



## Article

# Volatility Spillovers between Energy and Agricultural Markets: A Critical Appraisal of Theory and Practice

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**Abstract:** Energy and agricultural commodities and markets have been examined extensively, albeit separately, for a number of years. In the energy literature, the returns, volatility and volatility spillovers (namely, the delayed effect of a returns shock in one asset on the subsequent volatility or cointegration in another asset), among alternative energy commodities, such as oil, gasoline and ethanol across different markets, have been analysed using a variety of univariate and multivariate models, estimation techniques, data sets, and time frequencies. A similar comment applies to the separate theoretical and empirical analysis of a wide range of agricultural commodities and markets. Given the recent interest and emphasis in bio-fuels and green energy, especially bio-ethanol, which is derived from a range of agricultural products, it is not surprising that there is a topical and developing literature on the spillovers between energy and agricultural markets. Modelling and testing spillovers between the energy and agricultural markets has typically been based on estimating multivariate conditional volatility models, specifically the Baba, Engle, Kraft, and Kroner (BEKK) and dynamic conditional correlation (DCC) models. A serious technical deficiency is that the Quasi-Maximum Likelihood Estimates (QMLE) of a Full BEKK matrix, which is typically estimated in examining volatility spillover effects, has no asymptotic properties, except by assumption, so that no valid statistical test of volatility spillovers is possible. Some papers in the literature have used the DCC model to test for volatility spillovers. However, it is well known in the financial econometrics literature that the DCC model has no regularity conditions, and that the QMLE of the parameters of DCC has no asymptotic properties, so that there is no valid statistical testing of volatility spillovers. The purpose of the paper is to evaluate the theory and practice in testing for volatility spillovers between energy and agricultural markets using the multivariate Full BEKK and DCC models, and to make recommendations as to how such spillovers might be tested using valid statistical techniques. Three new definitions of volatility and cointegration spillovers are given, and the different models used in empirical applications are evaluated in terms of the new definitions and statistical criteria.

**Keywords:** energy markets; agricultural markets; volatility and cointegration spillovers; univariate and multivariate conditional volatility models; Baba, Engle, Kraft, and Kroner; dynamic conditional correlation; definitions of spillovers

**JEL Classification:** C22; C32; C58; G32; O13; Q42

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## 1. Introduction

Energy and agricultural commodities and markets have been examined extensively, albeit separately, for a number of years. In the energy literature, the returns, volatility and volatility spillovers (namely, the delayed effect of a returns shock in one financial asset on the subsequent volatility or cointegration in another asset), among alternative energy commodities, such as oil, gasoline and ethanol across different markets, have been analysed using a variety of univariate and multivariate models, estimation techniques, data sets, and time frequencies. A similar comment applies to the separate theoretical and empirical analysis of a wide range of agricultural commodities and markets.

Given the recent interest and emphasis in bio-fuels and green energy, especially bio-ethanol, which can be derived from a range of agricultural products, it is not surprising that there is a topical and developing literature on the spillovers between energy and agricultural markets, where the emphasis is on testing the magnitude and direction of the volatility spillovers between alternative commodities in these markets.

A related area of research is the relationship between energy prices, on the one hand, and food and fertilizer prices, on the other, as fertilizer prices have a direct and significant effect on the prices of agricultural products (see, for example [1,2]). However, there do not seem to be any published papers that have tested for volatility spillover effects between the energy and fertilizer markets as existing research has focused on univariate conditional volatility models rather than their multivariate counterparts.

Spillovers can be examined in the conditional means, that is, the financial returns on commodity prices, or the conditional volatility of the shocks to returns. When spillovers are analysed in the context of returns, such testing of spillover effects are based on the well-known Granger (non-) causality test in a vector autoregressive process. Estimation and testing are typically undertaken within a systems framework for purposes of efficiency in estimation and greater power of the associated tests.

Spillover effects can also be tested in terms of the conditional volatility. Modelling and testing spillovers between the energy and agricultural markets has typically been based on estimating multivariate conditional volatility models, specifically the Baba, Engle, Kraft, and Kroner (BEKK) model of Engle and Kroner [3] and the dynamic conditional correlation (DCC) model of Engle [4]. It has been shown in McAleer et al. [5] that BEKK can be derived from a vector random coefficient autoregressive model, and that the Quasi-Maximum Likelihood Estimates (QMLE) of the parameters in BEKK have the asymptotic properties of consistency and asymptotic normality, but only where the covariance matrix of the random coefficient is a diagonal matrix (or the associated special case of a scalar matrix). In practice, in the literature on testing for volatility spillovers between energy and agricultural markets, virtually all of the published papers seem to have estimated a Full BEKK matrix to test for spillover effects.

A serious technical deficiency and limitation is that the QMLE of a Full BEKK matrix has no asymptotic properties, except by assumption of the existence of multivariate eighth moments, which cannot be verified. Therefore, no valid statistical test of volatility spillover effects is possible within the context of a Full BEKK model. This is in contrast with the Diagonal BEKK counterpart, where the regularity conditions can be verified, so that the asymptotic properties of the QMLE allow valid statistical tests of volatility spillovers.

Some papers in the literature have used the DCC model to test for volatility spillovers using multivariate conditional covariances and conditional correlations. However, it is well known in the financial econometrics literature that the DCC model has no regularity conditions, and that the QMLE of the parameters of DCC has no asymptotic properties, except by assumption (see McAleer [6] for

further details). Therefore, volatility spillovers cannot be tested statistically using the associated conditional covariances and conditional correlations.

The purpose of the paper is to evaluate the theory and practice in testing for volatility spillovers between energy and agricultural markets using the BEKK and DCC models, and to make recommendations as to how such spillovers might be tested using valid statistical techniques. The published papers in the literature will be evaluated on the basis of countries, energy and agricultural commodities and markets, data sources, sample periods, data frequencies, analytical properties of the model specifications, statistical properties of the associated estimators, convergence of the associated estimation algorithms, number of parameters to be estimated, the hypotheses to be tested for volatility spillovers, significance of the associated estimators, magnitudes and signs of the estimators, use of univariate and multivariate conditional volatility models, the presence or otherwise of volatility spillovers, and an overall assessment of the empirical results in the literature based on misinterpretations of the models used in estimation.

The plan of the remainder of the paper is as follows: Section 2 presents the stochastic processes for the two most widely used univariate conditional volatility models in the first step of estimating the two multivariate conditional volatility models with spillover effects. The technical presentation is warranted as a precursor to a critical assessment of published empirical papers in leading international journals. Section 3 analyses 11 papers that have been published in international journals to evaluate volatility spillovers between energy and agricultural markets, and makes recommendations as to how such spillovers might be tested using valid statistical techniques. Three new definitions of volatility spillovers are given, specifically full volatility, full covolatility spillovers, and partial covolatility spillovers, the alternative multivariate models are evaluated in terms of the new definitions, and the different multivariate models used in empirical applications are evaluated in terms of the new definitions and relevant regularity conditions and statistical criteria. Section 4 gives a summary of the main results in the paper.

## 2. Stochastic Processes for Univariate and Multivariate Conditional Volatility Models: Full and Partial Volatility and Covolatility Spillovers

In order to accommodate volatility spillover effects, alternative multivariate volatility models of the conditional covariances are available. Examples include the diagonal model of Bollerslev et al. [7], the vech and diagonal vech models of Engle and Kroner [3], the Baba, Engle, Kraft, and Kroner (BEKK) multivariate GARCH (Generalized AutoRegressive Conditional Heteroskedasticity) model of Baba et al. [8] and Engle and Kroner [3], the constant conditional correlation (CCC) (specifically, multiple univariate rather than multivariate) GARCH model of Bollerslev [9], the Ling and McAleer [10] vector ARMA (Autoregressive moving average)-GARCH (VARMA-GARCH) model, and the VARMA-asymmetric GARCH (VARMA-AGARCH) model of McAleer et al. [11] the Engle [4] dynamic conditional correlation (DCC) (technically, dynamic conditional covariance rather than correlation model—See McAleer [6] and the Tse and Tsui [12] varying conditional correlation (VCC) model. For further details on most of these multivariate models see, for example, McAleer [13].

The first step in estimating multivariate models is to obtain the standardized shocks from the conditional mean returns shocks. For this reason, the three most widely used univariate conditional volatility models, namely GARCH, GJR (Glosten, Jagannathan, and Runkle) and EGARCH (exponential generalized autoregressive conditional heteroscedastic), will be presented briefly, followed by the two most widely estimated multivariate conditional covariance models, namely variations of BEKK and DCC.

Consider the conditional mean of financial returns as follows:

$$y_t = E(y_t | I_{t-1}) + \varepsilon_t \quad (1)$$

where the returns,  $y_t = \Delta \log P_t$ , represent the log-difference in financial commodity or agricultural prices ( $P_t$ ),  $I_{t-1}$  is the information set at time  $t - 1$ , and  $\varepsilon_t$  is conditionally heteroskedastic. In order to

derive conditional volatility specifications, it is necessary to specify the stochastic processes underlying the returns shocks,  $\varepsilon_t$ .

### 2.1. Univariate Conditional Volatility Models

Alternative univariate conditional volatility models are of interest in single index models to describe individual financial assets and markets. Univariate conditional volatilities can also be used to standardize the conditional covariances in alternative multivariate conditional volatility models to estimate conditional correlations, which are particularly useful in developing dynamic hedging strategies.

The three most popular univariate conditional volatility models are discussed below, together with the associated regularity conditions, the conditions required for asymmetry and leverage, and the conditions underlying the asymptotic properties of consistency and asymptotic normality, where they can be shown to exist.

#### 2.1.1. Random Coefficient Autoregressive Process and GARCH

Consider the random coefficient autoregressive process of order one:

$$\varepsilon_t = \phi_t \varepsilon_{t-1} + \eta_t \quad (2)$$

where  $\phi_t \sim iid(0, \alpha)$ ,  $\eta_t \sim iid(0, \omega)$ , and  $\eta_t = \varepsilon_t / \sqrt{h_t}$  is the standardized residual. Tsay [14] derived the ARCH (1) model of Engle [15] from Equation (2) as:

$$h_t = E(\varepsilon_t^2 | I_{t-1}) = \omega + \alpha \varepsilon_{t-1}^2 \quad (3)$$

where  $h_t$  is conditional volatility, and  $I_{t-1}$  is the information set available at time  $t - 1$ . The use of an infinite lag length for the random coefficient autoregressive process in Equation (2), with appropriate geometric restrictions (or stability conditions) on the random coefficients, leads to the GARCH model of Bollerslev [16]. From the specification of Equation (2), it is clear that both  $\omega$  and  $\alpha$  should be positive as they are the unconditional variances of two different stochastic processes.

The QMLE of the parameters of ARCH (Autoregressive conditional heteroscedasticity) and GARCH have been shown to be consistent and asymptotically normal in several papers. For example, Ling and McAleer [10] showed that the QMLE for GARCH( $p, q$ ) is consistent if the second moment is finite. Moreover, a weak sufficient log-moment condition for the QMLE of GARCH(1,1) to be consistent and asymptotically normal is given by:  $E(\log(\alpha \eta_t^2 + \beta)) < 0$ ,  $|\beta| < 1$ , which is not easy to check in practice as it involves two unknown parameters and a random variable. The more restrictive second moment condition, namely  $\alpha + \beta < 1$ , is much easier to check in practice.

In general, the proofs of the asymptotic properties follow from the fact that ARCH and GARCH can be derived from a random coefficient autoregressive process (see McAleer et al. [5] for a general proof of multivariate models that are based on proving that the regularity conditions satisfy the conditions given in Jeantheau [17] for consistency, and the conditions given in Theorem 4.1.3 in Amemiya for asymptotic normality).

#### 2.1.2. Random Coefficient Autoregressive Process and GJR

The ARCH and GARCH models are symmetric, that is, positive and negative shocks of equal magnitude have identical effects on conditional volatility. Consequently, there is no asymmetry, and hence no leverage, whereby negative shocks increase conditional volatility and positive shocks decrease conditional volatility (see Black [18]).

McAleer [19] showed that the GJR model of Glosten, Jagannathan and Runkle [20] could be derived as a simple extension of the random coefficient autoregressive process in Equation (2), with an

indicator variable  $I(\varepsilon_{t-1})$  that distinguishes between the different effects of positive and negative returns shocks on conditional volatility, namely:

$$\varepsilon_t = \phi_t \varepsilon_{t-1} + \psi_t I(\varepsilon_{t-1}) \varepsilon_{t-1} + \eta_t \quad (4)$$

where  $\phi_t \sim iid(0, \alpha)$ ,  $\psi_t \sim iid(0, \gamma)$ ,  $\eta_t \sim iid(0, \omega)$ ,  $I(\varepsilon_{t-1}) = 1$  when  $\varepsilon_{t-1} < 0$ ,  $I(\varepsilon_{t-1}) = 0$  when  $\varepsilon_{t-1} \geq 0$ ,  $\eta_t = \varepsilon_t / \sqrt{h_t}$  is the standardized residual, and the indicator functions,  $I(\varepsilon_{t-1})$ , are random variables.

The conditional expectation of the squared returns shocks in Equation (4), which is typically referred to as the GJR (alternatively, as the threshold or asymmetric GARCH) model, is an extension of Equation (3), as follows:

$$h_t = E(\varepsilon_t^2 | I_{t-1}) = \omega + \alpha \varepsilon_{t-1}^2 + \gamma I(\varepsilon_{t-1}) \varepsilon_{t-1}^2 \quad (5)$$

The use of an infinite lag length for the random coefficient autoregressive process in Equation (4), with appropriate restrictions on the random coefficients (namely, stability conditions), leads to the standard GJR model with lagged conditional volatility. From the specification of Equation (4), it is clear that all three parameters should be positive as they are the variances of three different stochastic processes.

A sufficient condition for the consistency of the QMLE of GJR(1,1) is the existence of the second moment, namely  $\alpha + \beta + \gamma/2 < 1$ . McAleer et al. [21] showed that the weaker sufficient log-moment condition for consistency of the QMLE of GJR(1,1) is given by:  $E(\log[(\alpha + \gamma I(\eta_t)) \eta_t^2 + \beta]) < 0$ ,  $|\beta| < 1$ , which involves three unknown parameters, an indicator function, and a random variable. As in the case of the log-moment condition for GARCH(1,1), the more restrictive second moment condition is much easier to check in practice.

As in the case of ARCH and GARCH, the proofs of the asymptotic properties follow from the fact that GJR can be derived from a random coefficient autoregressive process (see McAleer et al. [5] for a general proof of multivariate models that are based on proving that the regularity conditions satisfy the conditions given in Jeantheau [17] for consistency, and the general conditions given in Theorem 4.1.3 in Amemiya [22] for asymptotic normality).

As shown in McAleer [19], the GJR model is asymmetric, in that positive and negative shocks of equal magnitude have different effects on conditional volatility. Therefore, asymmetry exists for GJR if: Condition for Asymmetry for GJR:  $\gamma > 0$ .

A special case of asymmetry is leverage, which is the negative correlation between returns shocks and subsequent shocks to volatility (see Black [18]). The differences between asymmetry and leverage are frequently misunderstood and misinterpreted in practice, it is worth stating them explicitly. The conditions for leverage in the GJR model in Equation (5) are: Condition for Leverage for GJR:  $\alpha < 0$  and  $\alpha + \gamma > 0$ .

The second parametric condition for leverage is typically omitted in the literature on GJR. It is clear that leverage is not possible for GJR as both  $\alpha$  and  $\gamma$ , which are the variances of two stochastic processes, must be positive.

### 2.1.3. Random Coefficient Complex Nonlinear Moving Average Process and EGARCH

Another conditional volatility model that can accommodate asymmetry is the EGARCH model of Nelson [23,24]. McAleer and Hafner [25] showed that EGARCH could be derived from a random coefficient complex nonlinear moving average (RCCNMA) process, as follows:

$$\varepsilon_t = \phi_t \sqrt{|\eta_{t-1}|} + \psi_t \sqrt{\eta_{t-1}} + \eta_t \quad (6)$$



where  $\phi_t \sim iid(0, \alpha)$ ,  $\psi_t \sim iid(0, \gamma)$ ,  $\eta_t \sim iid(0, \omega)$ ,  $\sqrt{\eta_{t-1}}$  is a complex-valued function of  $\eta_{t-1}$ , and  $\eta_t = \varepsilon_t / \sqrt{h_t}$  is the standardized residual. McAleer and Hafner [25] show that the conditional variance of the squared returns shocks in Equation (6) is:

$$h_t = E(\varepsilon_t^2 | I_{t-1}) = \omega + \alpha |\eta_{t-1}| + \gamma \eta_{t-1} \quad (7)$$

where it is clear from the RCCNMA process in Equation (6) that all three parameters should be positive as they are the variances of three different stochastic processes.

Although the transformation of  $h_t$  in Equation (7) is not logarithmic, the approximation given by:  $\log h_t = \log(1 + (h_t - 1)) \approx h_t - 1$  can be used to replace  $h_t$  in Equation (7) with  $1 + \log h_t$ . The use of an infinite lag for the RCCNMA process in Equation (6) would yield the standard EGARCH model with lagged conditional volatility.

As EGARCH can be derived from a random coefficient complex nonlinear moving average (RCCNMA) process, it follows that there is no invertibility condition to transform the returns shocks to the standardized residuals. Therefore, there are as yet no asymptotic properties of the QMLE of the parameters of EGARCH.

Recently, Martinet and McAleer [26] showed that the EGARCH( $p, q$ ) model could be derived from a stochastic process, for which the invertibility conditions can be stated simply and explicitly. This theoretical result is likely to lead to the development of asymptotic properties for the QMLE of EGARCH.

McAleer and Hafner [25], among others, including Nelson [23], Engle and Figlewski [27], EVIEWS [28], and RATS [29], show that asymmetry exists for EGARCH if: Condition for Asymmetry for EGARCH:  $\gamma > 0$ , and that leverage exists for EGARCH model if: Condition for Leverage for EGARCH:  $\gamma < 0$  and  $\gamma < \alpha < -\gamma$ . Chang and McAleer [30,31] provide the correct interpretation of asymmetry for EGARCH, and show that the correct condition for asymmetry is actually: Condition for Asymmetry for EGARCH:  $\alpha \neq 0$ .

In virtually every empirical study where EGARCH is estimated, the QMLE of  $\alpha$  is statistically significant, so  $\alpha \neq 0$ . Thus, in practice, EGARCH always displays asymmetry, though not leverage.

The second parametric condition for leverage is typically omitted in the literature on EGARCH, without explanation. As in the case of the GJR model, it is clear that leverage is not possible for EGARCH as both  $\alpha$  and  $\gamma$ , which are the variances of two stochastic processes, must be positive.

## 2.2. Multivariate Conditional Volatility Models

The multivariate extension of univariate GARCH is given in Baba et al. [8] and Engle and Kroner [3], while the multivariate extension of univariate GJR is given in McAleer et al. [11]. A multivariate extension of the univariate EGARCH model has been considered in Kawakatsu [32], although no asymptotic properties have yet been established for the matrix exponential GARCH model (see Martinet and McAleer [26] for further details).

It would seem that the conditions for asymmetry and leverage for the GJR and EGARCH models should also be applicable to their multivariate counterparts, although this does not seem to be common in practice. The asymmetry conditions for multivariate GJR are given in the VARMA-AGARCH model of McAleer et al. [11]. Leverage has typically been presented for individual equations only, as defined by Black [18] for univariate processes using arguments based on the debt-to-equity ratio. The multivariate counterpart of leverage does not yet seem to have been defined, primarily because co-leverage across different assets does not have an unambiguous meaning in terms of the debt-equity ratio for a portfolio of assets.

In order to establish volatility spillovers in a multivariate framework, it is useful to define the multivariate extension of the relationship between the returns shocks and the standardized residuals, that is,  $\eta_t = \varepsilon_t / \sqrt{h_t}$ . The multivariate extension of Equation (1), namely  $y_t = E(y_t | I_{t-1}) + \varepsilon_t$ ,

can remain unchanged by assuming that the three components are now  $m \times 1$  vectors, where  $m$  is the number of financial assets. The multivariate definition of the relationship between  $\varepsilon_t$  and  $\eta_t$  is given as:

$$\varepsilon_t = D_t^{1/2} \eta_t \quad (8)$$

where  $D_t = \text{diag}(h_{1t}, h_{2t}, \dots, h_{mt})$  is a diagonal matrix comprising the univariate conditional volatilities. Define the conditional covariance matrix of  $\varepsilon_t$  as  $Q_t$ . As the  $m \times 1$  vector,  $\eta_t$ , is assumed to be iid for all  $m$  elements, the conditional correlation matrix of  $\eta_t$ , which is equivalent to the conditional correlation matrix of  $\eta_t$ , is given by  $\Gamma_t$ . Therefore, the conditional expectation of (8) is defined as:

$$Q_t = D_t^{1/2} \Gamma_t D_t^{1/2} \quad (9)$$

Equivalently, the conditional correlation matrix,  $\Gamma_t$ , can be defined as:

$$\Gamma_t = D_t^{-1/2} Q_t D_t^{-1/2} \quad (10)$$

Equation (9) is useful if a model of  $\Gamma_t$  is available for purposes of estimating  $Q_t$ , where as (10) is useful if a model of  $Q_t$  is available for purposes of estimating  $\Gamma_t$ .

Equation (9) is convenient for a discussion of volatility spillover effects, while both Equations (9) and (10) are instructive for a discussion of asymptotic properties. As the elements of  $D_t$  are consistent and asymptotically normal, the consistency of  $Q_t$  in (9) depends on consistent estimation of  $\Gamma_t$ , whereas the consistency of  $\Gamma_t$  in (10) depends on consistent estimation of  $Q_t$ . As both  $Q_t$  and  $\Gamma_t$  are products of matrices, neither the QMLE of  $Q_t$  or  $\Gamma_t$  will be asymptotically normal based on the definitions given in Equations (9) and (10).

### 2.3. Full and Partial Volatility and Covolatility Spillovers

Volatility spillovers are defined as the delayed effect of a returns shock in one asset on the subsequent volatility or covolatility in another asset. Therefore, a model relating  $Q_t$  to returns shocks is essential, and this will be addressed in the following sub-section. Spillovers can be defined in terms of full volatility spillovers and full covolatility spillovers, as well as partial covolatility spillovers, as follows, for  $i, j, k = 1, \dots, m$ :

- (1) Full volatility spillovers:

$$\partial Q_{iit} / \partial \varepsilon_{kt-1}, k \neq i \quad (11)$$

- (2) Full covolatility spillovers:

$$\partial Q_{ijt} / \partial \varepsilon_{kt-1}, i \neq j, k \neq i, j \quad (12)$$

- (3) Partial covolatility spillovers:

$$\partial Q_{ijt} / \partial \varepsilon_{kt-1}, i \neq j, k = \text{either } i \text{ or } j \quad (13)$$

Full volatility spillovers occur when the returns shock from financial asset  $k$  affects the volatility of a different financial asset  $i$ .

Full covolatility spillovers occur when the returns shock from financial asset  $k$  affects the covolatility between two different financial assets,  $i$  and  $j$ .

Partial covolatility spillovers occur when the returns shock from financial asset  $k$  affects the covolatility between two financial assets,  $i$  and  $j$ , one of which can be asset  $k$ .

When  $m = 2$ , only (1) and (3) are possible as full covolatility spillovers depend on the existence of a third financial asset.

As mentioned above, spillovers require a model that relates the conditional volatility matrix,  $Q_t$ , to a matrix of delayed returns shocks. The two most frequently used models of multivariate conditional

covariances are alternative specifications of the BEKK and DCC models, with appropriate parametric restrictions, which will be considered below.

#### 2.4. Diagonal and Scalar Baba, Engle, Kraft, and Kroner (BEKK)

The vector random coefficient autoregressive process of order one is the multivariate extension of Equation (2), and is given as:

$$\varepsilon_t = \Phi_t \varepsilon_{t-1} + \eta_t \quad (14)$$

where  $\varepsilon_t$  and  $\eta_t$  are  $m \times 1$  vectors, and  $\Phi_t$  is an  $m \times m$  matrix of random coefficients, and  $\Phi_t \sim iid(0, A)$ ,  $\eta_t \sim iid(0, QQ')$ .

Technically, a vectorization of a full (that is, non-diagonal or non-scalar) matrix  $A$  to  $vec A$  can have dimension as high as  $m^2 \times m^2$ , whereas vectorization of a symmetric matrix  $A$  to  $vech A$  can have dimension as low as  $m(m-1)/2 \times m(m-1)/2$ .

In a case where  $A$  is either a diagonal matrix or the special case of a scalar matrix,  $A = aI_m$ , McAleer et al. [5] showed that the multivariate extension of GARCH(1,1) from Equation (14), incorporating an infinite geometric lag in terms of the returns shocks, is given as the diagonal or scalar BEKK model, namely:

$$Q_t = QQ' + A\varepsilon_{t-1}\varepsilon_{t-1}'A' + BQ_{t-1}B' \quad (15)$$

where  $A$  and  $B$  are both either diagonal or scalar matrices.

McAleer et al. [5] showed that the QMLE of the parameters of the diagonal or scalar BEKK models were consistent and asymptotically normal, so that standard statistical inference on testing hypotheses is valid. Moreover, as  $Q_t$  in (15) can be estimated consistently,  $\Gamma_t$  in Equation (10) can also be estimated consistently. For further details, see Chang et al. [33,34].

In terms of volatility spillovers, as the off-diagonal terms in the second term on the right-hand side of Equation (15),  $A\varepsilon_{t-1}\varepsilon_{t-1}'A'$ , have typical  $(i,j)$  elements  $a_{ii}a_{jj}\varepsilon_{it-1}\varepsilon_{jt-1}$ ,  $i \neq j$ ,  $i, j = 1, \dots, m$ , there are no full volatility or full covolatility spillovers. However, partial covolatility spillovers are not only possible, but they can also be tested using valid statistical procedures.

#### 2.5. Triangular, Hadamard and Full BEKK

Without actually deriving the model from an appropriate stochastic process, Baba et al. [8] and Engle and Kroner [3] considered the Full BEKK model, as well as the special cases of triangular and Hadamard (element-by-element multiplication) BEKK models. The specification of the multivariate model is the same as the specification in Equation (15), namely:

$$Q_t = QQ' + A\varepsilon_{t-1}\varepsilon_{t-1}'A' + BQ_{t-1}B' \quad (16)$$

except that  $A$  and  $B$  are Full, Hadamard or triangular matrices, rather than diagonal or scalar matrices, as in (15).

Although estimation of the Full, Hadamard and triangular BEKK models is available in some standard econometric and statistical software packages, it is not clear how the likelihood functions might be determined. Moreover, the so-called “curse of dimensionality”, whereby the number of parameters to be estimated is excessively large, makes convergence of any estimation algorithm somewhat problematic.

Jeantheau [17] showed that the QMLE of the parameters of the Full BEKK model is consistent under a multivariate log-moment condition, while Comte and Lieberman [35] showed that the QMLE are asymptotically normal under the assumption of the existence of eighth moments. Unfortunately, the multivariate log-moment condition is more complicated than the counterparts for the GARCH(1,1) and GJR(1,1) models given in Sections 2.1.1 and 2.1.2, respectively. Specifically, the multivariate log-moment conditions are difficult to verify when the matrices  $A$  and  $B$  are neither diagonal nor



scalar matrices, and the eighth moment condition cannot be verified for a Full BEKK model. Therefore, there are as yet no verifiable asymptotic properties of the Full, Hadamard or triangular BEKK models.

The Full, Hadamard and triangular BEKK models have full volatility spillovers, full covolatility spillovers, and partial covolatility spillovers. However, any hypothesis testing relating to such spillovers is not possible as the QMLE do not possess any verifiable asymptotic properties. Moreover, as  $Q_t$  in (15) cannot be shown to be estimated consistently,  $\Gamma_t$  in Equation (10) also cannot be shown to be estimated consistently.

This is in sharp contrast to a number of published papers in the literature whereby volatility spillovers have been tested incorrectly based on the off-diagonal terms in the matrix  $A$  in Equation (16). This will be elaborated in Section 3 below.

## 2.6. Diagonal and Scalar Dynamic Conditional Correlation (DCC)

Another multivariate conditional volatility model has been suggested by Engle [4], who presented, without using any stochastic process for the underlying returns shocks, what is purported to be a dynamic conditional correlation (DCC) model. Without distinguishing between dynamic conditional covariances and dynamic conditional correlations, Engle [4] presented the scalar DCC specification as:

$$Q_t = (1 - \alpha - \beta)\bar{Q} + \alpha\eta_{t-1}\eta'_{t-1} + \beta Q_{t-1} \quad (17)$$

where  $\bar{Q}$  is assumed to be positive definite with unit elements along the main diagonal, the scalar parameters are assumed to satisfy the stability condition,  $\alpha + \beta < 1$ , the standardized shocks,  $\eta_t$ , have been defined previously.

As the matrix in Equation (17) does not satisfy the definition of a correlation matrix, specifically the off-diagonal terms are not necessarily positive or negative fractions, and the diagonal elements are not necessarily all equal to one, Engle [4] uses the following standardization:

$$\Gamma_t = (\text{diag}(Q_t))^{-1/2} Q_t (\text{diag}(Q_t))^{-1/2} \quad (18)$$

As discussed in McAleer [6], there is no clear explanation given in Engle [4] for the standardization in Equation (18) or, more recently, in Aielli [36]. The standardization in Equation (18) might make sense if the matrix  $Q_t$  in (17) were the conditional covariance matrix of  $\varepsilon_t$  or  $\eta_t$ , though this is also not made clear. Despite the title of the paper, Aielli [36] also does not provide any stationarity conditions for the DCC model, and does not mention invertibility. Indeed, in the literature on DCC, it is not clear whether Equation (17) refers to a conditional covariance or a conditional correlation matrix.

Similar comments also apply to the varying conditional correlation (VCC) model of Tse and Tsui [12], where the first stage is based on a standard GARCH(1,1) model using returns shocks. The second stage is slightly different from the DCC formulation as the dynamic conditional correlations are defined appropriately as correlations. However, no regularity conditions are presented, and hence no statistical properties are given. Some useful caveats regarding DCC and VCC are given in Caporin and McAleer [37].

McAleer [6] uses a vector random coefficient moving average process to derive a scalar DCC model, where it is shown that: (i) DCC is a dynamic conditional covariance model of the returns shocks rather than a dynamic conditional correlation model; (ii) provides the motivation for standardization of the conditional covariances to obtain the conditional correlations; and (iii) shows that the appropriate GARCH model for DCC is based on the standardized shocks rather than the returns shocks.

In what follows, the analysis of McAleer [6] is extended to derive a Diagonal DCC model, of which a special case is the standard DCC model. Specifically, let:

$$\varepsilon_t = \Theta_t \eta_{t-1} + \eta_t \quad (19)$$

where  $\varepsilon_t$  and  $\eta_t$  are  $m \times 1$  vectors, and  $\Theta_t$  is an  $m \times m$  matrix of random coefficients, and  $\Theta_t \sim iid(0, A)$ ,  $\eta_t \sim iid(0, \Gamma)$ . The conditional covariance matrix of (19) is given as:

$$Q_t = \Gamma + A\eta_{t-1}\eta'_{t-1}A' \quad (20)$$

As in the case of the derivation of the BEKK model, it is assumed that  $A$  is either a diagonal or scalar matrix, otherwise the derivation in (20) will not be possible because of non-conformity of the matrices in the matrix product.

A straightforward extension of (19) to a vector random coefficient moving average process of order infinity, with appropriate geometric restrictions, leads to an extension of (20), as follows:

$$Q_t = \Gamma + A\eta_{t-1}\eta'_{t-1}A' + BQ_{t-1}B' \quad (21)$$

where  $B$  is also a diagonal or scalar matrix. The scalar version of DCC in (21), in which  $A = \alpha^{1/2}$  and  $B = \beta^{1/2}$  gives the scalar DCC model in (17). The standardization of  $\Gamma_t$  given in (18) ensures that the elements of the standardized matrix satisfy the definition of a matrix of correlation coefficients.

The diagonal and scalar versions of DCC do not have full volatility or full covolatility spillovers, but partial covolatility spillovers are possible. However, it is well known that the QMLE of DCC have no regularity conditions or asymptotic properties (see, for example, Aielli [36] and Caporin and McAleer [37]). McAleer [6] demonstrates stationarity and invertibility of the DCC specification given in Equation (21), which is an important step in demonstrating the asymptotic properties of the QMLE of the parameters of scalar BEKK. It follows, therefore, that any statistical tests of volatility spillovers, specifically partial covolatility spillovers, will be invalid.

This is in marked contrast to a number of published papers in the literature whereby volatility spillovers have been tested incorrectly based on the off-diagonal terms in the matrix  $A$  in Equation (21). This will be elaborated in Section 3 below.

### 3. Critical Analysis of the Empirical Literature

The technical presentation given above is warranted as a precursor to a critical assessment of published empirical papers in leading international journals. A useful though not necessarily a technically accurate or critical analysis of the empirical literature on examining volatility spillovers has been presented in “The dynamic pattern of volatility spillovers between oil and agricultural markets” by Saucedo, Brümmer and Jaghdani [38]. The authors examined 23 published papers predominantly on the basis of univariate and multivariate conditional volatility models, as well as one paper on each of univariate stochastic volatility and univariate realized volatility. It is clear that conditional volatility models, as discussed in the previous section, dominate in any empirical analysis that tests for volatility spillover effects.

The papers discussed in Saucedo, Brümmer and Jaghdani [38] were analysed on the basis of products (or energy and agricultural commodities), region or country, model (specification), time frame (or sample period), (data) frequency, and empirical findings regarding spillovers. As discussed in the previous section, sensible analysis of volatility spillovers requires multivariate models to estimate and test for full volatility spillovers, full covolatility spillovers, and partial covolatility spillovers.

For this reason, in this paper we have chosen 11 of the 23 published empirical papers that have used the multivariate Full BEKK model (in one paper, the Diagonal BEKK model), and two papers that estimated both the Full BEKK and scalar DCC models. The scalar BEKK model was not used at all, and in some cases a univariate conditional model was presented in addition to the multivariate conditional volatility models.

The 11 papers that will be appraised in chronological order are: “Ethanol, corn, and soybean price relations in a volatile vehicle-fuels market” by Zhang, Lohr, Escalante and Wetzstein [39], “Volatility spillovers between food and energy markets: A semiparametric approach” by Serra [40], “Price volatility in ethanol markets” by Serra, Zilberman and Gil [41], “Volatility spillover effects and cross

hedging in corn and crude oil futures” by Wu, Guan and Myers [42], “Volatility spillovers in US crude oil, ethanol and corn futures markets” by Trujillo-Barrera, Mallory and Garcia [43], “Inside the Black Box: The price linkage and transmission between energy and agricultural markets” by Du and McPhail [44], “Do energy prices stimulate food price volatility? Examining volatility transmission between US oil, ethanol and corn markets” by Gardebroek and Hernandez [45], “Price volatility in food markets: can stock building mitigate price fluctuations?” by Serra and Gil [46], “Volatility spillovers in China’s crude oil, corn and fuel ethanol markets” by Wu and Li [47], “The influence of biofuels, economic and financial factors on daily returns of commodity futures prices” by Algieri [48], and “Dynamic spillovers among major energy and cereal commodity prices” by Mensi, Hammoudeh, Nguyen and Yoon [49].

The appraisal of the empirical literature in this section does not consider the empirical findings as these are already given in Saucedo, Brümmer and Jaghdani [38], albeit not critically from either a mathematical or statistical perspective. This paper is concerned with the statistical testing of volatility spillover effects, and will concentrate on the regularity conditions, statistical properties, hypothesis testing and statistical significance, as appropriate.

In addition to the energy commodities, agricultural commodities, countries, multivariate conditional volatility models, sample periods, and data frequencies that were discussed in Saucedo, Brümmer and Jaghdani [38], the paper also considers in Tables 1–3, the journals in which the papers were published, the energy and agricultural prices (namely spot or futures prices), data sources, software packages used in estimation and testing, the univariate conditional volatility models used in estimation as a first step in estimating their multivariate counterparts, the types of spillover effects considered (namely full volatility, full cointegration and partial cointegration spillovers), the analytical and statistical properties of the conditional volatility models, the purported hypothesis tests, the purported statistical significance of the tests, and an overall assessment of each of the published papers.

The 11 papers were published in some of the leading energy, agricultural and natural resource economics, and futures market journals, namely *Energy Economics* (three papers), *European Review of Agricultural Economics* (two papers), *Energy Policy* (two papers), and one paper in each of the *Journal of Agricultural and Resource Economics*, *Energy Journal*, *Energies*, and *Journal of Futures Markets*.

Also given in Table 1 are the countries for which the energy and agricultural products data are obtained, predominantly the USA for ethanol, fuel ethanol, crude oil, light crude oil, heating oil, biodiesel, gasoline, and heating oil. Other countries or regions considered include France for ethanol, crude oil and biodiesel, the European Union for oil, heating oil and gasoline, China for crude oil and fuel ethanol, international countries for crude oil and ethanol, and Brazil for crude oil and ethanol. The agricultural commodities include corn, rapeseed, soybeans, soybean oil, sugar and wheat for the USA and France, barley, corn, sorghum and wheat for the USA and European Union, sugar for the USA, Brazil and other international countries, and corn for the USA and China.

Table 1 also shows that the most frequently used data on prices were for spot (or cash) prices (five papers), futures prices (three papers), and one paper each for both spot and futures prices, both spot prices and index, and nominal prices. The sample periods ranged from 1989, 1990, 1992, 1997, 2000, 2003, 2005 and 2006 through to 2007, 2008, 2009, 2010, 2011, 2012 and 2013 for weekly data (seven papers), daily data (three papers, and one paper that used monthly data for ethanol and corn (see also Table 2)).

**Table 1.** Summary of Literature on Volatility between Energy and Agricultural Markets using Baba, Engle, Kraft, and Kroner (BEKK) and dynamic conditional correlation (DCC).

Author(s) (Year) [Reference]	Journals	Countries	Energy Commodities	Agricultural Commodities	Sample Periods	Prices
Algieri (2014) [48]	<i>Energy Policy</i>	USA, France	Ethanol, crude oil, biodiesel	Corn, rapeseed, soybeans, soybean oil, sugar, wheat	2005–2013	Futures
Du and McPhail (2012) [44]	<i>Energy Journal</i>	USA	Ethanol, gasoline, light crude oil	Corn	25 March 2005–25 March 2011	Futures
Gardebroek and Hemandez (2013) [45]	<i>Energy Economics</i>	USA	Crude oil, ethanol	Corn	1997–2011	Spot
Mensi, Hammoudeh, Nguyen and Yoon (2014) [49]	<i>Energy Economics</i>	USA, EU	Oil, gasoline, heating oil	Barley, corn, sorghum, wheat	2000–2013	Spot
Serra (2011) [41]	<i>Energy Economics</i>	International, Brazil	Crude oil, ethanol	Sugar	July 2000–November 2009	Spot
Serra and Gil (2013) [46]	<i>European Review of Agricultural Economics</i>	USA	Ethanol	Corn	January 1990–December 2010	Nominal
Serra, Zilberman and Gil (2011) [41]	<i>European Review of Agricultural Economics</i>	USA, Brazil	Crude oil, ethanol	Sugar	July 2000–February 2008	Spot
Trujillo-Barrera, Mallory and Garcia (2012) [45]	<i>Journal of Agricultural and Resource Economics</i>	USA	Crude oil, ethanol	Corn	2006–2011	Futures
Wu, Guan and Myers (2011) [42]	<i>Journal of Futures Markets</i>	USA	Crude oil	Corn	1992–2009	Spot, futures
Wu and Li (2013) [47]	<i>Energy Policy</i>	China	Crude oil, fuel ethanol	Corn	2003–2012	Spot, index
Zhang, Lohr, Escalante and Wetzstein (2009) [39]	<i>Energies</i>	USA	Ethanol, gasoline, oil	Corn, soybean	March 1989–December 2007	Spot

**Table 2.** Summary of Literature on Volatility between Energy and Agricultural Markets using BEKK and DCC.

Author(s) (Year) [Reference]	Data Frequency	Data Sources	Software Packages Used	Multivariate Models	Univariate Models	Spillovers
Algieri (2014) [48]	Daily	Bloomberg	Unstated	Diagonal BEKK	GARCH, EGARCH	Partial covolatility
Du and McPhail (2012) [44]	Daily	CME (corn, ethanol, gasoline, light crude oil), NYMEX (gasoline)	Unstated	Scalar DCC	GARCH	Partial covolatility
Gardebroek and Hernandez (2013) [45]	Weekly	EIA (oil), CBOT (ethanol), FAO (corn)	Unstated	Full BEKK, scalar DCC	GARCH	Full volatility, Full covolatility, Partial covolatility
Mensi, Hammoudeh, Nguyen and Yoon (2014) [49]	Daily	EIA (energy), IGC (cereal)	Unstated	Full BEKK, scalar DCC	GARCH	Full volatility, Full covolatility, Partial covolatility
Serra (2011) [41]	Weekly	CEPEA (ethanol, sugar), EIA (crude oil)	Unstated	Full BEKK	Semi-parametric GARCH	Full volatility, Full covolatility, Partial covolatility
Serra and Gil (2013) [46]	Monthly	Nebraska Government (ethanol), NASS (corn)	Unstated	Full BEKK	GARCH	Full volatility, Full covolatility, Partial covolatility
Serra, Zilberman and Gil (2011) [41]	Weekly	Center for Advanced Studies on Applied Economics (ethanol, sugar), EIA (crude oil)	WinRATS (v. 6.30)	Full BEKK	GARCH	Full volatility, Full covolatility, Partial covolatility
Trujillo-Barrera, Mallory and Garcia (2012) [45]	Weekly	NYMEX (WTI), CBOT (ethanol, corn)	Unstated	Full BEKK	Threshold GARCH	Full volatility, Full covolatility, Partial covolatility
Wu, Guan and Myers (2011) [42]	Weekly	USDA (corn cash), CBOT (corn futures), NYMEX (crude oil)	Unstated	Full BEKK	Threshold GARCH	Full volatility, Full covolatility, Partial covolatility
Wu and Li (2013) [47]	Weekly	National Bureau of Statistics of China	Unstated	Full BEKK	EGARCH	Full volatility, Full covolatility, Partial covolatility
Zhang, Lohr, Escalante and Wetzstein (2009) [39]	Weekly	<i>Ethanol &amp; Biodiesel News</i> (ethanol), EIA (gasoline, oil), USDA (corn, soybean)	Unstated	Full BEKK	GARCH	Full volatility, Full covolatility, Partial covolatility

Table 2 also shows that the primary data sources included Bloomberg, EIA (energy, oil, crude oil, gasoline), IGC (cereal), CBOT (ethanol, corn, corn futures), FAO (corn), National Bureau of Statistics of China, Nebraska Government (ethanol), NASS (corn), CME (corn, ethanol, gasoline, light crude oil), NYMEX (gasoline, WTI, crude oil), CEPEA (ethanol, sugar), Center for Advanced Studies on Applied Economics (ethanol, sugar), USDA (corn cash), *Ethanol and Biodiesel News* (ethanol), and USDA (corn soybean).

The same table shows that only one paper, namely Serra, Zilberman and Gil [41], stated the statistical, econometric or financial econometric software package, specifically, WinRATS version 6.30, that was used in estimation, whether for univariate or multivariate conditional volatility models. Consequently, there was no discussion of convergence of any algorithms that were used to estimate the models. This is a disappointing finding as it can be quite difficult to reproduce empirical results, especially for multivariate conditional volatility models, when the software package is not stated explicitly. Moreover, the “curse of dimensionality” cannot be determined when there is no discussion of the convergence of the algorithms, despite the fact it is well known that convergence is problematic when more than three financial assets are used to estimate the Full BEKK model.

The last three columns of Table 2 provide some useful insights regarding the types of univariate and multivariate conditional volatility models that are estimated, as well as the alternative volatility spillovers that can be considered. The Full BEKK model is estimated in seven papers, both the Full BEKK and scalar DCC models are estimated in two papers, and the Diagonal BEKK model and scalar DCC model are estimated separately in one paper each. As discussed in Section 2.3, the Full BEKK models incorporate full volatility, full covolatility and partial co-volatility spillovers, while the Diagonal BEKK and scalar DCC models allow only partial co-volatility spillovers. Valid statistical testing of such spillovers effects is discussed in Table 3 below.

As univariate models are necessary to obtain the standardized residuals for subsequent multivariate estimation and testing, each of the papers uses at least one, indeed usually only one, univariate conditional volatility model to initiate the estimation process. Of the 11 published papers, seven use only the GARCH model (including one semi-parametric GARCH model), two use the threshold GARCH model (also commonly known as GJR), one paper uses only EGARCH, and one uses both the GARCH and EGARCH models.

The analytical and statistical properties of the QMLE of the univariate and multivariate conditional volatility models are analysed in Table 3. Somewhat surprisingly and disappointingly, all 11 papers ignore any discussion of the analytical properties of the multivariate conditional volatility models, and nine of the papers also ignore the analytical properties of the univariate conditional volatility models as a precursor to estimating the multivariate models. Gardebroek and Hernandez [45] report that  $\alpha + \beta < 1$ , without explanation, but do not seem to appreciate that this is a sufficient but not necessary condition for the unconditional variance to be finite, and for the QMLE to be consistent. Wu and Li [47] discuss the conditions for asymmetry and leverage for the EGARCH model, but do so incorrectly by concentrating on the first condition, albeit incorrectly, namely rather than, and ignoring the second condition altogether, namely.

The papers purportedly test the hypotheses relating to volatility and covolatility spillovers without recognizing that such tests are invalid except for the Diagonal and scalar BEKK models, and not valid whatsoever for the scalar DCC models. Only one paper fails to provide any evidence of any purported hypothesis tests or diagnostic checks. The diagnostic checks include the standard Ljung-Box Q test for the absence of serial correlation in the residuals of the conditional mean equation (in two papers), normality tests of the returns shocks (in four papers), both unit root tests and cointegration tests (in six papers), tests of causality (in three papers), and a test for long memory (in one paper).



**Table 3.** Summary of Literature on Volatility between Energy and Agricultural Markets using BEKK and DCC.

Author(s) (Year) [Reference]	Analytical Properties of Models	Statistical Properties of Models	Purported Hypothesis Testing	Purported Significance of Estimates	Overall Assessment
Algieri (2014) [48]	Not addressed	Not addressed	Ljung-Box Q, normality, long memory	Yes	Questionable
Du and McPhail (2012) [44]	Not addressed	Not addressed	Unit root, cointegration	Yes	Questionable
Gardebroek and Hemandez (2013) [45]	$\alpha + \beta < 1$ for GARCH	Not addressed	Ljung-Box Q, unit root	Yes	Questionable
Mensi, Hammoudeh, Nguyen and Yoon (2014) [49]	Not addressed	Not addressed	Normality, unit root, causality	Yes	Questionable
Serra (2011) [41]	Not addressed	Not addressed	Unit root, cointegration	Yes	Questionable
Serra and Gil (2013) [40]	Not addressed	Not addressed	None	Yes	Questionable
Serra, Zilberman and Gil (2011) [41]	Not addressed	Not addressed	Unit root, cointegration	Yes	Questionable
Trujillo-Barrera, Mallory and Garcia (2012) [43]	Not addressed	Not addressed	Unit root, cointegration	Yes	Questionable
Wu, Guan and Myers (2011) [42]	Not addressed	Not addressed	Normality, unit root, cointegration	Yes	Questionable
Wu and Li (2013) [47]	Incorrect discussion of leverage effect for EGARCH	Not addressed	Normality, unit root, causality	Yes	Questionable
Zhang, Lohr, Escalante and Wetzstein (2009) [39]	Not addressed	Not addressed	Unit root, cointegration, causality	Yes	Questionable

As can be seen from Table 3, all 11 papers reported on the purported statistical significance of the estimated parameters, despite the fact that there is no proof that the statistical properties hold for diagnostic checks and statistical significance of estimated presence in the absence of asymptotic results for the multivariate conditional volatility models. These diagnostic checks are generally invalid in the presence of estimating volatility and cointegration spillovers, except under the null hypothesis that such spillovers do not exist, which would seem to destroy the primary purpose of the analysis.

As seven of the 11 papers used weekly data and one paper used monthly data, with the remaining three papers having used daily data, it is surprising that there were no tests conducted for seasonal unit roots or the possibility of seasonal cointegration. Having said that, there is no statistical proof that such diagnostic checks would be valid in the absence of any asymptotic theory underlying the Full BEKK and scalar DCC models.

The last column in Table 3 makes it clear that the overall assessment of the empirical literature in estimating and testing for volatility and cointegration spillovers between the energy and agricultural markets is one of disappointment. In short, the theoretical and empirical analyses in every paper are questionable.

The only tests that are valid asymptotically are for the scalar and Diagonal BEKK models. The Diagonal BEKK model was estimated only in the paper by Algieri, but without explanation or any statement to the effect of statistical validity. It can reasonably be presumed that the Diagonal BEKK model was estimated to overcome the “curse of dimensionality” that would otherwise have been faced in trying to obtain convergence in estimating the Full BEKK model. This raises serious questions and reservations about the unstated convergence in estimating the Full BEKK model in 9 of the 11 published papers in the literature on volatility spillovers between energy and agricultural markets.

The technical presentation of the published empirical papers in leading international journals is warranted as a precursor to a critical assessment and evaluation. Technical validity is essential, otherwise the mathematical regularity conditions, including invertibility, will not be satisfied. This means that the likelihood function either does not exist or cannot be specified algebraically, so that the likelihood equations required to present the first and second derivatives for purposes of deriving the asymptotic distribution cannot be obtained. In short, there will be no statistical properties for purposes of a valid statement of inferences regarding statistical significance.

#### 4. Concluding Remarks

The primary purpose of the paper was to specify, estimate and test for volatility and cointegration spillovers between the energy and agricultural markets. The paper showed that in the energy literature, the returns, volatility and volatility spillovers among alternative energy commodities have been analysed using a variety of univariate and multivariate conditional volatility models, the leading energy and agricultural economics journals in which the papers were published, estimation techniques, data sets, time frequencies, energy and agricultural prices, data sources, software packages used in estimation and testing, the univariate conditional volatility models used in estimation as a first step in estimating their multivariate counterparts, the types of volatility spillover effects that are considered (namely full volatility, full cointegration and partial cointegration spillovers), the analytical (regularity) conditions, statistical properties of the conditional volatility models, the purported hypothesis tests, the purported statistical significance of the tests, and an overall assessment of each of the published papers.

A similar comment applies to the separate theoretical and empirical analysis of a wide range of agricultural commodities and markets. The technical presentation above is warranted as a precursor to a critical assessment of published empirical papers in leading international journals. Each paper that was critically assessed failed to adhere to acceptable mathematical and statistical standards in terms of the regularity conditions that govern the model, including invertibility, which determines whether the likelihood function can be related to the data, as well as the likelihood equations that lead to the asymptotic distribution of the QMLE of the unknown parameters. Seen in this light, the empirical

estimates of volatility, covolatility, full volatility spillovers, and full and partial covolatility spillovers, can be based on a firm foundation when the technical conditions are satisfied.

Given the recent interest and emphasis in bio-fuels and green energy, especially bio-ethanol, which can be derived from a range of agricultural products, it is not surprising that there is a topical and developing literature on the volatility and covolatility spillovers between the energy and agricultural markets. Renewable energy sources (including wind, solar, hydro, wave, bio-mass, and agricultural commodities, such as corn and sugarcane, which can be converted into bio-ethanol as a renewable energy source), and nonrenewable fossil fuel energy sources (including coal, oil, gas and nuclear energy), can have full and partial covolatility spillovers. Such serious and significant possibilities emphasize how they are interconnected in terms of formulating portfolios of financial, energy and agricultural commodities to obtain optimal hedge strategies for insuring losses against profits elsewhere in the portfolio.

Modelling and testing spillovers between these two markets has typically been based on estimating multivariate conditional volatility models. A serious technical deficiency is that the Quasi-Maximum Likelihood Estimates (QMLE) of the two most popular multivariate conditional volatility models, namely the BEKK and DCC models, typically have no asymptotic properties, except by assumption or under appropriate parametric assumptions, so that no valid statistical test of volatility spillovers is possible.

The paper evaluated the theory and practice in testing for volatility spillovers between energy and agricultural markets using the multivariate BEKK and DCC models, and provided recommendations as to how such volatility and covolatility spillovers might be tested using valid statistical techniques. Three new definitions of volatility and covolatility spillovers were given, and the different models used in empirical applications were evaluated in terms of the new definitions and other criteria.

In an area as important as examining volatility and covolatility spillovers between the energy and agricultural markets, greater care and attention needs to be placed on the mathematical and statistical properties of the estimated univariate and especially multivariate conditional volatility models. Otherwise, the selected portfolios of renewable energy sources and nonrenewable fossil fuel energy sources will not be able to mitigate the financial risks against each other, and hence not be able to achieve optimal hedge ratios.

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