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TRADE UNCERTAINTY AND THE TWO-STEP PROCEDURE: THE CHOICE OF NUMERAIRE AND EXACT INDEXATION

BY

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1 INTRODUCTION

Uncertainty is a fact of life in international economics. High volatility of exchange rates and relative prices influences the decisions of private firms and consumers and consequently (the possibilities for) foreign trade (see *e.g.* Ruffin (1974) or Pomery (1984)). Uncertainty can also be quantitative, arising from quotas, boycotts, sanctions, 'voluntary' export restraints or strategic behavior by private competitors, see Van Bergeijk (1989, 1990 and 1991).[1] Recently, Van Marrewijk and Van Bergeijk (1990) formalized the case of trade disruption in a traditional neo-classical model of international trade. They show that (*i*) there is a one-to-one correspondence between the optimal production level and the possibility of trade disruption and (*ii*) that private production does not take place at the social optimum. In deriving this second result they followed the approach used in Batra and Russell (1974). For reasons that will become clear shortly we will refer to this approach as the 'two-step procedure.'

Welfare maximization for a small open economy in a deterministic international trade model is a two-step procedure, as illustrated in Figure 1. The small economy will be faced with given world prices that it cannot influence. Given the available supply of factors of production and the present state of technology the economy can produce any output combination within the production possibility curve $\phi(x)$. In the absence of uncertainty one can now *first* maximize national income measured in world prices, which results in the production point F_p, and *second* choose the consumption point optimally, given this level of national income, which results in the consumption point F_c. The

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1 Recently, Kofman, Viaene and De Vries (1990) performed an empirical investigation into exchange rate and quantitative uncertainty concerning eighteen primary commodities in the period 1957–88 to conclude that 'traditional analysis which takes only price uncertainty into account falls short of empirical facts.'

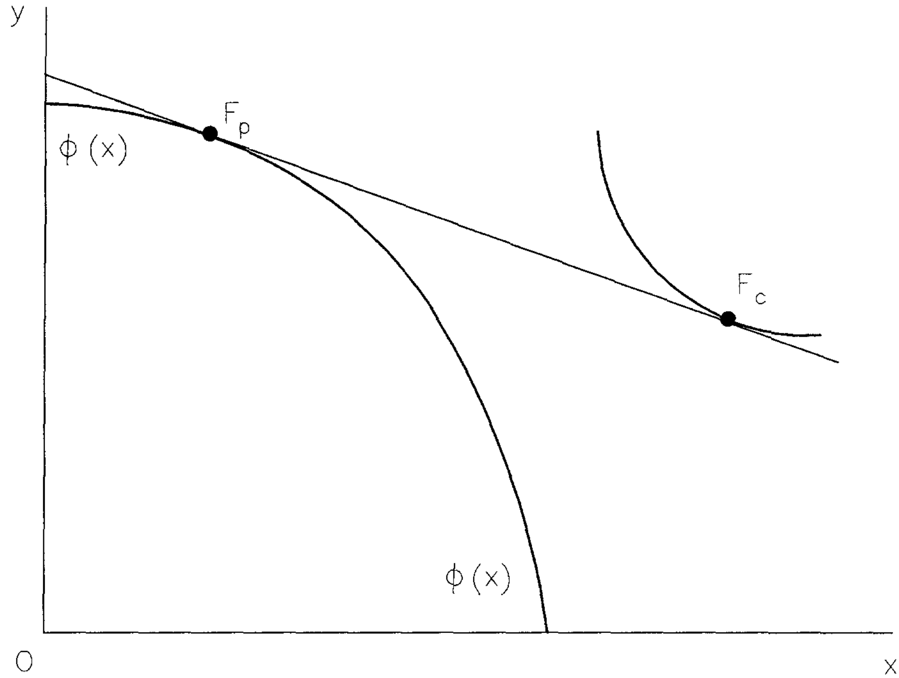

Figure 1 - Illustration of the two-step procedure

optimal trading level, exports and imports, then follows automatically from this two-step procedure as the difference between production and consumption. Under 'suitable' economic conditions (perfect competition, constant returns to scale, *etc.*) this optimal production and consumption-plan is the microeconomically-justified outcome of the economy. The two-step procedure has great analytic advantages, and is therefore frequently used, in modeling international economics in a deterministic world.

Using the two-step procedure in the presence of (price) uncertainty Batra and Russell (*ibid.*) arrived at some counterintuitive results, *e.g.* that free trade may be worse than autarky.[2] Their paper was subsequently critized by Kemp and Ohyama (1978) and Helpman and Razin (1978, chapter 4). The main objection to the two-step procedure in the presence of uncertainty is that production decisions (expected national income maximization) are not necessarily consistent with consumption decisions (the utility function that is maximized). Therefore, production does not take place at the social optimum. To overcome this shortcoming of the model Diamond (1967) introduced the stock market model, see also Helpman and Razin (*ibid.*). In the stock market model firms maximize their market value and firm owners bear risks and need to use their subjective

2 With quantitative uncertainty this is not possible, see Van Marrewijk and Van Bergeijk (*ibid.*).

probabilities in determining their portfolio structures. Firms planning for production to maximize stock market value, however, employ no probabilities in their calculations but rely on the prices inherent in the stock market.[3] The private economy outcome of the stock market model is again socially optimal. Proper microeconomic underpinnings therefore solve Batra and Russell's perverse outcomes at the apparent cost of losing the two-step procedure.

This paper extends the Van Marrewijk and Van Bergeijk (*ibid.*) paper in two ways. First, we show how social and private decisions diverge in the presence of uncertainty if the two-step procedure is used by showing that the outcome depends on the choice of numéraire. Second, the first result suggests a way to rectify the two-step procedure, without resorting to the introduction of a stock market, such that private production again takes place at the social optimum. This is done by looking at the appropriate prices, *i.e.* nominal prices deflated with the price index, instead of using one of the goods as numéraire. The next section introduces the model. Section 3 derives the numéraire dependency result of the two-step procedure. Section 4 shows that deflating with the proper price index is again socially optimal. Section 5 illustrates the results from the previous two sections by giving a numerical example. Section 6 concludes, while the basic result is generalized to an arbitrary number of commodities in the appendix.

2 THE MODEL

In order not to complicate the analysis unnecessarily we investigate a small country that cannot influence its terms of trade. Uncertainty arises through the possibility of a full trade disruption (with probability 1-π), in which case no trade at all takes place. This setting, used in Van Marrewijk and Van Bergeijk (*ibid.*), which is similar to Bhagwati and Srinivasan (1976), avoids the problems arising from distinguishing between *ex ante* and *ex post* trading decisions, see Helpman and Razin (*ibid.*). There are two goods, x and y. The strictly concave production possibility curve is given by $y=\phi(x)$. The utility function, U, is concave with non-negative cross-marginal utility ($U_{xy}\geq 0$). First, as in Chang, Ethier and Kemp (1980), the country has to decide on the production combination $(x, \phi(x))$; then it becomes clear whether trade at international prices will take place (with probability π) or not (with probability 1-π). We assume that the country has a comparative advantage in the production of good y and that $0<\pi<1$. Let x_f be the free trade production level of good x (optimal production if π were 1), x_a the autarky production level of good x (optimal production if π were 0) and, finally, let p be the given world relative price of good x in terms of good y, *i.e.* $p=p_x/p_y$.[4] Sub-indexes of functions are derivatives.

The problem facing the economy is illustrated in Figure 2. If the country

3 This parallels the contingent commodity approach, see Arrow (1964) and Debreu (1959).

4 Since the economy has a comparative advantage in the production of good y, we have $x_f<x_a$.

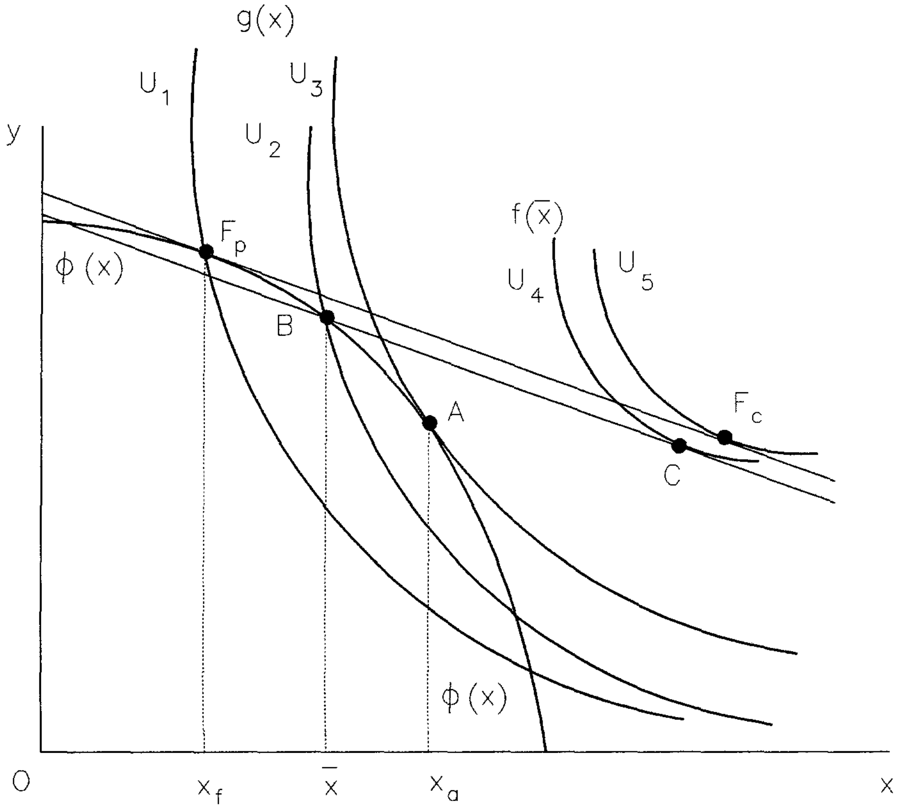

Figure 2 – Graphical illustration of the model

never engages in trade the best it can do is produce and consume at point A, reaching autarky utility level U_3. On the other hand, if trade is *never* disrupted the economy will produce at the free trade production point F_p, consume at the free trade consumption point F_c and reach utility level $U_5 > U_3$. Trade will, however, *sometimes* be disrupted and *sometimes* not. If the economy produces at the free trade production point F_p and trade becomes disrupted then the country can only reach utility level $U_1 < U_3$. Suppose, then, that the economy produces at the point $(\bar{x}, \phi(\bar{x}))$. If *no* trade takes place maximum attainable utility would be given by $U_2 = U(\bar{x}, \phi(\bar{x})) \equiv g(\bar{x})$, which is called the *isolation welfare function.*[5] If trade *does* take place maximum utility would be U_4, which is given by the indirect utility function $\vartheta(p, z(\bar{x})) \equiv f(\bar{x})$, where

5 We reserve the term 'autarky' for a country that freely chooses not to engage in trade. It optimally produces at the point x_a. The term 'isolation' then refers to an economy that wants to engage in trade but is a victim of a trade disruption.

$z(\bar{x}) \equiv p\bar{x} + \phi(\bar{x})$ is the real income function and f is called the *undisrupted trade welfare function*.[6] Both the isolation welfare function and the undisrupted trade welfare function are, under our assumptions, strictly concave, see Van Marrewijk and Van Bergeijk (*ibid.*). Obviously, an increase in the production of good x increases the undisrupted trade welfare function f (the isolation welfare function g) up to the free trade production point x_f (the autarky production point x_a) and decreases thereafter. When the economy (its government, an 'omniscient central planner' *etc.*) maximizes the Von Neumann-Morgenstern (1944) expected utility it must choose the production point $(x, \phi(x))$ so as to solve

Problem 1

$$\max_x \pi f(x) + (1-\pi)g(x)$$

$$\text{FOC } \pi f_x(x) + (1-\pi)g_x(x) = 0 \tag{1}$$

Equation (1) gives the necessary and sufficient condition for solving problem 1. Let x_0 be the optimal production level of good x. In this case we have $x_f < x_0 < x_a$.[7] Naturally, the optimal production level of good x approaches the free trade (autarky) production level if the probability of trade disruption approaches zero (one).

3 NUMERAIRE DEPENDENCY

With some exceptions, *e.g.* Anderson and Riley (1976), most trade uncertainty literature does not pay much attention to the choice of numéraire, see *e.g.* Pomery (1984) and Grinols (1987). As we will see, however, the choice of numéraire can be of vital importance. Suppose our private economy follows the two-step procedure in that it maximizes expected revenue using one of the goods as numéraire. Then it does not produce at the social optimum. In fact, it overproduces the non-numéraire good.

Let p^d denote the domestic relative price of good x if no trade takes place. If producers maximize expected revenue taking prices as given while good y is the numéraire they solve

Problem 2

$$\max_x \pi[px + \phi(x)] + (1-\pi)[p^d x + \phi(x)]$$

6 Not to be confused with the trade welfare function that has imports and exports as its arguments.

7 If the probability of trade disruption is not exogenous but depends on the level of trade it is possible for the optimal production of good x to exceed the autarky level, see Van Marrewijk (1990).

$$\text{FOC } \pi p + (1-\pi)p^d + \phi_x(x) = 0 \tag{2}$$

If no trade occurs domestic prices will have to adjust such that consumers want to consume at the point $(x, \phi(x))$, *i.e.* the domestic relative price equals the marginal rate of substitution: $p^d = U_x(x, \phi(x))/U_y(x, \phi(x)) \equiv p^d(x)$. Note that $p_x^d(x) < 0$. The private economy equilibrium is reached when producers want to produce at the point generated by the domestic prices. Define the function $\gamma(x|y) \equiv \pi p + (1-\pi)p^d(x) + \phi_x(x)$, then the private economy equilibrium when good y is the numéraire solves $\gamma(x|y) = 0$. Let x_p^y be the solution. Note that $\gamma_x(x|y) = (1-\pi)p_x^d(x) + \phi_{xx}(x) < 0$, so x_p^y is unique.[8]

Define the function $\gamma(x|x) \equiv [\pi/p + (1-\pi)/p^d(x)]\phi_x(x) + 1$ and let x_p^x be the private production of good x if producers maximize expected revenue taking prices as given while good x is the numéraire. Similar calculations as before show that $\gamma(x_p^x|x) = 0$. Note that $\gamma_x(x|x) = [\pi/p + (1-\pi)/p^d(x)]\phi_{xx}(x) - (1-\pi)\phi_x(x)p_x^d(x)/[p^d(x)]^2 < 0$, so x_p^x is also unique.[9]

Proposition 1

If the private economy maximizes expected revenue taking prices as given and using one of the goods as numéraire it produces too much of the non-numéraire good, *i.e.* $x_p^x \leq x_0 \leq x_p^y$.

Proof

Note that $f_x(x) = \vartheta_z(p, z(x))z_x(x) = U_y(C_x^*, C_y^*)(p + \phi_x(x))$, where (C_x^*, C_y^*) is the optimal consumption point at price p and income $z(x)$. This is because ϑ_z equals the Lagrange multiplier (of the utility maximization problem) which equals U_y since y serves as numéraire. Let, for notional convenience U_y (U_y^*) denote the marginal utility of good y evaluated at the point $(x, \phi(x))$ (at (C_x^*, C_y^*) respectively). If we divide the first-order condition of problem 1 by U_y, use the definition of domestic prices and rearrange terms, it can be shown, see Van Marrewijk and Van Bergeijk (*ibid*), that utility is maximized if and only if

$$\gamma(x|y) + \delta(x|y) = 0, \tag{3}$$

where $\delta(x|y) \equiv \pi(U_y^* - U_y)(p + \phi_x(x))/U_y$. Now for $x \geq x_p^y$ we have $\gamma(x|y) \leq 0$ and $\delta(x|y) < 0$, therefore such an x cannot solve equation (3). We therefore must have $x_0 < x_p^y$. Similar reasoning leads to $x_p^x < x_0$. □

Proposition 1 shows that over- or under-specialization in the economy depends on the choice of numéraire. It is therefore of vital importance for the government to know which good serves as numéraire for the private economy in order to use the appropriate policy instrument.

8 We have $x_f < x_p^y < x_a$ because $\gamma(x_f|y) > 0$ and $\gamma(x_a|y) < 0$.

9 We also have $x_f < x_p^x < x_a$.

4 PRICE INDEXATION

We have seen in section 3 that taking one of the goods as numéraire leads to overproduction of the other good. But why would producers take one of the goods as numéraire? One can assume that they are aware of the disadvantages of overemphasizing the importance of one of the goods. Should they not look at the real prices of both goods? General price indices are reported very frequently, so it is only natural to assume that producers deflate nominal prices with the price index in maximizing expected national income. This makes private decisions again optimal, as we will see shortly, without having to resort to the introduction of stock markets, as in Diamond (*ibid.*) and Helpman and Razin (*ibid.*). We will restrict attention to neo-classical utility functions to make the use of exact price indices possible. A quick refresher course on price indices now follows.

Let $\boldsymbol{x}=(x, y)$ be a quantity vector and $\boldsymbol{p}=(p_x, p_y)$ be a price vector, where p_x (p_y) is the nominal price of good x (y). A utility function is neo-classical if it is (*i*) positive, (*ii*) positively linearly homogeneous and (*iii*) concave. Expenditure minimization to reach utility U at prices $\boldsymbol{p}$, $E(U, \boldsymbol{p})$, solves

Problem 3

$$E(U, \boldsymbol{p})=\min_{x}\{\boldsymbol{px} \mid U(\boldsymbol{x}) \geq U, \boldsymbol{x} \geq 0\}=UE(1, \boldsymbol{p})=Ue(\boldsymbol{p}) \tag{4}$$

where e, the unit utility expenditure function, is a neo-classical function as well. The price index $PI({}_0\boldsymbol{p}, {}_1\boldsymbol{p}, {}_0\boldsymbol{x}, {}_1\boldsymbol{x})$ is called *exact*, see Diewert (1981), if and only if for all price and quantity pairs $({}_0\boldsymbol{p}, {}_0\boldsymbol{x})$ and $({}_1\boldsymbol{p}, {}_1\boldsymbol{x})$, which are utility maximizers, the following holds: $PI({}_0\boldsymbol{p}, {}_1\boldsymbol{p}, {}_0\boldsymbol{x}, {}_1\boldsymbol{x})=e({}_1\boldsymbol{p})/e({}_0\boldsymbol{p})$. Furthermore, we have

$$\begin{aligned} &{}_jp_x/E(U, {}_j\boldsymbol{p})=U_x({}_j\boldsymbol{x})/U({}_j\boldsymbol{x}), \text{ and} \\ &{}_jp_y/E(U, {}_j\boldsymbol{p})=U_y({}_j\boldsymbol{x})/U({}_j\boldsymbol{x}), \text{ for } i=0,1 \end{aligned} \tag{5}$$

Utility maximization (given income z and prices p_{x} and p_y) solves

Problem 4

$$\begin{aligned} &\max_{x, y} U(x, y) \text{ s.t. } p_x x+p_y y=z \\ &\text{FOC } U_x(x, y)=\lambda p_x \\ &\qquad U_y(x, y)=\lambda p_y \\ &\qquad p_x x+p_y y=z \end{aligned} \tag{6}$$

where λ is the Lagrange multiplier. Furthermore, the indirect utility function is given by $\vartheta(p_x, p_y, z)=U(x(p_x, p_y, z), y(p_x, p_y, z))$, with $\vartheta_z(p_x, p_y, z)=\lambda$, the

Lagrange multiplier of problem 4. Income is given by $z(x) = p_y\phi(x) + p_x x$, therefore $z_x(x) = p_y\phi_x(x) + p_x$. Hence

$$f(x) = \vartheta(p_x, p_y, z(x))$$

$$f_x(x) = \vartheta_z(p_x, p_y, z(x))z_x(x) = \vartheta_z(p_x, p_y, z(x))[p_y\phi_x(x) + p_x] \quad (7)$$

and

$$g(x) = U(x, \phi(x))$$

$$g_x(x) = U_x(x, \phi(x)) + U_y(x, \phi(x))\phi_x(x) \quad (8)$$

In the sequel partial derivatives, utility and expenditure levels evaluated at the trade (isolation) consumption point will be identified with a t, for trade, (d, for domestic) in parentheses. Because f and g are strictly concave the first-order conditions are necessary and sufficient and the optimal production point solves $\pi f_x(x) + (1-\pi)g_x(x) = 0$. Using equations (6), (7) and (8) this can be written as

$$\pi[U_y(t)\phi_x(x) + U_x(t)] + (1-\pi)[U_y(d)\phi_x(x) + U_x(d)] = 0 \quad (9)$$

Let the exact price index if trade occurs be given by PI_t and if trade is disrupted by PI_d. Domestic prices if trade is disrupted are given by ${}_dp_x$ and ${}_dpy$. Producers maximizing expected national income taking real prices as given solve

Problem 5

$$\max_x (\pi/PI_t)[p_y\phi(x) + p_x x] + [(1-\pi)/PI_d][{}_dp_y\phi(x) + {}_dp_x x]$$

$$\text{FOC } (\pi/PI_t)[p_y\phi_x(x) + p_x] + [(1-\pi)/PI_d][{}_dp_y\phi_x(x) + {}_dp_x] = 0 \quad (10)$$

Let $E(t)$ denote expenditures if trade occurs and $E(d)$ if not. The same holds for utility levels $U(t)$ and $U(d)$. Use equation (5) to rewrite (10)

$$(\pi E(t)/U(t)PI_t)[U_y(t)\phi_x(x) + U_x(t)] +$$

$$[(1-\pi)E(d)/U(d)PI_d][U_y(d)\phi_x(x) + U_x(d)] = 0 \quad (11)$$

Multiply both sides by $U(d)PI_d/E(d)$ and use the fact that the price index is exact, such that $E(t)U(d)PI_d/E(d)U(t)PI_t = 1$, to get

$$\pi[U_y(t)\phi_x(x) + U_x(t)] + (1-\pi)[U_y(d)\phi_x(x) + U_x(d)] = 0 \quad (12)$$

Equations (9) and (12) are the same, hence[10]

10 Recall that the solution to the optimization problem, and therefore the root of equation (9), is unique.

Proposition 2
If the private economy maximizes expected revenue taking nominal prices deflated with the exact price index as given it produces at the social optimum.

Proposition 2 is generalized to an arbitrary number of goods and illustrated for Cobb-Douglas preferences in the appendix below. It should be noted that the result derived in proposition 2 depends critically on the properties of an *exact* price index. This implies that the Lagrangian multiplier of the expenditure minimization problem is just $e(p)$. Hence, when we deflate by an ideal price index we are adjusting by something which is proportional to the marginal utility of income. Deflated prices then give the correct production signals to the economy. This is also true if there is no uncertainty because then the producers' objective function is just divided by some constant, which does not affect the outcome of the optimization process. Hence, under ideal situations we have shown equivalence between the two-step procedure and the introduction of stock markets.[11]

5 NUMERICAL EXAMPLE

This section will give a numerical example, taken from Van Marrewijk and Van Bergeijk, to illustrate the stated propositions. The results derived in Sections 3 and 4 also shed more light on Van Marrewijk and Van Bergeijk's (*ibid.*, p. 24) observation that

> 'One might be tempted to argue that the disoptimality problem arises from the fact that producers maximize expected profits, and hence are risk-neutral, whereas consumers are risk-averse. This is not true.'

They then give an example, also used below, with risk-neutral consumers (in the Arrow-Pratt sense) but divergent social and private production points. Proposition 1 in Section 3 showed that it is the choice of numéraire that causes the divergence, while proposition 2 in Section 4 showed that the use of a proper price index, if consumers are risk-neutral in the Arrow-Pratt sense, eliminates the non-optimality problem. Let the utility function be

$$U(x, y) = x^{1/2}y^{1/2} \tag{13}$$

and the production possibility frontier

$$\phi(x) = (1 - x^2)^{1/2} \tag{14}$$

Income measured in y at international prices is

11 In applications the two-step procedure might then be used as a first approximation.

$$z(x) = px + (1 - x^2)^{1/2} \tag{15}$$

Take the world relative price of good x in terms of good y to be 1/4, *i.e.* $p = 1/4$. This gives the indirect utility function

$$\vartheta(p_x, p_y, z) = z \tag{16}$$

Hence the undisrupted trade welfare function becomes

$$f(x) = (1 - x^2)^{1/2} + x/4 \tag{17}$$

with derivative

$$f_x(x) = -x(1 - x^2)^{-1/2} + 1/4 \tag{18}$$

The isolation welfare function is

TABLE 1 — OPTIMAL PRODUCTION OF GOOD x (x_0) AND PRIVATE PRODUCTION OF GOOD x IF GOOD x IS THE NUMÉRAIRE (x_p^x), IF GOOD y IS THE NUMÉRAIRE (x_p^y) AND IF NOMINAL PRICES ARE DEFLATED BY A PRICE INDEX (x_p) AS A FUNCTION OF THE PROBABILITY OF TRADE (π).

π	x_p^y	$x_0 = x_p$	x_p^x
0	.707	.707	.707
.05	.700	.693	.679
.1	.693	.679	.650
.15	.685	.664	.619
.2	.677	.648	.587
.25	.668	.631	.555
.3	.659	.613	.523
.35	.648	.595	.492
.4	.637	.575	.463
.45	.625	.554	.436
.5	.611	.533	.410
.55	.596	.511	.386
.6	.580	.487	.364
.65	.561	.462	.344
.7	.540	.437	.326
.75	.516	.409	.309
.8	.487	.381	.293
.85	.453	.350	.279
.9	.409	.318	.266
.95	.349	.282	.254
1	.243	.243	.243

$$g(x) = x^{1/2}(1 - x^2)^{1/4} \tag{19}$$

with derivative

$$g_x(x) = (1/2)x^{-1/2}(1 - x^2)^{-3/4}(1 - 2x^2) \tag{20}$$

The probability of free trade equals π, so the optimality problem solves

$$\pi f_x(x) + (1 - \pi)g_x(x) = 0 \tag{21}$$

Changes in π reflect changes in uncertainty. An active embargo policy, political instability, unbalanced capital flows, the reputation of the trading partners or an active 'voluntary' export restraint policy may decrease the trust in free trade and, consequently, reduce the potential for trade and shift x in the direction of x_a. Since ${}_dp_x/{}_dp_y \equiv p^d(x) = U_x(x, \phi(x))/U_y(x, \phi(x)) = (1 - x^2)^{1/2}x^{-1}$, the private economy equilibrium solves

$$\pi/4 + (1\text{-}\pi)(1 - x^2)^{1/2}x^{-1} - x(1 - x^2)^{-1/2}) = 0 \tag{22}$$

if good y is the numéraire and

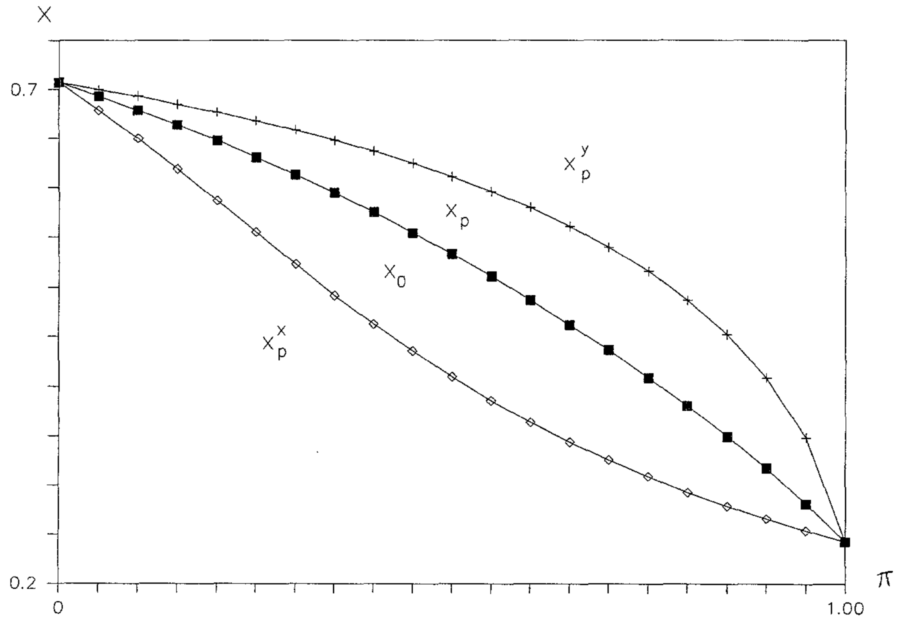

Figure 3 - Optimal production of good x (x_0) and private production of good x if good x is the numéraire (x_p^x), if good y is the numéraire (x_p^y) and if nominal prices are deflated by a price index (x_p) as a function of the probability of trade (π)

$$1 - x(1 - x^2)^{-1/2}(4\pi + (1 - \pi)x(1 - x^2)^{-1/2}) = 0 \tag{23}$$

if good x is the numéraire. The *MathCad*(c) program was used to solve numerically for x_0, x_p^x and x_p^y. The parameters chosen in this example imply that the free trade production level of good x equals 0.243 ($x_f = .243$) and the autarky production level of good x equals 0.707 ($x_a = .707$). The outcomes for various values of the probability of trade parameter π are reported in Table 1 and illustrated in Figure 3. Clearly, private decisions are not optimal if one of the goods is used as numéraire, since x_p^x and x_p^y deviate from x_0. The divergence between private and optimal decisions in those cases only becomes negligible in the neighborhood of extreme values for π, *i.e.* for π close to either zero or one. In contradistinction, proper deflating of prices leads to optimal outcomes, *i.e.* $x_0 = x_p$.

6 CONCLUSIONS

A small open economy facing given terms of trade and an uncertain volume of trade is studied. In a deterministic international trade model we can first maximize national income and then choose the consumption point optimally. The same two-step procedure, which is frequently used in a deterministic setting, has also been applied to the case of uncertainty. This paper derives two results. First, it is shown that the two-step procedure leads to suboptimal outcomes in the presence of uncertainty if one of the goods is used as numéraire. In reality, however, firms do not use a specific good as numéraire. Instead, they are confronted with a monetary world and nominal prices. Being trained economists, not blinded by money illusion, they realize that these nominal prices will have to be deflated by a price index. The second result in the paper shows that if producers maximize national output taking nominal prices deflated by the exact price index as given, then private production takes place at the social optimum. Hence, there is equivalence between the 'appropriate' two-step procedure and the introduction of a stock market, see Diamond (*ibid.*), under ideal circumstances.

APPENDIX

This appendix will first generalize proposition 2 derived in section 4 to an arbitrary number of goods. Then it will illustrate this generalization by giving an example with Cobb-Douglas utility. We use the following notation

$_0\boldsymbol{x}$, $_1\boldsymbol{x}$ and $\boldsymbol{x}$ – are strictly positive column vectors of n goods
$_0\boldsymbol{p}$, $_1\boldsymbol{p}$ and $\boldsymbol{p}$ – are strictly positive row vectors of n prices

Expenditure minimization problem 3 becomes

Problem 3'

$$E(U,p) = \min_x \{px \mid U(x) \geq U, x \geq 0\} = UE(1,p) = Ue(p) \tag{24}$$

where e, the unit utility expenditure function, is a neo-classical function as well. The definition of the exact price index remains the same and equation (5) becomes

$$_jp_i/(_jp_j x) = U_{xi}(_jx)/U(_jx), \text{ for } i = 0, 1 \tag{25}$$

Suppose there are $n+1$ goods. Let $X \subset \mathbb{R}^n_+$ contain the origin and be compact and convex. Let ϕ: $X \to \mathbb{R}_+$ be the strictly concave production possibility function, *i.e.* the production possibility set $\{(x_0, x) \in \mathbb{R}^{n+1}_+ | x \in X \text{ and } 0 < x_0 < \phi(x)\}$ is strictly convex. Let $U(x_0, x)$ be the neo-classical utility function, p_i be the (given) world price of good i and $p = (p_1, \ldots, p_n)$. Utility maximization solves

Problem 4'

$$\max_{x_0, x} U(x_0, x) \ s.t. \ p_0 x_0 + px = z$$

$$\text{FOC } U_{x0}(x_0, x) = \lambda p_0$$

$$U_x(x_0, x) = \lambda p \tag{26}$$

$$p_0 x_0 + px = z$$

where λ is the Lagrange multiplier. Furthermore, the indirect utility function is given by $\vartheta(p_0, p, z) = U(x_0(p_0, p, z), x(p_0, p, z))$, with $\vartheta_z(p_0, p, z) = \lambda$, the Lagrange multiplier of problem 4'. Income is given by $z(x) = p_0\phi(x) + px$, therefore $z_x(x) = p_0\phi_x(x) + p$. Hence

$$f(x) = \vartheta(p_0, p, z(x))$$

$$f_x(x) = \vartheta_z(p_0, p, z(x))z_x(x) = \vartheta_z(p_0, p, z(x))[p_0\phi_x(x) + p] \tag{27}$$

and

$$g(x) = U(\phi(x), x)$$

$$g_x(x) = U_{x0}(\phi(x), x)\phi_x(x) + U_x(\phi(x), x) \tag{28}$$

Again partial derivatives, utility and expenditure levels evaluated at the trade (isolation) consumption point will be identified with a t, for trade, (d, for domestic) in parentheses. Because f and g are strictly concave, see Van Marrewijk and Van Bergeijk, the first-order conditions are again necessary and sufficient and the optimal production point solves $\pi f_x(x) + (1\text{-}\pi) g_x(x) = 0$. Using equations (26), (27) and (28) this can be rewritten as

$$\pi[U_{x0}(t)\phi_x(x)+U_x(t)]+(1-\pi)[U_{x0}(d)\phi_x(x)+U_x(d)]=0 \tag{29}$$

Let the exact price index if trade occurs be given by PI_t and if trade is disrupted by PI_d. Domestic prices if trade is disrupted are given by ${}_dp_i$ and ${}_d\boldsymbol{p}=({}_dp_1,\ldots,{}_dp_n)$. Producers maximizing expected national income taking real prices as given solve

Problem 5'

$$\max_{x}\,(\pi/PI_t)[p_0\phi(x)+px]+[(1-\pi)/PI_d][{}_dp_0\phi(x)+{}_dpx]$$

$$\text{FOC}\ (\pi/PI_t)[p_0\phi_x(x)+p]+[1-\pi)/PI_d][{}_dp_0\phi_x(x)+{}_dp]=0 \tag{30}$$

Recall that $E(t)$ are expenditures if trade occurs and $E(d)$ if not. The same holds for utility levels $U(t)$ and $U(d)$. Use equation (25) to rewrite (30)

$$(\pi E(t)/U(t)PI_t)[U_{x0}(t)\phi_x(x)+U_x(t)]+$$
$$[(1-\pi)E(d)/U(d)PI_d][U_{x0}(d)\phi_x(x)+U_x(d)]=0 \tag{31}$$

Multiply both sides by $U(d)PI_d/E(d)$ and use the fact that the price index is exact, such that $E(t)U(d)PI_d/E(d)U(t)PI_t=1$, to get

$$\pi[U_{x0}(t)\phi_x(x)+U_x(t)]+(1-\pi)[U_{x0}(d)\phi_x(x)+U_x(d)]=0 \tag{32}$$

Equations (29) and (32) are the same, hence

Proposition 2'

If there is an arbitrary, but finite, number of goods and the private economy maximizes expected revenue taking nominal prices deflated with the exact price index as given, then the private economy produces at the social optimum.

Now we will look at an example. Let $U(x_0,x)$ be a Cobb-Douglas utility function with positive expenditure shares α_i, *i.e.* $U(x_0,x)=\prod_{i=0}^{n}(x_i^{\alpha_i})$ and $\sum_{i=0}^{n}(\alpha_i)=1$. *Let* p_i be the (given) world price of good i (normalized to one for good zero) and let $p=(p_1,\ldots,p_n)$. The exact price index P is given by $P=\prod_{i=0}^{n}(p_i^{\alpha_i})$. Income z is given by $z(x)=\phi(x)+px$. Define $\theta\equiv\prod_{i=0}^{n}(\alpha_i^{\alpha_i})$. Then the direct utility function can be written as

$$\vartheta(p,z)=\theta z/P \tag{33}$$

Therefore

$$f(x)=(\theta/P)[\phi(x)+px] \tag{34}$$

Clearly

$$g(x) = [\phi(x)]^{\alpha_0} \prod_{i=1}^{n} (x_i^{\alpha_i}) \tag{35}$$

So we get the following

Optimality Problem

$$\max_{x} \pi f(x) + (1-\pi) g(x)$$

$$\text{FOC } \pi f_{xi}(x) + (1-\pi) g_{xi}(x) = 0, \; i = 1, \dots, n \tag{36}$$

Equations (36) reduce to

$$(\pi\theta/P)[\phi_{xi}(x) + p_i] + (1-\pi) g(x)[(\alpha_0/\phi(x))\phi_{xi}(x) + \alpha_i/x_i] = 0 \tag{37}$$

for $i = 1, \dots, n$. Again let ${}_dp_i$ be the domestic price of good i if no trade takes place and let $P_d \equiv \prod_{i=0}^{n} [({}_dp_i)^{\alpha_i}]$ be the price index. Then we get the

Producers' Problem

$$\max_{x} (\pi/P)[\phi(x) + px] + (1-\pi)[({}_dp_0/P_d)\phi(x) + \textstyle\sum_{i=0}^{n} ({}_dp_i/P_d)x_i]$$

$$\text{FOC } (\pi/P)[\phi_{xi}(x) + p_i] + (1-\pi)[({}_dp_0/P_d)\phi_{xi}(x) + {}_dp_i/P_d] = 0 \tag{38}$$

for $i = 1, \dots, n$. The private economy equilibrium is reached when producers want to produce at the point generated by domestic prices.

$$(\pi/P)[\phi_{xi}(x) + p_i] +$$
$$(1-\pi)[({}_dp_0(x)/P_d(x))\phi_{xi}(x) + {}_dp_i(x)/P_d(x)] = 0 \tag{39}$$

for $i = 1, \dots, n$. From utility maximization we know ${}_dp_i/{}_dp_j = \alpha_i x_j/(\alpha_j x_i)$ for all $i, i = 0, \dots, n$. Using this and the definition of P_d we get

$${}_dp_i/P_d = \alpha_i g(x)/(\theta x_i), \text{ for } i = 0, \dots, n \tag{40}$$

Therefore, equations (39) reduce to

$$(\pi/P)[\phi_{xi}(x) + p_i] +$$
$$[(1-\pi) g(x)/\theta][(\alpha_0/\phi(x))\phi_{xi}(x) + \alpha_i/x_i] = 0 \tag{41}$$

for $i = 1, \dots, n$. Equations (41) are clearly equivalent to equations (37), so the private economy produces at the social optimum.

REFERENCES

Anderson, J.E. and J.G. Riley, 1976, 'International Trade with Fluctuating Prices,' *International Economic Review*, 17, pp. 76–97.

Arrow, K.J., 1964, 'The Role of Securities in the Optimal Allocation of Risk-Bearing,' *Review of Economic Studies*, 31, pp. 91-96.

Batra, R.N. and W.R. Russell, 1974, 'Gains from Trade under Uncertainty,' *American Economic Review*, 64, pp. 1040-1048.

Bergeijk, P.A.G. van, 1989, 'Success and Failure of Economic Sanctions,' *Kyklos*, 42, pp. 385-404.

Bergeijk, P.A.G. van, 1990, *Handel en diplomatie* (*Trade and Diplomacy*), Ph.D. Dissertation, University of Groningen.

Bergeijk, P.A.G. van, 1991, *Handel, politiek en handelspolitiek* (*Trade, Politics and Trade Politics*), Den Haag.

Bhagwati, J.N. and T.N. Srinivasan, 1976, 'Optimal Trade Policy and Compensation under Endogenous Uncertainty: The Phenomenon of Market Disruption,' *Journal of International Economics*, 6, pp. 317-336.

Chang, W.W., W.J. Ethier and M.C. Kemp, 1980, 'The Theorems of International Trade with Joint Production,' *Journal of International Economics*, 10, pp. 377-394.

Debreu, G., 1959, *Theory of Value*, New York.

Diamond, P.A., 1967, 'The Role of a Stock Market in a General Equilibrium Model with Technological Uncertainty,' *American Economic Review*, 57, pp. 171-179.

Diewert, W.E., 1981, 'The Economic Theory of Index Numbers: A Survey,' in: A. Deaton (ed.), *Essays in the Theory and Measurement of Consumer Behavior*, Cambridge *etc.*, pp. 163-208.

Grinols, E.L., 1987, *Uncertainty and the Theory of International Trade*, H. Chur.

Helpman, E. and A. Razin, 1978, *A Theory of International Trade under Uncertainty*, New York, *etc.*

Kemp, M.C. and M. Ohyama, 1978, 'The Gain from Free Trade under Conditions of Uncertainty,' *Journal of International Economics*, 8, pp. 139-141.

Kofman, P., J.-M. Viaene and C.G. de Vries, 1990, 'Primary Commodity Prices and Exchange Rate Volatility,' in: L.A. Winters and D. Sapsford (eds.), *Primary Commodity Prices: Economic Models and Policy*, Cambridge, *etc.*, pp. 213-239.

Marrewijk, C. van and P.A.G. van Bergeijk, 1990, 'Trade Uncertainty and the Volume of Trade,' *De Economist*, 138, pp. 15-32.

Marrewijk, C. van, 1990, 'Endogenous Trade Uncertainty and Specialization against Comparative Advantage,' Institute of Economic Research, Research Memorandum nr. 367, University of Groningen.

Neumann, J. von and O. Morgenstern, 1944, *Theory of Games and Economic Behavior*, Princeton.

Pomery, J., 1984, 'Uncertainty in Trade Models,' in: R.W. Jones and P.B. Kenen (eds.), *Handbook of International Economics*, Vol. I, Amsterdam, pp. 419-465.

Ruffin, R.J., 1974, 'International Trade under Uncertainty,' *Journal of International Economics*, 4, pp. 243-259.

Summary

TRADE UNCERTAINTY AND THE TWO-STEP PROCEDURE: THE CHOICE OF NUMÉRAIRE AND EXACT INDEXATION

In a small open economy it is optimal to first maximize national income and second choose the best consumption point.

The same two-step procedure under (quantitative) uncertainty is suboptimal if one of the goods is used as numéraire. Optimality is restored however, if nominal prices are deflated by the exact price index. Hence there is equivalence between the 'appropriate' two-step procedure and the introduction of a stock market under uncertainty (Diamond 1967) under ideal circumstances.