

Growth, Budget Deficits, and Fiscal Policies in an Overlapping Generations Model

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Analyzing an overlapping generations model with growing endowments and a government sector that is permitted to have a budget deficit forces the real interest factor to deviate from Samuelson's biological interest factor. Fiscal policies then affect the real interest factor, which in turn has consequences both for the direction and the effectiveness of those fiscal policies. These consequences depend upon the borrowing position of the young.

1. Introduction

The overlapping generations model introduced by Samuelson (1958) has been used extensively for the analysis of alternative government fiscal policies. Important extensions of the model were the introduction of storable output by Cass and Yaari (1966) and of production by Diamond (1965) and again by Cass and Yaari (1967). An important extension, the bequest motive, was introduced by Barro (1974) and generalized by Buiter (1979). Building on the exchange variety of the model the main objective of the paper is to analyze the implications of the presence of a government budget deficit on

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alternative fiscal policies and on changes in wealth and growth of the economy. Aggregate endowments grow due to population growth and growth of individual endowments. The existence of a government budget deficit forces the Samuelson biological rate of interest to deviate from the real interest rate. This deviation has remarkable consequences for the influence of different fiscal policies. These fiscal policies are analyzed in the presence of a constant government budget deficit and compared with the same fiscal policies in the case of a balanced budget. Section 2 introduces the model. Section 3 analyzes existence and stability. Section 4 discusses the influence of parametric changes on the real interest factor and section 5 discusses the influence of different fiscal policies. Section 6 concludes.

2. The Model

Consider a growing two-period overlapping generations exchange economy with a government sector. Growth occurs in two ways: the population grows at a rate of $(\pi - 1)$ every period and per capita endowments grow at a rate of $(\lambda - 1)$. Government expenditures and taxes are assumed to grow at a rate $(\gamma - 1)$. We will assume that the government never has a budget surplus. Any deficits, now or from the past, are financed by issuing bonds with a fixed positive nominal interest rate of $(R - 1)$. Bonds are the only store of value.¹ Taxes are lump sum and therefore non-distortionary. Each member of a generation is taxed identically.

The individual consumer maximizes his intertemporal utility function,² $U(c_1(t), c_2(t + 1))$, assumed to be the same for every consumer, through his choice of consumption and bonds, subject to the budget constraints³

$$p(t)c_1(t) + b(t) = p(t) \left(\omega_1(t) - \frac{T_1(t)}{\pi^t} \right), \quad (1)$$

$$p(t + 1)c_2(t + 1) = R b(t) + p(t + 1) \left(\omega_2(t + 1) - \frac{T_2(t + 1)}{\pi^t} \right), \quad (2)$$

¹ Goods are therefore, as in Samuelson's model, perishable.

² Utility from government expenditures is either absent or additive.

³ Without loss of generality we assume there is one consumer in the first period.

where:

$p(t)$ is price in period t ,

$c_1(t)$ is consumption by the young in period t ,

$c_2(t + 1)$ is consumption by the old in period $t + 1$,

$b(t)$ is government bonds bought or sold by the young in period t ,

$\omega_1(t)$ is the endowment received by the young in period t ,

$\omega_2(t + 1)$ is the endowment received by the old in period $t + 1$,

$T_1(t)$ is the tax paid by the young generation in period t ,

$T_2(t + 1)$ is the tax paid by the old generation in period $t + 1$.

Equation (1) indicates that the consumer can use his first period income to buy consumption goods or bonds.⁴ Equation (2) says that the consumer can spend his second period earnings on consumption goods in that period. Defining $\rho(t) \equiv (p(t)R/p(t + 1))$, the real interest factor, and using our assumptions [$\omega_1(t) = \lambda^t \omega_1$; $\omega_2(t + 1) = \lambda^t \omega_2$; $T_1(t) = \gamma^t \tau_1$; $T_2(t + 1) = \gamma^t \tau_2$], equations (1) and (2) can be written as

$$\rho(t) \frac{c_1(t)}{\lambda^t} + \frac{c_2(t + 1)}{\lambda^t} - \left(\rho(t) \omega_1 + \omega_2 - \rho(t) \left(\frac{\gamma}{\lambda \pi} \right)^t \tau_1 - \left(\frac{\gamma}{\lambda \pi} \right)^t \tau_2 \right) = 0. \quad (3)$$

Market equilibrium in period t holds if total demand equals total supply

$$\pi^t c_1(t) + \pi^{t-1} c_2(t) + G(t) = \pi^t \omega_1(t) + \pi^{t-1} \omega_2(t), \quad (4)$$

where $G(t)$ is government demand for consumption goods in period t , therefore $G(t) = \gamma^t g$. So demand comes from the young consumers, of whom there are π^t , from the old consumers, of whom there are π^{t-1} , and from the government. Supply only comes from the young and the old, not from the government. The government budget constraint is given by

$$B(t) = p(t)[G(t) - T_1(t) - T_2(t)] + RB(t - 1), \quad (5)$$

where $B(t)$ is the total amount of government bonds issued in period t . The difference between government expenditures and taxes this period

⁴ Note that if $b(t)$ is negative the consumer sells bonds when young.

plus repayment of last period's debt plus interest has to be financed this period by issuing bonds.⁵

It is well-known that stationarity of the model requires (i) homothetic preferences, see e.g. Buiter (1979), (ii) growth of consumption at the factor λ , and (iii) $\gamma = \lambda\pi$, i.e., public expenditures grow at the same rate as aggregate endowments of the economy. Therefore, since preferences are homothetic, the ratio $[c_2(t+1)/c_1(t)]$ is a function of $\rho(t)$ only:

$$c_2(t+1) = f[\rho(t)]c_1(t), \quad \text{with } f' > 0. \quad (6)$$

Moreover, the deficit ($d \equiv g - \tau_1 - \tau_2/\lambda\pi > 0$) is a constant fraction of total endowments in every period, and at stationarity $c_1(t) = \lambda^t c_1$ and $c_2(t+1) = \lambda^t c_2$. At a stationary state with a constant real interest factor equations (3), (4), and (6) become

$$\rho c_1 + c_2 - (\rho\omega_1 + \omega_2 - \rho\tau_1 - \tau_2) = 0, \quad (7)$$

$$\lambda\pi c_1 + c_2 - (\lambda\pi\omega_1 + \omega_2 - \lambda\pi g) = 0, \quad (8)$$

$$c_2 = f(\rho)c_1. \quad (9)$$

We will distinguish between two cases: (i) with and (ii) without a balanced budget.

Case (i): No current government budget deficit ($d = 0$).

Both equations (7) and (8) pass through the point $(\omega_1 - \tau_1, \omega_2 - \tau_2)$, which we will call *extended autarky* as the government steps in to take away τ_1 in the first and τ_2 in the second period. There are two possibilities.

First, that there has never been a government budget deficit or surplus. Therefore, there is no store of value, meaning that there is no intertemporal exchange possible, so consumption takes place at the extended autarky point. This will be an optimal solution for the utility maximizing consumer iff $\rho = f^{-1}[(\omega_2 - \tau_2)/(\omega_1 - \tau_1)] = \lambda\pi$.

Second, that there has been a government deficit in the past. This means that $B(t) \neq 0$ for all periods after the government managed to let the deficit vanish. Bonds are a store of value that allows intertemporal

⁵ If agents want to borrow when young, $-B(t)$ can be viewed as the total amount to be borrowed. As the government is assumed to be a net borrower savings can only come from the old who have to repay last period's debts. The government is an intermediary here and failure of the old to repay results in "death."

exchange to take place. For a stationary state with a constant real interest factor equations (7) and (8) have to coincide, i.e. the real interest factor (ρ) has to equal the product of the endowment growth factor (λ) and the population growth factor (π). Consumption will be at

$$c_1 = \frac{1}{f(\lambda\pi) + \lambda\pi} \left(\lambda\pi(\omega_1 - \tau_1) + \omega_2 - \tau_2 \right) \quad (10)$$

and

$$c_2 = \frac{f(\lambda\pi)}{f(\lambda\pi) + \lambda\pi} \left(\lambda\pi(\omega_1 - \tau_1) + \omega_2 - \tau_2 \right). \quad (11)$$

Proposition 1: If the current government budget is balanced then either the constant real interest factor equals the product of the population growth factor and the endowment growth factor ($\rho = \lambda\pi$) when there has been a government deficit in the past, or we are in *extended autarky*.

Since there is no government budget deficit equation (5) reduces to $B(t) = RB(t-1)$, and therefore the stock of bonds grows with the factor R , the nominal interest factor. Prices with a balanced budget change with a constant factor equal to $R/\lambda\pi$. Inflation is therefore possible if the nominal interest factor exceeds the growth rate of the economy. Case (i) has received wide attention in the literature even though most governments *do* have budget deficits. We are, therefore, more interested in case (ii) and will devote more time to it.

Case (ii): A government budget deficit ($d > 0$).

Equation (7) passes through the *extended autarky* point $(\omega_1 - \tau_1, \omega_2 - \tau_2)$ and equation (8) passes through the point $(\omega_1 - \tau_1, \omega_2 - \lambda\pi(g - \tau_1))$, below the *extended autarky* point since $d > 0$, see figure 1. Henceforth, we assume the public sector not to be “too big,” more specifically:

$d < \omega_1 - \tau_1$; the young generation is able to pay for the deficit,
 $d < (\omega_2 - \tau_2)/\lambda\pi$; the old generation is able to pay for the deficit,
 $\omega_i - \tau_i > 0$; the remaining net income of each generation is positive.

Some conclusions can immediately be drawn.

Proposition 2: If the government runs a budget deficit

- (a) the real interest factor cannot be equal to the product of the population growth factor and the endowment growth factor ($\rho \neq \lambda\pi$);
- (b) a stationary state in *extended autarky* is not possible;
- (c) if ρ , c_1 , and c_2 solve equations (7)–(9) then the real interest factor

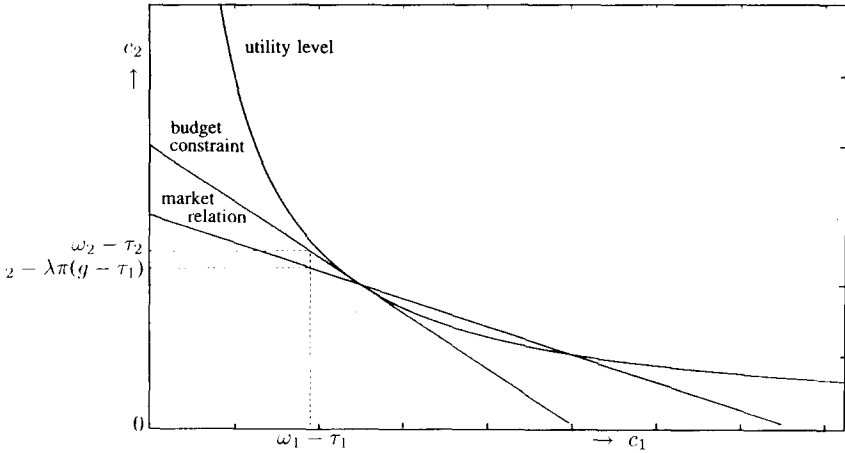


Fig. 1. The real interest factor (ρ) exceeds Samuelson's biological interest factor ($\lambda\pi$).

is smaller (greater) than the government growth factor if the agents save (borrow) when young, i.e.

$$\rho \begin{matrix} < \\ > \end{matrix} \gamma = \lambda\pi \text{ iff } \begin{cases} c_1 < \\ > \end{cases} \omega_1 - \tau_1 \\ \begin{cases} c_2 > \\ < \end{cases} \omega_2 - \tau_2 \end{cases} .$$

If ρ were to equal $\lambda\pi$, the budget relation and the market relation would be two non-intersecting parallel lines, hence proposition 2 (a) follows. The consequence of proposition 2 (a) is that the presence of the government deficit forces the real interest factor to deviate from the Samuelson biological interest factor. Proposition 2 (b) follows from the fact that the market relation does not pass through *extended autarky*. Finally, using equations (7) and (8) to solve for c_1 and c_2 in terms of ρ we get⁶

$$c_1 = \omega_1 - \tau_1 + \frac{\lambda\pi}{(\rho - \lambda\pi)} d , \tag{12}$$

$$c_2 = \omega_2 - \tau_2 - \frac{\rho\lambda\pi}{(\rho - \lambda\pi)} d , \tag{13}$$

From this proposition 2 (c) follows easily.

⁶ Note that equations (12) and (13) are well-defined because of proposition 2 (a).

3. Existence and Stability

We have already seen that a solution always exists if there is no government budget deficit, provided taxes are not too large, that is provided $\omega_1 > \tau_1$ and $\omega_2 > \tau_2$. Using equations (12) and (13) to determine the ratio c_2/c_1 we can define the following function

$$\begin{aligned}
 q(\rho) &\equiv \frac{\rho[\omega_2 - \lambda\pi(g - \tau_1)] - \lambda\pi(\omega_2 - \tau_2)}{\rho(\omega_1 - \tau_1) - \lambda\pi[\omega_1 - (g - \tau_2/\lambda\pi)]} \\
 &\equiv \frac{\rho\alpha_1 - \alpha_2}{\rho\alpha_3 - \alpha_4} = \frac{c_2}{c_1}
 \end{aligned}
 \tag{14}$$

with: $\alpha_1 \equiv \omega_2 - \lambda\pi(g - \tau_1) > 0$,
 $\alpha_2 \equiv (\omega_2 - \tau_2)\lambda\pi > 0$,
 $\alpha_3 \equiv (\omega_1 - \tau_1) > 0$,
 $\alpha_4 \equiv [\omega_1 - (g - \tau_2/\lambda\pi)]\lambda\pi > 0$.

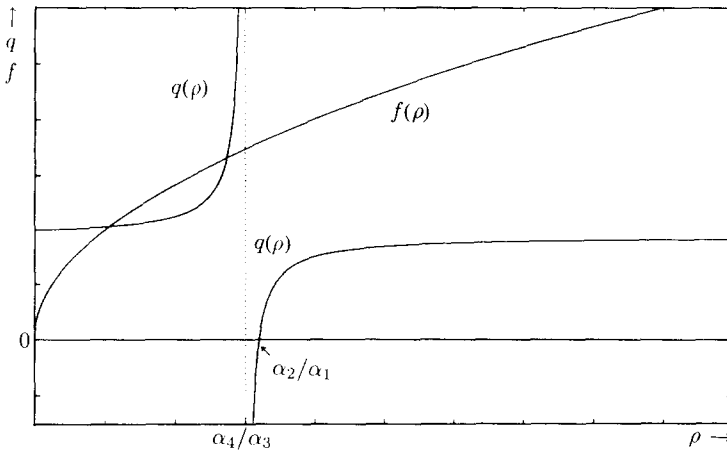


Fig. 2. The graph of $q(\rho)$ and $f(\rho)$.

It is easy to see that $\alpha_4/\alpha_3 < \lambda\pi < \alpha_2/\alpha_1$, and therefore the function $q(\rho)$, which is illustrated in figure 2, has the following properties:

$$\begin{aligned}
 q(0) &= \frac{\alpha_2}{\alpha_4} > 0 & \lim_{\rho \rightarrow \infty} q(\rho) &= \frac{\alpha_1}{\alpha_3} > 0 \\
 \lim_{\rho \downarrow \alpha_4/\alpha_3} q(\rho) &= -\infty & \lim_{\rho \uparrow \alpha_4/\alpha_3} q(\rho) &= \infty \\
 \text{and } q'(\rho) &= \frac{(\alpha_2\alpha_3 - \alpha_4\alpha_1)}{(\rho\alpha_3 - \alpha_4)^2} > 0 .
 \end{aligned}$$

If ρ is to be an equilibrium interest factor, $q(\rho)$ must be equal to $f(\rho)$. A fairly arbitrary $f(\rho)$ function is also drawn in figure 2. It is clear that if the $f(\rho)$ function passes through the *gap* left open by the $q(\rho)$ function a stationary state with constant real interest factor does not exist. It should be equally clear that any parametric changes do not affect the $f(\rho)$ function, but do affect the $q(\rho)$ function.

Proposition 3: If the government runs a budget deficit that is “too big” there does not exist a stationary state with constant real interest factor. Furthermore, if the function $f(\rho)$ has no inflexion points (for example if utility is of the CES type) there are at most two stationary states with constant real interest factor.

An increase in government expenditures decreases α_1 and α_4 and increases the *gap* left open by the $q(\rho)$ function without bound.⁷ The second part of proposition 3 is obvious.

Stability will be analyzed analogous to Grandmont (1985). Define modified net demands, $z_1(t)$ and $z_2(t+1)$, as follows:

$$\begin{aligned}
 z_1(t) &\equiv [c_1(t)/\lambda^t] - (\omega_1 - \tau_1) & \text{and} \\
 z_2(t+1) &\equiv (c_2(t+1)/\lambda^t) - (\omega_2 - \tau_2) .
 \end{aligned}$$

Use equations (3), (4), and (6) to get

$$\rho(t)z_1(t) + z_2(t+1) = 0 , \quad (15)$$

$$\lambda\pi z_1(t) + z_2(t) + \lambda\pi d = 0 , \quad (16)$$

$$z_2(t+1) = f[\rho(t)]z_1(t) + f[\rho(t)](\omega_1 - \tau_1) - (\omega_2 - \tau_2) . \quad (17)$$

⁷ On the other hand, a decrease in g such that d approaches 0 from above, drives α_2/α_4 to $(\omega_2 - \tau_2)/(\omega_1 - \tau_1)$ and α_4/α_3 to $\lambda\pi$. Any arbitrary $f(\rho)$ function will then have two solutions, except for the limiting case where $f(\lambda\pi) = (\omega_2 - \tau_2)/(\omega_1 - \tau_1)$, which has one solution. See case (i) in section 2.

These three equations, together with the initial conditions, determine the time path of the economy. Equations (15) and (17) can be combined to get the offer curve (optimal combinations of z_1 and z_2 for different values of ρ). For simplicity we will *not* investigate cases with *backward bending offer curves* as this gives rise to ambiguity and endogenous

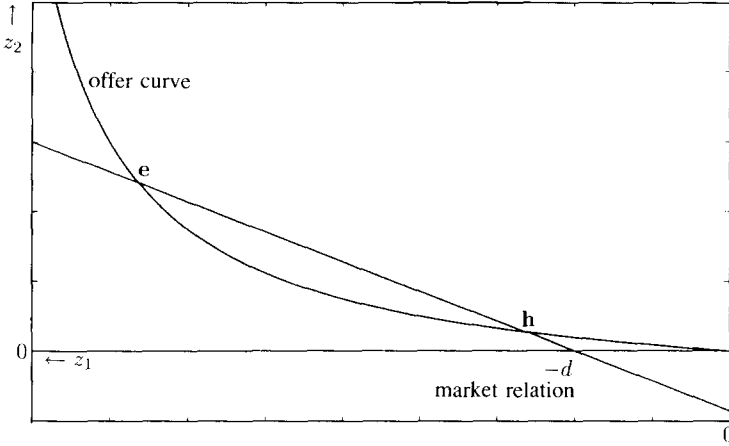


Fig. 3a: Offer curve and market relation if Samuelson's biological interest rate ($\lambda\pi$) exceeds the real interest factor (ρ); the case of two steady states.

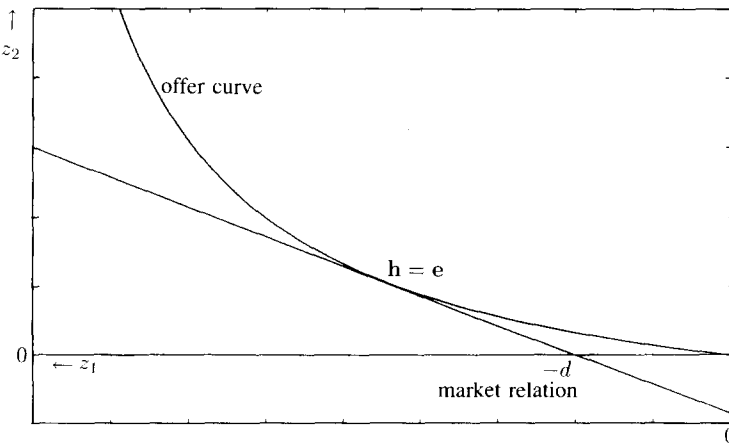


Fig. 3b: Offer curve and market relation if Samuelson's biological interest rate ($\lambda\pi$) exceeds the real interest factor (ρ); the case of one steady state.

business cycles.⁸ The interested reader is referred to Grandmont (1985). We can draw the offer curve and the market equation (16) together in one figure, see figures 3a and 3b for the case $\rho < \lambda\pi$. Suppose the market relation cuts the offer curve twice (figure 3a). Starting with any $z_2(1)$ point the dynamic path is determined as follows: go horizontal until the market relation determines $z_1(1)$, then go vertical until the offer curve determines $z_2(2)$, horizontal until the market relation determines $z_1(2)$, etc. Three cases are of particular interest:

- (i) $z_2(1) > z_2^e$. The solution path explodes and is ultimately infeasible. Perfect foresight excludes this possibility.
- (ii) $z_2(1) = z_2^e$. The economy will stay in the e stationary state with constant real interest factor forever.
- (iii) $z_2(1) < z_2^e$. The economy will converge (either asymptotically or directly if $z_2(1) = z_2^h$) to the h stationary state with constant real interest factor.

We see, therefore, that the h equilibrium is stable and the e equilibrium is unstable. Suppose that government expenditures (g) increase. This does not affect the offer curve, but shifts the market relation down.⁹ Ultimately we would end up in the situation depicted in figure 3b. The e and h equilibrium coincide making this point semi-stable. This means that the solution path with $z_2(1) > z_2^{e=h}$ explodes and with $z_2(1) < z_2^{e=h}$ converges to $z_2^{e=h}$. This shows once more that the budget deficit cannot be "too big." Obviously, the same type of reasoning can be used if $\rho > \lambda\pi$.

4. The Government Deficit and Parametric Changes

This section investigates the effect of a government budget deficit on parametric changes ($\lambda, \pi, \omega_1, \omega_2$). The next section will discuss different fiscal policies. The analysis will be in terms of net demands, z_1 and z_2 , which can be easily translated to consumption demands, c_1 and c_2 , using $z_i \equiv c_i - (\omega_i - \tau_i)$, $i = 1, 2$. We use net demands, because a change in z_1 (z_2) is directly equivalent to a change in the saving/borrowing position of the young (old) of each generation. This will be important in the results to follow. Furthermore, we restrict attention to the *stable* stationary state because changes in parameters

⁸ Despite homothetic preferences, backward bending offer curves are possible, for an example see Weddepohl (1990). Ruling out backward bending offer curves is equivalent to the assumption that $c_1(t)$ and $c_2(t+1)$ are gross substitutes.

⁹ Note that a change in $\omega_1, \omega_2, \tau_1$, or τ_2 would affect the offer curve.

or policies will bring us back to the stable stationary state. Whereas, starting from the unstable stationary state, a change in a parameter or policy would take us to the stable stationary state according to the global correspondence principle, see, e.g., Samuelson (1971) or Ide and Takayama (1990).

4a. A Government Budget Deficit

Consider the stationary state variant of equations (15)–(17)

$$\rho z_1 + z_2 = 0, \quad (15')$$

$$\lambda \pi z_1 + z_2 + \lambda \pi d = 0, \quad (16')$$

$$-f(\rho)z_1 + z_2 + (\omega_2 - \tau_2) - f(\rho)(\omega_1 - \tau_1) = 0. \quad (17')$$

From this it should be clear that the effect of a change in ω_1 on z_1 , z_2 and ρ equals $-f$ times the effect of a change in ω_2 on these variables. Similarly, the effect of a change in π equals λ/π times the effect of a change in λ . Therefore, we investigate only changes in ω_1 and π . The Jacobian of the system of equations (15')–(17') is given by

$$\begin{bmatrix} \rho & 1 & z_1 \\ \lambda \pi & 1 & 0 \\ -f & 1 & -c_1 f' \end{bmatrix}.$$

The Jacobian's inverse is

$$\Delta \begin{bmatrix} -c_1 f' & c_1 f' + z_1 & -z_1 \\ \lambda \pi c_1 f' & z_1 f - \rho c_1 f' & \lambda \pi z_1 \\ \lambda \pi + f & -(\rho + f) & \rho - \lambda \pi \end{bmatrix},$$

where $1/\Delta \equiv z_1(\lambda \pi + f) + (\lambda \pi - \rho)c_1 f'$.

With this information it is straightforward to construct the table of partial derivatives (table 1).¹⁰ The problem, then, is to determine the sign of Δ . Consider an increase in government expenditures, g . This will not affect the offer curve in figure 3a. It will, however, cause a parallel downward shift of the market relation. This means an *increase* in the real interest factor (which is the slope of the line from a point on the offer curve to the origin) for the *stable* stationary real interest

¹⁰ The reader should be aware that a change in λ or π implies an equiproportional change in γ .

Table 1. The influence of parameters on stationary state values in the case of a budget deficit

| parameter | z_1 | z_2 |
|------------|---|---|
| ω_1 | $-z_1 f \Delta$ | $\lambda \pi z_1 f \Delta$ |
| π | $-\lambda(c_1 f' + z_1)(z_1 + d)\Delta$ | $\lambda(\rho c_1 f' - z_1 f)(z_1 + d)\Delta$ |
| parameter | ρ | |
| ω_1 | $(\rho - \lambda \pi)f \Delta$ | |
| π | $\lambda(\rho + f)(z_1 + d)\Delta$ | |

factor. Therefore $\Delta > 0$ for the stable stationary real interest factor.¹¹ Note, furthermore, from figure 3a that $\rho < \lambda \pi$ implies $z_1 + d < 0$. This leads to the following proposition:

Proposition 4: In the presence of a government budget deficit a decrease in either population growth (π) or in first period endowments (ω_1) will increase (decrease) the stable real interest factor (ρ) if agents save (borrow) when young.

An increase in the real interest factor is always accompanied by an increase in second period net demand and a decrease in first period net demand, as agents are enticed to save more or borrow less. The results derived above are intuitively appealing. If, for example, agents save when young, a decrease in the population growth factor makes young people relatively more scarce. This decreases relative savings and increases the real interest rate.

4b. A Balanced Budget

The stationary state variant with a balanced budget can be written as¹²

$$\rho - \lambda \pi = 0, \quad (18)$$

$$\lambda \pi z_1 + z_2 = 0, \quad (19)$$

$$-f(\rho)z_1 + z_2 + (\omega_2 - \tau_2) - f(\rho)(\omega_1 - \tau_1) = 0. \quad (20)$$

¹¹ Because $\partial \rho / \partial g = \lambda \pi (\rho + f) \Delta$, see section 5.

¹² We exclude the trivial case of consumption in extended autarky, i.e., we assume $B(0) \neq 0$.

The Jacobian of this system is given by

$$\begin{bmatrix} 0 & 0 & 1 \\ \lambda\pi & 1 & 0 \\ -f & 1 & -c_1f' \end{bmatrix}.$$

This Jacobian's inverse is

$$\Gamma \begin{bmatrix} -c_1f' & 1 & -1 \\ \lambda\pi c_1f' & f & \lambda\pi \\ \lambda\pi + f & 0 & 0 \end{bmatrix},$$

with $1/\Gamma \equiv (\lambda\pi + f)$. Using this we construct the table of partial derivatives in the case of a balanced budget (table 2).

Table 2. The influence of parameters on stationary state values in the case of a balanced budget¹³

| parameter | z_1 | z_2 | ρ |
|------------|------------------------|----------------------------|-----------|
| ω_1 | $-f\Gamma$ | $\lambda\pi f\Gamma$ | 0 |
| π | $-c_1\lambda f'\Gamma$ | $c_1\lambda^2\pi f'\Gamma$ | λ |

Trivially, only a change in population growth leads to an equi-proportional change in the real interest factor. Endowment changes are changes in real income (there is no substitution effect since ρ is constant) leading to increased consumption in both periods. An increase in ω_1 , for instance, is only partly consumed in that period leading to a decrease in net demand when young.

4c. Comparison

The qualitative results of tables 1 and 2 are neatly summarized in table 3.

¹³ Note that a change in λ or π affects the government budget constraint ($d = g - \tau_1 - \tau_2 / \lambda\pi$). We have assumed that a change in λ or π is compensated by an appropriate change in g to keep the budget balanced.

Table 3. The signs of the partial derivatives

| | $d = 0$ | | | $d > 0$ and $\rho > \lambda\pi$ | | | $d > 0$ and $\rho < \lambda\pi$ | | |
|------------|---------|-------|--------|---------------------------------|-------|--------|---------------------------------|-------|--------|
| | z_1 | z_2 | ρ | z_1 | z_2 | ρ | z_1 | z_2 | ρ |
| ω_1 | - | + | 0 | - | + | + | + | - | - |
| π | - | + | + | - | + | + | + | - | - |

Proposition 5: A government budget deficit causes the parametric multipliers to be larger (in absolute value) and of the same sign if agents borrow when young and causes sign reversal when they save for the stable stationary state.

Inspection of tables 1 and 2 leads to this conclusion, but, as an example, consider the change in ρ caused by a change in π . Assume $\rho > \lambda\pi$, then

$$\begin{aligned} & \lambda(\rho + f)(z_1 + d)\Delta = \\ & = \left\{ \lambda(\rho + f)/[(\lambda\pi + f) + (\lambda\pi - \rho)c_1 f'/z_1] \right\} + \lambda(\rho + f)d\Delta > \\ & > [\lambda(\rho + f)/(\lambda\pi + f)] + \lambda(\rho + f)d\Delta > \lambda . \end{aligned}$$

On the left hand side we have the change in ρ with a budget deficit and on the right hand side the change in ρ with a balanced budget. The real interest factor therefore rises more in the presence of a budget deficit, causing an extra substitution effect away from period 1 consumption to period 2 consumption. Sign reversal if $\rho < \lambda\pi$ is obvious because in that case $z_1 + d < 0$, see section 4a.

Note further that an increase in ω_1 or π , if there is a budget deficit, decreases the intertemporal exchange that takes place in the stable stationary state. In the case of a balanced budget the outcome of these changes depends on the sign of z_1 . If z_1 is positive, that is if the agents borrow when young, the stable stationary state moves towards *extended autarky*. If z_1 is negative the stationary state moves away from the *extended autarky* point, thereby increasing the intertemporal exchange that takes place.

5. The Government Deficit and Fiscal Policy

We now turn to the effects of different fiscal policies. Our approach will be the same as in section 4. Note that a change in τ_1 does not necessarily equal minus one times a change in ω_1 , as would be suggested by

equations (15')–(17'), because τ_1 affects the budget deficit. Therefore, it is only true that the effect of a change in τ_1 on z_1 , z_2 and ρ equals $-f$ times the effect of a change in τ_2 on these variables if the budget deficit is kept constant, see tables 5 and 6 below. Otherwise, changes in taxes affect the budget deficit differently. Then, however, it is easy to see, from $\partial c_1/\partial \tau_1 = \partial z_1/\partial \tau_1 - 1$, $\partial c_2/\partial \tau_1 = \partial z_2/\partial \tau_1$ etc., that the effect of a change in τ_1 on consumption demand, c_1 and c_2 , and ρ equals ρ times the effect of a change in τ_2 on these variables, see table 4 below.

5a. A Government Budget Deficit

Analogous calculations as in section 4a lead to table 4.

Table 4. The influence of government instruments on stationary state values in the case of a budget deficit

| parameter | z_1 | z_2 | ρ |
|-----------|-----------------------------------|---|-------------------------------|
| g | $-\lambda\pi(c_1 f' + z_1)\Delta$ | $\lambda\pi(\rho c_1 f' - z_1 f)\Delta$ | $\lambda\pi(\rho + f)\Delta$ |
| τ_1 | $1 + \rho c_1 f' \Delta$ | $-\lambda\pi\rho c_1 f' \Delta$ | $-\rho(\lambda\pi + f)\Delta$ |
| τ_2 | $c_1 f' \Delta$ | $1 - \lambda\pi c_1 f' \Delta$ | $-(\lambda\pi + f)\Delta$ |

Proposition 6: In the presence of a budget deficit an increase in government expenditures (g) or a decrease in taxes (τ_1 or τ_2) leads to an increase in the stable real interest factor.

The government's demand for savings increases, therefore the interest rate rises, as was to be expected. An increase in taxes, either τ_1 or τ_2 , increases consumption when young and decreases consumption when old. This is caused by the substitution effect due to the decrease in the real interest factor. Note that the substitution effect on consumption of an increase in taxes when young dominates the income effect.

There is a problem, however, if we want to compare different policy effects with and without a budget deficit. In the present case we can independently vary taxes or the government expenditures. If the budget is balanced we cannot do this, as we would immediately leave the balanced budget regime. There are three basic possibilities: (a) shift taxes from one generation to the other, (b) increase government expenditures paid for by the young, or (c) increase government expenditures paid for by the old. Proper combinations of (a), (b) and (c) are, of course, also possible. To make comparisons between the two regimes possible

we give the results of these policy combinations for a budget deficit below, see table 5.

Table 5. The influence of fiscal policies with constant budget deficit on stationary state values

| parameter | z_1 | z_2 | ρ |
|-------------|-----------------------------|--|---|
| a. τ_1 | $z_1(\lambda\pi + f)\Delta$ | $-z_1\lambda\pi(\lambda\pi + f)\Delta$ | $(\lambda\pi - \rho)(\lambda\pi + f)\Delta$ |
| b. τ_1 | $z_1f\Delta$ | $-z_1\lambda\pi f\Delta$ | $(\lambda\pi - \rho)f\Delta$ |
| c. τ_2 | $-z_1\Delta$ | $z_1\lambda\pi\Delta$ | $-(\lambda\pi - \rho)\Delta$ |

- a. A decrease in τ_2 equal to $\lambda\pi$ times the increase in τ_1 ;
 b. An increase in g equal to the increase in τ_1 ;
 c. An increase in τ_2 equal to $\lambda\pi$ times the increase in g .

Proposition 7: In the presence of a budget deficit the stable real interest factor will *increase (decrease)* if agents *save (borrow)* when young in combination with (a) a shift of taxes from the old to the young, or (b) an expansion of government expenditures paid for by the young, or (c) a contraction of government expenditures benefiting the old.

Naturally, fiscal policy *b*, an equal increase in g and τ_1 , has the same effect as a decrease in ω_1 , which has the same effect as $-f$ times a decrease in ω_2 , which, in turn, has the same effect as fiscal policy *c*, an equal increase in $\lambda\pi g$ and τ_2 .

5b. A Balanced Budget

Similar calculations as in section 4b lead to table 6.

Table 6. The influence of fiscal policies with a balanced budget on stationary state values

| parameter | z_1 | z_2 | ρ |
|-------------|-----------|-----------------------|--------|
| a. τ_1 | 1 | $-\lambda\pi$ | 0 |
| b. τ_1 | $f\Gamma$ | $-\lambda\pi f\Gamma$ | 0 |
| c. τ_2 | $-\Gamma$ | $\lambda\pi\Gamma$ | 0 |

- a. A decrease in τ_2 equal to $\lambda\pi$ times the increase in τ_1 ;
 b. An increase in g equal to the increase in τ_1 ;
 c. An increase in τ_2 equal to $\lambda\pi$ times the increase in g .

Of course, since $\rho = \lambda\pi$, the real interest factor is not changed in response to government policies. This means that all effects are pure income effects. Fiscal policy *a* does not change real income, therefore $\partial c_1/\partial\tau_1 = \partial c_2/\partial\tau_1 = 0$ and only the borrowing position is affected. Fiscal policies *b* and *c* decrease real income, therefore consumption is changed equiproportionally; for fiscal policy *b* we have $\partial c_1/\partial\tau_1 = -\lambda\pi\Gamma = f\partial c_2/\partial\tau_1$, whereas for fiscal policy *c* we have $\partial c_1/\partial\tau_2 = -\Gamma = f\partial c_2/\partial\tau_2$.

5c. A Comparison

The qualitative results from tables 5 and 6 can be summarized in table 7.

Table 7. The signs of the effects of different fiscal policies

| parameter | $d = 0$ | | | $d > 0$ and $\rho > \lambda\pi$ | | | $d > 0$ and $\rho < \lambda\pi$ | | |
|-------------|---------|-------|--------|---------------------------------|-------|--------|---------------------------------|-------|--------|
| | z_1 | z_2 | ρ | z_1 | z_2 | ρ | z_1 | z_2 | ρ |
| a. τ_1 | + | - | 0 | + | - | - | - | + | + |
| b. τ_1 | + | - | 0 | + | - | - | - | + | + |
| c. τ_2 | - | + | 0 | - | + | + | + | - | - |

- a. A decrease in τ_2 equal to $\lambda\pi$ times the increase in τ_1 ;
- b. An increase in g equal to the increase in τ_1 ;
- c. An increase in τ_2 equal to $\lambda\pi$ times the increase in g .

Proposition 8: A government budget deficit causes the fiscal policy multipliers to be larger (in absolute value) and of the same sign if agents borrow when young and causes sign reversal when they save.

Inspection of tables 5 and 6 leads to this conclusion. Consider, for example, a shift of the tax burden from the old to the young ($-\partial\tau_2 = \lambda\pi\partial\tau_1$). If the budget is balanced z_1 increases by 1 unit and if there is a budget deficit the multiplier equals $z_1(\lambda\pi + f)\Delta$, which can be written as $1 + (\rho - \lambda\pi)c_1f'\Delta$. Clearly this is bigger than 1 if agents borrow when young ($\rho > \lambda\pi$). The difference between the two multipliers $[(\rho - \lambda\pi)c_1f'\Delta]$ is caused by the change in the real interest factor. Similar arguments can be applied to the other multipliers. The decrease in the real interest factor entices agents to borrow more when there is a budget deficit, thereby increasing first period net demand even more.

6. Conclusions

We investigate a two-period overlapping generations model with growing per capita endowments and a government sector. The existing literature has paid excessive attention to a government that does not run a budget deficit. If that is the case then either the economy is in *extended autarky*, or the real interest factor equals the product of the population growth factor and the endowment growth factor, which is called Samuelson's biological interest factor. Most governments, however, *do* run a budget deficit. Consumption at *extended autarky* is then impossible, as is the equality of the real interest factor and Samuelson's biological interest factor. Intergenerational exchange *has* to take place. Existence of a stationary state is only guaranteed as long as the public sector is not "too big." Under some conditions (e.g. CES preferences) there are at most two stationary states with constant real interest factor; one stable and one instable. The presence of a budget deficit causes changes in endowments or fiscal policies to affect the real interest factor, which is not the case if the budget is balanced. Three basic fiscal policies are analyzed: (a) shift taxes from one generation to the other, (b) increase government expenditures paid for by the young, or (c) increase government expenditures paid for by the old. Restricting attention to the stable stationary state fiscal policy is more effective if there is a budget deficit, than if the budget is balanced, and agents borrow when young. If the young save the multipliers reverse sign. It is, therefore, of critical importance for the government to know which regime applies to achieve the desired result.

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