The economics of violence in natural states

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A B S T R A C T

Violence is key to understanding human interaction and societal development. The natural state of societal organization is that a subset of the population, capable of mustering organized large-scale violence, forms an elite coalition that restrains both violence and coercive appropriation. We highlight key mechanisms underlying such natural states. Our results show that natural states either have a large elite coalition and a high tax rate, or a weak elite and a high level of appropriation by a large group of violence specialists outside the elite, termed warlords. When output elasticity of effort is high, it induces elite members to limit their tax rate, which in turn promotes warlordism. Only when the elite coalition is small but still able to control a sizeable share of production, as a result of its cooperative quality and a low decisiveness of conflict between elite and warlords, do we find comparatively high levels of production and producer welfare. Our results imply that almost all natural states experience continuous coercion exercised by elite members and violence between elite coalitions and warlords. We show that this is not a temporary out-of-equilibrium situation but a permanent phenomenon, as can most conspicuously be observed in parts of Sub-Saharan Africa. Our model thus illustrates the rigidity of natural states.

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1. Introduction

Violence is key to understanding human interaction and societal development. A society that is unable to contain violence will be disrupted and cannot be expected to sustain high levels of welfare, as is painfully illustrated by the current situation in Afghanistan, Libya or, perhaps most conspicuously, parts of Sub-Saharan Africa. Countries like Congo, Somalia, and Sudan are almost continuously torn up by extortion and coercion under the threat of violence, factional strife, and intermittent periods of open violence. Such conditions may destroy lives and capital goods, and deter interaction, exchange, investment, trade, and the benefits of specialization that come with trade, leading to significant welfare losses (Hirshleifer, 1988; Skaperdas and Syropoulos, 2002; North et al., 2013).

The ways in which societies cope with the threat of endemic violence are intrinsically linked up with, or embedded in, their economic and political systems. These systems structure the distribution of rents in order to contain the destructive effects of unconstrained violence on production, the economy, and society at large (North et al., 2009). One archetype of
such systems is formed by open, democratic governments, combined with competitive economies, and a state monopoly on violence which limits the incidence of violence. Few societies, however, conform to this pattern. Instead, most historical and many contemporary societies are dominated by a ruling elite that combines exclusive economic, political, and military advantages over the rest of society. Theoretically, such an elite is willing to maintain order – i.e. restrict violence within society – when its benefits of taxing society exceed the cost of maintaining order and the opportunity costs of restricting its own direct appropriation (McGuire and Olson, 1996; Acemoglu and Robinson, 2006). However, a high level of the required ‘tax rate’ and the difficulty of effectively limiting violence pose two possibly serious restrictions on economic development in these natural states.

These potential restrictions can be observed in many historical and contemporaneous societies that may be characterized as natural states, be it the feudal societies of mediaeval Europe and Japan, or present-day societies in much of Latin America, the Middle East and Africa. In such societies, the elite brings together their political parties, ethnic groups, patronage networks and associated organizations into a coalition, and commits to restrict violence among themselves and, possibly, towards the rest of society. This coalition, in such cases, forms ‘the state’ in natural states, according to North et al. (2009), and in contrast to for instance Bates (2008) who views the elite as separate from the state. Doing so, the coalition can create and distribute rents. Rents may be created by way of monopolies, exclusive rights to trade, subsidies, redistribution of taxes, or privileges to exploit natural resources. Especially in resource-rich areas such privileges are a crucial source of rents, and a potential source of conflicts, both in historical and present-day natural states, as in many parts of Sub-Saharan Africa, including the Democratic Republic of Congo, with its huge deposits of cobalt, oil, copper, diamond and tin (Olsson and Congdon Fors, 2004; Kaiser and Wolters, 2013). The latter case illustrates the potential problems for these natural states, as Congo is intermittently plagued by high levels of appropriation – through rent extraction and extortion – by the elite coalition and its rivals, as well as by outbreaks of open violence, as in the 1990s when various militias, political factions and ethnic groups militarily challenged the elite coalition (Bates et al., 2002; Bates, 2008; Kaiser and Wolters, 2013). Arguably, the problem of endemic violence and outbursts of open violence is most severe in countries with abundant natural resources, as the gains from violence are higher and there is more to be reaped (Sachs and Warner, 2001; van der Ploeg, 2011). Recent research indicates that the resource curse is not extrinsically linked to resource abundance, but can materialize in a context of low-quality institutions, where state competence, transparency and democratic accountability are weak; a situation found in natural states (Bulte et al., 2005; Mehlm et al., 2006).

In this paper we present a model to probe the existence of elite coalitions and the economic mechanisms of natural states. Thus, inspired by North et al. (2009), we address why elite members are unable or unwilling to effectively limit violence and, possibly, appropriation in natural states despite the evident welfare losses. We start from the assertion that there is no monopoly on violence in natural states, concentrated in a state. Instead, the capacity to exert violence – or coercion under the threat of violence – in an organized fashion and on substantial scale is concentrated in a small subset of the population, which opportunistically exploits this capacity to appropriate production from the producing population. In this we follow North et al. (2009) and other recent work. Francois et al. (2015) convincingly show that the ruling elite in countries in sub-Saharan Africa consists of a group of people who individually have substantial control over parts of the country’s population. According to the same authors, the consequence of not incorporating a wide enough range of such individuals is that they might instigate a rebellion through their patronage network – we will refer to such individuals with the capacity to organize and orchestrate large-scale violence as ‘violence specialists’. To North et al. (2009), a coalition of such violence specialists is the only possible social structure that limits violence in societies beyond the scope of foraging societies, and apart from modern open democracies.

There exists a substantial literature analysing incentives in terms of production and offensive or defensive violence activities in anarchical states – in the absence of a monopoly of violence – (cf. Hirshleifer, 1988; Skaperdas, 1992; Grossman and Kim, 1995). However, the capacity for violence is often dispersed over the entire population. And if hierarchical state structures are considered, then the violence capacities of ordinary producers are often contrasted to an individual or monolithic elite with the capacity to control the entire population that, as a result, has the opportunity to operate as a Leviathan-like source of order (Usher, 1989; Hirshleifer, 1995; Grossman, 2002; Bates et al., 2002; Acemoglu and Robinson, 2006; Konrad and Skaperdas, 2012). We stress that the nature of violence considered in this paper, however, is of a higher level than the banditry-type of violence commonly considered in the literature on the economics of conflict and appropriation (for an overview of this literature, see Garfinkel and Skaperdas, 2007; Konrad, 2009), and that ordinary producers cannot therefore decide to become specialists in violence. In addition, the capacity for large-scale violence is the domain of a small, but substantial group of violence specialists who do not necessarily work together, instead of a monolithic elite or elite group.

Organized and large-scale violence comprises the possibility to assemble small armies of loyal troops or mercenaries. More generally, violence specialists have the skills to generate and maintain a patronage network of followers that can, in turn, be used to exert violence – or coercion – on others. In our model, violence specialists face a choice either to join forces in a coalition of equals wherein they accept to respect each other’s privileges and income, or to individually exploit their violence capacities for appropriative activities as ‘warlords’. This coalition – subsequently termed the elite coalition – provides elite members with relative security over specific sources of income. This, in turn, induces the elite members to take account of the effects of their appropriation on production. Warlords, in contrast, lacking the support and accepted privileges characterizing the elite, do not incorporate the effect of their appropriation on production into their decision and, thus, appropriate all production under their control. Warlords may be the leaders of guerilla movements (e.g. Colombia), revolting militias (Somalia) or the leaders of independent clans or tribes (Afghanistan). The number of warlords relative to
elite members can vary over time. Examples of rapidly growing numbers of warlords are Liberia in the 1980s, as officers with their soldiers left the government army and turned predatory (Bates et al., 2002); or the earlier cited case of Congo in the 1990s (Bates et al., 2002; Bates, 2008; Kaiser and Wolters, 2013). Although warlords do not take into account how their appropriation affects production, their appropriation is not necessarily violent. Instead, their appropriation generally takes place as coercion under the threat of violence even though their rate of appropriation is unconstrained. The appropriation rate imposed by the elite is also based on coercion rather than violence, but is self-constrained – and, hence, referred to as a tax rate. Coercive appropriation could, of course, turn into open violence. However, the real source of open violence and instability in natural states is violent competition between violence specialists, that is between the elite coalition and warlords.\(^1\)

A direct implication of our discussion of violence specialists as capable of organizing large-scale violence is that entry and exit of violence specialists is exceptional. Depending on the specific setting, membership of the class of violence specialists is by inheritance, as with hereditary membership of a noble caste or the feudal nobility, or by a combination of wealth, inheritance, standing, and social networks. This is exactly one of the characteristics of natural states, where social mobility is highly restricted, in contrast to more open societies, including many present-day, Western ones (North et al., 2009). Sub-Saharan Africa is a case in point, as the chiefs and tribal leaders derive their vital positions in society from customary or hereditary sources, as most can claim a descent from elite lineages (Michalopoulos and Papaioannou, 2013; Herbst, 2014; Baldwin, 2016). Entry is therefore very exceptional, while exit is possible, but in principle not attractive in view of the time and means invested in acquiring this position and the possible gains it brings. As such, our model is closer to real-world situations and departs from models where agents can switch between production and (offensive or defensive) violence activities (cf. Hirschleifer, 1995; Grossman and Kim, 1995; Konrad and Skaperdas, 2012). In our model, producing agents have no means to engage in organized large-scale violence, and, simultaneously, they have no means to defend against such violence.

A key feature of the model we present is the interaction between the behaviour of violence specialists and production. Production is the domain of a separate class of actors in our model; the producers. Producers have no capacity for large-scale violence and, thus, are completely vulnerable to appropriation by both types of violence specialists. Producers do respond to appropriation by limiting their investment in production – which, in the model implies that they restrict ‘effort’, the sole input in the production function.\(^2\) Contrary to the above mentioned literature on elite behaviour in natural states, the appropriation rate faced by producers is the aggregate rate of appropriation – the combination of the tax rate of the elite coalition and the pure appropriation by warlords – determined by a group of individually optimizing violence specialists. That is, violence specialists compete over a single pool of production (as in Skaperdas, 2002) where the behaviour of each individual violence specialist affects the payoff to other violence specialists. This is in sharp contrast to the theoretical work closest to ours by Konrad and Skaperdas (2012) who model competition among elites that have a private resource pool and emphasize the welfare losses associated with ‘warlord states’ compared to states of self-organized producers.

In a nutshell our model is as follows. Violence specialists either join the elite coalition or become a warlord. Depending on the relative size of each group, the elite and warlords each control a share of the total population, and production. Both elite members and warlords use their violence capacity to appropriate production, but they do so in distinct ways. The elite resembles a ‘stationary bandit’, by enforcing a tax on production to support their coalition, taking into account – in the spirit of McGuire and Olson (1996) – that a high tax rate deters production. This tax rate should be interpreted broadly as encompassing all possible forms of rent extraction, and the optimal level of rent extraction may be very high, for instance when the marginal effect of taxation on production is low. Warlords do not levy taxes but instead they exploit their violence capacity to appropriate as much production as they can. This advantage of warlords over elite members in terms of rent extraction is partly mitigated by cooperation of the elite in the coalition, based on the mutual agreement to respect each other’s privileges and collectively defend these against warlords. In response to the expected level of appropriation, producers – modelled as one representative producer – decide how much effort to devote to production. The key outcomes of the model are the tax rate set by the elite, the ratio of elite members to warlords, and the production decision of the representative producer.

The model provides four main insights. First, the tax rate imposed by the elite depends on the output elasticity of effort. That is, the elite limits tax rates when output elasticity is high. This in turn, increases the tendency of violence specialists to become warlord, at least partially offsetting the effect of limited taxation on the aggregate rate of appropriation. Consequently, natural states generally face high rates of appropriation either by a strong elite coalition or, when a weak elite is willing to limit its appropriation, by a relatively high number of warlords. This result corroborates studies and theories highlighting the rigidity of natural states. Second, comparatively high levels of production and producer welfare occur only when output elasticity is high – inducing lower tax rates by the elite – in combination with strong cooperative quality of the elite and low decisiveness of conflict between the elite and warlords – allowing the small elite to control a relatively

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1. In our model we do not explicitly separate violence and coercion and take their effect on production to be similar. Of course, violence would lead to outright destruction and, thus, decrease production further than through producer incentives alone. However, including this effect simply decreases total production while not changing our results qualitatively.

2. ‘Effort’ is a stylized term to capture several mechanisms through which appropriation leads to reduced production.
large share of total production. Only then will producers benefit from the lower tax rates set by the elite. Third, we provide intuition on some of the internal mechanisms of the elite coalition. This we relate to the organizational and institutional development of the elite coalition, following North et al. (2009) who describe a maturation process of elite coalitions. The cooperative nature of controlling production and taxing the production under control could lead to conflict over distribution within the coalition, which we address in a first extension of the model. Furthermore, the collective nature of the coalition triggers questions over the optimal size of the coalition and possibly restrictions on the number of elite members, which we discuss in a second extension. The fourth main insight we provide, albeit more tentatively, is that our results suggest an ‘institutions-induced resource curse’, where low cooperative quality, and the context of the natural state more generally, can be equated with weak institutions, and high output elasticity of effort to the abundance of natural resources. In this interpretation, better institutions within the elite coalition (higher cooperative quality of the elite) allow society to benefit from natural resources (higher elasticity of output). However, the third necessary condition required for relatively high levels of production in our model is that the decisiveness of conflict remains low, which is difficult to reconcile with the point-based nature of natural resources as found in many regions of sub-Saharan Africa (Bulte et al., 2005; Mehlum et al., 2006).

2. Model

Consider a natural state with a population of fixed size. There are two subsets of individuals in this society. The first comprises producers, whose production decisions are aggregated in Section 2.3 into a single decision by a representative producer. The second comprises violence specialists, denoted by the set \( V \). Each violence specialist is either an elite member or a warlord, denoted by the sets \( E \) and \( V \), respectively. We have \( E \subset V \) and \( W = V \setminus E \). In our model there is no mobility between violence specialists and producers, with the exception of a model extension presented in Section 4.3. The two sets \( E \) and \( V \) are mutually exclusive and collectively exhaustive with respect to \( V \), but we explicitly allow for mobility between them. Members of each set are homogeneous in all relevant aspects. We will often refer to elites and warlords by their number: \( e = |E| \) and \( w = |W| \).

Violence specialists \( i \in V \) can appropriate production from the producers in two different ways, and this appropriation decision is determined by their choice of occupation. Elite members cooperate and appropriate by levying a jointly determined tax on their controlled production, while warlords appropriate by stealing all of their controlled production. This model feature is a stylized representation of the warlords’ uncertain power base which makes them myopic in their decision on how much to appropriate. The two occupations exert negative externalities, because the amount of production appropriated by warlords decreases the production available for appropriation by elite members, and vice versa. As a result, elite members and warlords compete over the share of total production either side controls. From the side of the elite members, this can be interpreted as either (i) the share of society whereon they effectively impose order, or (ii) the extent to which they succeed in establishing order over the entire population.

Our main simplifying assumption is that we consider violence specialists as individuals, and their capacities as homogeneous. Recall that we consider violence specialists as having the skills to generate and maintain a patronage network of followers. Our assumption of violence specialists as homogeneous individuals has three main implications. First, homogeneity removes any reason to discriminate between violence specialists of the same occupation. As a result, payoffs are equal across elite members as well as across warlords. Second, we can abstract from the specificities of the formation and size of patronage networks. Third, we need not explicitly model entry and exit into the elite coalition although we discuss this in more detail in Section 4.2. In reality, as a result of competition over the tax rents within the coalition, the composition of the coalition may be continuously changing, as power relations between members change, and because members with negligible contributions are weeded out and, potentially, substituted for new members.

Given this simplification, our model allows us to focus on the violence specialists’ choice whether or not to join the elite coalition, and its implications for production levels and producer welfare. This choice depends on the relative profitability of each occupation. Violence specialists can switch occupations without cost and do so until payoffs are equalized. As a result, each equilibrium features a specific distribution of elite members and war lords. The occupation choice by violence specialists is the first stage of a three-stage model. The stages are shortly described below and worked out in detail in Section 3. In addition, In Section 4 we present extensions of the model with an additional stage that features conflict within the coalition, a discussion of the propensity of elite members to limit the size of the coalition, and an extension on the endogenous size of the group of violence specialists.

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3 Following North et al. (2009) in terms of the maturity of natural states, our model may apply more closely to fragile and basic natural states than to mature natural states.

4 In addition, since warlords operate alone – and given a sufficiently large number of violence specialists – individual appropriation rates have only negligible impact on production so that myopic behaviour is not an assumption but an implication from the model introduced below.

5 The model guarantees internal solutions featuring equilibria with strictly positive amounts of warlords and elite members, as we will demonstrate below.
2.1. Occupation choice (Stage 1)

Each violence specialist \(i \in V\) decides to join the elite coalition or not. We denote this occupation choice by \(\mu_i \in \{1, 0\} \ \forall i \in V\). If \(\mu_i = 1\), the specialist joins the elite coalition. If \(\mu_i = 0\), the specialist becomes a warlord. The outcome of these decisions is a vector \(\mu = (\mu_i : i \in V)\) that partitions the violence specialists in two subsets: the elite coalition \(E = \{i : \mu_i = 1\}\) and its complement \(W = V \setminus E = \{i : \mu_i = 0\}\) comprising all warlords.

Control over producers by elite members and warlords is given deterministically by the ratio \(\frac{e}{w}\) (see (17)). We use the following function to determine the share of total production that is controlled by elites:

\[
\rho(e, w) = \frac{\theta e^m}{\theta e^m + w^m},
\]

(1)

with \(m \in (0, 1)\) and \(\theta \in [1, \infty)\). It follows that the share of total production that is controlled by warlords equals \(1 - \rho(e, w)\).

A few comments on (1) are appropriate here.

First, its functional form is borrowed from the rent seeking literature (Tullock, 1980). Our specification of \(\rho(e, w)\) is based on a modification of the ratio-form contest success function (CSF), inspired by the axiomatic characterization of group CSFs by Münster (2009). Parameter \(m\) is conventionally interpreted as the decisiveness of conflict, and here we interpret it as the decisiveness of group size, where group refers to either the elite coalition or the aggregate of warlords. Given \(m < 1\), there are diminishing marginal returns to group formation. Specifically, low \(m\) implies that a small elite coalition (group of warlords) is capable of controlling a relatively large share of production. Parameter \(\theta\) represents a fighting asymmetry (cf. Usher, 1989; Clark and Riis, 1998) in favour of the elite coalition that we consider to be better organized than warlords, because of their commitment to respect each other’s privileges. In the context of North et al. (2009) one could interpret \(\theta\) as the cooperative quality, or maturity, of the coalition, with more mature coalitions capable of organizing and coordinating power more efficiently.\(^6\) One implication of this functional form is that the elite may control a larger share of production, even if it is smaller in size than the aggregate of warlords.

Second, note that our interpretation of \(\rho(e, w)\) is non-probabilistic in the sense that it represents a share, rather than a winning probability (although the two interpretations are equivalent under the assumption of risk neutrality). For a detailed discussion of CSFs and their interpretations, see Hirshleifer (2000), Garfinkel and Skaperdas (2007), and Konrad (2009).

Third, although we talk freely about the share of production that is controlled by warlords, the degree of cooperation by warlords has not been specified yet. In the functional form chosen in (1), warlords do work together, but have a fighting disadvantage compared with the elite, through \(\theta\). The alternative approach is to model warlords as operating alone, using the term \(w^m \times 1^m\) rather than \(w^m\), which would imply

\[
\rho'(e, w) = \frac{\theta e^m}{\theta e^m + w^m} = \frac{\theta e^m}{\theta e^m + w^m}.
\]

(2)

This alternative specification, however, has two disadvantages: (i) It would give the coalition a fighting disadvantage for any \(m < 1\), so that the effects of \(m\) and \(\theta\) may cancel each other out. This disadvantage is reversed for \(m > 1\). In our model set-up, however, if \(\theta\) is sufficiently large, then \(m > 1\) leads to a corner solution where all specialists end up in the elite coalition.\(^7\) This is a standard feature of the ratio-form CSF, discussed in detail by Hirshleifer (1995) and employed by Skaperdas (1998) to assess coalition formation in a different setting. (ii) Using \(\rho(e, w)\) to calculate the equilibrium ratio of elite members to warlords, as we do for \(\rho(e, w)\) in (14), would lead to asymptotic behaviour of this ratio, including discontinuities and negative outcomes. Both features are undesirable and we stick to (1).

Fourth and final, our specification of \(\rho(e, w)\) deviates from the standard approach in the economic literature on conflict and appropriation. Most importantly, this contest is deterministic in the sense that the outcome of the contest depends only on the ratio \(\frac{e}{w}\). Notably, it does not depend on costly investments in violence capacity. That is, in the Stage 2 contest, elite members and warlords do not explicitly choose their violence level as is conventional in models that feature a trade-off between own production and appropriation (e.g. Hirshleifer, 1988, 1995; Skaperdas, 1992; Grossman and Kim, 1995) or in rent-seeking models (Nitzan, 1994). We focus, however, on organized large-scale violence and, in our model, the capacity for such violence is restricted to violence specialists, while production is the domain of the separate subset of producers. As a result, violence specialists are not confronted with this trade-off between own production and appropriation. In addition, motivated by the homogeneity of violence specialists, the only effect of adding costly investments in violence would be that payoffs of violence specialists would be reduced in the symmetric outcome of such a model, without any qualitative impacts on model results.

2.2. Tax (Stage 2)

Given the outcome of Stage 1, the elite controls a share \(\rho(e, w)\) and warlords jointly control a share \(1 - \rho(e, w)\), which each of them can appropriate as they wish. Following the main features of the natural state as discussed above, elite members

\(^6\) See van Bavel et al. (2016) who provide a historical exploration of the role of organizations on \(\theta\).

\(^7\) Specifically, a corner solution results if \((1 - \alpha)\theta \geq 1\), where parameter \(\alpha\) is the output elasticity to effort introduced in Section 2.3).
collectively determine their tax rate $\tau \in [0, 1]$, while warlords, by construction, choose to appropriate all production under their control.

2.3. Production (Stage 3)

In modelling production, we choose to aggregate production decisions by all producers into a single decision by a representative producer, although the same functional form would be obtained by assuming a set of homogeneous producers, each working a plot of land with symmetric concavity in effort. Given the outcome of Stages 1–2, the representative producer chooses its joint production level. We model production $Y$ as a single-input production function with effort $\phi$ as input.

$$Y(\phi) = \beta \phi^\alpha.$$  (3)

Parameter $\alpha \in (0, 1)$ denotes the output elasticity of effort. Parameter $\beta \in (0, \infty)$ reflects total factor productivity, which in the presence of only one factor of production, we will refer to as a technology parameter.

 Appropriation by elite members and warlords reduces the amount of produce available for consumption. Producers maximize utility $U$ which equals aggregate consumption – production net of appropriation – minus the cost of effort.

$$U = (1 - \tau)\rho(e, w)Y(\phi) - \gamma \phi.$$  (4)

with cost parameter $\gamma \in (0, \infty)$.

Given our main assumption that violence specialists are homogeneous, payoffs $\pi_i$ are equal across warlords as well as across elite members. Incorporating all decisions made in Stages 2–4, this implies the following payoff functions to violence specialists:

$$\pi_i = \left(\frac{1}{e}\right)\tau \rho(e, w)Y(\phi) \quad \forall i \in E;$$  (5)

$$\pi_j = \left(\frac{1}{w}\right)(1 - \rho(e, w))Y(\phi) \quad \forall j \in W.$$  (6)

The stability concept that we use to evaluate outcomes of the model is a simple equilibrating mechanism that equates payoffs to elite members and warlords. That is, in Stage 1 violence specialists choose the most profitable occupation. They make this choice whilst taking into account (i) the optimal tax rate that will be chosen by the elite, and (ii) the optimal response in terms of production by the representative producer. Hence, in equilibrium, payoffs to elite members and warlords are equal. If not, then a profitable switch of occupation could be made by at least one violence specialist, while taking into account that changing occupations shifts the balance of power between warlords and the elite coalition with subsequent impacts on the outcomes of Stages 2–3. This equilibrating mechanism is reminiscent of more advanced stability concepts applied in alliance models and non-cooperative models of coalition formation (cf. Skaperdas, 1998; Yi, 2003; Garfinkel, 2004).

3. Results

In this section we present the results of our model. Solving the model backwards, we analyse each of the four stages consecutively.

3.1. Production (Stage 3)

Given outcomes of Stages 1–2, the representative producer chooses $\phi$ to maximize utility (4):

$$\frac{\partial U}{\partial \phi} = (1 - \tau)\rho(e, w)\frac{\partial Y(\phi)}{\partial \phi} - \gamma = 0.$$  (7)

By the production function in (3) we have

$$\frac{\partial Y(\phi)}{\partial \phi} = \alpha \beta \phi^{\alpha - 1}.$$  (8)

Substituting this derivative into (7) and solving for $\phi$, we obtain:

$$\phi^* = \left(\frac{1 - \tau}{\beta}\right)\left(\frac{\alpha \beta}{\gamma}\right)^{1/\alpha}. $$  (9)

Substituting this equilibrium level of effort into (3) and solving for $Y$, we obtain:

$$Y^* = \beta \left(\frac{1 - \tau}{\beta}\right)\left(\frac{\alpha \beta}{\gamma}\right)^{1/\alpha}. $$  (10)

We will further assess $Y^*$ in Section 3.4.
3.2. Tax (Stage 2)

Given the outcome of Stage 1, the elite coalition chooses the tax rate $\tau$. Since violence specialists are homogeneous, there is no difference between choosing a tax rate that maximizes individual payoffs or one that maximizes the payoff to the coalition as a whole. Substituting (10) for $Y$, the coalition payoff $\pi^e$ equals $e$ times individual elite payoff (5):

$$\pi^e = e \left( \frac{1}{e} \right) \tau \rho(e, w) Y(\phi) = \tau \rho(e, w) \beta \left( 1 - \gamma \right) \frac{\alpha \beta^2}{\gamma}^{\frac{1}{1 - \gamma}}. \quad (11)$$

We can now maximize and solve for $\tau$ to find:

$$\tau^* = 1 - \alpha. \quad (12)$$

Our first result follows directly.

**Proposition 1.** In equilibrium, the optimal tax rate $\tau^*$ decreases linearly in $\alpha$, and is independent of $\beta$, $\gamma$, $\theta$, and $m$.

Given $\alpha < 1$, the equilibrium tax rate is strictly smaller than 1. The elite coalition abandons from fully taxing away its controlled production. When $\alpha$ is low the effect of the tax rate on production is small and hence, it is optimal to set a high tax rate. The opposite holds when $\alpha$ is high.

3.3. Occupation choice (Stage 1)

Each violence specialist chooses his occupation $\mu \in \{0, 1\}$ to maximize his payoff as given by (5) and (6), taking into account the effects of occupation choice on payoffs via (1) on the Stage 2 tax rate and Stage 3 production. Recall our equilibrating mechanism of equal payoffs to both occupations such that $\pi^e = \pi$ for all $i \in V$. Applying this mechanism, we equate (5) and (6) to find the equilibrium ratio of elite members to warlords, which is independent of production:

$$\frac{e}{w} = \frac{\tau \rho(e, w)}{1 - \rho(e, w)}. \quad (13)$$

By the specification of $\rho(e, w)$ in (1), the size of the elite-controlled production depends positively on the ratio of elites to warlords. Since our economy has a population of fixed size and does not allow mobility between violence specialists and producers, the number of violence specialists is also fixed at $e + w$: an increase in $e$ implies a decrease in $w$ of equal size and vice versa. We use this model feature and also substitute (1) for $\rho(e, w)$ in the equilibrium ratio (13). After simplification and substitution of (12) for $\tau$ we obtain the equilibrium elite–warlord ratio as a function of parameters $\alpha$, $m$, and $\theta$:

$$\frac{e^*}{w^*} = \left( \frac{1}{(1 - \alpha)\theta} \right)^{\frac{1}{1 - m}}. \quad (14)$$

By substituting $w^* = |V| - e^*$, we also obtain $e^*$ and $w^*$ separately:

$$e^* = \left( \frac{|V|}{\left( \frac{1}{(1 - \alpha)\theta} \right)^{\frac{1}{1 - m}} + 1} \right)^{\frac{1}{1 - m}} |V| \quad (15)$$

$$w^* = \left( \frac{1}{\left( \frac{1}{(1 - \alpha)\theta} \right)^{\frac{1}{1 - m}} + 1} \right)^{\frac{1}{1 - m}} |V|. \quad (16)$$

The elite–warlord ratio (14) increases with the tax rate. It also increases with $m$ when $(1 - \alpha)\theta > 1$ which implies $\frac{e^*}{w^*} > 1$. In that case, the elite has an advantage in generating rents as a combined effect of controlling and taxing production. Given $m < 1$, profit per occupation, in both occupations, decreases with the size of the group. Therefore, equilibrium group size can be interpreted as the relative profitability of an occupation, with the larger group having an advantage in generating rents.

For the limit case where $\theta = 1$, and since $\alpha \in (0, 1)$, the elite–warlord ratio is strictly smaller than 1, approaching unity only in the limit where the tax rate (i.e. $1 - \alpha$) approaches 1. Put differently, in absence of an elite fighting advantage, the number of elite members is never larger than the number of warlords. This cap on elite size follows from our model feature that warlords fully appropriate their controlled production, whereas elite members do not necessarily. Hence, elite members, by construction, have a disadvantage in terms of their capacity to generate rents, which can be offset only by their fighting advantage in case $\theta > 1$.

In Fig. 1 we plot (15), the equilibrium number of elite members $e^*$, and (14), the equilibrium elite–warlord ratio, as a function of $(1 - \alpha)\theta$ for different values of parameter $m$. This figure illustrates that the presence of a sizeable elite coalition in a natural state is not trivial. An (almost) empty elite coalition is possible for low values of $(1 - \alpha)\theta$. In contrast, an elite coalition that contains (nearly) all violence specialists is possible for high values of $(1 - \alpha)\theta$ and high $m$. Note that, by the exponent $\frac{1}{1 - m}$ in (15) and (16), these are limit results for $\theta\tau$ going to zero or infinity. Both $e^*$ and $w^*$ converge to, but will never reach, 0 or $|V|$, ruling out any corner solutions.

From (14) follows our next result.
Proposition 2. In equilibrium, the elite–warlord ratio $\frac{e}{w}$ is:

(i) decreasing with output elasticity $\alpha$;
(ii) independent of the technology-cost ratio $\frac{\beta}{\pi}$;
(iii) increasing with elite fighting advantage $\theta$;
(iv) decreasing with the decisiveness of group size $m$ if and only if $(1-\alpha)\theta < 1$.

Proof. The results follow directly from the relevant first order conditions to (14). □

3.4. Producer welfare (back to Stage 3)

The negative relation between output elasticity $\alpha$ and the equilibrium elite–warlord ratio naturally results in the question whether and, if so, under what conditions the representative producer benefits from the presence of a large elite coalition. Our results show three countervailing effects with respect to $\alpha$. First, there is a direct positive effect of $\alpha$ on production (see (8)). Second, there is an indirect positive effect of $\alpha$ on production via the tax rate (see Proposition 1). Third, there is an indirect negative effect of $\alpha$ on production via the elite–warlord ratio (see Proposition 2). We assess this combination of effects on production by evaluating the equilibrium production level through substitution of the equilibrium elite–warlord ratio and the equilibrium tax rate. From this equilibrium production level, we can then proceed to evaluate producer utility (4), our measure of producer welfare.

We first rewrite the CSF in (1) in terms of the elite–warlord ratio by multiplying both RHS fraction terms with $(\rho e^m)^{-1}$:

$$\rho(e, w) = \left( \frac{1}{1 + \frac{1}{\rho (\alpha \bar{\theta})^m}} \right).$$

We then proceed to substitute (14) for $\frac{e}{w}$ in order to obtain $\rho(e^*, w^*)$ in equilibrium:

$$\rho(e^*, w^*) = \left( \frac{1}{1 + \frac{(1-\alpha)}{(1-\alpha)^m}} \right).$$

Next, we substitute (12) for $r$ and (18) for $\rho(e, w)$ in the equilibrium production level (10), to obtain equilibrium production as a function of exogenous parameters only:

$$Y^* = \beta \left( \frac{\alpha^2 \beta}{Y} \right)^{\frac{m}{\alpha Y}} \left( \frac{1}{1 + \frac{(1-\alpha)}{(1-\alpha)^m}} \right)^{\frac{m}{1-\alpha}}.$$

From (19) follows our next result.
**Proposition 3.** In equilibrium, production $Y^*$ is:

(i) increasing with the technology-cost ratio $\frac{\bar{c}}{\gamma}$;
(ii) increasing with elite fighting advantage $\theta$;
(iii) decreasing with the decisiveness of group size $m$ if and only if $(1 - \alpha)\theta < 1$.

**Proof.** The results follow directly from the relevant first order conditions to (19). \(\square\)

Note that Proposition 3 does not cover the impact of $\alpha$ on optimal production. The relevant first order condition to (19) can be solved analytically using logarithmic differentiation, which yields the following expression:

$$
\frac{\partial Y^*}{\partial \alpha} = \beta \left( \frac{\alpha^2 \beta}{\gamma} \right)^{\frac{m}{\alpha}} \left( 1 - \alpha \right) \left( 2 - \frac{am}{(1 - m)^2 - \alpha \gamma (1 - \alpha) y} \right) + \ln(a^2 \beta) - \ln \left( \frac{1 + \frac{(1 - \alpha)}{(1 - \alpha) y}^{\frac{m}{\alpha}}}{(1 - \alpha)^2} \right)
$$

Clearly, this comparative static is ambiguous in sign and depends crucially on parameter values. Therefore, we evaluate (19) numerically for a wide range of values for $\beta$, $\gamma$, $\theta$, and $m$ below. First, however, a few observations on (20) can be made. The first term on the right-hand side – which is simply equilibrium output – is strictly positive and has a wide range that depends mostly on the technology-cost ratio $\frac{\bar{c}}{\gamma}$. The second term on the right-hand side of (20) features three additive elements, none of which has an unambiguous sign. The first logarithmic term is positive for sufficiently high $\beta$, while the second logarithmic term is positive – thus entering negatively in the total fraction – for sufficiently high $\gamma$. For high values of $\alpha$ and $m$, however, the first term turns negative and dominates the logarithmic terms.

To sum up, the comparative static effect of $\alpha$ on $Y^*$ is small when the technology-cost ratio is low, but can increase substantially with $\alpha$. However, it turns negative for high values of $\alpha$. Parameters $m$ and $\theta$ limit the magnitude of the comparative static effect and $m$ increases the likelihood of a negative sign as well. This suggest a hump-shaped effect of $\alpha$ that becomes more pronounced for low $m$ and $\theta$ and a high technology-cost ratio.

The numerical simulation reveals that, for most parameter combinations, production is not very sensitive to $\alpha$; the countervailing effects identified in the beginning of Section 3.4 cancel each other out. There is one exception that confirms our earlier observations. When $\beta$ is sufficiently large relative to $\gamma$ and $m$ is sufficiently small, production peaks for relatively high values of $\alpha$. Example plots for different values of $\beta$ and $m$ are provided in Fig. 2. The increase in production for sufficiently high $\alpha$ and $\beta$ is largely driven by a peak in optimal effort (9), which is subsequently offset (for even higher $\alpha$) by the effect of increasing appropriation. Fig. 2 also illustrates that for low values of $\alpha$, production may be decreasing in $\alpha$. This is the case when the optimal level of effort chosen by the representative producer is below 1.

---

To see this, note that the exponent tends to infinity for high values of $\alpha$, whereas the base of the exponent is smaller than 1 for many parameter values. The base is a fraction where the numerator has $\beta \in (0, \infty)$ multiplied by $\alpha^2 \in (0, 1)$, whereas the denominator has $\gamma \in (0, \infty)$ multiplied by a term that is strictly larger than one, although bounded by parameters $\theta$ and $m$. 

---

Fig. 2. Equilibrium production $Y^*$ as a function of $\alpha$; example plots for $\gamma = 1, \theta = 2$, and different values of parameters $\beta$ and $m$. 

\[\begin{align*}
\text{Fig. 2} & \quad \text{Equilibrium production } Y^* \text{ as a function of } \alpha; \text{ example plots for } \gamma = 1, \theta = 2, \text{ and different values of parameters } \beta \text{ and } m.
\end{align*}\]
Substituting (12) for \( r \), (18) for \( \rho(e, w) \), (19) for \( Y(\phi) \) and (9) for \( \phi \) in the representative producer’s utility function (4), we obtain, after substantial simplification:

\[
U^* = \left(1 - \alpha\right)^{\gamma/\alpha} \left(\alpha^2 \beta / \gamma\right)^{1/\gamma} \left(1 + \frac{1}{\left(\left(1 - \alpha\right) / \left(1 - \alpha \cdot \theta\right)\right)^{\gamma/m}}\right)^{1/\gamma},
\]

(21)

which is strictly positive under our parameter assumptions. Based on (21), the effects of model parameters on producer welfare are summarized in our next result.

**Proposition 4.** In equilibrium, producer utility \( U^* \) is:

(i) increasing with the technology-cost ratio \( \beta / \gamma \);
(ii) increasing with elite fighting advantage \( \theta \);
(iii) decreasing with the decisiveness of group size \( m \) if and only if \( 1 - \alpha \cdot \theta < 1 \);
(iv) hump-shaped in the output elasticity \( \alpha \).

**Proof.** The results for parts (i)–(iii) follow directly from the relevant first order conditions to (21). For part (iv), we derive the relevant first order condition to (21) using logarithmic differentiation, which yields the following expression:

\[
\frac{\partial U^*}{\partial \alpha} = \left(1 - \alpha\right)^{\gamma/\alpha} \left(\alpha^2 \beta / \gamma\right)^{1/\gamma} \left(1 + \frac{1}{\left(\left(1 - \alpha\right) / \left(1 - \alpha \cdot \theta\right)\right)^{\gamma/m}}\right)^{1/\gamma} \left(\frac{1 + ((1 - \alpha) / \theta)^{\gamma/m} + \frac{\alpha}{\alpha(1 - \alpha)}}{\alpha(1 - \alpha)((1 - \alpha) / \theta)^{\gamma/m} + (1 - \alpha)}\right).
\]

(22)

All terms in this expression are positive, except for \( \alpha/(m - 1) \theta \) in the numerator. As a result, negative values can only be obtained for high \( \alpha \) (relative to \( \theta \)), which implies a hump-shaped curve. \( \square \)

To illustrate the hump-shaped relation between \( \alpha \) and \( U^* \), we evaluated (21) numerically for a wide range of values for \( \beta \), \( \gamma \), \( \theta \), and \( m \). Example plots for different values of \( \beta \) and \( m \) are provided in Fig. 3, which is directly comparable to Fig. 2. This figure illustrates the combination of direct and indirect effects of \( \alpha \) on utility – as discussed in the beginning of this section. Utility is low for both low \( \alpha \) and, perhaps surprisingly, high \( \alpha \). Maximal welfare levels are reached for intermediate values of \( \alpha \). The explanation for this shape is largely found in the level of the appropriation rate and the return to investments in effort. The appropriation rate equals the sum of taxation by elite and full rent extraction by warlords. It can be expressed as \( \rho(e, w')(1 - \alpha) + (1 - \rho(e, w)) = 1 - \alpha \rho(e, w) \). From this expression follows immediately that the appropriation rate tends to 1 for low \( \alpha \). Also, by (18) we know that \( \rho(e^*, w^*) \) is decreasing in \( \alpha \), which implies increasing rent extraction by warlords. Jointly, these effects make that the appropriation rate follows a \( U \)-shape with minimal appropriation for intermediate levels of \( \alpha \).

All in all, producers are best off with intermediate levels of \( \alpha \), where their production peaks through substantial investments in effort and the level of appropriation is still relatively low. Combined with low \( m \) and substantial \( \theta \), the elite coalition is not very large (see Fig. 1), but is able to control the bulk of production. Lower levels of \( \alpha \) would decrease the return to investments in effort – e.g. higher tax and lower elasticity of output, while higher levels of \( \alpha \) would lead to increased appropriation by warlords. Both effects are detrimental to producer utility.
4. Extensions

In this section we consider three extension to our model. So far, we have assumed a frictionless elite coalition which stands in sharp contrast to some of the real world examples that were referred to in the introduction. Two main sources of friction within the elite coalition are the distribution of the collectively generated tax rent, and possible barriers to entry for new members. These are just two possible examples of frictions within the elite coalition, but they are important ones and discussed in the first two extensions. In Section 4.3 we endogenize the size of the group of violence specialists, dropping our assumption that violence specialists cannot switch to become producers – while maintaining that mobility in the opposite direction is restricted in natural states.

4.1. Within-coalition conflict

In the natural state, the elite emerges out of the pool of violence specialists, and cooperation of violence specialists in the elite coalition is not self-evident. Given the outcome of Stages 1–3, members of the elite coalition may engage in conflict over the tax rent. There are various ways to model such conflict and one could even argue that an appropriately designed sharing rule or voting procedure could eliminate the incentive for rent-seeking within the coalition. Indeed, the mere possibility of within-coalition conflict can be interpreted as characterizing an earlier state of development than the setting of our base model, where within-coalition conflict is absent by construction. One could, in the spirit of North et al. (2009), argue that the absence of conflict within the coalition is a result of a more developed ‘rule-of-law’ among the elite.9

We proceed with a simple conflict model that we include as Stage 4 of our model. This simple set-up is sufficient to demonstrate the impact of the prospect of conflict on elite size. We do so using a ratio-form CSF, similar to (1). As with (1), the CSF can be interpreted as probabilistic or, assuming risk-neutrality, as defining shares to each elite member. Here we stick to the non-probabilistic interpretation. Each elite member receives a share \( \sigma_i(s) \) of the tax rent, which depends on costly investments in conflict by all members, captured in the vector \( s = (s_i : i \in E) \):

\[
\sigma_i(s) = \frac{s_i^n}{\sum_{j \in E} s_j^n},
\]

with \( n \in (0, 1) \) being the decisiveness parameter for within-coalition conflict.

We update the payoff function (5) to elites:

\[
\pi_i = \sigma_i(s) \tau_p(e, w) Y(\phi) - s_i \quad \forall i \in E.
\]

(24)

Now, in Stage 1 violence specialists choose the most profitable occupation whilst taking into account not only the optimal tax rate and production, but also the severity of conflict within the coalition. Note that the combination of Stage 1 and Stage 4 resembles models of sequential inter- and intra-group resource contest (Wärneryd, 1998; Esteban and Sákovics, 2003; Garfinkel, 2004; Inderst et al., 2007). We add to this the interaction between violence specialists and producers.

Given outcomes of Stages 1–3, each elite member chooses \( s_i \) to maximize his payoff as given by (24):

\[
\frac{\partial \pi_i}{\partial s_i} = \frac{\partial \sigma_i(s)}{\partial s_i} \tau_p(e, w) Y(\phi) - 1 = 0 \quad \forall i \in E.
\]

(25)

Note that we exclude the peaceful outcome where \( s_i = 0 \) for each agent. Such a peaceful outcome cannot be an equilibrium to the conflict since one elite member \( j \) could secure the complete resource with a small investment in conflict \( s_j > 0 \) (Garfinkel and Skaperdas, 2007). This opportunity would not be left unexploited in equilibrium, which is why we exclude it from our analysis.

By (23) we have:

\[
\frac{\partial \sigma_i(s)}{\partial s_i} = \frac{ns_i^{n-1} \sum_{j \in E \setminus \{i\}} s_j^n}{\left( \sum_{j \in E} s_j^n \right)^2}.
\]

(26)

With homogeneous violence specialists, such that \( s_i = s \) for each \( i \in E \), we can simplify this derivative to

\[
\frac{\partial \sigma_i(s)}{\partial s_i} = \frac{n(e - 1)}{se^2}.
\]

(27)

Substituting this simplified derivative into (25) and solving for \( s \), we obtain:

\[
s = \left( \frac{e - 1}{e^2} \right) n \tau_p(e, w) Y(\phi).
\]

(28)

---

9 We thank a reviewer for making this observation.
Note that we cannot proceed by substituting our previously found equilibrium values, since these may be affected by the prospect of within-coalition conflict on Stage 1–3 decisions. Specifically, this prospect affects the elite–warlord ratio. It does not affect our result on the optimal tax rate and it also does not affect our results on optimal production or producer welfare, except through this ratio. To show the effect of within-coalition conflict on $\frac{e}{w}$, we equate (24) with (6) and we substitute (28) for $s_i$, to obtain:

$$
\frac{1 - n + n/e}{e} \tau \rho(e, w) = \frac{1}{w} (1 - \rho(e, w)).
$$

(29)

We can now rearrange terms to find the equilibrium elite–warlord ratio, which is again independent from production $Y$:

$$
\frac{e}{w} = \frac{1 - n + n/e}{1 - \rho(e, w)} \tau \rho(e, w).
$$

(30)

We substitute (1) for $\rho(e, w)$ and (10) for $\tau$ in the equilibrium ratio (30). After simplification we obtain the following elite–warlord ratio as a function of exogenous parameters and $e$:

$$
\frac{e}{w} = \left(\frac{1 - n + n/e}{1 - (1 - m)(1 - \alpha)}\right)^{\frac{1}{1 - \alpha}}.
$$

(31)

Comparing (30) with the related ratio in the standard version of our model (14), we see that an additional term $(1 - n + n/e)$ has entered the solution. This term is driven by the anticipation of conflict in the fourth stage of the model (recall $n$ is the decisiveness parameter for within-coalition conflict). For $n \to 0$, the effect of Stage 4 within-coalition conflict on Stage 1 occupation choice vanishes; the ratio $\frac{e}{w}$ converges to the ratio (14) of the standard version of our model.

We continue to assess the effect of within-coalition conflict on $\frac{e}{w}$ by substituting $|V| - e$ for $w$ in (31) and using implicit differentiation:

$$
\frac{d}{dt}e^* = \frac{(e - e^2)(|V| - e)^2}{n(|V| - e^2)^2 + (1 - m)(|V|e^2)(1 - n + n/e)^{\alpha-1}((1 - \alpha)\theta)^{-1}} < 0.
$$

(32)

All terms of (32) are positive, with the exception of the term $(e - e^2)$ in the numerator. Hence, $\frac{d}{dt}e^* < 0$. As a result, we find that the elite–warlord ratio $\frac{e^*}{w}$ is decreasing with the decisiveness of within–coalition conflict $n$. To illustrate this result we reproduce Fig. 1 (which features no within–coalition fighting, hence $n = 0$), for the case where $n = 1$. Values for $e^*$ and $\frac{e^*}{w}$ in Fig. 4 are computed by solving (31) numerically for $n = 1$ using the Newton–Raphson method. Comparison of both figures shows that the limit case of the model with $n = 1$ implies a substantially smaller elite coalition in equilibrium. Choosing the elite occupation has become less attractive compared to the standard version of our model, because of the prospect of within–coalition conflict. Obviously, this difference in elite size is mitigated if we allow for the possibility that warlords also engage in a conflict over their appropriative rents. Such warlord conflict would obviously decrease warlord payoffs. The combined effect of warlord conflict and within–coalition conflict on elite size will depend on the relative decisiveness of both conflicts. Note that we will need much of the above derivations to introduce our results in the remainder of this section, which do not depend on the existence or not of warlord conflict.

Our analysis up to here allows us to assess the impact of the number of contestants on investments in conflict. A standard result from rent–seeking models (cf. Garfinkel and Skaperdas, 2007) is that an increase in the number of contestants decreases individual investments in conflict. The intuition for this result is that in the presence of more competitors the expected return to investments in conflict decreases. In the context of our paper, this result would imply that the effect of an increase of $e$ on $s^*$ is negative. In conflict models with endogenous production, however, this result is reversed (Hirschleifer, 1995): an increase in the number of contestants increases individual investments in conflict. The intuition is that, as the number of contestants increases, a smaller fraction of own production can be retained and hence investments in conflict become more
attractive. Our next proposition shows that in the setting of our paper, where production is the domain of a separate subset of producers and subject to interaction with violence specialists, both results are combined. Specifically, an increase in the size of the elite coalition decreases the aggregate level of appropriation which provides incentives to increase production. Increased production, in turn, provides incentives to increase investments in conflict. This indirect positive effect may offset the direct negative effect of \( e \) on \( s' \).

**Proposition 5.** In equilibrium, within-coalition investments in costly conflict \( s' \) are hump-shaped in the size of the elite coalition \( e \) with a global maximum \( \hat{e} \in \mathbb{R} : \hat{e} = \frac{\hat{Z}(\hat{e})}{\partial s/\partial e} \), where \( \hat{Z}(e) = \frac{m}{1 - \rho(e, w)}(1 - \rho(e, w)) \).

**Proof.** Using (28), we first derive the first order condition \( \partial s/ \partial e \) and solve for \( e \) which yields the implicit function

\[
e = \frac{2(1 - \alpha)\left(\rho w^m + w^m\right) - mw^m}{(1 - \alpha)\left(\rho w^m + w^m\right) - mw^m} = \frac{2 - \hat{Z}(e)}{1 - \hat{Z}(e)}.
\]

where \( \hat{Z}(e) = \frac{m}{1 - \rho(e, w)}(1 - \rho(e, w)) \). The second derivative of (28) with respect to \( e \) is rather involved, so we confirmed concavity of \( \partial s/ \partial e \) numerically. Given the domains of parameters \( \alpha \) and \( m \) and the function \( \rho(e, w) \), we have that \( Z(e) > 0 \). By (33), \( Z(e) > 0 \) implies that \( \hat{e} \) is unbounded (e.g. consider cases with \( \alpha \) and \( m \) such that \( Z(e) \) is close to unity).

To gain further insights into the effect of \( e \) on \( s' \), notice that \( e \) not only affects \( s' \) directly but also through \( \rho(e, w) \) and \( \Omega(\phi) \), where \( \phi \) depends on \( e \) through \( \rho(e, w) \). To evaluate these effects separately we take the total derivative of (28) to \( e \) (to reduce notational clutter we write \( \rho(e, w) \) as \( \rho \) and \( \Omega(\phi) \) as \( \Omega \)):

\[
\frac{ds}{de} = \frac{\partial s}{\partial e} + \frac{\partial s}{\partial \rho} \frac{d\rho}{de} + \frac{\partial s}{\partial \Omega} \frac{d\rho}{d\Omega} \frac{d\rho}{de}.
\]

We find that these three effects are given by:

\[
\frac{\partial s}{\partial e} = \left[ -\left( \frac{e - 2}{e^2} \right) m \right];
\]

\[
\frac{\partial s}{\partial \rho} \frac{d\rho}{de} = \left[ \left( \frac{e - 1}{e^2} \right) m \right] \times \rho(1 - \rho) \left( \frac{m}{e} \right);
\]

\[
\frac{\partial s}{\partial \Omega} \frac{d\rho}{d\Omega} \frac{d\rho}{de} = \left[ \left( \frac{e - 1}{e^2} \right) m \right] \times \rho(1 - \rho) \left( \frac{m}{e} \right) \times \left( \frac{\alpha}{1 - \alpha} \right) \beta \left( (1 - \tau) \rho \frac{\alpha \beta}{\gamma} \right)^{\frac{\alpha - 1}{\beta}}.
\]

The direct effect of an increase of \( e \) on \( s' \) in (35) is negative (as long as \( e > 2 \)), while both indirect effects in (36) and (37) are positive for \( e > 1 \). An increase in \( e \) increases both the share of production controlled by elite members as well as (indirectly) the production level. Both of these contribute to a higher tax rent, which makes fighting more attractive. Depending on parameter values the positive or negative effect dominates as stated in the proposition. \( \square \)

4.2. Limiting access to the coalition

In this extension, we assess whether elite members have an incentive to limit entry into the coalition in order to avoid dilution of the tax rent. It seems reasonable to assume that access into the coalition should require consent of the coalition members. In the literature on coalition formation in games with externalities it is shown that the rules of coalition formation may impact the coalition size. Specifically, requiring consent on membership – called ‘exclusive membership’ in this literature – is one such rule. In a negative externality game (as we have here), it normally implies smaller coalitions with higher payoffs per member compared to ‘open membership’ (Yi, 1997).

Below we will illustrate that this result does not necessarily hold in our setting. We do not impose a specific membership rule but rather compare the impact of exogenous coalition size on individual elite member payoffs, and subsequently compare these payoffs to those under the endogenously determined coalition size derived in (15). Doing so, we find that elite payoffs are not necessarily maximized at the endogenously determined coalition size (15). Instead, members of the elite coalition may prefer an alternative elite size, which may be smaller or larger, depending on parameter combinations in the model. The rationale behind this result follows form our equilibrium mechanism. The elite coalition cannot attract more (less) violence specialists because in equilibrium the payoff to the marginal warlord is even higher (lower) than that of the elite members. This result is illustrated in Fig. 5 for different parameter combinations of \( \alpha \) and \( m \).

Fig. 5 shows the elite payoff function (5). Substitute (12) for \( \tau \) and (10) for \( \Omega(\phi) \) and rearrange to obtain elite payoffs as a function of exogenous parameters and \( e \):

\[
\pi_l = \left( \frac{(\rho(e, w))^{1 - \alpha}}{e} \right) (1 - \alpha) \beta \left( \frac{\alpha^2 \beta}{\gamma} \right)^{\frac{\alpha - 1}{\beta}}.
\]
Only the first RHS term of (38) depends on \( e \) and only this term was used to construct Fig. 5, the other terms being constant for given parameter values. The figure shows individual elite payoffs as a function of elite coalition size \( e \) for three values of \( m \). It also displays, for each \( m \), the equilibrium coalition size as the outcome of our model. Clearly, elite members could benefit from a change in the size of the elite coalition. For \( m = 0.3 \), elite members would benefit from a decrease in elite size. Conversely, for \( m = 0.7 \), elite members would benefit from an increase in elite size. For \( m = 0.5 \), they are largely indifferent. Only when elite members prefer a smaller coalition would they benefit from implementing exclusive membership. When they prefer a larger coalition, then apparently the equilibrium payoffs to warlords are sufficiently large to keep them out of the coalition; no warlord has an incentive to switch and become a member. In such a setting, exclusive membership would not affect coalition size.

The dependence of preferred elite size on \( m \) is due to the extent of diminishing marginal returns to group formation for any \( m < 1 \). If \( m \) is low, group size becomes less relevant for the share of production controlled by the elite than if \( m \) is large. Hence, for low \( m \), a smaller elite coalition would increase the tax rent per elite member. The opposite effect holds for high \( m \).

4.3. Endogenous exit

One question that may arise is how the profitability of warlordism and elite membership affects the equilibrium number of violence specialists. As a final extension, we scrutinize this question by endogenizing the number of violence specialists, allowing them to switch and become a producer themselves.\(^{10}\) Recall our argument in Section 1 that entry of violence specialists is exceptional for a variety of reasons while exit is possible though in principle not attractive. In assessing the impact of possible exit on model outcomes, we assume that the exiting violence specialist will earn a payoff equal to the representative producer. The alternative approach is to drop the assumption of a representative producer and, instead, explicitly model the size of the producing population. Both approaches yield qualitatively similar results, but one advantage of the current approach is that we need not make explicit assumptions on the size of the producing population. We assume that the number of violence specialists is small compared to the producing population, such that any switch will not affect aggregate production. As before, violence specialists choose their preferred occupation, elite member or warlord, but now the third option is to exit and become a producer. To assess the resulting equilibrium, we equate violence specialists’ payoff from (5) and (6) with the representative producer’s utility (4). In doing so, we account for the cost of maintaining a patronage network by subtracting cost parameter \( \eta \) from the specialists’ payoffs. The resulting functions, with superscript \( x \) indicating the possibility of exit, are:

\[
\pi_i^x = \pi_i - \eta \quad \forall i \in E;
\]

\[
\pi_j^x = \pi_j - \eta \quad \forall j \in W;
\]

\[
\pi_k^x = U \quad \forall k \notin E \cup W.
\]

An important observation is that these affine transformations of payoff functions do not affect the choice of effort \( \phi \) nor tax rate \( r \) in the base model of Section 2. In other words, for given parameter values, endogenizing the number of violence specialists does not change our model results. As a result, both equilibrium production \( \hat{Y} \) and the equilibrium elite–warlord ratio \( \hat{\rho} \) remain unchanged, which also fixes the controlled share \( \rho(e^*, w^*) \) and, finally, the representative producer’s utility

\(^{10}\) There is also a literature on endogenous alliance or coalition formation wherein multiple and competing coalitions can emerge. In our interpretation of the elite coalition as the state such an alternative coalition would in principle form an additional state. Co-existence of multiple coalitions is analysed by a.o. Skaperdas (1998) and Garfinkel (2004).
The only important question that remains is how the total number of violence specialists may be affected by key model parameters.

Consider a violence specialist that switches to become a producer. For illustrative purposes, assume this specialist belongs to the elite coalition. The exit implies that in the new situation we have $e^* = e - 1$. The new producer will earn the representative producer’s utility. Since production is not affected, by (41) each elite member will have a higher payoff as they can now share their appropriated production, whose level remains unchanged, among a smaller number of elite members; see (5) and (6) upon which (39) and (40) are based. The same result follows for warlords, since the equilibrium elite–warlord ratio remains unchanged and hence, in equilibrium, the number of warlords will adjust downward.

The equilibrium numbers of elite members and warlords can be derived by equating payoffs (39) and (40). We did so already for the ratio of elite members to warlords in (14). Next, we equate payoffs to elite members (39) and the representative producer (41), which yields the following implicit expressions for both $e^*$ and $w^*$ using (14) − $w^*$:

$$e^* = \frac{(1 - \alpha)\rho(e^*, w^*)Y^*}{(1 - \alpha)\rho(e^*, w^*)Y^* + \eta};$$

$$w^* = \frac{(1 - \alpha)\rho(e^*, w^*)Y^*}{\left(\frac{1}{(1 - \alpha)\theta} \right) \left(\frac{1}{\eta}\right)};$$

where $\rho(e^*, w^*)$ and $Y^*$ take their equilibrium values from (18) and (19). Combining (14) with (42), we can calculate the numbers of elite members and warlords under endogenous exit. As an illustration we limit ourselves to assessing the impact of two parameters on these numbers: patronage costs $\eta$ and cooperative quality $\theta$. Starting with $\eta$, since all terms in (42) and (43) are positive, we find that $\frac{\partial e^*}{\partial \eta} > 0$ and $\frac{\partial w^*}{\partial \eta} < 0$. Given the unchanged equilibrium elite–warlord ratio from (14), this implies that both the number of elite members and warlords decrease in patronage costs. The impact of $\theta$ is more nuanced, because it affects the equilibrium elite–warlord ratio. Since both $\frac{\partial e^*}{\partial \theta} > 0$ and $\frac{\partial w^*}{\partial \theta} > 0$, we have that $\frac{\partial e^*}{\partial \eta} > 0$. The number of elite members increases with cooperative quality $\theta$. The number of warlords increases with $\theta$ too, but only for sufficiently high $\alpha$ and low $\theta$; it decreases otherwise because the second term in (43) decreases in $\theta$.

The effects of endogenous exit are illustrated in Fig. 6 which displays the impact of patronage costs $\eta$ as well as cooperative quality $\theta$ on the equilibrium number of elite members and warlords. As shown, when $\eta$ increases it becomes more attractive for violence specialists to exit and become a producer. Note that the equilibrium elite–warlord ratio, equal to 1/4 for the parameter values in Fig. 6, is independent of $\eta$. When $\theta$ increases, the elite coalition grows, but the impact on the number of warlords is ambiguous. The combined effect is that the number of violence specialists is first increasing, then decreasing in $\theta$.

5. Discussion and conclusion

In this paper we provide economic intuition for the natural state by analysing the role of organized large-scale violence and how it is contained. We start from the observation that natural states lack a monopoly on violence concentrated in ‘the state’, but assert that the capacity for large-scale violence is concentrated in the hands of a small subset of the population that we term ‘violence specialists’ (North et al., 2009). These violence specialists choose between exploiting their violence capacities for pure appropriative purposes or to impose a degree of order, where we refer to the former category as ‘warlords’ and the latter as ‘elites’. The elite join forces in a coalition of equals where they agree to respect each other’s privileges and rents, and to jointly fight off warlords. This relative security in turn allows the elite coalition to incorporate the consequences of their appropriative behaviour on total production in their appropriation rate − resembling the behaviour of stationary bandits in the fashion of McGuire and Olson (1996) and Usher (1989). Production is generated by a representative producer that responds to appropriation by limiting its investment in effort − the only variable input in our production function. In contrast,
warlords operate alone and face continuous pressure on their rents from other violence specialists, which induces them to fully appropriate all production they control. The share of production controlled by warlords and elite members is determined by a deterministic standard contest success function, featuring diminishing returns in group size and a cooperative advantage to the elite coalition relative to warlords (Garfinkel and Skaperdas, 2007; Konrad, 2009). As a result, warlords have a natural advantage in appropriation rates over elite members, whereas elite members have a contest-advantage over warlords.

Our model illustrates the rigidity of natural states. We find that elite members are responsive to the economic consequences of their appropriation on production, thus restricting their tax rate, and more so the higher the output elasticity of effort. However, lower tax rates imposed by the elite increase the appropriative advantage of warlords over elite members. As a result, more violence specialists will opt to become warlord, in turn increasing the general appropriation rate imposed on the violence specialists at large, and at least partially offsetting the elite members’ appropriation decline following limited taxation. Due to this trade-off, our results indicate that the total appropriation rate of warlords and elite members combined is rather stable for most parameter combinations. Only when the output elasticity of effort is high – inducing low tax rates – combined with strong cooperative quality of the elite and a low decisiveness of conflict – allowing the relatively small elite coalition to control a relatively large share of total production – do we find comparatively high levels of production and producer welfare. These results point to a delicate balance between welfare and order as also observed by Bates et al. (2002).

Dal Bó et al. (2015) analyse this balance as a pre-institutional process, arguing that institutions play no role in explaining different outcomes across states. Their setting is different – i.e. a monolithic incumbent with given defence and growth capabilities owns a productive asset across the states and is challenged by a predatory competitor – and we place our analysis in the specific institutional context of a natural state, but their conclusion is similar to ours: order is a necessary condition in stimulating production and thus welfare, but welfare, in turn, decreases the likelihood of order. Like Dal Bó et al. (2015), our results challenge existing explanations for welfare that focus on the intentional design of institutions by the elite fostering or limiting production and appropriation (cf. Grossman, 2002; Acemoglu et al., 2005).

Thus, our model is capable to answer, at least partially, why societies with an extractive elite emerge and persist. The traditional answer is that the elite is better off in an extractive and exclusive regime and powerful enough to maintain it (Sokoloff and Engerman, 2000; Grossman, 2002; Acemoglu and Robinson, 2008). In contrast, we start from the assertion by North et al. (2009) that violence is an endemic threat to the stability of societies, arguing that a society with a small and extractive elite coalition is the natural social order since it guarantees a certain degree of order and stability, through a system of coercive rent extraction rather than by unconstrained, and potentially violent, appropriation. To this, we add that violence specialists have a potentially lucrative alternative option when they operate as warlords outside – and in competition with – the elite coalition. We view this as a permanent situation where the fragile elite coalition forms the de facto state while many violence specialists remain tempted by the warlord-option. Thus, even though incorporating a broad range of violence specialists into the elite coalition may be optimal for the leaders of the elite coalition – as is convincingly shown by Francois et al. (2015) – and for society at large, an elite coalition that completely eradicates warlords is unlikely in natural states.

In our extensions we further probe the relation between the average payoff for elite members and the size of the coalition. There are two alternative results put forward in the literature. Francois et al. (2015) argue that increasing the size of the elite coalition generally increases the average payoff to elite members, because the marginal elite member requires the highest reward for joining. In contrast, North et al. (2009) suggest that the elite coalition generally has an incentive to restrict its own size because this increases the average payoff per elite member. We provide two further insight into these results. First, in Section 4.2 we show that the relation between the average payoff per elite member and the size of the elite coalition is ambiguous, and crucially depends on the decisiveness of group size in conflict. When the decisiveness of group size is low (high) the endogenous ratio of elite members to warlords is too high (low) for the average elite member. That is the average elite might be better off limiting the number of coalition members when decisiveness is low. A reduction in the number of elite is problematic, because the option of becoming warlord at such an equilibrium is even less lucrative such that no elite members will willingly leave the coalition. Our second insight follows from the introduction of distributive conflict in the elite coalition discussed in Section 4.1. While a standard result in the literature on conflict and appropriation is that investment in conflict decreases with the number of contenders (Garfinkel and Skaperdas, 2007), we show that this effect within the coalition is mitigated by the fact that an increase in the size of the coalition results in an increase of the contested resource pool – i.e. output under control of the elite coalition. This in turn stimulates investment in conflict. Which effect dominates depends on parameter values.

Our results imply that almost all natural states experience continuous coercion exercised by elite members and violence between elite coalitions and warlords, as is most conspicuously observed in parts of Sub-Saharan Africa. We show that this is not a temporary out-of-equilibrium-situation but a permanent phenomenon. There is an inherent tendency in natural states to have rebellions and competing factions, not necessarily because of ethnic or religious rivalries, ideological struggles or social injustice, but because of the fundamental economic mechanisms of the natural state resulting in numerous violence specialists opting for the ‘warlord-option’ (see for the role of economic factors on rebellions and civil strife also Collier and Hoeffler, 2004; Blattman and Miguel, 2010; van der Ploeg, 2011). Our model provides insights on how these economic mechanisms work. Specifically, that we show that unrest is exacerbated when the output elasticity of effort is high, because it induces elite members to limit their tax rate, which in turn results in a strong tendency towards warlordism. We have also shown that this mechanism is partly mitigated by the cooperative quality of the elite coalition for two reasons. First, its cooperative quality allows a coalition of given size to control more production, inducing more violence specialists to join to
elite coalition, further increasing its control over production. Second, in our last extension, we show that increases in the cooperative quality may even induce some warlords to abandon their capacities as violence specialist completely and turn to production instead.

More tentatively, our results on the interaction between order and production carry implications for the resource-curse literature that suggests that institutional quality determines to what extent natural resources are a curse – when institutions are inferior – or a blessing (Bulte et al., 2005; van der Ploeg, 2011). Output elasticity can be related to the availability of rich natural resources. Since the natural state can be equated with the weak or inferior institutions of the resource curse literature, this result underpins Bulte et al. (2005) and Mehlum et al. (2006) and others in showing that the effect of abundant resources on economic development is mediated by the quality of the institutional framework. Furthermore, the cooperative quality of the elite coalition can be related to the development of a rule-of-law among elite members and the development of organizations within the coalition, as discussed in North et al. (2009) as part of the maturation process of natural states; we have probed this maturation process elsewhere using historical case material (van Bavel et al., 2016). Indeed, our results suggest that enhancing institutional quality mitigates the resource curse. However, the third condition required to obtain comparatively high production in our model is that the decisiveness of conflict is low, which seems at odds with the availability of natural resources. Especially, natural resources can be geographically concentrated – often referred to as point-based natural resources – which brings about strong benefits for the party that controls the particularly endowed geographical areas. Consequently, we suggest that the point-based nature of natural resources increases the decisiveness of conflict and, thus, forms an important determinant of the resource curse. The more concentrated natural resources are in specific geographic areas, as is the case with many mineral resources in Sub-Saharan Africa, the more decisive conflict is, and the more unlikely a state is to escape the resource curse (Bulte et al., 2005; van der Ploeg, 2011).

Using the model developed in this paper, there are several promising avenues for further research. One concerns the spatial dimension of production and conflict in natural state, including the point-based nature of some natural resources, migratory behaviour of producers in the face of coercion and violence, and the spread of violence and coercion itself. The second concerns the formation of patronage networks and potential heterogeneity across violence specialists. The third concerns the link between the current paper’s insights and the big question of how to escape the situation of violence and coercion in natural states towards open democracies. Our results suggest that such an escape may be even more problematic than conventionally thought, because of the economic mechanisms underpinning the rigidity of natural states.

References


