they show that even $m = 1$ may be a good choice. The question is to what extent this critical m -value also applies in automated and other manual systems. However, research that systematically quantifies these benefits is lacking. Obviously, the factor would depend on the type of system, and some other factors, such as the storage strategies deployed in the forward zone, the steepness of the demand curve, and the reorder quantity.

Forward-Reserve (FR) storage can be applied in different systems: the forward and reserve stock are stored in different racks or the forward and reserve stock are stored in the same rack (we regard two identical racks on the left-hand and right-hand sides of the same aisle as one rack). A further distinction can be made by: (i) items (or products) in the reserve area are replenished to the forward zone either manually or automatically; and (ii) items in the forward zone are picked using either a manual picker-to-parts or an automated parts-to-picker system, i.e., automated retrieval followed by manual picking at an order picking station. When the bulk stock is stored in wide-aisle pallet racks, replenishment from bulk to pick stock is often done manually, using a manned forklift truck. The bulk locations are then often located above the pick locations, in the same rack. A popular automated system where both bulk and pick locations can be found in the same rack uses automated cranes to replenish the pick locations in the lower levels. The Dynamic Picking System (DPS) (Witron, 2019) is an example (see also Yu and de Koster (2010)). The pickers

Table 1. Overview of FR storage systems.

Forward and reserve storage area	Replenishment from reserve to forward area	Picking system in forward zone	Typical instance	Literature
Different racks	Manual	Picker-to-parts	Pallet racks for reserve storage + Flow racks for picking	Hackman <i>et al.</i> (1990), Frazelle <i>et al.</i> (1994), Van den Berg <i>et al.</i> (1998), Bartholdi and Hackman (2008), Gu <i>et al.</i> (2010), Bartholdi and Hackman (2016)
		Parts-to-picker	Pallet racks for reserve storage + Miniload Automated Storage/Retrieval (AS/R) system with order picking station	
	Automated	Picker-to-parts	Unit-load AS/R system for reserve storage + Flow racks for picking	—
		Parts-to-picker	Unit-load AS/R system for reserve storage + Miniload AS/R system with order picking station	—
	Manual	Picker-to-parts	Pallet rack. Replenished from higher levels by forklift trucks + Lower tiers for picking (sometimes different racks)	Van den Berg <i>et al.</i> (1998), Thomas and Meller (2015), Bartholdi and Hackman (2016)
		Parts-to-picker	Not common	
Same rack	Automated	Picker-to-parts	Tote rack. Replenished from higher levels by AS/R cranes + Flow channels in the lower tiers for picking (e.g., Witron DPS.)	Yu and de Koster (2010), Ramtin and Pazour (2014, 2015), Schwerdfeger and Boysen (2017)
		Parts-to-picker	AS/R system with order picking station	Our paper

walk along the pick face of the racks and pick the orders by a picker-to-parts system, e.g., using a pick-by-light system. Another possible system is to combine an automated retrieval system for replenishing the forward zone, with an automated retrieval system for retrieving loads from the forward zone and bringing them to an order picking station.

Table 1 gives an overview of different FR storage systems, as well as literature that studies performance aspects of these systems.

This article focuses on FR storage systems where forward and reserve stock are stored in the same rack and where the replenishments and retrievals for picking are automated. We systematically analyze the benefits of using FR storage in Automated Storage/Retrieval (AS/R) systems (i.e., a parts-to-picker system) with order picking.

AS/R systems comprise a variety of automated warehousing systems (Johnson and Brandeau, 1996). Such systems consist of aisle-bound cranes serving storage racks with unit loads (e.g., storage totes or pallets). Often, they are combined with order picking stations where units are picked from the loads to fill customer orders (e.g., miniload systems). Typically, a load containing multiple units of an item is retrieved and returned to the storage rack several times before it is depleted. Figure 1 shows a typical miniload AS/R system with order picking stations.

Recently, other types of automated warehouse systems (such as shuttle-based systems) have emerged. Shuttle-based systems form economical solutions in environments with very small inventories per item and a large throughput capacity requirement (e.g., some e-commerce warehouses). However, for larger inventories per item and a lower throughput capacity requirement, crane-based systems are still the cheaper solution. Crane-based AS/R systems (both pallet and miniload) still form the backbone in many newly realized warehouses. We surprisingly find that FR storage in AS/R systems with order picking has not yet been studied

**Figure 1.** A miniload AS/R system (Source: Mecalux 2019).

(see Table 1). However, FR storage can be very efficient in AS/R systems with order picking. Figure 2(a) divides the rack into two areas (forward and reserve area), such that each item is assigned to either the forward or the reserve area. If an item is assigned to the forward zone, only a few loads of that item need to be stored in the forward zone, and the remaining loads are stored in the reserve area. If the item is assigned to the reserve area, all of its loads are stored in the reserve area. A load in the forward zone containing multiple units can be retrieved and returned to the storage rack many times until it is empty, after which it is removed from the rack. This then triggers a replenishment from the reserve area to fill the empty location in the forward zone. Multiple retrievals of a load stored close to the Input/Output (IO) point in the forward zone can result in substantial savings in response time, and a few replenishments do not lead to much increase in the total response time.

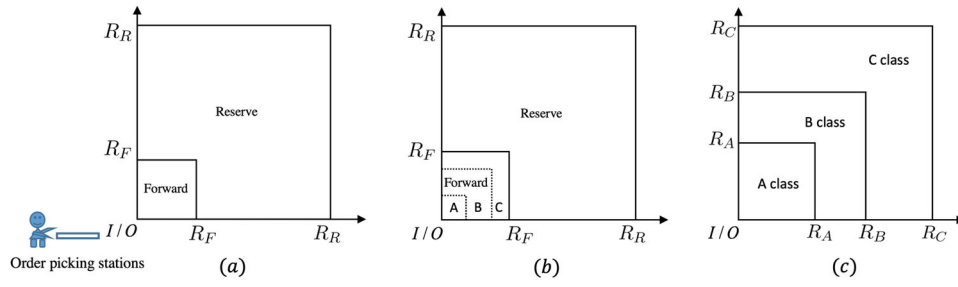


Figure 2. Different storage systems (side view of the rack): (a) FR storage, (b) FR-ABC storage, (c) ABC class-based storage.

The average response time may substantially decrease compared with other storage strategies.

We focus on the following two research questions:

1. How can the response time of the crane be evaluated in an AS/R system with FR storage and order picking?
2. Under what circumstances (i.e., for which parameters in what range) does it pay off to use FR storage?

In order to answer research question 2, we compare the response time of FR storage with ABC class-based storage. ABC class-based storage is a class-based storage strategy that divides the items into three groups. As shown in Figure 2(c), a few high-demanded items (the A class items) are stored in the region closest to the I/O point. Low-demand items, grouped in the C class, are stored in the region farthest from the I/O point. ABC class-based storage is the preferred storage strategy to compare with FR storage, because: (i) dividing items into only three turnover-frequency classes yields a near-optimal solution to minimize the expected retrieval time for class-based storage (random storage has only one class whereas in full turnover-based storage, each item has its designated class) (Yu *et al.*, 2015); (ii) ABC class-based storage can be implemented easily and there is no need to frequently reconfigure the storage assignment.

In this study, we show that, in automated FR storage systems where forward and reserve stock are stored in the same rack, combined with order picking, FR storage usually pays off, as long as the ratio of picks per replenishment, m , is sufficiently larger than one. The response time savings can go up to 50% when m is larger than 10 and the average annual demand per item is more than 10 loads. We validate these results using real data from a wholesale distributor, which again shows substantial (up to 46%) response time savings using FR storage.

The remainder of this article is organized as follows. In Section 2, we include a literature review of related studies. Section 3 establishes the travel time model for FR storage and provides the optimal solution for the model. In Section 4, we extend the FR storage with ABC class-based storage in the forward zone as FR-ABC storage. In Section 5, we use numerical experiments to evaluate the response time of FR storage and FR-ABC storage, and find under which circumstances it pays off to use FR storage (and FR-ABC storage) instead of using ABC class-based storage. Section 6 uses data from a case study in the analytical models. Section 7 concludes this article.

2. Literature review

In this section, we review two literature streams:

1. Papers that analyze the impact of storage strategies on performance in general AS/R systems and papers that focus on performance analysis of AS/R systems in conjunction with order picking stations, with emphasis on the storage strategies used.
2. Papers that study or compare FR storage strategies.

Storage strategies and their impact on performance in AS/R systems have been studied widely. We only review a selection of key papers. Three storage strategies have received most attention: (i) random storage, where each load is equally likely to be stored in any location; (ii) full turnover-based storage, where a load with higher turnover is assigned to a location closer (in crane travel time) to the I/O point; and (iii) class-based storage, which divides items into different classes based on their turnover frequency and places higher turnover class in locations closer to the I/O point. Items of the same turnover class are stored randomly in the same storage zone. Hausman *et al.* (1976) formulate a travel time model for random storage, full turnover-based storage, and two and three class-based storage. Rosenblatt and Eynan (1989) and Eynan and Rosenblatt (1994) extend this travel time model to n classes for both square-in-time racks and non-square-in-time racks and show that the average retrieval time decreases when the number of classes increases. Following the results of these papers, most research on class-based storage implicitly or explicitly assumes that the number of items in each class is infinite and the required space of each storage class is fully shared between the items. This implies that it equals the average total inventory level of all items in the class (Eynan and Rosenblatt, 1994; Park *et al.*, 2006). Yu *et al.* (2015) point out that with a finite number of items, the storage space in a zone cannot be fully shared. The required space for each class is larger than the average total inventory level of all items in the class. The fewer items that share a class, the more space each item needs. This leads to a trade off between the effects of more classes leading to less space sharing and therefore a larger required rack, and more classes leading to more accurate storage leading to shorter travel time. They formulate a travel time model for a class-based storage strategy, explicitly considering space sharing with a finite number of items in the zones. They find that the optimal number of classes minimizing travel time is

Table 2. Overview of research on FR storage strategies.

Article	Forward and reserve areas		Function of forward zone		Replenishment quantity			Objective		Decision variables			Method
	Different racks	Same rack	Storage and picking	Picking	Suffices to replenish an item	One unit load	Minimize total picking and replenishment cost	Minimize replenishment cost		Size of the forward zone	Items to be stored in the forward zone	Quantity per item stored in the forward zone	
Hackman <i>et al.</i> (1990)	•		•		•		•				•	•	Greedy heuristic
Frazelle <i>et al.</i> (1994)	•		•		•		•				•	•	Greedy heuristic
Van den Berg <i>et al.</i> (1998)	•	•	•			•					•	•	Greedy heuristic
Bartholdi and Hackman (2008)	•		•		•			•				•	Analytical, optimal
Gu <i>et al.</i> (2010)	•		•		•		•				•	•	Branch-and-bound, optimal
Yu and de Koster (2010)													Analytical, optimal
Ramtin and Pazour (2015)		•		•		•	•			•			Analytical, optimal
Thomas and Meller (2015)		•		•		•		•			•		Scenarios, numerical
Schwerdfeger and Boysen (2017)		•		•		•		•					Branch-and-bound, optimal
Our paper		•	•			•	•			•			Analytical, optimal

small and ABC class-based storage (three classes) is near optimal.

AS/R systems in conjunction with end-of-aisle or remote order picking stations have not been studied widely. Bozer and White (1990) were the first to combine an AS/R system with order picking stations. They analyze the performance of end-of-aisle order picking systems assuming items are randomly stored in the rack. Following the research of Bozer and White (1990), several papers study the performance of an AS/R system with end-of-aisle or remote order picking stations by adopting the three widely studied storage strategies into the system: random storage (Foley and Frazelle, 1991; Claeyss *et al.*, 2016; Tappia *et al.*, 2019), full turnover-based storage (Park *et al.*, 2003) and class-based storage (Park *et al.*, 2006), or assuming the travel time of the crane follows a general distribution, so that the results can be applied for all the three storage strategies (Park *et al.*, 1999; Koh *et al.*, 2005).

The FR storage strategy has been studied in only a few papers, which are summarized and compared in Table 2. Three main decision problems can be distinguished related to sizing the forward zone (see Bartholdi and Hackman, 2016): (i) determining the size of the forward zone, (ii) determining the items to be stored in the forward zone, and (iii) determining the quantity per item to be stored in the forward zone. Hackman *et al.* (1990) provide a cost model where the forward and reserve areas are located in different racks. They assume that one replenishment from the reserve area suffices to replenish all loads of an item in the forward zone and propose a heuristic algorithm to minimize the total costs for picking and replenishing. Following this paper, several papers have studied forward zone sizing and assignment of items to the forward zone, where forward and reserve areas are in different racks. Frazelle *et al.* (1994) extend this research by considering the size of the forward zone as a decision variable. Whereas Hackman *et al.* (1990) and Frazelle *et al.* (1994) assume that within a single replenishment multiple loads of one item can be replenished, Van den Berg *et al.* (1998) assume that at each replenishment only one unit-load of an item can be replenished. Bartholdi and Hackman (2008) extend the model of Hackman *et al.* (1990) by assuming that items have already been pre-selected for storage in the forward zone. They derive the optimal storage quantity per item in the forward zone minimizing annual restocks. Gu *et al.* (2010) give an optimal branch-and-bound algorithm for the problem of Hackman *et al.* (1990), to maximize the savings in picking and restocks.

Other papers study systems in which the forward and reserve stocks are stored in a single rack. The lower tiers serve as pick positions in combination with a picker-to-parts order picking method and the upper levels are used for automated replenishment by AS/R cranes. Yu and de Koster (2010) optimize the picking order batch size to maximize the throughput capacity of such a system, under the assumption that not all items have a position in the forward zone. This implies that a new item needed in the forward zone must be swapped for an old item, which has to be brought back to the reserve stock. Schwerdfeger and Boysen (2017) give a heuristic decomposition approach and an exact

branch-and-bound procedure in order to minimize the maximum number of such load swaps to be executed by the crane, between any pair of successive orders. Ramtin and Pazour (2015) focus on the minimization of the expected replenishment travel time. To achieve this, they optimize the assignment of items to pick positions.

Thomas and Meller (2015) study total labor time for manual, parallel-aisle, picker-to-parts case-picking warehouse designs. They use different routing heuristics, such as traversal and return heuristics to calculate approximate travel times for order picking and also calculate replenishment and put-away time, for different rack layouts (detailed calculations are based on a paper by Thomas and Meller (2014)). They also compare FR storage with random storage for some settings. They claim that in their setting “a forward area is preferred for even slightly skewed ABC curves”. Although they do not explicitly study the impact of the number of picks per replenishment, their numerical results suggest that FR splitting may pay off for $m \geq 1.6$. We choose a different approach, by obtaining the optimal solutions for the design of the FR storage analytically through closed-form equations for the travel time. In addition, we compare FR storage with ABC class-based storage.

According to Table 1, FR storage in AS/R systems combined with order picking has not yet been studied. However, as argued in the Introduction, substantial travel time savings may be achieved by applying such a strategy.

3. Travel time model for FR storage

In this section, we derive the travel time model for FR storage, assuming a crane-based system operating in the reserve area. The objective is to minimize the expected response time, i.e., the expected travel time of the crane, to retrieve a load and bring it to the I/O point (from where it is conveyed to an order picking station). We do not explicitly model the total cycle time, i.e., including the order picking and conveying time. Instead, we follow travel time literature (e.g., Yu *et al.* (2015)) and focus only on crane travel time, as cranes are expensive and consume large amounts of (expensive) space, their number and capacity are fixed. In a situation with fluctuating demand they are therefore often a critical resource. Knowing and being able to minimize the cycle time of a crane system is pivotal to obtain the minimum number of required cranes and enhance the system throughput.

In an AS/R system with order picking, two types of operating modes for the crane can be distinguished: single-command and dual-command cycles. In a Single-Command (SC) cycle, either a storage or retrieval is performed in a single travel cycle. In a Dual-Command (DC) cycle, a storage operation is paired with a retrieval, which reduces total travel time for the crane compared with SC cycles.

We distinguish two operating policies of the crane, while assuming that the crane dwells at the I/O point:

1. Policy P_1 : The crane only carries out SC cycles. In this policy, the crane performs several retrievals to satisfy a

batch of customer orders before returning any of the loads.

2. Policy P_2 : The crane carries out DC cycles, with an occasional SC cycle when a pick load has been depleted. In this policy, the crane returns a previously picked load, which has not been depleted, before it retrieves a next load.

Policy P_1 is treated in Section 3.3 and Policy P_2 is treated in Section 3.4. We first present model assumptions in Section 3.1 and derive the size of the areas in Section 3.2.

3.1. Assumptions and definitions

The following assumptions are made throughout this article:

1. (a) The rack is continuous in space and Square-In-Time (SIT). However, it is possible, albeit at the expense of more involved calculations, to generalize results for non-SIT racks. (b) All storage locations are one-unit wide SIT, which implies that each rack part of 1×1 time units (length \times height) can precisely store one load. We also neglect crane acceleration and deceleration. These assumptions simplify calculations without too much loss of generality. They are reasonably accurate for not too small racks. In Section 6, we include an example for loads with different size (non-SIT).
2. The AS/R crane can move simultaneously and independently in both vertical and horizontal directions. This is based on reality and it means that the travel time is determined by the maximum of the driving and lifting time (a Chebyshev metric).
3. The crane cannot carry more than one load at a time.
4. The constant load pick-up and drop-off time for the crane are ignored (i.e., we focus on travel time only).
5. The number of picks per load of item i until depletion of the load is m_i , which depends on the capacity of the load and the number of units the picker picks from the load each time.
6. The demand in unit loads per unit time of each item follows a stationary stochastic distribution.
7. Inventory restocking. Item inventories are restocked using continuous review (r, Q) reorder policies with back-ordering. A service level, $1 - \alpha$, is required, which is the probability of not stocking out during an inventory restocking cycle. The inventories in the reserve area are restocked in the spare time of the crane, i.e., in time during which no load is required for order picking. A depleted item in the forward zone is replenished by the bulk stock, retrieved from the reserve area. We assume that new incoming inventory is stored in the reserve area.

Table 3 presents some notations which are introduced in the following sections.

In FR storage, we divide the rack into two areas. The forward zone is SIT and located in the lower-left corner of the rack next to the I/O point (see Figure 2(a)). The loads are stored randomly in both the forward and reserve area. We consider a single aisle of the system with one order picker

Table 3. Main notations.

N	Total number of items in the AS/R rack
i	Index of the i th item. An item with higher expected demand in unit loads has a smaller index from the index set: $\{1, 2, 3, \dots, N\}$
\mathcal{F}	The set of items chosen to be stored in the forward zone
\mathcal{U}	The set of all items
N_F	Number of items chosen to be stored in the forward zone
m_i	Number of picks per unit load of item i
D	Expected total demand in unit loads per unit time of all the items
$D(i)$	Expected demand in unit loads per unit time of item i
R_F	The one-way travel time for storing or retrieving a load at the farthest boundary of the forward zone in FR storage
R_R	The one-way travel time for storing or retrieving a load at the farthest boundary of the reserve area in FR storage
s	Shape factor of the ABC demand curve; represents the skewness of the demand curve
ε	Space-sharing factor; represents the space-sharing effect among items in the same class
K_i	The ratio of order cost to holding cost per load of item i per unit time
r_i	Reorder point of item i in unit loads
Q_i	Order quantity of item i in unit loads
l_i	Mean lead time for the orders of item i
$F_i(\cdot)$	Cumulative demand distribution of item i in unit loads during lead time l_i
$1 - \alpha_i$	Service level of item i
ss_i	Safety stock of item i in unit loads

(not explicitly modeled) and one I/O point at the end of the aisle and a single crane serving the racks at both aisle sides.

In general, two types of replenishments from the reserve to the forward area can be distinguished based on whether or not the replenishment process is carried out outside the picking period: (i) advance replenishments, if carried out outside the picking period; and (ii) concurrent replenishments, i.e., a replenishment is performed whenever an item in the forward zone is depleted during the picking period (see, e.g., Van den Berg *et al.* (1998)). With advance replenishment, assigning multiple loads of an item to the forward zone can reduce the number of concurrent replenishments in the picking period, and therefore help reduce the expected response time.

In class-based storage, advance replenishments do not occur. In order to allow a fair comparison of the performance of FR storage and class-based storage, we assume that also in FR storage, advance replenishments are not allowed. With this assumption, assigning multiple loads of an item to the forward zone cannot significantly reduce the number of concurrent replenishments. In addition, it would also increase the size of the forward zone, which leads to a longer response time. Therefore, we assume that:

- There are no advance replenishments and if an item is assigned to the forward zone, only one load of that item is stored in the forward zone. When an item has been depleted in the forward zone and it is needed for order picking, a crane retrieves a load from the reserve area and brings it to the I/O point for order picking. After picking, the crane then returns the load from the I/O point to the forward zone as a replenishment.

We next need to find which items should be stored in the forward zone (with additional reserve stock in the reserve area), and which items should only be stored in the reserve area (to be retrieved from there), so that the expected response time for FR storage is minimized.

3.2. Sizing the forward and reserve areas

Let N be the number of items stored in the rack and D be the expected total demand in unit loads per unit time of all

the items. We assume the items $i = 1, \dots, N$ are sorted on decreasing unit-load demand per time unit. The expected demand in unit loads per unit time of item i is continuous and is described by the classic cumulative ABC demand curve, which can be approximated by the function (Hausman *et al.*, 1976):

$$G(i) = \left(\frac{i}{N}\right)^s = \sum_{j=1}^i D(j)/D,$$

for some shape parameter s , with $0 < s \leq 1$, $i = 1, \dots, N$.

Therefore, the expected demand in unit loads per unit time of item i is:

$$\begin{aligned} D(i) &= D \times (G(i) - G(i-1)) \\ &= D \times \left[\left(\frac{i}{N}\right)^s - \left(\frac{i-1}{N}\right)^s \right] \text{ for } 0 < s \leq 1, i = 1, \dots, N. \end{aligned} \quad (1)$$

Item inventories are restocked using continuous review (r , Q) reorder policies with backordering. The mean lead time for restocking item i is l_i . Let $F_i(\cdot)$ be the cumulative demand distribution of item i in unit loads during the lead time. Since a service level, $1 - \alpha_i$, is required, which is the probability of not stocking out during an inventory restocking cycle, we can derive the reorder point of item i :

$$r_i = F_i^{-1}(1 - \alpha_i).$$

Then we can get the safety stock of item i :

$$ss_i = r_i - l_i D(i) = F_i^{-1}(1 - \alpha_i) - l_i D(i). \quad (2)$$

The optimal order quantity for each inventory restocking cycle can be obtained as the Economic Order Quantity (EOQ):

$$Q_i^* = \sqrt{2D(i) \times K_i}, \quad (3)$$

where K_i is the ratio of order cost to holding cost per unit time per load of item i .

In the forward zone, each load occupies only one location without space sharing. Space sharing means that different items stored in a particular storage zone share the same slots. That is, if the load of one item has been depleted, the

empty position can be occupied by a load of a different item assigned to that storage zone. Let N_F be the number of items chosen to be stored in the forward zone, where each item occupies only one location. So the space required in the forward zone is N_F . The boundary of the forward zone (measured in travel time units) is therefore:

$$R_F = \sqrt{N_F}. \quad (4)$$

In the reserve area, N items are stored in total. At the beginning of the inventory restocking cycle, Q_i^* loads are stored in the reserve area sharing the same space for item $i \in \mathcal{U}$ while ss_i loads of safety stock are maintained without space sharing, where \mathcal{U} denotes the set of all items. Thus, according to Yu *et al.* (2015), the required space in unit loads in the reserve area for item $i \in \mathcal{U}$, when N items share total space, is:

$$a_i(N) = 0.5(1 + N^{-\varepsilon_i})Q_i^* + ss_i, \quad i \in \mathcal{U}, \quad (5)$$

where $0 \leq \varepsilon_i \leq 1$ is the space factor for item i . According to Yu *et al.* (2015), $\varepsilon_i = \varepsilon = 0.22$ (independent of i) is a good choice.

Then the boundary of total required shared space in the reserve area is (see Figure 2(a)):

$$R_R = \sqrt{N_F + \sum_{i \in \mathcal{U}} a_i(N)}. \quad (6)$$

3.3. SC cycles

In this section, we derive the expected travel time model for operating policy P_1 , where the crane only carries out SC cycles. Since we aim to minimize the expected response travel time, we only focus on the retrieval processes. Let set \mathcal{F} denote the items chosen to be stored in the forward zone. To retrieve item i , two possibilities have to be distinguished:

1. If item $i \in \mathcal{F}$, the load is retrieved from the forward zone with a probability p_i^F . However, with a probability $1 - p_i^F$, the load in the forward zone is depleted and the item must be retrieved from the reserve area.
2. If item $i \in \mathcal{U} \setminus \mathcal{F}$, the item must be retrieved from the reserve area.

From a single load of item i , m_i picks can be carried out meaning that it can be retrieved m_i times before it is empty. For item $i \in \mathcal{F}$, a load can be retrieved from the reserve area to the I/O point for the first time picking (Assumption 8). Since the load retrieved from the reserve area is then returned to the forward zone, still $m_i - 1$ retrievals are carried out from the forward zone. Thus, when a load of item i needs to be retrieved for an order, the probability that it is retrieved from the forward zone is:

$$p_i^F = \begin{cases} \frac{m_i - 1}{m_i} & i \in \mathcal{F} \\ 0 & i \in \mathcal{U} \setminus \mathcal{F}. \end{cases} \quad (7)$$

The expected number of retrievals of item i per unit time is $D(i) \times m_i$. When a load needs to be retrieved for an order,

we can derive the probability that it is retrieved from the forward zone:

$$p^F = \frac{\sum_{i \in \mathcal{F}} D(i) \times (m_i - 1)}{\sum_{i \in \mathcal{U}} D(i) \times m_i}. \quad (8)$$

The probability that a load is retrieved from the reserve area is then:

$$p^R = 1 - p^F. \quad (9)$$

According to Hausman *et al.* (1976) and Rosenblatt and Eynan (1989), the expected one-way retrieval time in the forward zone and reserve area are:

$$\bar{T}_F = \frac{2}{3} R_F, \quad (10)$$

$$\text{and } \bar{T}_R = \frac{2}{3} \frac{R_R^3 - R_F^3}{R_R^2 - R_F^2}, \quad (11)$$

with R_F obtained from Equation (4) and R_R from Equation (6).

We can now obtain the minimum expected response time, i.e., expected travel time, for FR storage in a SC cycle, T_{SC} , by solving model M_1 :

$$\begin{aligned} M_1 : \min T_{SC} &= 2 \times (p^F \times \bar{T}_F + p^R \times \bar{T}_R). \\ \text{Subject to } &\mathcal{F} \subseteq \mathcal{U}, \end{aligned} \quad (12)$$

with the choice of items in set \mathcal{F} as decision variables. In order to solve model M_1 , we use Proposition 1.

Proposition 1: Define $w_i = D(i)(m_i - 1)$. For an optimal assignment of items to the forward zone for Policy P_1 with SC cycles, it holds that $\forall i \in \mathcal{F}$ and $j \in \mathcal{U} \setminus \mathcal{F}$, $w_i \geq w_j$.

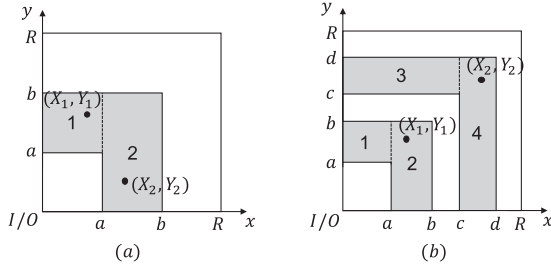
Proof. Given the number of items to be stored in the forward zone, N_F , both the size of the forward and the total storage space are fixed. Therefore, R_F and R_R are given (see Equations (4) and (6)), which determines \bar{T}_F and \bar{T}_R from Equations (10) and (11). Note that $\bar{T}_F < \bar{T}_R$. To minimize Equation (12) and optimize the assignment, p^F should be maximized. w_i is the number of retrievals from the forward zone of item $i \in \mathcal{F}$ per unit time. Item i with larger w_i is retrieved from the forward zone with a higher probability. It therefore has higher priority to be assigned to the forward zone. \square

Model M_1 can now be solved as follows:

1. Order the items by decreasing w_i , where $w_i = D(i)(m_i - 1)$. This number can be interpreted as the total number of picks of item i from the forward zone per unit time, while the remaining $D(i)$ picks are retrieved from the reserve area.
2. Given N_F (running from 1 to N), the optimal solution is to choose the N_F items with the highest w_i .
3. Calculate the space needs (Equations (4) and (6)), and find the optimal result for given N_F (Equation (12)).
4. Compare the optimal results for all subproblems (N_F varies from 1 to N) to obtain the overall optimal solution.

Table 4. Operating process of DC cycles mixed with SC cycles.

Load of item i_1 after picking	Storage item i_1 (probability)	Retrieval item i_2 (probability)	Operating Mode (Expected travel time, with equations)
Not depleted	Forward zone (p_i^{SF})	Forward zone (p_i^F)	DC ($2 \times \bar{T}_F + \bar{T}_{FF}$, Equations (10) and (18))
		Reserve area (p_i^R)	DC ($\bar{T}_F + \bar{T}_{FR} + \bar{T}_{RR}$, Equations (10), (11) and (20))
	Reserve area (p_i^{SR})	Forward zone (p_i^F)	DC ($\bar{T}_R + \bar{T}_{RF} + \bar{T}_F$, Equations (10), (11) and (20))
		Reserve area (p_i^R)	DC ($2 \times \bar{T}_R + \bar{T}_{RR}$, Equations (11) and (19))
Depleted	Not return ($1 - p_i^{SF} - p_i^{SR} = p_i^{SCC}$)	Forward zone (p_i^F)	SC ($2 \times \bar{T}_F$, Equation (10))
		Reserve area (p_i^R)	SC ($2 \times \bar{T}_R$, Equation (11))

**Figure 3.** Two random locations in an SIT rack: (a) in the same SIT L-shaped region, (b) in different SIT L-shaped regions.

3.4. DC cycles mixed with SC cycles

In this section, we derive the expected response travel time model for operating policy P_2 , where the crane works in DC cycles.

Let item i_1 and i_2 be successive requests in a sequential retrieval request list, where item i_1 has been picked by the worker at the order picking station. Let p_i^{SF} be the probability that item i must be returned to the forward zone and p_i^{SR} be the probability that item i must be returned to the reserve area. If item $i_1 \in \mathcal{F}$ (or $i_1 \in \mathcal{U} \setminus \mathcal{F}$), then the crane stores the load in the forward zone (or reserve area) with a probability p_i^{SF} (or p_i^{SR} , respectively) and moves without load to the location of item i_2 to retrieve it, which completes a DC cycle. However, with a probability $1 - p_i^{SF}$ (or $1 - p_i^{SR}$), the load of item i_1 has been depleted and should not be returned to the rack. Thus, the crane carries out a SC round-trip cycle for retrieving item i_2 . Item i_2 must be retrieved from the forward zone with probability p_i^F (Equation (8)) or from the reserve area with probability p_i^R (Equation (9)). Table 4 gives an overview of the operating process.

Since a load of item i can be retrieved m_i times before it is depleted, we find:

$$p_i^{SF} = \begin{cases} \frac{m_i - 1}{m_i} & i \in \mathcal{F} \\ 0 & i \in \mathcal{U} \setminus \mathcal{F} \end{cases}$$

$$p_i^{SR} = \begin{cases} \frac{m_i - 1}{m_i} & i \in \mathcal{U} \setminus \mathcal{F} \\ 0 & i \in \mathcal{F}. \end{cases}$$

The probability that a load is returned to the forward zone is therefore:

$$p^{SF} = \frac{\sum_{i \in \mathcal{F}} D(i) \times (m_i - 1)}{\sum_{i \in \mathcal{U}} D(i) \times m_i}, \quad (13)$$

and the probability that a load is returned to the reserve area is:

$$p^{SR} = \frac{\sum_{i \in \mathcal{U} \setminus \mathcal{F}} D(i) \times (m_i - 1)}{\sum_{i \in \mathcal{U}} D(i) \times m_i}. \quad (14)$$

The probability that the crane operates a SC cycle is:

$$p^{SCC} = 1 - p^{SF} - p^{SR} = \frac{\sum_{i \in \mathcal{U}} D(i) \times 1}{\sum_{i \in \mathcal{U}} D(i) \times m_i}. \quad (15)$$

Four subcases can be distinguished when executing a DC to pick up a load of item i_2 :

1. The crane travels from the forward zone to the reserve area, with travel between time T_{FR} .
2. The crane travels from the reserve area to the forward zone, with travel between time T_{RF} . Note that $\bar{T}_{RF} = \bar{T}_{FR}$.
3. The crane travels inside the forward zone, with travel between time T_{FF} .
4. The crane travels inside the reserve area, with expected travel between time T_{RR} . The resulting operating modes with travel time are summarized in Table 4.

In order to find \bar{T}_{FF} , \bar{T}_{FR} , \bar{T}_{RR} , we derive the general formulas for the expected travel time between two random locations in an SIT AS/R rack. We distinguish two situations, based on whether the two locations (X_1, Y_1) and (X_2, Y_2) are randomly located in the same SIT L-shaped region or not for both class-based storage and FR storage.

Situation 1: The two locations are randomly located in the same SIT L-shaped region like the shaded area in Figure 3(a). In this figure, the rack boundary is $R \in \mathbb{R}$ and the region boundaries are $a, b \in \mathbb{R}$ where $R \geq b > a \geq 0$. The expected travel time between two random locations in this situation (TB_1) is a function of a, b , that is to say

$$\overline{TB}_1(a, b). \quad (16)$$

Situation 2: The two locations are randomly located in different SIT L-shaped regions like the shaded areas in Figure 3(b). In this figure, the rack boundary is $R \in \mathbb{R}$ and the region boundaries are $a, b, c, d \in \mathbb{R}$ where $R \geq d > c \geq b > a \geq 0$. The expected travel time between two random locations in this situation (TB_2) is a function of a, b, c, d , that is to say

$$\overline{TB}_2(a, b, c, d). \quad (17)$$

Functions (16) and (17) are worked out further in online Appendix A.

Then we get:

$$\bar{T}_{FF} = \overline{TB}_1(0, R_F). \quad (18)$$

$$\bar{T}_{RR} = \overline{TB}_1(R_F, R_R). \quad (19)$$

$$\bar{T}_{FR} = \overline{TB}_2(0, R_F, R_R). \quad (20)$$

Recall the probability of retrieving a load from the forward zone (p^F) and from the reserve area (p^R) from Equations (8) and (9). We can then derive the minimum expected response time, i.e., expected travel time, for the process of DC cycles mixed with SC cycles, T_{DC} , by solving model M_2 (see Table 4):

$$\begin{aligned} M_2 : \min T_{DC} = & 2 \times (p^F \times \bar{T}_F + p^R \times \bar{T}_R) \times p^{SCC} \\ & + (\bar{T}_F + \bar{T}_{FR} + \bar{T}_R)(p^{SF} \times p^R) \\ & + (\bar{T}_R + \bar{T}_{RF} + \bar{T}_F)(p^{SR} \times p^F) \\ & + (\bar{T}_F + \bar{T}_{FF} + \bar{T}_F)(p^{SF} \times p^F) \\ & + (\bar{T}_R + \bar{T}_{RR} + \bar{T}_R)(p^{SR} \times p^R). \end{aligned} \quad (21)$$

Subject to $\mathcal{F} \subseteq \mathcal{U}$,

with the choice of items in set \mathcal{F} as decision variables.

In order to solve model M_2 , we use Proposition 2.

Proposition 2: Define $w_i = D(i)(m_i - 1)$. For an optimal assignment of items to the forward zone for Policy P_2 , it holds that $\forall i \in \mathcal{F}$ and $j \in \mathcal{U} \setminus \mathcal{F}$, $w_i \geq w_j$.

The proof of Proposition 2 can be found in online Appendix B.

Model M_2 can now be solved as follows:

1. Order the items by decreasing w_i , where $w_i = D(i)(m_i - 1)$.
2. Given N_F (running from 1 to N), the optimal solution is to choose the N_F items with the highest w_i .
3. Calculate the space needs (Equations (4) and (6)), and find the optimal result for given N_F (Equation (21)).
4. Compare the optimal results for all subproblems (N_F varies from 1 to N) to obtain the overall optimal solution.

4. Travel time model for FR-ABC storage

In this section, we extend the FR storage with ABC class-based storage in the forward zone (FR-ABC storage). Like the solutions for FR storage (for both Policy P_1 and P_2), items are ordered by decreasing the w_i value and items with larger w_i are chosen to be stored with one load in the forward zone. All the other loads are stored randomly in the reserve area. The items in the forward zone are divided into three groups (A,B,C classes) based on their pick frequency per unit space (i.e., one location), $w_i = D(i)(m_i - 1)$. As shown in Figure 2(b), the forward zone (with SIT shape) is divided into three zones. Items of the same turnover class are stored randomly in the same storage zone. We derive the travel time model for FR-ABC storage for both Policy P_1 and Policy P_2 . The derivation is similar to FR storage, but more complex. The details of the derivation of the travel time model for FR-ABC storage are shown in online Appendix C.

The assignment solution for FR-ABC storage (for both Policy P_1 and P_2) is as follows:

1. Order the items by decreasing w_i , where $w_i = D(i)(m_i - 1)$.

2. Given N_F (running from 3 to N), choose the N_F items with the highest w_i to store in the forward zone.
3. Let $\mathcal{A}, \mathcal{B}, \mathcal{C}$ denote the sets of items chosen to be stored in the A, B, C class in the forward zone, respectively. Assign the N_F items into A, B, C classes so that $w_{i_1} \geq w_{i_2} \geq w_{i_3}$, where $i_1 \in \mathcal{A}, i_2 \in \mathcal{B}, i_3 \in \mathcal{C}$. Enumerate all the possible $\binom{N_F - 1}{2}$ assignment options, calculate the resulting minimum time according to Policy P_1 or P_2 and find the optimal allocation for given N_F .
4. Compare the optimal results for all subproblems (N_F varies from 3 to N) to obtain the overall optimal allocation.

5. Numerical results

In this section, we use our analytical model to evaluate the response time of FR storage and FR-ABC storage, and compare the performance of FR and FR-ABC storage with ABC class-based storage in numerical experiments.

In ABC class-based storage, the items are ordered by their pick frequency per used location, i.e.

$$f_i = \frac{D(i) \times m_i}{Q_i^*},$$

and divided into three groups (A,B,C classes). We find the optimal assignment of items to different classes for both operating policy P_1 and P_2 as follows: Let $\mathcal{A}, \mathcal{B}, \mathcal{C}$ denote the sets of items to be stored in the A, B, C class, respectively. Assign the items into A, B, C classes so that $f_{i_1} \geq f_{i_2} \geq f_{i_3}$, where $i_1 \in \mathcal{A}, i_2 \in \mathcal{B}, i_3 \in \mathcal{C}$. Enumerate all the possible $\binom{N - 1}{2}$ assignment options, calculate the resulting minimum time according to Policy P_1 or P_2 and find the optimal allocation. Note that the optimal allocation may be different for Policies P_1 and P_2 , since for an increasing m_i value, it may be beneficial to increase the A zones for Policy P_2 . (In fact, the optimal size of the A zone using DC cycles is larger than that with SC cycles. Since the travel-between time in the A zone is short, it is beneficial to increase the A zone when using DC cycles.)

For all storage methods, the items have to be sorted on decreasing w_i (for FR and FR-ABC storage) or f_i (ABC class-based storage), which requires $N \log N$ steps. After sorting, the remaining time can be found as follows:

1. For FR storage, the expected travel time must be calculated for $O(N)$ subproblems. Finding the minimum value of these therefore takes $O(N)$ time and the total complexity is $O(N \log N)$.
2. For FR-ABC storage, the expected travel time must be calculated for $O(N^3)$ subproblems to find the minimum. Therefore, the total complexity is $O(N^3)$.
3. For ABC class-based storage, the expected travel time must be calculated $O(N^2)$ subproblems to find the minimum. Therefore, the total complexity is $O(N^2)$.

Table 5. Response time of different storage strategies for various parameters⁽¹⁾.

$N^{(2)}$	D / N	m_i	$\bar{Q}_i^{(3)}$	$T_{ABC}^{(4)}$		T_{FR}		T_{FR-ABC}		$T_{FR}^{sav(7)}$		$T_{FR-ABC}^{sav(8)}$		Best Strategy	
				$P_1^{(5)}$	$P_2^{(6)}$	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
50	0.5	1	1.30	8.31	8.31	9.37	9.37	9.85	9.85	-12.70	-12.70	-18.46	-18.46	ABC	ABC
		2	1.30	8.31	9.92	8.62	10.11	8.66	10.09	-3.75	-1.89	-4.18	-1.75	ABC	ABC
		3	1.30	8.31	10.45	8.33	10.36	8.26	10.29	-0.18	0.85	0.58	1.54	FR-ABC	FR-ABC
		10	1.30	8.31	11.20	7.83	10.73	7.60	10.56	5.84	4.15	8.55	5.72	FR-ABC	FR-ABC
	1	1	1.85	9.89	9.89	11.06	11.06	11.47	11.47	-11.78	-11.78	-15.97	-15.97	ABC	ABC
		2	1.85	9.89	11.81	10.10	11.8	10.05	11.72	-2.08	0.07	-1.53	0.73	ABC	FR-ABC
		3	1.85	9.89	12.44	9.65	11.97	9.49	11.78	2.47	3.75	4.07	5.34	FR-ABC	FR-ABC
		10	1.85	9.89	13.33	8.87	12.17	8.10	10.71	10.33	8.68	18.16	19.60	FR-ABC	FR-ABC
	2	1	2.61	11.78	11.78	13.09	13.09	13.45	13.45	-11.10	-11.10	-14.14	-14.14	ABC	ABC
		2	2.61	11.78	14.06	11.77	13.67	11.65	13.48	0.06	2.73	1.12	4.11	FR-ABC	FR-ABC
		3	2.61	11.78	14.81	11.08	13.64	10.74	12.77	5.92	7.95	8.83	13.80	FR-ABC	FR-ABC
		10	2.61	11.78	15.87	9.87	13.24	8.23	10.85	16.26	16.58	30.11	31.63	FR-ABC	FR-ABC
	20	1	8.25	21.16	21.16	23.23	23.23	23.45	23.45	-9.80	-9.80	-10.80	-10.80	ABC	ABC
		2	8.25	21.16	25.24	18.44	19.66	17.27	18.30	12.88	22.12	18.38	27.50	FR-ABC	FR-ABC
		3	8.25	21.16	26.59	15.47	17.24	13.84	15.45	26.89	35.18	34.58	41.91	FR-ABC	FR-ABC
		10	8.25	21.16	28.49	11.24	14.04	9.04	11.65	46.88	50.73	57.27	59.09	FR-ABC	FR-ABC
100	0.5	1	1.30	11.39	11.39	12.78	12.78	13.15	13.15	-12.22	-12.22	-15.43	-15.43	ABC	ABC
		2	1.30	11.39	13.61	11.94	14.00	11.90	13.95	-4.84	-2.85	-4.50	-2.49	ABC	ABC
		3	1.30	11.39	14.35	11.55	14.37	11.44	14.27	-1.38	-0.17	-0.44	0.56	ABC	FR-ABC
		10	1.30	11.39	15.38	10.88	14.91	10.57	14.68	4.52	3.04	7.25	4.50	FR-ABC	FR-ABC
	1	1	1.84	13.56	13.56	15.15	15.15	15.46	15.46	-11.70	-11.70	-14.02	-14.02	ABC	ABC
		2	1.84	13.56	16.20	14.00	16.36	13.91	16.24	-3.21	-0.97	-2.58	-0.27	ABC	ABC
		3	1.84	13.56	17.08	13.39	16.63	13.17	16.38	1.23	2.63	2.88	4.11	FR-ABC	FR-ABC
		10	1.84	13.56	18.30	12.36	16.96	11.43	15.13	8.88	7.34	15.73	17.33	FR-ABC	FR-ABC
	2	1	2.60	16.15	16.15	17.98	17.98	18.25	18.25	-11.32	-11.32	-12.98	-12.98	ABC	ABC
		2	2.60	16.15	19.29	16.33	18.98	16.15	18.73	-1.13	1.59	-0.03	2.90	ABC	FR-ABC
		3	2.60	16.15	20.34	15.41	18.99	14.96	17.96	4.59	6.64	7.36	11.67	FR-ABC	FR-ABC
		10	2.60	16.15	21.80	13.78	18.64	11.62	15.32	14.64	14.49	28.07	29.73	FR-ABC	FR-ABC
	20	1	8.23	29.02	29.02	32.07	32.07	32.23	32.23	-10.49	-10.49	-11.04	-11.04	ABC	ABC
		2	8.23	29.02	34.66	25.71	27.50	24.12	25.57	11.41	20.67	16.90	26.21	FR-ABC	FR-ABC
		3	8.23	29.02	36.53	21.68	24.18	19.37	21.64	25.32	33.82	33.25	40.75	FR-ABC	FR-ABC
		10	8.23	29.02	39.15	15.84	19.79	12.73	16.42	45.44	49.45	56.15	58.06	FR-ABC	FR-ABC
150	0.5	1	1.30	13.72	13.72	15.40	15.40	15.71	15.71	-12.21	-12.21	-14.47	-14.47	ABC	ABC
		2	1.30	13.72	16.40	14.47	16.96	14.41	16.90	-5.43	-3.39	-5.04	-3.01	ABC	ABC
		3	1.30	13.72	17.30	14.00	17.42	13.86	17.29	-2.01	-0.72	-1.02	0.02	ABC	FR-ABC
		10	1.30	13.72	18.54	13.20	18.09	12.83	17.83	3.82	2.45	6.50	3.84	FR-ABC	FR-ABC
	1	1	1.84	16.34	16.34	18.27	18.27	18.53	18.53	-11.84	-11.84	-13.47	-13.47	ABC	ABC
		2	1.84	16.34	19.53	16.96	19.83	16.85	19.69	-3.82	-1.53	-3.15	-0.83	ABC	ABC
		3	1.84	16.34	20.59	16.24	20.17	15.98	19.88	0.56	2.02	2.20	3.44	FR-ABC	FR-ABC
		10	1.84	16.34	22.07	15.01	20.60	13.98	18.52	8.12	6.64	14.41	16.09	FR-ABC	FR-ABC
	2	1	2.60	19.45	19.45	21.70	21.70	21.93	21.93	-11.56	-11.56	-12.73	-12.73	ABC	ABC
		2	2.60	19.45	23.26	19.80	23.02	19.59	22.73	-1.76	0.99	-0.67	2.25	ABC	FR-ABC
		3	2.60	19.45	24.52	18.70	23.06	18.18	21.94	3.89	5.95	6.56	10.50	FR-ABC	FR-ABC
		10	2.60	19.45	26.28	16.77	22.73	14.21	18.74	13.80	13.51	26.97	28.69	FR-ABC	FR-ABC
	20	1	8.23	34.98	34.98	38.79	38.79	38.92	38.92	-10.91	-10.91	-11.28	-11.28	ABC	ABC
		2	8.23	34.98	41.79	31.26	33.48	29.35	31.13	10.63	19.89	16.10	25.51	FR-ABC	FR-ABC
		3	8.23	34.98	44.06	26.42	29.48	23.60	26.38	24.47	33.08	32.54	40.12	FR-ABC	FR-ABC
		10	8.23	34.98	47.22	19.36	24.20	15.55	20.07	44.66	48.76	55.55	57.50	FR-ABC	FR-ABC

⁽¹⁾ $s = 0.431$, $K_i = 2$, $l_i = 0.02$ year, $1 - \alpha_i = 95\%$, $cv_i = 0.2$.⁽²⁾ N is the number of items per aisle side.⁽³⁾ \bar{Q}_i^* is the average EOQ over all the items.⁽⁴⁾ T_{ABC} , T_{FR} , T_{FR-ABC} are the expected response time of ABC class-based storage, FR storage and FR-ABC storage, respectively.⁽⁵⁾ Policy P_1 , where the crane operates SC cycles.⁽⁶⁾ Policy P_2 , where the crane operates DC cycles with an occasional SC cycle.⁽⁷⁾ T_{FR}^{sav} is the relative expected response time savings of FR storage compared to ABC class-based storage (%).⁽⁸⁾ T_{FR-ABC}^{sav} is the relative expected response time savings of FR-ABC storage compared to ABC class-based storage (%).

In the numerical experiments, we assume the demand of each item over the lead time follows a lognormal distribution with a homogeneous coefficient of variation $cv_i = 0.2$ (the ratio of standard deviation to the mean of demand in unit loads of item i) for all i . In Section 6, inhomogeneous values of cv_i are studied. The required service level is $1 - \alpha_i = 95\%$, for all i .

5.1. Results for homogeneous m_i

We first set m_i identical for all items. Table 5 compares the performance of different storage strategies for Policies P_1 and P_2 . The parameters are set as follows: $s = 0.431$, $K_i = 2$ in year, $l_i = 0.02$ year, for all i . The expected response time (i.e., crane travel time) and the average EOQ over all the items

$$\overline{Q_i^*} = \frac{\sum_{i=1}^N Q_i^*}{N}$$

for different levels of the input variables (N , D/N and m_i) are shown in the table. The relative response travel time savings of using FR storage

$$T_{FR}^{sav} = \frac{T_{ABC} - T_{FR}}{T_{ABC}} \times 100$$

and FR-ABC storage

$$T_{FR-ABC}^{sav} = \frac{T_{ABC} - T_{FR-ABC}}{T_{ABC}} \times 100$$

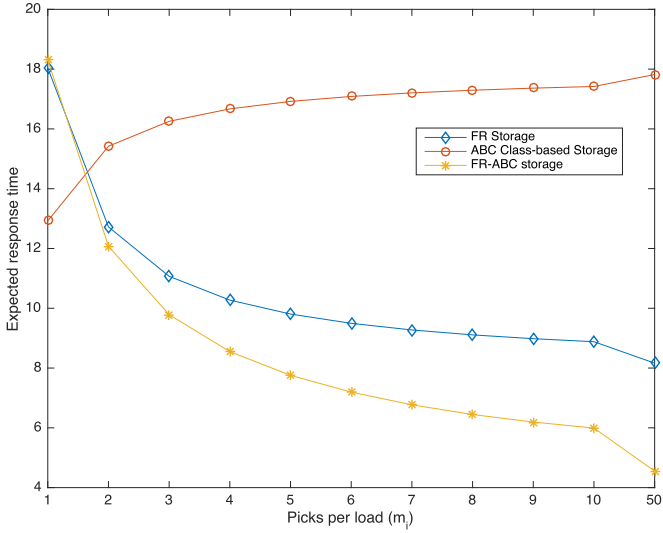
compared with ABC class-based storage are also shown in the table (T_{ABC} , T_{FR} , T_{FR-ABC} denote the expected response time of ABC class-based storage, FR storage and FR-ABC storage, respectively). The best storage strategy with the shortest response time is indicated in the last two columns. From the table we see that variables m_i and D/N are the

main factors that affect the performance of FR and FR-ABC storage compared with ABC class-based storage.

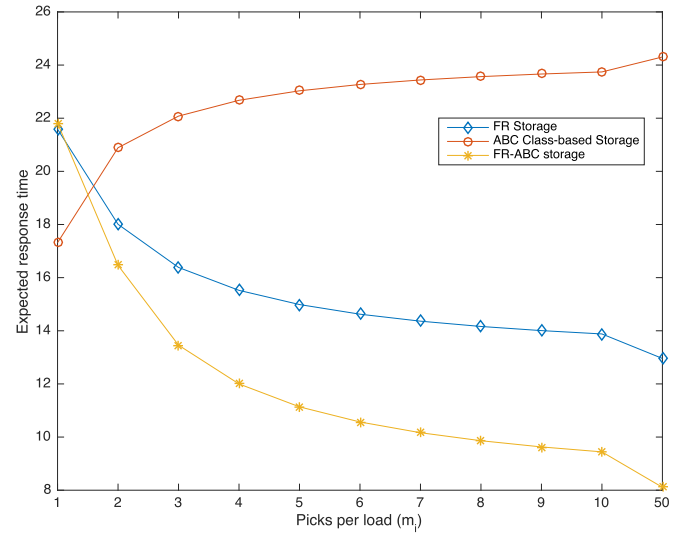
Figure 4 shows the expected response time for different storage strategies as a function of m_i , for $s = 0.065$, $s = 0.222$, $s = 0.431$, $s = 0.748$, when $N = 50$, $D = 1000$, $K_i = 2$, $l_i = 0.02$. The crane operates Policy P_2 . Results for other operating policies and other values of N and D show similar patterns and are shown in online Appendix D. When m_i increases, the response time savings of FR and FR-ABC storage compared to ABC class-based storage increase.

Figure 5 uses contour maps to show the relative expected response time savings of using FR and FR-ABC storage, compared with ABC class-based storage for varying values of m_i and D/N , for $s = 0.065$, $s = 0.222$, $s = 0.431$, $s = 0.748$, when $N = 50$, $K_i = 2$, $l_i = 0.02$. The values on the color bar indicate the time savings

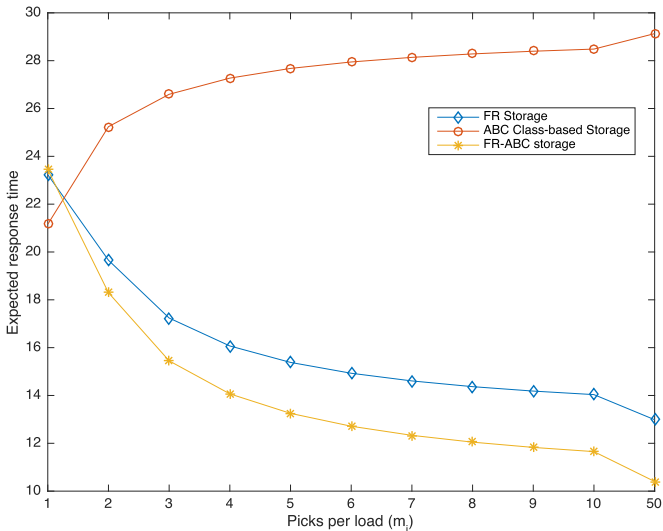
$$T^{sav} = \frac{T_{ABC} - \min\{T_{FR}, T_{FR-ABC}\}}{T_{ABC}} \times 100.$$



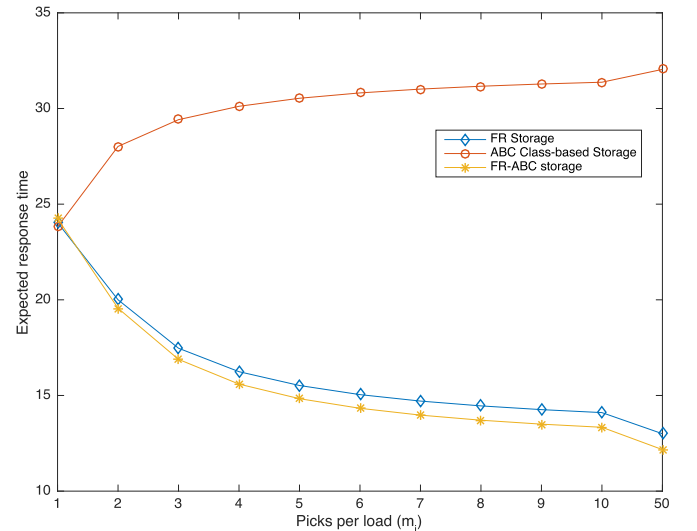
(a)



(b)



(c)



(d)

Figure 4. Performance of different storage strategies for various s and m_i in DC cycles with an occasional SC cycle: (a) $s = 0.065$, (b) $s = 0.222$, (c) $s = 0.431$, (d) $s = 0.748$. $N = 50$, $D = 1000$, $K_i = 2$, $l_i = 0.02$ year, $1 - \alpha_i = 95\%$, $cv_i = 0.2$.

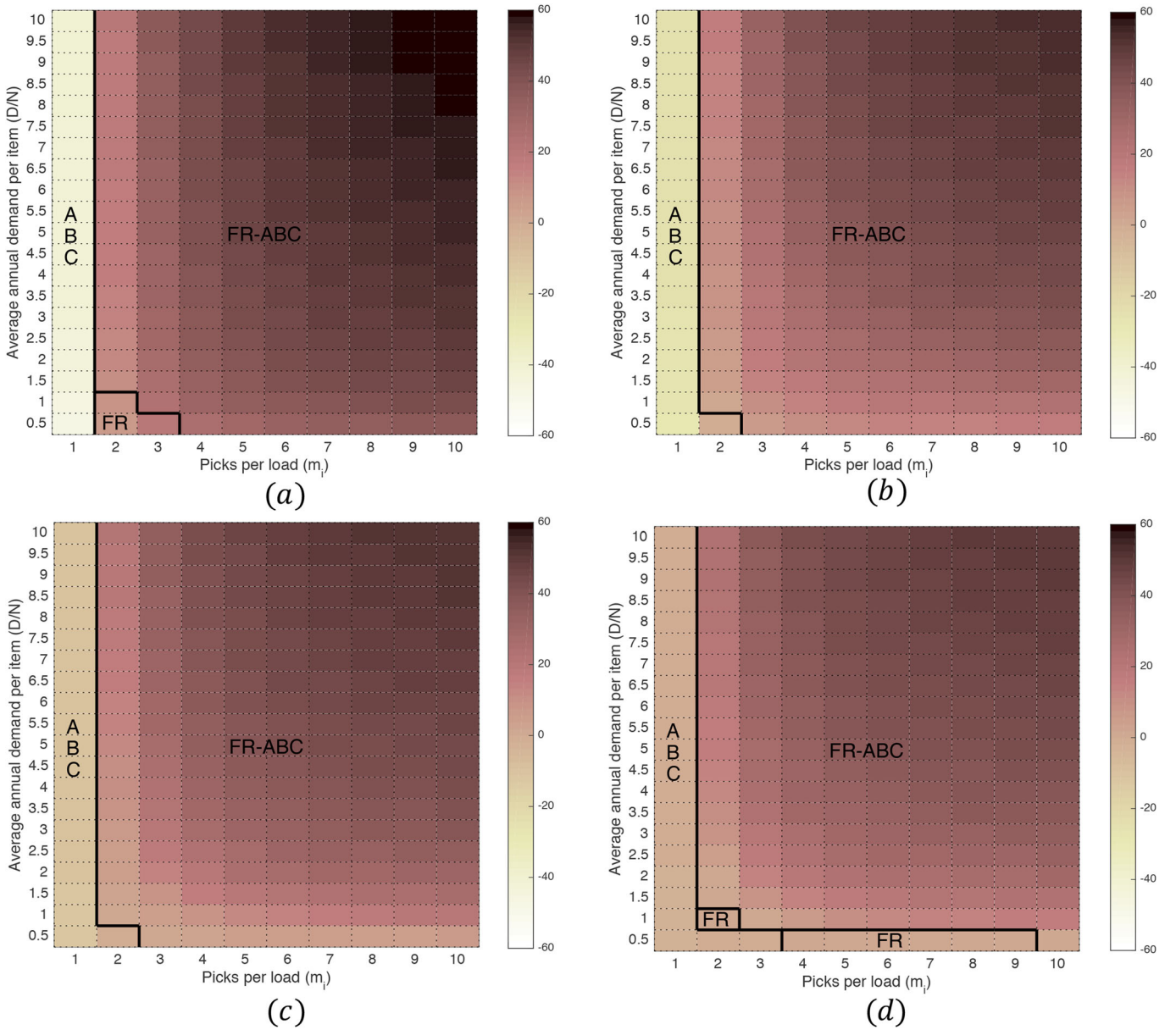


Figure 5. Performance comparison between different storage strategies (T^{sav}) in DC cycles with an occasional SC cycle: (a) $s = 0.065$, (b) $s = 0.222$, (c) $s = 0.431$, (d) $s = 0.748$. $N = 50$, $K_i = 2$, $l_i = 0.02$ year, $1 - \alpha_i = 95\%$, $cv_i = 0.2$.

A darker color indicates a larger time saving. The boundaries indicate which storage strategy is best. Results for Policy P_1 and for other values of N show similar patterns and are shown in online Appendix E. From Figure 5, we see that FR-ABC storage is even better than FR storage for most of the parameter settings.

The performance comparison for varying K_i and l_i are shown in online Appendix F. The response time savings of FR and FR-ABC storage are insensitive to the value of K_i and l_i . When K_i and l_i increases, the response time savings increase slightly.

From the results in the numerical experiments, we make the following observations:

1. Variables m_i and D/N are the main factors that affect the performance of FR storage compared with ABC class-based storage. According to Table 5, we see that when

m_i and D/N increase, the response time savings of FR storage (T_{FR}^{sav}) increases. From Figure 5, we see that the color becomes more and more darker from the left bottom to the right top of each contour map, for a given value of s . This also shows that the response time savings of FR and FR-ABC storage become larger when m_i and D/N increase, for all the values of s shown in the figure.

According to Table 5, FR storage is better than ABC class-based storage when $m_i > 1$ for most of the cases, which means, as long as the picks are not unit loads, using FR storage pays off. When m_i increases, the number of replenishments from the reserve area becomes less important compared with the large number of picks from the forward zone, so the response time savings becomes larger. However, from Figure 4, we see that when m_i is bigger than 10, the increase in response time savings becomes negligible. When m_i is large, the

retrievals occur mainly in the forward zone, so further increasing m_i does not save more time.

From Table 5, we see that, when D/N increases, the average reorder quantity increases. ABC class-based storage assigns more loads per item to the A zone. This means the travel time benefit of ABC class-based storage compared with FR storage reduces. (At some point it will offset the extra replenishment travel time in FR storage, when a pallet depletes in the forward zone.) Note that even when $m_i > 1$, ABC class-based storage can outperform FR storage. This can happen in particular when D/N and m_i are small ($D/N \leq 2, m_i \leq 3$). When D/N becomes large, FR storage benefits substantially, due to a small forward zone. The response time savings of FR storage (T_{FR}^{sav}) can be up to 50% when $D/N = 20$ and $m_i = 10$.

2. According to Figure 5, we see that FR-ABC storage is even better than FR storage for most of the parameter settings. The response time savings of FR-ABC storage (T_{FR-ABC}^{sav}) can be up to 50% for $D/N = 10$ and $m_i = 10$. In some extreme cases where m_i and D/N are very small ($m_i = 1, 2, D/N = 0.5$), FR is better than FR-ABC storage. The optimal number of items assigned to the forward zone for FR storage can be less than three. However, in FR-ABC storage, there are at least three items in the forward zone (one for each class). In this case, assigning three items to the forward zone enlarges the size of the forward zone, which increases the expected response time. From Figure 4, we see that, when s becomes large (e.g., $s = 0.748$), there is not too much difference between FR storage and FR-ABC storage in response time savings, because the extra benefit of dividing items into three classes in the forward zone is negligible.
3. The response time savings of FR and FR-ABC storage are quite insensitive to the value of s when $m_i > 1$. From Figure 5, we see that for the column where $m_i = 1$, the color becomes darker when the value of s increases. When $m_i = 1, s = 0.065$, ABC class-based storage performs much better than FR and FR-ABC storage. However, when $m_i = 1, s = 0.748$, the benefit of using ABC class-based storage becomes negligible. When $m_i > 1$, the color for $s = 0.065, s = 0.222, s = 0.431$, or $s = 0.748$ does not differ too much.
4. The response time savings of FR and FR-ABC storage are quite insensitive to the precise value of N . According to Table 5, we see that the influence of N to the response time savings of FR and FR-ABC storage are not too large. When N increases, the response time savings decrease slightly. With more items to be considered in the system, the number of items to be assigned to the forward zone may increase, which costs additional space in the forward zone. This may increase the expected response time for the FR and FR-ABC storage.
5. According to Table 5, we see that the response time savings of FR and FR-ABC storage are even better for Policy P_2 than Policy P_1 for most of the parameter settings, since a small-sized forward zone can not only

reduce the retrieval time, but also reduce the travel-between time.

5.2. Results for inhomogeneous m_i

In this section, we assume the m_i are inhomogeneous and follow a discrete truncated normal distribution, with means (approximately) equal to 1, 2, 3, or 10, similar to the homogeneous values taken in Table 5. Figure 6 shows an example distribution with mean = 3 and standard deviation = 1.

For a given distribution, for each item i , a random m_i value is drawn from the distribution. This is repeated 100 times. The 95% Confidence Interval (CI) for the response time, for both Policy P_1 and P_2 can be found in Table 6. The relative response travel time savings of using FR storage

$$T_{FR}^{sav} = \frac{T_{ABC} - T_{FR}}{T_{ABC}} \times 100$$

and FR-ABC storage

$$T_{FR-ABC}^{sav} = \frac{T_{ABC} - T_{FR-ABC}}{T_{ABC}} \times 100$$

compared with ABC class-based storage are also shown in the table.

Table 6 shows that the results for inhomogeneous m_i hardly differ from those of homogeneous m_i values (i.e., with m_i standard deviation equalling zero).

6. Evaluation of FR storage: A case study

In this section, we use the order and product data of a wholesale distributor of nonfood products to show the applicability of our model (courtesy of “warehouse-science.com”). The customers of this warehouse are retail stores. Most of them are relatively small and order much of their stock in piece quantities. The data cover the history of piece-picks over a period of 5 months (from January to May) for around 1800 items. The data show the total order quantities in units (d_i) and number of orderlines (o_i) for different items for the whole period. Also the three-dimensional size data of each item can be obtained from the data set (the volume of a unit then can be derived as V_i). We choose items that fit in the storage totes of a miniload AS/R system with order picking to determine the performance of FR and FR-ABC storage, and compare it with ABC class-based storage.

We assume a rack is 30 meters long and 7.5 meters high. The speed of the crane is 4 meter per second horizontally and 1 meter per second vertically. So the rack is SIT. The totes stored in the rack are European standard totes, which are 0.4 meter wide, 0.4 meter high and 0.6 meter deep. A location for one tote in the side view of the rack occupies a square slot, which is 0.5 meter wide and 0.5 meter high (i.e., $0.5 \times 0.125 = 0.0625$ square seconds). A single rack can therefore store a maximum of $\frac{30 \times 7.5}{0.5 \times 0.5} = 900$ totes, or 1800 totes per aisle (two racks).

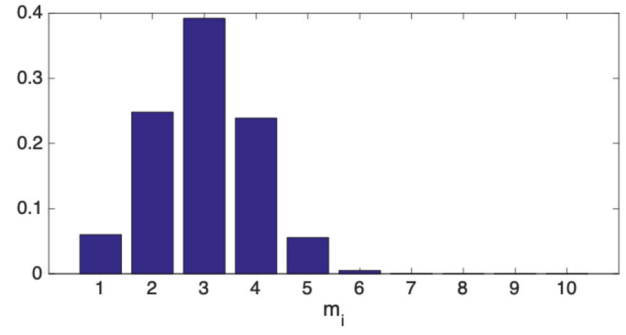
The number of units, totes, locations and picks are all integers. The number of units per tote for each item then can be derived as

Table 6. Response time (95% CI) of different storage strategies for inhomogeneous $m_i^{(1)}$.

m_i			$T_{ABC}^{(4)}$		T_{FR}	T_{FR-ABC}	$T_{FR}^{sav(7)}$	$T_{FR-ABC}^{sav(8)}$
Mean	Std ⁽²⁾	Mode ⁽³⁾	$P_1^{(5)}$	$P_2^{(6)}$	P_1	P_2	P_1	P_2
1	0	1	21.16±0.00	21.16±0.00	23.23±0.00	23.23±0.00	23.45±0.00	23.45±0.00
1.1	0.3	1	20.72±0.10	20.94±0.07	22.06±0.16	22.11±0.13	22.04±0.17	22.09±0.14
1.3	0.5	1	20.06±0.13	21.15±0.09	19.87±0.18	20.23±0.17	19.54±0.21	19.86±0.21
2	0	2	21.16±0.00	25.24±0.00	18.44±0.00	19.66±0.00	17.27±0.00	18.30±0.00
2	0.5	2	20.59±0.09	24.34±0.10	17.82±0.09	19.07±0.07	16.68±0.12	17.72±0.11
2	1	1	19.57±0.14	22.56±0.15	16.81±0.16	17.84±0.14	15.82±0.20	16.59±0.21
3	0	3	21.16±0.00	26.59±0.00	15.47±0.00	17.24±0.00	13.84±0.00	15.45±0.00
3	0.5	3	20.92±0.06	26.11±0.06	15.39±0.03	17.19±0.03	13.77±0.06	15.29±0.06
3	1	3	20.24±0.13	25.07±0.10	15.02±0.10	16.86±0.06	13.47±0.15	14.93±0.10
10	0	10	21.16±0.00	28.49±0.00	11.24±0.00	14.04±0.00	9.04±0.00	11.65±0.00
10	0.5	10	21.13±0.02	28.45±0.02	11.24±0.00	14.04±0.00	9.04±0.01	11.66±0.01
10	1	10	21.03±0.04	28.35±0.04	11.23±0.01	14.04±0.00	9.01±0.02	11.65±0.02

⁽¹⁾ $N=50$, $D=1000$, $s=0.431$, $K_i=2$, $l_i=0.02$ year, $1-\alpha_i=95\%$, $cv_i=0.2$.⁽²⁾Std is the standard deviation of the distribution of m_i .⁽³⁾Mode is the mode of the distribution of m_i .

Notation (4)/(5)/(6)/(7)/(8) see Table 5.

Figure 6. Distribution of m_i (mean = 3, standard deviation = 1).

$$q_i = \left\lfloor \frac{0.4 \times 0.4 \times 0.6 \text{ m}^3}{V_i} \right\rfloor.$$

The demand in totes in the period of 5 months for each item is then $D(i) = \lceil d_i/q_i \rceil$. The number of picks per tote is $m_i = \lceil o_i/D(i) \rceil$. Assume the ratio of order cost to holding cost per period is four. The EOQ for each item is then $Q_i^* = \lceil \sqrt{2 \times 4 \times D(i)} \rceil$. Assuming the lead time is one week, we can obtain the coefficient of variation cv_i of the demand over the lead time from the daily demand data set. We assume the demand of each item over lead time follows a lognormal distribution and the service level is 0.95 for all the items, to be able to calculate safety stock (ss_i). The number of required locations for item i , sharing space with another $N-1$ items, can be derived as $a_i(N) = \lceil 0.5(1 + N^{-0.5})Q_i^* + ss_i \rceil$ (see Equation (5)).

Normally, a miniload AS/R system only stores items that do not have too much inventory (not too fast-moving products). Thus, we select items for which $o_i \leq 45$ and $Q_i^* \leq 15$. There are 248 such items. To assign these items to one aisle (two racks), we order the items by decreasing o_i and assign items with odd index to the left rack and items with even index to the right rack. Since one location occupies 0.0625 square seconds in the rack, we can find the optimal expected response time for FR storage, FR-ABC storage and ABC class-based storage for both Policy P_1 and P_2 , using the same optimal allocation methods as in the numerical experiments (Section 5), using the empirical data for e.g., demand. Table 7 shows the results averaged over the two racks. In this case, $D/N = 5.25$, $\overline{cv} = 2.06$ and the ratio m of average picks per replenishment is six.

From Table 7, we can see that FR and FR-ABC storage strategies require more space than ABC class-based storage. However, the response time savings of FR and FR-ABC storage compared with ABC class-based storage can reach up to 46% in this case. The last row of Table 7 shows that the savings of FR storage are similar to those found from our analytical model using a homogeneous value of $m_i = 6$ for all items. The real savings of FR-ABC storage are overestimated by 5 percentage points. This difference is largely explained by a small number of items with relatively low demand in totes, but a very large m_i value, leading to a large standard deviation in m_i ($cv_m = 1.34$).

Table 7. Results for the case study.

Operating policy of the crane	Policy P1			Policy P2		
	FR	FR-ABC	ABC	FR	FR-ABC	ABC
Optimal expected response time (seconds)	4.76	4.26	7.40	5.72	5.12	9.59
Time savings compared with ABC class-based storage (%)	35.65	42.44	0	40.35	46.61	0
Required locations (one tote per location)	839	847	739	843	847	742
Model time savings ($s=0.353$, $m_i=6$, $cv_i=2.06$) ⁽¹⁾ (%)	32.13	46.45	0	38.25	50.14	0

⁽¹⁾The demand curve in totes per period in this case can be modeled by $G(i) = (\frac{i}{N})^s$, with $s=0.353$ (with 95% confidence bounds of (0.341, 0.364) and R-square of 0.94, using the Matlab Curve Fitting Tool). Other parameters are $m_i=6$, $N=124$ (for a single rack), $D/N=5.25$, $K_i=4$, $l_i=0.046$ (one week), $cv_i=2.06$, $1-\alpha_i=95\%$.

7. Conclusion

In this article, we calculate under what circumstances it pays off to use FR storage compared with ABC class-based storage for parts-to-picker systems. By using FR storage, extra replenishments from the reserve to the forward area need to be carried out, which cost extra time for retrieving the loads. We develop response travel time models for FR storage in an AS/R system combined with order picking.

Our results show that, in an AS/R system with order picking, FR storage pays off for many parameter settings, as long as the ratio of picks per replenishment, m , is sufficiently larger than one. The crucial factors that affect the response time savings in such systems are m and the average annual demand per item D/N . As m and D/N increase, the response time savings increase, which can go up to 50% when m is larger than 10 and D/N is larger than 10 unit loads. We validate these results using real data from a wholesale distributor, which again shows substantial (up to 46%) response time savings using FR storage. Our results are supported by those of Thomas and Meller (2015), who study total labor time for manual, parallel-aisle, picker-to-parts case-picking warehouse designs. Although they do not explicitly study the impact of the number of picks per replenishment, their numerical results suggest that FR splitting may pay off for $m \geq 1.6$. This also shows that our results can be a reference to different kinds of warehouse systems.

Table 1 shows that many automated warehouse systems have not been studied for the impact of FR storage strategies on performance. This article focuses on AS/R systems only, where replenishments are carried out in the aisle, combined with order picking. This leaves ample room for research on the application of FR storage strategies for different automated warehouse systems, particularly those used in e-commerce environments, where orders require piece picking, which leads to very high values of m . In addition, also for manual, picker-to-parts warehouses, more research on the impact of FR storage is required, in particular for different types of storage systems and racks.

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