



The measurement of environmental economic inefficiency with pollution-generating technologies

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ARTICLE INFO

Article history:

Received 1 July 2019

Received in revised form 9 April 2020

Accepted 8 June 2020

Available online 20 June 2020

JEL classification:

D20

D24

N52

Q50

Keywords:

Environmental economic inefficiency

Pollution-generating technologies

Technical and allocative efficiency measurement

Data envelopment analysis

US agriculture

ABSTRACT

This study introduces the measurement of environmental inefficiency from an economic perspective. We develop our proposal using the latest by-production models that consider two separate and parallel technologies: a standard technology generating good outputs, and a polluting technology for the by-production of bad outputs. While research into environmental inefficiency incorporating undesirable or bad outputs from a technological perspective is well established, no significant attempts have been made to extend it to the economic sphere. Based on the definition of net profits, we develop an economic inefficiency measure that accounts for suboptimal behavior in the form of foregone private revenue and environmental cost excess. We show that economic inefficiency can be consistently decomposed according to technical and allocative criteria, considering the two separate technologies and market prices, respectively. We illustrate the empirical implementation of our approach using a dataset on agriculture at the level of US states.

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1. Introduction

Measuring the environmental inefficiency of production units is an increasingly important topic of recent economic research. Environmental inefficiency assessment integrates marketed (desirable, intended, or good) outputs with negative environmental externalities into production modeling (the production of so-called undesirable, unintended, detrimental, or bad outputs). Such analysis is important from the perspective of sustainable production because it provides valuable insights for firms and industry stakeholders on how to adopt environmentally friendly strategies, and for policy makers to improve the design of pollutant-abatement instruments, accounting for environmental challenges.

However, the existing environmental efficiency models lack the economic inefficiency dimension of the analysis; that is, a comprehensive measure that also considers the foregone profits, in the form of lower revenues and/or higher costs, that are not only related to a technological inefficient behavior, but also to allocative inefficiency. This implies that firms should not only pursue being technically efficient by exploiting the potential of the production frontier, but they should also

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use the optimal amounts of (good and bad) outputs and inputs consistent with profit maximization (extensible to revenue maximization and/or cost minimization); i.e., they should be allocative efficient by supplying and demanding the optimal bundles (or mixes) of output and inputs.

Following the literature on economic efficiency measurement and its decomposition into technical and allocative components initiated by Farrell (1957), we bring this theoretical framework to the field of environmental economics. In this framework, economic efficiency analysis not only considers the production of good outputs (associated to private revenue), but must also account for the economic cost of producing bad outputs, which is proxied through the so-called damage cost functions; and since bad outputs are externalities, they are also related in the literature to social cost functions, e.g., see Bretschger and Pattakou (2019). From the perspective of the firm, the possibility of simultaneously increasing market goods while reducing environmentally damaging outputs is economically appealing. On one hand, there is a clear (private) incentive to increase revenue, but also current sustainability and corporate social responsibility concerns are increasingly brought into firms' decision making. This means that inefficient firms, falling short from the best practice frontier, can improve its environmental efficiency at no private cost, and therefore it is possible to reduce the externalities caused by bad outputs by matching best practice standards of efficiency. Nowadays firms routinely advertise environmental achievements in their annual reports; e.g., for the airline industry the reduction of the carbon footprint is becoming increasingly important.

From the perspective of environmental economics, the above framework can be related to the notion of net profits (or net revenues if we consider productive inputs as given, as we do for simplicity in our model). This means that the objective function of the firm corresponds to the maximum of its market revenues net of the environmental costs that it causes through the inevitable by-production of bad outputs (i.e., the materials balance principle associated to the first law of thermodynamics). Of course, the reduction of bad outputs entails abatement costs if the firm is efficient, thereby producing at the technological frontier (and this is reflected by the substitutability characteristics of the technology between the good and the bad outputs, see Murty et al., 2012). But if the firm is technically inefficient, both good and bad outputs can be freely increased and reduced, respectively. Consequently, inefficient firms can increase private revenue while reducing environmental damage. In sum, the environmental economic efficiency model that we proposed in the vein of Farrell (1957) extends the existing technological models for environmental efficiency measurement to account for these economic dimensions by postulating an objective function that aims at maximizing private revenue net of the environmental damage.

The determination of economic efficiency is important from a managerial standpoint focused on market-oriented performance, but also for other stakeholders and decisions makers such as, for example, politicians, local and state governments, or regulators (e.g., environmental protection agencies). For example, managers are interested in increasing performance not only in physical terms by taking advantage of the best technology available, but also by realizing the economic gains associated with allocative efficiency improvements; that is, the choice of optimal output and input mixes, leading to either maximum profit, revenue or minimum cost. The information on allocative efficiency that our economic model yields is also relevant for the above economic agents as improving allocative inefficiency is arguably cheaper and easier for firms to achieve than improving their technical inefficiency. In this sense, being aware of the level of this inefficiency and related potential for revenue increases or cost savings enables firms to "reap a low-hanging fruit". For regulators minimizing the environmental cost of production from an allocative perspective is also critical, as the cost of carbon dioxide emissions (e.g., related to respiratory illnesses) could be lower than those associated to the use of pesticides (e.g., related to cancer treatments). How the polluting inputs causing both damages should be regulated in terms of their economic costs can now be addressed thanks to our new framework.

Considering only a technological perspective, the literature on modeling production technologies that account for bad outputs has developed following two approaches mainly: one involving parametric methods (such as stochastic frontier analysis, SFA; Aigner et al., 1977), and one based on nonparametric methods (such as data envelopment analysis, DEA; Charnes et al., 1978; Banker et al., 1984). Common to both methods, many different approaches have been proposed to assess environmental efficiency of production units. Lauwers (2009) classified these approaches into three groups. The first group concerns environmentally adjusted production efficiency models, in which undesirable outputs are incorporated into the production technology. In general, two main branches of studies within this group can be distinguished: (i) treating bad outputs as strong (free) disposable inputs (Haynes et al., 1993; Hailu and Veeman, 2001) or (ii) treating bad outputs as weekly disposable outputs and assuming the null-jointness of both bad and good outputs (Färe et al., 1986, 1989). The second group of studies consists of frontier eco-efficiency models (Korhonen and Luptacik, 2004; Kuosmanen and Kortelainen, 2005), which do not follow axiomatic production efficiency frameworks, but relate aggregate ecological outcomes with economic outcomes only. In other words, eco-efficiency is measured either through minimization of environmental outcomes given economic outcomes (e.g., value added) or the alternative maximization of economic outcomes given the environmental outcomes. The third group of studies is based on the introduction of the materials balance principle into production models (Lauwers and Van Huylenbroeck, 2003; Coelli et al., 2007; Førsund, 2009). The materials balance principle states that flows into and out of the environment are equal, linking the raw materials used in the production system to outputs, both intended and residual ones.

Dakpo et al.'s (2016) recent survey of environmental efficiency studies extended the Lauwers (2009) classification into the fourth, most recent, category of by-production models, which are based on the idea of defining two subtechnologies in parallel: one that generates good outputs and a second that generates bad outputs. This approach was introduced by Murty et al. (2012) and, as a consistent and relatively new approach, its empirical applications are flourishing (e.g., Dakpo et al., 2017; Arjomandi et al., 2018; Ray et al., 2018), as are its extensions (e.g., Serra et al., 2014; Lozano, 2015; Dakpo, 2016;

Førsund, 2018). We rely on the novel by-production approach to introduce our economic environmental efficiency model. Nevertheless, it could be easily particularized for previous approaches.¹ We also consider recent qualifications of the original by-production model by Dakpo (2016) and Førsund (2018).²

As anticipated, regardless the modeling approach—parametric or non-parametric—under the four listed categories, a common feature of all previous studies is that they are only capable of measuring technical efficiency by focusing on the technological side of the production process, thereby neglecting the measurement of environmental efficiency from an economic perspective. This allows us to summarize what our model does; i.e., enhancing the existing approaches through the introduction of a measure of environmental *economic* inefficiency that, grounded on the theoretical framework proposed by Farrell (1957), considers both good and bad outputs, and enables its decomposition into technical and allocative components. To fill in the gap in the literature we postulate a comprehensive framework that is consistent with the economic behavior of organizations in their attempt to maximize revenue, but also accounts for the environmental inefficiency that results from the failure to minimize the economic costs associated to environmental damage. As previously remarked, this results in the definition of a net economic function that maximizes the difference between private (market) revenue less environmental (social) cost, using prices of good and bad outputs.³ In this regard, our framework is capable of balancing private gains (revenue) and environmental damage (cost) into a measure of economic inefficiency that can be decomposed according to technical and allocative criteria.

From an applied perspective we rely on DEA techniques because most existing empirical applications follow this approach: they are flexible, do not impose restrictive assumptions on the parametric specification of the technology, nor on the distribution of environmental inefficiency.⁴ Nevertheless, the drawbacks of DEA should be also highlighted and these include its deterministic nature and the sensitivity to outliers (for a comprehensive exposition of strengths and weaknesses of DEA see, for example, Stolp (1990), Berg (2010)). Aware of these caveats, which can be eventually addressed through, e.g., bootstrapping and other resampling techniques (see the methods introduced by Simar and Zelenyuk (2006) employed in the empirical section), we define the DEA programs that allow the empirical implementation of our novel approach.⁵ Our point of departure is the by-production model introduced by Murty et al. (2012), as it represents the most recent extension of previous approaches and can arguably be seen as a generalization that, by considering two independent technologies for desirable and undesirable outputs, avoids some of their inconsistencies (namely, the multiplicity of optimal combinations of desirable and undesirable outputs for a given level of inputs, and erroneously signed marginal rates of transformation – shadow prices – between outputs and inputs).

We demonstrate the practical usefulness of our newly developed methodology through an application to state-level data of the United States agricultural sector. Agriculture involves the production of not only good outputs such as primary food commodities, but also of bad outputs related with, for example, the need for fuel, the usage of pesticides, fertilizers and other agriculture chemicals, or the management of manure (Skinner et al., 1997; Reinhard et al., 1999). Examples of bad outputs associated to these polluting inputs in agriculture are greenhouse gas emissions, pesticide and nitrogen leaching and runoff, risk to human health and fish from exposure to pesticides and fertilizers, etc. (see Ball et al., 2001; Kellogg et al., 2002; Dakpo et al., 2017). In the empirical application we are capable of considering two of these bad outputs: CO₂ emissions and pesticide exposures.

The remainder of this paper is structured as follows. The next section reviews the by-production models of technical inefficiency and introduces their mathematical underpinnings. The subsequent section develops our extension allowing the measurement of economic (profit) inefficiency. We then discuss our empirical application, briefly commenting the dataset and presenting the results. Conclusions are drawn in the final section.

2. The by-production models

Pittman (1983) and Färe et al. (1986) initiated the asymmetric modeling of outputs when measuring efficiency depending on their nature, increasing those that are market-oriented while reducing those that are detrimental to the environment. A key question is how to axiomatically model the production technology when calculating technical efficiency through distance functions. Most particularly, as commented in the introduction to this paper, should the axioms underlying the production

¹ Details on the characteristics of the by-production approach are presented in the next section.

² Although we are aware of other methodological developments that rely on the by-production model, such as Serra et al. (2014) or Lozano (2015), we have not considered them since their general idea is to mix the by-production approach with other efficiency frameworks, and not the modification of the model *per se*. Hence, if applied, their results would not be comparable to those of the original by-production model.

³ The model can be easily enhanced to include the minimization of inputs cost, but instead we keep the definition of “environmental profit inefficiency” as a trade-off between private revenue and environmental cost.

⁴ See Zofio and Prieto (2001) for an exposition of early models within the non-parametric approach based on the output, input, and hyperbolic distance functions, which were subsequently implemented in a parametric framework by Cuesta et al. (2009). Du et al. (2016) rely on the latter approach to estimate carbon abatements costs through shadow prices.

⁵ Brännlund et al. (1995) measured profit inefficiency under a quota system and the production of undesirable outputs by DEA models. However, they did not use prices for weighting the negative externalities and do not decompose profit inefficiency into its drivers, something that we will do in this paper. Additionally, we note that Pham and Zelenyuk (2018) define revenue inefficiency in the banking industry accounting for nonperforming loans (NPLs), which are modeled as undesirable outputs under the approach of weak disposability. However, the model is internal to the firm (that is, private revenue), as it does not include environmental indicators, while they do not implement it empirically.

technology reflect their strong or weak disposability, and eventually, be modeled as outputs or as if they were inputs? Among the existing approaches for dealing with undesirable outputs and efficiency, the by-production model introduced by [Murty and Russell \(2002\)](#) and [Murty et al. \(2012\)](#) is currently considered a preferred option.

The by-production approach posits that complex production systems are made up of several independent processes ([Frisch, 1965](#)). In this model, the technology can be separated into sets of sub-technologies; one for the production of good outputs and one for the generation of bad outputs. The “global” technology implies interactions between several separate sub-technologies. [Førsund \(2018\)](#) and [Murty and Russell \(2018\)](#) recently classified the by-production approach among the multi-equation modeling approaches and argued that an important advantage of this approach is that it represents pollution-generating technologies by accounting for the Material Balance Principle, thereby satisfying the laws of thermodynamics. Additionally, as [Murty et al. \(2012\)](#) remarked, the by-production model avoids two inconsistencies of previous approaches. In particular, several technical efficiency combinations of good and bad outputs, with varying levels of bad output, could be possible when holding (polluting and non-polluting) input quantities fixed. However, in the absence of abatement activities implemented by the firm, this type of combination is contrary to the phenomenon of by-production, since by-production implies that, at fixed levels of inputs, there is only one level of pollution at the frontier of the production possibility set. Moreover, it is possible to observe a negative trade-off between the inputs associated with pollution, like fuel, and their associated bad output, such as CO₂, which represents a clear inconsistency (more fuel but less CO₂). These are the reasons why the by-production approach is utilized in the current study to introduce the concept of environmental economic inefficiency taking market prices into account.

In order to review the standard by-production approach, let us formally define $x \in R_+^n$ as a vector of inputs, $y \in R_+^m$ as a vector of good outputs, $z \in R_+^{m'}$ as a vector of bad outputs (e.g., pollutants), and let us assume that p DMUs have been observed. [Murty et al. \(2012\)](#) presented their model by splitting the input vector into two groups: non-polluting inputs, $x_1 \in R_+^{n_1}$ and pollution-generating inputs, $x_2 \in R_+^{n_2}$, with $n_1 + n_2 = n$.^{6,7} The first set could comprise land, labor, and so on, while the second set, in the context of our empirical application on agriculture, consists of inputs like fuel, fertilizers, and pesticides, which produce certain pollutants as by-products, such as CO₂ emissions and pesticide exposures. In this way, the ‘global’ technology, denoted by T , is the intersection of two sub-technologies, T_1 and T_2 . Whereas T_1 is the standard production technology with only good outputs, T_2 represents the production of bad outputs. In the model by [Murty et al. \(2012\)](#), both technologies are linked through the level of the polluting inputs. In more detail, [Murty et al. \(2012\)](#) define in general terms the technology as:

$$T = T_1 \cap T_2, \quad (1)$$

where

$$T_1 = \{(x_1, x_2, y, z) \geq 0 : f(x_1, x_2, y) \leq 0\}, \quad (2)$$

$$T_2 = \{(x_1, x_2, y, z) \geq 0 : z \geq g(x_2)\} \quad (3)$$

and f and g are continuously differentiable functions. The set T_1 is a standard technology set, reflecting the ways in which the inputs can be transformed into the intended outputs. The standard free-disposability properties may be imposed by assuming that $\frac{\partial f(x_1, x_2, y)}{\partial x_1} \leq 0$, $\frac{\partial f(x_1, x_2, y)}{\partial x_2} \leq 0$ and $\frac{\partial f(x_1, x_2, y)}{\partial y} \geq 0$. Note also that T_1 imposes no constraint on z ; that is, it is implicitly assumed that the by-product does not affect the production of bad outputs. On the other hand, T_2 reflects a residual-generation mechanism. It is worth mentioning that, in the formulation of T_2 , pollution is really treated as an output. In particular, [Murty et al. \(2012\)](#) assume that $\frac{\partial g(x_2)}{\partial x_2} > 0$. This expression and the formulation of T_2 capture the fact that pollution is an output of the production process for which disposal is not free. This property was called “costly disposability” of residuals. In words of [Murty et al. \(2012\)](#): “Costly disposability implies the possibility of inefficiencies in the generation of pollution (e.g., if a given level of coal generates some minimal level of smoke then inefficiency in the use of coal may imply that this level of coal can also generate a greater amount of smoke)”.

In the non-parametric framework of DEA, the two sub-technologies may be expressed mathematically under variable returns to scale (VRS) as:

$$T_1 = \left\{ (x_1, x_2, y, z) \geq 0 : \sum_{d=1}^p \lambda_d x_{1d} \leq x_1, \sum_{d=1}^p \lambda_d x_{2d} \leq x_2, \sum_{d=1}^p \lambda_d y_d \geq y, \sum_{d=1}^p \lambda_d = 1, \lambda_d \geq 0 \right\}, \quad (4)$$

$$T_2 = \left\{ (x_1, x_2, y, z) \geq 0 : \sum_{d=1}^p \mu_d x_{2d} \geq x_2, \sum_{d=1}^p \mu_d z_d \leq z, \sum_{d=1}^p \mu_d = 1, \mu_d \geq 0 \right\}. \quad (5)$$

⁶ Ayres and Kneese (1969) proposed these two same groups when introducing the materials balance principle to economists.

⁷ As [Murty et al. \(2012\)](#), we assume that Decision Making Units apply uniform abatement factors and, consequently, these factors are not explicitly considered in the analysis.

T_1 in (4) is the representation of the general set T_1 in (2) under several postulates as convexity and minimal extrapolation (see Banker et al., 1984). The same can be said for T_2 in (5) with respect to the general expression of T_2 in (3). Note that the sub-technologies are defined, in this framework, with two different intensity variables: λ and μ . Otherwise, we will have a confusion between these variables in the above two production possibility sets.

Regarding the measurement of technical efficiency, Murty et al. (2012) showed that some conventional approaches, like the hyperbolic and directional distance function defined on $T = T_1 \cap T_2$, are inadequate in the context of by-production. We use the term “output-oriented” in this context because these distance functions measure efficiency with respect to both good and bad outputs simultaneously. In this way, the weakness is due to the fact that the two aforementioned measures use the same coefficient (decision variable) for determining efficiency both in T_1 for the good outputs and T_2 for the bad outputs. This implies that it is possible to reach the efficiency frontier for some of the sub-technologies, but the observation can fall short of achieving the frontier of the other one. For consistency, efficiency in the by-production approach requires models that project the assessed observations onto *both* the efficient frontier of T_1 and the efficient frontier of T_2 .

The abovementioned drawbacks of standard approaches motivated Murty et al. (2012) to propose a different measure for dealing with good and bad outputs under by-production. For DMU_0 , this measure is good-output-specific and bad-output-specific, and is based on the index previously defined by Färe et al. (1985):

$$\begin{aligned}
 \min \quad & \frac{1}{2} \left[\underbrace{\frac{1}{m} \sum_{j=1}^m \theta_j}_{\text{standard efficiency}} + \underbrace{\frac{1}{m'} \sum_{k=1}^{m'} \gamma_k}_{\text{environmental efficiency}} \right] \\
 \text{s.t.} \quad & \sum_{d=1}^p \lambda_d x_{id} \leq x_{i0}, \quad i = 1, \dots, n \\
 & \sum_{d=1}^p \lambda_d y_{jd} \geq y_{j0}/\theta_j, \quad j = 1, \dots, m \\
 & \sum_{d=1}^p \lambda_d = 1, \\
 & \sum_{d=1}^p \mu_d x_{id} \geq x_{i0}, \quad i = n_1 + 1, \dots, n \\
 & \sum_{d=1}^p \mu_d z_{kd} \leq \gamma_k z_{k0}, \quad k = 1, \dots, m' \\
 & \sum_{d=1}^p \mu_d = 1, \\
 & \theta_j \leq 1, \quad j = 1, \dots, m \\
 & \gamma_k \leq 1, \quad k = 1, \dots, m' \\
 & \lambda_d \geq 0, \mu_d \geq 0, \quad d = 1, \dots, p
 \end{aligned} \tag{6}$$

Model (6) projects the assessed $DMU_0 (x_{10}, x_{20}, y_0, z_0)$ onto the efficient frontier of the technology T by increasing good outputs and reducing bad outputs. These changes are variable-specific, using a different decision variable for each dimension: $\theta_j, j = 1, \dots, m$, and $\gamma_k, k = 1, \dots, m'$. The constraints of model (6) coincide with the restrictions that define the DEA production possibility sets T_1 and T_2 in (4) and (5). Additionally, the optimal value of (6) coincides with the mean of the standard good-output-oriented efficiency and the environmental bad-output-oriented efficiency. Note also that the above model is

separable. In this case, this means that the optimal value can be determined as the mean of a model that minimizes $\frac{1}{m} \sum_{j=1}^m \theta_j$

on T_1 and a model that minimizes $\frac{1}{m'} \sum_{k=1}^{m'} \gamma_k$ on T_2 :

$$\begin{array}{ll}
 \min & \frac{1}{m} \sum_{j=1}^m \theta_j \\
 \text{s.t.} & \sum_{d=1}^p \lambda_d x_{id} \leq x_{i0}, \quad i = 1, \dots, n \\
 & \sum_{d=1}^p \lambda_d y_{jd} \geq y_{j0}/\theta_j, \quad j = 1, \dots, m \\
 & \sum_{d=1}^p \lambda_d = 1, \\
 & \theta_j \leq 1, \quad j = 1, \dots, m \\
 & \lambda_d \geq 0, \quad d = 1, \dots, p \\
 \min & \frac{1}{m'} \sum_{k=1}^{m'} \gamma_k \\
 \text{s.t.} & \sum_{d=1}^p \mu_d x_{id} \geq x_{i0}, \quad i = n_1 + 1, \dots, n_2 \\
 & \sum_{d=1}^p \mu_d z_{kd} \leq \gamma_k z_{k0}, \quad k = 1, \dots, m' \\
 & \sum_{d=1}^p \mu_d = 1, \\
 & \gamma_k \leq 1, \quad k = 1, \dots, m' \\
 & \mu_d \geq 0, \quad d = 1, \dots, p
 \end{array} \tag{7}$$

It is worth mentioning that the recent paper by [Førsund \(2018\)](#) argued that non-pollution causing inputs should also be included in technology T_2 given that substitution between the two groups of inputs can help mitigate the pollution. [Dakpo et al. \(2017\)](#) indicated that some additional constraints must be added to the by-production approach of [Murty et al. \(2012\)](#) in order to guarantee that the projection points for the input dimensions are the same in T_1 and T_2 . In particular, the condition

that should be incorporated to model (6) would be: $\sum_{d=1}^p \lambda_d x_{id} = \sum_{d=1}^p \mu_d x_{id}, \forall i$. Hereafter, we use T^M to denote the production

possibility set defined as the intersection of T_1 and T_2 in (1) and (2), respectively, as a way of highlighting that the definition of this technology corresponds to the original proposal of [Murty et al. \(2012\)](#). In the same way, we use T^D to denote the

production possibility set defined from the original by-production approach but incorporating the constraints $\sum_{d=1}^p \lambda_d x_{id} =$

$\sum_{d=1}^p \mu_d x_{id}, \forall i$, as pointed out by [Dakpo et al. \(2017\)](#). Finally, we will utilize T^{MF} to denote the production possibility set defined by [Murty et al. \(2012\)](#) but incorporating non-polluting inputs in technology T_2 . Likewise, T^{DF} denotes the production possibility set à la [Dakpo et al. \(2017\)](#) but again considering non-polluting inputs in the definition of technology T_2 .

To introduce our economic inefficiency model, we extend the state-of-the-art of by-production approach ([Murty et al., 2012](#); [Dakpo et al., 2017](#) and [Førsund, 2018](#)) by incorporating information on market prices. To do that, we resort to duality theory following [Chambers et al. \(1998\)](#), and, more recently, [Aparicio et al. \(2015\)](#), [Aparicio et al. \(2016a\)](#), and [Aparicio et al. \(2016b\)](#). In particular, we recall relevant duality results concerning the directional distance function⁸. Consequently, we start out by defining this type of measure from an output-oriented perspective in the context of by-production. Under the viewpoint introduced by [Murty et al. \(2012\)](#), we need a measure that allows us to project the assessed observations onto

⁸ Although the directional distance function is well-known due to its flexibility and because it encompasses the Shephard distance functions, it presents some drawbacks. This measure neglects slacks and, therefore, it does not take into account all sources of technical inefficiency (see, e.g., [Ray, 2004](#)). Additionally, the use of the directional distance function could result in infeasibilities under certain conditions, which are analysed in detail in [Briec and Kerstens \(2009\)](#).

the efficient frontiers of T_1 and T_2 simultaneously. In this way, the “by-production” directional output-oriented distance function for the Murty et al. (2012) approach with directional vector $g = (0, y_0, z_0)$ is defined as follows:

$$\begin{aligned} \vec{B}(x_0, y_0, z_0; T^M) = \max \quad & \delta^{T_1} \beta^{T_1} + \delta^{T_2} \beta^{T_2} \\ \text{s.t.} \quad & \sum_{j=1}^p \lambda_{j0} x_{ij} \leq x_{i0}, \quad i = 1, \dots, n_1 \quad (8.1) \\ & \sum_{j=1}^p \lambda_{j0} x_{ij} \leq x_{i0}, \quad i = n_1 + 1, \dots, n_2 \quad (8.2) \\ & -\sum_{j=1}^p \lambda_{j0} y_{rj} + \beta^{T_1} y_{r0} \leq -y_{r0}, \quad r = 1, \dots, m \quad (8.3) \\ & \sum_{j=1}^p \lambda_{j0} = 1, \quad (8.4) \\ & -\sum_{j=1}^p \mu_{j0} x_{ij} \leq -x_{i0}, \quad i = n_1 + 1, \dots, n_2 \quad (8.5) \\ & \sum_{j=1}^p \mu_{j0} z_{kj} + \beta^{T_2} z_{k0} \leq z_{k0}, \quad k = 1, \dots, m' \quad (8.6) \\ & \sum_{j=1}^p \mu_{j0} = 1, \quad (8.7) \\ & \beta^{T_1}, \beta^{T_2}, \lambda_{j0}, \mu_{j0} \geq 0 \quad (8.8) \end{aligned} \quad (8)$$

Model (8) projects the assessed DMU₀ (x_{10}, x_{20}, y_0, z_0) onto the efficient frontier of the technology T by increasing good outputs and reducing bad outputs. In this model, the changes in outputs are not variable-specific, i.e., it does not utilize a different decision variable for each dimension. Instead, it uses the same expansion factor for the good outputs, β^{T_1} , and the same reduction factor for the bad outputs, β^{T_2} . Moreover, the constraints of model (8) coincide with the restrictions that define the DEA production possibility sets T_1 and T_2 in (4) and (5). Additionally, the exogenous coefficients $\delta^{T_1} \geq 0$ and $\delta^{T_2} \geq 0$, $\delta^{T_1} + \delta^{T_2} = 1$, which appear in the objective function, are weights that are fixed exogenously by the corresponding decision maker (manager, politician, regulator, etc.) to reflect the relative importance of the standard (traditional) way of producing versus the new and clean paradigm for generating goods and services. Additionally, its linear dual is:

$$\begin{aligned} \vec{B}(x_0, y_0, z_0; T^M) = \min \quad & \sum_{i=1}^{n_1} v_{i0}^1 x_{i0} + \sum_{i=n_1+1}^{n_2} v_{i0}^1 x_{i0} - \sum_{r=1}^m u_{r0}^1 y_{r0} + \alpha_0^1 + \\ & - \sum_{i=n_1+1}^{n_2} v_{i0}^2 x_{i0} + \sum_{k=1}^{m'} u_{k0}^2 z_{k0} + \alpha_0^2 \\ \text{s.t.} \quad & \sum_{i=1}^{n_1} v_{i0}^1 x_{ij} + \sum_{i=n_1+1}^{n_2} v_{i0}^1 x_{ij} - \sum_{r=1}^m u_{r0}^1 y_{rj} + \alpha_0^1 \geq 0, \quad j = 1, \dots, p \quad (9.1) \\ & \sum_{r=1}^m u_{r0}^1 y_{r0} \geq \delta^{T_1}, \quad (9.2) \\ & - \sum_{i=n_1+1}^{n_2} v_{i0}^2 x_{ij} + \sum_{k=1}^{m'} u_{k0}^2 z_{kj} + \alpha_0^2 \geq 0, \quad j = 1, \dots, p \quad (9.3) \\ & \sum_{k=1}^{m'} u_{k0}^2 z_{k0} \geq \delta^{T_2}, \quad (9.4) \\ & v_{i0}^1, v_{i0}^2, u_{r0}^1, u_{k0}^2 \geq 0, \quad (9.5) \\ & \alpha_0^1, \alpha_0^2 \text{ free} \quad (9.6) \end{aligned} \quad (9)$$

Finally, to complete this opening section, we recall the first additive measure and decomposition of economic inefficiency proposed in the literature. We refer to the Nerlovian profit inefficiency measure, which can be decomposed into technical

inefficiency (the directional distance function) and a residual term interpreted as allocative inefficiency (Chambers et al., 1998).⁹

In the standard production context, considering private revenue and cost only, and given a vector of input and output prices $(\omega, q) \in R_+^{n+m}$ and technology T , the profit function Π is defined as $\Pi_T(\omega, q) = \max_{x,y} \left\{ \sum_{r=1}^m q_r y_r - \sum_{i=1}^n \omega_i x_i : (x, y) \in T \right\}$. Profit inefficiency à la Nerlove for DMU₀ is defined as optimal profit (that is, the value of the profit function at market prices) minus observed profit, both normalized by the value of a reference vec-

tor $g = (g^x, g^y) \in R_+^{n+m}$:
$$\frac{\Pi_T(\omega, q) - \left(\sum_{r=1}^m q_r y_{r0} - \sum_{i=1}^n \omega_i x_{i0} \right)}{\sum_{r=1}^m q_r g_r^y + \sum_{i=1}^n \omega_i g_i^x}$$
. Additionally, Chambers et al. (1998) showed that profit inefficiency

may be decomposed into technical inefficiency and allocative inefficiency, where technical inefficiency corresponds to the directional distance function $\vec{D}_T(x_0, y_0; g^x, g^y) = \max \{ \beta : (x_0 - \beta g^x, y_0 + \beta g^y) \in T \}$:

$$\frac{\Pi_T(\omega, q) - \left(\sum_{r=1}^m q_r y_{r0} - \sum_{i=1}^n \omega_i x_{i0} \right)}{\sum_{r=1}^m q_r g_r^y + \sum_{i=1}^n \omega_i g_i^x} = \vec{D}_T(x_0, y_0; g^x, g^y) + AI_T^N(x_0, y_0; \omega, q; g^x, g^y) \quad (10)$$

In model (10), the left-hand side corresponds to a measure of profit inefficiency, defined as the normalized difference between maximum profit and actual profit at observed market prices. This may be decomposed into technical inefficiency, i.e., the value of the directional distance function $\vec{D}_T(x_0, y_0; g^x, g^y)$, and price or allocative inefficiency $AI_T^N(x_0, y_0; \omega, q; g^x, g^y)$.

3. Measuring economic inefficiency with by-production models in DEA

3.1. Economic inefficiency model considering Murty et al.'s (2012) technology

We first introduce some notation and definitions. Given a fixed level of input $x_0 = (x_{10}, \dots, x_{n0}) \in R_+^n$ and a fixed level of bad output $z_0 = (z_{10}, \dots, z_{m'0}) \in R_+^{m'}$, let us also define as $r(x_0, z_0, q, T)$ the maximum feasible revenue given the output price vector $q = (q_1, \dots, q_m) \in R_{++}^m$:

$$r(x_0, z_0, q, T) = \sup_y \left\{ \sum_{r=1}^m q_r y_r : (x_0, y, z_0) \in T \right\} = \sup_y \left\{ \sum_{r=1}^m q_r y_r : (x_0, y, z_0) \in [T_1 \cap T_2] \right\}. \quad (11)$$

Eq. (11) represents a generic formulation for expressing how to determine the maximum revenue $\sum_{r=1}^m q_r y_r$ for good outputs that can be obtained given a technology $T = T_1 \cap T$ and fixed quantities of inputs x_0 and bad outputs z_0 .

Under Murty et al.'s (2012) approach, this optimization problem can be always solved independently on T_1 and T_2 . Therefore, as for T_1 , maximum feasible revenue given the output price vector $q = (q_1, \dots, q_m) \in R_{++}^m$ may be determined by:

$$r(x_0, z_0, q, T^M) = \sup_y \left\{ \sum_{r=1}^m q_r y_r : (x_0, y, z_0) \in T^M \right\} = \sup_y \left\{ \sum_{r=1}^m q_r y_r : (x_0, y, z_0) \in T_1 \right\}. \quad (12)$$

Again, (12) represents a general formulation for stating how to determine the maximum revenue $\sum_{r=1}^m q_r y_r$ for good outputs that can be obtained given the sub-technology T_1 and fixed quantities of inputs x_0 and bad outputs z_0 .

⁹ See also Koop and Diewert (1982) and Zieschang (1983) for earlier decompositions of economic (cost) efficiency into technical and allocative components. These authors implement Farrell's (1957) decomposition based on the radial input measure within a parametric (Cobb-Douglas) deterministic approach.

Next, we explicitly show how the value of $r(x_0, z_0, q, T^M)$ can be calculated in DEA under the by-production framework (see Ray, 2004):

$$\begin{aligned}
 r(x_0, z_0, q, T^M) &= \max_{\lambda, y} \sum_{r=1}^s q_r y_r \\
 \text{s.t.} \quad &\sum_{j=1}^p \lambda_j x^{ij} \leq x^{i0}, \quad i = 1, \dots, n_1 \quad (13.1) \\
 &\sum_{j=1}^p \lambda_j x^{ij} \leq x^{i0}, \quad i = n_1 + 1, \dots, n_2 \quad (13.2) \\
 &-\sum_{j=1}^p \lambda_j y^{rj} + y_r \leq 0, \quad r = 1, \dots, m \quad (13.3) \\
 &\sum_{j=1}^p \lambda_j = 1, \quad (13.4) \\
 &\lambda_j \geq 0, \quad j = 1, \dots, p \quad (13.5) \\
 &y_r \geq 0, \quad r = 1, \dots, m \quad (13.6)
 \end{aligned} \tag{13}$$

Model (13) is the DEA implementation of the general expression in (12) for determining maximum revenue. The objective function is the same, while the constraints coincide with those that define T_1 in (4).

The dual program of (13) is (14):¹⁰

$$\begin{aligned}
 \min_{c, d, \psi} &= \sum_{i=1}^{n_1} c_{i0} x^{i0} + \sum_{i=n_1+1}^{n_2} c_{i0} x^{i0} + \psi \\
 \text{s.t.} \quad &\sum_{i=1}^{n_1} c_{i0} x^{ij} + \sum_{i=n_1+1}^{n_2} c_{i0} x^{ij} - \sum_{r=1}^m d_{r0} y^{jr} + \psi_0 \geq 0, \quad j = 1, \dots, p \quad (14.1) \\
 &d_{r0} \geq q_r, \quad r = 1, \dots, m \quad (14.2) \\
 &c_{i0} \geq 0 \quad i = 1, \dots, n \quad (14.3)
 \end{aligned} \tag{14}$$

Being (14) the dual problem of (13), the decision variables c_{i0} , $i = 1, \dots, n$, and d_{r0} , $r = 1, \dots, m$, can be interpreted as shadow prices while the decision variable ψ_0 may be interpreted as shadow profit (see Aparicio et al., 2015).

If revenue maximization is assumed, as is the case here, the firm faces exogenously determined market output prices. Following this line, we may suppose that the objective of the DMU is to choose the outputs combination that yield the maximum revenue at the applicable prices. In this sense, revenue inefficiency measures how close is the observed revenue of the DMU under evaluation to the maximum feasible revenue. To evaluate economic loss due to revenue inefficiency, in the context of the directional output distance functions, Färe and Primont (2006) proved that a normalized measure of

revenue inefficiency, in particular the ratio $\frac{r(x_0, q, T) - \sum_{r=1}^m q_r y_{r0}}{\sum_{r=1}^m q_r g_r}$, may be decomposed into technical inefficiency, $\vec{D}_o(x^0, y^0; g)$,

plus a residual term interpreted as allocative inefficiency in the Farrell tradition, where $r(x_0, q, T)$ and $\vec{D}_o(x^0, y^0; g)$ denote the 'standard' revenue function and directional output distance function, respectively, and g is the corresponding reference directional vector.

Likewise, we can introduce cost efficiency following the same rationale, and based on the cost function. However, in our context we are interested in environmental cost functions rather than private costs, representing a measure of the (monetary) minimal damage caused by the production of undesirable outputs. The environmental cost function represents a "monetized metric" of the ecological footprint of the bad outputs; see, for example, Pearce et al. (1996) who relate the damage per ton of CO₂ with the social cost of carbon (SCC). Correspondingly, an observation is economically inefficient in environmental terms if, given the amount of undesirable outputs produced, it causes larger damage than that represented

¹⁰ Actually, the dual program of model (13) has an additional set of non-negativity constraints for the decision variables d_{r0} , $r = 1, \dots, m$. However, this set of constraints is redundant if we consider (14.2) and $q_r > 0$, $r = 1, \dots, m$.

by the minimum environmental cost function (either as a result of technical or allocative inefficiencies). Let us assume that it is possible to observe or estimate prices for the undesirable outputs: $w = (w_1, \dots, w_{m'}) \in R_{++}^{m'}$. Under Murty et al.'s (2012) approach, the eco-damage function will be non-parametrically determined directly from T_2 as follows.

$$\begin{aligned}
 D(x_0, y_0, w, T^M) = & \min_{\mu, z} \sum_{k=1}^{m'} w_k z_k \\
 \text{s.t.} & \\
 & \sum_{j=1}^p \mu_j x^{ij} \geq x^{i0}, \quad i = n_1 + 1, \dots, n_2 \quad (15.1) \\
 & - \sum_{j=1}^p \mu_j z^{kj} + z_k \geq 0, \quad k = 1, \dots, m' \quad (15.2) \\
 & \sum_{j=1}^p \mu_j = 1, \quad (15.3) \\
 & \mu_j \geq 0, \quad j = 1, \dots, p \quad (15.4) \\
 & z_k \geq 0, \quad k = 1, \dots, m' \quad (15.5)
 \end{aligned} \tag{15}$$

Model (15) minimizes the environmental cost, $\sum_{k=1}^{m'} w_k z_k$, associated with the production/emission of z (bad outputs) given a fixed quantity of pollution-generating inputs. The constraints in (15) coincide with the restrictions that define the sub-technology T_2 in (5).

The dual program of (15) is (16):

$$\begin{aligned}
 \max_{e, f, \chi} & \sum_{i=n_1+1}^{n_2} e_{i0} x^{i0} - \chi_0 \\
 \text{s.t.} & \\
 & \sum_{i=n_1+1}^{n_2} e_{i0} x^{ij} - \sum_{k=1}^{m'} f_{k0} z^{kj} - \chi_0 \leq 0, \quad j = 1, \dots, p \quad (16.1) \\
 & f_{k0} \leq w_k, \quad r = 1, \dots, m \quad (16.2) \\
 & e_{i0}, f_{k0} \geq 0 \quad (16.3)
 \end{aligned} \tag{16}$$

Being (16) the linear dual of model (15), it is well-known that the optimal values of both models are related.

We now derive, by duality, a normalized measure of economic inefficiency and show how it can be decomposed into (desirable) revenue inefficiency and eco-damage inefficiency. In order to do that, we first prove the following technical proposition.

Proposition 1. Let $\delta^{T_1}, \delta^{T_2} > 0$.

Then,

$$\inf_{t, h} \left\{ r(x_0, z_0, t, T^M) - \sum_{r=1}^m t_r y_{r0} + \sum_{k=1}^{m'} h_k z_{k0} - D^{T_2}(x_0, y_0, h, T^M) : \min \left\{ \frac{\sum_{r=1}^m t_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} h_k z_{k0}}{\delta^{T_2}} \right\} \geq 1 \right\} \geq \vec{B}(x_0, y_0, z_0; T^M).$$

Proof. Let $x_0 \in R_+^n$, $y_0 \in R_+^m$, $z_0 \in R_+^{m'}$ and let $t \in R_+^m$, $h \in R_+^{m'}$ such that $\min \left\{ \frac{\sum_{r=1}^m t_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} h_k z_{k0}}{\delta^{T_2}} \right\} \geq 1$. Let

(c_0^*, d_0^*, ψ_0^*) be an optimal solution of (14) and let (e_0^*, f_0^*, χ_0^*) be an optimal solution of (16) when $x_0 \in R_+^n$, $y_0 \in R_+^m$, $z_0 \in R_+^{m'}$ and $t \in R_+^m$ (acting as q), $h \in R_+^{m'}$ (acting as w) are taken as arguments. We will prove that

$(v_0^1, u_0^1, \alpha_0^1, v_0^2, u_0^2, \alpha_0^2) = (c_0^*, t, \psi_0^*, e_0^*, h, \chi_0^*)$ is a feasible solution of (9). Constraints (9.5) and (9.6) are trivially satisfied. Regarding (9.1), $\sum_{i=1}^{n_1} c_{i0}^* x_{ij} + \sum_{i=n_1+1}^{n_2} c_{i0}^* x_{ij} - \sum_{r=1}^m t_r y_{rj} + \psi_0^* \underset{\text{by (14.2)}}{\geq} \sum_{i=1}^{n_1} c_{i0}^* x_{ij} + \sum_{i=n_1+1}^{n_2} c_{i0}^* x_{ij} - \sum_{r=1}^m d_{r0}^* y_{rj} + \psi_0^* \underset{\text{by (14.1)}}{\geq} 0$. As

for (9.2), $\frac{r=1}{\delta^{t_1}} \geq 1$ since $\frac{r=1}{\delta^{t_1}} \geq \min \left\{ \frac{r=1}{\delta^{t_1}}, \frac{k=1}{\delta^{t_2}} \right\} \geq 1$. Therefore, $\sum_{r=1}^m t_r y_{r0} \geq \delta^{t_1}$. In the same way,

it is possible to prove that (9.3) and (9.4) are also satisfied. In particular, constraint (9.3) holds by (16.1) and (16.2). Consequently, $(c_0^*, t, \psi_0^*, e_0^*, h, \chi_0^*)$ is a feasible solution of (9). Regarding the objective function of

(9) evaluated at this point, $\vec{B}(x_0, y_0, z_0; T^M) \leq \sum_{i=1}^{n_1} c_{i0}^* x_{i0} + \sum_{i=n_1+1}^{n_2} c_{i0}^* x_{i0} - \sum_{r=1}^m t_r y_{r0} + \psi_0^* - \sum_{i=n_1+1}^{n_2} e_{i0}^* x_{i0} + \sum_{k=1}^{m'} h_k z_{k0} + \chi_0^* = r(x_0, z_0, t, T^M) - \sum_{r=1}^m t_r y_{r0} + \sum_{k=1}^{m'} h_k z_{r0} - D(x_0, y_0, h, T^M)$, since models (13) and (14) have the same optimal

value and models (15) and (16) also have the same optimal value. In this way, $\vec{B}(x_0, y_0, z_0; T^M)$ is a lower

bound of the set $\left\{ \sum_{i=1}^{n_1} c_{i0}^*(t) x_{i0} + \sum_{i=n_1+1}^{n_2} c_{i0}^*(t) x_{i0} - \sum_{r=1}^m t_r y_{r0} + \psi_0^*(t) - \sum_{i=n_1+1}^{n_2} e_{i0}^*(h) x_{i0} + \sum_{k=1}^{m'} h_k z_{k0} + \chi_0^*(h) : \forall (t, h) \in S_0 \right\}$,

where $(c_0^*(t), d_0^*(t), \psi_0^*(t))$ is any optimal solution of (14) when $q = t$ and $(e_0^*(h), f_0^*(h), \chi_0^*(h))$ is any optimal

solution of (16) when $w = h$. Note that $\left\{ \sum_{i=1}^{n_1} c_{i0}^*(t) x_{i0} + \sum_{i=n_1+1}^{n_2} c_{i0}^*(t) x_{i0} - \sum_{r=1}^m t_r y_{r0} + \psi_0^*(t) - \sum_{i=n_1+1}^{n_2} e_{i0}^*(h) x_{i0} + \sum_{k=1}^{m'} h_k z_{k0} + \chi_0^*(h) : \forall (t, h) \in S_0 \right\} = \left\{ r(x_0, z_0, t, T^M) - \sum_{r=1}^m t_r y_{r0} + \sum_{k=1}^{m'} h_k z_{r0} - D(x_0, y_0, h, T^M) : \forall (t, h) \in S_0 \right\}$, with $S_0 =$

$\left\{ (q, w) \in R_+^{m+m'} : \min \left\{ \frac{r=1}{\delta^{t_1}}, \frac{k=1}{\delta^{t_2}} \right\} \geq 1 \right\}$. Now, given that the infimum of a set is the greatest lower bound

of that set, we see that $\inf_{t,h} \left\{ r(x_0, z_0, t, T^M) - \sum_{r=1}^m t_r y_{r0} + \sum_{k=1}^{m'} h_k z_{r0} - D(x_0, y_0, h, T^M) : (t, h) \in S_0 \right\} \geq \vec{B}(x_0, y_0, z_0; T^M)$,

which is the inequality that we were seeking. ■

Let $(q, w) \in R_+^{m+m'}$ be market prices for good and bad outputs, respectively. Then, $(\tilde{q}, \tilde{w}) = \frac{(q, w)}{\min \left\{ \frac{r=1}{\delta^{t_1}}, \frac{k=1}{\delta^{t_2}} \right\}} \in S_0 =$

$\{(q, w) \in R_+^{m+m'} : \min \left\{ \frac{r=1}{\delta^{t_1}}, \frac{k=1}{\delta^{t_2}} \right\} \geq 1\}$.

Consequently, applying Proposition 1, we get

$$\begin{aligned} & r(x_0, z_0, \tilde{q}, T^M) - \sum_{r=1}^m \tilde{q}_r y_{r0} + \sum_{k=1}^{m'} \tilde{w}_k z_{r0} - D(x_0, y_0, \tilde{w}, T^M) \geq \\ & \inf_{t,h} \left\{ r(x_0, z_0, t, T^M) - \sum_{r=1}^m t_r y_{r0} + \sum_{k=1}^{m'} h_k z_{r0} - D(x_0, y_0, h, T^M) : (q, h) \in S_0 \right\} \\ & \geq \vec{B}(x_0, y_0, z_0; T^M). \end{aligned} \quad (17)$$

This inequality will be useful for stating the desired relationship between economic environmental inefficiency and $\vec{B}(x_0, y_0, z_0; T^M)$.

Finally, given that $r(x_0, z_0, t, T^M)$ is a function homogeneous of degree +1 in t and $D(x_0, y_0, h, T^M)$ is a function homogeneous of degree +1 in h , then

$$\frac{\left[r(x_0, z_0, q, T^M) - \sum_{r=1}^m q_r y_{r0} \right] + \left[\sum_{k=1}^{m'} w_k z_{k0} - D(x_0, y_0, w, T^M) \right]}{\min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}} \right\}} \geq \vec{B}(x_0, y_0, z_0; T^M). \quad (18)$$

Note that the left-hand side of (18) may be interpreted as a (normalized) measure of economic environmental inefficiency. Additionally, following Farrell's tradition, the right-hand side can be interpreted as (environmental) technical inefficiency and the residual term associated with closing the inequality could be interpreted as allocative inefficiency. Moreover, it is possible to decompose the left-hand side of (18) into

$$\begin{aligned} & \frac{r(x_0, z_0, q, T^M) - \sum_{r=1}^m q_r y_{r0} + \sum_{k=1}^{m'} w_k z_{k0} - D(x_0, y_0, w, T^M)}{\min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}} \right\}} = \\ & \underbrace{\frac{r(x_0, z_0, q, T^M) - \sum_{r=1}^m q_r y_{r0} + \sum_{k=1}^{m'} w_k z_{k0} - D(x_0, y_0, w, T^M)}{\min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}} \right\}}}_{\text{Overall Inefficiency}} = \\ & = \underbrace{\frac{r(x_0, z_0, q, T^M) - \sum_{r=1}^m q_r y_{r0}}{\min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}} \right\}}}_{\text{(Good)Revenue Inefficiency}} + \underbrace{\frac{\sum_{k=1}^{m'} w_k z_{k0} - D(x_0, y_0, w, T^M)}{\min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}} \right\}}}_{\text{Eco-Damage Inefficiency}}. \end{aligned} \quad (19)$$

However, note that the normalization term used in (18) and (19) – that is, $\min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}} \right\}$ – depends on two different terms, in contrast to what happens with respect to the Nerlovian profit inefficiency measure in (10). This means that,

depending on the observed data for each DMU, overall inefficiency is normalized by either $\frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}$ or $\frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}}$. In other words, in the same sample, the DMUs could use different normalization factors for their measure of overall inefficiency. Something that makes difficult the comparison of results. Hence, by analogy with the standard approach based on the directional distance function, we suggest resorting to an endogenous value for δ^{T_1} and, therefore, also for $\delta^{T_2} = 1 - \delta^{T_1}$, such

that $\frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}} = \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}}$. In other words, the value of this endogenous δ^{T_1} makes the two components be equal. It is easy to check that this value is $\delta^{T_1*} = \frac{\sum_{r=1}^m q_r y_{r0}}{\left(\sum_{r=1}^m q_r y_{r0} + \sum_{k=1}^{m'} w_k z_{k0} \right)}$.

3.2. Economic inefficiency model considering [Dakpo et al.'s \(2017\)](#) approach

We now turn to [Dakpo et al.'s \(2017\)](#) approach. In this case, the projection points in the two subtechnologies for the input dimensions must coincide. The “by-production” directional output distance function under this approach is as follows:¹¹

$$\begin{aligned} \vec{B}(x_0, y_0, z_0; T^D) = \max \quad & \delta^{T_1} \beta^{T_1} + \delta^{T_2} \beta^{T_2} \\ \text{s.t.} \quad & \sum_{j=1}^p \lambda_{j0} x_{ij} \leq x_{i0}, \quad i = 1, \dots, n_1 \quad (20.1) \\ & \sum_{j=1}^p \lambda_{j0} x_{ij} \leq x_{i0}, \quad i = n_1 + 1, \dots, n_2 \quad (20.2) \\ & -\sum_{j=1}^p \lambda_{j0} y_{rj} + \beta^{T_1} y_{r0} \leq -y_{r0}, \quad r = 1, \dots, m \quad (20.3) \\ & \sum_{j=1}^p \lambda_{j0} = 1, \quad (20.4) \\ & -\sum_{j=1}^p \mu_{j0} x_{ij} \leq -x_{i0}, \quad i = n_1 + 1, \dots, n_2 \quad (20.5) \\ & \sum_{j=1}^p \mu_{j0} z_{kj} + \beta^{T_2} z_{k0} \leq z_{k0}, \quad k = 1, \dots, m' \quad (20.6) \\ & \sum_{j=1}^p \mu_{j0} = 1, \quad (20.7) \\ & -\sum_{j=1}^p \lambda_{j0} x_{ij} + \sum_{j=1}^p \mu_{j0} x_{ij} \leq 0, \quad i = n_1 + 1, \dots, n_2 \quad (20.8) \\ & \beta^{T_1}, \beta^{T_2}, \lambda_{j0}, \mu_{j0} \geq 0 \quad (20.9) \end{aligned} \quad (20)$$

Model (20) is like model (8), where the [Dakpo et al.'s \(2017\)](#) approach has been considered through constraint (20.8), which interconnects the projections of the pollution-generating inputs in T_1 and T_2 .

¹¹ Constraints (20.2) and (20.5) imply that $-\sum_{j=1}^p \lambda_{j0} x_{ij} + \sum_{j=1}^p \mu_{j0} x_{ij} \geq 0$, for all $i = n_1 + 1, \dots, n_2$. This inequality, together with (20.8), implies $\sum_{j=1}^p \lambda_{j0} x_{ij} = \sum_{j=1}^p \mu_{j0} x_{ij}$ for all $i = n_1 + 1, \dots, n_2$, which coincides with the constraint related to [Dakpo et al.'s \(2017\)](#) approach. We prefer to include (20.8) instead of $\sum_{j=1}^p \lambda_{j0} x_{ij} = \sum_{j=1}^p \mu_{j0} x_{ij}$, for all $i = n_1 + 1, \dots, n_2$, because, in this way, the corresponding dual decision variables in model (21) are directly non-negative.

Its linear dual is:

$$\begin{aligned} \vec{B}(x_0, y_0, z_0; T^D) = \min & \sum_{i=1}^{n_1} v_{i0}^1 x_{i0} + \sum_{i=n_1+1}^{n_2} v_{i0}^1 x_{i0} - \sum_{r=1}^m u_{r0}^1 y_{r0} + \alpha_0^1 + \\ & - \sum_{i=n_1+1}^{n_2} v_{i0}^2 x_{i0} + \sum_{k=1}^{m'} u_{k0}^2 z_{k0} + \alpha_0^2 \\ \text{s.t.} & \sum_{i=1}^{n_1} v_{i0}^1 x_{ij} + \sum_{i=n_1+1}^{n_2} v_{i0}^1 x_{ij} - \sum_{r=1}^m u_{r0}^1 y_{rj} + \alpha_0^1 + \\ & - \sum_{i=n_1+1}^{n_2} v_{i0}^2 x_{ij} \geq 0, j = 1, \dots, p \end{aligned} \quad (21.1)$$

$$\sum_{r=1}^m u_{r0}^1 y_{r0} \geq \delta^{T_1}, \quad (21.2)$$

$$- \sum_{i=n_1+1}^{n_2} v_{i0}^2 x_{ij} + \sum_{k=1}^{m'} u_{k0}^2 z_{kj} + \alpha_0^2 + \sum_{i=n_1+1}^{n_2} v_{i0}^1 x_{ij} \geq 0, j = 1, \dots, p \quad (21.3)$$

$$\sum_{k=1}^{m'} u_{k0}^2 z_{k0} \geq \delta^{T_2}, \quad (21.4)$$

$$v_{i0}^1, v_{i0}^2, u_{r0}^1, u_{k0}^2, \gamma^{i0} \geq 0, \quad (21.5)$$

$$\alpha_0^1, \alpha_0^2 \text{ free} \quad (21.6)$$

Models (20) and (21) are related by the theory of Linear Programming.

In this context we now define a new support function, representing profit in Dakpo et al.'s model, as $\Gamma(x_0, q, w, T^D)$:

$$\Gamma(x_0, q, w, T^D) = \max \sum_{r=1}^m q_r y_r - \sum_{k=1}^{m'} w_k z_k$$

$$\text{s.t.} \quad \sum_{j=1}^p \lambda_{j0} x_{ij} \leq x_{i0}, \quad i = 1, \dots, n_1 \quad (22.1)$$

$$\sum_{j=1}^p \lambda_{j0} x_{ij} \leq x_{i0}, \quad i = n_1 + 1, \dots, n_2 \quad (22.2)$$

$$- \sum_{j=1}^p \lambda_{j0} y_{rj} + y_r \leq 0, \quad r = 1, \dots, m \quad (22.3)$$

$$\sum_{j=1}^p \lambda_{j0} = 1, \quad (22.4)$$

$$- \sum_{j=1}^p \mu_{j0} x_{ij} \leq -x_{i0}, \quad i = n_1 + 1, \dots, n_2 \quad (22.5)$$

$$\sum_{j=1}^p \mu_{j0} z_{kj} - z_k \leq 0, \quad k = 1, \dots, m' \quad (22.6)$$

$$\sum_{j=1}^p \mu_{j0} = 1, \quad (22.7)$$

$$- \sum_{j=1}^p \lambda_{j0} x_{ij} + \sum_{j=1}^p \mu_{j0} x_{ij} \leq 0, \quad i = n_1 + 1, \dots, n_2 \quad (22.8)$$

$$y_r, z_k, \lambda_{j0}, \mu_{j0} \geq 0, \quad (22.9)$$

(22)

which maximizes the difference between private revenue and eco-damage costs in our by-production context. Note that the [Dakpo et al.'s \(2017\)](#) approach has been considered in model (22) through constraint (22.8).

The linear dual of (22) is:

$$\begin{aligned}
 \Gamma(x_0, q, w, T^D) = \min & \quad \sum_{i=1}^{n_1} c^{i0} x_{i0} + \sum_{i=n_1+1}^{n_2} c^{i0} x_{i0} + \psi^0 + \\
 & - \sum_{i=n_1+1}^{n_2} e^{i0} x_{i0} + \chi^0 \\
 \text{s.t.} & \quad \sum_{i=1}^{n_1} c^{i0} x_{ij} + \sum_{i=n_1+1}^{n_2} c^{i0} x_{ij} - \sum_{r=1}^m d^{r0} y_{rj} + \psi^0 - \sum_{i=n_1+1}^{n_2} a^{i0} x_{ij} \geq 0, \quad (23.1) \\
 & \quad j = 1, \dots, p, \quad d_{r0} \geq q_r, \quad (23.2) \\
 & \quad - \sum_{i=n_1+1}^{n_2} e^{i0} x_{ij} + \sum_{k=1}^{m'} f^{k0} z_{kj} + \chi^0 + \sum_{i=n_1+1}^{n_2} a^{i0} x_{ij} \geq 0, \quad (23.3) \\
 & \quad j = 1, \dots, p, \quad f_{k0} \leq w_k, \quad (23.4) \\
 & \quad c^{i0}, d_{r0}, e_{i0}, f_{k0}, a_{i0} \geq 0 \quad (23.5) \\
 & \quad \psi_0, \chi_0 \text{ free.} \quad (23.6)
 \end{aligned}
 \tag{23}$$

By Linear Programming, the optimal values of model (22) and (23) are related.

Next, we show a relationship between $\Gamma(x_0, q, w, T^D)$ and $\vec{B}(x_0, y_0, z_0; T^D)$.

Proposition 2. Let $\delta^{T_1}, \delta^{T_2} > 0$.

Then,

$$\inf_{t, h} \left\{ \Gamma(x_0, t, h, T^D) - \sum_{r=1}^m t_r y_{r0} + \sum_{k=1}^{m'} h_k z_{k0} : \min \left\{ \frac{\sum_{r=1}^m t_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} h_k z_{k0}}{\delta^{T_2}} \right\} \geq 1 \right\} \geq \vec{B}(x_0, y_0, z_0; T^D).$$

Proof. Following the same steps than in Proposition 1, we get the desired result. ■

Applying Proposition 2, with market prices (q, w) , we get the following inequality.

$$\frac{\Gamma(x_0, q, w, T^D) - \left(\sum_{r=1}^m q_r y_{r0} - \sum_{k=1}^{m'} w_k z_{k0} \right)}{\min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}} \right\}} \geq \vec{B}(x_0, y_0, z_0; T^D). \tag{24}$$

The left-hand side in (24) may be interpreted as a measure of economic environmental inefficiency, which could be decomposed into technical inefficiency (the right-hand side in (24)) and a residual term, interpreted as allocative inefficiency.

3.3. Economic inefficiency model considering Førsund's (2018) proposal

Finally, it is possible to incorporate Førsund's (2018) proposal, adapting Murty et al. (2012) and Dakpo et al. (2017). To do this, it is sufficient to include the non-polluting inputs in the sub-technology T_2 . The results of Proposition 1 and 2 are valid for $\vec{B}(x_0, y_0, z_0; T^{MF})$ and $\vec{B}(x_0, y_0, z_0; T^{DF})$. Hence, we have model (25).

$$\begin{aligned} \vec{B}(x_0, y_0, z_0; T^{MF}) = \max \quad & \delta^{T_1} \beta^{T_1} + \delta^{T_2} \beta^{T_2} \\ \text{s.t.} \quad & \sum_{j=1}^p \lambda_{j0} x_{ij} \leq x_{i0}, \quad i = 1, \dots, n_1 \quad (25.1) \\ & \sum_{j=1}^p \lambda_{j0} x_{ij} \leq x_{i0}, \quad i = n_1 + 1, \dots, n_2 \quad (25.2) \\ & -\sum_{j=1}^p \lambda_{j0} y_{rj} + \beta^{T_1} y_{r0} \leq -y_{r0}, \quad r = 1, \dots, m \quad (25.3) \\ & \sum_{j=1}^p \lambda_{j0} = 1, \quad (25.4) \\ & -\sum_{j=1}^p \mu_{j0} x_{ij} \leq -x_{i0}, \quad i = 1, \dots, n \quad (25.5) \\ & \sum_{j=1}^p \mu_{j0} z_{kj} + \beta^{T_2} z_{k0} \leq z_{k0}, \quad k = 1, \dots, m' \quad (25.6) \\ & \sum_{j=1}^p \mu_{j0} = 1, \quad (25.7) \\ & \beta^{T_1}, \beta^{T_2}, \lambda_{j0}, \mu_{j0} \geq 0, \quad (25.8) \end{aligned} \quad (25)$$

Model (25) is like model (8), where the Førsund's (2018) approach has been considered by changing $i = n_1 + 1, \dots, n_2$ by $i = 1, \dots, n$ in constraint (25.5). And

$$\begin{aligned} D(x_0, y_0, w, T^{MF}) = \min_{\mu, z} \quad & \sum_{k=1}^{m'} w_k z_k \\ \text{s.t.} \quad & \sum_{j=1}^p \mu_j x^{ij} \geq x^{i0}, \quad i = 1, \dots, n \quad (26.1) \\ & -\sum_{j=1}^p \mu_j z^{kj} + z_k \geq 0, \quad k = 1, \dots, m' \quad (26.2) \\ & \sum_{j=1}^p \mu_j = 1, \quad (26.3) \\ & \mu_j \geq 0, \quad j = 1, \dots, p \quad (26.4) \\ & z_k \geq 0, \quad k = 1, \dots, m' \quad (26.5) \end{aligned} \quad (26)$$

with

$$\begin{aligned} \frac{r(x_0, z_0, q, T^M) - \sum_{r=1}^m q_r y_{r0} + \sum_{k=1}^{m'} w_k z_{r0} - D(x_0, y_0, w, T^{MF})}{\min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}} \right\}} \geq \vec{B}(x_0, y_0, z_0; T^{MF}). \end{aligned} \quad (27)$$

Model (26) allows to determine the damage function when the [Murty et al. \(2012\)](#) approach is adapted through [Førsund's \(2018\)](#) proposal. Additionally, the left-hand side in (27) may be interpreted as a measure of economic environmental inefficiency. In particular, it is possible to decompose it into

$$\begin{aligned}
 & \frac{r(x_0, z_0, q, T^M) - \sum_{r=1}^m q_r y_{r0} + \sum_{k=1}^{m'} w_k z_{k0} - D(x_0, y_0, w, T^{MF})}{\min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}} \right\}} = \\
 & \underbrace{\hspace{10em}}_{\text{Overall Inefficiency}} \\
 & = \underbrace{\frac{r(x_0, z_0, q, T^M) - \sum_{r=1}^m q_r y_{r0}}{\min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}} \right\}}}_{\text{(Good) Revenue Inefficiency}} + \underbrace{\frac{\sum_{k=1}^{m'} w_k z_{k0} - D(x_0, y_0, w, T^{MF})}{\min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}} \right\}}}_{\text{Eco-Damage Inefficiency}}.
 \end{aligned} \tag{28}$$

Regarding [Dakpo et al.'s \(2017\)](#) model, including [Førsund's \(2018\)](#) extension, we have:

$$\begin{aligned}
 \tilde{B}(x_0, y_0, z_0; T^{DF}) &= \max \quad \delta^{T_1} \beta^{T_1} + \delta^{T_2} \beta^{T_2} \\
 \text{s.t.} \quad & \sum_{j=1}^p \lambda_{j0} x_{ij} \leq x_{i0}, \quad i = 1, \dots, n_1 \tag{29.1} \\
 & \sum_{j=1}^p \lambda_{j0} x_{ij} \leq x_{i0}, \quad i = n_1 + 1, \dots, n_2 \tag{29.2} \\
 & -\sum_{j=1}^p \lambda_{j0} y_{rj} + \beta^{T_1} y_{r0} \leq -y_{r0}, \quad r = 1, \dots, m \tag{29.3} \\
 & \sum_{j=1}^p \lambda_{j0} = 1, \tag{29.4} \\
 & -\sum_{j=1}^p \mu_{j0} x_{ij} \leq -x_{i0}, \quad i = 1, \dots, n \tag{29.5} \\
 & \sum_{j=1}^p \mu_{j0} z_{kj} + \beta^{T_2} z_{k0} \leq z_{k0}, \quad k = 1, \dots, m' \tag{29.6} \\
 & \sum_{j=1}^p \mu_{j0} = 1, \tag{29.7} \\
 & -\sum_{j=1}^p \lambda_{j0} x_{ij} + \sum_{j=1}^p \mu_{j0} x_{ij} \leq 0, \quad i = n_1 + 1, \dots, n_2 \tag{29.8} \\
 & \beta^{T_1}, \beta^{T_2}, \lambda_{j0}, \mu_{j0} \geq 0 \tag{29.9}
 \end{aligned} \tag{29}$$

And

$$\Gamma(x_0, q, w, T^{DF}) = \max \sum_{r=1}^m q_r y_r - \sum_{k=1}^{m'} w_k z_k$$

$$s.t. \sum_{j=1}^p \lambda_{j0} x_{ij} \leq x_{i0}, \quad i = 1, \dots, n_1 \quad (30.1)$$

$$\sum_{j=1}^p \lambda_{j0} x_{ij} \leq x_{i0}, \quad i = n_1 + 1, \dots, n_2 \quad (30.2)$$

$$-\sum_{j=1}^p \lambda_{j0} y_{rj} + y_r \leq 0, \quad r = 1, \dots, m \quad (30.3)$$

$$\sum_{j=1}^p \lambda_{j0} = 1, \quad (30.4)$$

$$-\sum_{j=1}^p \mu_{j0} x_{ij} \leq -x_{i0}, \quad i = 1, \dots, n \quad (30.5)$$

$$\sum_{j=1}^p \mu_{j0} z_{kj} - z_k \leq 0, \quad k = 1, \dots, m' \quad (30.6)$$

$$\sum_{j=1}^p \mu_{j0} = 1, \quad (30.7)$$

$$-\sum_{j=1}^p \lambda_{j0} x_{ij} + \sum_{j=1}^p \mu_{j0} x_{ij} \leq 0, \quad i = n_1 + 1, \dots, n_2 \quad (30.8)$$

$$y_r, z_k, \lambda_{j0}, \mu_{j0} \geq 0, \quad (30.9)$$

(30)

which results in the following inequality:

$$\frac{\Gamma(x_0, q, w, T^{DF}) - \left(\sum_{r=1}^m q_r y_{r0} - \sum_{k=1}^{m'} w_k z_{k0} \right)}{\min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{I1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{I2}} \right\}} \geq \vec{B}(x_0, y_0, z_0; T^{DF}). \quad (31)$$

Inequalities (27) and (31) make it possible to define technical and allocative terms as drivers of the corresponding measure of economic environmental inefficiency. In the empirical application we solve the models corresponding to [Murty et al. \(2012\)](#) and [Dakpo et al. \(2017\)](#), enhanced with Førsund's (2018) proposal. This represents a total of four models.

4. Empirical application

4.1. Dataset and variables

The empirical illustration relies on state-level data in the United States that comes from multiple agencies. The dataset consists of aggregated firm data for each state as, unfortunately, and due to statistical confidentiality reasons, we do not have access to the individual microdata.¹² The main source of data is the [U.S. Department of Agriculture \(USDA\)](#) Economic

¹² We perform a macro-level analysis in our empirical application, assuming that the DMUs (the states) can be compared. A more suitable analysis would consist in estimating a meta-frontier ([O'Donnell et al., 2008](#); [Battese et al., 2004](#)) using the data for all the firms in all the states and then, decomposing inefficiency into within-state inefficiency and a gap between the technology of each state and the global frontier. This line could be a good avenue for further research.

Table 1
Descriptive statistics of input-output data (implicit real quantities), 2004.

Variable	Mean	SD	Coefficient of variation
Non-pollution-generating inputs			
Capital services (million \$)	541.068	450.804	0.833
Land service flows (million \$)	650.678	738.002	1.134
Labor services (million \$)	1,292.827	1,186.855	0.918
Pollution-generating inputs			
Energy (million \$)	166.647	144.484	0.867
Pesticides (million \$)	164.921	163.858	0.994
Good outputs			
Livestock and products (million \$)	2,103.076	1,997.443	0.950
Crops (million \$)	2,819.480	3,419.130	1.213
Bad outputs			
CO ₂ emissions (tons of CO ₂ equivalents)	996,394.520	914,820.595	0.918
Pesticide exposures (number)	2,458.708	2,256.445	0.918

Notes: SD = Standard deviation.

Research Service (ERS), which compiled the data necessary to calculate agricultural productivity in the US, and, in particular, the price indices and implicit quantities of farm outputs and inputs for each of the 48 continental states for 1960 – 2004. The dataset has been validated and used extensively in previous research (for example, in Ball et al., 1999; Zofio and Knox Lovell, 2001; Huffman and Evenson, 2006; Sabasi and Shumway, 2018). A critical review of the data in light of recent developments can be found in Shumway et al. (2015, 2016). To illustrate our models, we consider the most recent year available in the dataset (2004) and assume that the production process is characterized by the following three non-polluting inputs (capital services excluding land, land service flows, and labor services), two polluting inputs (energy and pesticides), and two good outputs (livestock and crops). The prices for these variables were directly obtained from the ERS dataset. Prices indices in the ERS dataset are constructed using the Törnqvist formulation.¹³

As for the undesirable output production generated by energy consumption, we consider carbon dioxide (CO₂) emissions from the agricultural sector associated with fuel combustion, also for 2004 (expressed in tons of CO₂ equivalents), obtained from the U.S. Environmental Protection Agency (EPA).¹⁴ The price of CO₂ emissions is proxied by the market clearing price set in the state of California (price of carbon emissions expressed in thousands of dollars per ton of CO₂ equivalents), since a general market for CO₂ for the whole US does not exist. In particular, this is the price of carbon for tradable allowances with a futures contract that originates from the Californian greenhouse gases trading market under the California Cap and Trade Program, 2019. We consider the average 2012 price and deflate it to 2004 using the consumer price index in absence of a suitable deflator (US Bureau of Labor Statistics).¹⁵ The measure of bad output related to pesticides is the number of pesticide exposures per state for 2004 obtained from the Centers for Disease Control and Prevention at the US Department of Health & Human Services. As the approximation of the price of this bad output we use the cost of hospitalized treatment of pesticide-related poisonings (in thousands of dollars) as estimated in Pimentel (2005). Because this cost is provided for 1995, we further express it to 2004 prices using the price index for medical services as obtained from the U.S. Bureau of Labor Statistics.¹⁶

Tables 1 and 2 summarize the descriptive statistics of input-output quantities and their corresponding prices, respectively, for the US states in 2004. Appendix A.1. of the online supplemental material accompanying the paper presents this data for each state.

4.2. Results

4.2.1. Technical, allocative and profit frontiers

When solving our four reference economic models – that is, Murty et al. (2012) and Dakpo et al. (2017), each complemented with Førsund's (2018) proposal – it is relevant to determine, from a technological perspective, the number of

¹³ The details on the method of construction of all variables are contained in the following webpage of the USDA-ERS: <https://www.ers.usda.gov/data-products/agricultural-productivity-in-the-us/methods/>.

¹⁴ Since these data are given in overall terms for the whole country, we further disaggregate it by state, using for that purpose the share that each state has in farm production expenses for gasoline, fuels, and oils, as reported by the U.S. Department of Agriculture, expressed in thousands of dollars.

¹⁵ California's GHG emissions program is the fourth largest in the world after the European Union's Emissions Trading System, South Korean Emissions Trading Scheme and the Emission Trading System in the Chinese province of Guangdong. From its beginning in 2012 it covered the power and industrial facilities, and it expanded to natural gas and transportation fuels in 2015, allowing it to cover approximately 85 percent of California's GHG emissions.

¹⁶ The consideration of these market prices for the bad outputs, which can be considered as the opportunity costs of environmental damage, enables the analysis of allocative inefficiency with respect to the environmentally efficient frontier. Market prices can be compared to the shadow prices corresponding to the marginal rate of transformation of bad outputs for good outputs, which are observed at the optimal projection of the firm on the technological frontier. Comparing both sets of prices can help regulators to adopt specific polluting abatement instruments such as taxes, permits, or emission standards. On this topic, a recent and comprehensive study comparing shadow price calculations for carbon dioxide emissions in China, using both parametric and non-parametric techniques, can be found in Ma et al. (2019).

Table 2

Descriptive statistics of prices for input-output data, 2004.

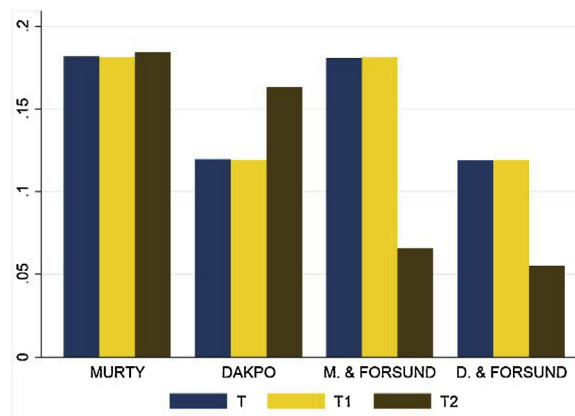
Variable	Mean	SD	Coefficient of variation
Non-pollution-generating inputs			
Capital services (indices relative to Alabama)	1.105	0.028	0.026
Land service flows (indices relative to Alabama)	1.134	0.636	0.560
Labor services (indices relative to Alabama)	1.158	0.325	0.280
Pollution-generating inputs			
Energy (indices relative to Alabama)	1.441	0.166	0.115
Pesticides (indices relative to Alabama)	1.142	0.252	0.220
Good outputs			
Livestock and products	1.245	0.197	0.158
Crops (indices relative to Alabama)	1.067	0.156	0.146
Bad outputs			
CO ₂ emissions (thousand \$ per ton of CO ₂ equivalents)	0.011	0	0
Pesticide exposures (thousand \$)	6.222	0	0

Notes: SD = Standard deviation. Prices of inputs and good outputs vary across states and are expressed in relative terms with respect to the first state, Alabama, considering 1996 as the base year. Prices of bad outputs are unique for all states and are deflated to the 2004 reference year.

Table 3

Number of efficient US states.

Model	Technical			Allocative	Profit
	T	T_1	T_2		
Murty et al.	6	19	11	4	4
Dakpo et al.	13	30	16	6	6
Murty & Førsund	10	19	24	4	4
Dakpo & Førsund	19	30	28	15	15

**Fig. 1.** Average technical inefficiencies by models (absolute values).

observations that are efficient, thereby defining the frontier of the global *by-production* technology T , consisting of both the *intended production* T_1 technology, (4) (hereafter, conventional or standard technology) and the *pollution-generating* technology T_2 , (5) (hereafter, polluting technology). Table 3 shows that the number of observations defining the production frontier is greater in the conventional technology T_1 than in the polluting technology T_2 , except in the case of the Murty et al. (2012) model incorporating Førsund's proposal. Clearly, not all technically efficient states are allocative-efficient and thereby achieve profit efficiency. California, Delaware, Iowa and Vermont are efficient in all models. Approximately 10 percent of US states (four or six out of 48) are fully efficient, except in Dakpo et al.'s approach enhanced with Førsund's assumption, where the number of profit-efficient states increases to 31.25 percent¹⁷.

4.2.2. Technical inefficiency: results within and between models

Departing from this general portrait of inefficiency frequencies at the technical, allocative, and overall profit inefficiency levels, we now focus on the technological side, with Fig. 1 portraying the average absolute technical efficiency values in T_1 , T_2 and their global *by-production* aggregate T , across the four models.

¹⁷ Detailed results on inefficiencies per state are presented in Appendix A.2. of the online supplemental material accompanying the paper.

Table 4

Spearman's correlation matrix between technical inefficiencies.

		Conventional technology (T_1)				Polluting Technology (T_2)			
		Murty T1	Dakpo T1	M. & F. T1	D. & F. T1	Murty T2	Dakpo T2	M. & F. T2	D. & F. T2
Conventional technology (T_1)	Murty T1	1.000*							
	Dakpo T1	0.764*	1.000*						
	M. & F. T1	1.000*	0.764*	1.000*					
	D. & F. T1	0.764*	1.000*	0.764*	1.0000*				
Polluting technology (T_2)	Murty T2	−0.142 ^{ns}	0.090 ^{ns}	−0.142 ^{ns}	0.090 ^{ns}	1.000*			
	Dakpo T2	−0.079 ^{ns}	0.108 ^{ns}	−0.080 ^{ns}	0.108 ^{ns}	0.901*	1.000*		
	M. & F. T2	−0.132 ^{ns}	−0.133 ^{ns}	−0.132 ^{ns}	−0.133 ^{ns}	0.557*	0.549*	1.000*	
	D. & F. T2	0.012 ^{ns}	−0.006 ^{ns}	0.012 ^{ns}	−0.006 ^{ns}	0.430*	0.542*	0.871*	1.000*

Notes: Murty et al.: (8), Dakpo et al.: (20), Murty et al. & Førsund: (25), Dakpo et al. & Førsund: (29).

* $p < 0.01$; ^{ns} Non-significant at 10 % level.

Several features are worth highlighting:

- Technical inefficiency in the conventional technology T_1 differs substantially on average between Murty et al.'s models (8) and (25), and Dakpo et al.'s models (20) and (29), but it is equal *within* each type of model. That is, as Førsund's assumption includes non-polluting inputs in T_2 , the characterization of T_1 is the same and results are unaffected.
- Average T_1 inefficiency in Murty et al.'s models is about 50 percent greater than Dakpo et al.'s models: 0.181 vs. 0.119. This finding is also expected since the introduction of the additional constraint in Dakpo et al.'s models, ensuring that optimal polluting input quantities are the same in T_1 and T_2 , results in a tighter envelopment of the observed data, and hence in lower inefficiency values.
- Technical inefficiency in the residual (polluting) technology T_2 differs across the four models. However, while there are not significant differences between Murty et al.'s and Dakpo (20) et al.'s (8) models (0.184 vs. 0.163), what makes a difference is the introduction of Førsund's proposal including non-polluting inputs in T_2 . Indeed, the average technical inefficiency in Murty et al.'s model (8) is three times greater than that for the same model enhanced with Førsund's assumption (25): 0.184 vs. 0.066. The difference between Dakpo et al.'s model (20) and that enhanced with Førsund's proposal (29) is similar to the difference above: 0.163 vs. 0.055. Thus, the inclusion of non-polluting outputs in T_2 has remarkable effects on reducing technical inefficiency.

The correlations between the technical efficiency scores using Spearman's definition show that the conventional and environmental efficiency performance are weakly or even negatively correlated in most cases (see Table 4). This should come as no surprise given that observations do have market incentives to perform better in the conventional side of the production process T_1 (that is, to maximize output revenue), but these incentives are weak or absent in the case of environmental cost minimization. Since the production of undesirable outputs (CO₂ emissions and pesticide exposures) is not normally internalized by the economic system, productive efficiency in T_2 is not tightly pursued, which means that a negative correlation between both rankings is a likely outcome.

This can be seen clearly in the Murty et al. and Dakpo et al. models, where T_2 inefficiencies are greater on average. Nevertheless, we note that the variability in the rankings is so high that none of these correlations are significant at the standard confidence levels. It is also worth remarking that the correlations between models' rankings for the polluting technology T_2 are generally smaller than for the standard technology T_1 (except for the Murty et al. and Dakpo et al. models enhanced with Førsund's proposal, whose correlation is $\rho(M\&F^{T_2}, D\&F^{T_2}) = 0.871$).

A visual comparison of the values of the technical inefficiency scores for T_1 and T_2 is presented in Fig. 2, where box-plots of the different distributions make it possible to identify extreme values. The different boxes (grouped in pairs by models) represent the intervals between the first and third quartiles of the ranking distribution (that is, the interquartile range (IQR) between Q1 and Q3), with its median represented by the horizontal line within it (the median can then be compared to the mean values presented in Fig. 1). The dispersion in the rankings within this interquartile range is relatively low, particularly for the polluting technologies T_2 , incorporating Førsund's assumption. It is also reassuring that only a few outliers were identified. In particular, for the Murty et al. model the states of Louisiana (0.994) and Montana (1.066) are the most inefficient, lying outside the area below the whisker equal to one a half times the IQR. In Dakpo et al.'s model, Montana (0.970) is the worst-performing state. As for these extreme states, the ranking and values do not change when Førsund's assumption is considered, suggesting robustness in the results.

Still focusing on the box-plots, it is relevant to test whether the distributions of the conventional and polluting technologies, T_1 and T_2 , are equal *within* each one of the four models. For this purpose, we have performed the test proposed by Simar and Zelenyuk (2006). Their method adapts the nonparametric test for the equality of two densities developed by Li (1996). For this test we use algorithm II with 1000 replications, which computes and bootstraps the Li statistic on the DEA scores, and where the null values of the efficient firms (resulting in the truncation of the efficiency scores) are smoothed by adding

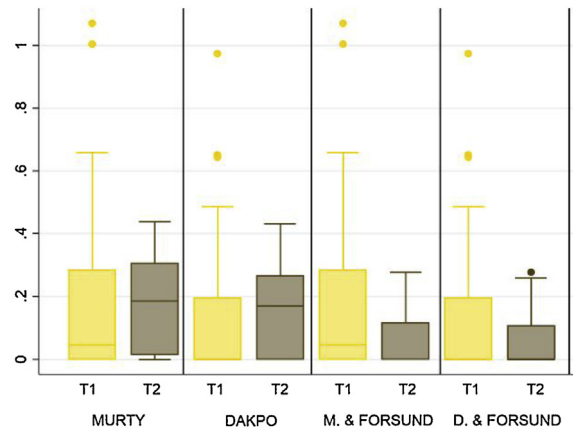


Fig. 2. Box plots of the technical inefficiency distributions by models.

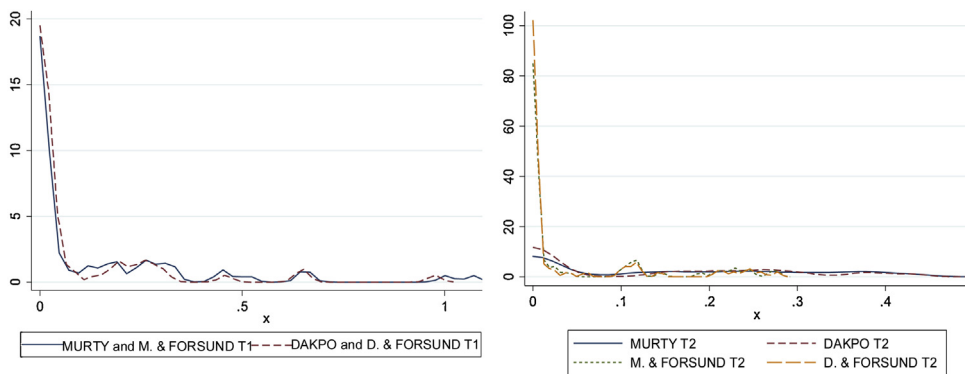


Fig. 3. Technical inefficiencies kernel distributions (T_1 and T_2) by models.

a small noise.¹⁸ The obtained results reject the null hypothesis of equality of densities for all models at the 5 percent level of significance, which means that T_1 and T_2 inefficiencies are statistically different in each of the four models.¹⁹

Alternatively, it is also interesting to test if the T_1 and T_2 inefficiency distributions are different *between* models. Fig. 3 depicts their kernel distributions in each one of the four models (left and right panels for T_1 and T_2 , respectively). When plotting these distributions we follow the procedure proposed by Simar and Zelenyuk (2006), which in short: (i) uses Gaussian kernels, (ii) employs the reflection method to overcome the issue of a zero-bounded support of the inefficiency scores (Silverman, 1986), and (iii) determines the bandwidths using Sheather and Jones's (1991) method. As commented, T_1 distributions are the same across pairwise models; that is, Murty et al.'s model (8), and Dakpo et al.'s model (20), are equal to their respective Førsund's extensions, (25) and (29) (that is, $M^{T_1} = M\&F^{T_1}$, $D^{T_1} = D\&F^{T_1}$). When comparing Murty et al.'s and Dakpo et al.'s models (M^{T_1} vs. D^{T_1}) and Murty et al.'s enhanced with Førsund's assumption model and Dakpo et al.'s model ($M\&F^{T_1}$ vs. D^{T_1}), the null hypotheses of the equality of densities cannot be rejected at the 10 percent level, so these models are not statistically different. However, the comparison between Murty et al.'s and Dakpo et al.'s models, both enhanced with Førsund's assumption ($M\&F^{T_1}$ vs. $D\&F^{T_1}$), and Murty et al.'s model and Dakpo et al.'s model enhanced with Førsund's assumption (M^{T_1} vs. $D\&F^{T_1}$), shows that the null hypotheses of equality of densities is rejected at the 5 percent and 10 percent levels, respectively.²⁰

As for the differences in the polluting technology T_2 (right panel), the adapted Li test returns that the Murty et al.'s (8) and Dakpo et al.'s models (20) are not statistically different among themselves (M^{T_2} vs. D^{T_2}). This result extends to their Førsund's versions: (25) and (29) ($M\&F^{T_2}$ vs. $D\&F^{T_2}$). However, when comparing Murty et al.'s model to its Førsund's extension (M^{T_2} vs. $M\&F^{T_2}$), Dakpo et al.'s model to its Førsund's extension (D^{T_2} vs. $D\&F^{T_2}$), Murty et al.'s model to Dakpo et al.'s in its Førsund's

¹⁸ Simar and Zelenyuk (2006) develop their algorithm for radial distance functions, in which the efficiency values equal to one are smoothed. We adapt their algorithm to our additive context by smoothing the inefficiency scores equal to zero.

¹⁹ The level of significance changes to 1 percent when the hypothesis is tested for Dakpo et al.'s model enhanced with Førsund's assumption.

²⁰ Detailed results on these and subsequent bilateral tests can be found in Appendix A.3. of the online supplemental material accompanying the paper.

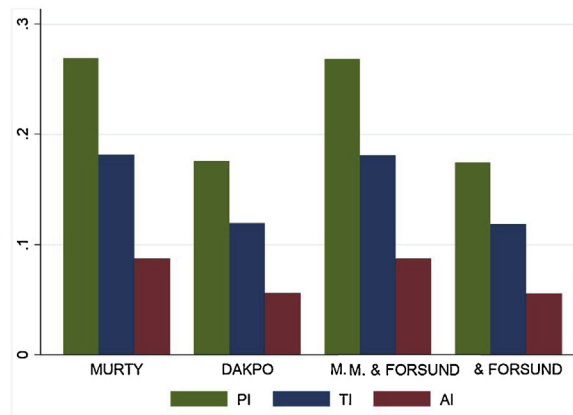


Fig. 4. Average values of technical and allocative inefficiencies (AI and TI) by models.

version (M^{T_2} vs. $D\&F^{T_2}$), as well as Murty et al.'s in its Førsund's version model to Dakpo et al.'s model ($M\&F^{T_2}$ vs. D^{T_2}), the results of the adapted Li test show that that null hypotheses of the equality of densities are rejected at the 1 percent level. Therefore, as for technical inefficiencies, we conclude that T_1 and T_2 results are statistically different within models, but in many cases not between models.

4.2.3. Profit inefficiency: technical and allocative inefficiencies between models

We now discuss the economic efficiency dimension of the agricultural sector at the state level. Fig. 4 portrays absolute average values of the profit, technical, and allocative inefficiencies of the four technological models (denoted by PI, TI and AI). Average profit inefficiency is about 50 percent greater in Murty et al.'s model (18) than in Dakpo et al.'s model (24): 0.269 vs. 0.176, respectively. Despite the different technological characterization of the polluting technology when incorporating Førsund's extensions to the profit inefficiency definition, (28) and (31), the difference in results between the former and the latter models is marginal. Consequently, while the difference *within* each type of model is minimal (that is, M^{PI} vs. $M\&F^{PI}$, and D^{PI} vs. $D\&F^{PI}$), the differences *between* models remain the same at the 50 percent level. Regarding the difference between technical and allocative inefficiencies in absolute terms, the former doubles the latter in absolute terms on average.

We have also calculated Spearman's correlations between the different pairs of models. Table 5 reports the coefficients for the profit inefficiencies (PIs), allocative inefficiencies (AIs), and the aggregate by-production technical inefficiencies (TIs), thus complementing Table 4's presentation of the correlations for the standard and polluting technologies, T_1 and T_2 .

The ranking correlation *between* Murty et al. and Dakpo et al.'s profit inefficiencies is rather high at $\rho(M^{PI}, D^{PI}) = 0.705$, similar to that between their Førsund extensions: $\rho(M\&F^{PI}, D\&F^{PI}) = 0.749$. Nevertheless, the ranking compatibility is much greater *within* models. Indeed, the correlation between profit inefficiency defined under each type of technological model (either *à la* Murty et al. or Dakpo et al.) and their corresponding Førsund variations is almost perfect at $\rho(M^{PI}, M\&F^{PI}) = 0.999$ and $\rho(D^{PI}, D\&F^{PI}) = 0.969$. Also, reading vertically the profit inefficiency columns (PIs) in Table 5, we learn that they correlate more with their technical inefficiencies components (TIs) than with their allocative inefficiencies (AIs), particularly for the Murty et al. models.

The magnitudes of these correlation coefficients extend to the aggregate by-production technical inefficiencies (TIs), as the correlation *between* the Murty et al. and Dakpo et al. models is $\rho(M^{TI}, D^{TI}) = 0.681$, increasing to $\rho(M\&F^{TI}, D\&F^{TI}) = 0.757$ if their Førsund specifications are considered. Correlation increases again *within* models. That is, the coefficient for the Murty et al. model and its Førsund extension is $\rho(M^{TI}, M\&F^{TI}) = 0.983$, while that for the Dakpo et al. approach and Førsund extension is $\rho(D^{TI}, D\&F^{TI}) = 0.932$. Similarly, the alternative allocative inefficiency rankings (AIs) present rather comparable results, both *between* and *within* models.

Finally, as presented in the lower central panel of Table 5, technical and allocative inefficiencies correlate mildly, albeit positively. Particularly for the Dakpo et al. models: $\rho(D^{TI}, D^{AI}) = 0.777$ and $\rho(D\&F^{TI}, D\&F^{AI}) = 0.697$. This suggests that the economic performance of US states from both technological and allocative perspectives go hand by hand. Indeed, as none of the coefficients are negative, it is possible to dismiss the idea that realized technological and allocative behavior follow opposite ways. This result is expected because profit inefficiency includes marketed outputs, and farms have an incentive to perform well technologically by attaining the maximum feasible quantities of desirable outputs given the technology (that is, livestock and crops), and also to choose their optimal relative quantities (output mix) in order to maximize revenue at market prices. On the contrary, although the environmental side of the profit inefficiency definition, represented by the damage (monetary) cost function, is less binding (to the extent that farmers do not explicitly aim to minimize CO₂ emissions and pesticide exposures, and therefore their environmental cost), its economic values are notably smaller than their marketed private revenue counterparts. Hence the technical and allocative efficiency levels correlate positively, as they are both dominated by the private side of economic performance, actively pursued by the economic agents. We also

Table 5
Spearman's correlation matrix between profit inefficiencies (PI, TI and AI).

		Profit inefficiency (PI)				Technical inefficiency (TI)				Allocative inefficiency (AIs)			
		Murty PI	Dakpo PI	M. & F. PI	D. & F. PI	Murty TI	Dakpo TI	M. & F. TI	D. & F. TI	Murty AI	Dakpo AI	M. & F. AI	D. & F. AI
Profit Inefficiency (PI)	Murty PI	1.000*											
	Dakpo PI	0.705*	1.000*										
	M. & F. PI	0.999*	0.700*	1.000*									
	D. & F. PI	0.749*	0.969*	0.749*	1.000*								
Technical Inefficiency (TI)	Murty TI	0.900*	0.677*	0.900*	0.702*	1.000*							
	Dakpo TI	0.625*	0.945*	0.620*	0.893*	0.681*	1.000*						
	M. & F. TI	0.896*	0.665*	0.899*	0.722*	0.983*	0.657*	1.000*					
	D. & F. TI	0.684*	0.881*	0.683*	0.907*	0.740*	0.932*	0.757*	1.000*				
Allocative Inefficiency (AI)	Murty AI	0.648*	0.416*	0.648*	0.452*	0.397*	0.265**	0.392*	0.280**	1.000*			
	Dakpo AI	0.585*	0.898*	0.580*	0.869*	0.500*	0.777*	0.491*	0.696*	0.579*	1.000		
	M. & F. AI	0.639*	0.409*	0.639*	0.449*	0.390*	0.258**	0.386	0.278**	0.998*	0.579*	1.000*	
	D. & F. AI	0.592*	0.842*	0.593*	0.875*	0.489*	0.702*	0.512	0.697	0.624*	0.963*	0.630*	1.000*

Notes: Murty et al.: (18), Dakpo et al.: (24), Murty et al. & Førsund: (28), Dakpo et al & Førsund: (31).

* p < 0.01; ** p < 0.1.

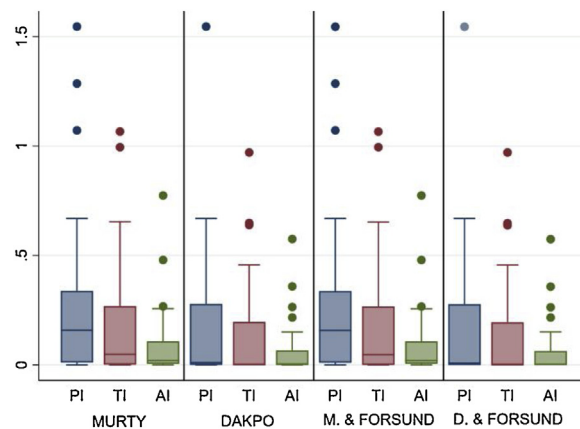


Fig. 5. Box plots of the profit inefficiency distributions by models.

stress that, in this case, most of the correlations are statistically significant at the 1 percent level, particularly within models. Finally, all these results suggest that increased revenue efficiency (associated to the increase in good outputs that higher technically efficiency brings) is compatible with lower environmental costs (associated to the contemporary reduction in bad outputs).

Also, since profit inefficiency is the aggregate resulting from adding technical and allocative inefficiencies ($PI = TI + AI$), it is relevant to highlight its sources in percentage terms. Fig. 4 shows that, on average, TI doubles AI in value. Specifically, in the model characterizing the technology following Murty et al., average TI amounts to 0.182, while AI amounts to 0.087. This 50 percent difference is also observed for the economic model based on Dakpo et al.'s technological characterization: 0.119 vs. 0.056. The same holds for the difference between both types of models assuming the Førsund extension in the polluting technology.

We also study the characteristics of these distributions by resorting to their box-plot representations. One can corroborate the differences *between* Murty et al.'s and Dakpo et al.'s models, and the similarities *within* the same type of model when comparing the former to their Førsund extensions. Focusing initially on the three US states with the worst economic performance, lying above one and half times the interquartile range (IQR) of Murty et al.'s model, Montana presents a profit inefficiency value of 1.545 (five times greater than the mean at 0.260), resulting from the addition of technical inefficiency, 1.066 (whose mean value is 0.182), and allocative inefficiency, 0.479 (0.087). The second and third worst-performing states are North Dakota ($1.285 = 0.511 + 0.774$) and Louisiana ($1.071 = 0.994 + 0.077$), respectively. These results illustrate the high variability in the relative values of the technical and allocative components of overall profit inefficiency across the sample. Nevertheless, it is observed that the IQR for allocative inefficiency, $Q_3^A - Q_1^A = 0.104$, is about half of that observed for technical inefficiency, $Q_3^T - Q_1^T = 0.284$. As for the worst-performing states outside one and a half times the IQR in the Dakpo et al. model, only one (Montana again) incurs the highest profit inefficiency ($1.545 = 0.970 + 0.575$); in this case, about nine times greater than the profit inefficiency mean at 0.167. A similar gap between the technical and allocative IQRs can also be observed in this model. We do not comment further on the results of each type of model enhanced with Førsund's proposal since they closely follow those already presented.

We conclude the empirical section by checking whether these distributions are statistically different. As before, we first test whether the technical and allocative efficiencies components of profit inefficiency are different from each other *within* the same model (M^T vs. M^A , D^T vs. D^A , etc.) following the method proposed by Simar and Zelenyuk (2006). In all models except Dakpo et al.'s, the null hypothesis testing the equality of the densities cannot be rejected, even at the 10 percent level of significance. Hence, technical and allocative inefficiencies are not statistically different in the three models. This can be visually corroborated by comparing the TI and AI distributions in the box-plots corresponding to each model in Fig. 5, or comparing the technical and allocative distributions presented in the left and central panels of Fig. 6.

As for the differences in profit, technical and allocative inefficiencies *between* models (M vs. D, M vs. M&F, etc.), the test of Simar and Zelenyuk (2006) returns varied results on the (in)existence of statistical differences between these distributions, which can be visually anticipated in Fig. 6. The comparison of profit, technical, and allocative inefficiencies for Murty et al.'s model with Dakpo et al.'s model reveals significant differences between them (depending on the models being compared at the 5 percent or 10 percent level of significance) (M^{PI} vs. D^{PI} , M^T vs. D^T , and M^A vs. D^A); this result extends to their Førsund's versions ($M\&F^{PI}$ vs. $D\&F^{PI}$, $M\&F^T$ vs. $D\&F^T$, and $M\&F^A$ vs. $D\&F^A$). Hence we conclude that the choice of model is not neutral when comparing economic performance across states. On the contrary, it turns out that these distributions are the same for Murty et al.'s model and its counterpart extended with Førsund assumption (M^{PI} vs. $M\&F^{PI}$), as well as for Dakpo et al.'s model and its Førsund extension (D^{PI} vs. $D\&F^{PI}$). This is also a remarkable result, implying that, for the whole US agricultural sector, profit inefficiency, including its technical and allocative terms, can be



Fig. 6. Profit inefficiency kernel distributions (PI, TI and AI). Map 1. Profit Inefficiency: Murty et al.'s (2012) model.

equally measured *irrespective* of whether the global underlying production technology T incorporates Førsund's proposal or not.²¹

Finally, Fig. 6 illustrates the profit, technical, and allocative inefficiency results for all 48 states in the sample. As all four models are equally representative, we map the results of the economic model characterizing the global technology following Murty et al. (2012). We visually confirm the existence of several geographical clusters, particularly of large profit inefficiencies in the Pacific Northwest states of Washington, Oregon, Montana, and the Dakotas. This suggests that the agricultural characteristics of their production processes, mainly focused in livestock production, along with market prices, are hampering their economic performance. On the contrary, we could not observe a significant clustering of the economically efficient states: California, Delaware, Iowa and Vermont. While there seems to be visual evidence calling for the application of spatial regression analyses on the inefficiency results, as well as their explanation in terms of the technological specialization and market orientation of the different states, these extensions fall beyond the scope of the current application, which is intended to illustrate the new economic models.

5. Conclusions

This paper introduces the theory and practice of environmental *economic* inefficiency measurement. Environmental economic inefficiency represents the ability of firms to maximize the difference between private revenue and environmental cost given the production technology and market prices. This objective can be likened to a "profit" function that economically weighs private gains and environmental losses by internalizing the damage associated with the production of undesirable outputs. Resorting to duality theory enables us to demonstrate how this (supporting) economic function relates to a technical counterpart represented by the directional distance function, effectively extending the analytical framework of Chambers et al. (1998) to the field of environmental economics. Since the directional distance function can be regarded a measure of technical efficiency, the gap between technical and optimal economic performance can be attributed to allocative inefficiencies. Hence, profit inefficiency can be consistently decomposed into its technical and allocative sources.

²¹ Again, detailed results on the test for profit, technical, and allocative inefficiencies can be found in Appendix A.4. of the online supplemental material accompanying the paper.

The new model builds upon one of the most recent proposals characterizing the production technology in the presence of undesirable outputs; the so-called by-production model put forward by Murty et al. (2012). This analytical framework differentiates between two separate sub-technologies, one corresponding to the conventional (privately oriented) approach and one characterizing the production of pollutants only. Our model makes it possible to assign different weights to each technology in order to account for the modeler or stakeholder preferences (managerial, political, legal/regulatory, etc.). Although the new economic model could be developed adopting other technological characterizations, the by-production approach overcomes prior limitations and is becoming increasingly popular among practitioners. Moreover, it is subject to continuous qualifications such as those recently introduced by Dakpo et al. (2017) and Førsund (2018).

We develop our new model within the data envelopment analysis framework, which allows us to illustrate its empirical viability using a real-life data set on US agriculture for 2004. We implement four models corresponding to the original proposal by Murty et al. (2012), a modified version corresponding to Dakpo et al.'s (2017) qualification that ensures that the projections points for input dimensions are the same in the conventional and polluting technologies, and their corresponding modifications that incorporate Førsund's (2018) proposal of bringing non-contaminating inputs into the polluting technology, so as to allow for substitution effects. The main empirical findings are the following:

- 1) Technical inefficiencies between the Murty et al. and Dakpo et al. models do not generally differ in the case of the conventional technology, either looking at Spearman's correlations or Li tests. On the contrary, although they also yield similar results regarding the polluting technology, they differ statistically from their Førsund extensions. We confirm that technical inefficiencies in the conventional and environmental technologies are unrelated, with the latter being larger than the former. This simply reflects the fact that farmers do not have market incentives to perform better in the environmental side of the production process (that is, reducing environmental costs), as opposed to the conventional side, where falling short from the production frontier results in lower (private) revenue.
- 2) As for the new economic inefficiency framework, statistical differences can be found across the alternative models. Profit inefficiency is generally larger in Murty et al.'s model than in Dakpo et al.'s. This result extends to their technical and allocative components. Our results show that technical inefficiency is generally larger than allocative inefficiency, suggesting that there is more room for economic improvements by taking advantage of the existing technology than by reallocating the relative demand for inputs and outputs given their market prices (that is, the relative specialization in input usage and output production). As for the extension of these two models with Førsund's proposal, no statistically significant differences emerge.

We conclude from these results that, as expected, the analytical approach chosen to evaluate environmental economic efficiency is highly dependent on the technological model upon which it is based. Choosing alternative models leads to significant differences in the magnitude of technical and allocative inefficiency, which may question the credibility of results given their lack of robustness, and lead to contradictions and faulty managerial and policy decision making. Therefore, caution should be exerted when implementing the new analytical framework, which nevertheless opens the door to a whole new range of models capable of internalizing the social cost of environmental damage when assessing economic performance. This is a key extension in the measurement of environmental efficiency that was not available until now.

CRedit authorship contribution statement

Juan Aparicio: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Writing - original draft. **Magdalena Kapelko:** Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Writing - original draft. **José L. Zofio:** Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Writing - original draft, Writing - review & editing.

Acknowledgements

The authors are grateful to the Spanish Ministry for Economy and Competitiveness (*Ministerio de Economía, Industria y Competitividad*), the State Research Agency (*Agencia Estatal de Investigación*), and the European Regional Development Fund (*Fondo Europeo de Desarrollo Regional*) under grants PID2019-105952GB-I00 and ECO2017-82449-P (AEI/FEDER, UE), as well as the National Science Centre in Poland (grant number 2016/23/B/HS4/03398), for providing financial support for this article. The calculations of adapted Li test were made at the Wrocław Centre for Networking and Supercomputing (www.wcss.wroc.pl), grant no. 286.

Appendix A. Supplementary data

Supplementary material related to this article can be found, in the online version, at doi:<https://doi.org/10.1016/j.reseneeco.2020.101185>.

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