An introduction to time-varying lag autoregression

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Abstract

This paper introduces a new autoregressive model, with the specific feature that the lag structure can vary over time. More precise, and to keep matters simple, the autoregressive model sometimes has lag 1, and sometimes lag 2. Representation, autocorrelation, specification, inference, and the creation of forecasts are presented. A detailed illustration for annual inflation rates for eight countries in Africa shows the empirical relevance of the new model. Various potential extensions are discussed.

JEL codes: C22; C53

Key words: Autoregression; Time-varying lags; Forecasting

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1. Introduction

Time series models are frequently used to create out-of-sample forecasts. Commonly applied time series models are the autoregression, the moving average model, or a combination of these two. In general, these models are linear in the parameters and variables. For example, an autoregression of order 1 for a time series y_t (with acronym AR(1)) reads as

$$y_t = \mu + \alpha y_{t-1} + \varepsilon_t$$

where μ and α are unknown parameters, and where ε_t is a standard white noise process with mean zero and constant variance σ^2.

There are many extensions of this basic time series model. The order can be higher than 1 like *p* (AR(p)), and the error term can include lags of ε_t (a moving average model). Additionally, one can relax the assumption of a constant variance σ^2, and allow for time dependence for σ_t^2 as in the well-known ARCH model (Engle, 1982). One may also allow the function *f* in

$$y_t = \mu + f(y_{t-1}; \alpha) + \varepsilon_t$$

to be a nonlinear function, thereby allowing for jumps, thresholds, and changing regimes in the data, see De Gooijer (2017) and Granger and Teräsvirta (1993) for overviews of many nonlinear time series models.

In this paper I introduce yet another class of time series models. This class allows the lag structure to vary over time. In its simplest form, such a model reads as

$$y_t = \mu + \alpha y_{t-lag_t} + \varepsilon_t$$

where lag_t is a variable. I discuss representation and autocorrelations in Section 2, whereas parameter estimation, and the creation of forecasts appear in Section 3. In Section 4, an illustration to annual inflation rates for eight African countries shows the merits of this new model. Various potential extensions are proposed in the concluding Section 5.

2. Representation

Consider a time series y_t, where there are $t = -1, 0, 1,2, \dots, T$ observations[1], and assume it can be described by the following autoregression

$$y_t = \mu + \alpha y_{t-lag_t} + \varepsilon_t \tag{1}$$

where ε_t is a standard white noise process with mean zero and constant variance σ^2, and where α is an unknown parameter with $|\alpha| < 1$, and where μ is the intercept. The lag_t is a dummy-type variable, $t = 1,2, \dots, T$, which can take the values either 1 or 2. Various other choices can be made, of course.

Special cases of (1) appear when $lag_t = 1$ for all $t = 1, \dots, T$, and then the familiar first order autoregression (AR(1) appears, that is,

$$y_t = \mu + \alpha y_{t-1} + \varepsilon_t \tag{2}$$

Another special case appears when $lag_t = 2$ for all $t = 1, \dots, T$, which is

$$y_t = \mu + \alpha y_{t-2} + \varepsilon_t \tag{3}$$

and which can be called a subset autoregression of order 2 (Subset AR(2). Indeed, the familiar second order autoregression (AR(2) is represented by

$$y_t = \mu + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \varepsilon_t \tag{4}$$

[1] This notation entails that there are two pre-sample observations, and that the effective sample for parameter estimation starts at $t = 1$.

which includes both y_{t-1} and y_{t-2}.

The model in (1) describes a time-varying lag structure, in the sense that for T_1 observations the predictive model for y_t is (2), while for T_2 observations the predictive model for y_t is (3), where $T_1 + T_2 = T$.

Figure 1 presents 100 artificial observations from the models in (1) and (2), where $lag_t = 1, 2, 1, 2, 1, 2, ...$, $\alpha = 0.9$, $y_{-1} = y_0 = 0$, and $\varepsilon_t \sim N(0,1)$, which is held the same across the two models. The model in (1) gets the acronym TVLAR for time-varying lags autoregression. Next, Figure 2 presents artificial data from the models in (1) and (3), where again $lag_t = 1, 2, 1, 2, 1, 2, ...$, $\alpha = 0.9$, $y_{-1} = y_0 = 0$, and $\varepsilon_t \sim N(0,1)$, which is held the same across the two models. It can be seen from these two graphs that the TVLAR model allows for data that sometimes show sawtooth type patterns.

The unconditional mean of y_t when it follows (1) can be derived from the unconditional mean of y_t when it follows either (2) or (3). In both cases, the mean of y_t follows from rearranging model (2) as

$$y_t - \frac{\mu}{1-\alpha} = \alpha \left(y_{t-1} - \frac{\mu}{1-\alpha} \right) + \varepsilon_t$$

and for (3) as

$$y_t - \frac{\mu}{1-\alpha} = \alpha \left(y_{t-2} - \frac{\mu}{1-\alpha} \right) + \varepsilon_t$$

In both cases the mean is equal to $\frac{\mu}{1-\alpha}$. As (1) is either (2) or (3) depending on the value of lag_t, the unconditional mean of y_t in (1) is also $\frac{\mu}{1-\alpha}$.

The unconditional variance of y_t in case (2) and (3) is

$$\frac{\sigma_\varepsilon^2}{1-\alpha^2}$$

and hence the unconditional variance γ_0 for the time-varying lag autoregression in (1) is also equal to $\frac{\sigma_\varepsilon^2}{1-\alpha^2}$.

For the first order autocorrelation of the time-varying lag autoregression, matters are a bit more complicated. It all depends on the sequence of lags 1 and 2 in the lag_t variable. Consider the following table with transition events

		t	
		1	2
$t-1$	1	$T_{1,1}$	$T_{1,2}$
	2	$T_{2,1}$	$T_{2,2}$

where for example $T_{2,1}$ is the number of observations for which at time t holds that the lag is 1, while at $t-1$ it is lag 2. Naturally, $T_{1,1}+T_{1,2}+T_{2,1}+T_{2,2}=T$. For the observations $T_{1,1}$ the autocorrelation is α, and the same holds for the observations in $T_{2,1}$. For the observations $T_{1,2}$ the first order autocorrelation is α^2. For the $T_{2,2}$ observations, matters are bit more involved. If the lags sequence is 2, 2, 1, then the autocorrelation is α^3. If it is 2, 2, 2, 1, the autocorrelation becomes α^5, and so on. In sum, the first order autocorrelation for (1) is

$$\rho_1 = \frac{T_{1,1}\alpha + T_{1,2}\alpha^2 + T_{2,1}\alpha + \sum \beta_j T_{2,2}\alpha^{3+2j}}{T-1}$$

where β_j is a fraction of $T_{2,2}$, with $j = 0,1,2,\ldots$, and $T-1$ as there are $T-1$ transitions in an effective sample of size T. .Below, for the case of Kenya, this will be illustrated.

The first order partial autocorrelation is equal to the first order autocorrelation. The third order partial autocorrelation is equal to 0, and this helps to specify the model. So, the partial autocorrelation function may suggest the potential usefulness of the AR(2) model in (4) and also the TVLAR model in (1). The autocorrelation function has a pattern that looks like the familiar AR(1) or AR(2) model.

For the simulated data in Figures 1 and 2, the estimated autocorrelations are presented in Table 1. As the lags alternate between 1 and 2 in the variable lag_t , we have that $T_{1,1} = T_{2,2} = 0$. Hence, we have

$$\rho_1 = \frac{T_{1,2}\alpha^2 + T_{2,1}\alpha}{T_{1,2} + T_{2,1}} = \frac{\alpha^2 + \alpha}{2}$$

With $\alpha = 0.9$, we have $\rho_1 = 0.855$, and the empirical estimate in Table 1 is slightly below that. For this alternating lag autoregression, we also have that

$$\rho_2 = \rho_1$$

which seems to be reflected in Table 1. And, due to this specific alternating structure, we have

$$\rho_4 = \rho_3 = \frac{\alpha^4 + \alpha^3}{2} = \alpha^2\rho_1$$

In the last two columns of Table 1, this pattern is visible, although of course higher order autocorrelations are estimated with increasingly less observations.

3. Inference and forecasts

Parameter estimation for the model

$$y_t = \mu + \alpha y_{t-lag_t} + \varepsilon_t$$

can simply be done using Ordinary Least Squares (OLS), given the availability of the variable lag_t. With the lag_t, one can create the variable y_{t-lag_t}. A simple specification strategy for that lag variable amounts to estimating (using OLS) the two models

$$y_t = \mu + \alpha y_{t-1} + \varepsilon_{1,t}$$

and

$$y_t = \mu + \alpha y_{t-2} + \varepsilon_{2,t}$$

and to use the rule

$$lag = 1 \; if \; abs(\varepsilon_{2,t}) \geq abs(\varepsilon_{1,t})$$
$$lag = 2 \; if \; abs(\varepsilon_{2,t}) < abs(\varepsilon_{1,t})$$

With the lag_t variable and the estimated parameter, one can create forecasts. The observation at time $T + 1$ is

$$y_{T+1} = \mu + \alpha y_{T-lag_{T+1}} + \varepsilon_{T+1}$$

Clearly, the one-step ahead forecast from origin T depends on the value of lag_{T+1}, which is unknown at time *T*. When it is 1, the forecast is

$$y_{T+1|T} = \mu + \alpha y_T$$

and when it is 2, the forecast is

$$y_{T+1|T} = \mu + \alpha y_{T-1}$$

In the absence of knowledge on lag_{T+1}, it seems sensible to take an equal weighted combination of the two forecasts, that is

$$y_{T+1|T} = \mu + \frac{1}{2}\alpha(y_T + y_{T-1})$$

If the true lag at $T+1$ is 1, the forecast error is

$$y_{T+1} - y_{T+1|T} = \alpha y_T - \frac{1}{2}\alpha(y_T + y_{T-1}) + \varepsilon_{T+1} = \frac{1}{2}\alpha(y_T - y_{T-1}) + \varepsilon_{T+1}$$

Likewise, when the true lag at $T+1$ is 2, the forecast error is

$$y_{T+1} - y_{T+1|T} = \frac{1}{2}\alpha(y_{T-1} - y_T) + \varepsilon_{T+1}$$

Hence, the average forecast error is ε_{T+1}. Following the same notion of averaging, the two-steps ahead forecast is then

$$y_{T+2|T} = \mu + \frac{1}{2}\alpha(y_{T+1|T} \; + \; y_T)$$

which becomes

$$y_{T+2|T} = (1 + \frac{1}{2}\alpha)\mu + (\frac{1}{2}\alpha + \frac{1}{4}\alpha^2)y_T + \frac{1}{4}\alpha^2 y_{T-1}$$

The true observation at $T+2$ is

$$y_{T+2} = \mu + y_{T-lag_{T+2}} + \varepsilon_{T+2}$$

and there are four types of outcomes. At $T+2$, the lag can be 1 or 2, and at $T+1$ it can be 1 or 2. It is easy to derive that the two-steps ahead forecast error is

$$y_{T+2} - y_{T+2|T} = \varepsilon_{T+2} + \frac{1}{2}\alpha\varepsilon_{T+1}$$

with variance equal to

$$(1 + \frac{1}{4}\alpha^2)\sigma^2$$

which is smaller than the two-steps ahead forecast error from an AR(1) model, which would be equal to $(1 + \alpha^2)\rho^2$.

Misspecification

There are two potential cases of misspecification. The first is that the proper model is an AR(1) but a TVLAR is specified. Setting $\mu = 0$ for convenience, the data generating process (DGP) is

$$y_t = \alpha y_{t-1} + \varepsilon_t$$

and therefore

$$y_t = \alpha^2 y_{t-2} + \varepsilon_t + \alpha\varepsilon_{t-1}$$

The subset AR(2) model included in the TVLAR model specification

$$y_t = \alpha y_{t-2} + \varepsilon_t$$

is clearly mis-specified as it does not include the MA term $\varepsilon_t + \alpha\varepsilon_{t-1}$ and because the parameter at lag 2 is not α but α^2, in case of an AR(1). So, the subset AR model shall not give a good fit, and the absolute residuals will be larger than those of an AR(1), although of course at random moments it can become smaller.

A second type of misspecification is that the data are TVLAR and one specifies an AR(1). Ignoring time-variation in the lags can lead to spurious ARCH effects. Consider the artificial data in Figure 3. The data are generated from (1) with a specific pattern in the lag structure. Around the middle of the sample the lag switches from 1 to 2 and after 10 observations it switches back from 2 to 1. In Figure 3 it is already visible that the middle 10 observations show some sawtooth pattern. When an AR(1) model is fitted to these data, the residuals take over that sawtooth pattern, as can be seen from Figure 4. Here, an LM test for ARCH effects obtains the value 25.030 for these hypothetical data. The parameter for the squared residuals one period lagged is estimated as 0.503, and it associated standard error is 0.088.

If there are several models to be compared, against the TVLAR model, it seems best to make a model selection using the familiar information criteria AIC and BIC. When comparing out-of-sample forecasts, one can rely on the familiar root mean squared prediction error or any of the many forecast evaluation criteria.

4. Illustration

In this section the TVLAR model will be fitted to annual inflation rates series for eight African countries. The data can be found in Franses and Janssens (2018). These eight countries are those with the longest time series available, where $t = 1960, \ldots, 2015$. I will compare the TVLAR model with an AR(1), an AR(2), and a subset AR(2) model. Hence, the pre-sample observations are 1960 and 1961. Graphs of the data appear in Figure 5 and 6. A casual look at these graphs suggests some jagged patterns sometimes, and given the graphs in Figures 3 and 4, it may be that the TVLAR model is useful.

Table 2 presents the estimated autocorrelations and partial autocorrelations for the first six lags. Clearly all third order partial autocorrelation are not significant, when evaluated against the interval ± 0.268. All first and second order autocorrelations are significant, as is sometimes the second order partial autocorrelation (Burkina Faso, Morocco and Sudan).

These correlations seem to suggest that four models can be considered, and their estimation results appear in Tables 3 and 4. Except for Burkina Faso, the α parameter in the AR(1) model is estimated as significant. For all eight variables, the α parameter in the AR(1) model is estimated as significant. The second parameter in the AR(2) model, α_2 is only significant for Burkina Faso, Morocco and Sudan, as expected, given the significant partial autocorrelations.

The α parameter in the TVLAR model is estimated as significant in all eight cases. The lag_t variables are presented in Figure 7. There are no evident common lag structures across the series, as is also indicated by pairwise correlations, of which the largest is for the pair Burkina Faso and South Africa with a value of 0.295.

The AIC and BIC values are the smallest for the TVLAR model for all eight cases.

The α parameter for Kenya is estimated 0.724. For Kenya the obtained lag_t variable implies $T_{1,1} = 22$, $T_{1,2} = 10$ and $T_{2,1} = 10$. Furthermore, there are seven cases with the sequence 2, 2, 1, there are two cases with 2, 2, 2, 1, and there is each one case for 2, 2, 2, 2, 1 and 2, 2, 2, 2, 2, 1, respectively. Given this, the first order autocorrelation can be computed as

$$\rho_1 = \frac{T_{1,1}0.724 + T_{1,2}0.724^2 + T_{2,1}0.724 + 7(0.724)^3 + 2(0.724)^5 + (0.724)^7 + (0.724)^9}{T - 1}$$

which equals 0.592. The denominator is $T - 1$ because there are only $T - 1$ transitions. The estimated first order autocorrelation is 0.582, see Table 3.

A special case of the TVLAR model is

$$y_t = \mu + \alpha y_{t-1} + \varepsilon_t \; if \; y_{t-1} > \tau$$
$$y_t = \mu + \alpha y_{t-2} + \varepsilon_t \; if \; y_{t-1} \leq \tau$$

which assumes that the lag structure varies with the past value of the variable. In words, and for this illustration the subset AR(2) model is preferred in case a recent inflation observation is larger than a threshold. It could indeed be a sensible strategy to skip an outlier at the forecast origin and move to one year earlier. To see if such is the case for the inflation series, I estimate a logit model (lag 2 is 1, lag 1 is 0), and include one-year lagged inflation as the regressor. The estimation results are in Table 5 and it can be seen that only for Egypt higher one-year lagged inflation indicates a preference for lag 2. The fit is not high, as can be learned from the McFadden R^2 values.

Table 6 provides the forecasts for 2016 for each of the eight countries, and the actual values. When comparing the forecasts with those from an AR(1) model, it can be seen that for six out of the eight countries, the TVLAR forecast is closer to the realization.

All in all, it seems that the TVLAR model describes the inflation rates rather well. And, it seems also that more accurate forecasts can be obtained, at least for the eight series studied.

5. Extensions and conclusion

The specification strategy considered in the empirical analysis is based on comparing the absolute residuals of an AR(1) and a subset AR(2) model. A more subtle strategy could be to allow for some threshold values, like

$$lag = 1 \; if \; abs(\varepsilon_{2,t}) \geq \tau_1 abs(\varepsilon_{1,t})$$
$$lag = 2 \; if \; abs(\varepsilon_{2,t}) < \tau_2 abs(\varepsilon_{1,t})$$

where τ_1 and τ_2 are certain thresholds.

The basic TVLAR model in (1) can be extended in various dimensions. For example, a distributed lag version of the model, with time-varying lag, can look like

$$y_t = \beta_0 x_t + \beta_1 x_{t-lag_t} + \varepsilon_t$$

Such a combination is also possible for pure time series models, like

$$y_t = \mu + \alpha_1 y_{t-1} + \alpha_2 y_{t-lag_t} + \varepsilon_t$$

where for seasonal (like quarterly) data the lag_t variable can for example contain either 4 or 5.

An autoregressive distributed lag version of the model can read as

$$y_t = \mu + \alpha y_{t-lag\ y_t} + \beta x_{t-lag\ x_t} + \varepsilon_t$$

with $lag\ y_t$ is either 1 or 2, and with $lag\ x_t$ is either 0 and 1, or 1 and 2. Vector autoregressive versions of the time-varying lag model seem also possible.

In this paper I introduced a new and simple time series model, where the lag structure can vary over time. Inference is easy, and the creation of forecasts too. When evaluating the new model for eight example series to close competitors. it was found that the model fits better in sample, and also seems to deliver more accurate forecasts. Of course, more empirical experience should be gained to see whether this model, or any extensions of it, can be useful for other variables as well.

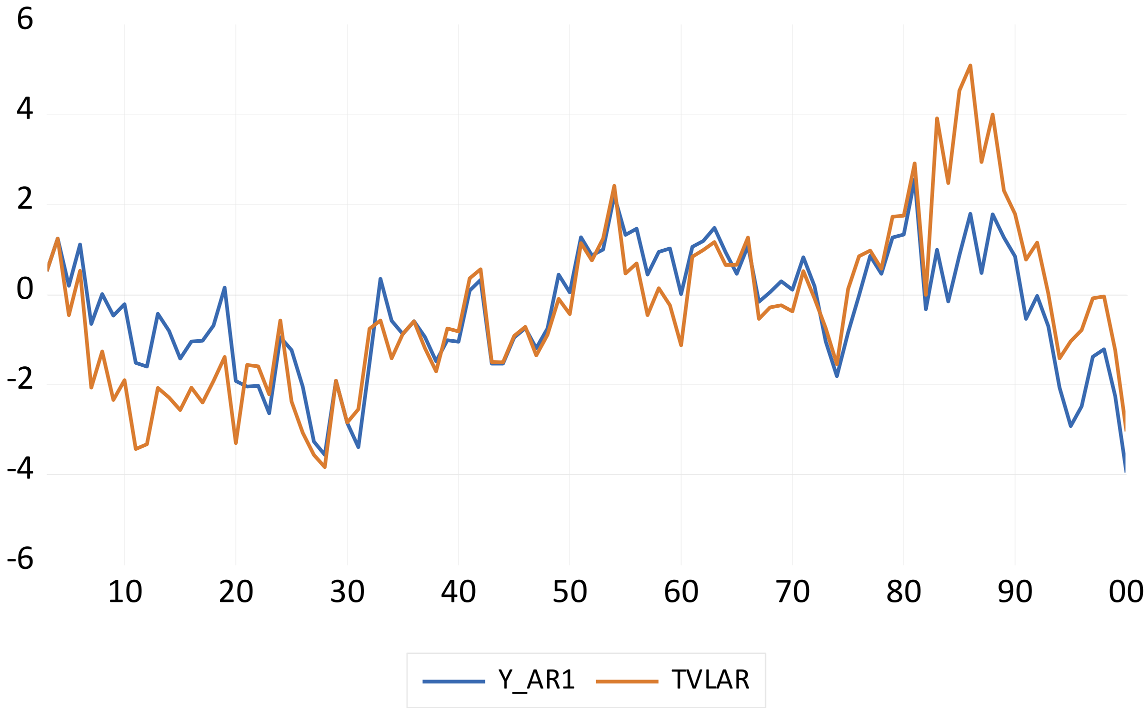

Figure 1: Artificial data from the models in (1) and (2), where $lag_t = 1, 2, 1, 2, 1, 2, ...$, $\alpha = 0.9$, $y_{-1} = y_0 = 0$, and $\varepsilon_t \sim N(0,1)$, which is held the same across the two models.

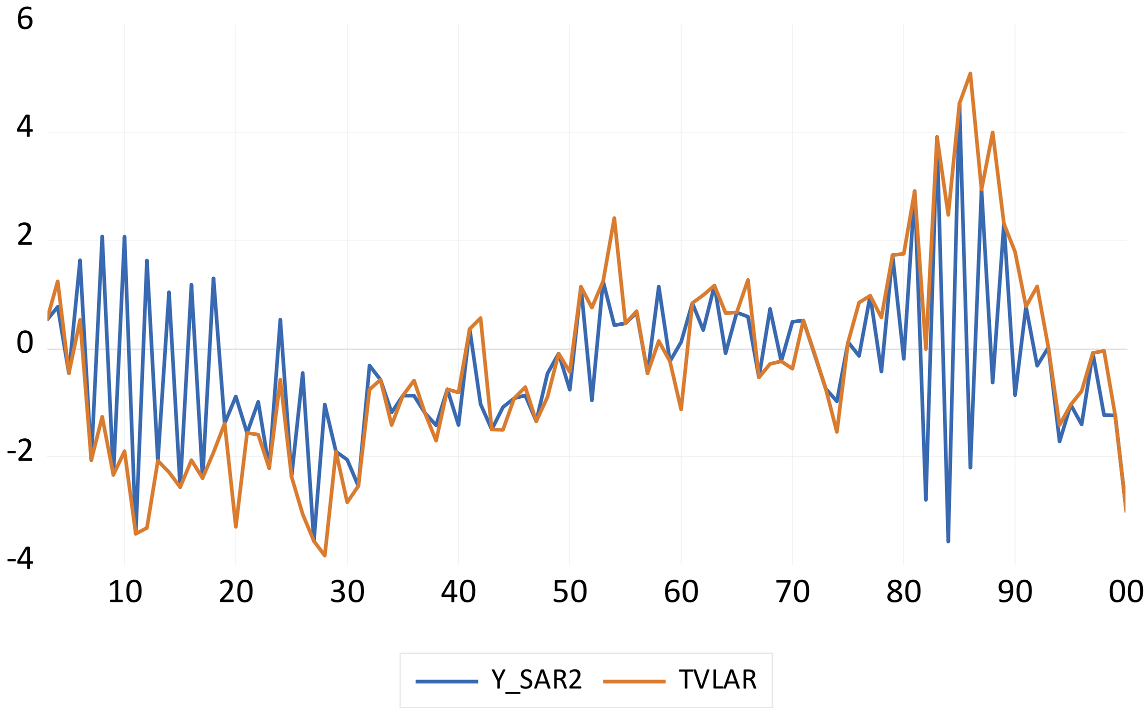

Figure 2: Artificial data from the models in (1) and (3), where $lag_t = 1, 2, 1, 2, 1, 2, \ldots$, $\alpha = 0.9$, $y_{-1} = y_0 = 0$, and $\varepsilon_t \sim N(0,1)$, which is held the same across the two models.

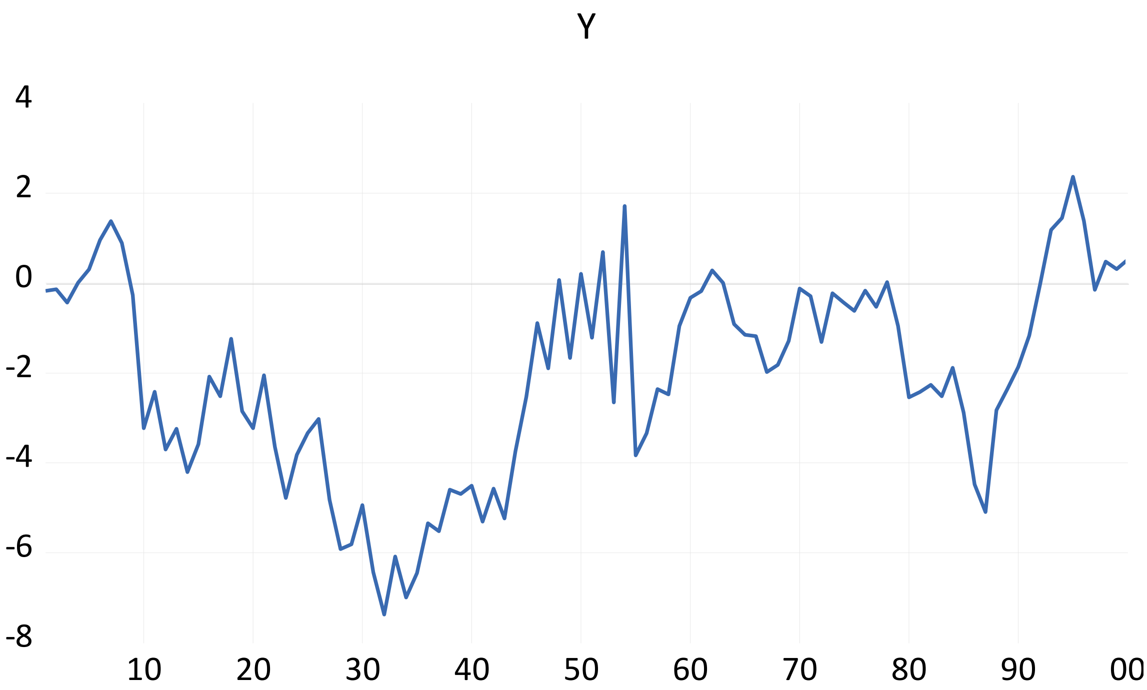

Figure 3: Artificial data for the model in (1), where $lag_t = 1$ for observations 1, 2, …, 45, where $lag_t = 2$, for observations 46, …, 55, and where $lag_t = 1$ for observations 56 to 100. $\alpha = 0.9$, $y_0 = 0$, and $\varepsilon_t \sim N(0,1)$.

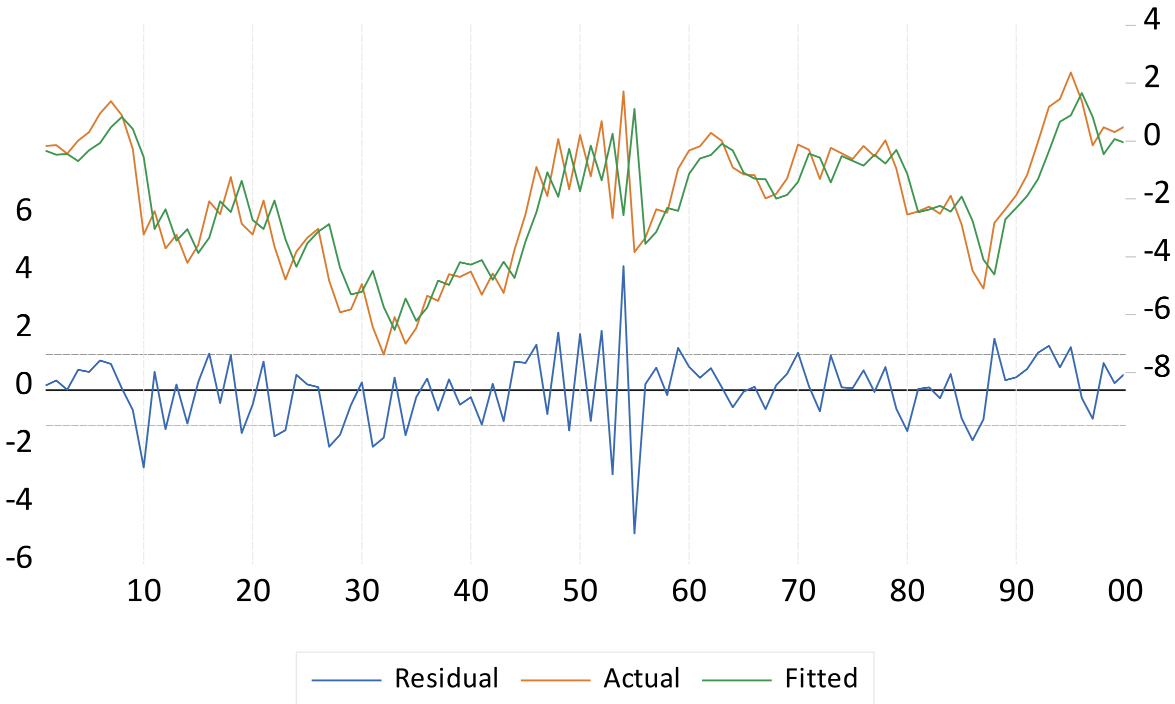

Figure 4: Fit and estimated residuals when an AR(1) model is fitted to the data in Figure 3.

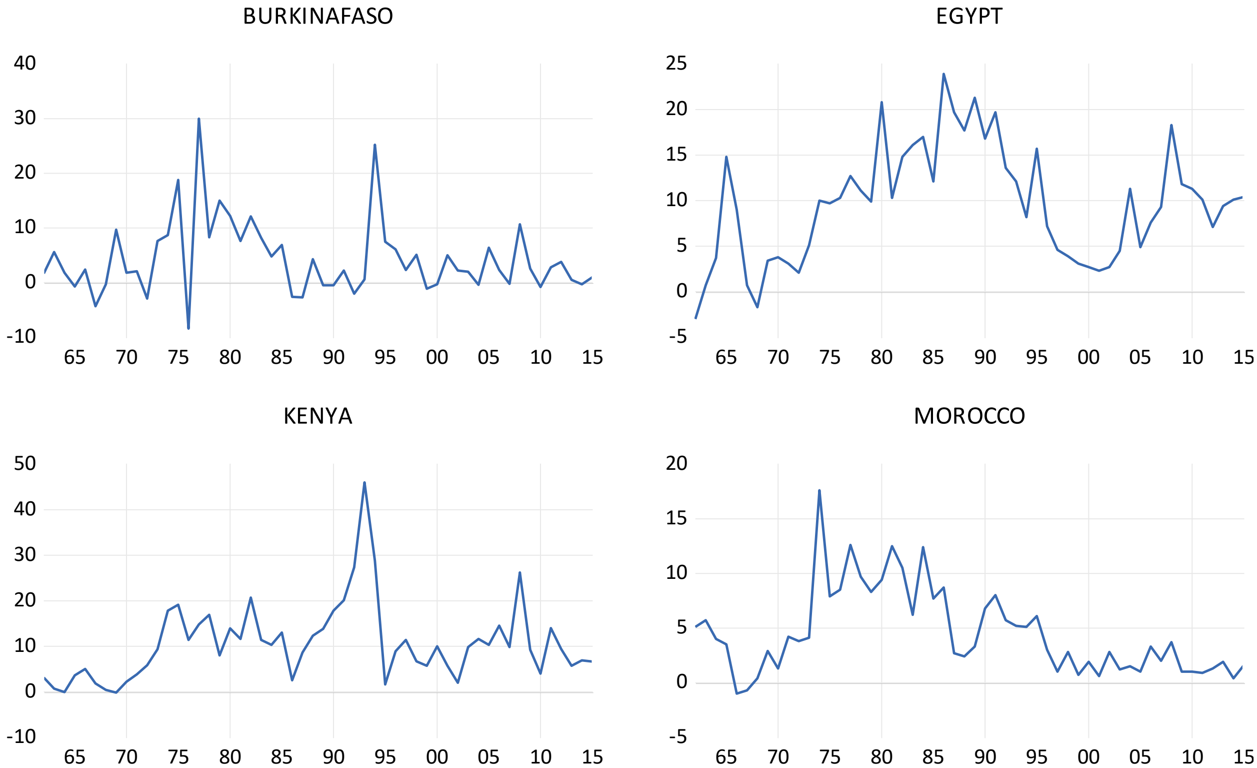

Figure 5: Annual inflation rates for Burkina Faso, Egypt, Kenya and Morocco, 1960-2015. Data source: Franses and Janssens (2019) and World Bank

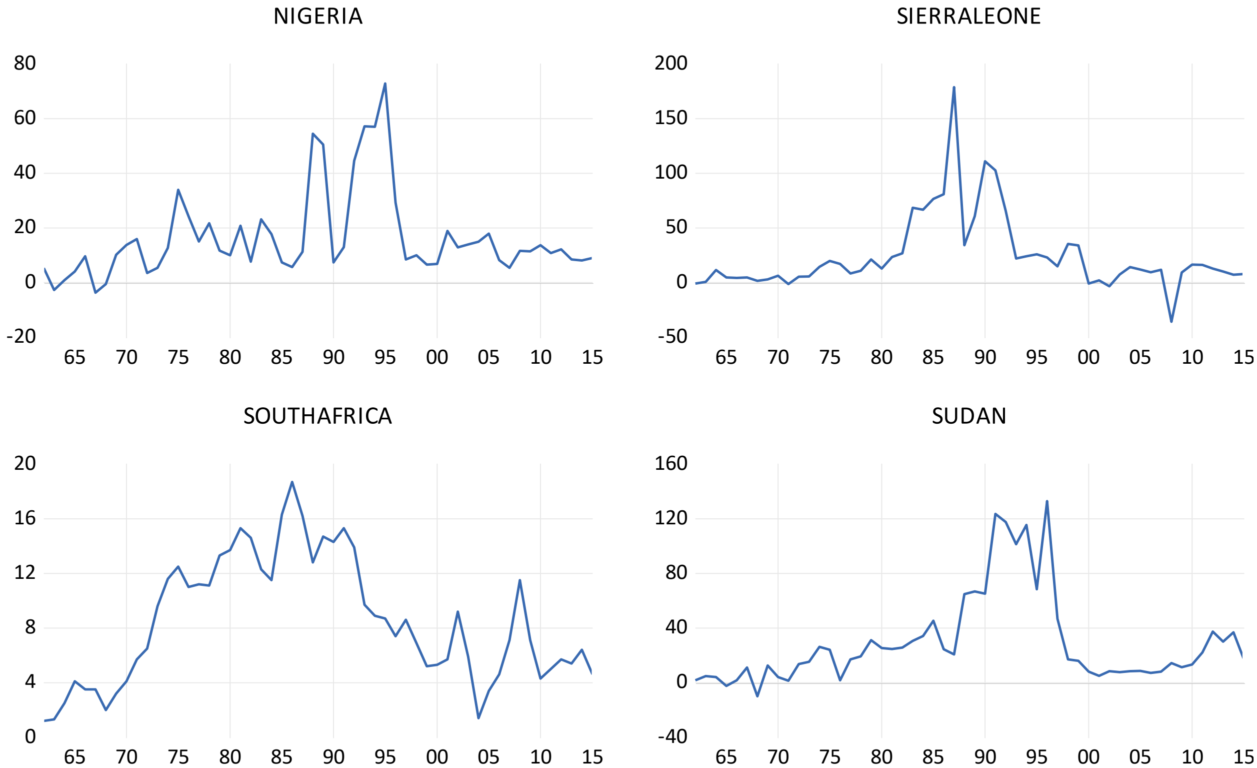

Figure 6: Annual inflation rates for Nigeria, Sierra Leone, South Africa, and Sudan, 1960-2015. Data source: Franses and Janssens (2019) and World Bank.

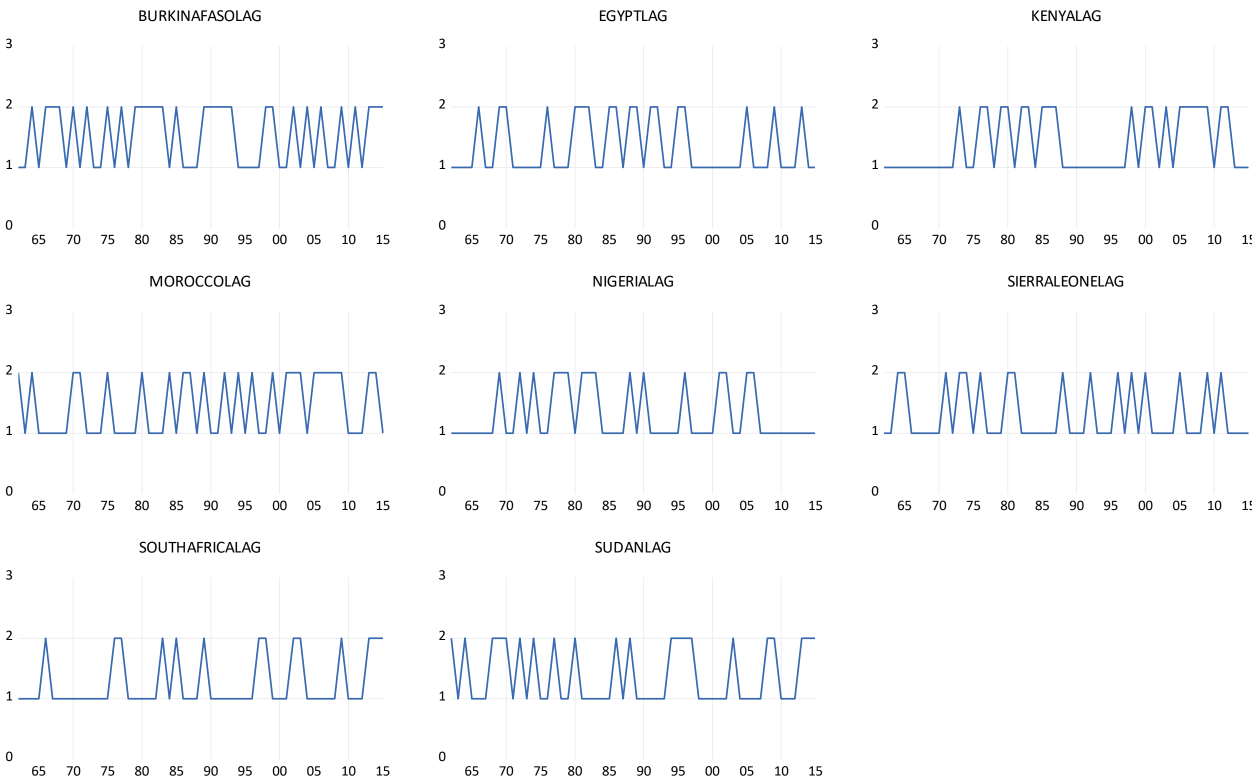

Figure 7: Estimated lags in time-varying lag models for annual inflation rates for eight countries in Africa

Table 1: Estimated autocorrelations and partial autocorrelations for the artificial data (T = 100) in Figures 1 and 2.

	AR(1)		Subset AR(2)		TVLAR	
	AC	PAC	AC	PAC	AC	PAC
Lags						
1	0.727	0.727	-0.406	-0.406	0.787	0.787
2	0.596	0.142	0.792	0.751	0.749	0.340
3	0.517	0.092	-0.336	0.135	0.667	0.028
4	0.476	0.094	0.636	0.050	0.585	-0.064
5	0.416	0.002	-0.228	0.154	0.543	0.053
6	0.290	-0.158	0.477	-0.033	0.435	-0.141

Table 2: Autocorrelations (AC) and partial autocorrelations (PAC) for annual inflation rates for eight countries in Africa, 1962-2015. Standard error is $\frac{1}{\sqrt{56}} = 0.134$.

	Burkina Faso		Egypt		Kenya		Morocco	
	AC	PAC	AC	PAC	AC	PAC	AC	PAC
Lags								
1	0.017	0.017	0.704	0.704	0.582	0.582	0.659	0.659
2	0.279	0.278	0.601	0.209	0.291	-0.073	0.591	0.276
3	0.136	0.138	0.489	0.014	0.260	0.184	0.548	0.161
4	0.139	0.070	0.458	0.116	0.118	-0.158	0.439	-0.043
5	0.007	-0.070	0.337	-0.115	-0.007	-0.033	0.351	-0.068
6	0.016	-0.067	0.318	0.077	-0.110	-0.152	0.248	-0.110

	Nigeria		Sierra Leone		South Africa		Sudan	
	AC	PAC	AC	PAC	AC	PAC	AC	PAC
Lags								
1	0.630	0.630	0.635	0.635	0.867	0.867	0.790	0.790
2	0.244	-0.253	0.536	0.223	0.717	-0.140	0.729	0.280
3	0.153	0.213	0.539	0.235	0.637	0.203	0.627	-0.024
4	0.144	-0.030	0.521	0.142	0.608	0.123	0.461	-0.274
5	0.192	0.180	0.368	-0.145	0.588	0.056	0.350	-0.086
6	0.212	-0.003	0.263	-0.138	0.510	-0.184	0.181	-0.186

Table 3: Parameter estimates and information criteria for AR(1), AR(2), subset AR(2) and time-varying lag AR models for Burkina Faso, Egypt, Kenya and Morocco. Estimation sample is 1962-2015. Estimated standard errors are in parentheses.

	μ	α (α_1)	α_2	AIC	BIC
Burkina Faso					
AR(1)	4.247 (1.119)	0.002 (0.134)		6.733	6.807
AR(2)	3.013 (1.246)	-0.001(0.130)	0.264 (0.130)	6.692	6.803
Subset AR(2)	3.008 (1.088)	0.264 (0.129)		6.655	6.729
TVL-AR	2.712 (1.047)	0.308 (0.114)		6.601	6.675
Egypt					
AR(1)	3.053 (1.092)	0.694 (0.096)		5.875	5.949
AR(2)	2.507 (0.116)	0.525 (0.136)	0.232 (0.133)	5.854	5.964
Subset AR(2)	4.032 (1.176)	0.602 (0.104)		6.074	6.148
TVL-AR	2.557 (0.852)	0.774 (0.076)		5.472	5.545
Kenya					
AR(1)	4.378 (1.492)	0.591 (0.111)		6.725	6.799
AR(2)	4.574 (1.605)	0.620 (0.140)	-0.049 (0.138)	6.760	6.870
Subset AR(2)	7.251 (1.734)	0.320 (0.129)		7.049	7.122
TVL-AR	3.112 (1.363)	0.724 (0.104)		6.505	6.579
Morocco					
AR(1)	1.544 (0.625)	0.658 (0.105)		5.069	5.142
AR(2)	1.070 (0.644)	0.473 (0.134)	0.287 (0.135)	5.021	5.132
Subset AR(2)	1.780 (0.677)	0.598 (0.113)		5.203	5.277
TVL-AR	1.175 (0.540)	0.780 (0.095)		4.799	4.873

Table 4: Parameter estimates and information criteria for AR(1), AR(2), subset AR(2) and time-varying lag AR models for Nigeria, Sierra Leone, South Africa and Sudan. Estimation sample is 1962-2015. Estimated standard errors are in parentheses.

	μ	α (α_1)	α_2	AIC	BIC
Nigeria					
AR(1)	5.982 (2.435)	0.635 (0.107)		7.932	8.006
AR(2)	7.358 (2.517)	0.786 (0.136)	-0.236 (0.136)	7.912	8.022
Subset AR(2)	12.03 (3.037)	0.264 (0.133)		8.379	8.452
TVL-AR	5.845 (2.404)	0.662 (0.108)		7.909	7.983
Sierra Leone					
AR(1)	8.837 (4.490)	0.642 (0.106)		9.460	9.534
AR(2)	6.818 (4.560)	0.491 (0.136)	0.235 (0.136)	9.440	9.551
Subset AR(2)	11.14 (4.880)	0.550 (0.115)		9.630	9.704
TVL-AR	2.641 (3.886)	0.951 (0.107)		9.066	9.140
South Africa					
AR(1)	0.986 (0.578)	0.888 (0.060)		4.302	4.376
AR(2)	1.080 (0.581)	1.038 (0.140)	-0.163 (0.137)	4.312	4.422
Subset AR(2)	2.168 (0.803)	0.756 (0.084)		5.009	5.083
TVL-AR	1.165 (0.497)	0.878 (0.052)		4.076	4.150
Sudan					
AR(1)	6.126 (3.713)	0.796 (0.084)		8.921	8.994
AR(2)	4.679 (3.648)	0.560 (0.135)	0.292 (0.134)	8.869	8.979
Subset AR(2)	8.309 (4.056)	0.738 (0.092)		9.123	9.196
TVL-AR	3.234 (2.508)	0.929 (0.058)		8.151	8.224

Table 5: Logit models for 1 (lag is 2) versus 0 (lag is 1) with one-year lagged inflation as explanatory variable. Estimated standard errors are in parentheses.

	Counts		μ	β	McFadden R^2
	Lag = 2	Lag = 1			
Country					
Burkina Faso	29	25	0.286 (0.329)	-0.030 (0.040)	0.008
Egypt	18	36	-2.134 (0.663)	0.141 (0.054)	0.119
Kenya	21	33	-0.661 (0.451)	0.020 (0.033)	0.005
Morocco	24	30	-0.135 (0.419)	-0.019 (0.071)	0.001
Nigeria	16	38	-1.160 (0.434)	0.017 (0.018)	0.014
Sierra Leone	16	38	-0.944 (0.368)	0.003 (0.008)	0.002
South Africa	14	40	-1.398 (0.668)	0.040 (0.067)	0.006
Sudan	21	33	-0.617 (0.374)	0.006 (0.008)	0.007

Table 6: Forecasts for 2016 form a TLVAR and an AR(1) model, and the realizations, all rounded at one decimal.

	Forecasts		Realization
	TVLAR	AR(1)	
Burkina Faso	2.8	4.2	-0.2
Egypt	10.5	10.3	13.8
Kenya	8.0	8.3	6.3
Morocco	2.0	2.6	1.6
Nigeria	11.5	11.7	15.7
Sierra Leone	9.9	14.0	10.9
South Africa	6.0	5.1	6.6
Sudan	28.2	19.6	17.8

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