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An auction for collaborative vehicle routing: Models and algorithms

Vincentius C.G. Karels *, Lucas P. Veelenturf, Tom Van Woensel

School of Industrial Engineering, Eindhoven University of Technology, Eindhoven, the Netherlands

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ABSTRACT

Increasing competition and expectations from customers pressures carriers to further improve efficiency. Forming collaborations is essential for carriers to reach their targeted efficiency levels. In this study, we investigate an auction mechanism to facilitate collaboration amongst carriers while maintaining autonomy for the individual carriers. Multiple auction implementations are evaluated. As the underlying decision problem (which is a traditional vehicle routing problem) is known to be NP-hard, this auction mechanism has an important inherent complexity. Therefore, we use fast and efficient algorithms for the vehicle routing problem to ensure that the auction can be used in operational decision making. Numerical results are presented, indicating that the auction achieves a savings potential better than the thus far reported approaches in the literature. Managerial insights are discussed, particularly related to the properties of the auction and value of the information.

1. Introduction

Customers expect faster and more reliable deliveries and increased flexibility, which compels carriers to increase the efficiency and accuracy of their transportation services. Cooperation has the potential to develop synergistic approaches and reduce costs. This is particularly the case for small carriers, who individually cannot achieve appropriate economies of scale. Collaboration between carriers is then almost becoming a necessity to survive, certainly given the increased competition. Ultimately, collaboration results in fewer on-road vehicles, reduced empty mileage drives, and lower carbon emissions, as shown in Pérez-Bernabeu et al. (2015).

Following the sharing and platform economy, there is a growing interest in sharing resources. These platforms make it easy to rent out underutilized resources in a peer-to-peer but also business-to-business (B2B) settings. Examples for peer-to-peer are: AirBnB (renting out your home when at holiday) or Snappcar (renting out your car when not used). In a B2B setting, especially for our problem under consideration, also many initiatives take place. For example, *Convoy* is a digital (auctioning) platform that matches carriers with shippers, currently valued at 2.75 billion dollars (Ohnsman (2019)). Utilizing the work of Berger and Bierwirth (2010), the platform auctions pickup and delivery requests from shippers to carriers. The largest competitor of Convoy is Uber Freight, which utilizes a dynamic pricing system to match demand from shippers with supply from carriers (or individuals). This dynamic pricing algorithm is not open source, and therefore its effectiveness is hard to assess. On the other hand, scientific research on auctioning systems in exchange transport requests is abundant (Gansterer and Hartl (2018b)). Also in practice, horizontal collaboration is slowly increasing between competitors, as seen in Gerdes (2014). In our paper, we investigate whether it is also interesting for carriers and logistic service providers (LSPs) to put requests on these auctioning systems.

Linking our suggested auctioning system to the planning software of the carriers makes it possible for them to determine the cost (gain) of adding (removing) the requests to (from) their existing pool of requests. With the results of the real-life based case, discussed in Section 6.2.5, we show that algorithms exist that allow for enough precision to estimate these costs to allow for a effective run of these auctions. While non-truthful bidding is a possibility, we assume that there is collective rationality.

In this paper, we consider distribution carriers that daily solve a (capacitated) vehicle routing problem (VRP, see e.g. Laporte et al. (1986)), possibly including time windows (VRPTW). These carriers prefer to collaborate horizontally, as large savings can be achieved in horizontal collaboration by carriers (Cruijssen et al. (2007)). However, importantly, carriers prefer to remain autonomous such that they obtain the benefits of collaboration without completely merging their activities. Therefore, we introduce a collaborative environment that extends and generalizes the work of Berger and Bierwirth (2010). Two important benchmarks are used to evaluate the collaborative environment. First, we compare the performance of the collaboration versus an environment without collaboration. Second, we compare collaboration versus a

* Corresponding author.
E-mail addresses: v.c.g.karels@tue.nl (V.C.G. Karels), l.p.veelenturf@tue.nl (L.P. Veelenturf), t.v.woensel@tue.nl (T. Van Woensel).

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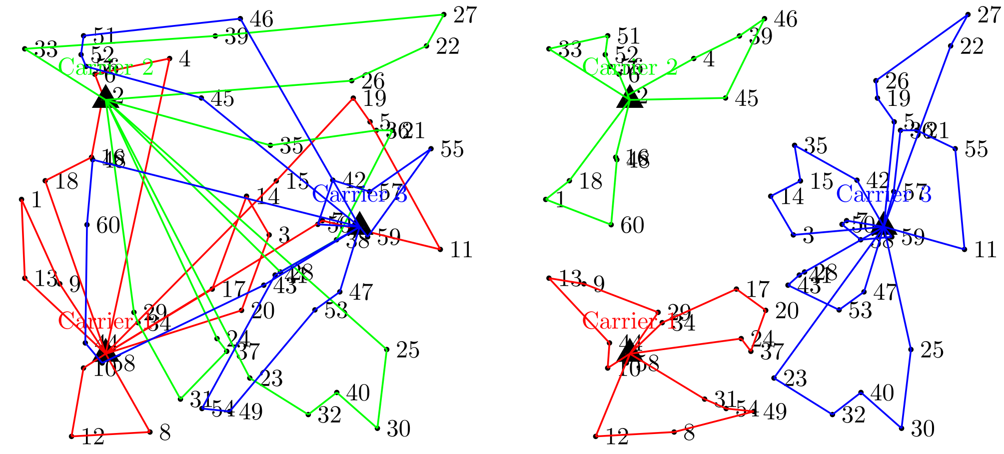


(a) No collaboration (b) Central plan

Fig. 1. Abstract examples of routing on networks.

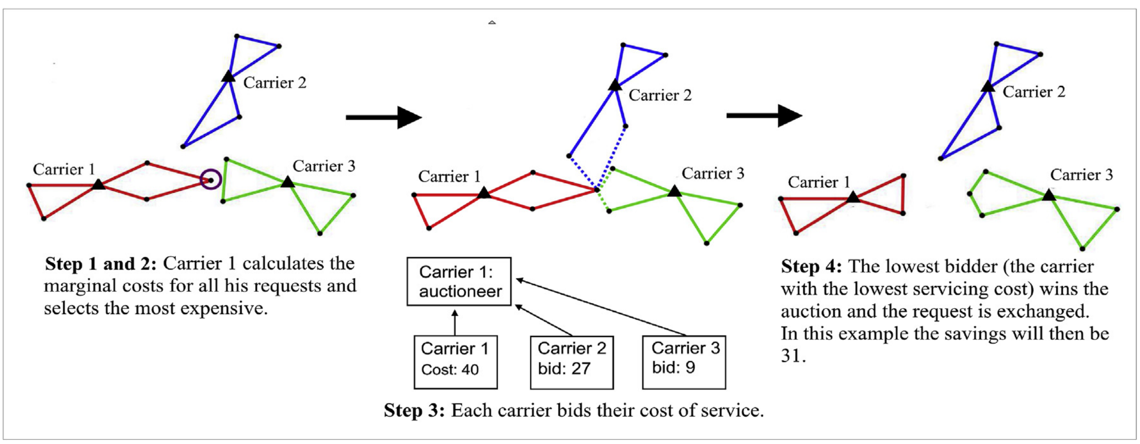


Fig. 2. Example of our single item Limited Reassignment Auction.

Table 1
An example of cash flows.

Carrier	1	2	3
Cash out (−)	9	0	0
Cash in (+)	0	0	9
Old routing costs (+)	90	70	80
New routing costs (−)	50	70	89
Savings (=)	31	0	0

Table 2
An overview of the auctions.

	Limited	Combinatorial
Single Request selection	LRA(SB)	CRA(SB)
Route-based selection	LRA(RB)	CRA(RB)
Cluster-based selection	LRA(CB)	–

Note that LRA(SB) is equal to the SRRA method of Berger and Bierwirth (2010) and the CRA(SB) method equal to the BRRA method of Berger and Bierwirth (2010).

situation where all carriers give up their autonomy (i.e. merge) and act as a single company. In this specific setup, a single delivery plan is formulated, referred to as the *central plan* in the remainder of this paper. Examples of a plan without collaboration and a central plan are shown in Fig. 1.

Consider a carrier who owns a distribution center and a fleet of trucks to service a set of customers. These customers, usually chains of stores under a particular brand, operate locations around the country. In an effort to minimize inventory space at the stores, these customers have most of their inventory at a distribution center of a logistic service provider (carrier). Periodically goods are delivered to the stores by the carrier. Suppose two carriers exist according to the aforementioned description, carriers A and B. Carrier A's customers are in the market segments: 'shoes', 'sports clothing' and 'pet food', while carrier B customers are in the market segments: 'electronics equipment', 'sports equipment' and 'cosmetic products'. Note that while in this example these market segments are non-overlapping, multiple customers exist within any segment and market segments for logistics service providers could potentially overlap. The competing carriers have most likely customers in the same cities, but for both the demand to a single city center is less than a full truckload. In such a situation savings can be obtained in merging the customers of both carriers in the same city into a single route. Note, to achieve this, products have to be re-positioned a priori. To make this a possibility, the carriers should start a collaboration and should be willing to share (partial) information about their customers.

Sharing information is sensitive. As carrier A receives information about customers from carrier B, they might realize that some locations from 'electronics equipment' can be very easily absorbed in their network. This might result in carrier A producing an offer to this customer of carrier B, undercutting them. This, in turn, results in a loss of revenue for carrier B. This possibility generates fear and mistrust for the

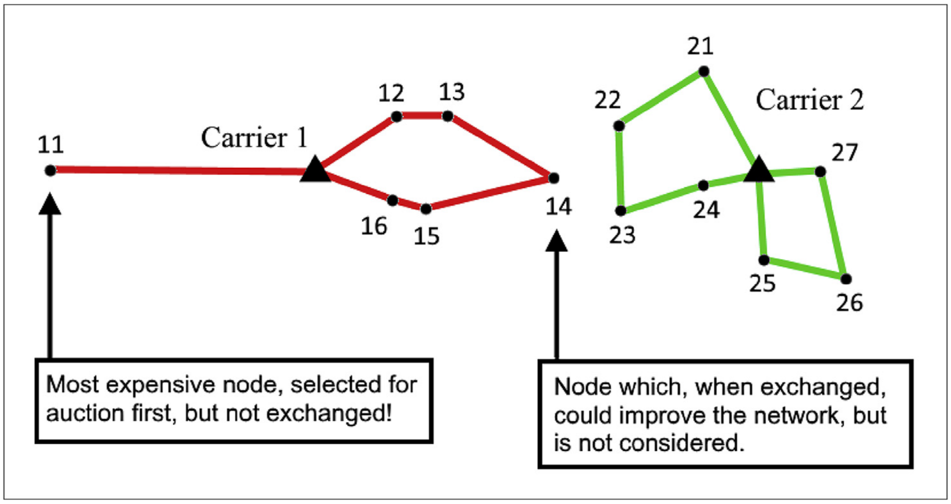


Fig. 3. Example of clustering.

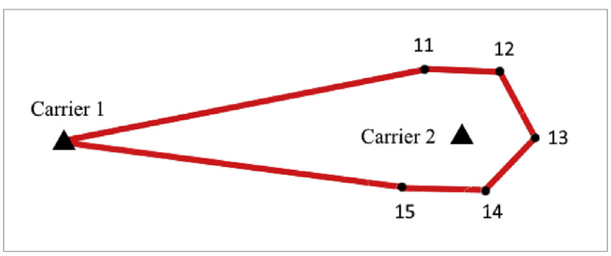


Fig. 4. Example of clustering.

sharing of information. The challenge is to let carrier A and B collaborate in a setting which they trust. The central plan leads to the lowest total transportation cost for the carriers, but also requires carriers to share complete information of their customers. This complete information is rarely, if ever, shared between competing carriers because of the aforementioned fears. For our collaborative environment, we propose an auction mechanism to exchange requests which we call the *Reassignment Auction*. Such an auction can be implemented as a physical, or electronical auction. An advantage of an auction is the fact that the participating carriers can decide themselves which information to reveal (e.g. they can decide to not share some very sensitive high profit customers). This, in turn, increases trust (Pomponi et al. (2015)). Note that, for collaboration to exist in this form, at least some information *must* be shared among the participants.

The main contributions of this paper are as follows:

- The *Reassignment Auction* given in Berger and Bierwirth (2010) that reallocates requests among participating carriers, is *generalized* in two ways. Compared to Berger and Bierwirth's two auctioning mechanisms, we test an additional three.
  1. We introduce the *Limited Reassignment Auction* (*LRA*) allowing for sets of requests to be submitted to the auction, to be transferred in its entirety. The *Single Request Reassignment Auction* (SRRA) of Berger and Bierwirth (2010) is a special case of this auction. This auction is also extended with a tabu list. Additionally, we show the strength of using routes and clusters as submitted sets. For the latter, we introduce a novel cluster recognition algorithm.
  2. We introduce the *Combinatorial Reassignment Auction* (*CRA*) where the *Combinatorial Auction Problem* (CAP) is solved for the reassignment problem. Participants are not only allowed to bid on the offered set, but also on all potential subsets. As a result, the *Bundle Request Reassignment Auction* (BRRA) of Berger and Bierwirth

f = 1
while $\sum_f |T_f| < \sum_f |N_f|$ **do**
  Formulate L_f from N_f based on the selection of a.
  $I^* = \arg\min\{I \subseteq L_f : m_f(I)\}$ with $m_f(I) = c[S[N_f]] - c[S[N_f \setminus I]]$. ▷ *Step 1.*
  Put I^* in the auction, and add to revealed. ▷ *Step 2.*
  $g^* = \arg\min\{g \in F : b_g(I^*)\}$ with $b_g(I) = c[S[N_g + I]] - c[S[N_g]]$. ▷ *Step 3.*
  Reassignment of requests: $N_{g^*} = N_{g^*} \cup I^*$ and $N_f = N_f \setminus I^*$. ▷ *Step 4.*
  if $f \neq g^*$ **then**
    $T_f = \emptyset \quad \forall f \in F.$
  else
    $T_f = T_f \cup I^*$
  end if
  f = f + 1.
  if f $> |F|$ **then**
    f = 1.
  end if
end while

Fig. 5. The Limited Reassignment Auction with tabu, LRA(tabu).

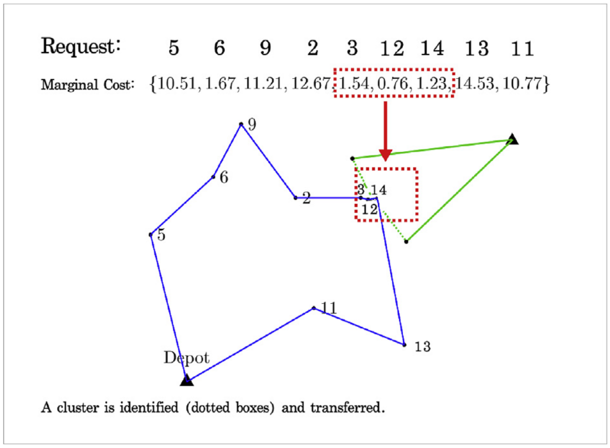


Fig. 6. Example of the cluster auction.

(2010) is a special case of this auction. In our numerical results, we illustrate the effect of submitting routes as bundles.

- The Reassignment Auction variants are applied to carriers who solve the VRP(TW), an auction variant that has received little attention in the literature. Applying auctioning mechanisms to pick-up and delivery problems is easier compared to more traditional vehicle routing problems, as such problems require the re-positioning of requests a priori.
- We use fast routing algorithms that allow the Reassignment Auction to be applied to problems of real-life sizes (3 carriers with 2967 requests in total).
- Valuable insights based on the outcomes of the reassignment auction are discussed. This includes the understanding with regards to the value of information sharing in relation to cost savings. It is also shown that during the auction, the marginal value of information decreases.

The structure of this paper is as follows. The following section briefly discusses the literature on horizontal collaboration. In Section 3, a conceptual collaboration model is introduced, which is formalized in Section 4. In Section 5, the computational results are discussed. The last section concludes this paper with the main insights and possible future work.

2. Literature research

In this section, the literature regarding carrier collaboration on routing services is discussed. Consider the review paper by Gansterer and Hartl (2018b), where this collaboration is subdivided into centralized versus decentralized planning. Central planning usually involves the participating carries to have perfect information with regards to all request portfolios. In such collaborations, profit sharing occurs after the central plan has been formulated, usually through game-theoretical principles. In decentralized planning the participants have imperfect to no information about the request portfolios of the other carriers. The method for exchanging requests can be exceedingly complex however, and auctioning has currently been the dominant method in the literature. We note that there exists a lot of research focused on horizontal collaborations. In this section we delineate only the most relevant literature for the positioning of our paper.

$iterator = 0$
while $iterator < |r|$ **do**
  $p \leftarrow r[iterator]$
  if $(\sum_{i\in p} m_f(i)/|p|)/(\sum_{i\in u} m_f(i)/|r-p|) < \alpha$ **then**
    Potential Cluster $= p$
  else
    if Potential Cluster $\neq \emptyset$ & $|$Potential Cluster$| > 2$ **then**
      Set of Clusters $\leftarrow$ Potential Cluster
    end if
    $p = \emptyset$
  end if
  $iterator = iterator + 1$
end while

Fig. 7. The algorithm for cluster detection.

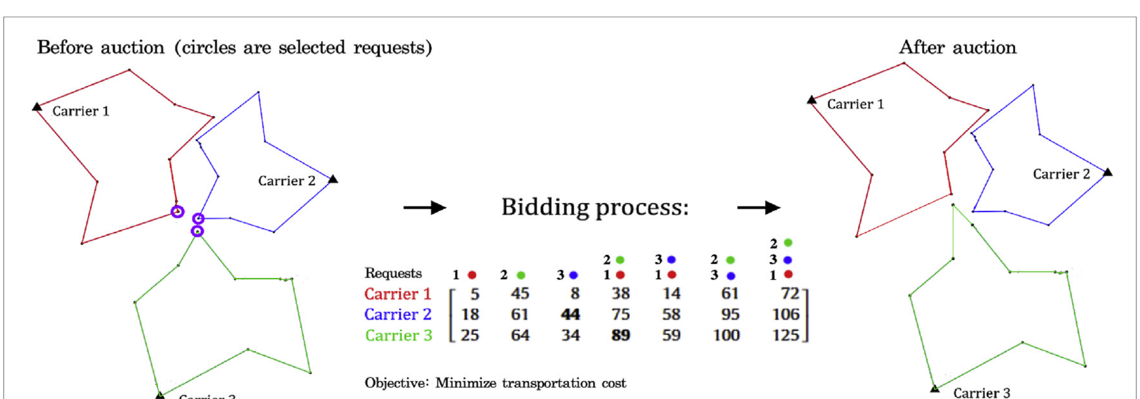


Fig. 8. Example of the combinatorial auction.

Table 3
Results for the vehicle routing problem.

	Gap to central plan		Savings		Savings with repositioning		Requests revealed	
	μ	σ	μ	σ	μ	σ	μ	σ
1	50.52%	19.44%	21.95%	9.52%	6.14%	4.46%	29.96%	11.88%
2	17.31%	8.48%	39.01%	6.74%	22.40%	3.52%	100%	–
3	13.05%	14.14%	41.38%	7.25%	23.16%	5.21%	100%	–
4	10.96%	12.43%	42.07%	6.87%	23.63%	5.17%	100%	–
5	16.44%	7.86%	35.97%	6.33%	21.30%	3.41%	61.44%	7.74%
6	0.003%	0.01%	47.60%	2.49%	28.73%	1.30%	100%	–

[1] $LRA(SB, 100\%)$ = SRRA (Berger and Bierwirth, 2010).
[2] .$LRA(tabu, SB, 100\%)$
[3] .$LRA(tabu, RB, 100\%) \rightarrow LRA(tabu, SB, 100\%)$
[4] .$LRA(tabu, RB, 100\%) \rightarrow LRA(tabu, CB, 100\%) \rightarrow LRA(tabu, 100\%)$
[5] $CRA(SB, 100\%)$ = BRRA (Berger and Bierwirth, 2010).
[6] .$CRA(tabu, RB, 100\%)$

Table 4
Results for the vehicle routing problem with time windows.

	Gap to central plan		Savings		Savings with repositioning		Requests revealed	
	μ	σ	μ	σ	μ	σ	μ	σ
1	60.67%	18.69%	28.99%	9.12%	8.32%	4.23%	24.96%	9.61%
2	21.87%	7.99%	46.11%	5.23%	24.56%	4.01%	100%	–
3	17.55%	16.81%	48.43%	9.03%	25.01%	6.89%	100%	–
4	13.34%	15.95%	49.67%	8.55%	26.27%	6.81%	100%	–
5	22.24%	8.36%	47.34%	3.94%	24.84%	3.95%	59.116%	6.97%
6	0.006%	1.55%	55.98%	4.13%	28.02%	3.67%	100%	–

[1] $LRA(SB, 100\%)$ = **BRRA** (Berger and Bierwirth, 2010).
[2] .$LRA(tabu, SB, 100\%)$
[3] .$LRA(tabu, RB, 100\%) \rightarrow LRA(tabu, SB, 100\%)$
[4] .$LRA(tabu, RB, 100\%) \rightarrow LRA(tabu, CB, 100\%) \rightarrow LRA(tabu, SB, 100\%)$
[5] $CRA(SB, 100\%)$ = **BRRA** (Berger and Bierwirth, 2010).
[6] .$CRA(tabu, RB, 100\%)$

Table 5
Legend for Figure 11.

1$LRA(SB, 100\%)$ = **BRRA** (Berger and Bierwirth, 2010)
2$LRA(tabu, SB, 100\%)$
3$LRA(tabu, RB, 100\%) \rightarrow LRA(tabu, SB, 100\%)$
4$LRA(tabu, RB, 100\%) \rightarrow LRA(tabu, CB, 100\%) \rightarrow LRA(tabu, SB, 100\%)$
5$CRA(SB, 100\%)$ = **BRRA** (Berger and Bierwirth, 2010)
6$CRA(tabu, RB, 100\%)$

2.1. Collaboration by central planning with profit sharing

Early research on horizontal collaboration between independent freight carriers was performed by Kopfer and Pankratz (Kopfer et al., 1999). They investigated a groupage system, and first coined the term collaborative transport planning (CTP). Krajewska and Kopfer (2006) introduced an exchange mechanism for CTP consisting of three phases: preprocessing, exchange mechanism, and profit sharing. The requests to be exchanged are identified and valued during the preprocessing phase. Subsequently, the requests are exchanged in such a way that it matches the central plan, potentially resulting in savings. The fair allocation of the savings is later determined during the profit sharing phase. This fair allocation of savings is firmly nested in the field of cooperative game theory, where the participating carriers form a coalition. One possible fair allocation is the Shapley value introduced by Shapley (1953) that uniquely distributes the savings among the participants. Further literature in this research direction is often similar to these ideas. Wang and Kopfer (2011) discussed the opportunities and challenges of each of these phases.Cruijssen and Salomon (2004) showed that order sharing between companies can lead to remarkable savings in the order of 5–15%. They also mentioned that trust is a challenge for achieving collaboration, which makes simple order sharing difficult. Cruijssen et al. (2007) also investigated the opportunities and obstacles carriers face in horizontal collaborations. Topics such as a fair allocation of the savings and carrier differentiation have been discussed. Trust and extent of cooperation was also discussed by Pomponi et al. (2015), who presented an evolutionary mechanism to generate trust. Krajewska et al. (2008) subsequently introduced an implementation of the mechanism from Krajewska and Kopfer (2006) for the pickup and delivery problem with time windows (PDPTW). They showed that collaboration results in a significant cost decreases and that efficient profit allocation is possible. Hezarkhani et al. (2016) introduce a solution for the sharing of savings where logistic providers perform joint planning of truckload deliveries, which enables the reduction of empty kilometers. They compare their solution to several existing solutions and show that their solution has some desirable properties. Vornhusen et al. (2014) also investigated the PDPTW, extending it with transshipment points for the collaborating carriers. Their model was translated into an MIP model and realized reasonable cost reductions. Finally, Wang et al. (2017) investigated the capacitated VRP with the above-mentioned mechanism. Finally, Cuervo et al. (2016) performed simulation studies on the effects of partner characteristics on the collaborative coalition. They conclude that coalitions with similar order sizes achieve the best savings. Futhermore, the larger the order portfolios, the more gains can be achieved through forming collaborative coalitions. Savings generated by centralized planning could be a benchmark for collaboration by auctions. It shows the maximal savings potential that could be generated, even with imperfect information. Most literature show savings in the order of 20%–30%.

2.2. Decentralized collaboration by auctions

Berger and Bierwirth (2010) introduced a new direction related to

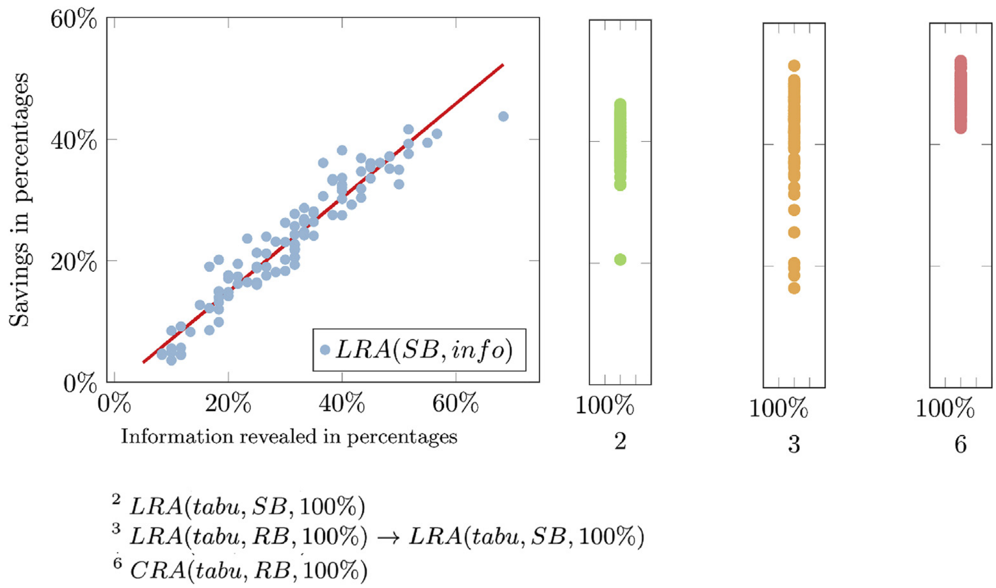


Fig. 9. Relation between the savings and the percentage of requests shared as information.

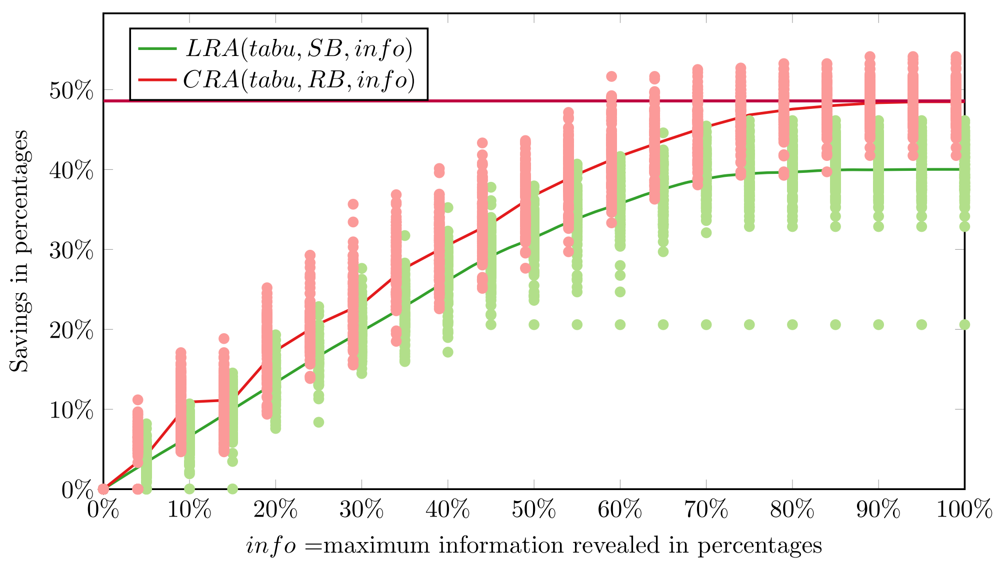


Fig. 10. Savings as a result of information revealed.

exchanging requests. They did not consider the final phase of the mechanism formulated by Krajewska (Krajewska et al., 2007), i.e., profit sharing, but instead focused on the exchange mechanism. Berger and Bierwirth (2010) utilized the ideas from the work of Song and Regan (2005), which initiated the concept of examining requests separately. A substantial part of their work was on determining whether an order should be exchanged, which is a complex decision for a carrier depending on several factors. Each bundle of items auctioned generates savings for the auctioneer and profits for the bidder, negating the necessity for profit sharing after the exchange phase. Each participating carrier is both a bidder and an auctioneer, resulting in extremely localized auctions. The auctioning of bundles of requests results in an NP–hard combinatorial auctioning problem (CAP). Wang and Kopfer (2014) noted that by utilizing their approach the cost savings potential was on average 18.2%–64.8%, which was lower compared to the savings from the central plan.The underlying routing problem defined by Berger and Bierwirth (2010) is the traveling salesman problem with pickup and delivery. Dai and Chen (2011) used the same concepts as formulated by Berger and Bierwirth (2010) and applied them to a pickup and delivery problem for a reduced truckload freight. They introduced an iterative auctioning system with some distinct properties. Multiple auctions can happen simultaneously. They showed that significant gains could be incurred using this system. Wang and Kopfer (2014) discussed the PDPTW and applied a route-based bidding mechanism, where the bids were based on the routes they generated. Li et al. (2015) formulated a request exchange approach based on a single request and introduced four profit allocation strategies. They included new exchange approaches that avoid local optima and showed some promising results. In general though, the literature agrees that clusters affect the performance of single item auctions in a negative way. Jacob and Buer (2018) investigated the effects of bidding non-truthfully. They showed that non-truthful bidding is not collectively rational, but individually it is, and it directly results in something similar to the famous prisoner's dilemma. Gansterer and Hartl (2016) investigated several request evaluation strategies for the auction mechanism proposed by Berger and Bierwirth (2010). By replacing the exact approach with a heuristic, they could solve larger instances for the

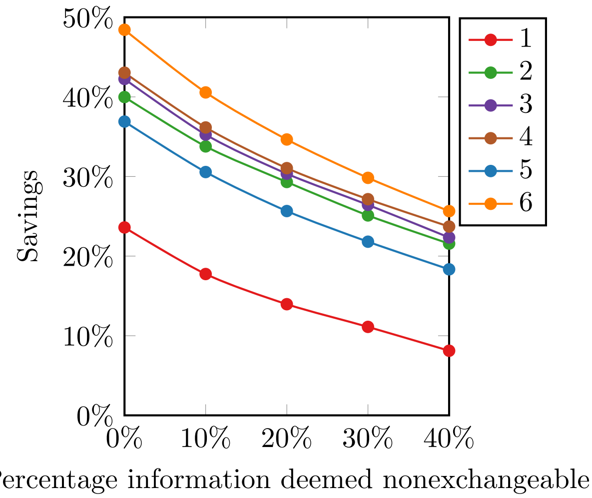


Fig. 11. Savings in relation to non–exchangeable requests. See Table 5 for the Legend.

TSP with precedence constraints. They showed that strategies that consider the geographical location dominate other strategies. When submitting bundles to the auction, Gansterer and Hartl (2018a) show that very attractive subsets of these bundles can be effectively identified. This greatly reduces the computation complexity of auctions involving sets, as well as all possible subsets. More recently frontiers were pushed by Gansterer et al. (2019) where it was shown that the auctioneer should bundle requests. Furthermore, a new formula for the profit sharing mechanic was introduced, which is computationally tractable and guarantees individual rationality.

This paper is rooted in the literature regarding decentralized collaboration based through auctions. In a lot of research the pickup and delivery problem is investigated. The reason for this is obvious, requests are then independent between carriers. In our problem, we consider distribution networks. The result is that the goods have to be re–positioned among the depots post–auction. Which makes the auction only profitable if the savings made by redistributing the customers is larger than the extra re-positioning cost. This issue is more deeply investigated in the remainder of the paper.

3. Conceptual environment

A set of independent carriers is considered, with each carrier providing similar transportation services to their customers. Each carrier has a sufficiently large and homogeneous fleet of vehicles, and each vehicle has a known maximum capacity. They also have their own single depot, and all the requests need to be served (potentially within a certain time window). The travel and service times are considered to be deterministic. The objective of the carriers is to maximize profit. Assuming that the price of the service paid by the customers is fixed and given, as specified by their contract, the decision problem is actually a cost minimization problem. Goods have to be re-positioned a priori among the depots. In this paper, carriers collaborate and apply the Reassignment Auction to exchange requests.

3.1. Collaborative environment

In this section, the collaborative environment, including the trade mechanism, is discussed in detail. Carriers may choose to serve only their own requests, but they can also subcontract requests to other carriers, or accept requests offered by these carriers. This trading of requests is performed by an auction, denoted as the Reassignment Auction.

In each iteration, a *selling* carrier submits a *candidate set* of requests to the auction. The size of the candidate set of requests depends on the selection method used. In this paper, a few selection methods are discussed. These are selection methods based on single requests, a specific combination of requests based on certain criteria (clusters), or complete routes (see Section 3.2.1 for more details).

The selling carrier selects the candidate set to be submitted to the auction based on marginal cost (i.e., the cost difference between the

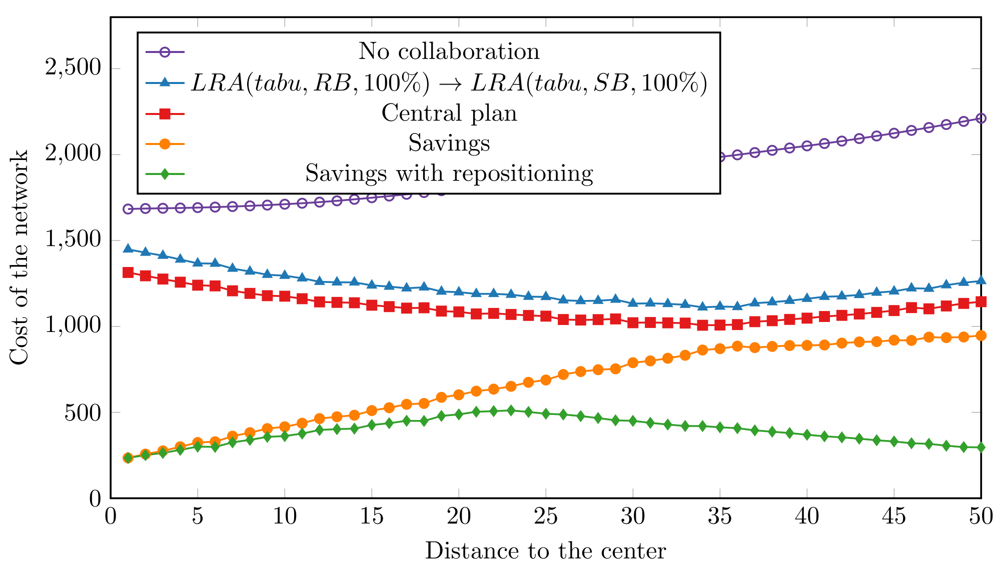


Fig. 12. Cost of the network as a result of distance.

Table 6
Savings as a function of α.

α	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45
Savings	9.3%	15.2%	17.3%	19.2%	21.2%	18.9%	16.7%	14.2%	12.6%
Average cluster size	2	2.1	2.5	3	3.6	4.6	5.3	5.9	6.5

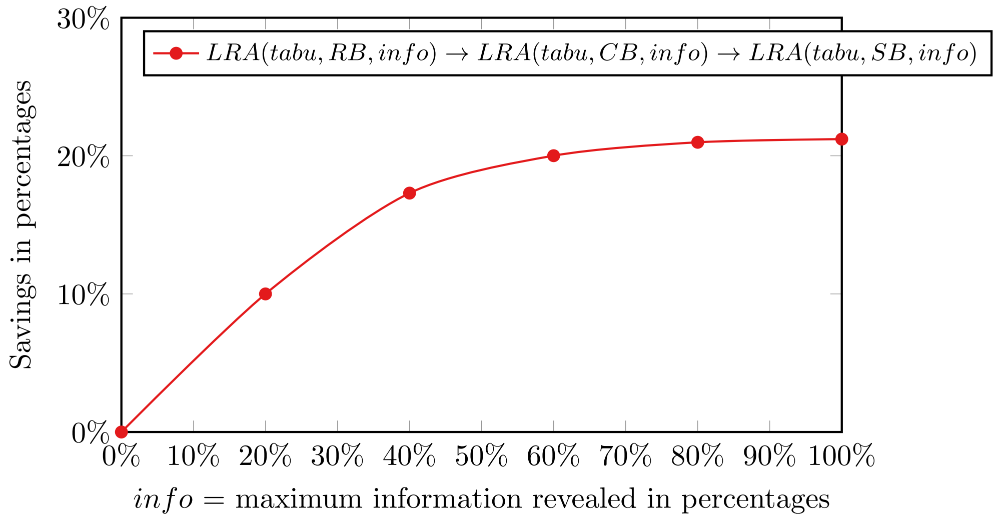


Fig. 13. Savings if controlled for information revealed for the real–life case.

routing plan including this set of request and the cost of the routing plan excluding this set of requests). This concept of the marginal cost is formalized in Section 4. The set with the largest marginal cost is presented at the auction.

All participants, excluding the selling carrier, subsequently bid on this set of requests and become *bidding* carriers. In our auction, each bidding carrier bids their true cost, defined as the marginal cost incurred by including the extra requests in their service. The participant with the lowest bid has the lowest marginal cost of servicing the request of the candidate set, and thus, wins the auction so that the auctioned set is added to his requests. In this manner, the overall savings are maximized. Note that to evaluate the bids, the selling carrier also needs to evaluate its own marginal cost for the offered request. If his own marginal costs are lower than the bids, he rejects the bids and keeps the requests in his own portfolio.

Whenever a carrier submits a candidate set to the auction, information of this request is shared with the other carriers. This information consists of the location and quantity of the request. For any form of horizontal collaboration to exist between these competitors, information must be shared. The participants can however decide how much information to share. Carriers can limit the sharing of information in two ways. There can be set a maximum amount of information to be shared or carriers can select a predefined set of requests that they will not share with the other carriers, fearing it will be used to obtain a competitive edge.

The relationship between savings and the information revealed is very dependent on the distribution of goods geographically and among the participants. For example, if the initial solution to the instance is already the optimal solution, no request will be exchanged while some information will be shared. Furthermore, if all requests are always positioned very close to the depot of another participant, sharing information will result in the maximum amount of savings. Both are extreme instances. However, in general there will be a positive correlation between information shared and the savings.

In summary, the Reassignment Auction follows these steps (see Section 4 for details):

- Step 1. The selling carrier computes the marginal costs for all the candidate subsets in his request set.
- Step 2. Among the candidate subsets, the most expensive subset of requests is selected as the candidate set to be submitted to the auction.
- Step 3. Each bidding carrier bids on this subset.
- Step 4. The candidate set is reassigned to the buying carrier(s).

During step 4, in the *Limited Reassignment Auction* (LRA) a candidate set is only allowed to be reassigned in its entirety. In the *Combinatorial Reassignment Auction* (CRA), participants can bid for the full set, but also for all possible subsets. The candidate set can then be reassigned to multiple carriers whose combined bid wins. This is referred to as the *Combinatorial Auction Problem*, discussed in Section 4.3.

It is required to re-position the goods between the depots prior to the execution of the plan. This re-positioning can in general be performed by less costly transportation methods, such as using Longer Heavier Vehicles or rail. Alternatively, vertical collaboration can cause the suppliers to directly deliver goods to the right depot. As such, we assume transportation costs between depots to be lower per kilometer than the actual delivery itself. The savings can only be determined when the auction is completed, and information is revealed. In our experiment we will focus on this specific situation as well. Moreover, the decision for re-positioning can also be made during the auction, as an example when enough requests have been exchanged to fill a truck. However, making the decision during the auction can be a research topic in itself, weighing the potential future cost savings of continuing the auction against the ordering of a extra truck.

Note that there is the topic of who pays for the re-positioning cost and the implementation and maintenance of the online auctioning system. While several game-theoretical possibilities here exist, it holds that the auctioneer using the "re-positioning" space the most is also the participant who has gained the most savings through the auction.

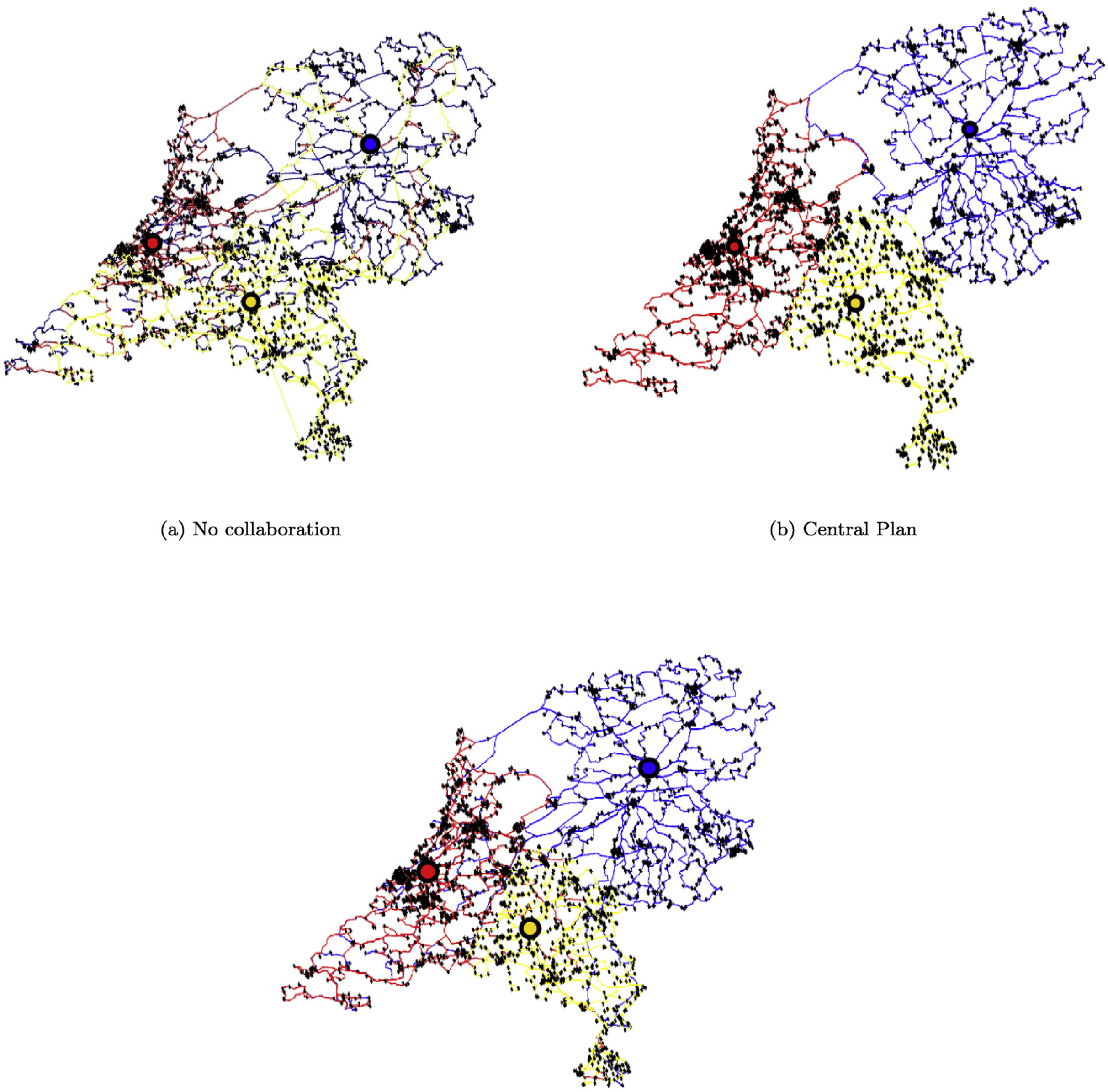
3.2. Candidate set selection

We consider several request selection methods for the Reassignment Auction. These are, as mentioned in section 3.1, one request, complete routes, or a specific combination of requests based on clusters. We first introduce the candidate sets of single requests, before considering the multi-item sets.

3.2.1. Candidate sets of single requests

Whenever a candidate set is submitted consisting of a single request, it works differently for the LRA compared to the CRA.

Limited Reassignment Auction. In this part, we discuss the investigation of the *Limited Reassignment Auction* (LRA) with the special case of a candidate set of size 1. This is equivalent to the *Single Request Reassignment Auction* (SRRA) as mentioned in Berger and Bierwirth (2010). See Fig. 2 for an illustration of this auction. Consider the three participating carriers prior to commencing the auction, who will participate in the Limited



(a) No collaboration

(b) Central Plan

(c) Result after $LRA(tabu, RB, 100\%) \rightarrow LRA(tabu, CB, 100\%) \rightarrow LRA(tabu, SB, 100\%)$

Fig. 14. Results for a real–life based case.

Table 7
Cash-flows for the real–life based case.

Carrier	1	2	3
Cash out (−)	2.945	1.210	2.875
Cash in (+)	1.363	3.894	1.773
Old Routing costs (+)	19.769	6.095	7.562
New Routing costs (−)	11.187	8.447	6.460
Difference (=)	8.582	−2.352	1.102

Table 8
Performance for the real–life based case with *alpha* = 0.25.

	Savings	Savings with re-positioning	Gap
Results for the real–life based case	21.2%	16.8%	12.9%

Reassignment Auction with a single request selection method. Each participating carrier offers a candidate request to the auction by turn. A *round* is completed when each participating carrier has taken his turn. The auction stops when no requests are reassigned during a round. This is the original stopping criterion of Berger and Bierwirth (2010). Carrier 1 starts, and offers his request 14 with the highest marginal costs, 40, on the auction. During the round Carrier 2 bids 27 and carrier 3 bids 9 for this request. Carrier 3 is subsequently the winner, adds request 14 to his request set, and receives 9 in payment from carrier 1. This results in savings of 31 for carrier 1.

Suppose the initial total costs for carriers 1 through 3 was, 90, 70 and 80 respectively. Table 1 reflects all cash flows. Multiple strategies can be implemented to divide savings among the participants. However, redistribution of these savings is the topic of game theory, which is not within the scope of this research.

It is improbable for the Limited Reassignment Auction with candidate set size 1 to reach the cost savings potential realized by the central plan, as illustrated in Fig. 3. In this example, selling carrier 1 offers node 11, but this node is not exchanged. However, node 14 could be exchanged, and thereby, reduce the cost of servicing the network even further. The auction will, however, terminate before this occurs. Thus, there could be requests leading to savings if put on the auction, but the auction stops before that could happen.

To improve the performance of the LRA, we introduce a tabu list. A request is added to the tabu list when it is not reassigned during a round. Accordingly, in the next round, the requests on the tabu list are not considered. This allows for considering other candidates in the request set besides the one with the largest marginal cost. Any time a request is exchanged, the tabu list is emptied. Thus, in this manner, a larger number of requests is considered. Note that when the tabu list is enabled, the auction will only terminate after all information which is allowed to be shared, is actually shared.

Combinatorial Reassignment Auction. In the Combinatorial Reassignment Auction, *each* participant submits a single request at the same time, which forms a set. All participants can then bid on all possible subsets of this set. The combination of participants with the lowest bid have the requests reassigned to them. Without a tabu list this variant is equivalent to the *Bundled Request Reassignment Auction* (BRRA) as mentioned in Berger and Bierwirth (2010). Note that, if only a *single* participant would submit a single request, this variant would be identical to the LRA.

3.2.2. Multi-item auction sets

Intuitively, two requests positioned close to each other (either in space or time) will most probably be also on the same route. Additionally, the marginal cost of such an individual request is low. Clearly, when one of the requests is already being served, it takes minimal effort to serve the other request in close vicinity. The difference in the cost of the routing plan and considered request is almost equal to the cost of the original routing plan. The sets of requests with these properties are defined as clusters. Similarly, Gansterer and Hartl (2016) stated that the geographical location of the requests affects the performance of an auction.

Clustered requests have a low individual marginal cost. The cluster as a complete entity, however, may have a significant marginal cost. In the LRA with candidate set size 1, the most expensive request is selected, but not the most expensive cluster. In Fig. 4, a route serviced by selling carriers 1 {11, 12, 13, 14, 15} is visualized. As bidding carrier 2 is closer to all the nodes in the route, it could perform the route at a reduced cost. However, if a single request is transferred to bidding carrier 2, the total cost of the network increases. Carrier 1 bids lower than carrier 2 for this request because it still has to service the requests for the remainder of the route. The additional effort is then minimal. Routes similar to Fig. 4 will not effectively be reallocated by the Limited Reassignment Auction with candidate set size 1.

Based on the above insight, it is apparent that to maximize the cost savings potential of an auction, the candidate set should be comprised of bundled requests rather than individual requests.

Limited Reassignment Auction. For the Limited Reassignment Auction it means that multi-item candidate sets will be submitted to the auction, where entire sets are exchanged from one participant to the other. The two selection methods that we investigate are the route-based request set selection method and the cluster-based request set selection method. In the route-based request set selection method a participant submits the route with the highest marginal costs in his current plan to the auction. In the cluster-based request set selection method, a participant identifies clusters within routes, and subsequently submits the cluster with the highest marginal cost. More details of these selection methods can be found in Section 4.2.

Combinatorial Reassignment Auction. For the Combinatorial Reassignment Auction we only introduce the route-based request set selection method, in which a *single* participant submits a *single* route as the candidate set. Compared to the Limited Reassignment Auction, the cluster–based selection method will not be investigated. One would expect that a dense cluster of requests will not be reassigned among multiple participants. Since, similar to the logic as seen in Section 3.2.2, it takes minimal effort to service request in close vicinity from each other. Furthermore, as in the Combinatorial Reassignment it is possible to bid on subsets of request.

Summary of the auctions. Summarizing, we investigate the five different auctioning variants as indicated in Table 2.

4. Formal models

In this section we formally introduce the *Limited Reassignment Auction* (*LRA*) and the *Combinatorial Reassignment Auction* (*CRA*). We generalize the auctions introduced by Berger and Bierwirth (2010).

In what follows, *SB* indicates that the single request method is used (equivalent to the SRRA Berger and Bierwirth (2010)), *RB* implies that the route-based method is used, and *CB* implies that the cluster-based method is used. Formal definitions of these are given in Section 4.2. All auction variants with *tabu* indicate that the tabu list is enabled and *info* indicates the maximum percentage of information which may be revealed. For example, $LRA(tabu, RB, 100\%)$ indicates that the Reassignment Auction is executed with the route based request selection method, the tabu list enabled and 100% of the information is allowed to be shared. If $info = 50\%$ maximally 50% of the requests may be revealed in the Reassignment Auction. If multiple Reassignment Auctions are performed sequentially, $info = 50\%$ implies that if in the first Reassignment Auction already 30% information is revealed, in the following Reassignment Auction only 20% additional requests may be revealed. The request already revealed in an earlier Reassignment Auction could always be revealed in a following Reassignment Auction, as this information has already become available to the other carriers.

Each carrier $f \in F$ has its own request set, N_f. Individual requests are denoted as i, and a candidate set is denoted as I. $S[Q]$ is a solution of the routing plan over a set of requests, Q. Let $c[S[Q]]$ be the costs associated with this solution.

Let L_f contain all possible candidate sets from N_f, based on the chosen request set selection method. Consider a candidate set, $I \subseteq L_f$; then marginal cost, $m_f(I)$, is equal to $c[S[N_f]] - c[S[N_f \setminus \{I\}]]$. Bid $b_f(I)$ of carrier f is then equal to $c[S[N_f \cup \{I\}]] - c[S[N_f]]$, which is the marginal cost of adding candidate set I to the request set.

We reintroduce and formalize the steps from Section 3.1, where f is the selling carrier:

- Step 1. For all candidate sets $I \subseteq L_f$, the marginal costs are computed, $m_f(I) = c[S[N_f]] - c[S[N_f \setminus \{I\}]]$.
- Step 2. From these sets, we select the most expensive set, I^*, of requests, $I^* = \arg\max\{I \subseteq L_f : mf(I)\}$.
- Step 3. Each participating carrier, $g \in F$, bids on set I^* with price $b_g(I^*) = c[S[N_g \cup \{I^*\}]] - c[S[N_g]]$. In the case of a combinatorial auction, bids are also made for all subsets of I^*, where the formula for the bid price is equivalent.
- Step 4. Reassignment of request set I^* to the winning carrier, $g^* = \arg\min\{g \in F : b_g(I^*)\}$, takes place. The winning carrier, g, obtains an updated set of requests, $N_{g^*} = N_{g^*} \cup I^*$.

Each of these steps is discussed in detail in the following section.

4.1. The reassignment auction

As mentioned in 3.2.1, each participating carrier offers a candidate set in the auction by turns. The exception to this is the $CRA(SB)$, where *each* participant offers a single request at the same time. A round is completed when each participating carrier has had his turn. The Reassignment Auction stops when no requests are exchanged during a round. As mentioned in section 3.1, for each carrier, $f \in F$, a tabu list can be introduced, denoted by set T_f. Whenever a carrier, f, wins his own candidate set, I, it is added to the tabu list, T_f. The auction terminates as no further improvement can be made (i.e., when all the requests are on one of the tabu lists), or if the maximum percentage (*info*) of requests is revealed. The basic algorithm for the Limited Reassignment Auction, $LRA(tabu)$, is presented in Fig. 5. Based on how the information allowed to be revealed is limited, this algorithm can be adapted in two ways:

- In the first, the maximum amount of information to be revealed is equal to a percentage of the requests. A request is seen as revealed if it is placed at least once in the auction, and thus, have become visible to the other participants. In the algorithm of Fig. 5, this means that information revealed is recorded, and the auction only continues while this amount is smaller than parameter *info*.
- In the second option carriers have a predetermined set of requests that are not allowed to be auctioned; they are designated non-exchangeable. All candidate sets containing one of these requests are a priori removed from L_f. The results of this are presented in Section 6.2.3.

The Limited Reassignment Auction with tabu, $LRA(tabu)$.

4.2. Request set selection methods

Determining the set L_f of all possible candidate sets of a single request (i.e. the SB method) is trivial. However, as discussed earlier the marginal cost for each individual request in a cluster may be low. In contrast, the marginal cost for the cluster as a whole could be very high. Therefore, we also introduced candidate sets containing multiple requests. In this section, we discuss in detail the request selection methods RB and CB.

Route-based request set selection method (RB)

One potential request selection method is to select candidate set I on the basis of routes, denoted as $r \subseteq S[N_f]$. Running the Reassignment Auction with only routes will not explore the full cost savings potential. Auctioning routes only allows for approximate improvements to the network. Therefore, in our experiments a Limited Reassignment Auction using the RB method is followed by a Limited Reassignment Auction with single requests. When this set selection method is used, an additional parameter *RB* is allotted to the RA.

Cluster-based request set selection method (CB)

In real-life and depending on the carrier, routes can contain numerous requests. For routes above a certain size (> 4) of requests, the cluster detection algorithm is introduced. This algorithm is based on the concept that individual requests on a route that form a cluster have lower marginal costs. Allow us to first present an example of this concept.

Consider the following route, as visualized in Fig. 6:

$$route = \{5, 6, 9, 2, 4, 14, 12, 3, 11\}$$

With the following associated marginal cost:

$$m = \{10.51, 1.67, 11.21, 12.67, 1.54, 0.76, 1.23, 14.53, 10.77\}$$

Note that requests 6, 14,12 and 3 have far lower marginal costs than average marginal cost for the route. Marginal costs of a certain request are intuitively the difference in cost between the route including and excluding that request. Thus, if one already has to go to a certain location, the marginal cost of additional requests at that location will be lower. Requests 14, 12 and 3 appear in sequence, which might (in the Figure it *does*) indicate that they form a cluster. Request 6 might coincidentally be on the way from 5 to 9. We can now formulate a methodology that generates clusters on the basis of this information.

Suppose there is a vector, p, of sequential requests within a route, r, for which the average marginal cost is significantly lower than for the rest of the route, r. Then, this vector, p, is identified as a cluster. Formally, vector p is defined as a cluster if the following holds:

$$\frac{\frac{\sum_{i \in p} m_f(i)}{|p|}}{\frac{\sum_{i \in r: i \notin p} m_f(i)}{|r| - |p|}} < \alpha$$

Of course, the size of such a cluster depends on the choice of α. To this end, to determine the best value for α, we will do multiple tests with different values. Note that because this is a cluster detection mechanism, vector p needs to be of minimum size 2. Vector p is determined by sequentially checking the route for the existence of a pair of linked requests for which the above statement holds. This set is then extended with requests linked to this pair until the statement no longer holds. As a result, all clusters are disjoint, and no cluster can contain the same request. Vector p could then be considered a potential cluster and can be an effective candidate set for the auction. Note that even when vector p is not a cluster, a transfer of the set only happens when a competitor can serve the set with less cost. Any route r can contain multiple clusters. During each round, all potential clusters are determined. Once again, the cluster with the highest marginal cost is selected for submission to the Reassignment Auction. For all Reassignment Auctions the average cluster size will be reported. When this set selection method is used, an additional parameter *CB* is allotted to the RA. The above-mentioned method is processed in the algorithm in Fig. 7.

4.3. Combinatorial auctioning problem

The Combinatorial Auctioning Problem can be considered as a replacement of *Step 4* of the Reassignment Auction mentioned in section 3.1. When this is implemented, it results in the *CRA*. In the reallocation of subsets, it is possible to reallocate parts of the candidate set to different carriers. Only the RB set selection method is applied to *CRA*, and not the CB. By the definition of a cluster, it is in general only attractive to a single participant. Also, since each cluster is a subset of a route, the participants will in the $CRA(RB)$ already bid on these clusters as they need to bid on each possible subset of the route. As a result, the CB set selection method is not applied to *CRA*.

Given a set of carriers F, candidate set I and bids $b_f(I_k)$ that carriers $f \in F$ are willing to pay for a subset $I_k \subseteq I$, the objective is to minimize the cash flow of the auction (i.e., to determine the reallocation of requests with a minimum cost). The restriction is that every individual request i in candidate set I is uniquely allocated to a carrier. All subsets of candidate set I are generated. This results in the following integer programming formulation, where $x(I_k, f)$ is 1 if the bundle I_k is allocated to carrier f and 0 otherwise.

$$\text{minimize}: \quad \sum_{f \in F} \sum_{I_k \subseteq I} b_f(I_k) x(I_k, f)$$

$$\text{subject to}: \quad \sum_{I_k \subseteq I : i \in I_k} \sum_{f \in F} x(I_k, f) = 1, \quad \forall i \in I$$

$$x(I_k, f) \in \{0, 1\}, \forall f \in F \quad \forall i \in I$$

This formulation assures that each individual request i is uniquely allocated to a carrier f and that the total marginal cost is minimized. This problem is known as the *CAP* and is categorized as NP-hard (De Vries and Vohra (De Vries and Vohra, 2003)). In our experiments, however, the Combinatorial Auctioning Problem is solved using complete enumeration because of the small instances, which are determined by the length of a single route. Note that the time-consuming part of the method is not actually solving the CAP, but determining the bid prices and costs over all possible subsets. The result is that the CAP is not computationally tractable for large instances. Because the Combinatorial Auctioning Problem is an enhancement to step 4 of the reallocation of requests, more solution space is considered in the *CRA* compared to the *LRA*. Consequently, the increase in savings resulting from this auction is expected to be more than from the other implementations of the auction. A visual example is presented in Fig. 8. For a more detailed explanation we refer to (Berger and Bierwirth, 2010).

5. Routing algorithms

Our methodology will require many VRP calculations for which we will use a state of the art methodology as discussed in 5.1. Next to that, to discuss the performance of our approach, we need to know the optimal solution for the central plan. The way to estimate the objective function value for the central plan is discussed in 5.2.

5.1. Vehicle routing sub problems

To calculate $S[N_f]$ a solution to the vehicle routing problem VRP(TW) needs to be known. For the small instances, where carriers service approximately 20 requests, the intermediate VRP(TW) computations can be solved optimally using Gurobi 8.1 with short computation times. For the huge instances the ALNS algorithm as mentioned in Pisinger and Ropke (2007) is used as a heuristic for intermediate VRP(TW) computations. For the quality of this heuristic we refer to their paper. In the next section the algorithm to compute the central plan is discussed.

5.2. Central plan

To asses the quality of the auction it is required that the central plan is approximated, which is the solution to a Multi Depot Vehicle Routing Problem (MDVRP). The MDVRP differs significantly from the CVRP in that requests also have to be assigned to the depots. Finding exact solutions for the problem sizes of our instances is not possible. Therefore, we use a Genetic Hybrid Algorithm that follows the work of Ho et al. (2008). The algorithm works similar to local search techniques, some offspring (neighbours) are produced through a variety of genetic operations of the current considered parent ("best found") solution. The algorithm starts with an initial solution, which in our algorithm is generated with the ALNS algorithm, while in Ho et al. (2008) it is generated through Clarke and Wright. Subsequently, the following steps are iterated:

1. *Improvement* Here different kind of inter-route and inter-depot swaps are performed to obtain offspring.
2. *Evaluate* These offspring are subsequently evaluated according to the fitness function. Our fitness function is different as a result of the difference in objectives. Whereas Ho et al. (2008) has the objective of minimizing the maximum delivery time among all routes, we have the objective of minimizing the total travel distance. We use as fitness function simply the sum of the driving distances over all used arcs, which is the traditional MDVRP objective function, $Eval(X_h) = \sum_{i=0}^{n+f}\sum_{j=0}^{n+f} c_{ij}x_{ij}$.
3. *Selection*From the offspring a roulette wheel selection is used to select offspring that undergo mutations.
4. *Genetic Operations* Mutations are then applied to the selected offspring .

This algorithm provides good results in reasonable computation times compared to the best known solutions in literature, which we show in Table 9 in the Appendix. The average gap of the solution of our central plan algorithm to the best known central plan is 4.9%, with a standard deviation of 2.2%. This makes it accurate enough to provide valid insights for our auctioning algorithms.

6. Results

First we discuss the instances in more detail, and then, an extensive numerical analysis is discussed.

6.1. Instances

Two types of instances are considered in this study: small and a real–life based case. The small instances contain 60 requests, the real–life based instance contains 2967 requests. The real–life based instance is based on real-life data, contains a number of requests similar to real-life carriers, and is discussed in more detail in Section 6.2.5. Additionally, the Cordeau et al. (1997) instances are used to test the effectiveness of the Limited Reassignment Auction and to validate our approach for approximating the central plan. This is performed by randomly assigning requests to each of the depots and subsequently testing the best performing (on our self-generated instances) LRA auctioning variant. Independent of how the requests are allocated to the carriers, the solution of the central plan remains the same. Therefore we consider the best known solution for the original instance in Cordeau et al. (1997) as the optimal solution for the central plan.

Note that instances with 60 requests are not considered normal for real-life vehicle routing problems, but they are used to validate the presented methodology. During the auction the participant has to determine the marginal costs for each separate requests. As an example, if a carrier has a set of 20 requests, the vehicle routing problem needs to be solved 20 times. When 20 requests are exchanged then a vehicle routing problem needs to be solved 400 times, only for determining the marginal costs. Both the bidding and effect of the tabu list is not included here, which could even raise the amount more substantially.

For the small test cases, random instances are generated on a surface of 100×100. Unless otherwise stated, the depots of the carriers are positioned on the vertices of the triangle $\{\{20,20\},\{20,80\},\{80,50\}\}$. The choice of these vertices is based on dividing the surface into three similar hinterlands. Subsequently, 60 requests are randomly positioned on the surface and randomly allocated to the carriers. Unless otherwise stated, 100 instances are generated. The capacity of the vehicles is set as 5 requests. The re-positioning costs are chosen to be 70% less costly than normal daily operations.

6.2. Experiments

In this section, we first present the results of the Reassignment Auction on the randomly generated small instances ($n = 60$) in Section 6.2.1. Additional experimental setups were chosen to provide insight into the effects of several potentially performance varying externalities, such as, the presence of time windows (Section 6.2.2), the amount of information allowed to be revealed (Section 6.2.3) and the positions of the depots of the carriers (Section 6.2.4). For each implementation, the percentage deviation from the central plan is presented as well as the savings with respect to the case where there is no collaboration and the amount of information (i.e., requests) revealed. Note that the auctioning mechanisms are based on servicing costs. We assume that the three carriers have similar costs per distance measure and therefore the savings are interpreted as a reduction in the total distance driven by the 3 carriers. In all tables, the average of the performance over 100 instances is represented as μ and the standard deviation is represented as σ.

In principle we have 6 implementations of the Reassignment Auction which we test in our experiments:

1. $LRA(SB, info)$, the Limited Reassignment Auction of single requests, which is equivalent to the Single Request Reassignment Auction of Berger and Bierwirth (2010).
2. $LRA(tabu, SB, info)$, the Limited Reassignment Auction of single requests with an enabled tabu list.
3. $LRA(tabu, RB, info) \rightarrow LRA(tabu, SB, info)$, the Route Based Limited Reassignment Auction followed by a Limited Reassignment Auction of single requests with an enabled tabu list.

4. $LRA(tabu, RB, info) \rightarrow LRA(tabu, CB, info) \rightarrow LRA(tabu, SB, info)$ are three Limited Reassignment Auctions applied sequentially that are route based, cluster detection based, and single item versions, in which the tabu list is enabled.
5. $CRA(SB, info)$, the Combinatorial Reassignment Auction of single requests, equivalent to the Bundle Request Reassignment Auction of Berger and Bierwirth (2010).
6. $CRA(tabu, RB, info)$, the Route Based Combinatorial Reassignment Auction.

In most experiments, we allow all information to be shared (i.e. $info = 100\%$. However in Section 6.2.3 we discuss the effects of different values for *info*. Also for the real–life based case, Section 6.2.5, different values of *info* are tested. Note that *info* is not shared among participants, but simply an evaluation measurement.

Of these Reassignment Auction variants $CRA(tabu, RB, 100\%)$ is expected to be the best performing auction. Since the Combinatorial Reassignment Auction is computationally very slow, we have only additionally included the variant $CRA(SB, 100\%)$. This also allows for a comparison with the Bundle Request Reassignment Auction of Berger and Bierwirth (2010) which is the same.

6.2.1. Results of the reassignment auction

The results of the Reassignment Auction are presented in Table 3. For each implementation of the auction, the average and standard deviation (over the 100 instances) of the deviation (gap) from the central plan, savings with respect to no collaboration, and the percentage of requests revealed to all participants are stated in the table. Note that setting $info = 100\%$ allows for revelation of all the requests on the auction. The effect of varying this parameter is discussed in the following section, as well as marking requests as nonexchangeable. Note that $LRA(tabu, RB, 100\%) \rightarrow LRA(tabu, CB, 100\%) \rightarrow LRA(tabu, SB, 100\%)$ is only a marginal improvement over $LRA(tabu, RB, 100\%) \rightarrow LRA(tabu, SB, 100\%)$. To maintain parity with the real–life based case of Section 6.2.5, an α of 0.25 was chosen for the cluster detection algorithm. The average size of a cluster in $LRA(tabu, RB, 100\%) \rightarrow LRA(tabu, CB, 100\%) \rightarrow LRA(tabu, SB, 100\%)$ was 2.01, though it should be noted that a cluster was rarely transferred in these instances. $CRA(tabu, RB, 100\%)$ is always the best performer. The difference in the performances between $CRA(tabu, RB, 100\%)$ and $LRA(tabu, RB, 100\%) \rightarrow LRA(tabu, SB, 100\%)$ is significant, indicating that routes are not necessarily good indicators for clusters.

Although $LRA(SB, 100\%)$ may reveal all requests on the auction, it mostly terminates very fast (after approximately 29% of the requests are placed on the reassignment auction). This is expected as the tabu list is not enabled. However, sharing this 30% of the requests already leads to 22% savings on average, or 6.1% savings when re-positioning of the goods amongst the depots is necessary. Note that the savings when re–positioning is taken into account is significantly lower. This implies that the difference in distance between the carriers is of significant effect on the actual realized savings.

6.2.2. Results for the reassignment auction applied to the VRPTW

The results for the carriers which solve the VRP with Time Windows are presented in Table 4. The VRPTW is in general a routing problem that for the considered carriers has the most practical relevance. It can be observed that the performance does not deviate much from the performance of the auctions applied without the time windows, though the savings increase somewhat. These results support the conjecture that the reassignment auction also achieves a good performance on instances with time windows. Time windows create additional restrictions in the routing problem. This results in a greater difference between the costs of the single and central plan. As the Reassignment Auction still approaches the central plan, though being less effective, the savings increase substantially as a consequence.

6.2.3. Value of information

The relation between the percentage of requests shared and savings for the 100 instances is presented in a scatter plot in Fig. 9 for 4 of the Reassignment Auction variants. For the methods with tabu list enabled, all information is shared and the figures provide an indication about the spread of the savings. For $LRA(SB, info)$ it can be seen that the amount of savings gained by revealing extra information is quite stable as all dots are positioned along a straight line. However, remember that in this situation we did not control the information shared (i.e. the auction stopped because in one round no request was exchanged).

For methods with the tabu list enabled we could control the amount of information shared. In Fig. 10, the effect of the different values of *info* on the $LRA(tabu, SB, info)$ and $CRA(tabu, RB, info)$ implementations is shown. The maximum percentage of requests that is allowed to be revealed on the auction (*info*) is presented on the horizontal axis. The carriers cannot reveal any new requests at the auction after *info*% of the total number of requests have already been revealed on the reassignment auction. Hundred instances are tested for the settings of *info* from 0 to 100% in steps of 5%. The thick horizontal red line is the average savings achieved by implementing the central plan. This allows us to illustrate the gain in savings when a certain maximum percentage of information is revealed per carrier. On average the $CRA(tabu, RB, info)$ leads to larger savings than the $LRA(tabu, SB, info)$ implementation while revealing the same amount of information. One can observe that the trend lines over the observations have a decreasing slope. This can be attributed to the fact that the requests with the highest marginal costs are auctioned off first, and that most of the savings potential is gained in the first rounds of the auction. Moreover, the slope at the tail end of the trend line is equal to zero for most instances. Here no more savings are made. This implies that not all the information has to be revealed to obtain the maximum savings potential. After about 70% of information has been revealed the auctions more or less reached the maximum level. This is intuitively logical as we would expect the Reassignment Auction to approach the central plan. As expected, 1/3 of the request set of each carrier is in its own area of service, and thus, does not need to be revealed.

Furthermore, we have tested the effects of having a predefined subset of requests being determined non–exchangeable. This has been performed with values 10%, 20%, 30% and 40% of the requests being not available for exchange. In these experiments, we did not update the central plan benchmark. The central plan still allows for all requests to be exchanged. The results can be seen in Figure 11. Note that as expected, the gap to the central plan is increasing and the savings (with or without re-positioning), are decreasing. Note also that the incline for the best performing Reassignment Auction, $CRA(tabu, RB, 100\%)$ is higher than for the least performing Reassignment Auction, $LRA(SB, 100\%)$. This could be due to the fact that the best potential solution gets further removed from the best known solution. The savings with re-positioning become 0 for $LRA(SB, 100\%)$ when 40% of requests become non-exchangeable. With the cluster-based request set selection method more savings are achieved with less information shared.

6.2.4. Results with regards to the sensitivity of the depot locations

In this section, the sensitivity of the performance of the Reassignment Auction to the depot locations is tested and the results are presented.

The analysis leaves the service regions of the carriers equal to $1/f$. Each carrier has to serve 20 nodes, and there are $f = 3$ carriers. The depots of the three carriers are located on a circle, whose radius is a control variable. This implies that only the distance between the depots is increased, but the shape of the service regions remains unchanged. $LRA(tabu, RB, 100\%) \rightarrow LRA(tabu, SB, 100\%)$ is used as the auction mechanism. The graph corresponding to this method for the three carriers is shown in Fig. 12. Note that as the distance between the facilities increases, the effects of the cost of re-positioning become greater.

6.2.5. Results for a real–life based case

A case with an more practical problem size is introduced based on the *actual* orders of a specific carrier on a specific day in the Netherlands. The order set consists of 2967 individual orders. The location of two depots of their competitors is known. A portion of the known order set of the first depot was assigned to the other depots, in order to make the example more realistic. This was performed based on the notion that a depot is probably close to his own customer portfolio. Note that only the assignment of requests to the depots has changed.

On a route there are on average 15 requests, depending on the capacity and demand. The capacity of each truck is 30 m^2, and requests use 2 m^2 on average. The result of sequentially performing a reassignment auction with the selection methods of routes, clusters, and single item (all with a tabu list enabled) is investigated ($LRA(tabu, RB, 100\%) \rightarrow LRA(tabu, CB, 100\%) \rightarrow LRA(tabu, SB, 100\%)$). The savings are based on kilometers driven. The algorithm to solve the routing problem is the ALNS algorithm, as discussed in Section 5.

The cluster detection method requires a value for α between 0 and 0.5. The results for certain values of α are presented in Table 6, based on the which $\alpha = 0.25$ was ultimately chosen for our example.

Based on the results of the real–life based case (Table 8, Figure 14), it can be observed that the performance of the auction mechanism is still very good even for problems of this size, where the road distance instead of the Euclidean distance is used. The number of kilometers driven is 33.426 with no collaboration, 26.094 with the Limited Reassignment Auction, and 23.094 with the central plan. Kilometers driven reaches 27.810, when re-positioning of the goods is taken into account. If there is a maximum percentage *info* allowed to be shared (Fig. 13), the value of information follows a similar pattern as in Fig. 9. Sharing 20% of information leads to more than 10% savings. The cash-flows are visible in Table 7. Note that carrier 2 has a negative difference, indicating that it has absorbed more than it has sold. This is compensated by the other participants. Again, a fair redistribution of the savings will be a research topic on its own.

7. Conclusions

Collaboration between carriers is desirable to reduce carbon emissions, kilometers driven and cost. Carriers, however, are reluctant to share information with their competitors. Joint planning, which requires perfect information, thus, becomes infeasible. If carriers would be willing to share small amounts of information iteratively, an auction becomes a possibility for exchanging requests. Such an auction could be implemented on an electronic platform. In this study, we generalize the *Reassignment Auction* as introduced by Berger and Bierwirth (2010) to the *Limited Reassignment Auction* and *Combinatorial Reassignment Auction*. Additional extensions are introduced: adding a tabu list and introducing a cluster detection algorithm for the LRA. A set of auction variants is subsequently applied to carriers that solve a VRP(TW) as their routing problem. The algorithms are designed to solve large-scale problems. The results from the instances generated by the auction provide us with the following valuable insights:

1. Initially, large savings are gained by sharing little information. As such, the mechanisms performs well in environments with little trust.
2. By including a tabu list, the Limited Reassignment Auction performs relatively well, though ultimately it does not reach the cost savings potential of the central plan. However, the participants can easily control the amount of information they share to achieve savings. The results are independent of whether or not time windows are present for the requests.
3. The Reassignment Auction performs best when bundles of requests are auctioned. The selection methods for these bundles are important for the performance. Two variants of the selection method were tested for the LRA, namely, route based and cluster detection based. Both selection methods performed excellently with the randomly generated instances we tested. However, the Combinatorial Reassignment Auction variants performed better, almost achieving the central plan. Finally, a sequence of Limited Reassignment Auctions were applied to a reallife size case, and this variant performed very well.
4. The Combinatorial Reassignment Auction solves the reassignment of requests and approaches the central plan in terms of the cost savings potential, but it is computationally expensive and reveals significant information. By implementing three successive Limited Reassignment Auctions, route-based, followed by cluster detection based, and single item (all with an enabled tabu list), already most of the savings potential is achieved, while requiring a considerably lower computation time. The Reassignment Auction yields cost savings potential in close agreement with that of the central plan for all our test cases.
5. There is a balance in the distance between the depots. Initially, as the distance between the depots increases, more of the region is covered by the depots, yielding higher savings. However, after a certain distance the cost of re-positioning also start to increase substantially.
6. The real–life based case shows that the savings made through the Request Reassignment Auction method can be varying for the participants, depending on their original allocation of requests.

Summarized, the Reassignment Auction is a very viable collaborative environment for carriers with which large savings can be obtained.

The following areas are identified for future research:

- In our research, we used complete solutions to the routing problem to obtain the shadow prices (marginal cost) of the requests. This is computationally expensive, and better methods could exist. Further research could be performed for estimating the shadow prices of the requests in the network.
- In our work, a transaction takes place each time a request is reassigned. Savings are gained from this reassignment, but the allocation of these savings was not in the scope of this research. Similar topics related to game theory, such as strategic bidding, were not investigated. This opens the door for interesting research on the player behavior in such auctions.

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Appendices.

Table 9
Results for the cordeau instances.

Inst	Size	BKR	CP	Gap	Inst	Size	BKR	CP	Gap
p01	n50k4	576.87	598.16	3.69%	**p18**	n240k6	3702.85	3913.84	5.70%
p02	n50k4	473.53	492.43	3.99%	**p19**	n240k6	3827.06	3977.57	3.93%
p03	n70k5	641.19	648.24	1.10%	**p20**	n240k6	4058.07	4439.09	9.39%
p04	n100k2	1001.59	1056.68	5.50%	**p21**	n360k9	5474.84	5841.61	6.70%
p05	n100k2	750.04	792.79	5.70%	**p22**	n360k9	5702.16	6083.80	6.69%
p06	n100k3	876.50	931.72	6.30%	**p23**	n360k9	6095.46	6475.90	6.24%
p07	n100k4	885.80	954.01	7.70%	**pr01**	n48k4	861.32	906.44	5.24%
p08	n249k2	4437.68	4584.12	3.30%	**pr02**	n96k4	1307.61	1395.65	6.73%
p09	n249k3	3900.22	4216.14	8.10%	**pr03**	n144k4	1806.60	1885.45	4.36%
p10	n249k4	3663.02	3857.16	5.30%	**pr04**	n192k4	2072.52	2145.05	3.50%
p11	n249k5	3554.18	3792.31	6.70%	**pr05**	n240k4	2385.77	2428.08	1.77%
p12	n80k2	1318.95	1332.36	1.02%	**pr06**	n288k4	2723.27	2788.84	2.41%
p13	n80k2	1318.95	1332.36	1.02%	**pr07**	n72k6	1089.56	1135.84	4.25%
p14	n80k2	1360.12	1459.41	7.30%	**pr08**	n144k6	1666.60	1755.84	5.35%
p15	n160k2	2505.42	2610.65	4.20%	**pr09**	n216k6	2153.10	2265.06	5.20%
p16	n160k4	2572.23	2654.54	3.20%	**pr10**	n288k6	2921.85	2979.68	1.98%
p17	n160k4	2709.09	2763.27	2.00%					

[Inst]Instance name.
[Size]Problem Size: The number after *n* denotes # of requests, the number after *k* denotes # of depots. [BKR]Best Known Result (http://neo.lcc.uma.es/vrp/vrp-instances/multiple-depot-vrp-instances/visited September 4, 2019).
[CP]Our Central Plan approximation (Section 5.2).
[Gap]Percent difference between CP and BKR.

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