Essays on Firm Heterogeneity and Quality in International Trade

Eddy Bekkers
This book is No. 427 of the Tinbergen Institute Research Series, established through cooperation between Thela Thesis and the Tinbergen Institute. A list of books which already appeared in the series can be found in the back.
Essays on Firm Heterogeneity and Quality in International Trade

Essays over heterogeniteit van bedrijven en kwaliteit bij internationale handel

Proefschrift

ter verkrijging van de graad van doctor aan de
Erasmus Universiteit Rotterdam
op gezag van de rector magnificus
Prof.dr. S.W.J. Lamberts
en volgens besluit van het College voor Promoties.

De openbare verdediging zal plaatsvinden op
18 september 2008 om 11.00 uur door

Eddy Henricus Gijsbertus Bekkers

geboren te 's-Hertogenbosch
Promotiecommissie

Promoter: Prof. dr. Joseph F. Francois

Overige leden: Prof. dr. Charles van Marrewijk
               Prof. dr. Jean-Marie Viaene
               Prof. dr. Doug Nelson
Preface

Writing a PhD thesis seems a long process. Basically, you start with a topic, change the topic and then you work until you have four papers about the same topic. In the meantime you have to teach, supervise students and you try to help colleagues if they have a challenging problem.

A problem with research at the university is that once you go into a certain direction with your research, you become more and more focused on it. Up to the point that you are not able to explain anymore what you are doing to friends. After a while you are able to explain what you are doing, but then the big question props up: is this relevant? And you also ask yourself: what on earth am I doing? What is the relevance of showing that there is a unique equilibrium in a firm heterogeneity model under oligopoly not only in the short-run but also in the long-run? But the next day you work even more fanatic on deriving the welfare effects of trade liberalization in the same model. And it really gives satisfaction if you are able to establish these effects clearly and give also an intuitive account for it.

And what about relevance for society? I would say that the research questions we pose are important and interesting. In the end they help us to understand the world better and to conduct a better policy. Often the academic community only develops abstract theories and tools. But without these theories and tools policy researchers could not work the way they do right now. As an academic there is the danger that you focus too much on detailed debates with other people in the field working on exactly the same models. Therefore, I will widen the scope of my work in the future and work on different topics. And switch from theory work to empirics.

Writing the thesis, I got a lot of help from my supervisor Joe Francois. He is always full of new and creative ideas, which were very useful in developing the different chapters of the thesis. Joe helped me to stay focused on the core questions that the different chapters address. And although he always posed several new questions that could also be dealt with within the framework of a certain model, he stimulated me to finish up the different chapters. Almost against his own nature, Joe said to me that certain questions were really important and interesting, but only for discussion in new and future papers. I also want to think the members of the committee, Charles van Marrewijk, Jean-Marie Viane and Doug Nelson for their comments on the different chapters.

I also benefited a lot from the help of my colleagues, first the PhD colleagues in Amsterdam, then the PhD colleagues in Rotterdam and finally the colleagues at the department of economics in Rotterdam. Of my PhD colleagues, I want to thank Matthijs and Felix in particular. Matthijs was of great help with the stimulating debates we had on both our research fields. Felix helped me with many things. I have a strong preference for proving results analytically with pen and paper. Felix made me enthusiastic to make more use of the computer and helped me a lot with the computer simulations and with finalizing the layout of the thesis. He also kept me sharp in the debates about the merits of formal economic models with the formidable assumptions we always make. I enjoyed teaching together with my colleagues from the department. The cooperation with Charles, Leon and Annette was very efficient and I learned a lot about supervising student-assistants, organizing a course and dealing with students. I also want to thank Leon for his help
with finalizing the layout of the thesis.
My friends supported me to go on with my research when I thought I was at a dead-end road and also gave me inspiration for new research ideas. In particular I want to thank Veysel for his general support and all the discussions with reflections on my research.
Finally I want to thank my brother and my parents. With my brother I always had very interesting and fruitful discussions about my research, which is often surprisingly much related to his research. My parents enabled me to study as extensive and as long as I did. I was stimulated to develop my talents as I wanted. And my parents supported me to go on with the thesis and were always there to help me.
# Contents

**Preface**

**Contents**

1. **Introduction**
   1.1 Firm Heterogeneity in Trade .............................................. 1
   1.2 Contribution of the Thesis ............................................... 2
     1.2.1 Heterogeneous Popularity and Exporting Uncertainty ............... 2
     1.2.2 Firm Heterogeneity in An Oligopoly Model of Trade ............... 3
     1.2.3 Firm Heterogeneity and Endogenous Quality ....................... 4
     1.2.4 Within-Sector Specialization in a Monopolistic Competition Model of Trade ......................................................... 4
   1.3 Outline ............................................................................. 4

2. **Imperfect Competition and Trade**
   2.1 Introduction ...................................................................... 7
   2.2 Gains from Trade in Imperfect Competition ............................. 8
     2.2.1 Variety Effect of International Trade ................................ 8
     2.2.2 Scale Effect of International Trade .................................. 10
     2.2.3 Enhanced Competition Effect of International Trade ............. 12
     2.2.4 Labor Division Effect of International Trade .................... 13
     2.2.5 Empirical Evidence on the Gains from Trade in Early Monopolistic Competition Models ......................................................... 14
   2.3 Three Firm Heterogeneity Models ........................................... 16
     2.3.1 Melitz (2003) ................................................................ 16
     2.3.2 Bernard, Eaton, Jensen and Kortum (2003) .......................... 23
     2.3.3 Melitz and Ottaviano (2008) ............................................ 25
   2.4 Empirics Firm Heterogeneity .................................................. 32
   2.5 Comparison Firm Heterogeneity Models .................................... 36
   2.6 Concluding Remarks ........................................................... 43

3. **Heterogeneous Popularity**
   3.1 Introduction ...................................................................... 45
   3.2 Heterogeneous Popularity ..................................................... 48
   3.3 Closed Economy Model ...................................................... 49
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4 Open Economy Model</td>
<td>50</td>
</tr>
<tr>
<td>3.4.1 Solving The Open Economy Model</td>
<td>51</td>
</tr>
<tr>
<td>3.4.2 Effects of Changed Trade Costs</td>
<td>54</td>
</tr>
<tr>
<td>3.5 Concluding Remarks</td>
<td>57</td>
</tr>
<tr>
<td>A Unique Equilibrium Open Economy</td>
<td>57</td>
</tr>
<tr>
<td>B Derivation Number of Firms</td>
<td>59</td>
</tr>
<tr>
<td>C Effect of Changed Trade Costs</td>
<td>60</td>
</tr>
<tr>
<td>C.1 Changed Iceberg Trade Costs</td>
<td>60</td>
</tr>
<tr>
<td>C.2 Changed Sunk Export Costs</td>
<td>61</td>
</tr>
<tr>
<td>C.3 Changed Fixed Export Costs</td>
<td>62</td>
</tr>
<tr>
<td>4 Firm Heterogeneity and Oligopoly</td>
<td>65</td>
</tr>
<tr>
<td>4.1 Introduction</td>
<td>65</td>
</tr>
<tr>
<td>4.2 Basic Model without Trade</td>
<td>66</td>
</tr>
<tr>
<td>4.3 Trade in The Short Run</td>
<td>71</td>
</tr>
<tr>
<td>4.4 Trade in The Long Run</td>
<td>77</td>
</tr>
<tr>
<td>4.5 Trade between Unequal Countries</td>
<td>81</td>
</tr>
<tr>
<td>4.6 Concluding Remarks</td>
<td>88</td>
</tr>
<tr>
<td>A Basic Model</td>
<td>88</td>
</tr>
<tr>
<td>B Free Exit Model</td>
<td>89</td>
</tr>
<tr>
<td>C Free Entry Model</td>
<td>95</td>
</tr>
<tr>
<td>D Unequal Countries Model</td>
<td>97</td>
</tr>
<tr>
<td>5 Endogenous Quality</td>
<td>99</td>
</tr>
<tr>
<td>5.1 Introduction</td>
<td>99</td>
</tr>
<tr>
<td>5.2 Closed Economy Model</td>
<td>101</td>
</tr>
<tr>
<td>5.3 Open Economy Model</td>
<td>106</td>
</tr>
<tr>
<td>5.4 Concluding Remarks</td>
<td>108</td>
</tr>
<tr>
<td>A Uniqueness of the Closed and Open Economy</td>
<td>108</td>
</tr>
<tr>
<td>B Number of Firms</td>
<td>109</td>
</tr>
<tr>
<td>C More General Marginal Cost Function</td>
<td>110</td>
</tr>
<tr>
<td>D Effect of Trade Liberalization</td>
<td>111</td>
</tr>
<tr>
<td>6 Within Sector Specialization</td>
<td>113</td>
</tr>
<tr>
<td>6.1 Introduction</td>
<td>113</td>
</tr>
<tr>
<td>6.2 Closed Economy Model</td>
<td>114</td>
</tr>
<tr>
<td>6.3 Open Economy Model</td>
<td>118</td>
</tr>
<tr>
<td>6.4 Concluding Remarks</td>
<td>120</td>
</tr>
<tr>
<td>7 Concluding Remarks</td>
<td>121</td>
</tr>
<tr>
<td>Bibliography</td>
<td>129</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Firm Heterogeneity in International Trade Models

Traditional trade theory features comparative advantage and factor abundance to account for inter-industry trade in models of perfect competition. New trade theory emerged at the end of the 1970s to provide an explanation for the ever larger amount of intra-industry trade using models of imperfect competition. Whereas the gains from trade in traditional models are due to specialization, the new trade theory adds four different gains from trade. First, the monopolistic competition model of Krugman (1980) displays a variety effect with trade enabling the consumption of more different varieties. Second, the model of Krugman (1979) features a scale effect besides a variety effect: trade implies a larger market and the possibility to produce at a larger scale leading to efficiency gains. Third, the model by Ethier (1982) on intermediate goods trade contains a labor division effect: trade enables the use of more intermediate varieties in production. Fourth, the oligopoly model of Brander and Krugman (1983) illuminates the pro-competitive effects from trade. A larger market raises the number of competitors and drives down consumer prices.

The imperfect competition models in new trade theory assume that all firms are equal. Relaxing this assumption leads to a fifth gain from trade, the reallocation effect. Trade enables more productive exporting firms to gain market share at the expense of less productive firm who only produce for the domestic market. The reallocation effect can be driven by enhanced product market competition or enhanced labor market competition.

International trade leads to tougher competition on the product market in the former case, driving the least productive firms out of the market. In the latter case international trade leads to larger profit opportunities for the most productive firms, creating more demand for labor and driving up wages. This squeezes the least productive firms out of the market.

Empirical work finds wide support for the reallocation effect of trade already in the 1990s. Tybout (1991) uses data from Chile, Colombia and Morocco to show the importance of reallocation effects and Bernard and Jensen (2004a) do the same with data from the US. Theory on firm heterogeneity is developed only in 2000. Various
different models are proposed. Melitz (2003) includes heterogeneous productivity in a monopolistic competition model with CES demand. Bernard et al. (2003) propose a model of Bertrand competition with firm heterogeneity. Melitz and Ottaviano (2008) create a firm heterogeneity model with monopolistic competition and linear demand. In Melitz (2003) the reallocation effect works through enhanced labor market competition, whereas in the other two papers the reallocation effect works through tougher product market competition. The first contribution of this thesis is to propose a model of firm heterogeneity in an oligopoly setting with Cournot competition. This model displays an intuitive reallocation effect through tougher product market competition in a basic and parsimonious model. It is a natural extension to the oligopoly model of Brander and Krugman (1983) and also nests the Ricardian comparative advantage model as a special case.

With the growing availability of firm-level datasets recent years have seen a huge increase in the number of empirical studies on firm heterogeneity. Besides the oligopoly model mentioned in the previous paragraph, the work in this thesis provides theory work to account for various findings in the empirical literature on firm heterogeneity. A first chapter takes as starting point the empirical finding that many firms quit the exporting market shortly after entry. It generalizes the firm heterogeneity model of Melitz (2003) adding exporting uncertainty. Firms are heterogeneous with respect to the popularity of their good, the popularity varies across markets and firms are uncertain about the popularity of their good in each market. Therefore, a considerable fraction of firms that start exporting have to leave the export market, because they cannot sell profitably.

A second and third chapter takes the empirical findings by Schott (2004) as a starting point. Schott (2004) finds that within detailed product categories goods from richer and more capital and skill abundant countries display higher unit values. Two models are proposed to account for these findings. First, a firm heterogeneity model with endogenous quality is put forward, where more productive firms sell higher quality and higher priced goods. Second, a monopolistic competition model with equal firms and capital abundance dependent on the quality of goods is created to account for the fact that more capital and skill abundant countries sell higher quality higher priced goods.

### 1.2 Contribution of the Thesis

This section discusses the models put forward in the thesis into more detail and addresses in particular the contribution these models make to the literature.

#### 1.2.1 Heterogeneous Popularity and Exporting Uncertainty

The model on exporting uncertainty and heterogeneous popularity starts from the empirical finding that many exporting firms leave the export market shortly after entry. Eaton et al. (2007) find for example in a sample of Colombian firms that more than two-third of the exporters drop out of the export market in the first year. Besedes and Prusa (2006) show that around one third of the imports in a certain product category stop within one year. The model presented to account for this finding is a firm het-
1.2 CONTRIBUTION OF THE THESIS

The heterogeneity model where firms are heterogeneous with respect to the popularity of their variety. The popularity varies across markets but is correlated. Firms don’t know their popularity before they enter a specific market and have to pay a sunk entry cost to get to know it. So, when they know the popularity of their good on the domestic market they don’t know how successful they will be on the exporting market. The implication is that a fraction of firms tries to enter the exporting market, but has to leave shortly after entry because it cannot sell profitably.

There are three types of trade costs, iceberg trade costs, sunk export costs and fixed export costs. Comparative statics show that lower trade costs lead for each of the three types to a higher domestic cutoff popularity level and hence a reallocation effect towards firms with a higher taste parameter. But the effects of lower fixed export costs and lower sunk export costs on exporting success (the probability of profitable sales in the export market conditional upon entry in the export market) are opposite. Lower fixed export costs raise the export success rate, whereas lower sunk export costs decrease the probability of export success.

The contribution of the model is threefold. First, the model provides a natural interpretation for the large fraction of exporting firms that quit the export market shortly after entry. Two competing explanations for the relatively large fraction of (quick) exit from the export market are a network model of trade put forward in Rauch and Watson (2003) and a model with shocks to variables affecting exporting profitability featuring in Irarrazabal and Opromolla (2006). Second, the model generalizes the firm heterogeneity model of Melitz (2003) in a non-trivial way to account for exporting uncertainty. Third, the model generates interesting comparative statics results on the probability of exporting and the exporting success rate.

1.2.2 Firm Heterogeneity in An Oligopoly Model of Trade

The model on firm heterogeneity under oligopoly is a natural extension of the Brander and Krugman (1983) reciprocal dumping model. There is CES demand across sectors and Cournot competition within each sector between firms with different productivities. Production is constant returns to scale and firms have to pay a sunk entry cost to get to know their productivity. There is a short-run and long-run analysis depending on the presence of a free entry condition. All results are derived for a general distribution of productivities. The chapter finds several interesting results and thus makes several interesting contributions. In a basic and parsimonious set-up the model generates a reallocation effect of (freer) trade. Both in the short-run and the long-run the least productive firms are squeezed out of the market and market prices decrease. The model nests the reciprocal dumping model and the Ricardian comparative advantage model as special cases. As in Brander and Krugman (1983) the welfare effect of lower trade costs can be negative in the short-run because of increased cross-hauling. But the analysis in the present model can make more precise statements on when the welfare effect is positive depending on the distribution of productivities. The model also contains predictions on the effect of trade costs and importer country size on the probability of zero trade flows and importer unit values. A final interesting result is that unilateral liberalization leads
to lower prices in the short-run but higher prices in the long-run in the import liberalizing country. This is due to relocation effects.

1.2.3 Firm Heterogeneity and Endogenous Quality

The model on firm heterogeneity and endogenous quality takes the empirical paper by Schott (2004) as its starting point. Schott (2004) relates US unit values to exporter characteristics. He finds at a detailed product level that richer countries export goods with higher unit values. To the extent that in richer countries firms are on average more productive, these findings are at odds with the standard firm heterogeneity model of Melitz (2003). In this model more productive firms have a lower marginal cost and given the fixed markup thus charge a lower price.

A model is put forward to bring the firm heterogeneity model of Melitz (2003) in line with the empirical findings by Schott (2004). It extends the firm heterogeneity model with endogenous quality. Each firm has a different productivity to produce quality and more productive firms will choose to produce higher quality goods which also involve higher fixed and marginal costs. The implication is that more productive firms charge higher prices. The contribution of this chapter is straightforward. It tries to bring the firm heterogeneity model of Melitz (2003) in line with empirical findings by Schott (2004) on unit values and exporter characteristics. Besides that, various empirically testable implications are derived.

1.2.4 Within-Sector Specialization in a Monopolistic Competition Model of Trade

The model on within-sector specialization also starts from Schott’s (2004) empirical findings. Another important finding in Schott (2004) is that within detailed product categories more skill-abundant and more capital-abundant countries export goods with higher unit values. A monopolistic competition model with non-homothetic production is proposed to account for the empirical findings of Schott. Higher quality goods require relatively more skilled labor in production. The implication is that more skill abundant countries propose higher quality goods. Because marginal costs increase with quality in the model, also unit values rise in relative skill-abundance. The contribution of this model is similar to the contribution of the previous model. It aligns a very influential model in international trade with important empirical findings on unit values and relative factor-abundance.

1.3 Outline

The thesis is organized as follows. Chapter 2 contains a survey of the three most influential models on firm heterogeneity and of the most important empirical work on firm heterogeneity. The chapter starts with a brief review of the homogeneous productivity imperfect competition literature. Chapter 2 finishes with a comparison of the three most
influential models of firm heterogeneity and the oligopoly model put forward in the thesis. Chapter 3 addresses exporting uncertainty under heterogeneous popularity. Chapter 4 contains the chapter on firm heterogeneity under oligopoly. Chapter 5 constitutes the models on firm heterogeneity and endogenous quality. Chapter 6 points out the within-sector specialization model. Chapter 7 addresses the effect of importer characteristics on unit values and the role of markups and quality to explain this effect. Chapter 8 concludes.
Chapter 2

Models of Imperfect Competition and Firm Heterogeneity in International Trade

2.1 Introduction

Rising intra-industry made the inclusion of imperfect competition in trade models an urgent task in the 1970s. The development of the Dixit and Stiglitz (1977) model of monopolistic competition allowed the introduction of imperfect competition and increasing returns to scale with proper microfoundations in international trade models. This gave rise to the so-called new trade theory explaining intra-industry trade. Krugman (1979, 1980) and Ethier (1982) were the first to use the Dixit Stiglitz framework in international trade. International trade creates various gains from trade in their models. Gains manifest themselves through an increasing number of varieties available to consumers (Krugman (1980)), a larger scale of production (Krugman (1979)), a larger degree of international labor division (Ethier (1982)) and more competition driving down markups (Brander and Krugman (1983)). Recently, Melitz (2003), Bernard et al. (2003) and Melitz and Ottaviano (2008) among others add a fourth gain of international trade assuming heterogeneous productivity of firms in the so-called new new trade theory. More productive firms can expand through exporting reducing the market share of the less productive ones. Average productivity rises in the economy through this composition effect.

This chapter provides an overview of the most important theoretical models in the new new trade theory on firm heterogeneity. It starts with a brief review of the new trade theory addressing variety, scale, competition and labor division effects. This part also contains a summary of empirical evidence on the new trade theory. Following, three models of the new new trade theory on firm heterogeneity are outlined in detail. The seminal work of Melitz (2003) is discussed as well as the model by Bernard et al. (2003) (BEJK in the remainder) and Melitz and Ottaviano (2008) (MO in the remainder). The exposition of these models is followed by a discussion of empirical work on firm heterogeneity and reallocation effects. This chapter closes with a comparison of
the different models containing both an outline of the differences in modeling setup and a confrontation of the modeling outcomes with the findings in empirical work.

2.2 Gains from Trade in Imperfect Competition Models

This section presents the different gains from trade in the older literature on monopolistic competition and international trade and discusses empirical evidence on the importance of these gains. Subsection 2.2.1 presents the basic Dixit-Stiglitz model used by Krugman (1980) displaying a variety effect. 2.2.2 points out various ways in which there can be a beneficial scale effect from international trade. Subsection 2.2.3 lays out the basics of the Brander and Krugman (1983) model where trade generates a beneficial competition effect. 2.2.4 discusses Ethier (1982) with gains from trade through labor division/specialization. Subsection 2.2.5 presents empirical work on the different gains from trade that show up in the older monopolistic competition models.

2.2.1 Variety Effect of International Trade

Krugman (1980) uses a standard version of the Dixit and Stiglitz (1977) love of variety model leading to gains from trade because of a larger variety of products in the economy. Dixit and Stiglitz (1977) formalized the concept of monopolistic competition. In their model a monopolistic competition industry is characterized by firms having market power through the supply of a unique variety, free entry and exit driving economic profits to zero, the absence of strategic interaction between different suppliers and increasing returns to scale in production. In the Dixit Stiglitz model used by Krugman (1980) all consumers have the same CES-utility over the different potential varieties $V$ in the economy with elasticity of substitution:

$$U = \left( \sum_{v \in V} c_v^{\frac{\sigma-1}{\sigma}} \right)^\frac{\sigma}{\sigma-1} \tag{2.1}$$

As the CES-utility function is homothetic one can calculate a unique price index defined formally as the minimum expenditure to attain a level of utility of 1:

$$P = \left( \sum_{v=1}^{N} p_v^{1-\sigma} \right)^\frac{1}{1-\sigma} \tag{2.2}$$

$p_v$ is the price of variety $v$ and $N$ is the number of products actually produced. Maximizing utility subject to the budget constraint, $\sum_{v=1}^{N} p_v c_v = w$, direct demand for a variety $c_v$ is given by:

$$c_v = \left( \frac{P_v}{P} \right)^{-\sigma} \left( \frac{w}{P} \right) \tag{2.3}$$
All firms use the same production technology displaying increasing returns to scale with a fixed cost of production \( f \) and a marginal cost of production \( a \). There is only one factor of production, labor \( l \), which is homogeneous in the economy. There are \( L \) workers in the economy. Therefore the total demand facing a firm is \( x_v = Lc_v \). Total cost of producing \( x_v \) units of output is given by:

\[
C(x_v) = w(f + ax_v)
\]  

Each firm produces a unique variety to benefit optimally from the market power this generates. Assuming that the number of varieties \( N \) is large, there is no impact of the price a firm sets on its demand through the price index \( P \). Hence it faces a constant demand elasticity equal to the elasticity of substitution \( \sigma \). Consequently, the optimal markup of price over marginal cost is also constant:

\[
p_v = \frac{\sigma}{\sigma - 1}aw
\]  

Note that price is independent of output or the number of firms, because of the fixed mark-up. Consequently, output per firm is also constant.

As all firms face the same market demand and have the same technology, they all set the same price, so the subscript \( v \) can be dropped in the remainder. The revenues of an arbitrary firm are given by:

\[
r(x) = \left( \frac{\sigma}{\sigma - 1}aw \right)^{1-\sigma} wLP^{\sigma - 1}
\]  

By free entry and exit economic profits are driven down to zero:

\[
\pi(x) = \frac{r(x)}{\sigma} - f = px - awx - wf = 0
\]  

Two equilibrium conditions can be added to the model. First, with a fixed labor supply \( L \), full employment leads to:

\[
N(f + ax) = L
\]  

Equilibrium in the goods market is given by:

\[
x = Lc
\]  

Equation (2.5) and (2.7) can be combined to produce an expression for output per firm which is fixed because of the fixed mark-up:

\[
x = \frac{f(\sigma - 1)}{a}
\]  

Combining (2.8) and (2.10) one finds the number of varieties in the economy:

\[
N = \frac{L}{\sigma f}
\]
The variety gains of international trade in this model can be seen in an easy way. Assume the modelled economy starts to trade with an identical economy of the same size and abstract from trade costs. The absence of trade costs (or the identical size) guarantees that the countries have equal wages. Again all firms produce distinct varieties to use their market power optimally. As there are no trade costs, all domestic and foreign varieties will be consumed at equal amounts by all consumers. So, the amount of varieties available to a consumer will double generating a welfare gain. The scale of production remains constant as the elasticity of substitution is constant, cf. equation (2.10). Each consumer reduces its consumption of each variety by 50% to double the number of varieties it consumes. In this way total demand for a single variety remains constant. Formally, one can see these results by considering a doubling of the labor force, which is equivalent to international trade as the countries are identical in all respects. Equation (2.10) shows that production per firm remains constant. Therefore, from labor market equilibrium equation (2.8) one finds that the amount of varieties $N$ rises proportionally with the labor force. The product market equilibrium equation (2.10) requires that consumption per consumer declines proportionally with the labor force. The next section considers different ways to generate a beneficial scale effect from international trade.

### 2.2.2 Scale Effect of International Trade

There are various ways to create a scale effect from international trade. This section discusses three of them. They all lead to a price elasticity of demand that is dependent on the number of competitors. International trade increases the number of competitors, increasing the price elasticity, which in turn decreases the price. Because the markup of firms declines, the scale of production has to rise to restore the zero profit condition. As a consequence some firms disappear. The first approach is to generalize the CES-utility function in the model of 2.2.1 to create a non-fixed elasticity of substitution dependent on the number of firms. Second, one can change the preference structure, using Lancaster’s (1979) ideal variety approach leading as well to a non-fixed elasticity of substitution. Third, one can relax the assumption that a price change has no impact on demand through the price index leading to a price elasticity depending on the number of firms.

Krugman (1979) proposes a model identical to the model in the previous section, with one exception: the utility function is not CES, but is left unspecified:

$$U = \sum_{v \in V} u(c_v)$$  \hspace{1cm} (2.12)

The first order condition of the consumer can be expressed with the Lagrange multiplier of the budget constraint $\lambda$:

$$u'(c_v) = \lambda p_v$$  \hspace{1cm} (2.13)

Plugging the goods market equilibrium equation (2.8) into (2.13) one finds an implicit expression for the demand facing an individual firm. As the number of goods is large, its pricing policy has a negligible impact on the marginal utility of income $\lambda$. Therefore,
2.2 GAINS FROM TRADE IN IMPERFECT COMPETITION

one finds the following price elasticity of demand:

\[ \varepsilon_v = - \frac{u'(c_v)}{u''(c_v) c_v} \quad (2.14) \]

Krugman (1979) assumes that this price elasticity declines in consumption per consumer \( c_v \), which is crucial to create a scale effect. The remainder of the model is equal to the model in 2.2.1, so again the variety subscript can be dropped as all firms have equal technology. One finds the same pricing equation, zero profit equation, goods market equilibrium equation and full employment equation as in 2.2.1 with the elasticity of substitution substituted by the price elasticity of demand from (2.14). Thus, a combination of the first two gives:

\[ x = \frac{f(\varepsilon(c) - 1)}{a} \quad (2.15) \]

Again the effects of international trade can be seen most easily by considering trade with an identical country in all respects abstracting from any trade costs. International trade is then like doubling the labor force. Using equation (2.9) and (2.15) one can see the impact on the scale of production. In the model of 2.2.1 a doubling of \( L \) leads to a proportional decline of \( c \), as can be seen from (2.10). With a variable elasticity of demand the decline of \( c \) causes production per firm \( x \) to rise, confront equation (2.15). When firms face less consumption per consumer the price elasticity rises and the markup declines. The profit level of firms declines and to restore zero profit the scale of production has to rise. The result is that the increasing labor force and so the larger market is now absorbed by a rise in the amount of varieties \( N \), but also in the scale of production of each firm, \( x \). This effect is caused by the non-fixed price elasticity of demand.

The weakness of the first approach is that the elasticity of demand declines in by assumption, to create reasonable results. A second way to get a non-fixed elasticity of demand is to make a small-group assumption. There is only a small group of firms, implying an effect of price on demand through the price index. The model is identical to the model in 2.2.1 using a CES-utility function again, but the price elasticity of demand changes:

\[ \varepsilon_v = \sigma + (1 - \sigma) \frac{p_v^{1-\sigma}}{N} = \sigma + \frac{1 - \sigma}{N} \quad (2.16) \]

The second equality sign in (2.16) is justified when all firms are identical. Equation (2.16) shows that the elasticity of demand rises when the number of firms in the market rises; firms get less market power and their pricing decision has less impact through the price index when \( N \) rises. Combining the pricing equation and the zero profit equation shows that output per firm rises in the number of firms through a competition effect:

\[ x = \frac{f}{a} (\sigma - 1) \left(1 - \frac{1}{N}\right) \quad (2.17) \]

Again the effect of international trade can be addressed by an increase in the labor force. Combining full employment equation (2.8) with equation (2.17) it can be seen that both
the number of varieties $N$ and the scale of production $x$ rise when $L$ grows. A larger market leading to more varieties makes competition fiercer, lowers profit margins and thus requires a larger scale of production.

A third way to create a scale effect is to leave the love of variety approach to preferences and switch to Lancaster’s (1979) ideal variety approach. In this approach consumers do not appreciate variety, but each consumer has an ideal variety. All consumers prefer different varieties and are distributed uniformly over a unit circle. This model set-up leads to demand equations similar to the love for variety approach above. The difference is that the elasticity of substitution depends positively on the number of varieties (see Helpman and Krugman (1985), chapter 6 for details). This model generates again a scale and variety effect of international trade. An increasing labor force leads to more varieties. Because the elasticity of substitution rises in the number of varieties this in turn enhances competition, lowers profit margins and increases the scale of production.

2.2.3 Enhanced Competition Effect of International Trade

Brander and Krugman (1983) propose an oligopoly model of international trade to show the beneficial pro-competitive effects of international trade. In a setting with Cournot competition between equal firms, international trade increases the number of competitors and drives down the market price. In the long-run (with a free entry condition) this will unambiguously raise welfare. In the short-run the welfare effect of (freer) trade can be negative, because more trade also involves a larger amount of costly transport of goods. Brander and Krugman (1983) show that the welfare effect of freer trade is positive in the short run when trade costs are negligible and that the welfare effect is negative when trade costs decline from a prohibitive level. The intuition is that there are no welfare costs of cross-hauling (pointless shipping of goods) when trade costs are negligible. And the beneficial effect on prices is only marginally different from zero when tariffs decline from a prohibitive level, because the (exporting) entrants on the market have a negligible market share, whereas the negative welfare effect from increased cross-hauling is larger, because trade costs are large.

The model of Brander and Krugman (1983) consists of two identical countries and one sector with Cournot competition. There are $n$ firms in each country. A domestic firm produces output $x$ in the domestic market and output $x^*$ in the foreign market. A foreign firm produces $y$ in the domestic (exporting) market and output $y^*$ in its own foreign market. Total market sales in the domestic and foreign market are equal to $Z$ and $Z^*$, respectively. Firms have equal marginal costs $c$, fixed costs $f$ and face iceberg trade costs $\tau$. Profits of a domestic and foreign firm are equal to:

$$\pi = xp(Z) + x^* p^*(Z) - c(x + \tau x^*) 0 - f$$ \hspace{1cm} (2.18)

$$\pi^* = yp(Z) + y^* p^*(Z^*) - c(y^* + \tau y) - f$$ \hspace{1cm} (2.19)

One can concentrate on one of the markets as the two countries are equal The first order conditions for the domestic firm and for the exporting firm for sales in the domestic
2.2 GAINS FROM TRADE IN IMPERFECT COMPETITION

The market are equal to:

\[ \pi_x = xp' + p - c = 0 \]  \hspace{1cm} (2.20)
\[ \pi_y = yp' + p - \tau c = 0 \]  \hspace{1cm} (2.21)

The first order conditions can be solved to find an expression for the market price:

\[ p = c(\tau + 1) \frac{\varepsilon n}{2n\varepsilon - 1} \]  \hspace{1cm} (2.22)

\( \varepsilon \) is the price elasticity of domestic demand. Brander and Krugman (1983) show that the welfare effect of lower trade costs is always positive in the long-run, i.e. imposing a zero profit condition. Furthermore they derive the intuitive result mentioned before that the welfare effect of trade liberalization in the short-run is always positive when trade costs are negligible and is always negative when trade costs are prohibitive.

2.2.4 Labor Division Effect of International Trade

Ethier (1982) introduces a love for variety into the production function of the manufactures sector. The more varieties of intermediate goods a final goods producer of manufactures can use, the more productive he/she will be. International trade enlarges the number of different intermediate goods varieties available and thus raises productivity of the final goods producers. Different countries produce different varieties, so it is international labor division that raises productivity in the manufactures sector.

Ethier (1982) considers a two sector model of wheat and manufactures with two factors of production, capital and labor. This is unnecessary for the purposes of this paper and a simplified version is outlined. With two sectors in the model, Ethier’s model is considerably more difficult than the presentation below, in particular due to the external economies from the amount of resources available in the manufactures sectors in both/all countries. For the basic goals of this section, to present the labor division effect of international trade and to show that marginal costs decline with international trade in Ethier’s model, the simplified version suffices.

Consider thus a model with one sector in the economy, manufactures, and one factor of production, labor. Final goods \( M \) are produced with a constant returns to scale technology using varieties of intermediate goods \( x_v \). The production function is CES and displays a love of variety effect:

\[ M = \left( \sum_{v=1}^{N} (x_v)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \]  \hspace{1cm} (2.23)

\( N \) is the number of available varieties and \( \sigma \) the elasticity of substitution between the different varieties. It is assumed like in 2.2.1 that \( \sigma > 1 \). With this assumption there are constant returns to scale in each intermediate \( x_v \), but increasing returns in the amount of varieties \( N \). These increasing returns are external to an individual producer of final goods. Intermediate goods are produced under (internal) increasing returns to scale.
using labor with the same parameters as in 2.2.1. All firms have an identical technology:

\[ l_v = ax_v + f \]  

(2.24)

All producers of intermediate goods choose to produce a unique variety like before. They face a demand curve with price elasticity equal to elasticity of substitution \( \sigma \). Combining their optimal pricing behavior with a zero profit equation leads to equation (2.10) of 2.2.1. Assuming that labor supply is given by \( L \), full employment equation (2.8) of 2.2.1 also applies.

In this simplified version of Ethier’s model international trade with an equal country can again be modeled by a rise in the labor force. Equation (2.11) shows that the amount of intermediate varieties available rises in the labor force. The difference with the models before is that the rise in the number of varieties does not affect consumer welfare, but the productive capacity of the economy. International trade raises the number of varieties available to producers, a process of specialization in the (world) economy. An important result is that marginal cost in the final goods sector declines with international trade. This can be seen from the cost function given by:

\[ C(M) = MN^{\frac{1}{1-\sigma}} p \]  

(2.25)

This result shows that the novelty of heterogeneous productivity models where marginal costs decline with international trade also appears in Ethier’s model.

### 2.2.5 Empirical Evidence on the Gains from Trade in Early Monopolistic Competition Models

There is no abundance of empirical work on the gains from trade through scale, variety, competition and labor division. Welfare gains through an increase in variety are studied by an examination of the impact of new varieties on the import price index. The most recent and complete study is Broda and Weinstein (2005, 2006). They use the framework by Feenstra (1994) to calculate the welfare gains from an increased number of imported varieties for the USA between 1972 and 2001. Feenstra (1994) derives the impact of a changing composition of varieties (with disappearing and emerging varieties) on an exact price index assuming CES-utility. This price index is based on changes in market shares of disappearing and emerging varieties instead of on changes in the number of available varieties. The latter could overstate the decline in the price index when many new varieties are included with a small market share. Broda and Weinstein (2006) calculate different elasticities of substitution between varieties from different countries. They find that a rise of the number of varieties by 251% leads to a drop in the import price index of 28.1%. Assuming Cobb-Douglas preferences for imported and domestic goods this represents a welfare gain of about 2.8%. Only using the increase in the number of consumed varieties instead of Feenstra’s approach would lead to a welfare gain of 6.52% and thus overstate it. Assuming that there is only one elasticity of substitution with a value of 2 for all imported goods as Romer (1994) does, generates a welfare gain of 8.47% and is thus a strong exaggeration.
Broda and Weinstein (2006) note that 2/3 of the varieties included in their sample are intermediates or capital goods. The gains from a larger variety of intermediates are theoretically gains from labor division. But in a monopolistic competition model with homothetic production, the drop in the prices of imported intermediates is fully passed through to lower output prices. So, the amount of welfare gain calculated by Broda and Weinstein (2006) remains valid.

There are many CGE-models featuring imperfect competition and economies of scale calculating the gains from trade through scale effects. The gains depend on the size of scale effects assumed. Tybout and Westbrook (1995) review the CGE-studies and find that most studies assume returns to scale between 1.1 and 1.33. This leads to gains from trade between 1% and 5%. Tybout and Westbrook (1995) argue that these gains are too large, because engineering studies find usually smaller returns to scale than 1.1. Econometric estimates of returns to scale are even smaller. Returns to scale as large as used in CGE-models ‘may in fact describe the large number of small plants in a typical industry, but these plants account of only a small fraction of sectoral output (Tybout and Westbrook (1995), p.133).’ Furthermore, empirical work shows that scales of production in import-competing sectors do not rise after trade liberalization and in some studies even decline (Tybout (2001)). Roberts and Tybout (1996) explore the gains through scale expansion in another way. They make a decomposition of productivity changes in Mexico’s manufacturing industry between 1984 and 1990, a period of trade liberalization. Productivity change is decomposed into a composition effect, a scale effect and a residual term. The scale effect leads only to a 0.55% productivity growth, which is less than 5% of the total productivity growth. So, estimates of returns to scale and the scarce empirical work on trade liberalization and scale suggest that the gains from trade through scale effects are modest.

There are various empirical studies on the pro-competitive effect of trade. Roberts and Tybout (1996) find that larger import competition goes along with lower markups in studies on Mexico, Colombia, Chile and Morocco. Tybout (2001) reviews the work of other authors and comes to the same findings that markups decline in the amount of import competition. Despite these beneficial effects on markups, more imports could still have a negative welfare effect when the adverse effects of cross-hauling dominate. Friberg and Ganslandt (2006) address this possibility. They estimate a structural model of the bottled water market in Sweden. Using the estimated coefficients they perform a counterfactual simulation without trade to see if welfare increases. They do not find evidence that in the no-imports case welfare would increase. A final study to mention on this topic is a simulation study of the effects of trade liberalization in Cameroon by Devarajan and Rodrik (1991) featuring pro-competitive effects. They find that the positive welfare effects from trade liberalization are driven by the pro-competitive effects.

Empirical work on the gains from trade through increased division of labor is scarce. The study by Broda and Weinstein (2005) discussed above shows that a large part of the increased number of imported varieties are intermediate goods. Thus the gains they calculate from a lower import price index are actually gains from labor division. Another study by Feenstra et al. (1992) uses the same framework as Broda and Weinstein (2005) to calculate the gains from a larger variety of intermediate inputs in Korean business
groups, chaebols. Chaebols are strongly vertically integrated, thus intermediate inputs from a new member of a chaebol will be used by the other members of the chaebol. Feenstra et al. (1992) calculate the impact of new members in a chaebol on its total factor productivity (TFP). They find that new members have a significant impact on TFP of a chaebol. Tybout (2001) criticizes the result for an obvious reason: unobserved heterogeneity could drive both TFP and the inclusion of new members into a chaebol. Feenstra et al. (1992) include deviation from long-term growth of a chaebol, its capital labor ratio, the imports over consumption ratio and R&D expenditure as controls. So, for example the development of a new product not accounted for by R&D expenditure could be an unobserved variable driving both TFP and new members into a chaebol.

2.3 Three Firm Heterogeneity Models

Heterogeneous productivity of firms creates another gain from trade. Trade changes the composition of firms. More productive firms gain market share at the expense of less productive ones, raising the average productivity in the economy. Two mechanisms can create a composition effect. The first one, appearing in Melitz (2003), is fiercer competition on the labor market as a result of trade. Assuming exporting requires fixed beachhead costs to enter a market, only more productive firms can export. The possibility to trade raises their demand for labor, driving up real wages. As a consequence less productive firms disappear changing the composition of firms: more productive firms expand their production and less productive firms disappear. The second mechanism, featuring in Bernard et al. (2003) and Melitz and Ottaviano (2008), is stronger competition on the product market as a result of trade. International trade extends the number of competitors. This drives the less competitive firms out of the market. The models differ in their set-up to create this effect. The next three subsections describe the three different models. Section 2.4 discusses empirical evidence on firm heterogeneity and reallocation effects of trade and section 2.5 contains a comparison of the three modeling approaches.

2.3.1 Melitz (2003)

The model appearing in Melitz (2003) is identical to the Dixit-Stiglitz model in Krugman (1980) with the difference that firms have heterogeneous productivity. So, there is CES-demand with constant elasticity of substitution, each firm produces a unique variety creating monopoly power, firms use only labor and there are increasing returns to scale with a fixed cost of production. Krugman’s model is extended by explicitly modeling entry and exit of new firms creating a distribution of firm productivities. The main equations solving the model are changed in two ways compared with Krugman. First, entry is costly, creating a different free entry condition and also a different labor market equilibrium equation as labor is used as an input into the entry costs sector. Second, average productivity becomes endogenous depending on the distribution of productivities. First, the basic model in a closed economy is considered followed by the extension to an open economy with trade.
2.3 THREE FIRM HETEROGENEITY MODELS

The Model in a Closed Economy

Utility of a representative consumer is given by equation (2.1), the price index by equation (2.2) and the demand facing a firm by (2.3) with the notion that summations should be replaced by integrals. Costs of production, the optimal price, revenues and profits are given by respectively (2.4), (2.5), (2.6) and (2.7) with the notion that marginal cost \( a_i \) varies per firm \( i \). So, heterogeneity in productivity is modeled by a difference in the marginal cost of production. Fixed costs are equal for all firms. In the remainder wages are normalized at 1. The ratio of revenues of two firms can be expressed as a function of the ratio of marginal costs. This turns out to be convenient when relating average productivity to the cutoff productivity, the productivity of the firm making zero profit.

\[
\frac{r(a_i)}{r(a_j)} = \left( \frac{a_j}{a_i} \right)^{\sigma-1}
\]

(2.26)

Next assumptions about the distribution of productivities are needed. Melitz assumes that there are an unbounded number of potential firms that can enter the market by incurring a sunk entry cost \( f_e \). Before entering potential firms do not know their productivity. After having paid the entry cost, they draw a productivity level at random from a distribution function \( G(a) \). This is a reasonable assumption, as it is very difficult for an entrepreneur to know how a new product will be received in the market without a market study.

When firms know their productivity, they will decide to enter the market or not depending on whether their productivity level can generate positive profit. Productivity levels do not change over time. The productivity at which a firm makes zero profit is the cutoff cost level, \( a^* \). Below this marginal cost level firms make positive profit and stay in the market and above this level firms leave the market immediately. The zero cutoff profit (ZCP) condition leads to a useful expression for revenues of a firm with cutoff marginal cost level:

\[
\pi(a^*) = 0 \Rightarrow r(a^*) = \sigma f
\]

(2.27)

To create a steady state productivity level of firms in the market, Melitz assumes that firms with positive profits also leave the market with same fixed per period probability \( \delta \). Assuming that there is no time discounting, the value of a firm with marginal cost lower than cutoff marginal cost will be \( \sum_{t=0}^{\infty} (1-\delta)^t \pi(a) = \frac{\pi(a)}{\delta} \). Melitz shows that a steady state can be found where the number of entering and leaving firms is equal and the distribution of marginal costs of producing firms (to be defined below) does not change over time.

From the distribution function of initial marginal costs, one can define a probability density function of marginal costs of producing firms \( \mu(a) \). This is the truncated distribution of:

\[
\mu(a) = \begin{cases} 
\frac{g(a)}{G(a^*)} & \text{if } a \leq a^* \\
0 & \text{if } a \geq a^*
\end{cases}
\]

(2.28)

The probability of producing is thus given by \( p_{in} = G(a^*) \). The distribution of cost
levels of producing firms $\mu(a)$ can be used to express the price index $P$ from equation (2.2) as a function of cost levels. The price index becomes a function of marginal cost $a$ instead of a function of varieties $v$ in this way:

$$P = \left[ \int_0^\infty p(a)^{1-\sigma} N\mu(a) da \right]^{1/\sigma}$$  \hspace{1cm} (2.29)

$N$ is the number of firms. Substituting pricing equation (2.5) one finds the following expression for the price index:

$$P = \frac{\sigma}{\sigma - 1} N^{1/\sigma} \tilde{a}$$  \hspace{1cm} (2.30)

$\tilde{a}$ is a measure for average marginal costs:

$$\tilde{a} = \left[ \int_0^\infty a^{1-\sigma} \mu(a) da \right]^{1/\sigma}$$  \hspace{1cm} (2.31)

Next, equation (2.28) can be substituted into equation (2.31) to get a relation between average cost levels and cutoff cost levels. The cutoff marginal cost will then comprise all information about the price index and thus welfare:

$$\tilde{a} = \left[ \frac{1}{G(a^*)} \int_0^\infty a^{1-\sigma} g(a) da \right]^{1/\sigma}$$  \hspace{1cm} (2.32)

The cutoff marginal cost can be determined by two conditions: the zero cutoff profit condition and the free entry condition. Both conditions are a relation between average profit and cutoff marginal cost. The zero cutoff profit condition imposes the condition that a firm with cutoff marginal cost has a profit of 0. It can be found by first writing average profit as a function of average revenues, then writing average revenues as a function of the revenues of the firm with cutoff marginal cost and the ratio of average marginal cost and cutoff productivity using equation (2.26) and finally imposing zero profit for the cutoff cost level. These steps are shown below:

$$\tilde{\pi} = \pi(\tilde{a}) = \frac{r(\tilde{a})}{\sigma} - f$$

$$r(\tilde{a}) = r(a^*) \left( \frac{a^*}{\tilde{a}} \right)^{\sigma-1}$$

$$\pi(a^*) = 0 \Rightarrow r(a^*) = \sigma f$$

$$\tilde{\pi} = \left[ \left( \frac{a^*}{\tilde{a}(a^*)} \right)^{\sigma-1} - 1 \right] f$$ \hspace{1cm} (2.33)

The free entry condition is found by equating the expected value of the firm to the fixed entry cost. The expected value of the firm is equal to the present discounted value of average profits (discounted by the exit probabilities) times the probability that marginal cost is below cutoff marginal cost. So, one gets:

$$\frac{G(a^*)}{\delta} \tilde{\pi} = f_e \Leftrightarrow \tilde{\pi} = \frac{\delta f_e}{G(a^*)}$$ \hspace{1cm} (2.34)
The zero cutoff profit condition and the free entry condition can be combined to find a unique cutoff marginal cost level. Melitz shows that for a general distribution of initial cost levels, the ZCP condition is upward sloping in \((a, \pi)\) space and the FE-condition downward sloping. The slopes reflect the mechanisms in the model. When the average profit is larger, the cutoff marginal cost should be larger as well. Suppose average profit rises. A higher average profit implies a higher profit for the firm with cutoff marginal cost as well. Therefore, the firm with the old cutoff marginal cost will make positive profit. So, the cutoff marginal cost rises. More firms can make positive profits. A lower marginal cost cutoff requires a higher average profit to satisfy FE. Suppose the cutoff marginal cost is smaller. This implies that the probability of a good draw of productivities is smaller and therefore expected profit, the value when one has a good draw, should be larger too.

The upward sloping ZCP-line and the downward sloping FE-line determine a unique cutoff marginal cost level. Note that the slopes of the lines are different from the slopes in Melitz, because he uses productivity instead of costs.

As remarked before, Melitz shows that one can find a stationary equilibrium of entry and exit where the distribution of productivities stays constant. As there is free entry, in equilibrium all income will go to labor. Labor is either used for production or the fixed costs to set up a new firm. Therefore aggregate expenditures (or revenues) \(R\), are equal to the total amount of labor as wages are normalized at 1. The number of produced varieties \(N\) is equal to total revenue divided by average revenue per firm:

\[
N = \frac{R}{\bar{r}} = \frac{L}{\sigma (\bar{\pi} + f)} \quad (2.35)
\]

As wages are normalized at 1, welfare per worker is equal to the inverse of the price index:

\[
W = \frac{1}{P} = \frac{\sigma - 1}{\sigma} \frac{N^{\frac{1}{\sigma - 1}}}{\bar{\pi}} \quad (2.36)
\]

So, like in Krugman’s model welfare can increase as a result of trade when the number of varieties rises. The difference is that in this model welfare can also rise, because of an decrease of average cost levels due to the change of the composition of producers. Before turning to the introduction of trade, it is important to note that welfare can be expressed as a function of some parameters and the cutoff marginal cost. By equation (2.26) average marginal cost can be expressed as a function of the cutoff marginal cost and average and cutoff revenues. From equation (2.35) an expression for average revenue is substituted and from equation (2.27) an expression for ‘cutoff revenues.’ Combining gives:

\[
\bar{a} = a^* \left( \frac{r(a^*)}{\bar{r}(a^*)} \right)^{\frac{1}{\sigma - 1}} = a^* \left( \frac{\sigma f L/N}{\sigma f a^*} \right)^{\frac{1}{\sigma - 1}} \quad (2.37)
\]
The Model in an Open Economy

Melitz makes four assumptions when international trade is introduced in the model. First, a firm knows its productivity on the foreign market before it has to decide whether to enter the foreign market or not. Second, there are per unit ‘transport costs’ of the iceberg type with. So, \( \tau > 1 \) units have to be exported to make one unit arrive at the destination. Third, it is assumed that the different countries in the model are identical. In the absence of a homogeneous good sector, equal size of the countries is necessary to create equal wages (Krugman, 1980). Fourth, it is assumed that there are sunk costs to enter a foreign market, \( f_{ex} \). Fixed entry costs in export markets are a reasonable assumption, as firms have to gather all kinds of information before they can enter a foreign market and a distribution channel has to be set up. These so-called beachhead costs to enter a foreign market are introduced in the literature by Baldwin (1988) and Baldwin and Krugman (1989) and there is empirical support for its existence (Roberts and Tybout, 1997). Because a firm’s information does not change after it has entered the foreign market the one time investment costs can also be expressed as per period fixed costs using the ‘discount rate’ \( \delta \). This gives: \( f_x = \delta f_{ex} \). The assumption is crucial, because without fixed entry costs, trade liberalization would mean nothing more than just an extension of the market without any impact on productivity levels.

Prices charged abroad are still a mark-up over marginal cost, now including transport costs. So, export prices and export revenues are given by:

\[
p_x = \frac{\sigma}{\sigma - 1} \tau a = \tau p_d \tag{2.38}
\]

\[
r_x = \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} U P^\sigma = \tau^{1-\sigma} r_d \tag{2.39}
\]

Variables with an index \( x \) denote export variables and with a \( d \) domestic variables. Profits from domestic and exporting sales are equal to, respectively:

\[
\pi_d(a) = \frac{r_d(a)}{\sigma} - f \tag{2.40}
\]

\[
\pi_x(a) = \frac{r_x(a)}{\sigma} - f_x = \frac{\tau^{1-\sigma} r_d}{\sigma} - f_x \tag{2.41}
\]

The probability that a firm is exporting conditional on profitably selling in the domestic market is given by \( p_x = \frac{G(a_x)}{G(a)} \). Using this probability the number of exporting firms can be expressed as a fraction of the total number of producing firms, \( N_x = p_x N \). Thus with countries \( m + 1 \) countries, the total number of varieties available in every country is \( N_i = N + mN_x = N(1 + mp_x) \). Like in equation (2.32) for the model without trade, average cost levels of domestically selling firms and exporting firms can be written as a function of their cutoff cost levels. These average cost levels can be used to write an expression for overall average marginal cost as a function of average marginal costs of
domestic producers and exporting producers, which is given by:

$$\tilde{a}_t = \left\{ \frac{1}{N_t} \left[ N\tilde{a}^{1-\sigma} + mN_x \left( \frac{\tilde{a}_x}{\tau} \right)^{1-\sigma} \right] \right\}^{\frac{1}{1-\sigma}}$$

Like in the closed economy, the price index (and welfare) is a function of this average marginal cost and thus of cutoff marginal cost:

$$W = 1/P = \frac{\sigma - 1}{\sigma} N_t^{\frac{1}{\sigma-1}} \frac{1}{\tilde{a}_t} = \frac{\sigma - 1}{\sigma} \left( \frac{L}{\sigma f} \right)^{\frac{1}{\sigma-1}} \frac{1}{\tilde{a}^*}$$

Cutoff marginal cost is again at the intersection of the free entry condition and zero cutoff profit condition. The free entry condition remains the same, given in equation (2.34). The zero cutoff profit condition does change however, because the cutoff cost level is not related to average profit only through the domestic market, but also through the foreign market. Average profit is found by adding up domestic profit and exporting profit, correcting for the number of countries and the probability of exporting.

$$\tilde{\pi} = \pi_d(\tilde{a}) + p_x m \pi_x(\tilde{a}_x)$$

Average domestic and exporting profits are both related to their cutoff cost level by zero cutoff profit conditions:

$$\tilde{\pi}_d = \left[ \left( \frac{a^*}{\tilde{a}} \right)^{\sigma-1} - 1 \right] f$$

$$\tilde{\pi}_x = \left[ \left( \frac{a^*_x}{\tilde{a}_x} \right)^{\sigma-1} - 1 \right] f_x$$

Combining equations (2.26) and (2.39) one can express the domestic cutoff marginal cost as a function of the exporting cutoff productivity:

$$a^*_x = a^* \left( \frac{f}{f_x} \right)^{\frac{1}{\sigma-1}}$$

Crucial in Melitz’ model to create a composition effect is that only relatively more productive firms can export. This requires that the exporting cutoff marginal cost level is smaller than the domestic cutoff marginal cost level. Condition (41) implies that this condition is satisfied when:

$$\tau^{\sigma-1} f_x > f$$

Melitz argues that this condition is likely to be satisfied as not all firms are exporting. This is an instrumentalist way to justify an assumption: not by the realism of the assumption itself, but by the truth of the predictions it generates (Friedman (1953)). The realism of the assumption can be questioned. $f_x$ is the fixed per period cost of introducing a certain variety abroad and $f$ is the fixed cost of production at home. So,
f includes the fixed costs to produce like overhead costs as well as the costs to introduce a new variety at home, the beachhead costs in the domestic market consisting of for example marketing costs and setting up a distribution network. \( f_x \) includes only the latter type of beachhead costs for the exporting market. Baldwin and Forslid (2004) interpret the fixed costs of production as only the beachhead costs \( f \). In this reading of the model, there are no fixed overhead costs and condition (2.48) requires that exporting beachhead costs are larger than domestic beachhead costs. This assumption seems fair: beachhead costs abroad are larger than beachhead costs in the domestic market, because of information advantages in setting up a distribution network in the domestic market for example. The problem with this reading of the model is that somehow fixed overhead costs are assumed away. There are only fixed costs involved in setting up a new variety, \( f_e \), and in entering a market with a new variety, the domestic and exporting beachhead costs, \( f \) and \( f_x \). Including overhead costs, satisfying equation (36) requires that the fixed overhead costs are small relative to the difference between domestic and exporting beachhead costs.\(^1\)

So, average profits in (38) are related to the cutoff productivity \( a^* \), giving the ZCP-condition. The open economy ZCP-condition lies above the closed economy one, because a higher cutoff productivity does now affect total average profit through average domestic profit and average exporting profit. When the ZCP-line shifts upward the new cutoff marginal cost declines. Cutoff marginal cost declines, because real wages rise implying that the least productive firms cannot make positive profits anymore. Real wages rise for two reasons. First, in an open economy the very productive firms can not only sell at home, but also abroad. They expand their production and therefore need more labor. So, labor demand rises driving up real wages. Second, the prospect for a very productive firm that enters is better in the open economy. Therefore, more firms will try to set up a new firm and enter. The fixed costs involved in entering also require labor.

As productivity changes are fully reflected in changed prices the high productive firms charge lower prices and sell more. Therefore, large exporting firms replace small domestically producing firms. As a result, the number of consumed varieties can be expected to decline.\(^2\) Despite the possible decline in the number of consumed varieties, welfare always rises as a result of trade liberalization because the rise in average productivity always dominates a possible decline in the number of varieties. Melitz (2003), p.1722 proves formally that cutoff marginal cost declines as a result of liberalization. Equation (2.43) shows that a lower cutoff cost level leads to higher welfare.

To summarize, the effect of trade is a replacement of low-productive domestic firms who cannot afford the higher real wages anymore by high productive exporting firms which have more than average productivity. As a result average productivity rises. The composition effect is caused by the fact that the cutoff marginal cost to export lies at a lower level than the overall cutoff marginal cost. The possibility to export allows only the more productive firms to expand their size of production at the expense of less

---

\(^1\) Defining fixed costs, \( f \), as the sum of fixed overhead costs, \( f_o \), and domestic beachhead costs, \( f_{db} \), equation (2.48) requires: \( f_o < f_x \frac{1}{\kappa - 1} f_{db} \).

\(^2\) Baldwin and Forslid (2004) show that the number of consumed varieties declines as a result of trade liberalization when \( f_x < f \) assuming a Pareto distribution of productivities.
productive domestic producing ones. It is important to emphasize that this result relies on the assumption that the fixed beachhead costs to export multiplied by a trade barriers term, $\tau f_x$, are larger than the fixed costs to produce domestically, $f$. As discussed above, this assumption is not evident.

### 2.3.2 Bernard, Eaton, Jensen and Kortum (2003)

Bernard et al. (2003), BEJK, propose a model with CES-preferences and Bertrand competition between different (potential) producers of the same variety. There are constant returns to scale. Different producers of a certain variety have different marginal costs. So, each country has a lowest cost producer. Trade allows the replacement of some domestic lowest cost producers of a certain variety by exporting firms who have lower marginal costs inclusive of trade barriers. Productivity rises, because the composition of firms changes as a result of trade: less productive firms are competed out by foreign firms and more competitive firms can expand selling on export markets. Bernard et al. (2003) include the use of intermediates in production. International trade decreases the prices of intermediates leading to another reason for the rise in productivity.

In BEJK there is a continuum 1 of varieties and on each variety there is Bertrand competition over the price. So, utility is given by equation (2.1) with the notion that there is a continuum of varieties of size 1. A potential producer of a certain variety faces demand $c_v$ given in equation (2.3). There are constant returns to scale in production. In each country there are several potential producers for each variety with varying marginal cost levels. Potential producers in country $i$ can transform one unit of inputs $w_i$ into $\varphi_i(v)$ units of variety $v$. The $k$–th most efficient producer of variety $v$ in country $i$ has a productivity of $\varphi_{ki}(v)$. Products can be shipped from country $i$ to country $n$. For 1 unit to arrive in country $n$, $\tau_{ni}$ units have to be shipped from country $i$. So, there are standard iceberg transport costs like in the previous model. Combining productivity and transport costs, the $k$–th most efficient producer in country $i$ has the following unit cost to deliver variety $v$ in country $n$:

$$C_{kni}(v) = \left( \frac{w_i}{\varphi_{ki}(v)} \right) \tau_{ni}$$

The lowest cost producer of variety $v$ in country $n$ is the one with the lowest cost among the lowest cost producers in all different countries. It has unit cost:

$$C_{1n}(v) = \min_i \{C_{1ni}(v)\} \tag{2.49}$$

As noted, the model assumes Bertrand competition. So, the lowest cost seller in country $n$ can set a price equal to the unit cost of the second most efficient supplier in that country. This leads to the following upper limit on the price of the most efficient producer:

$$C_{2n}(v) = \min_i \left\{ C_{2ni}(v), \min_{i \neq i^*} \{C_{1ni}(v)\} \right\} \tag{2.50}$$
Here, $i^*$ is the country with the most efficient producer. So, the price the most efficient producer can set is either the unit cost of the second most efficient producer from the same country or the unit cost of the most efficient producer from another country. Still sometimes the most efficient producer will set an even lower price. This is the case when the monopoly price of variety $v$ determined by the elasticity of substitution is lower than the lowest price of potential competitors. So, the price of variety $v$ in country $n$ is given by:

$$p_n(v) = \min \left\{ C_{2n}(v), \frac{\sigma}{\sigma - 1} C_{1n}(v) \right\}$$

(2.51)

Equations (2.50) and (2.51) show that more efficient firms with a larger productivity parameter $\varphi$ can charge a larger mark-up, because the difference of their unit cost with the unit cost of the second most efficient producer will be larger, an artifact of the Fréchet-distribution which does make sense.

To continue, BEJK assume that productivities are probabilistic and drawn from some distribution function. To solve the model the productivities of the most efficient and second most efficient producers from each country are needed. The authors assume a Fréchet distribution given by:

$$F_i(\varphi_{i1}, \varphi_{i2}) = \frac{1}{1 + T_i (\varphi_{i2} - \varphi_{i1})} \exp \left\{ -T_i \varphi_{i2} \right\}$$

In this expression $\theta$ determines how heterogeneous the efficiency of the two most efficient firms is with smaller values implying more variability. In fact, this parameter determines how much trade gains can be attained because of comparative advantage. $T_i$ is a parameter for absolute advantage of country $i$. BEJK solve in a web-appendix for the cost distribution of the most efficient and second most efficient supplier in a certain country. It is given by:

$$G_n(c_1, c_2) = \Pr (C_{1n} \leq c_{1n}, C_{2n} \leq c_{2n}) = 1 - \exp \left\{ -\Phi_n c_1^\theta - \Phi_n c_1^\theta \exp \left\{ -\Phi_n c_2^\theta \right\} \right\}$$

(2.53)

$\Phi_n = \sum_{i=1}^{N} T_i (w_i d_{ni})^{-\theta}$ is a cost parameter and depends on input costs, the two trade parameters and trade costs.

Two important outcomes of the model can be seen easily. First, exporting firms display higher productivity. A best potential producer of good $j$ from country $i$ can sell at home when he/she is cheaper than its competitors in its home market:

$$\frac{w_i}{\varphi_{1i}} \leq \frac{w_k \tau_{ik}}{\varphi_{1k}} \quad \forall k \neq i$$

(2.54)

It can export to a country $n$ when it is cheaper than any other supplier to market $n$:

$$\frac{w_i \tau_{ni}}{\varphi_{1i}} \leq \frac{w_k \tau_{nk}}{\varphi_{1k}} \quad \forall k \neq i$$

(2.55)

Using the triangle inequality that shipping a good directly to a country is cheaper than
shipping it through another country, \( \tau_{nk} \leq \tau_{ni} \tau_{ik} \), implies that \( \frac{w_i \tau_{nk}}{w_k \tau_{nk}} \leq \frac{w_i}{w_k} \). So, the inequality in (2.55) is harder to satisfy than the one in (2.54) and thus exporting requires a higher productivity. This result simply says that exporting requires on average higher productivity, because of trading costs when there is competition over the price.

A second result of the model is that more productive firms are bigger. A firm either sets a Dixit-Stiglitz monopoly mark-up where lower unit costs imply a lower price and thus more sales. Or if potential competitors are close, a firm sets a price equal to the unit cost of the second most efficient firm. More efficient firms also face more efficient competitors assuming a Fréchet-distribution.\(^3\) BEJK prove six other analytic results for their model on among others the mark-up, the price index, market shares and the share of variable costs in aggregate revenues, which are not of interest for the basic description of the model and the comparison with the other models.

BEJK calibrate their model on aggregate trade shares and expenditures of 47 leading US export destinations (including the US itself) in 1992 and on US plant level micro data from the US Census of Manufactures in the Longitudinal Research Database of the Bureau of Census. The two parameters \( \sigma \) and \( \theta \) are used to create a match between the productivity and size gains of exporters in the simulation and in the data. Other results, on the fraction of firms that export and the variability in productivity and in size for example, are generated by the model and are subsequently compared to real-world data to assess the fit of the model.

To conduct policy experiments, BEJK close their model by including a tradable non-manufactured good produced in each country and serving as the numéraire. Labor supply in each country is assumed to be fully elastic with wages equal to labor productivity in the tradable non-manufactured good sector. Production uses labor and tradable intermediate inputs. BEJK perform a simulation of the effects of a 5% fall in worldwide geographic barriers and show that it raises productivity by 4.7%. The main effect is through the cost declines of inputs for continuing firms (3.9%). Exit of less productive firms contributes 0.8% and expansion of high productive firms 0.2%.

To summarize the main mechanisms in the model, average productivity rises in this model because the composition of firms changes towards the more efficient ones as a result of competition on the product market and because inputs become cheaper. Freer trade lowers the transport costs and thus foreign firms will be more often the most competitive ones. Therefore, exporting high productive firms can expand and low productive domestic selling firms disappear.

### 2.3.3 Melitz and Ottaviano (2008)

The model by Melitz and Ottaviano (2008), MO, unites the heterogeneous productivity approach in Melitz with the linear demand system in Ottaviano et al. (2002). The linear demand system displays horizontal product differentiation and love for variety. Pricing behavior depends on the number of competitors and the average price level in

---

\(^3\)Together with a result discussed before, this result shows that the Fréchet distribution displays two properties: a more productive firm is more ahead of its first rival and also has a rival that is more productive. Both results are motivated by Bernard et al. (2003), p.1278, by a sports analogy.
contrast to standard CES-preferences where prices are fixed mark-ups over marginal cost. Like in Melitz there are sunk set-up costs to start a new variety, but production is constant returns to scale. There are no beachhead costs in domestic or exporting markets. Labor supply in the differentiated goods sector is fully elastic through the inclusion of a homogeneous goods sector. Therefore, the composition effect of trade appears because of tougher product market competition and not because of factor market competition. A description of the basic model in a closed economy is followed by a discussion of the open economy model. The effect of trade liberalization is shown in the model, followed by a verbal discussion of the effect of other types of trade liberalization.

The Model in a Closed Economy

Utility depends on a homogeneous good, \( c_0 \), that serves as the numeraire and on a continuum of differentiated goods, \( c_v \):

\[
U = c_0 + \alpha \int_{v \in V} c_v dv - \frac{1}{2} \gamma \int_{v \in V} c_v^2 dv - \frac{1}{2} \eta \left( \int_{v \in V} c_v dv \right)^2 \tag{2.56}
\]

The parameters \( \alpha \) and \( \eta \) measure the relative attractiveness of differentiated goods. \( \gamma \) measures the love for variety or equivalently the ease of substitution between the differentiated goods. A larger \( \gamma \) corresponds with a lower substitution elasticity. Given that the demand for the numeraire good is positive, the inverse demand function is given by:

\[
p_v = \alpha - \gamma c_v - \eta C \tag{2.57}
\]

\( C = \int_{v \in V} c_v dv \) is consumption of all varieties. Upon inversion and multiplication by the number of consumers \( L \), one finds the demand function facing the firm\(^4\):

\[
x_v = \frac{\alpha L}{\gamma + \eta N} - \frac{L \eta N}{\gamma + \eta N} \bar{p} \tag{2.58}
\]

\( \bar{p} = \frac{1}{N} \int_{v \in V^*} p_v dv \) is the average price level and \( N \) is the number of consumed varieties. The set of varieties \( V^* \) for which there is positive demand is given by:

\[
V^* = \{ v \mid c_v \geq 0 \} = \left\{ v \mid p_v \leq \frac{1}{\eta N + \gamma} (\gamma \alpha + \eta N \bar{p}) \right\} \tag{2.59}
\]

Condition (2.59) shows that a larger number of firms or a lower price level decreases the price at which there is positive demand. MO characterize this as a ‘tougher competitive environment.’ It can be shown that a larger number of firms or a lower price level also increases the price elasticity of demand.

There are constant returns to scale in both the homogeneous goods sector and the

\(^4\)Write the inverse demand function for another variety with index \( w \), subtract the two inverse demands, and integrate the result over \( w \). This gives an expression for \( C, C = N c_w - \frac{1}{2} \int_{w} (p_w - p_v) dw \), that can be substituted into the inverse demand function giving the demand function.
differentiated goods sector. Labor is the only input of production, is homogeneous and can move freely between the two sectors. This implies that competition on the labor market cannot drive less competitive firms out of the market like in Melitz. A unit labor requirement in the homogeneous goods sector implies a unit wage. Firms in the differentiated goods sector can enter upon drawing a marginal cost from a known distribution. They have to incur a sunk entry cost, \( f_e \), before they know their marginal cost level like in Melitz. All firms making positive or zero profits stay in business, the others exit. Firms differentiate their goods to get market power and choose the following pricing rule:

\[
x(a) = \frac{L}{\gamma} (p(a) - a)
\]  

(2.60)

There is a firm with cutoff marginal cost, \( a^* \), earning zero profit. (2.60) shows that its demand is zero. Therefore, equation (2.59) with equality sign gives a relation between the cutoff cost level \( a^* \) and the number of firms \( N \):

\[
a^* = \frac{1}{\eta N + \gamma} (\gamma \alpha + \eta N \bar{p})
\]  

(2.61)

The optimal price of a firm can be expressed as a function of its own cost level and the cutoff cost level. By solving equation (2.61) for \( \bar{p} \) and substituting the resulting expression in the demand equation (2.58) one finds an expression for market demand \( x(a) \) that can be plugged into (2.60) to give:

\[
p(a) = \frac{1}{2} (a^* + a)
\]  

(2.62)

Convenient expressions for output, the absolute markup, revenues and profits follow from the pricing equation:

\[
x(a) = \frac{L}{2\gamma} (a^* - a)
\]  

(2.63)

\[
m(a) = \frac{1}{2} (a^* - a)
\]  

(2.64)

\[
r(a) = \frac{L}{4\gamma} ((a^*)^2 - a^2)
\]  

(2.65)

\[
\pi(a) = \frac{L}{4\gamma} (a^* - a)^2
\]  

(2.66)

To close the model, a free entry condition is imposed. Expected profit should be equal to the sunk set-up cost:

\[
\int_0^{a^*} \pi(a) dG(a) = \frac{L}{4\gamma} \int_0^{a^*} (a^* - a)^2 dG(a) = f_e
\]  

(2.67)

\( G(a) \) is the distribution of potential unit costs, from which firms draw before entrance. Firms produce only for one period. This assumption is equivalent to an exit probability \( \delta \) of 1 in Melitz (2003). The number of entrants is then given by \( N_E = \frac{N}{a^*(a^*)} \). The free
entry condition determines the cutoff cost level \(a^*\). The ZCP-condition in (2.61) can be used to determine the number of firms as a function of the cutoff cost level and average costs:

\[
N = \frac{2\gamma}{\eta} \frac{a - a^*}{a^* - \bar{a}}
\]  

(2.68)

\[\tilde{a} = \frac{1}{G(a^*)} \int_0^{a^*} adG(a)\] is the average cost of surviving firms and a measure of average productivity in the economy. Equation (2.67) shows that a larger market \(L\) leads to a lower cutoff marginal cost. As a result average cost declines and from (2.67) the number of firms rises. Like in the models of section 2.2, a larger market is in this model equivalent to the introduction of costless international trade with an equal country. The intuitive story of a larger market size is that more firms enter the market, making competition tougher by lowering the average price level. As a result, the cutoff cost level declines. Some less competitive firms cannot survive. It follows that average productivity in the economy rises. A firm with a certain cost level \(a\) sets lower absolute and relative markups in response to tougher competition, cf. equation (2.64).

MO assume a Pareto distribution of initial productivities to solve explicitly for the cutoff cost level from the free entry condition. Initial productivities \(1/a\) are distributed according to a Pareto distribution with lower bound \(1/a_M\) and shape parameter \(k \geq 1\):

\[G(a) = \left(\frac{a}{a_M}\right)^k \]  

(2.69)

The truncated distribution of firms with a productivity above \(1/a^*\) is also Pareto with the same shape parameter and with lower bound equal to the cutoff productivity:

\[\mu(a) = g(a \mid a \geq a^*) = \frac{ka^{k-1}}{(a^*)^k} \]  

(2.70)

A Pareto distribution makes calculations tractable in this model. The question is whether it is a reasonable assumption. Baldwin (2005), p.7, states that ‘the empirical literature on firm size distribution suggests that a Pareto distribution is a reasonable approximation [for the size distribution of firms].’ In Melitz with CES-preferences size and productivity vary inversely and proportionally (cf. equation (2.6)), so a Pareto distribution for productivity seems fair in that setting. But in the linear demand system size does not vary proportionally with productivity, cf. equation (2.65). So, the fact that the size distribution is Pareto does not imply that the productivity distribution is Pareto in MO.

Plugging equation (2.69) into the free entry condition, equation (2.67), leads to an explicit expression for \(a^*\)\(^5\) and from the ZCP-condition for \(N\):

\[a^* = \left[\frac{2f_L(k + 1)(k + 2)a_M^k}{L}\right]^\frac{1}{k+2} \]  

(2.71)

\(^5\)Use repeated integration by parts.
2.3 THREE FIRM HETEROGENEITY MODELS

\[ N = \frac{2\gamma (k + 1) \alpha - a^*}{\eta a^*} \]  

(2.72)

The assumed Pareto distribution allows some more comparative statics exercises on average values. Average cost, price, absolute markup, output, revenues and profits are given by:

- \[ \bar{a} = \frac{k}{k + 1} a^* \]
- \[ \bar{\pi} = \frac{2k + 1}{2k + 2} a^* \]
- \[ \bar{m} = \frac{1}{2k + 1} a^* \]
- \[ \bar{x} = \frac{L}{2\gamma k + 1} a^* \]
- \[ \bar{r} = \frac{L}{2\gamma k + 2} (a^*)^2 \]
- \[ \bar{\pi} = \frac{L}{2\gamma (k + 1) (k + 2)} (a^*)^2 \]

A larger market size leads to larger average firm size and higher average profits, because the direct market size effect dominates the effect of lower prices and lower markups. Also, average absolute markups decline, because the direct effect of tougher competition dominates the composition effect towards more productive firms with larger markups. To summarize, as a result of the larger market-size the cutoff cost level decreases, average productivity rises, the number of firms and varieties rises, the average absolute markup declines and average firm size rises. MO show formally that a larger market raises welfare.

MO consider a short-run equilibrium to address the short-run effects of liberalization. In a short run equilibrium there is no entry or exit of firms. Firms decide whether to produce or not, depending on the ability to make profit. In the model this implies that the ZCP-condition, equation (2.67), remains, but the free entry condition is replaced by another equation. The other equation gives the number of firms that remain in business as a function of the pre-existing firms in business (the incumbents), \( N \), the distribution of costs of these firms \( G(a) \) (with support between 0 and the old cutoff cost level, \( \bar{\pi}_M \)) and the new cutoff cost level \( a^* \):

\[ N = \overline{NG}(a^*) = \overline{N} \left( \frac{a^*}{\bar{\pi}_M} \right)^k \]  

(2.73)

So, when the cutoff cost level in a short-run equilibrium remains equal following a change in some parameter then the number of firms stays equal and the cutoff cost level does not change. When the cutoff cost-level does change, combining (2.73) with the ZCP-condition in (2.67) gives the new cutoff cost level:

\[ \frac{(a^*)^{k+1}}{\alpha - a^*} = \frac{2 (k + 1) \gamma (\bar{\pi}_M)^k}{\eta \overline{N}} \]

In the short-run equilibrium a larger market does not change the distribution of producing firms; all firms adjust production levels in proportion to market size. The reason is that only a change in the number of firms can change the toughness of competition leading to a change in the cutoff cost level.
The Model in an Open Economy

Without trade barriers the open economy is just an extension of the market like in section 2.2 leading to more firms, tougher competition, a lower cutoff cost level, higher average productivity, lower average markups, lower average prices and larger average firm size and profit. Proportional bilateral liberalization generates equal effects as an extension of the market. Still the inclusion of trade barriers is useful to study different ways of trade liberalization.

A model with two countries is considered, indexed by \(s, t = H, F\). The two countries can differ along two dimensions, their size \(L^s\) and the iceberg trade costs \(\tau^s > 1\) to import into a country \(s\). As there are constant returns to scale, the optimal pricing rules for domestic and export supply are independent:

\[
x^s_d(a) = \frac{L^s}{\gamma} (p^s_d(a) - a)
\]

\[
x^s_x(a) = \frac{L^t}{\gamma} (p^s_x(a) - \tau^t a)
\]

Prices are always defined as delivery prices, so including trade costs. The domestic cutoff cost level in country \(s\), \(a^{ss}_s\) is related to the exporting cutoff cost level in country \(t\), \(a^{ts}_x\), as they are both determined by the delivery price \(s\) in at which a firm just sells a non-negative amount:

\[
\tau^s a^{ts}_x = a^{ss} = \frac{1}{\eta N + \gamma} (\gamma \alpha + \eta \bar{N} \bar{p})
\]

Using the ZCP-condition in (2.76) and the market demand equation, equation (2.58), one finds the price of a firm as a function of its own cost level and the cutoff cost and all the implied expressions on revenues, markups, etc. Only the expressions for profits are displayed.

\[
p^s_d(a) = \frac{1}{2} (a^{ss} + a)
\]

\[
p^s_x(a) = \frac{\tau^t}{2} (a^{ss} - a)^2
\]

\[
\pi^s_d(a) = \frac{L^s}{4\gamma} (a^{ss} - a)^2
\]

\[
\pi^s_x(a) = \frac{L^t}{4\gamma} (\tau^t)^2 (a^{ss} - a)^2
\]

The free entry condition is given by equality of average profits from domestic and exporting sales and the sunk entry cost:

\[
\int_0^{a^{ss}} \pi^s_d(a) \, dG(a) + \int_0^{a^{ss}} \pi^s_x(a) \, dG(a) = f_e
\]

In country \(t\) there is a similar free entry condition. Using the fact that exporting cutoff cost levels can be expressed as a function of domestic cutoff cost levels as in equation
2.3 THREE FIRM HETEROGENEITY MODELS

2.76, and imposing a Pareto distribution, expressions for the cutoff cost levels can be found:

\[
a^{ss} = \left[ \frac{\gamma \phi}{L^s} \frac{1 - \rho^l}{1 + \rho_l \rho^s} \right]^{\frac{1}{1+2}} \tag{2.82}
\]

\(\rho^s\) measures trade barriers into country \(s\), \(\rho^s = (\tau^s)^{-k}\) and \(\phi = 2 f_e (k + 1) (k + 2) (a_M)^k\) is a technology index. The cutoff cost level for exporting firms inclusive of trade costs is equal to the cutoff cost level of domestic producers. By the assumption of cost distributions that are equal and Pareto, this implies that the price distributions of domestic and exporting firms supplying the same market are equal. Therefore, one can solve for the average price level as a function of the cutoff cost level and average cost, using equation (2.77). Using this expression in the ZCP-condition, equation (2.76), and assuming a Pareto distribution, one gets the same expression for the number of firms as in the closed economy:

\[
N^s = \frac{2 (k + 1)}{\eta} \frac{\alpha - a^{ss}}{a^{ss}} \tag{2.83}
\]

Equation (2.76) provides some interesting insights. First, opening an economy to trade has equal effects as an extension of the market. The cutoff cost level becomes smaller.\(^6\) The reasons for this effect are equal: more firms and tougher competition. In the short-run equilibrium trade has different effects than an extension of the market. The reason is that trade increases the number of competitors in the short-run, whereas an extension of the market does not raise the number of competitors in the short-run. Opening up to trade thus decreases the cutoff marginal cost in the short-run, because of import competition. Second, bilateral liberalization between two countries with equal trade barriers has similar though not equal effects as an increase of the market. This is seen by reducing equation (2.76) to the following expression when trade barriers are equal.

\[
a^{ss} = \left[ \frac{\gamma \phi}{L^s} \frac{1}{1 + \rho_l} \right]^{\frac{1}{1+2}} \tag{2.84}
\]

A decrease in trade barriers, i.e. a rise in \(\rho\), decreases the cutoff cost level and thus raises average productivity. But average firm size declines, whereas an increasing market size increased average firm size. Total average revenues are equal to:

\[
\bar{r}^s = \bar{r}^s_d (a) + \bar{r}^s_x (a) = \frac{L^s}{2\gamma k + 2} (a^{ss})^2 + \frac{L^l}{2\gamma l} \frac{1}{k + 2} (a^{ss}_x)^2 \tag{2.85}
\]

As the exporting cutoff cost level is related to the foreign domestic cutoff cost level by \(a^{ss}_x = \frac{a^{ss}}{\tau}\), one can write:

\[
\bar{r}^s = \frac{1}{2\gamma} \frac{1}{k + 2} \left( (a^{ss})^2 L^s + (a^{ss}_x)^2 L^l \right)
\]

\(^6\) Compare equation (2.82) with the ZCP-condition in the closed economy, equation (2.71), and note that \(\frac{1 - \rho^l}{1 - \rho^l \rho^s}\).
Trade liberalization reduces the cutoff cost levels in both countries, confront equation (2.84), hence average firm size declines. Section 2.5 contains a discussion of this seemingly strange result.

Third, market size of the domestic market is relevant for cutoff cost levels and thus for all other important variables in the model. The market size of the trading partner has no impact in the model. The latter is due to some offsetting effects caused by the functional form specification of the model (see for a discussion MO). Fourth, unilateral liberalization has adverse consequences in the long-run in this model. Equation (2.82) shows that a rise in $\rho^s$, a decline of trade barriers, increases the cutoff cost level. Average cost rises as well, markups rise and the number of firms declines with adverse welfare effects. The reason is a phenomenon discussed in earlier literature (Venables (1987)): there is a delocation of firms towards the other country, because they can enter the liberalizing country at lower trade costs. In the short-run firms cannot delocate. Therefore, the liberalizing country gains in the short-run as a result of the increased import competition causing tougher competition and thus a lower cutoff cost level.

To summarize, trade leads to more firms resulting in tougher competition. The markup of a firm with a certain cost level declines, driving the less competitive firms out of the market. The cutoff cost level declines. As a result average cost levels decline, i.e. average productivity rises. Welfare rises for two other reasons as well: an increase in the number of varieties and a decrease in average absolute markups. The composition effect is driven by tougher competition on the product market, like in BEJK and not by competition on the labor market like in Melitz, because labor supply is fully elastic.

### 2.4 Empirical Evidence on Heterogeneous Productivity Models

Although theoretical models of international trade with heterogeneous productivity and gains from trade through a composition effect are only developed recently, empirical work on composition effects dates back a longer time. This section starts with some relevant facts on exporting and productivity followed by a discussion of the direction of causality between exporting and productivity. Then evidence on composition effects from different countries is discussed. The section closes by listing empirical results on the importance of sunk costs in exporting, the effect of import competition on firm size and the reaction of markups to trade liberalization.

BEJK list five empirical regularities on exporting and productivity used as test for the model they develop. As in all the papers cited by Bernard and various coauthors they use plant-level and firm-level data from the United States collected in the Longitudinal Research Database of the Bureau of the Census. The basic facts reported here are from 1984, 1987 and 1992 and taken from Bernard and Jensen (1999). The first fact is that there is substantial dispersion in productivity of all firms, exporting and non-exporting. The standard deviation of the log of value added per worker within four digit industries is 0.66 for 1992, although measurement error could lead to an upward bias. Second, exporting firms are more productive. Within four digit industries and within states
2.4 EMPIRICS FIRM HETEROGENEITY

labor productivity is 12%-24% larger among exporters and TFP is 4%-18% larger in the three years explored. Third, exporting firms are much larger. Controlling again for four digit industries and states exporters are about twice as larger as non-exporters. Fourth, only a small fraction of firms export. In the US-dataset used 21% of the firms report exporting. Fifth, exporting firms earn only a small fraction of their revenues from exporting. Fewer than 5% of the exporting plants export more than 50% of their production.

The correlation between exporting and productivity raises the issue of causality. Do more productive firms become exporters or does exporting increase productivity? Bernard and Jensen (1999, 2004b) answer the first question with yes and the second question with no. The impact of productivity on exporting can be examined in various ways. First, non exporters who become exporters are compared with non exporters who do become exporters. Bernard and Jensen (1999) find that future exporters are 20% to 45% larger in terms of employment, 27% to 54% larger in terms of shipments, 7%-8% larger in terms of labor productivity and 2%-4% larger in terms of total factor productivity, although the last difference is not significant. Second, Bernard and Jensen (1999) consider a small dynamic model of the decision to export including sunk entry costs. The binary choice of exporting is estimated with a linear probability model including fixed effects. GMM is applied to the model in first differences. The estimation shows that a larger productivity, a larger total employment and a higher level of wages increase the probability of exporting significantly. One should keep in mind that this estimation could suffer from weak instruments as the number of time periods is only five (Bun and Kiviet (2006)). Third, one can conduct Granger causality tests of productivity growth and export growth. Bernard and Jensen (2004b) find both on aggregate and on industry level that productivity growth has a positive and significant impact on export growth, whereas export growth has a negative and significant impact on productivity growth.

The impact of exporting on productivity can be addressed by regressing growth rates in employment, shipments, TFP and value added per worker on initial export status. Controlling for initial size and other plant characteristics, Bernard and Jensen (1999) find that exporting firms show faster growth in shipments and employment but slower growth in TFP over annual horizons. Over longer horizons only employment grows significantly faster, shipments are not significantly affected by exporting and productivity (TFP) grows more slowly. Bernard and Jensen (2004b) explain the distinction between long run and short run effects by the fact that there is a considerable amount of entry into and exit out of exporting. So, quite a large fraction of firms that are exporters in some year might not be exporters anymore five years later. Average entry is 10% (as a fraction of the non-exporters) and average exit is 17% (as a fraction of the exporters).

To account for entry and exit Bernard and Jensen (2004b) compare the performance of continued exporters, entrants, exiting firms and non-exporters. They show that exiting firms grow significantly slower than non-exporters on all measures. In contrast, entrants grow significantly faster on all measures in the year of entry. Continued exporters have significantly larger employment and shipments growth than non-exporters on the long run, but TFP grows slower, albeit non-significantly.

To summarize, the US-data show that a higher productivity leads to exporting, al-
though the impact of size on exporting is much larger. In the other direction, exporting
does not promote productivity growth at the plant or firm-level. Through composition
effects trade and exporting can have impact on aggregate productivity. When exporting
firms are more productive on average and exporting allows a reallocation of resources
towards more productive firms, exporting raises aggregate productivity. Bernard and
Jensen (2004b) consider this possibility. One of the previous paragraphs reported that
exporters have 4%-18% higher TFP. Bernard and Jensen (2004a) make a more detailed
comparison by exploring productivity levels of non-exporters, continued exporters, en-
trants into exporting, exiting firms and switching firms (entering or exiting more than
once). They find that exporters are 8%-9% more productive than non-exporters con-
trolling for industry and year effects. Entrants move towards the productivity level of
exporters and exiting firms move towards the productivity of non-exporters. The next
step is to examine whether the size of exporting firms grows faster than of non-exporters
implying a reallocation of resources towards exporting, more productive firms. The
findings are that employment growth at exporters is 0.79%-1.08% larger and shipments
growth 0.57%-1.32% larger. Comparing never exporters with always exporters shows
even differences of 2%-4%. Entrants grow slower than always exporters. So, together
these findings imply a reallocation of resources towards exporting more productive firms.
To shed light on the importance of the composition effects, Bernard and Jensen (2004a)
decompose productivity growth at the aggregate and industry level into within and be-
tween plant effects. They find that reallocation effects account for 42% of productivity
growth between 1983 and 1992. More than half of the reallocation effect takes place
within sectors: 22.5% of productivity growth is accounted for by reallocations within
sectors. Exporting contributes 70% to the total reallocation effect.

Tybout (2001) reviews empirical work on composition effects in developing countries.
Tybout (1991) uses revenue per worker as a measure for productivity. He finds in studies
of Chile (data year: 1979), Colombia (1977-1987) and Morocco (1984-1987) that ‘market
share reallocations contribute to productivity growth among tradable goods, but his data
span periods of major macro shocks rather than major trade liberalization episodes, so
it is difficult to argue that the gains are trade induced (p.15).’ Pavcnik (2000) uses
the same Chilean data set and works with TFP instead of labor productivity. He gets
the same results as Tybout, but can neither prove a clear link between the reallocations
and increased foreign competition. The same studies find that exiting plants are less
productive then surviving plants, but again there is no link with trade liberalization.

Tybout and Westbrook (1995) use data for Mexico for the unilateral liberalization
episode 1984-1989. They find that liberalization was associated with productivity gains
due to a reallocation effect. They decompose productivity growth (11.17%) in that
period into a scale effect (0.55%) a reallocation effect (1.02%) and a residual effect
(9.60%). Tybout (2001) emphasizes in his survey that ‘they do not find strong evidence
that rationalization effects were concentrated in the tradable goods industries (p. 16).’

An indirect way to examine reallocation effects raising average productivity is to
look at the distribution of the size of firms. The economic models presented in previous
sections all contain a link between productivity and size: more productive firms have
larger revenues. So, when liberalization would change the size distribution towards larger
firms, there would also be a shift towards more productive firms. A robust finding in all research is that larger firms have a larger probability of exporting. This suggests that more access to foreign markets enables larger firms to expand, implying an increase in average productivity in the economy. But the studies discussed by Tybout (2001) that link changes in trade protection with changes in the intra industry size distribution of firms do not give this clear picture. Head and Ries (1999) find that large Canadian firms grew as a result of US tariff reductions but shrank as a result of Canadian tariff reductions. This suggests just a reallocation towards exporting away from importing as makes economically sense with liberalization. Roberts and Tybout (1996) find a relatively large shrinkage in response to import competition among large firms in Chile and Colombia instead of among small firms. Dutz (1996) finds that the dismantling of NTBs during the ‘80s in Morocco lead to a relatively large decrease in size of small plants and relatively large increase in exit probabilities of small plants. Tybout et al. (1991) find that the whole size distribution of employment decreased in Chile in response to liberalization between 1967 and 1979. So, with the exception of the study by Dutz (1996) on Morocco, the evidence on the effect of trade liberalization on the size distribution of firms does not provide support for the reallocation theory. The findings on size distributions conflict with those on productivity distributions. Studies on productivity distribution do find reallocation towards more productive firms, whereas studies on size distribution do not find reallocation towards larger firms. These two findings can only be reconciled when there is no perfect correlation between size and productivity, i.e. part of the size of firms cannot be accounted for by their productivity. Of course, other possibilities are that the different datasets used lead to the conflicting results or that there are measurement problems with productivity and/or size.

In the reallocation studies focusing on productivity there is evidence for reallocation, but Tybout (2001) repeatedly emphasizes that the reallocation effect cannot convincingly be linked with trade liberalization. The decomposition in Bernard and Jensen (2004a) does neither contain an explicit link with trade liberalization. But they do show that the reallocation towards more efficient firms is towards exporting firms. As trade liberalization can be expected to lead to more trade and exporting, their reallocation effects can logically be linked with trade liberalization.

Empirical work at the plant level on trade has produced three other empirical findings that are of interest in the comparison of the different theoretical models considered. The first finding is on the importance of sunk costs in exporting. Different authors have estimated the decision to export as a function of various explanatory variables and lagged export participation. Bernard and Jensen (2004a) find that lagged export participation raises the probability of exporting by 36%. Tybout (2001) summarizes the evidence in various papers arguing that the probability of exporting increases by up to 70% when a firm exported last period taking into account serial correlation in error terms. The hysteresis in exporting is evidence for the existence of sunk entry costs in the export market: firms already in the export market face another decision problem of participating in the export market than firms that are new on the export market.

The second finding is on the reaction of markups of firms to an increase in import competition. The main finding in the literature is that markups decline with increases
CHAPTER 2 IMPERFECT COMPETITION AND TRADE

in import competition. Import competition is measured in this literature by import penetration rates, effective protection rates or license coverage ratios. Roberts and Tybout (1996) study the impact of import competition in Mexico, Colombia, Chile and Morocco controlling for plant level market shares and industry dummies and find that ‘in every country studied relatively high industry wide exposure to foreign competition is associated with lower margins and the effect is concentrated in larger plants (p. 196).’ Explanation for the last finding is that large firms have most market power and their prices are most responsive to foreign competition. Tybout (2001) discusses other evidence based on the approach in Hall (1988). Hall (1988) writes log output growth rates as a function of log growth rates in inputs and productivity growth, noting that the coefficient on input growth is a function of markups. By allowing this coefficient to vary over time and with changes in trade policy, one can test whether markups vary with trade policy. Although this approach is subject to various econometric problems, the evidence confirms other results that markups decline with an increase in import competition.

The third finding is on the impact of trade liberalization on average firm size. Tybout (2001) argues that a general conclusion in the literature on this topic is a declining average firm size in response to increasing import competition. Tybout et al. (1991) use panel data and find in a study of Chile that plants in sectors with relatively large declines in import protection reduced employment levels. Tybout and Westbrook (1995) find in their study on Mexico that firms facing relatively large declines in license coverage ratios grow relatively slow. These empirical findings are problematic for the early monopolistic competition models where liberalization leads to welfare gains through an increase in the scale of firms. Empirical work shows that average firm size declines in response to liberalization. When there are increasing returns, the scale effects generated lead to welfare losses. An obvious possibility is that all the empirical work only considers short-term responses, whereas in the long run firms are squeezed out of the market leading to a larger firm size. Furthermore, the empirical work focuses on changes in import competition, whereas decreasing costs of exporting should also be taken into account.

2.5 Comparison of Firm Heterogeneity Models

This section contains a comparison of the three theoretical models presented in previous sections and the oligopoly model discussed in the next chapter which is based upon Bekkers and Francois (2008) (BF in the remainder). The models are compared with respect to (i) the modeling assumptions and mechanisms and (ii) the impact of liberalization in the models on various variables like the average size of firms, the markup of firms and the productivity of firms. The modeling mechanisms and outcomes are confronted with the empirical evidence available.

Table 2.1 contains a summary of the basic modeling setups of the three models. Preferences, production structure, the existence of sunk setup costs and sunk export costs, entry and exit into the market, labor market structure, product market structure and type of composition effect are addressed. Melitz, BEJK and BF work with CES preferences. MO use a linear demand system. The difference between Melitz and MO on the one hand and BEJK and BF on the other hand is that in Melitz and MO there
is only one producer for each variety and competition is between producers of different varieties. In BEJK and BF on the other hand there are many (potential) producers of each variety. In BEJK there are different potential producers of each variety and the cheapest takes the whole market, whereas in BF there are different producers of each variety.

In Melitz there are increasing returns to scale through fixed costs of production. BEJK, MO and BF use constant returns to scale in production. In Melitz, MO and BF labor is the only factor of production. In BEJK labor and intermediates are the inputs of production. Melitz, MO and BF assume that firms have to incur sunk setup costs to develop a new variety. BEJK abstracts from setup costs which is logically linked with the fact that there are several potential producers of a certain variety. The omission of setup costs in BEJK raises the question how varieties can be developed.

In Melitz, MO and BF of all the firms that incurred the setup costs, only those firms with sufficient productivity to make positive operating profits really enter the market. Firms exit the market with a certain fixed exit probability or when the cutoff cost level declines as a result of liberalization or an extension of the market. In BEJK the entrance of firms into the domestic market is not modeled. The number of varieties is fixed in BEJK. Firms enter the export market when they become the lowest cost producer of a certain variety. Firms exit the domestic and/or export market when they are no longer the lowest cost producer of a certain variety in a market. Only Melitz and BF include sunk costs to start exporting. Empirical evidence shows that sunk entry costs are important, so this adds realism to Melitz’ model. A problem of Melitz is the required assumption on the size of the sunk entry costs. To guarantee that only more productive firms export, per period sunk export costs multiplied by a measure of trade costs should be larger than the fixed costs to produce. Or when one assumes that the fixed costs to produce consist of on the one hand overhead costs and on the other hand per period sunk costs to enter the domestic market, these two type of costs should be lower than the sunk costs to export (multiplied by a measure of trade costs).

The structure of the labor market is modeled in two different ways. Melitz assumes that total labor supply is fully inelastic. This assumption is responsible for the reallocation effect in Melitz. Trade liberalization increases production possibilities for the most efficient firms. They hire more workers driving up the real wage in the economy as labor supply is fully inelastic. Entry becomes also more attractive, as a low cost draw enables more exports. More productive workers move away from the less productive firms who cannot afford the increased wages. As a result the less productive firms disappear. In BEJK, MO and BF labor supply is fully elastic by including a homogeneous goods sector from which additional workers can be hired without affecting the real wage rate. As a result there is no reallocation effect of these models because of labor market competition. The reallocation effect of these models takes place through increased competition on the product market.
Table 2.1: Model Setup of Melitz (2003), Bernard, Eaton, Jensen and Kortum (2003), Melitz and Ottaviano (2008) and Bekkers and Francois (2008)

<table>
<thead>
<tr>
<th>Preferences</th>
<th>Melitz</th>
<th>BEJK</th>
<th>MO</th>
<th>BF</th>
</tr>
</thead>
<tbody>
<tr>
<td>CES</td>
<td>CES</td>
<td>Linear demand</td>
<td>CES</td>
<td>Constant returns to scale</td>
</tr>
<tr>
<td>Production</td>
<td>Increasing returns with constant marginal costs and fixed costs</td>
<td>Constant returns to scale</td>
<td>Constant returns to scale</td>
<td>Constant returns to scale</td>
</tr>
<tr>
<td>Structure</td>
<td>Yes</td>
<td>No, raises question of how varieties are developed</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Entry and exit</td>
<td>Steady state of entry and exit; firms enter without knowing their productivity; firms exit with constant probability or after a change in cutoff cost level</td>
<td>Entry and exit are not modeled as a steady state; firms enter when competitors for same variety become more productive</td>
<td>Idem as in Melitz</td>
<td>Idem as in Melitz</td>
</tr>
<tr>
<td>Labor market structure</td>
<td>Inelastic supply of labor. Labor can move freely between different firms</td>
<td>Fully elastic supply of labor through inclusion of homogeneous goods sector</td>
<td>Idem as in BEJK</td>
<td>Idem as in BEJK</td>
</tr>
<tr>
<td>Product market structure</td>
<td>Monopolistic competition with constant price elasticity of demand. Number of firms does not affect pricing policy</td>
<td>Bertrand price competition between several potential producers of each variety</td>
<td>Monopolistic competition with price elasticity increasing in the number of competitors</td>
<td>Cournot competition within each sector</td>
</tr>
<tr>
<td>Composition effect</td>
<td>The possibility to export increases the market for high productive firms and makes entry more attractive. This raises labor demand. The increased real wage drives less competitive firms out of the market</td>
<td>The possibility to export increases price competition for a certain variety. High productive exporting firms replace domestic suppliers by undercutting their price</td>
<td>The possibility to export increases the number of competitors in a market. The general price level declines and high cost producers disappear.</td>
<td>International trade increases the number of competitors, leads to a lower market price and drives therefore the least productive firms out of the market</td>
</tr>
</tbody>
</table>
2.5 COMPARISON FIRM HETEROGENEITY MODELS

In MO an increase in the number of firms decreases the average price level. The effect is that the less productive firms cannot sell positive amounts of output anymore and vanish. In BEJK more competitors also leads to the disappearance of less productive firms, as there is a larger probability that one of the competitors has a lower cost level. In BF international trade leads to more firms in the market and lower market prices driving the least productive firms out of the market. In Melitz in contrast an increase in the number of firms and increased product market competition does not drive less productive firms out of the market as the markup of firms is fixed.

A third way to model the labor market would be by including transition costs of workers between firms. This would slow down the movement of workers towards the more productive firms and could be able to account for certain empirical regularities on the short-run.

In the comparison of the setups and mechanisms of the model, the way the reallocation of resources works is an important characteristic. So far, there is no conclusive empirical evidence on the relative importance of the two types of reallocation effects. An interesting empirical finding in this respect is that markups decline as a result of increased import competition. This fact is support for the product market competition view of MO, where the markups decline. Work by Menezes-Filho and Muendler (2007) using firm-level and worker-level data from Brazil in the 1990s seems to falsify the labor market view and support the product market competition view. Menezes-Filho and Muendler (2007) find that more import competition leads to a net lay-off of workers in comparative advantage sectors and a growth of employment in the informal sector. These findings do not support the mechanism proposed in Melitz that trade liberalization leads to increased competition for scarce labor resources driving up real wages.

The impact of trade liberalization on various variables in the four models can be confronted with the empirical evidence available. Melitz, BEJK only consider a symmetric fall in per unit trade costs, i.e. bilateral liberalization. MO and BF discuss also unilateral liberalization, BF for the short-run. When relevant the outcomes of this exercise are included in the comparison. Table 2.2 contains an overview of the impact of trade liberalization on average productivity, average markup, average firm size and the number of consumed varieties in the three models and in empirical work. First, consider the impact of trade liberalization on productivity. In all four models average productivity rises as a result of trade liberalization. Less productive firms get squeezed out by the different mechanisms discussed above and as more productive firms gain market share, average productivity in the economy rises. As discussed, in the empirical evidence from developing countries it is difficult to relate the rise in average productivity through reallocation to trade liberalization. The evidence in Bernard and Jensen (2004a) from the US shows rising productivity levels, because of increased exporting, which could obviously be linked to trade liberalization.

A second variable to consider is the markup of firms. Trade liberalization has different impacts on this variable in the three models. MO consider the absolute markup,
price minus marginal costs instead of the more familiar relative markup, price divided by marginal costs. Average absolute markups decline as a result of trade liberalization in MO. Trade liberalization has two conflicting effects on average markups. On the one hand, tougher competition reduces the markups of all firms. On the other hand, the least productive firms with the lowest markups are squeezed out leading to higher average markups. In MO the first effect dominates and average markups decline, although this conclusion can only be drawn when a Pareto distribution of productivities is assumed. Average relative markups are constant in MO with a Pareto distribution of productivities.\textsuperscript{8} The two effects cancel out.

In Melitz the relative markup is fixed, because of the CES preferences and the large group assumption implying a fixed price elasticity. A small group assumption in this model would make markups sensitive to trade liberalization. The sign of the effect is not a priori clear, because the composition effect and competition effect would work in opposite directions, just like in MO.

In BEJK average relative markups rise as a result of trade liberalization. Apparently, the composition effect towards more efficient firms dominates the competition effect. BEJK equalize the relative markup to measured productivity of a firm. The reasoning is as follows: measured productivity is equal to value added divided by the value of inputs. Dividing by the level of output gives price divided by average costs, equal to marginal costs because of constant returns to scale. BEJK criticize Melitz because its fixed markup would imply a constant measured productivity. But this critique is not entirely correct.

\begin{align*}
\int_0^a \frac{p(a)}{a} G(a) \, da &= \frac{1}{G(a^\gamma)} \int_0^{a^\gamma} \frac{1}{2} a^a + \frac{a}{2} \, a G(a) \, da = \frac{1}{G(a^\gamma)} \int_0^{a^\gamma} \frac{1}{2} a^a + \frac{a}{2} G(a) \, da \\
&= \int_0^{a^\gamma} \frac{1}{2} a^a \left( \frac{k}{a^\gamma} \right)^{k-1} + \frac{1}{2} = \frac{2k-1}{2k-2}
\end{align*}

\textsuperscript{8}The average relative markup in MO is equal to:
as a fixed markup only implies constant measured productivity as defined by BEJK when there are constant returns to scale implying that average costs are equal to marginal costs. Increasing returns to scale as in Melitz imply that measured productivity as defined by BEJK increases in revenues of the firm.\footnote{BEJK define measured productivity as price over average costs. This gives for Melitz:} As discussed below, average revenues (firm size) rises with trade liberalization in Melitz, so measured productivity as well. The critique of BEJK would apply to MO, because in their model the average relative markup is constant and production is CRS.

In BF there is a distinction between the short-run and the long-run (when a free entry condition is added). In the short-run average markups from domestic sales decline with lower trade costs and average markups from exporting sales rise. Hence, assuming that in import-competing sectors firms only produce for the domestic market average markups decline in these sectors. In the long-run the effect on average markups cannot be determined without specifying a distribution of costs. The competition effect and composition effect described above work in opposite directions.

Empirical evidence shows that (relative) markups decline in response to increased import competition. To the extent that these effects are short-run effects, BF is in line with the findings in empirical work. In MO markups also decline, but only absolute markups and not relative markups as used in empirical work.

A third variable to consider is average firm size (average revenues). Empirical work shows clearly that average firm size declines in response to increased import competition, although the effects measured could be only short-run effects. Furthermore, it does not take into account the impact of liberalization on the size of exporting firms. In Melitz average firm size rises when per unit trade costs decline.\footnote{Trade liberalization leads to an increase in the average profit level. The denominator of equation (2.35) shows that average revenues rise in average profits, } Firms have to share the market with more foreign competitors. But the less productive firms are squeezed out of the market and the high productive firms can export more. On balance, average firm size rises. In BF the effect on average firm size cannot be determined. In the long-run the balance of competition and composition effect cannot be ranked as with the effect on average markups and in the short-run calculations are complicated by the fact that profit income is endogenous.

In MO average firm size declines with bilateral trade liberalization. Applying the logic of the standard Krugman model with variable price elasticity this is a strange result. In that model trade liberalization makes competition tougher decreasing profit margins. This requires an increase in average firm size to satisfy the zero profit condition. So, one would expect an increase in firm size in this model as well. But in MO firms have heterogeneous productivity. Trade liberalization makes competition tougher and reduces profit margins of firms. But the less productive firms are squeezed out of the market which already raises the expected profit of firms. The increase in size is not needed to compensate for the declining profit margins as a result of tougher competition. Therefore,
the size of firms can go either way. The declining size of firms in MO apparently depends on the model setup with linear demand preferences.\footnote{In Melitz with CES preferences, the average size of firms rises. In Melitz there is no initial decline in profitability, because competition becomes tougher as markups are fixed. The increase in profitability as a result of the squeezing of less productive firms is counteracted by the declining probability of a good productivity draw and is supported by the rising average firm size. It is likely that a variable markup initially decreasing profitability of firms will also increase firm size to counteract the negative effect on profitability.} The model results of MO seem to be well in accordance with the empirical evidence, although the evidence presented is mainly about increased import competition and could reflect only short-run effects. MO also present model results of increased import competition and distinguish between the short-run and long-run. In the short-run increased unilateral liberalization leads to tougher competition. Some plants shut down production and the other firms reduce their scale of production. The mechanism here is different than above. Domestic firms just have to share the market with more competitors and therefore their size shrinks. Profit levels decline as a result, but they are still positive for the operating firms. In the long run unilateral liberalization leads to a delocation of firms towards the other country. Competition becomes less tough, cutoff cost levels rise and the average firm size increases for the same reason as with the declining firm size in reaction to bilateral liberalization.

BEJK do not report explicitly the impact of trade liberalization on average firm size and their model is relatively burdensome, so calculating the effect is not as easy as in MO. It seems that average firm size rises in the model. There is no free entry condition that imposes conditions on the size of the firm. Preferences are CES, so revenues of a firm are given by equation (2.6). Integrating this expression over all firms, suggests that average firm size is equal to nominal income divided by the number of firms. As the number of firms declines, because some firms are competed out of the market by foreign firms, average firm size has to rise.

A fourth variable to consider is the number of varieties available to consumers. In BEJK and BF the number of varieties available to consumers is exogenous. So, trade liberalization has no impact on it. In Melitz trade liberalization has an ambiguous impact on the number of consumed varieties. More foreign varieties are imported increasing the number of consumed varieties, but average firm size rises as well having a negative impact on the number of consumed varieties. Baldwin and Forslid (2004) show that trade liberalization decreases the number of consumed varieties with a Pareto distribution of initial productivities when per period sunk export costs are larger than the fixed cost of production, \( f_x > f \). The likeliness of this condition is discussed before. When the distribution of productivities is not Pareto, a stronger condition is needed to get a decrease in the number of varieties with trade liberalization.\footnote{Technically, the reason is that the ZCP-condition is horizontal in cutoff marginal cost, average profit-space with a Pareto distribution, whereas it slopes up for a general distribution. Trade liberalization shifts the ZCP-condition up. The new intersection with the FE-condition is at a larger profit level with a Pareto distribution than with a general distribution. Therefore, the average firm size rises more with a Pareto distribution and so the number of producers displays a stronger decline.} Melitz shows that welfare still increases unambiguously with trade liberalization, despite the possible decline in the number of consumed varieties. The increase in average productivity always dominates...
the possible decline in varieties.

In MO (bilateral) trade liberalization unambiguously increases the number of consumed varieties. The increase in the number of firms (and thus varieties) is a basic part of the model where liberalization makes competition tougher squeezing the less productive firms out of the market. It also corresponds logically with the decline in average firm size with bilateral liberalization. Unilateral liberalization decreases the number of varieties, because of the delocation of firms.

2.6 Concluding Remarks

This chapter gave a review of the most influential work on firm heterogeneity models of international trade. First the early work on imperfect competition models in international trade was exposed briefly, that provided an explanation for the ever more prominent intra-industry trade. This early literature, developed in the beginning of the 1980s, proposed four gains from international trade: an increasing availability of varieties, efficiency gains because of a larger scale of production and because of a larger division of labor and lower prices because of pro-competitive effects from trade. The early imperfect competition features equal firms. This gave in the beginning of 2000 rise to an extension to heterogeneous firms. Firm heterogeneity was not solely an exotic theoretical exercise, as it constituted a description of the empirically important reallocation effect of trade, where more productive exporting firms gain market share at the expense of domestic producing less productive firms. The firm heterogeneity models discussed in this chapter were the monopolistic competition CES model of Melitz (2003), the Bertrand competition model of Bernard et al. (2003) and the monopolistic competition linear demand model of Melitz and Ottaviano (2008). The next chapter describes a firm heterogeneity model of oligopolistic (Cournot) competition.

The three models discussed in this chapter and the oligopoly model were compared with each other with respect to modeling assumptions and type of reallocation mechanisms and the results of their models were confronted with empirical findings. The conclusion that can be drawn from this comparison is that none of the models performs superior. The setup of each model contains strong and weak points and in none of the models all predictions are in line with empirical findings. The main weak point in Melitz is its fixed price elasticity implying that markups are fixed, whereas empirical evidence shows that markups decline with increased import competition. The fixed price elasticity also implies that a composition effect through increased product market competition is absent. Another point of concern is the rising average firm size with trade liberalization; although the empirical evidence suggesting declining average firm size could mainly catch short run effects. Finally, the condition on fixed costs being larger than per period sunk entry costs times a measure of trade costs is necessary to get sensible results in the model, although the realism of this assumption is questionable. Strong points of Melitz are and its inclusion of sunk entry costs and in particular its use of a very tractable model that easily be extended to apply it to a wide range of topics.

Strong points of BEJK are the inclusion of intermediates as an input into production and the fact that BEJK is the only model that has applied real world data to its model.
to run a simulation. Also, the mechanism generating the composition effect is simple and intuitive. A weak point is that the details of the model are cumbersome. Other weak points in BEJK are the absence of sunk setup costs and the absence of a dynamic entry and exit equilibrium. This raises the issue of the origin of new product varieties. Another point of concern is that average markups increase with trade liberalization whereas empirical work shows a decline of markups. Finally, there is no reallocation effect through the labor market.

An important strong point of MO is that it produces an impact of trade liberalization that seems in accordance with the data. Average (absolute) markups and average firm size decline in response to trade liberalization. The declining average firm size can also be seen as a weak point for those who believe that the empirical evidence on the impact of trade liberalization on this variable reflects short-run effects. Another weak point of the model is its reliance on the Pareto assumption for the distribution of (potential) productivities.

Pre-dating the exposition in chapter 4 of the oligopoly model, strong points of BF are that it produces a reallocation effect of trade in a basic model with a parsimonious setup; that it provides a natural extension of the Brander and Krugman (1983) reciprocal dumping model and that it can be nested as a special case of the Ricardian comparative advantage model. A final strong point is that its short-run effects of trade liberalization on average markups seem in line with empirical work. An important weak point is that the effect on average firm size cannot be derived analytically.

Considering the applications of heterogeneous productivity models to various topics, it seems that Melitz is the winner. Baldwin and Okubo (2006) apply Melitz model to economic geography claiming that with heterogeneous productivity agglomeration effects could be overstated in empirical work; Baldwin and Robert-Nicoud (2004) address the growth implications of Melitz arguing that heterogeneous productivity which produces positive level effects because of the composition effect, can lead to negative growth effects; Bernard et al. (2007) merge Melitz with a traditional Heckscher-Ohlin model showing that scarce factors of production can still gain from trade liberalization, because of the decline in price levels generated by the composition effect; Helpman et al. (2004) consider the choice between exporting and FDI in a Melitz model. These are just a few applications. The literature is exploding and numerous other applications are appearing and have appeared in the recent past, making it impossible to give a complete overview of all applications. This thesis also features two chapters where the Melitz model serves as the starting point.
Chapter 3

Heterogeneous Popularity and Exporting Uncertainty

3.1 Introduction

Empirical work shows that a significant fraction of firms quit the export market soon after entrance. Bernard and Jensen (2004b) report that 15% of the exporting firms leave the export market every year in a sample of American firms between 1984 and 1992. Almost 5% of the firms have left the export market already the year after entry and do not return again on the export market afterwards (calculations from table 5 of Bernard and Jensen (2004b)). Irarrazabal and Opromolla (2006) find that 16% of the exporting firms leave the export market every year in a sample of Chilean firms between 1990 and 1996. They do not calculate which fraction of quitters from the export market had just entered the export market. Other evidence on quick exit can be found in Eaton et al. (2007). They show in a panel of Colombian firms between 1996 and 2005 where each single export transaction is recorded that ‘the survival rate among first-year exporters is typically around one-third, and in some cases is much lower (Eaton et al. (2007), p.19).’ Hence two-third of the starting exporters drop out within a year. Indirect evidence on the quick exit of firms from the export market after entry comes from duration analysis on detailed export categories. Besedes and Prusa (2006) show with a US dataset between 1992 and 2001 of detailed product categories that around one third of imports (depending on the data) in a product category for differentiated products stop again within one year. Nitsch (2007) finds in a sample of German imports between 1995 and 2005 that 40% of the newly emerging import product categories stop within one year.\footnote{These studies correct for censoring as a result of redefining product categories. As these data are on product categories, the hazard rates are a lower bound for the hazard rates for individual firms given that more firms could trade in one product category. On the other hand continuing exports of a certain variety could be recorded as exit from a product category when firms switch product categories with their variety.}

So, empirics show that there is a lot of exit from the export market shortly after entry. Broadly, two reasons for exit from the export market can be identified. On the one hand,
variables affecting profitability in the export market display variation. Negative shocks to such variables can induce firms to drop out of the export market. Baldwin (1988) and Baldwin and Krugman (1989) for example model fluctuations in exchange rates. Their model shows that sunk export costs lead to persistent trade effects of exchange rate shocks. Irarrazabal and Opromolla (2006) make productivity a stochastic process in a Melitz-type model of trade to explain that firms can leave the export market again after entrance.

On the other hand, firm exit from the export market can be due to the fact that firms simply do not have (enough) information about profitability in the export market. The model presented in this chapter follows this view. Firms cannot assess the popularity of their good in the export market before they enter the export market. After entrance, the popularity of their good could be too low to produce profitably. An example of this type of exporting uncertainty is the withdrawal of Wal-Mart from the German and South-Korean export market. After a presence of some years on these foreign markets incurring huge losses, Wal-Mart gathered enough information to realize that their ‘product-type’ was not popular enough in these foreign markets.

The model in this chapter is a firm heterogeneity model (in the spirit of Melitz (2003)) taking into account exporting uncertainty. There is heterogeneity in the taste parameter (the CES-weight) of a variety instead of in productivity and the taste parameters of the same variety are different in the domestic and exporting market. Firms are uncertain about the popularity of their good, reflected in the taste parameter, before they start producing and before they start exporting. Sunk costs have to be paid to learn the popularity of the good both in the domestic and exporting market. Hence there is uncertainty before a firm starts producing and before it starts exporting.

Heterogeneity in the taste parameter instead of heterogeneity of productivity leads to a more realistic interpretation of the sunk entry costs. These costs have to be incurred to explore the desirability of a variety in the market. It is also more realistic to assume that taste parameters are different on the domestic and exporting market than to assume different productivities in both markets. The same product is produced, so why should the productivity change? Differences in taste parameters have a more natural interpretation: preferences are different across countries. It is assumed that the taste parameters on the different markets correlate, albeit imperfectly. The model’s solution contains an additional parameter, the cutoff taste parameter below which a firm does not consider exporting. A firm with a taste parameter above this cutoff value starts exporting but can be unsuccessful when its product is not popular enough on the foreign market. This generates the exporting uncertainty. The model generates three cutoff taste parameters, a domestic cutoff taste parameter, a cutoff taste parameter to start exporting and a cutoff taste parameter to be successful in exporting.

The effects of changes in the different types of trade costs are interesting. The effect of lower iceberg trade costs is as expected: the domestic cutoff value increases raising average popularity, whereas both exporting cutoff levels decrease. But the effects of lower sunk export costs and of lower fixed export costs are different on the two exporting cutoff values. Lower fixed export costs decrease the cutoff level for successful exporting and increase the cutoff level to start exporting, whereas lower sunk export costs raise the
cutoff level for successful exporting and decrease the cutoff level to start exporting. The implication is that lower sunk export costs decrease the probability of success in the export market, whereas lower fixed export costs raise the probability of successful entry into the export market.

This chapter contributes to the literature on models with both exporting uncertainty and firm heterogeneity. Crozet et al. (2007) model uncertainty about the political environment to explain that less productive firms who were ‘lucky’ not to face bribes can enter a market whereas certain more productive firms cannot. In the already mentioned work of Irarrazabal and Opromolla (2006) firms’ productivities are a stochastic process. Some exporting firms will have to leave the exporting market because their productivity experienced a negative shock. Finally, there is structural estimation empirical work proving the importance of sunk export costs where firms are heterogeneous and face shocks to various variables (see for example Roberts and Tybout (1997), Das et al. (2007)).

The chapter differentiates itself from other models of exporting uncertainty and firm heterogeneity by starting from the view that uncertainty is due to a lack of information on exporting profitability. There is ample empirical evidence that both sources of uncertainty, lack of information on, and shocks to, variables affecting profitability are important. Firm level data show that a considerable fraction of exit from the export market is shortly after entry, although estimates differ across studies. Also, not only firms that leave the export market immediately after entrance are support for the approach in this chapter. Firms can stay in the market for some years before they know whether their product is popular enough and whether they can be profitable (firms could incur losses in a foreign market for some years until they are convinced they cannot be profitably like the Wal-Mart example mentioned). Moreover, data on duration of detailed export categories show that a large fraction the disappearance of non-zero export flows follows quickly after entry.

This chapter is also related to the literature on the role of networks in trade. Rauch and Watson (2003) propose a model where buyers are involved in costly search for potential importers. After being matched with an importer they get to know the cost of the importer. They can either buy a small amount from the importer generating zero surplus or invest a lump-sum amount to get to know whether the importer can process a large order. If the answer is yes a larger surplus from trade can be generated. This model can also explain a large fraction of exiting firms from the export market soon after entry. The model in this chapter is similar, the difference is that in this chapter sellers take the initiative to try and sell a product profitably. Another difference is that in this chapter, exports will be larger to countries that are culturally closer having more similar preferences, whereas in Rauch and Watson (2003) exports to countries with closer business ties and denser networks will be larger.

Rauch (1999) dismisses a type of model similar to the one in this chapter where producers produce varieties that are most popular in the home market and their popularity in other markets declines with (cultural) distance. He puts forward two pieces of evidence. First, he shows that a dummy for colonial ties between countries raise trade whereas a dummy for common language has hardly a significant effect. In his reading
colonial ties proxy business ties whereas a common language proxies similar tastes. This implies that the empirical evidence supports the network view in his work and dismisses the tastes view in this chapter. But it is disputable whether business ties are exclusively proxied by colonial ties whereas similar tastes are exclusively proxied by common language. A common language also eases business ties and colonial ties are likely to lead to more similar tastes as well as to more intense business ties. Second, he refers to a study by Gould (1994) who finds that immigration into the USA raises bilateral trade with the country of origin of the immigrants. The effect is stronger on exports from the USA than on imports, however, favoring a network view over a shared tastes view. But this evidence does not prove that the shared tastes view is not important. Indeed Felbermayr and Toubal (2006) use empirical evidence to show that cultural proximity of countries raises trade both because this leads to lower trade costs (implying closer business ties emphasized by Rauch (1999)) and because it implies more similar tastes. Hence, both channels seem to be important.

The next section points out preferences, demand, revenues and profits when the CES-model contains taste parameters. Section 3.3 and section 3.4 solve the closed economy model and open economy model, respectively. Section 3.5 contains some concluding remarks.

### 3.2 Heterogeneous Popularity

Assume that all goods in the economy belong to the differentiated goods sector. Utility of a representative consumer is CES with CES weights or taste parameters that differ across goods:

\[
U = \left[ \int_{v \in V} \alpha_v^{\frac{1}{\sigma}} c_v^{\sigma-1} dv \right]^{\frac{\sigma}{\sigma-1}}
\]  (3.1)

\(\alpha_v\) is the taste parameter of variety \(v\) and differs across varieties. Raising it to the power \(1/\sigma\) is an innocuous assumption. It will turn out to be computationally convenient later on. The demand facing a firm is given by:

\[
x_v = Lc_v = \alpha_v p_v^{1-\sigma} P^{\sigma-1} L
\]  (3.2)

\(P\) is the price index defined by:

\[
P = \left[ \int_{v \in V} \alpha_v p_v^{1-\sigma} dv \right]^{\frac{1}{1-\sigma}}
\]  (3.3)

The price index can be expressed as a function of the average taste parameter \(\alpha\) and the mass of firms \(N\):

\[
P = \left[ \int_{\alpha}^\infty p^{1-\sigma} \alpha N \mu(\alpha) d\alpha \right]^{\frac{1}{1-\sigma}} = p N^{\frac{1}{1-\sigma}} \bar{\alpha}^{\frac{1}{1-\sigma}}
\]  (3.4)
\( \tilde{a} \) is the average taste parameter and defined as:

\[
\tilde{a} = \int_{\alpha^*}^{\infty} \alpha \mu(\alpha) \, d\alpha
\]  

(3.5)

\( \mu(\alpha) \) is the distribution of taste parameters of producing firms. Note that variables with \( ^{\sim} \) are averages conditional upon successful entry.

Production is increasing returns with a fixed cost of production. Wages are normalized at 1 and it is assumed that productivity is homogeneous.\(^2\) The cost function is given by:

\[
C(x_v) = ax_v + f
\]  

(3.6)

A firm can only start to produce when it has incurred sunk entry costs \( f_e \). Paying these sunk costs reveals a firm’s taste parameter \( \alpha_v \). This seems a more natural interpretation of the sunk entry costs than in Melitz (2003). Firms have to explore the desirability of their product in the market and this requires sunken investments.

The pricing equation of a firm is independent from the taste parameter:

\[
p_v = \frac{\sigma}{\sigma - 1} a
\]

Revenues and profits of a firm do depend on the taste parameter:

\[
r(\alpha_v) = p_v x_v = \alpha_v p_v^{1-\sigma} P^{\sigma-1} L = \alpha_v \left( \frac{\sigma}{\sigma - 1} a \right)^{1-\sigma} P^{\sigma-1} L
\]  

(3.7)

\[
\pi(\alpha_v) = \frac{r(\alpha_v)}{\sigma}
\]  

(3.8)

### 3.3 Closed Economy Model

Immediately after entry firms decide whether to stay in business or not. When they can make positive profit they stay, otherwise they exit. The taste parameter value at which they are just indifferent is the cutoff taste parameter \( \alpha^* \). There is an initial distribution of taste parameters \( G(\alpha) \) from which the truncated distribution of taste parameters of producing firms follows:

\[
\mu(\alpha) = \frac{1}{1 - G(\alpha^*)} g(\alpha)
\]  

(3.9)

With this expression the average taste parameter from equation (3.5) can be expressed as a function of the cutoff taste parameter:

\[
\tilde{\alpha}(\alpha^*) = \frac{1}{1 - G(\alpha^*)} \int_{\alpha^*}^{\infty} \alpha g(\alpha) \, d\alpha
\]  

(3.10)

\(^2\)This can be generalized, making marginal costs dependent on the CES-weight, like in chapter 5 or in Baldwin and Harrigan (2007). But such a generalization would not add anything to the analysis and would not change the main results. Therefore, for computational simplicity the marginal cost is kept equal across firms.
To solve the model one has to find the cutoff taste parameter. The cutoff taste parameter
determines the average taste parameter. With the average taste parameter one can
express all aggregate variables. To find the cutoff taste parameter a zero cutoff profit
condition (ZCP) and a free entry condition (FE) are needed, analogous to Melitz. The
ZCP is given by:
\[ \pi (\alpha^*) = 0 \]  
(3.11)
The ZCP can be rewritten as a function of average profit and the cutoff taste parameter:
\[
\begin{align*}
    r (\alpha^*) &= \sigma f \\
    \frac{r(\alpha^*)}{r(\alpha)} &= \frac{\alpha^*}{\alpha} \\
    r (\tilde{\alpha}) &= \sigma \left( \pi (\tilde{\alpha}) + f \right)
\end{align*}
\]  
(3.12)
Note that profit and revenue of the firm with average taste parameter are equal to average
profit and average revenue. The FE is given by:
\[ \tilde{\pi} = \frac{\delta \pi_e}{1 - G(\alpha^*)} \]  
(3.13)
The equilibrium cutoff taste parameter is found by combining the ZCP, the FE and the
definition for the average taste parameter, equations (3.12), (3.13) and (3.10) respec-
tively.

### 3.4 Open Economy Model

To model the open economy, uncertainty in the exporting decision is included. To this
end, the taste parameter abroad is assumed to be different from the taste parameter
domestically. So, success on the domestic market is no guarantee for success on the
foreign market. Empirically this is important as many firms leave the foreign market
shortly after they entered it.

A different productivity abroad from the productivity at home would be strange,
as the production process does not change. This is the main reason to introduce the
heterogeneity in the taste parameter. The foreign taste parameter is assumed to be
related to the domestic taste parameter in the following way:
\[ \alpha_v^F = \beta \alpha_v^H + \varepsilon_v \]  
(3.14)
\( \varepsilon_v \) is a random variable with mean 0 and variance \( \sigma_v^2 \). \( 0 \leq \beta \leq 1 \). When \( \sigma_v = 0 \) uncertainty disappears. The superscripts \( H \) and \( F \) indicate home and foreign variables
respectively. According to equation (3.14) popularity in the domestic market tells some-
thing about popularity on the foreign market, but not everything. The value of \( \beta \) can
be seen as a measure for ‘psychic distance,’ a concept from the marketing literature.
Psychic distance is an indicator of for example differences in language, consumer beh-
avior and cultural standards between markets (Stoettinger and Schlegelmilch (1998)).
Equivalently it can be seen as a measure for ‘cultural proximity,’ a similar concept as psychic distance applied in Felbermayr and Toubal (2006). These authors show that countries that are culturally more proximate trade more because trade costs are lower, but also because preferences are more similar.

The remaining assumptions on the open economy keep the economy as simple as possible. There are two countries, the distribution of taste parameters is equal in both countries and the size of the economy is equal to ensure equal wages. There are three types of export costs, iceberg trade costs $\tau$, fixed (per period) export costs $f_x$ and sunk export costs $f_{ex}$. The sunk export costs are paid when a firm starts exporting. But the firm is uncertain whether it will be successful on the export market. When there is no uncertainty in exporting all firms that start exporting continue to export, so it does not matter whether one expresses the sunk costs as real sunk costs or as the amortized per period equivalents. But when the exporting decision is uncertain, once firms have incurred the sunk costs they don’t take them into account anymore in deciding whether to continue exporting or not. So, the sunk costs cannot be expressed as the per period amortized equivalents. Instead, to account for exporting uncertainty, there are fixed export costs $f_x$ in the model as well. Due to the nature of demand (CES) all firms face at least some demand for their variety. And without fixed export costs, each firm that would start exporting would continue exporting.

The existence of sunk export costs is widely discussed in the literature (Baldwin (1988); Baldwin and Krugman (1989)). Exporting is very persistent implying the existence of sunk export costs. Roberts and Tybout (1997) present empirical evidence on the existence of sunk export costs in a sample Colombian industries. On the existence of fixed export costs there is less empirical work. Some other papers feature the presence of fixed exports costs in their trade model. In particular, Venables (1994), Jean (2002) and Medin (2003) assume fixed export costs in their models.

### 3.4.1 Solving The Open Economy Model

To solve for the equilibrium of the model, three cutoff taste parameters have to be determined, the domestic cutoff taste parameter $\alpha^*$, the cutoff taste parameter that defines the firm that just starts exporting $\bar{\alpha}$, and the exporting cutoff taste parameter, $\alpha^*_x$, that determines the firm that just continues exporting. These three variables will be related as in figure 3.1. There is a fraction of firms with a taste parameter below the domestic cutoff value, which immediately exit after they tried to enter the domestic market. There is a fraction of firms that produces only for the domestic market. There is a fraction of firms that enters the export market but leaves immediately, because its taste parameter is not large enough and finally there is a fraction of firms that produces both for the domestic and the exporting market. The zero profit condition for exporting to be defined in equation (3.23), makes it clear that one cannot determine a priori whether $\bar{\alpha}$ is larger or smaller than $\alpha^*_x$.

Solving the model requires finding the cutoff values. Given the cutoff values, one can determine the number of firms active in the different markets from the steady state equations and the labor market equilibrium equations. One starts from the domestic
and exporting ZCP:

\[ \tilde{\pi}_d = f \left[ \frac{\tilde{\alpha}}{\alpha^*_x} - 1 \right] \]
\[ \tilde{\pi}_x = f_x \left[ \frac{\tilde{\alpha}_x}{\alpha^*_x} - 1 \right] \]

\( \tilde{\pi}_x \) is average profit from exporting conditional upon successful entry in the exporting market. \( f_x \) are the fixed costs in exporting. \( \tilde{\alpha}_x (\alpha^*_x, \tilde{\alpha}) \) is the average exporting taste parameter, defined as:

\[ \tilde{\alpha}_x (\alpha^*_x, \tilde{\alpha}) = \frac{1}{1 - H (\alpha^*_x, \tilde{\alpha})} \int_{\alpha_x}^{\infty} h (\alpha_x, \tilde{\alpha}) d\alpha_x \]

\( h (\alpha_x, \tilde{\alpha}) \) is the distribution of taste parameters in the exporting market conditional upon entering the export market and can be found from the density functions of \( \alpha \), \( g (\alpha) \) and \( \varepsilon \), \( f (\varepsilon) \) as follows:

\[ h (\alpha_x, \tilde{\alpha}) = \frac{1}{1 - G (\tilde{\alpha})} \int_{\alpha_x}^{\infty} \frac{1}{\beta} g \left( \frac{x}{\beta} \right) f (\alpha_x - x) dx \]

Without fixed export costs, the cutoff taste parameter in exporting \( \alpha^*_x \) would be equal to 0. All firms that start exporting would also continue exporting (until they die according to the death probability \( \delta \)). So, without fixed exporting costs, there would be no exporting uncertainty.

The following equation gives the relation between the exporting cutoff taste parameter and the domestic one:

\[ \alpha^*_x = \frac{f_x}{f} \tau^{\sigma - 1} \alpha^* \]

To continue solving the model, the average profits in the two ZCP-conditions should be
3.4 OPEN ECONOMY MODEL

added to get an equation in average profit. The latter is the average profit conditional upon entry in the domestic market. This complicates things as the domestic taste parameter is correlated with the exporting taste parameter. Average profit conditional upon domestic entry is given by:

$$\bar{\pi} = \bar{\pi}_d + \Pr (\alpha \geq \bar{\alpha} | \alpha \geq \alpha^*) [\Pr (\alpha_x \geq \alpha_x^* | \alpha \geq \bar{\alpha}) \bar{\pi}_y - \delta f_{ex}]$$ (3.20)

So, (3.20) says that the average profit from exports conditional upon entry into the domestic market is equal to the probability a firm starts exporting, \(\Pr (\alpha \geq \bar{\alpha} | \alpha \geq \alpha^*)\), times the expected profits of an exporting firm, \(\Pr (\alpha_x \geq \alpha_x^* | \alpha \geq \alpha^*) \bar{\pi}_y - \delta f_{ex}\). The conditional probability in the latter term is equal to \(1 - H (\alpha_x^*, \bar{\alpha})\).

Combining (3.15), (3.16) and (3.20) one can express expected profit as a function of the domestic cutoff and the zero exporting profit taste parameters:

$$\bar{\pi} = f \left[ \frac{\bar{\alpha}}{\alpha^*} - 1 \right] + \frac{1 - G (\bar{\alpha})}{1 - G (\alpha^*)} \left[ (1 - H (\alpha_x^*, \bar{\alpha})) f_x \left[ \frac{\alpha_x}{\alpha_x^*} - 1 \right] - \delta f_{ex} \right]$$ (3.21)

The free entry equation is like in the closed economy, as defined in equation (3.13). To find the equilibrium values a zero profit equation from exporting has to be added, i.e. the firm with domestic taste parameter that makes zero profit from exporting inclusive of sunk export costs should be defined. This firm’s taste parameter \(\bar{\alpha}\) is defined by the following zero expected profit condition:

$$\Pr (\alpha_x \geq \alpha_x^* | \alpha = \bar{\alpha}) E (\pi_x | \alpha_x \geq \alpha_x^* \cap \alpha = \bar{\alpha}) = \delta f_{ex}$$ (3.22)

Equation (3.22) can be rewritten as follows:

$$\Pr (\beta \alpha + \varepsilon \geq \alpha_x^* | \alpha = \bar{\alpha}) E (\pi_x | \alpha_x \geq \alpha_x^* \cap \alpha = \bar{\alpha}) = \delta f_{ex}$$

$$\Pr (\varepsilon \geq \alpha_x^* - \beta \bar{\alpha}) \left[ E \left( \frac{f_x}{\sigma} | \alpha_x \geq \alpha_x^* \cap \alpha = \bar{\alpha} \right) - f_x \right] = \delta f_{ex}$$

$$\left( 1 - F (\alpha_x^* - \beta \bar{\alpha}) \right) f_x \left[ \frac{\alpha_x (\alpha_x^*, \bar{\alpha})}{\alpha_x^*} - 1 \right] = \delta f_{ex}$$ (3.23)

With \(\bar{\alpha}_x (\alpha_x^*, \bar{\alpha})\) the expected exporting taste parameter for a firm that just enters the export market, so with domestic taste parameter \(\bar{\alpha}\):

$$\bar{\alpha}_x (\alpha_x^*, \bar{\alpha}) = \frac{1}{\int_{\alpha_x^*}^{\infty} f (\alpha_x - \beta \bar{\alpha}) \ d\alpha_x} \int_{\alpha_x^*}^{\infty} (\varepsilon + \beta \bar{\alpha}) f (\varepsilon) d\alpha_x$$

$$= \beta \bar{\alpha} + \frac{\int_{\alpha_x^*}^{\infty} \varepsilon f (\varepsilon) d\alpha_x}{1 - F (\alpha_x^* - \beta \bar{\alpha})}$$ (3.24)

The three cutoff taste parameters \(\alpha^*, \alpha_x^*, \bar{\alpha}\) and average profit can be found by combining
 equations (3.13), (3.19), (3.21), and (3.23). Appendix 3.A shows that there is a unique equilibrium solution of the model.

The number of firms producing for the domestic market follows from the steady state equations and the labor market equilibrium equations (derivation in appendix 3.B):

\[
N = \frac{L}{\sigma (\bar{\alpha} + f + \Pr (\alpha \geq \bar{\alpha} \mid \alpha \geq \alpha^*) [\Pr (\alpha_x \geq \alpha^*_X \mid \alpha \geq \bar{\alpha}) f_x + \delta f_{ex}])}
\]  

(3.25)

Using equation (3.25) the number of firms can be calculated as all the cutoff parameters necessary to calculate the variables on the RHS of (3.25) are known once equation (3.13), (3.19), (3.21), and (3.23) are solved.

Welfare per worker is equal to the inverse of the price index. The price index is defined as:

\[
P = \left[ N \int_{\alpha^*}^{\infty} \alpha^{1-\sigma} \mu (\alpha) d\alpha + N_x \int_{\alpha^*_x}^{\infty} \left( \frac{a T}{\sigma - 1} \right)^{1-\sigma} h (\alpha) d\alpha \right]^{\frac{1}{1-\sigma}}
\]

And it can be rewritten as:

\[
P = N_t^{\frac{1}{1-\sigma}} \bar{\alpha}_t^{1-\sigma} a^{\frac{\sigma}{\sigma - 1}}
\]  

(3.26)

With average popularity \(\bar{\alpha}_t\) and the number of available varieties \(N_t\) in a country equal to:

\[
\bar{\alpha}_t = \frac{1}{N_t} \left[ N \bar{\alpha} + N_x \tau^{1-\sigma} \bar{\alpha}_x \right]
\]  

(3.27)

\[
N_t = N + N_x
\]  

(3.28)

For welfare evaluation, the price index can also be written as:

\[
P = \left( \frac{L}{\sigma f} \right)^{\frac{1}{1-\sigma}} a^{\frac{\sigma}{\sigma - 1}} (\alpha^*)^{1-\sigma}
\]  

(3.29)

### 3.4.2 Effects of Changed Trade Costs

There are three types of trade costs, the iceberg trade costs \(\tau\), sunk export costs \(f_{ex}\) and fixed export costs \(f_x\). This section discusses the effects of changes in each of these types of trade costs on the different cutoff popularity levels \(\alpha^*, \alpha^*_x, \bar{\alpha}\), on the probability of entering the export market conditional upon entering the domestic market, \(\Pr (\alpha \geq \bar{\alpha} \mid \alpha \geq \alpha^*)\), and on the probability of profitable exporting conditional upon entry in the export market (success rate in the export market), \(\Pr (\alpha_x \geq \alpha^*_X \mid \alpha \geq \bar{\alpha})\). Most results are derived analytically without specifying a distribution for \(\alpha\) and \(\varepsilon\). But some effects cannot be determined analytically and the results from a simulation will be reported. In the simulation a truncated normal distribution for domestic popularity \(\alpha\) (truncated at 0) and a normal distribution for \(\varepsilon\) are assumed. Simulation results are

\[3\text{Using } \bar{\alpha}_t = \frac{1}{N_t} \left[ N \bar{\alpha} + N_x \tau^{1-\sigma} \bar{\alpha}_x \right] = \frac{L/N}{\sigma f} \]
### Table 3.1: Baseline Simulation: Chosen Parameters and Results Endogenous Variables

<table>
<thead>
<tr>
<th>Baseline parameters</th>
<th>Endogenous variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>marginal costs</td>
<td>$a$ 0.5</td>
</tr>
<tr>
<td>fixed costs</td>
<td>$f$ 1</td>
</tr>
<tr>
<td>fixed export costs</td>
<td>$f_x$ 1</td>
</tr>
<tr>
<td>sunk entry costs</td>
<td>$f_e$ 10</td>
</tr>
<tr>
<td>sunk export costs</td>
<td>$f_{ex}$ 5</td>
</tr>
<tr>
<td>iceberg trade costs</td>
<td>$\tau$ 1.5</td>
</tr>
<tr>
<td>death probability</td>
<td>$\delta$ 0.05</td>
</tr>
<tr>
<td>substitution elasticity</td>
<td>$\sigma$ 2</td>
</tr>
<tr>
<td>Parameter indicating relation CES-weights</td>
<td>$\beta$ 0.05</td>
</tr>
<tr>
<td>standdev alpha</td>
<td>$\sigma_\alpha$ 2</td>
</tr>
<tr>
<td>standdev epsilon</td>
<td>$\sigma_\varepsilon$ 1</td>
</tr>
<tr>
<td>number of workers</td>
<td>$L$ 100</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Endogenous variables</strong></td>
</tr>
<tr>
<td></td>
<td>---------------------------------------</td>
</tr>
<tr>
<td></td>
<td>$\alpha^*$ 0.90</td>
</tr>
<tr>
<td></td>
<td>$\bar{\alpha}$ 1.88</td>
</tr>
<tr>
<td></td>
<td>$\alpha_x^*$ 1.35</td>
</tr>
<tr>
<td></td>
<td>$\Pr(Exp)$ 0.53</td>
</tr>
<tr>
<td></td>
<td>$\Pr(Succ)$ 0.54</td>
</tr>
<tr>
<td></td>
<td>$\bar{\alpha}_t$ 1.45</td>
</tr>
<tr>
<td></td>
<td>$N$ 23</td>
</tr>
<tr>
<td></td>
<td>$N_x$ 7</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Parameter indicating price index</strong></td>
</tr>
<tr>
<td></td>
<td>$P$ 0.023</td>
</tr>
</tbody>
</table>

Robust to different choices of parameter values. A Table 3.1 contains the parameter values used for the baseline simulations and the outcomes of the main endogenous variables.

First, the effect of lower iceberg trade costs $\tau$ is addressed. It can be shown analytically that lower iceberg trade costs lead to a higher domestic cutoff popularity level $\alpha^*$ and lower cutoff popularity levels for entrance in the export market $\bar{\alpha}$ and success in the export market $\alpha_x^*$. The probability of entrance into the export market rises $Pr(\alpha \geq \bar{\alpha} | \alpha \geq \alpha^*)$. Appendix 3C1 contains the proofs for these results. The probability of success in the export market, $Pr(\alpha_x \geq \alpha_x^* | \alpha \geq \bar{\alpha})$, cannot be determined analytically. Table 3.2 displays the effects of changed iceberg trade costs on the probability of exporting and the success rate of exporting calculated from simulations using the baseline values. The probability of success in the export market rises with lower trade costs. If a larger distance between trading partners would exclusively affect the iceberg trade costs, this model would imply that trade over a larger distance should have a lower success rate. However, it is questionable that distance only affects variable trade costs and not fixed and sunk export costs as well.

Next, the effect of lower sunk export costs $f_{ex}$ are evaluated. In appendix 3C2 it is proved that lower sunk export costs lead to a higher domestic cutoff level $\alpha^*$, a lower cutoff level to start exporting $\bar{\alpha}$, but a higher cutoff level for successful entry into the export market $\alpha_x^*$. The implication is that the probability to start exporting rises with lower sunk export costs, but the success rate of exporting declines. Lower sunk export costs raise expected profit. To restore the ex ante zero profit condition, the cutoff cost

---

4The robustness check consists of varying all the parameters that affect the equilibrium separately, i.e. varying $f$, $f_e$, $\delta$, $\beta$, $\sigma_\alpha$, $\sigma_\varepsilon$ and $f_x$, $f_{ex}$, $\tau$ one at a time. Results and the R-code of the simulation are available upon request.
level of both domestic and exporting production will have to increase. At the same time it becomes cheaper to start exporting. More firms start to export, but a lower fraction of them will be successful.

Finally, the effect of lower fixed export costs $f_x$ can be calculated. They lead to a higher domestic popularity cutoff level $\alpha^*$, a higher cutoff level to start exporting $\bar{\alpha}$ and a lower cutoff level for successful exporting $\alpha_x^*$. These effects imply that the success rate of exporting rises with lower fixed export costs, whereas the effect on the probability of exporting cannot be determined analytically. The simulation results reported in table 3.2 make clear that the probability of exporting increases with lower fixed export costs.

Applying equation (3.29), the welfare effects of lower trade costs are positive for all three types of trade costs, as for all types of trade costs the domestic cutoff popularity level $\alpha^*$ rises with lower costs. The effects of the different types of trade liberalizations are summarized in table 3.3. Signs with a * are determined in simulations. The effects of lower iceberg trade costs on the different variables are as expected. It becomes easier to export and ex ante expected profit rises leading to a reallocation effect that shifts up the cutoff popularity in the domestic market. Comparing the effects of lower fixed exports costs and sunk export costs provides interesting insights. Both lead to higher ex ante expected profit leading to more entry, more competition for scarce labor resources and therefore a higher cutoff popularity level. But the effects on the two exporting cutoff levels are opposite. Lower fixed exports costs decrease the cutoff level of successful exporting but raise the cutoff level of entrance into the export market, whereas lower sunk export costs lead to a higher cutoff level of successful exporting and a lower cutoff level of export entrance. The implication is that lower fixed export costs increase the probability of success in the export market, whereas lower sunk export costs decrease the probability of successful exporting. The reason behind these differences is that fixed export costs do affect operational profits from exporting, whereas sunk export costs have no impact on operational profits.
3.5 Concluding Remarks

This chapter modeled export uncertainty due to lack of information among firms about the popularity of their variety in the export market to explain the large amount of firms exiting the export market shortly after entry. In other work (by Baldwin (1988) and Irarrazabal and Opromolla (2006) for example) shocks to variables affecting profitability like productivity are used to account for exit from the export market. The explanation put forward in this chapter is supported by empirical work that shows that a large fraction of exit from the export market is shortly after entry. The network view on international trade as proposed by Rauch (1999) and Rauch and Watson (2003) provides an alternative explanation for quick exit of firms from the export market: buyers switch suppliers often and the old seller drops out of the export market.

Work in the present chapter can be extended in various directions. Firstly, an n-country version of the model could be used to estimate the parameters of the model with firm-level data on exporting. Secondly, the empirical work by Eaton et al. (2007) shows that most Colombian firms start with small amounts of exports. This suggests that firm cannot only learn about the popularity of their variety by incurring sunk export costs, but also by small sales. An extended model could allow for the possibility to approach a foreign market in two ways: through market experimentation along with small-scale sales and through market flooding along with incurring large market research costs, product development and marketing costs. A third possible extension is empirical and could try to find ways to differentiate empirically between the tastes cum lack of information view and the network view, which both provide an explanation for exporting uncertainty and quick exit from the export market.

A Unique Equilibrium Open Economy

The equilibrium in the model is found by combining equations (3.13), (3.19), (3.21), and (3.23) and solving for the three unknowns $\alpha^*$, $\alpha_x^*$, $\bar{\alpha}$. Merging equations (3.13) and (3.21) and rewriting, the model consists of the following 3 equations in 3 unknowns:
\[ \alpha_x^* = \frac{f_x}{f} \tau^{-1} \alpha^* \quad (A.1) \]

\[ f_j (\alpha^*) + (1 - G(\bar{\alpha})) [f_x j_x (\alpha_x^*, \bar{\alpha}) - \delta f_{ex}] = \delta f_e \quad (A.2) \]

\[ f_x i_x (\alpha_x^*, \bar{\alpha}) = \delta f_{ex} \quad (A.3) \]

With \( j(\alpha), j_x (\alpha_x, \bar{\alpha}) \) and \( i_x (\alpha_x, \bar{\alpha}) \) defined as:

\[ j(\alpha) = (1 - G(\alpha)) \left[ \frac{\bar{\alpha}(\alpha)}{\alpha} - 1 \right] \]

\[ j_x (\alpha_x, \bar{\alpha}) = (1 - H(\alpha_x, \bar{\alpha})) \left[ \frac{\bar{\alpha}_x (\alpha_x, \bar{\alpha})}{\alpha_x} - 1 \right] \]

\[ i_x (\alpha_x, \bar{\alpha}) = (1 - F(\alpha_x - \beta \bar{\alpha})) \left[ \frac{\bar{\alpha}(\alpha_x, \bar{\alpha})}{\alpha_x} - 1 \right] \]

Uniqueness of the equilibrium is shown as follows. Equations (A.1)-(A.3) are log differentiated with respect to the three unknowns \( \alpha^*, \bar{\alpha} \) and \( \alpha_x^* \). Variables with a hat will indicate relative changes. It will become clear that (A.1) and (A.2) are independent of the level of \( \bar{\alpha} \). The log differentiation of (A.1) is then substituted into (A.2) to show that the LHS of (A.2) rises monotonically in \( \alpha^* \). This implies a unique solution. The log differentiation of (A.3) makes clear that there is a unique corresponding value of \( \bar{\alpha} \).

Log differentiating equations (A.1) and (A.2) with respect to \( \alpha^*, \alpha_x^* \) and \( \bar{\alpha} \) gives:

\[ \hat{\alpha}_x^* = \hat{\alpha}^* \quad (A.4) \]

\[ f \frac{\partial j (\alpha^*)}{\partial \alpha^*} - g(\bar{\alpha}) [f_x j_x (\alpha_x^*, \bar{\alpha}) - \delta f_{ex}] \hat{\alpha} \hat{\alpha} + f_x \frac{\partial j_x (\alpha_x^*, \bar{\alpha})}{\partial \alpha_x^*} \hat{\alpha}_x^* \hat{\alpha}_x^* + \]

\[ + (1 - G(\bar{\alpha})) f_x \frac{\partial j_x (\alpha_x^*, \bar{\alpha})}{\partial \bar{\alpha}} \hat{\alpha} \hat{\bar{\alpha}} = 0 \quad (A.5) \]

It can be proved that \( \frac{\partial j(\alpha)}{\partial \alpha} = -\frac{1}{\alpha} [j(\alpha) + (1 - G(\alpha))] < 0 \) and that \( \frac{\partial j_x (\alpha_x, \bar{\alpha})}{\partial \alpha_x} = -\frac{1}{\alpha_x} [j_x (\alpha_x, \bar{\alpha}) + (1 - H(\alpha_x, \bar{\alpha}))] < 0 \). The derivation is similar to the derivation in appendix B of Melitz (2003) and available upon request. Furthermore, it can be proved that the two terms in \( \hat{\alpha} \) cancel each other out. The derivation is lengthy but straightforward and available upon request. Therefore, using equation (A.4), equation (A.5) can be rewritten as:

\[ -f [j(\alpha^*) + (1 - G(\alpha^*))] \hat{\alpha}_x^* - (1 - G(\bar{\alpha})) f_x [j_x (\alpha_x^*, \bar{\alpha}) + (1 - H(\alpha_x^*, \bar{\alpha}))) \hat{\alpha}_x^* = 0 \quad (A.6) \]

Equation (A.6) shows that the LHS of (A.2) is monotonically increasing in \( \alpha_x^* \). Furthermore it can be shown that \( \lim_{\alpha \to 0} j(\alpha) = \infty \). This implies a unique solution for \( \alpha^* \) and \( \alpha_x^* \).

Log differentiating equation (A.3) with respect to \( \alpha_x^* \) and \( \bar{\alpha} \) gives the following ex-
pression (derivation available upon request):\
\[
(1 - F(\alpha^* - \beta \hat{\alpha})) \beta \hat{\alpha} \frac{\beta \hat{\alpha}}{\alpha^*_x} - [i_x (\alpha^*_x, \hat{\alpha}) + (1 - F(\alpha^*_x - \beta \hat{\alpha}))) \hat{\alpha}^*_x = 0
\] (A.7)

Equation (A.7) shows that rises monotonically in implying a unique equilibrium value for \( N^*_x \) corresponding to the equilibrium value \( \hat{\alpha}^* \). Equation (A.7) can be simplified to generate an expression useful for the comparative statics later on:

\[
\hat{\alpha} = \frac{\hat{\alpha}_x}{\beta \hat{\alpha}} \hat{\alpha}^*_x = \left( 1 + \frac{\int \varepsilon f(x) d\alpha_x}{\frac{\alpha^*_x - \beta \hat{\alpha}}{1 - F(\alpha^*_x - \beta \hat{\alpha})} \hat{\alpha}^*_x} \right) \hat{\alpha}^*_x
\] (A.8)

## B Derivation Number of Firms

Labor can be allocated to four different tasks, domestic production, domestic entry, exporting production and exporting entry listed in this sequence in equation (B.1):

\[
L = L_p + L_e + L_{p,x} + L_{e,x}
\] (B.1)

The labor market equilibria for production in the domestic and exporting markets are defined as:

\[
L_p = N \overline{\pi}_d - N \overline{\pi}_d
\] (B.2)
\[
L_{p,x} = N_x \overline{\pi}_x - N_x \overline{\pi}_x
\] (B.3)

\( N \) and \( N_x \) are the number of firms producing for respectively the domestic and the exporting market. The labor market equilibria for market exploration to enter the domestic market and the exporting market are given respectively by:

\[
L_e = f_e N_e
\] (B.4)
\[
L_{e,x} = f_{ex} N_{e,x}
\] (B.5)

\( N_e \) and \( N_{e,x} \) are the number of firms trying to enter the domestic market and the export market, respectively. Entry and exit on both the domestic and the exporting market has to be equal in steady state. This implies:

\[
N_e \Pr (\alpha \geq \alpha^*) = \delta N
\] (B.6)
\[
N_{e,x} \Pr (\alpha_x \geq \alpha^*_x \mid \alpha \geq \hat{\alpha}) = \delta N_x
\] (B.7)
Adding the four labor market equilibrium conditions, i.e. substituting equations (B.2)-(B.7) into equation (B.1), gives:

\[
L = N\tilde{r}_d - N\tilde{\pi}_d + N\tilde{\pi} + N_x\tilde{\pi}_x - N_x\tilde{\pi}_x + \frac{f_{ex}\delta N_x}{\Pr(\alpha_x \geq \alpha_x^* | \alpha \geq \tilde{\alpha})} \\
= N\left(\tilde{r}_d - \tilde{\pi}_d + \tilde{\pi} + \frac{N_x}{N} \left(\tilde{\pi}_x - \tilde{\pi}_x + \frac{f_{ex}\delta}{\Pr(\alpha_x \geq \alpha_x^* | \alpha \geq \tilde{\alpha})}\right)\right) \\
= N\left(\tilde{r}_d - \tilde{\pi}_d + \tilde{\pi} + \frac{1 - G(\tilde{\alpha})}{1 - G(\alpha^*)} \left(1 - H(\alpha_x^*, \tilde{\alpha})\right) \left(\tilde{\pi}_x - \tilde{\pi}_x + \frac{f_{ex}\delta}{1 - H(\alpha_x^*, \tilde{\alpha})}\right)\right) \\
= N\left(\tilde{r}_d + \frac{1 - G(\tilde{\alpha})}{1 - G(\alpha^*)} \left(1 - H(\alpha_x^*, \tilde{\alpha})\right) \tilde{\pi}_x\right) = N\tilde{r} = N_t\tilde{r}(\tilde{\alpha}_t) \tag{B.8}
\]

\(\tilde{\alpha}_t\) and \(N_t\) are defined in equations (3.27) and (3.28). Average revenues in equation (B.8) can be rewritten to find the following expression for the number of domestic producing firms:

\[
N = \frac{L}{\sigma \left(\tilde{\pi} + f + \frac{1 - G(\tilde{\alpha})}{1 - G(\alpha^*)} \left(\delta f_{ex} + (1 - H(\alpha_x^*, \tilde{\alpha})) f_x\right)\right)} \tag{B.9}
\]

\textbf{C  Effect of Changed Trade Costs}

This appendix derives the effects of changes in the three types of trade costs \(\tau\), \(f_x\) and \(f_{ex}\) on the three cutoff values \(\alpha^*_x\), \(\alpha^{**}_x\), \(\tilde{\alpha}\) and on the probabilities of exporting and the success rate of exporting, \(\Pr(\alpha \geq \tilde{\alpha} | \alpha \geq \alpha^*_x)\) and \(\Pr(\alpha_x \geq \alpha^*_x | \alpha \geq \tilde{\alpha})\) respectively.

\textbf{C.1 Changed Iceberg Trade Costs}

Equations (A.1)-(3.22) can be log differentiated with respect to \(\tau\), \(\alpha^*_x\), \(\alpha^{**}_x\), \(\tilde{\alpha}\). Only (A.1) contains \(\tau\), so the log differentiated equations of (A.2) and (3.22) are given in equations (A.6) and (A.8) respectively. The log differentiation of (A.1) becomes:

\[
\hat{\alpha}^*_x = (\sigma - 1) \tilde{\tau} + \hat{\alpha}^* \tag{C.1}
\]

Combining equations (C.1), (A.6) and (A.8), one can solve for \(\hat{\alpha}^*, \tilde{\alpha}, \hat{\alpha}^{**}_x\) as a function \(\tilde{\tau}\):

\[
\hat{\alpha}^* = \frac{(\sigma - 1) f_x \left(1 - H(\alpha_x^*, \tilde{\alpha})\right) \frac{\tilde{\alpha}}{\alpha_x^*}}{f \left(1 - G(\alpha^*)\right) \frac{\alpha}{\alpha^*} + f_x \left(1 - H(\alpha_x^*, \tilde{\alpha})\right) \frac{\tilde{\alpha}}{\alpha_x^*}} \tilde{\tau} \\
\hat{\alpha}^{**}_x = \frac{(\sigma - 1) f \left(1 - G(\alpha^*)\right) \frac{\tilde{\alpha}}{\alpha^*}}{f \left(1 - G(\alpha^*)\right) \frac{\alpha}{\alpha^*} + f_x \left(1 - H(\alpha_x^*, \tilde{\alpha})\right) \frac{\tilde{\alpha}}{\alpha_x^*}} \tilde{\tau} \\
\tilde{\alpha} = \frac{\hat{\alpha}_x}{\beta \tilde{\pi} \left(1 - G(\alpha^*)\right) \frac{\tilde{\alpha}}{\alpha^*} + f_x \left(1 - H(\alpha_x^*, \tilde{\alpha})\right) \frac{\tilde{\alpha}}{\alpha_x^*}} \tilde{\tau}
\]
Hence, $\varepsilon_{\alpha^*, \tau} < 0$, $\varepsilon_{\alpha_x^*, \tau} > 0$ and $\varepsilon_{\tilde{\alpha}, \tau} > 0$. The probability of exporting and the success rate of exporting can be log differentiated with respect to the three cutoff values:

$$d \ln \Pr (\alpha \geq \tilde{\alpha} | \alpha \geq \alpha^*) = -g (\tilde{\alpha}) \tilde{\alpha} \tilde{\alpha} + g (\alpha^*) \alpha^* \tilde{\alpha}$$  \hspace{1cm} (C.2)

$$d \ln \Pr (\alpha_x \geq \alpha_x^* | \alpha \geq \tilde{\alpha}) =$$

$$- h (\alpha_x^*, \tilde{\alpha}) \alpha_x^* \tilde{\alpha} + \frac{g (\tilde{\alpha})}{1 - G (\tilde{\alpha})} ((1 - H (\alpha_x^*, \tilde{\alpha})) - (1 - F (\alpha_x^* - \beta \tilde{\alpha}))) \tilde{\alpha}$$  \hspace{1cm} (C.3)

Notice that in equation (C.3), $(1 - H (\alpha_x^*, \tilde{\alpha})) \geq (1 - F (\alpha_x^* - \beta \tilde{\alpha}))$, because the probability of successful exporting is larger for firms with a larger domestic taste parameter. Substituting the relative changes in cutoff parameters into equations (C.2) and (C.3) shows that lower iceberg trade costs leads to a larger probability of exporting. The sign of the effect on the success rate of exporting cannot be determined:

$$\frac{\partial \ln \Pr (\alpha \geq \tilde{\alpha} | \alpha \geq \alpha^*)}{\partial \ln \tau} = -g (\tilde{\alpha}) \tilde{\alpha} \varepsilon_{\tilde{\alpha}, \tau} + g (\alpha^*) \alpha^* \varepsilon_{\alpha^*, \tau} > 0$$

$$\frac{\partial \ln \Pr (\alpha_x \geq \alpha_x^* | \alpha \geq \tilde{\alpha})}{\partial \ln \tau} =$$

$$- h (\alpha_x^*, \tilde{\alpha}) \alpha_x^* \varepsilon_{\alpha_x^*, \tau} + \frac{g (\tilde{\alpha})}{1 - G (\tilde{\alpha})} ((1 - H (\alpha_x^*, \tilde{\alpha})) - (1 - F (\alpha_x^* - \beta \tilde{\alpha}))) \varepsilon_{\tilde{\alpha}, \tau}$$

### C.2 Changed Sunk Export Costs

Equations (A.1)-(A.3) can be log differentiated with respect to $\alpha^*$, $\alpha_x^*$, $\tilde{\alpha}$ and $f_{ex}$. The log differentiation of equation (A.1) is given by (A.4). The log differentiations of (A.2) and (A.3) are given respectively by:

$$- f (1 - G (\alpha^*)) \frac{\tilde{\alpha}}{\alpha^*} \frac{\tilde{\alpha} \alpha^*}{\alpha_x^*} - f_x (1 - G (\tilde{\alpha})) (1 - H (\alpha_x^*, \tilde{\alpha})) \frac{\tilde{\alpha} \alpha_x^*}{\alpha_x^*} - (1 - G (\tilde{\alpha})) \delta f_{ex} f_{ex} = 0$$  \hspace{1cm} (C.4)

$$f_x (1 - F (\alpha_x^* - \beta \tilde{\alpha})) \frac{\beta \tilde{\alpha} \alpha_x^*}{\alpha_x^*} - f_x (1 - F (\alpha_x^* - \beta \tilde{\alpha})) \frac{\tilde{\alpha} \alpha_x^*}{\alpha_x^*} = \delta f_{ex} f_{ex}$$  \hspace{1cm} (C.5)

Solving for $\alpha^*$, $\alpha_x^*$ and $\tilde{\alpha}$ from equations (A.4), (C.4) and (C.5) generates:

$$\tilde{\alpha} = \frac{(1 - G (\tilde{\alpha})) \delta f_{ex}}{f (1 - G (\alpha^*)) \frac{\tilde{\alpha}}{\alpha^*} + f_x (1 - G (\tilde{\alpha})) (1 - H (\alpha_x^*, \tilde{\alpha})) \frac{\alpha_x^*}{\alpha_x^*} f_{ex}}$$  \hspace{1cm} (C.6)
\[\tilde{\alpha} = \frac{f (1 - G (\alpha^*)) \tilde{\alpha} f_x (1 - G (\tilde{\alpha})) \left(1 - H (\alpha^*_{x}, \tilde{\alpha})\right) \tilde{\alpha}_x - (1 - F (\alpha^*_{x} - \beta \tilde{\alpha})) \frac{\tilde{\alpha}_x}{\alpha^*_{x}}}{f_x (1 - F (\alpha^*_{x} - \beta \tilde{\alpha}))} \left[\frac{\tilde{\alpha}^2}{\alpha^*_{x}} \right] \tilde{f}_x \]

(C.7)

Equations (C.6) and (C.7) show that \(-1 < \varepsilon_{\alpha^*_{x}, f_x} = \varepsilon_{\alpha^*_{x}, f_{ex}} < 0\) and \(\varepsilon_{\tilde{\alpha}, f_x} > 0\). Using equations (C.2) and (C.3) this implies that the probability of exporting rises with lower sunk export costs whereas the success rate of exporting declines:

\[
\frac{\partial \ln \text{Pr} (\alpha \geq \tilde{\alpha} | \alpha \geq \alpha^*)}{\partial \ln f_{ex}} = -g (\tilde{\alpha}) \tilde{\alpha} \varepsilon_{\tilde{\alpha}, f_{ex}} + g (\alpha^*) \alpha^* \varepsilon_{\alpha^*, f_{ex}} < 0
\]

\[
\frac{\partial \ln \text{Pr} (\alpha_x \geq \alpha^*_{x} | \alpha \geq \tilde{\alpha})}{\partial \ln f_{ex}} = - h (\alpha^*_{x}, \tilde{\alpha}) \alpha^*_{x} \varepsilon_{\alpha^*_{x}, f_{ex}} + \frac{g (\tilde{\alpha})}{1 - G (\tilde{\alpha})} \left((1 - H (\alpha^*_{x}, \tilde{\alpha})) - (1 - F (\alpha^*_{x} - \beta \tilde{\alpha}))\right) \varepsilon_{\tilde{\alpha}, f_{ex}} > 0
\]

### C.3 Changed Fixed Export Costs

Equations (A.1)-(A.3) can be log differentiated with respect to \(\alpha^*, \alpha^*_{x}, \tilde{\alpha}\) and \(f_x\). This generates the following three equations:

\[
\tilde{\alpha} = \tilde{f}_x + \tilde{\alpha}^*
\]

(C.8)

\[- f (1 - G (\alpha^*)) \frac{\tilde{\alpha}}{\alpha^*} \tilde{\alpha} - f_x (1 - G (\tilde{\alpha})) (1 - H (\alpha^*_{x}, \tilde{\alpha})) \frac{\tilde{\alpha}_x}{\alpha^*_{x}} \tilde{\alpha}_x
\]

\[+ f_x (1 - G (\tilde{\alpha})) j_x (\alpha^*_{x}, \tilde{\alpha}) \tilde{f}_x = 0 (C.9)\]

\[- (1 - F (\alpha^*_{x} - \beta \tilde{\alpha})) \frac{\tilde{\alpha}^2}{\alpha^*_{x}} \tilde{\alpha} - (1 - F (\alpha^*_{x} - \beta \tilde{\alpha})) \frac{\tilde{\alpha}_x}{\alpha^*_{x}} + \delta f_{ex} \tilde{f}_x = 0 (C.10)\]

Solving for \(\tilde{\alpha}^*\) and \(\tilde{\alpha}^*_{x}\) from equations (C.8) and (C.9) leads to:

\[
\tilde{\alpha}^* = - \frac{f_x (1 - G (\tilde{\alpha})) (1 - H (\alpha^*_{x}, \tilde{\alpha}))}{f (1 - G (\alpha^*)) \frac{\tilde{\alpha}}{\alpha^*} + f_x (1 - G (\tilde{\alpha})) (1 - H (\alpha^*_{x}, \tilde{\alpha}))} \tilde{f}_x
\]

(C.11)

\[
\tilde{\alpha}^*_{x} = \frac{f (1 - G (\alpha^*)) \frac{\tilde{\alpha}}{\alpha^*} + f_x (1 - G (\tilde{\alpha})) (1 - H (\alpha^*_{x}, \tilde{\alpha})) \left[\frac{\tilde{\alpha}_x}{\alpha^*_{x}} - 1\right]}{f (1 - G (\alpha^*)) \frac{\tilde{\alpha}}{\alpha^*} + f_x (1 - G (\tilde{\alpha})) (1 - H (\alpha^*_{x}, \tilde{\alpha}))} \tilde{f}_x
\]

(C.12)
And from (C.10) and (C.12), \( \widehat{\alpha} \) can be calculated:

\[
\widehat{\alpha} = -\frac{\alpha^*_x f (1 - G (\alpha^*)) \tilde{\alpha} + f_x (1 - G (\tilde{\alpha})) (1 - H (\alpha^*_x, \tilde{\alpha})) \frac{\tilde{\alpha} - \alpha^*_x}{\alpha^*_x}}{\beta \tilde{\alpha} f (1 - G (\alpha^*)) \frac{\tilde{\alpha}}{\alpha^*_x} + f_x (1 - G (\tilde{\alpha})) (1 - H (\alpha^*_x, \tilde{\alpha})) \frac{\tilde{\alpha} - \alpha^*_x}{\alpha^*_x}} f_x
\]

Hence, \(-1 < \varepsilon_{\alpha^*_x f_x} < 0, 0 < \varepsilon_{\alpha^*_x f_x} < 1 \) and \( \varepsilon_{\tilde{\alpha} f_x} < 0 \). This implies, using (C.2) and (C.3), that the probability of exporting cannot be determined without specifying a distribution of taste parameters, whereas the success rate of exporting rises with lower fixed export costs:

\[
\frac{\partial \ln \Pr (\alpha \geq \tilde{\alpha} | \alpha \geq \alpha^*)}{\partial \ln f_x} = -g (\tilde{\alpha}) \tilde{\alpha} \varepsilon_{\tilde{\alpha} f_x} + g (\alpha^*) \alpha^* \varepsilon_{\alpha^* f_x}
\]

\[
\frac{\partial \ln \Pr (\alpha_x \geq \alpha^*_x | \alpha \geq \tilde{\alpha})}{\partial \ln f_x} = -h (\alpha^*_x, \tilde{\alpha}) \alpha^*_x \varepsilon_{\alpha^*_x f_x} + \frac{g (\tilde{\alpha})}{1 - G (\tilde{\alpha})} ((1 - H (\alpha^*_x, \tilde{\alpha})) - (1 - F (\alpha^*_x - \beta \tilde{\alpha}))) \varepsilon_{\tilde{\alpha} f_x} < 0
\]
Chapter 4

Heterogeneous Productivity and Trade under Oligopoly

4.1 Introduction

Several different models of heterogeneous productivity have been proposed in the last years in the trade literature. Heterogeneous productivity leads to a beneficial reallocation effect of trade liberalization. Less efficient firms producing for the domestic market are replaced by exporting more efficient firms. For example, Melitz (2003) introduces heterogeneous productivity in a monopolistic competition framework with CES-preferences and Bernard et al. (2003) include heterogeneous productivity in a model with Bertrand competition. Empirical work has shown that reallocation effects of trade are important. Bernard and Jensen (2004a) show that almost half of the rise of manufacturing TFP in the USA between 1983 and 1992 is due to a reallocation effect of resources towards exporting more productive firms. Also episodes of liberalization in developing countries show the importance of composition effects (Tybout (2001)).

This chapter explores heterogeneous productivity in a model with Cournot competition. The goal of this exercise is to show that a relatively easy model can generate the reallocation effect that characterizes other heterogeneous productivity models of trade. The model is a two-country, multi-sector model. The effects of trade liberalization are studied in a model with free entry and a model without free entry. In both models, freer trade leads to lower prices. In the free entry model this raises welfare unambiguously. In the model without free entry welfare rises as well under certain conditions on the distribution of productivities. This is an extension of the Brander and Krugman (1983) result, who only show that welfare rises with trade liberalization in the short run when trade costs are negligible. Another interesting result of the extension to heterogeneous firms is that the decrease in market price as a result of trade liberalization can go along with a declining number of firms when enough inefficient firms are squeezed out of the market. Assuming unequal countries, the model nests the Ricardian comparative advantage model when heterogeneity in productivity disappears. The model also generates clear predictions on the probability of zero trade flows within a sector and export prices.

1Based on Bekkers and Francois (2008)
A larger distance between countries leads to a higher probability of zero trade flows and lower FOB export prices. A larger size of the importer country also raises the probability of zero trade flows and decreases the FOB export price. A last remarkable feature of the unequal country model is the presence of delocation effects: unilateral liberalization leads to higher market prices in the liberalizing country. All results are derived without specifying a specific distribution of costs. Preferences are CES.

The chapter is an extension of Brander and Krugman (1983) who examine reciprocal dumping in a Cournot model of international trade without heterogeneous productivity. They showed the possibility of cross-hauling and reciprocal dumping. The present chapter is related to other studies of heterogeneous productivity and international trade like Melitz (2003), Bernard et al. (2003), and Melitz and Ottaviano (2008). The value added of the present model compared to other firm heterogeneity models is threefold. First, the model is parsimonious: in a very basic model of heterogeneous productivity, the Cournot model, trade generates welfare gains through a reallocation effect. Moreover, no specific distribution of productivities is assumed like in Bernard et al. (2003) and Melitz and Ottaviano (2008) to generate the results. Second the model nests the Brander and Krugman reciprocal dumping model and the Ricardian model as special cases. Third, the model generates clear testable predictions on the probability of zero trade flows and exporting prices. Raff et al. (2007) also address firm heterogeneity in an oligopoly model. Their paper is focused on the interaction of trade and R&D. Section 2 lays out the basic model. Section 3 goes into trade without free entry and section 4 addresses trade with free entry. Section 5 exposes the unequal country model. Section 6 concludes.

## 4.2 Basic Model without Trade

This section lays out the basics of the model without trade. There are \( Q + 1 \) sectors in the economy, \( Q \) sectors producing \( q_j \) with Cournot competition and 1 sector producing \( z \) under conditions of perfect competition. In the first sections it is assumed that the Cournot sectors are symmetric. Later on this assumption is relaxed when asymmetries in technology, size, and policy are explored. Throughout it is assumed that there are sufficient sectors in the economy so that the effect of a price change on demand through the price index is negligible for firms (there is no numeraire problem). There are \( L \) equal agents each supplying 1 unit of labor. All profit income from the Cournot sectors goes to the economic agents. The utility function of each agent is CES. The optimization problem of the consumer generates the following market demand functions in the Cournot sectors \( q_j \) and in the perfect competition sector \( z \):

\[
q_j = \frac{I P_U^{\sigma - 1}}{p_j^\sigma} \tag{4.1}
\]

\[
z = I P_U^{\sigma - 1} \tag{4.2}
\]
4.2 BASIC MODEL WITHOUT TRADE

The price of good \( z \) is normalized at 1 and \( I \) is the endogenous income of all agents, the sum of labor and profit income. \( P_U \) is the consumer price index, corresponding to one unit of utility:

\[
P_U = \left[ \sum_{j=1}^{Q} p_j^{1-\sigma} + 1 \right]^{\frac{1}{1-\sigma}} = [Qp^{1-\sigma} + 1]^{\frac{1}{1-\sigma}} \tag{4.3}
\]

In the remainder one Cournot sector is studied as they are all symmetric. Therefore the sector index \( j \) is omitted. Labor is the only factor of production and there is a labor force of size \( L \). One unit of labor is needed to produce one unit of the perfect competition good \( y \). Therefore the wage is equal to 1. In the \( q \) sectors productivity is heterogeneous. One unit of labor can be transformed into \( 1/c_i \) units of \( q \) for the \( i \)-th firm which has marginal cost of production \( c_i \). There are no fixed costs of production. Therefore the cost function of firm \( i \) is given by

\[
C_i(q_i) = c_i q_i \tag{4.4}
\]

There is Cournot competition between the different firms in the \( q \) sectors. So, firms maximize profits towards quantity supplied, taking the quantity supplied by other firms as given. Profit of firm \( i \) is given by:

\[
\pi_i = pq_i - c_i q_i \tag{4.5}
\]

The first order condition is defined as:

\[
\frac{\partial \pi_i}{\partial q_i} = p \left[ 1 - \frac{1}{\sigma} \frac{q_i}{q} \right] - c_i = 0 \tag{4.6}
\]

With \( q = \sum_{i=1}^{n} q_i \), \( n \) is the number of firms in the market. Using the first order condition, the second order condition can be written as follows (derivation in appendix A):

\[
-\frac{1}{\sigma} \frac{p}{q} \left[ (\sigma + 1) c_i - (\sigma - 1) p \right] < 0 \tag{4.7}
\]

Using the definition for market share, \( \theta_i = \frac{q_i}{q} \), the first order condition can be rewritten as:

\[
p \left( 1 - \frac{\theta_i}{\sigma} \right) = c_i \tag{4.8}
\]

\[
\theta_i = \frac{\sigma p - c_i}{p} \tag{4.9}
\]

The marginal revenues on the LHS of equation (4.9) should be at least as large as the marginal costs on the RHS. The larger is market share \( \theta_i \), the lower is marginal revenue. So, for positive sales \( \theta_i \geq 0 \) which are implicitly imposed, a firm can satisfy the FOC by just reducing its market share as long as its marginal cost is smaller than the market price. There is a cutoff cost level \( c^* \) with which a firm would just stay in the market.
This cutoff cost level \( c^* \) is equal to the market price \( p \). The highest cost firm staying in the market has a cost level equal or just below the cutoff cost level and selling an amount just above zero.

The equilibrium price and quantities sold can be found for a given number of firms. Below a free entry condition is added to endogenise the number of firms. Suppose for now there are \( n \) firms. Combining the demand equation in (4.1) with \( n \) first order conditions in equation (4.6) and with the equation for the sum of market shares, one can find the following solutions for the market price \( p \), total sector sales \( q \) and sales of an individual firm \( q_i \):

\[
p = \frac{\sigma}{\sigma n - 1} \sum_{i=1}^{n} c_i = \frac{\sigma n}{\sigma n - 1} \bar{c} \tag{4.10}
\]

\[
q = IP^\sigma_U \left( \frac{\sigma n - 1}{\sigma n} \right)^\sigma \tag{4.11}
\]

\[
q_i = \sigma IP^\sigma_U \bar{c} - c_i \left( \frac{\sigma n - 1}{\sigma n} \right)^{\sigma-1} \tag{4.12}
\]

with \( \bar{c} \) the average cost of firms, \( \bar{c} = \frac{1}{n} \sum_{i=1}^{n} c_i \).

Using the fact that the price is equal to the cutoff cost level, the price equation (4.10) can be rewritten to solve for the number of firms as a function of the cutoff cost level and average cost:

\[
n = \frac{1}{\sigma} \frac{c^*}{c^* - \bar{c}} \tag{4.13}
\]

Equation (4.13) gives rise to the following observation.

**Observation 1** The cost structure and market structure of industries are related in the model. In more competitive industries with more firms, the cost heterogeneity is smaller.

Equation (4.13) shows that an increase in the number of firms implies that the firm with the highest cost needs to have a cost parameter ever closer to average cost. Therefore, the cost levels of firms become ever closer to each other with more firms in the market. Observation 1 illuminates that the market structure and the cost structure in this model are interrelated. In more competitive industries with more firms, the cost levels of firms should be closer to each other.\(^2\)

Next, free entry is added to the model. This will endogenise the number of firms \( n \). Free entry is introduced like in Melitz (2003). Firms have to pay a sunk entry cost \( f_e \)

\(^2\)Van Long and Soubeyran (1997) find similar results. They show that the variance of the cost distribution and the Herfindahlindex of industry concentration are positively related in a model with Cournot competition: a larger variance leads to more industry concentration. The results in the present paper are an extension to the results found by Van Long and Soubeyran (1997), because the number of firms is endogenous in the present paper modeling entry and exit.
to draw a cost parameter $c_i$ randomly from a certain distribution of costs $F(c_i)$. Hence, uncertainty about productivity is a barrier to entry for firms. They start to produce when they can make positive operating profits. When they cannot make positive profits, they take their loss and leave the market immediately. Producing firms leave the market with a certain fixed death probability $\delta$ in each period or when market conditions have changed such that they cannot make positive profits anymore. The sunk entry costs use labor. As free entry leads to zero expected profits, all profit income on average is used to pay labor in the entry sector. Therefore, total income in the economy is fixed and equal to the amount of labor (with wages normalized at 1).

The entry and exit process described leads to a zero cutoff profit condition (ZCP) and a free entry condition (FE). Together these two conditions can be added to the 'no free entry' equilibrium equations, equations (4.10)-(4.12). The number of firms $n$ can then be determined.

The ZCP can be derived from the fact that zero profit implies that price should be equal to marginal cost. The FOC in equation (4.6) shows that this firm will reduce market share to (just above) zero, to satisfy the first order condition and make non-negative profit. One finds as ZCP:

$$p = c^*$$  \hspace{1cm} (4.14)

The FE is given by equalizing the ex ante expected profits from entry with the sunk entry cost:

$$F(c^*) \sum_{j=0}^{\infty} (1-\delta)^j \pi = f_e$$  \hspace{1cm} (4.15)

$$\hat{\pi} = \frac{\delta f_e}{F(c^*)}$$  \hspace{1cm} (4.16)

$\hat{\pi}$ is the expected profit conditional upon entry. It can be written as:

$$\hat{\pi} = \int_0^{c^*} [p(c)q(c) - cq(c)] \mu(c) \, dc = q \int_0^{c^*} \theta(c)(p-c) \mu(c) \, dc$$  \hspace{1cm} (4.17)

$\mu(c)$ is the truncated pdf of all firms producing, $\mu(c) = \frac{1}{F(c^*)} f(c)$. Continuing the derivation, leads to the following FE:

$$\hat{\pi} = \frac{LP_u^{\sigma-1}}{(c^*)^\sigma} \int_0^{c^*} \sigma \left(c^* - 2c + \frac{c^2}{c^*} \right) \mu(c) \, dc = \frac{\delta f_e}{F(c^*)}$$  \hspace{1cm} (4.18)

$$\frac{LP_u^{\sigma-1}}{(c^*)^\sigma} \sigma \left(c^* - 2Ec + \frac{E(c)^2}{c^*} \right) = \frac{\delta f_e}{F(c^*)}$$  \hspace{1cm} (4.19)

The expectation appearing in equation (4.19) is a truncated expectation, i.e. $Ec = E(c|c \leq c^*)$. All expectations appearing in the remainder of the chapter are the ap-

---

3Remember that $q$ is the sum of sales of all firms in a Cournot sector.
propriate truncated expectations. Combining the FE and ZCP generates the following equation:

\[ \frac{LP^{\sigma-1}}{(p)^{\sigma-\sigma}} \left( p - 2Ec + \frac{E(c)^2}{p} \right) = \delta f_e F(p) \]  

(4.20)

The combined FE/ZCP equilibrium condition generates a stable price equilibrium. Rewriting equation (4.18) using average profit unconditional upon entry \( \bar{\pi} = F(p) \bar{\pi} \), gives:

\[ \bar{\pi} = \frac{LP^{\sigma-1}}{(p)^{\sigma-\sigma}} \int_0^p \left( p - 2c + \frac{c^2}{p} \right) f(c) dc = \delta f_e \]  

(4.21)

Appendix A shows that the LHS of equation (4.21) rises in the market price from 0 to \( \infty \) when the SOC is imposed, implying that there is a unique equilibrium.

The FE and ZCP can be used to solve for the cutoff cost level. An explicit solution requires the choice of an initial cost distribution. Once the cutoff cost level \( c^* \) is known the number of firms can be determined.

In steady state average cost is equal to expected cost, \( \bar{c} = E(c | c \leq c^*) \). So, the number of firms is equal to:

\[ n = \frac{p}{p - E(c | c \leq c^*)} = \frac{1}{1 - \frac{E(c \leq p)}{p}} \]  

(4.22)

Equation (4.22) can be log differentiated with respect to the market price \( p \) and the number of firms \( n \):

\[ \hat{\pi} = -\frac{p - E(c | c \leq p)}{p} \frac{1}{1 - \varepsilon_{Ec,p}} \hat{n} \]  

(4.23)

\( \varepsilon_{Ec,p} \) is the elasticity of average costs with respect to the market price. Equation (4.23) implies the following:

**Observation 2** When the average cost decreases less than proportionally in response to a lower market price, a decreasing market price goes along with a rising number of firms. Or equivalently, a lower market price goes along with more firms when the average markup declines as a result of a lower market price. When the average cost decreases more than proportionally with a lower market price, a decreasing market price goes together with less firms.

Observation 2 follows from equation (4.22). As the market price is equal to the cutoff cost level, the relation between the market price and the number of firms depends on the distribution of costs.\(^4\) Intuitively, a lower market price can either be caused by more firms in the market or by more efficient firms in the market. When average costs respond less

\(^4\)With a Pareto distribution, the truncated mean is linear in the truncation point. Therefore, the number of firms will be fixed. Simulations show that with a lognormal distribution sensible results can be generated with a reasonable number of firms. Moreover, with a lognormal distribution the number of firms declines in the market price.
than proportionally to the market price, a lower market price is caused by more firms in the market. When average costs respond more than proportionally to the market price, a lower market price is caused by more efficient firms in the market. In this situation a lower market price can go along with less firms, because the least efficient firms are squeezed out of the market. A related result can be derived on the effect of market size \( L \) on the number of firms.

**Observation 3** The number of firms rises in the size of the market \( L \) when a lower market price leads to a less than proportional decline of average costs

The combined FE/ZCP in equation (4.21) can be totally differentiated towards \( p \) and \( L \):

\[
\frac{dp}{dL} = -\frac{L}{p} \int_0^p \frac{(p-c)^2}{p} f(c) dc < 0
\]  

(4.24)

The denominator is positive by the SOC in equation (4.7). Equation (4.24) shows that a larger market leads to a lower market price. From observation 2 it is known that the number of firms rises as a result of a lower market price when the decline in average cost is less than proportional than the decline in the market price, which implies observation 3. Intuitively, a larger market can be served in two ways: through more firms or through an increase in the sales per firm. Depending on the distribution of costs one or the other dominates.\(^5\) The number of firms can increase with a larger market, but the number of firms can also decrease with a larger market, when enough of the least efficient firms are squeezed out of the market. This result contrasts with the monopolistic competition model of Melitz, where the number of firms is linear in market size and the increase in market size is served through a proportional increase in the number of firms. Hence, the present model is more flexible.

### 4.3 International Trade without Free Entry: The Short Run

There is international trade between two countries \( s, r = H, F \). There are iceberg trade costs \( \tau \) in the Cournot sectors. There are no fixed trade costs. The perfect competition sector does not display any trade costs, an assumption often made in international trade models. The equilibrium market price in the representative Cournot sector becomes:

\[
p_s = \frac{\sigma n_s}{\sigma n_s - 1} \bar{c}_s
\]  

(4.25)

with \( \bar{c}_s = \frac{1}{n_s} \left[ \sum_{i=1}^{n_{ds}} c_{ids} + \sum_{i=1}^{n_{xs}} \tau c_{ixs} \right] \) and \( n_s = n_{ds} + n_{xs} \)

\(^5\)With a Pareto distribution the number of firms is fixed, so the increase in sales as a result of the larger market is fully realized through more sales per firm. With a lognormal distribution, also the number of firms changes considerably.
This section addresses the effects of trade liberalization without imposing the free entry condition. So, the short-run effects of trade liberalization are examined. This can be seen as a free exit case, because firms will leave the market as soon as they make negative operating profits. Total differentiation can be used to make several observations on the effect of trade liberalization in the model. The first observation is on the impact of trade liberalization on the market price, i.e. the effect of a decline of $\tau$ on the market price $p_s$. In equation (4.25), there is a direct effect of trade liberalization on the market price: exporting firms have lower costs and therefore average costs decline. And there is an indirect effect, because firms producing for the domestic market can disappear and exporting firms can appear on the market. It can be shown that this indirect effect is 0 at the margin (see appendix B). Therefore, the relative change in the market price is equal to:

\[
\hat{P}_s = \frac{\sum_{i=1}^{n_{is}} \tau c_{is}}{\sum_{i=1}^{n_{is}} c_{is} + \sum_{i=1}^{n_{is}} \tau c_{is}} \hat{\tau} \tag{4.26}
\]

Variables with a hat denote relative changes, $\hat{x} = \frac{dx}{x}$. The elasticity of the market price with respect to trade costs, $\varepsilon_{p,\tau}$, is between 0 and 1:

\[
\varepsilon_{p,\tau} = \frac{\sum_{i=1}^{n_{is}} \tau c_{is}}{\sum_{i=1}^{n_{is}} c_{is} + \sum_{i=1}^{n_{is}} \tau c_{is}} \tag{4.27}
\]

So, one finds that with trade liberalization:

**Observation 4** *the market price decreases*

Equation (4.26) shows that a decline of trade costs $\tau$ decreases the market price. The domestic cutoff marginal cost is equal to the market price, so it also declines.

Several other observations can be made on the effect of trade liberalization.

**Observation 5** *Some of the least productive firms are squeezed out of the market*

How many firms are squeezed out of the market depends on the price distribution of the firms, i.e. it depends on how far the highest cost firms are from the old market price.

**Observation 6** *More firms can export*

More firms can enter the export market, as the exporting cutoff marginal cost rises when $\tau$ declines:

\[
c_{XR}^* = \frac{P_s}{\tau} \tag{4.28}
\]

\[
\hat{c}_{XR}^* = \hat{p}_s - \hat{\tau} = -(1 - \varepsilon_{p,\tau}) \hat{\tau} \tag{4.29}
\]
4.3 TRADE IN THE SHORT RUN

Observation 7 (Average) markups from domestic sales decline and (Average) markups from exporting sales rise

Markups of all domestic sales decline, as the costs of the firms remain equal, whereas the market price declines. Markups of the exporting firms rise with trade liberalization, as the effect of the declining trade costs dominates the effect of the decrease in market price in the exporting market. Using the letter $m$ to indicate markup, the following can be derived:

$$m_{ixs} = \frac{P_r}{\tau c_i K}$$  \hspace{1cm} (4.30)

$$\hat{m}_{ixs} = \hat{P}_r - \hat{\tau} = (\varepsilon_{p,\tau} - 1) \hat{\tau}$$  \hspace{1cm} (4.31)

The effect on average domestic and exporting markups can be calculated as well. The markups of firms are weighted by market shares in calculating average markups, so as to give more weight to larger firms$^6$:

$$\hat{m}_{ids} = \sum_{i=1}^{n_{ds}} \frac{p_s}{c_{is}} \theta_{ids} = \sum_{i=1}^{n_{ds}} \sigma \frac{p_s - c_{is}}{c_{is}}$$ \hspace{1cm} (4.32)

$$\hat{m}_{ixs} = \sum_{i=1}^{n_{xs}} \frac{p_r}{\tau c_{is}} \theta_{ixs} = \sum_{i=1}^{n_{xs}} \sigma \frac{p_r - \tau c_{is}}{\tau c_{is}}$$ \hspace{1cm} (4.33)

Relative changes of the average exporting and domestic markups are equal to:

$$\hat{\Delta} m_{ids} = \sum_{i=1}^{n_{ds}} \frac{p_s}{c_{is}} \theta_{ids} = \sum_{i=1}^{n_{ds}} \sigma \frac{p_s - c_{is}}{c_{is}}$$ \hspace{1cm} (4.34)

$$\hat{\Delta} m_{ixs} = -\sum_{i=1}^{n_{xs}} \frac{p_r}{\tau c_{is}} \theta_{ixs} = -\sum_{i=1}^{n_{xs}} \sigma \frac{p_r - \tau c_{is}}{\tau c_{is}}$$ \hspace{1cm} (4.35)

So, average markups from domestic sales decline and average markups from exporting sales rise.$^7$ Declining markups in the domestic market fit well with empirical findings reported in Tybout (2001) from developing countries. Various studies find that more import competition goes along with declining markups.

As in almost any model of international trade (for example Armington) firms increase their market share on the exporting market and their market share is reduced in domestic

---

$^6$Implicitly it is assumed that the probability of firms to be in the market is equal for all marginal costs, i.e. that the distribution of costs is uniform.

$^7$Indirect effects because domestic producing firms disappear from the market and exporting firms enter the market are 0, because the averages are weighted by market shares and market shares are zero for entering and exiting firms.
markets. But the relative gain and loss of exporters and domestic producers displays an interesting pattern:

**Observation 8** *Large low cost firms lose less market share on the domestic market than small high cost firms and small high cost exporting firms gain more market share on the export market than large low cost firms*

Observation 8 follows from totally differentiating the expressions for market shares:

\[
\frac{d\theta_{ids}}{\sigma c_{ids} p \varepsilon_{p,\tau}} = \frac{d\theta_{ixs}}{\sigma c_{ixs} p (\varepsilon_{p,\tau} - 1)}
\]

Therefore small firms lose relatively more market share on the domestic market and small firms gain relatively more market share on the exporting market than large firms. So, more efficient big firms do not gain more from trade liberalization than less efficient small firms. Apparently the model works such that big firms already have a strong position in an exporting market, so they cannot grow as much as a result of trade liberalization as small firms.

Next, the welfare effect of trade liberalization can be calculated. This is complicated by the fact that income is endogenous as it depends on profit income in the imperfect competition sector. With free entry profit income is driven to zero, but in the no free entry case profit income is non-zero and varies.

Welfare in country \(s\) is equal to utility in that country:

\[
W_s = U_s = \frac{I_s}{P_{Us}} = \frac{L_s + \Pi_s}{P_{Us}}
\]

\(\Pi_s\) is total profit income in the economy. Elaborating upon this equation (see appendix B) assuming that both countries are equal, one arrives at the following expression:

\[
W = \frac{L (Qp + p^\sigma)}{p^\sigma + Q\tilde{c}} \frac{1}{P_U}
\]

\(\tilde{c}\) are the market share weighted average costs, \(\tilde{c} = \sum_{i=1}^{n_d} c_i \theta_{id} + \sum_{i=1}^{n_x} \tau c_i \theta_{ix}\). Log-linearizing welfare towards trade costs \(\tau\) from equation (4.39) and treating the price and the market share weighted average costs as endogenous, one finds (derivation in appendix B):

\[
\dot{W} = \left[ \frac{\sigma p^\sigma}{Qp + p^\sigma} - \frac{\sigma p^\sigma}{p^\sigma + Q\tilde{c}} \right] \dot{p} - \frac{Q}{p^\sigma + Q\tilde{c}} \dot{d\tilde{c}}
\]

The first term in (4.40) is the welfare gain through a decline in price. As expected the gain for the consumer from lower prices outweighs the loss of a lower profit income with lower prices. The second term measures the possible gain from trade liberalization of lower costs leading to a higher profit income. Elaborating on the cost effect, \(d\tilde{c}\), one gets:
4.3 TRADE IN THE SHORT RUN

\[ \hat{W} = - \left[ \frac{\sigma \rho^\sigma}{Q + \rho^\rho} - \frac{\sigma \rho^\sigma}{p \rho^\rho + Q \hat{c}} \right] \hat{p} \]

\[ - \frac{Q}{p^\rho + Q \hat{c}} \left[ \sum_{i=1}^{n_d} c_i \theta_{id} \hat{\theta}_{id} + \sum_{i=1}^{n_x} \tau c_i \theta_{ix} \hat{\theta}_{ix} + \sum_{i=1}^{n_x} \tau c_i \theta_{ix} \right] \]

(4.41)

\[ \hat{W} = - \left[ \frac{\sigma \rho^\sigma}{Q + \rho^\rho} - \frac{\sigma \rho^\sigma}{p^\rho + Q \hat{c}} \right] \varepsilon_{p,\hat{\tau}} \hat{\tau} \]

\[ - \frac{Q}{p^\rho + Q \hat{c}} \left[ \sum_{i=1}^{n_d} \sigma c_i^2 \frac{\varepsilon_{p,\hat{\tau}}}{p} - \sum_{i=1}^{n_x} \sigma c_i^2 \frac{\tau^2 c_i^2}{p} (1 - \varepsilon_{p,\hat{\tau}}) + \sum_{i=1}^{n_x} \tau c_i \theta_{ix} \right] \hat{\tau} \]

(4.42)

Equation (4.41) and (4.42) can be interpreted as follows. In both equations is the first term on the RHS again the welfare gain from a lower market price. The second term on the RHS measures the effect on profit income through changed costs. In both (4.41) and (4.42) the first term between the second brackets measures the gain from the declining market share of domestic producing firms. The second term between the second brackets measures the loss from the rising market share of exporting firms. The third term measures the welfare gain from lower trade costs with trade liberalization.

**Observation 9** Like in Brander and Krugman (1983) the welfare effect of trade liberalization can be negative when the tariff is reduced from a prohibitive level due to the increased costs of cross-hauling. However, the welfare effect can also be positive when the tariff is reduced from a prohibitive level.

Unlike in the model of Brander and Krugman (1983) the welfare effect of trade liberalization when the tariff is reduced from a prohibitive level is ambiguous. It depends on the cost structure of firms whether the welfare effect is positive or negative. It can be shown under what condition the welfare effect is negative in general, but this condition is cumbersome and does not lend itself to any interpretation. Therefore, two examples are used to show that the welfare effect can go both ways. First an example of a negative welfare effect from trade liberalization. Suppose there are two identical countries with each three firms. They have marginal costs of 1, 1 and 2. The autarky market price will be 2. The iceberg trade costs are equal to 2. This implies that 2 firms can export, but with a market share of 0. Substitution elasticity \( \sigma \) is equal to 1. Equation (4.42) can be applied to show that a marginal reduction of the tariff decreases welfare with \( \frac{1}{2} \frac{Q}{1+Q} \).

An example where the welfare effect is positive is the following. Again there are two identical countries with each three firms. Marginal costs are 1, 2 and 3. The autarky market price is 3. Iceberg trade costs are 3. So, only one firm can export. Furthermore, the substitution elasticity \( \sigma \) is 1, so utility is Cobb-Douglas. There are two sectors in the economy and the Cournot sector has CES-weight (Cobb-Douglas parameter) \( \alpha \). When the tariff is reduced from the prohibitive level, the welfare effect from equation (4.42) is equal to \( (1 - \alpha) \frac{5}{6} - \frac{4}{5} \). So, when the Cournot sector is small enough \( \alpha < 1/5 \), the welfare effect of trade liberalization is positive.
Observation 10  The welfare effect of trade liberalization is unambiguously positive when the tariff is negligible, like in Brander and Krugman (1983)

Observation 10 follows immediately from equation (4.42). When the tariff is equal to 1, the first two terms between brackets in equation (4.42) are equal. So, only negative terms are left and therefore the welfare effect from trade liberalization is positive. Brander and Krugman (1983) only show that the welfare effect is positive when the tariff is negligible. In the present heterogeneous productivity model one can say more on when the welfare effect is positive. Elaborating upon equation (4.42), the following expression can be derived for the welfare effect of trade liberalization (see appendix B):

\[ \hat{W} = - \left[ \frac{\sigma_p^\sigma}{Qp + p^\sigma} - \frac{\sigma_p^\sigma}{p^\sigma + Q\tilde{c}} \right] \hat{\rho} \]

\[ - \frac{Q}{p^\sigma + Q\tilde{c}p^2 (\sigma n - 1)} \sum_{i=1}^{n_x} \tau c_i \left[ n\mu_c (\mu_c + p - 2\tau c_i) + (n - 1) \text{Var} (c_i) \right] \hat{\tau} \]  \hspace{1cm} (4.43)

In equation (4.43) \( \mu_c \) and \( \text{Var} (c_i) \) are respectively the mean and variance of the marginal costs of domestic and exporting firms,

\[ \mu_c = \frac{1}{n} \left[ \sum_{i=1}^{n_d} c_i + \sum_{i=1}^{n_x} \tau c_i \right] \]  \hspace{1cm} (4.44)

\[ \text{Var} (c_i) = \frac{1}{n-1} \left( \sum_{i=1}^{n_d} c_i^2 + \sum_{i=1}^{n_x} \tau^2 c_i^2 - n\mu_c^2 \right) \]  \hspace{1cm} (4.45)

Note that the summation in equation (4.43) is over all the terms between brackets. It can also be shown that the welfare effect is positive when the following condition is satisfied:

\[ \frac{\text{Var} (c_i)}{\mu_c^2} \geq \frac{n}{(\sigma n - 1)(n - 1)} \]  \hspace{1cm} (4.46)

From equation (4.43) and (4.46) the following statements can be made:

Observation 11  The welfare effect of trade liberalization is positive when the exporting firms are efficient enough. In particular, the welfare effect is unambiguously positive when all exporting firms have marginal costs inclusive of trade costs lower than the average of market price and average costs.

Observation 12  The welfare effect of trade liberalization is positive when the coefficient of variation is larger than the square root of \( \frac{n}{(\sigma n - 1)(n - 1)} \).

Observation 11 follows from equation (4.43). When \( \mu_c + p \) is larger than \( 2\tau c_i \) all terms in equation (4.43) will be negative and hence the welfare effect of trade liberalization will be positive. Intuitively, when the exporting firms are productive, their gain in
market share at the expense of domestic producing firms represents a welfare gain. More productive firms replace less productive firms. But when the exporting firms’ marginal costs inclusive of trade costs are larger than the marginal costs of the domestic producing firms, the shift in market share towards exporting firms can represent a loss. In some cases this loss can be larger than the welfare gain due to lower prices and lower trade costs, as shown by the example above.

Observation 12 follows from (4.46). It can be interpreted as follows. When the variance of trade costs is large relative to average trade costs, the fraction of relatively inefficient exporting firms will be small. So, the welfare loss from an increasing market share of relatively inefficient exporting firms will be smaller than the welfare gain from a decreasing market share of domestic producing inefficient firms. The next section shows that the welfare effect from trade liberalization is unambiguously positive with free entry.

4.4 International Trade with Free Entry: The Long Run

In the free entry case, the welfare effect of trade liberalization depends entirely on the effect of liberalization on the market price as profit income remains zero. Showing that trade liberalization leads to a lower market price is sufficient to show that liberalization raises welfare. In this section an expression for the equilibrium of the model from the ZCP and FE is derived. Some observations are made on the equilibrium outcome and then this equilibrium condition is log differentiated to show that trade liberalization leads to a lower market price and thus to higher welfare. Also some comparative statics results are derived.

Firms can make profits from domestic and exporting sales, if they are productive enough to export. Average profit is defined as:

$$\tilde{\pi}_s = \tilde{\pi}_{ds} + \frac{F(c^*_x)}{F(\overline{c}_d)} \tilde{\pi}_{xs}$$  \hspace{1cm} (4.47)

$\tilde{\pi}_{ds}$ and $\tilde{\pi}_{xs}$ are the expected profits from respectively domestic and exporting sales, conditional upon entry. There are two ZCP for domestic and exporting sales:

$$c^*_{ds} = p_s$$  \hspace{1cm} (4.48)

$$c^*_{xs} = \frac{p_r}{\tau}$$  \hspace{1cm} (4.49)

Elaborating on expected profits as in the closed economy model, generates the following equilibrium equations deriving from the ZCP and FE:

$$\frac{L_s P^*_{Us}}{P_s} \int_0^{p_s} \left( p_s - 2c + \frac{c^2}{P_s} \right) f(c) \, dc + \frac{L_r P^*_{Ur}}{p_r} \int_0^{p_r} \left( p_r - 2c + \frac{c^2}{p_r} \right) f(c) \, dc = \delta f_e$$  \hspace{1cm} (4.50)
\[
\frac{L_s P_{U_s}^{\sigma-1}}{p_s^\sigma} \left[ p_s - 2EC_{ds} + \frac{E(c_{ds})^2}{p_s} \right] + \\
F \left( \frac{p_r}{p_s} \right) \frac{L_r P_{U_r}^{\sigma-1}}{p_r^\sigma} \left[ p_r - 2\tau EC_{xs} + \tau^2 \frac{E(c_{xs})^2}{p_r} \right] = \frac{\delta f_e}{F(c_{ds})} \quad (4.51)
\]

To determine the impact of trade costs on the market price, one can totally differentiate the free entry condition in equation (4.50) towards the cutoff cost level (which equals the market price) and trade costs. Both the impact of sectoral trade liberalization and trade liberalization in all Cournot sectors can be addressed. The effect of sectoral trade liberalization on the market price is larger. Totally differentiating towards \( p \) and \( \tau \) one finds the following expressions for the effect on market price of sectoral and economywide liberalization respectively (derivation in appendix C):

\[
\hat{p} = \frac{2 \int_0^\frac{\varepsilon}{\tau} \tau c \left( 1 - \frac{\tau c}{p} \right) f(c) dc}{A + B} \quad \hat{\tau} = \varepsilon_{p,\tau,sect,FEE} \hat{\tau} \quad (4.52)
\]

\[
\hat{p} = \frac{2 \int_0^\frac{\varepsilon}{\tau} \tau c \left( 1 - \frac{\tau c}{p} \right) f(c) dc}{A + B + C} \quad \hat{\tau} = \varepsilon_{p,\tau,nat,FEE} \hat{\tau} \quad (4.53)
\]

With:

\[
A = \int_0^p \theta_d(c) ((\sigma + 1)c - (\sigma - 1)p) f(c) dc \\
B = \int_0^\frac{\varepsilon}{\tau} \theta_x(c) ((\sigma + 1)\tau c - (\sigma - 1)p) f(c) dc \\
C = \frac{Qp^{1-\sigma}}{Qp^{1-\sigma} + 1} (\bar{\pi}_d + \bar{\pi}_x)
\]

\( \varepsilon_{p,\tau,sect,FEE} \) and \( \varepsilon_{p,\tau,nat,FEE} \) are the elasticities of the market price with respect to trade costs in the free entry case with sectoral and nationwide trade liberalization respectively. By the SOC in equation (4.7) the denominator in both equations is positive and hence the fraction is positive as well. This gives rise to the following observation:

**Observation 13** Trade liberalization leads to a lower market price and higher welfare in the free entry model

Welfare rises when the market price of \( q \) declines as income is fixed with free entry.\(^8\) Hence, welfare rises in this model as a result of trade liberalization. By observation 2 a lower market price goes along with more or less firms in the market depending on how

\[
\begin{align*}
\hat{U} &= -\frac{Qp^{1-\sigma}}{1 + \tau Qp^{1-\sigma}} \hat{p} \\
\end{align*}
\]

\(^8 U = \frac{L}{P}\)

\[
\hat{U} = -\frac{Qp^{1-\sigma}}{1 + \tau Qp^{1-\sigma}} \hat{p}
\]
much average costs decline when the market price declines. This result can be combined with observation 13. The implication is that the lower market price as a result of trade liberalization can go along with more but also with less firms in the market, depending on how many of the least efficient firms are squeezed out of the market. Hence, the conventional insight of the reciprocal dumping model that trade liberalization leads to lower market prices, because there are more firms in the market has to be relaxed. Trade liberalization can also lead to less firms in the market and still decrease market prices, because enough of the least efficient firms are squeezed out of the market.

The various effects of trade liberalization described in the section on ‘no free entry’ can also be examined in the free entry model. The following effects of trade liberalization are found:

**Observation 14** *The least productive firms get squeezed out of the market*

Observation 14 follows from the fact that the cutoff cost level is equal to the market price. A lower market price implies that the highest cost producers have to leave the market. Next, the effect on market shares from domestic and exporting sales is calculated.

**Observation 15** *Market shares from domestic sales decline and market shares from exporting sales rise*

Log-differentiating the expressions for market shares, defined implicitly in equations (4.9) and using equation (4.52) gives:

\[
\begin{align*}
\hat{\theta}_{id} &= \frac{1}{\sigma p - c_i} \hat{p} = \frac{1}{\sigma p - c_i} \varepsilon_{p,\tau,FE} \hat{\tau} \\
\hat{\theta}_{ix} &= \frac{1}{\sigma p - \tau c_i} (\hat{p} - \hat{\tau}) = -\frac{1}{\sigma p - \tau c_i} (1 - \varepsilon_{p,\tau,FE}) \hat{\tau}
\end{align*}
\]  

(4.54)

(4.55)

The market share from domestic sales declines for all firms. Therefore, the market share from exporting sales should rise, either because more firms can export or because the market share of firms that already exported should rise. The market share of firms that enter the exporting market is zero. Therefore, the market share of firms already exporting should rise. Equation (4.55) implies that the elasticity of the market price with respect to iceberg trade costs in equations (4.52) and (4.53) is smaller than 1. This result is useful in the remainder.

**Observation 16** *The elasticity of the market price with respect to trade costs is between 0 and 1.*

Observation 16 can immediately be used in the following two observations.

**Observation 17** *More firms can export.*
CHAPTER 4 FIRM HETEROGENEITY AND OLIGOPOLY

The exporting cutoff cost level $c^*_x$ is equal to $\frac{p}{c}$. Log-differentiating shows that the exporting cutoff cost level declines with trade liberalization, implying that more firms can export:

$$\hat{c}_x = \hat{p} - \hat{\tau} = (\varepsilon_{p,\tau,FE} - 1) \hat{\tau} \quad (4.56)$$

**Observation 18** Markups from domestic sales decline and markups from exporting sales rise.

Markups from domestic sales and exporting sales are equal to $\frac{p}{c}$ and $\frac{p}{\tau c}$, respectively. Log differentiating shows that markups from domestic sales decline and markups from exporting sales rise with trade liberalization:

$$\hat{m}_d = \hat{p} = \varepsilon_{p,\tau,FE} \hat{\tau} \quad (4.57)$$

$$\hat{m}_x = \hat{p} - \hat{\tau} = (\varepsilon_{p,\tau,FE} - 1) \hat{\tau} \quad (4.58)$$

The effects on average markups can be calculated as well. Average markups from domestic and exporting sales weighted by market shares are defined, respectively, as:

$$\bar{m}_d = \int_0^p \frac{p}{c} \frac{p-c}{p} \mu(c) \, dc - \int_0^p \frac{p-c}{c} f(c) \frac{F(p)}{F(p)} \, dc \quad (4.59)$$

$$\bar{m}_x = \int_0^p \frac{p}{\tau c} \frac{p-\tau c}{p} \mu(c) \, dc = \int_0^p \frac{p-\tau c}{\tau c} f(c) \frac{F(p)}{F(\frac{p}{\tau})} \, dc \quad (4.60)$$

Log-differentiating these expressions shows that average markups from domestic sales decline and average markups from exporting sales rise:

$$\hat{\bar{m}}_d = \frac{\sigma}{\tau} \left[ \int_0^p \frac{p}{c} \mu(c) \, dc - \frac{f(p)}{F(p)} \int_0^p \frac{p-c}{p} \mu(c) \, dc \right] \varepsilon_{p,\tau,FE} \quad (4.61)$$

$$\hat{\bar{m}}_x = \frac{\sigma}{\tau} \left[ \int_0^p \frac{p}{\tau c} \mu(c) \, dc - \frac{f(p)}{F(p)} \int_0^p \frac{p-\tau c}{p} \mu(c) \, dc \right] (\varepsilon_{p,\tau,FE} - 1) \quad (4.62)$$

The terms in $\frac{f(p)}{F(p)}$ represent the increased probability weight of all firms in the market, when the cutoff point declines as a result of trade liberalization. Hence, the effect of trade liberalization on average domestic and exporting markups are ambiguous and depend on the cost distribution of productivities. When there is a lot of probability mass at the cutoff cost levels the probability weight terms could dominate. This would imply that average markups from domestic sales could actually rise and average markups from exporting sales decline. The intuition for the possibility of a lower markup from domestic sales is the following: trade liberalization could squeeze many low productive firms with
low markups out of the market. The remaining firms all face lower prices and a lower markup, but they get a larger probability weight, so on average the markup could rise.

For the effect on average firm size we find a similar result. Average firm size from domestic sales and exporting sales are defined, respectively, as:

\[
\bar{r}_d = \int_0^p pq_d(c) \mu(c) dc = \frac{P_U^{\sigma-1}L}{p^{\sigma-1}} \int_0^p \theta_d(c) \mu(c) dc
\]

\[
= \frac{P_U^{\sigma-1}L}{p^{\sigma-1}} \int_0^p \sigma \frac{p - c}{p} \mu(c) dc = \frac{P_U^{\sigma-1}L}{p^{\sigma-1}} \int_0^p \left( \frac{1}{p^{\sigma-1}} - \frac{c}{p^\sigma} \right) \mu(c) dc \quad (4.63)
\]

\[
\bar{r}_x = \int_0^x pq_x(c) \mu(c) dc = \frac{P_U^{\sigma-1}L}{p^{\sigma-1}} \int_0^x \theta_x(c) \mu(c) dc
\]

\[
= \frac{P_U^{\sigma-1}L}{p^{\sigma-1}} \int_0^x \sigma \frac{p - \tau c}{p} \mu(c) dc = \frac{P_U^{\sigma-1}L}{p^{\sigma-1}} \int_0^x \left( \frac{1}{p^{\sigma-1}} - \frac{\tau c}{p^\sigma} \right) \mu(c) dc \quad (4.64)
\]

Differentiating these expressions towards trade costs generates:

\[
\frac{\partial \bar{r}_d}{\partial \tau} = \tau \frac{P_U^{\sigma-1}L}{p^{\sigma-1}} \left[ \sigma \int_0^p (1 - \theta_d(c)) \mu(c) dc - \frac{f(p)}{F(p)} \int_0^p \sigma \frac{p - c}{p} \mu(c) dc \right] \epsilon_{p, \tau, FE} \quad (4.65)
\]

\[
\frac{\partial \bar{r}_x}{\partial \tau} = -\frac{P_U^{\sigma-1}L}{p^{\sigma-1}} \frac{1}{\tau} \left[ \int_0^x \left[ (1 - \theta_x(c)) (1 - \epsilon_{p, \tau}) + \frac{\sigma - 1}{\sigma} \theta_x(c) \right] \mu(c) dc \right]
\]

\[
+ \frac{P_U^{\sigma-1}L f(p)}{p^{\sigma-1}} \frac{p}{F(p)} \int_0^x \sigma \frac{p - \tau c}{p^\sigma} \mu(c) dc \left[ 1 - \epsilon_{p, \tau, FE} \right] \quad (4.66)
\]

So, like with average markups, the effect on average domestic and exporting firm sales is indeterminate and depends on the distribution of costs.

### 4.5 International Trade with Free Entry and Unequal Countries

In the free entry model all results were derived assuming equal countries. In this section the equal countries assumption is Relaxed. Three set of results are derived. First, it is shown that unilateral liberalization leads to higher prices in the liberalizing country in
the long run. Second, the impact of country size and distance on the probability of zero trade and on exporting unit values are derived. Third, it is shown the Ricardian model with productivity differences can be seen as a nested model of the present framework.

The setup in this section is as follows. There are two countries, \( s, r = H, F \). The countries display differences in country size, in trade costs and in productivity. Productivity differences are modeled by a different lower frontier for the marginal cost distribution, \( c_s \neq c_r \). The combined FE/ZCP in both countries become:

\[
\frac{L_s P_s^{\sigma-1}}{P_s^\sigma} \int_{c_s}^{p_s} \sigma \left( p_s - 2c + \frac{c^2}{p_s} \right) f_s(c) dc + \frac{L_r P_r^{\sigma-1}}{P_r^\sigma} \int_{c_r}^{p_r} \sigma \left( p_r - 2\tau_{sr} c + \frac{\tau_{sr}^2 c^2}{p_r} \right) f_s(c) dc = \delta f_e
\]

(4.67)

\[
\frac{L_r P_r^{\sigma-1}}{P_r^\sigma} \int_{c_r}^{p_r} \sigma \left( p_r - 2c + \frac{c^2}{p_r} \right) f_r(c) dc + \frac{L_s P_s^{\sigma-1}}{P_s^\sigma} \int_{c_s}^{p_s} \sigma \left( p_s - 2\tau_{sr} c + \frac{\tau_{sr}^2 c^2}{p_s} \right) f_r(c) dc = \delta f_e
\]

(4.68)

Unilateral liberalization can be studied using the above two equations. Assuming that the two countries are equal in all respects except their trade costs, one can log-linearize the above system of equations towards market prices \( p_s, p_r \) and trade costs \( \tau_{sr}, \tau_{rs} \). Appendix D shows that this leads to the following result:

\[
\hat{p}_r = \frac{2}{\sigma} \left( \frac{q_s}{q_r} dc_{ds} dc_{xtr} \hat{\tau}_{rs} - dc_{xtr} dc_{xrs} \hat{\tau}_{sr} \right)
\]

(4.69)

\[
\hat{p}_s = \frac{2}{\sigma} \left( \frac{q_s}{q_r} dc_{ds} dc_{xrs} \hat{\tau}_{sr} - dc_{xrs} dc_{xtr} \hat{\tau}_{rs} \right)
\]

(4.70)

dc_{ds}, dc_{dr}, dc_{xs}, dc_{xr}, dc_{xrs} and dc_{xtr} are respectively the marginal effects on expected profit from domestic and exporting price changes and from trade liberalization in country \( s \) and \( r \), defined as follows:

\[
dc_{ds} = \int_{c_s}^{p_s} \theta_{ds} ((\sigma + 1) c - (\sigma - 1) p_s) f(c) dc
\]

(4.71)

\[
dc_{xs} = \int_{c_r}^{p_r} \theta_{xs} ((\sigma + 1) c - (\sigma - 1) p_r) f(c) dc
\]

(4.72)

\[
dc_{xrs} = \int_{c_r}^{p_r} \theta_{xrs} (c) \tau_{sr} c f(c) dc
\]

(4.73)

The variables in \( r \) are defined accordingly. The marginal effects from domestic price changes on expected profit \( dc_{ds} \) and \( dc_{dr} \) are larger than the marginal effects from ex-
4.5 TRADE BETWEEN UNEQUAL COUNTRIES

porting prices on expected profit $dc_{xs}$ and $dc_{xr}$, because the domestic market shares $\theta_d$ are larger than the exporting market shares $\theta_x$ and the integration frontier is larger for the domestic cost variables than for the exporting cost variables.

Equation (4.26) shows that in the short run unilateral liberalization leads to a lower market price in the importing country. Equations (4.69) and (4.70) show that unilateral liberalization in country $s$, i.e. a negative $\tau_{rs}$, decreases the market price in the exporting country $s$ and increases the market price in the importing country $r$ in the long run. This gives rise to the following observation:

**Observation 19** Unilateral liberalization causes a decreasing market price in the liberalizing (importing) country in the short run. In the long run, however, the market price in the importing country increases and the market price in the exporting country decreases. Hence, the welfare effect of unilateral liberalization is negative in the importing country and positive in the exporting country.

The short-run effect of unilateral liberalization is as one would expect. The long-run effect is due to relocation effects like in Melitz and Ottaviano (2008) for example. Due to unilateral liberalization in country $s$, expected profit rises in country $r$. Therefore, there will be more entry in country $r$. At the same time, the decreasing market price in country $s$ reduces entry in that country. The effect of this entry and exit is that the market price in the exporting country $s$ declines and the market price in the importing country $r$ rises.\footnote{Mathematically, the reason for the declining market price in the exporting country $s$ and the rising market price in the importing country $r$ is the following: the marginal effect on expected profit of a changing domestic price, as represented by $c_{ds}$ and $c_{dr}$, is larger than the effect on expected profit of a changing price in the export market, represented by $c_{xr}$ and $c_{xs}$. Therefore, when the expected profit from exports in country $s$ rise due to unilateral liberalization in country $r$, the FE can be restored by decreasing prices in the export market $r$ and/or in the domestic market $s$. The prices in the two markets should go in opposite directions, however, because the FE in foreign should also be satisfied. Because the marginal effect of domestic price changes is larger, the domestic price (in $s$) has to decrease and the exporting price (in $r$) has to rise. With decreasing export prices (in $r$) and rising domestic prices (in $s$), the FE’s could never be satisfied.}

In empirical work there is considerable attention for the determinants of zero trade flows (Baldwin and Harrigan (2007)). Below the impact of distance and importer country size on the probability of zero trade and on export prices is derived. We concentrate on country $r$ as the importer country. First consider the effect of a change in distance. We take as proxy a change in trade costs. Equations (4.69) and (4.70) show the effect of lower trade costs on market prices. Equalizing the change in trade costs, i.e. $\hat{\tau}_{rs} = \hat{\tau}_{sr} = \hat{\tau}$, one finds:

$$\hat{p}_r = \frac{2}{\sigma} \frac{\partial c_{ds} c_{xtr}}{\partial \hat{\tau}} - \frac{\partial c_{xtr} dc_{xrs}}{\partial \hat{\tau}} = \varepsilon_{p_r, UC} \hat{\tau}$$

(4.74)

$$\hat{p}_s = \frac{2}{\sigma} \frac{\partial c_{ds} c_{xtr}}{\partial \hat{\tau}} - \frac{\partial c_{xtr} dc_{xrs}}{\partial \hat{\tau}} = \varepsilon_{p_s, UC} \hat{\tau}$$

(4.75)

Unless country sizes differ a lot leading to strong delocation effects, market prices decline with lower trade costs in the importing country $r$. Using the same reasoning as in the
equal country case, one can prove that the elasticity of price wrt trade costs, $\varepsilon_{pr,\tau,UC}$, has to be between 0 and 1. Market shares of domestic producers in country $r$ and exporters from country $s$ can be log differentiated to get:

$$\hat{\theta}_{idr} = \frac{1}{\sigma p_r - c_{ir}} \hat{p}_r = \frac{1}{\sigma p_r - c_{ir}} \varepsilon_{pr,\tau,UC} \hat{\tau}$$

$$\hat{\theta}_{ixs} = \frac{1}{\sigma p_r - \tau c_{is}} (\hat{p}_r - \hat{\tau}) = -\frac{1}{\sigma p_r - \tau c_{is}} (1 - \varepsilon_{pr,\tau,UC}) \hat{\tau}$$

When distance becomes smaller, the market price in country $r$, $p_r$, declines (if there are no strong delocation effects). As a result the domestic market shares in the importing country, $\theta_{idr}$, decline. Hence, the $\theta_{ixs}$ have to rise to get a total market share of 1 and therefore $0 < \varepsilon_{pr,\tau,UC} < 1$. This implies that $p_r/\tau$ will decline, as the denominator $\tau$ declines at a larger rate than the numerator $p_r$. $p_r/\tau$ is both the export price and the cutoff cost value for exports from country $s$ to country $r$. When the cutoff value of exports declines, the probability of zero trade rises. It becomes more likely that no firm is able to export profitably. Therefore, we have the following result:

**Observation 20** A lower distance between trading partners leads to a lower probability of zero trade flows and a lower fob export price.

Second, the effect of importing country size on the probability of zero trade flows and export prices are addressed. The combined FE/ZCP equations, (4.68) and (4.67), are log differentiated wrt $p_r, p_s$ and $L_r$, leading to:

$$\hat{p}_r = -\frac{d_{csx} \bar{\pi}_{dr} - d_{crx} \bar{\pi}_{sx}}{(d_{cdr} d_{cds} - d_{crx} d_{csx}) q_r} \hat{L}_r$$

$$\hat{p}_s = -\frac{d_{cdr} \bar{\pi}_{dr} - d_{csx} \bar{\pi}_{sx}}{(d_{cdr} d_{cds} - d_{crx} d_{csx}) q_s} \hat{L}_r$$

$d_{cdr}$, $d_{crx}$, $d_{cds}$ and $d_{csx}$ are respectively the marginal effects on expected profit from domestic and exporting price changes in country $r$ and country $s$, as defined in equations (4.71) and (4.72) for country $s$. As the effect of domestic price changes on expected profit are larger, because market shares in the domestic market are larger, the denominator in both equations (4.78) and (4.79) is positive. When productivity differences between the two countries are not too large, expected profits from domestic sales of producers in country $r$, $\bar{\pi}_{dr}$, are larger than expected profits from exporting sales of exporters from country $s$, $\bar{\pi}_{sx}$. This implies that the numerator is also positive. Hence, the market price in country $r$ decreases in its market size. The fob price of exporters from country $s$, $p_r/\tau$, also decreases. Therefore, we have the following result:

**Observation 21** A larger market size of the importing country leads to a higher probability of zero trade flows and lower fob export prices.

\footnote{Derivation available upon request. The derivation is similar to the log differentiation wrt $p_r, p_s$ and $\tau$ discussed in appendix D.}
Baldwin and Harrigan (2007) compare different models of international trade on their predictions of the effect of distance and importing country size on the probability of zero trade flows and fob prices. From table 1 in their paper it is clear that the Cournot model in this chapter generates the same predictions as the Melitz and Ottaviano (2008) model. The predictions are different from the model proposed by Baldwin and Harrigan (2007), which seems to align with the empirical findings presented in their paper. However, whereas the model of Baldwin and Harrigan (2007) contains product differentiation and quality differences, the oligopoly model in this chapter describes a setting with homogeneous products. Therefore, the predictions from this model should be tested with data from homogeneous goods sectors and not with a dataset of all sectors as Baldwin and Harrigan (2007) do. Intuitively, the different predictions can be clearly explained from the different modeling setups. Baldwin and Harrigan (2007) adopt the Melitz firm heterogeneity model to allow for quality differences. More productive firms charge higher instead of lower prices, because they sell higher quality products involving also higher marginal costs. The probability of zero trade flows rises with distance in our model and in Baldwin and Harrigan (2007). A larger distance makes it in both models more likely that trade costs are too high and that no firm is productive enough to sell profitably in the export market. The probability of zero trade flows rises in importing country size in our model and declines in importing country size in Baldwin and Harrigan (2007). The intuition in our model is that a larger market leads to tougher competition, more entry of firms and lower prices. Henceforth, it becomes harder to export to that market. The model of Baldwin and Harrigan (2007) features fixed export costs. In a larger market it is easier to earn these fixed costs back and therefore also the less productive firms with lower quality and lower price can sell in the market profitably.\footnote{A larger market also implies a lower price index and therefore less sales for an individual firm, making it more difficult to sell profitably in the export market. Apparently the direct effect of market size dominates. An effect of market size on profit margins is absent in the model of Baldwin and Harrigan (2007), because they work with CES and thus fixed markups.}

A larger distance leads to higher fob export prices in Baldwin and Harrigan (2007) and lower export prices in our model. In both models a larger distance makes it harder to export and therefore only more productive firms can export. In our model with homogeneous goods more productive firms charge lower prices, whereas in Baldwin and Harrigan (2007) they charge higher prices, because the quality of the good is larger. Finally, the export price declines in both models in the importing country size. The reason is different, however. In our model prices are lower in a larger market due to intenser competition and for given trade costs this leads to lower export prices as well. In Baldwin and Harrigan (2007) it is easier to earn back the fixed export costs in a larger market. Therefore, also lower quality, lower price exporters can sell profitably and the average export price will be lower. It could be an interesting exercise to see if the predictions of Baldwin and Harrigan (2007) on the probability of trade zeros and export zeros carry through in a sample of sectors with homogeneous goods or if our model of oligopoly predicts better.

The remainder of this section shows that Ricardian comparative advantage can be modeled as a nested case of the model in this chapter. Comparative advantage is intro-
duced in the model as follows. There are two types of sectors, country $s$ has a comparative advantage in the $A$ sectors and country $r$ has a comparative advantage in the $B$ sectors. Comparative advantage is modeled by the integration frontiers of the initial distribution of productivities. As only the lower integration frontiers $\zeta$ appears in the relevant ZCP and FE equations, attention can be restricted to these. The following assumptions are made to define comparative advantage:

$$c_{sA} < c_{rA}$$  \hspace{1cm} (4.80)  

$$c_{sB} > c_{rB}$$  \hspace{1cm} (4.81)  

$c_{sA}$ is the lower integration frontier in country $s$ in the $A$ sectors, i.e. in the sectors in which country $s$ has a comparative advantage.

To show that Ricardian comparative advantage is a nested case of the model, the distribution of productivities within a country is squeezed, i.e. the heterogeneity of firms is reduced. The productivity differences between countries remain. When the within country distribution of productivities collapses to a single point, the model converges either to a Ricardian model with perfect competition or a Brander and Krugman (1983) Cournot model with specialization, depending on whether the sunk entry costs disappear or not.

Before the distribution of productivities is narrowed, the following relations between the lower integration frontiers, market prices and trade costs apply:

$$c_{sA} < c_{rA} < p_{rA}/\tau < p_{rA}$$  \hspace{1cm} (4.82)  

$$c_{sA} < c_{rA} < p_{sA}/\tau < p_{sA}$$  \hspace{1cm} (4.83)  

The focus in the discussion is on sector $A$, because sector $B$ is just its mirror image with a comparative advantage for country $r$. Equation (4.82) ensures that at least some firms in country $s$ can export in their comparative advantage sector $A$ and that at least some firms in country $r$ can produce for the domestic market. Equation (4.83) guarantees that some firms in country $r$ can also export in their comparative disadvantage market $A$ and that firms in country $s$ can sell in their domestic market in their comparative advantage sector $A$. Hence, there is two-way trade in sector $A$.

Next, suppose that the distribution of productivities becomes more homogeneous. This can be seen as a narrowing of the distribution of productivities. The lower integration frontier moves up and the upper integration frontier moves down. However, only the lower integration frontier appears in the combined ZCP/FE condition, so mathematically a more homogeneous productivity distribution comes down to an increase in the lowest cost.

Uncertainty about productivity is a barrier to entry for firms. The sunk entry costs are dependent on uncertainty about the prospective productivity. Firms have to incur research costs to get rid of the uncertainty about their productivity. This interpretation of the sunk entry costs implies that a squeezing of the productivity distribution decreases the sunk entry costs. The combined ZCP/FEs in a $S$ sector are given in equations (4.67) and (4.68) (with symmetric trade costs). Log differentiating these expressions
towards market prices, the lower integration frontiers and the sunk entry costs shows what happens when the distributions of productivities become more homogeneous:

\[
q_s \int_{\xi_s} p_s \theta_{ds} \left( (\sigma + 1) c - (\sigma - 1) p_s \right) f_s d\hat{p}_s + q_r \int_{\xi_r} p_r \theta_{xs} \left( (\sigma + 1) \tau c - (\sigma - 1) p_r \right) f_s d\hat{p}_r \\
- \sigma q_s c_s \left( p_s - 2c_s + \frac{c_s^2}{p_s} \right) f(c_s) \hat{c}_s + q_r c_r \left( p_r - 2\tau c_r + \frac{\tau^2 c_r^2}{p_r} \right) f(c_r) \hat{c}_r = \delta f_e \hat{f}_e \tag{4.84}
\]

\[
q_r \int_{\xi_r} p_r \theta_{dr} \left( (\sigma + 1) c - (\sigma - 1) p_r \right) f_r d\hat{p}_r + q_s \int_{\xi_s} p_s \theta_{xs} \left( (\sigma + 1) \tau c - (\sigma - 1) p_s \right) f_r d\hat{p}_s \\
- \sigma q_r c_r \left( p_r - 2c_r + \frac{c_r^2}{p_r} \right) f(c_r) \hat{c}_r + q_s c_s \left( p_s - 2\tau c_s + \frac{\tau^2 c_s^2}{p_s} \right) f(c_s) \hat{c}_s = \delta f_e \hat{f}_e \tag{4.85}
\]

The effect of squeezing the distribution of productivities on market prices depends on the size of the change in the sunk entry cost \(f_e\). When this change is small, the market prices will have to rise to keep on satisfying the free entry condition.

Suppose that the distribution of productivities becomes concentrated in one point. Then two questions remain. First, does the model converge to a Ricardian comparative advantage model with perfect competition or a Brander and Krugman Cournot model? Second, will there be full specialization across countries? To address the first question, where the model converges to depends on what happens with sunk entry costs. When some sunk entry costs remain, because uncertainty about productivity is not the only source of the sunk costs, the model remains Cournot. The market price becomes higher than marginal costs to cover the sunk entry costs and the number of firms is limited. When uncertainty is the only source of sunk costs and so when there are no sunk costs left when the distribution of productivities collapses to a single point, the model converges to a perfect competition Ricardian model. Marginal cost will be equal to the market price and the number of firms becomes infinite as is clear from equation (4.13).

**Observation 22** When the distribution of productivities becomes concentrated in one point the model either converges to a Brander & Krugman Cournot model or a Ricardian perfect competition model depending on the presence of sunk (or fixed) costs. Two-way trade emerges either from cost heterogeneity or the presence of sunk (or fixed) entry costs.

It should be observed that there are no wage differences in the present model like in most Ricardian models. Modeling wage differences constitutes a possible extension of the present model. Whether there will be full specialization depends on the relation between market prices and marginal cost levels that emerges. There will be full specialization when:

\[
\zeta r_A < \frac{p_A}{\tau} < \zeta s_A \tag{4.86}
\]
The model converges either to a Cournot model or a Ricardian perfect competition model depending on the presence of sunk costs. There is no strict link between the appearance of full specialization and the type of market competition that emerges. There can be full specialization with Cournot competition when productivity differences are large enough. Also, the Ricardian model does not imply full specialization. A country could still produce for its own market in the Ricardian model in its comparative disadvantage sector when trade costs are large enough. But two way trade is only possible with Cournot competition. Moreover, full specialization is more likely in the Ricardian model without fixed costs, because market prices become equal to marginal costs (inclusive of trade costs) in that case.

**Observation 23** When the distribution of productivities collapses to a single point, full specialization is more likely with lower trade costs, a larger cost difference between countries and the absence of sunk costs.

### 4.6 Concluding Remarks

Introducing heterogeneous productivity in a trade model of Cournot competition leads to results familiar from other heterogeneous productivity models. Market prices decline, the least productive firms get squeezed out of the market and exporting firms gain market share when trade is liberalized. These results are found in models with and without free entry. Welfare rises in both variants of the model with trade liberalization, unless the trade barriers decline from a prohibitive level in the short run. The model with unequal countries shows that the Brander and Krugman (1983) reciprocal dumping model and the Ricardian comparative advantage model can be nested as special cases. Furthermore, delocation effects are present in the unequal country model: unilateral liberalization leads in the long run to higher prices in the liberalizing country, because firms relocate to the other country. Finally, it is shown that the probability of zero trade flows rises with distance and with the size of the importer country and that fob export prices decrease with distance and with the size of the importer country. Possible extensions of the model are the introduction of wage differences between the two countries and specifying a distribution of costs enabling simulations with the model with more countries and more sectors.

### A Basic Model

The appendices show how to derive equations from the main text.

**Equation 4.7: SOC**

Differentiating the FOC in equation 4.6 with respect to firm sales \( q_i \) leads to:

\[
\frac{\partial^2 \pi_i}{\partial q_i^2} = -\frac{2p}{\sigma q} + \frac{(\sigma + 1)p}{\sigma^2 q} \theta_i
\]  

(A.1)
Substituting the first order condition, \( \theta_i = \frac{p - c_i}{\bar{p}} \) into (A.1), generates equation (4.7) in the main text:

\[
\frac{\partial^2 \pi_i}{\partial q_i^2} = -\frac{1}{\sigma q} \left[ 2 - \frac{\sigma + 1}{\sigma} \theta_i \right] = -\frac{1}{\sigma q} \left[ 2 - \frac{\sigma + 1}{\sigma} \frac{p - c_i}{\bar{p}} \right] = -\frac{1}{\sigma q} \frac{2p - (\sigma + 1)p + (\sigma + 1)c_i}{p} = -\frac{1}{\sigma q} \frac{p(\sigma + 1)c_i - (\sigma - 1)p}{p}
\]

Combined FEZCP leads to stable equilibrium

Average profit unconditional upon entry in equation (4.21) can be differentiated with respect to the market price:

\[
\frac{\partial \bar{\pi}}{\partial p} = \sigma L P^{\rho - 1} \int_0^\rho \left( -\frac{(\sigma - 1)}{p^\rho} + \frac{2c}{p^{\rho + 1}} - (\sigma + 1) \frac{c^2}{p^{\rho + 2}} \right) f(c) dc
\]

\[
= \frac{\sigma L P^{\rho - 1}}{p^\rho} \int_0^\rho \left( -\frac{(\sigma - 1)}{p} + \frac{2c}{p} - (\sigma + 1) \frac{c^2}{p^2} \right) f(c) dc
\]

\[
= \frac{\sigma L P^{\rho - 1}}{p^\rho} \int_0^\rho \left( 1 - \frac{c}{p} \right) \left( \frac{(\sigma + 1)c - (\sigma - 1)p}{p} \right) f(c) dc > 0 \quad (A.2)
\]

The integrand in (A.2) is positive by the SOC in equation (4.7), hence average profit unconditional upon entry rises in the market price. This reflects two opposite forces: firstly, a decline in the market price leads to larger market sales in the entire industry and thus a larger profit conditional upon entry. Secondly, a decline in market price decreases the average profit margin (weighted by the market share \( \theta \) and by the probability \( \mu \)). This is due to a decline in the profit margin \( p - c \) and to the declining market share. The second effect dominates the first effect. Hence, the model generates a stable equilibrium market price.

**B Free Exit Model**

**Equation 4.26: Direct and indirect effect of trade liberalization in short-run free exit model**

The market price is defined in equation (4.25)

\[
p_s = \frac{\sigma}{\sigma (n_{ds} + n_{xr}) - 1} \left( \sum_{i=1}^{n_{ds}} c_{ids} + \sum_{i=1}^{n_{xr}} \tau c_{ixr} \right) \quad (B.1)
\]
Totally differentiating equation (B.1) with respect to $p$ and $\tau$, one finds:

$$dp_s = \sum_{i=1}^{n_xr} c_{ixr} d\tau + \sum_{i=1}^{n_ds} c_{ids} + \sum_{i=1}^{n_xr} \tau c_{ixr} + \sum_{j=1}^{\Delta \exp} \tau c_{jxr} - \sum_{j=1}^{\Delta \text{dom}} c_{jdr}$$

$$n_s - 1 + \exp - \Delta \text{dom}$$

$$= \frac{n_ds}{n_s - 1} + \frac{n_xr}{n_s - 1}$$

$\Delta \exp$ is the number of exporting firms that are entering the market because of the change in tariffs and $\Delta \text{dom}$ is the number of domestic producing firms that have to leave the market. These firms that are entering the export market and leaving the domestic market all have marginal costs (inclusive of trade costs for the exporters) equal to the market price. Therefore, equation (B.2) can be written as:

$$dp_s = \sum_{i=1}^{n_xr} c_{ixr} d\tau + \sum_{i=1}^{n_ds} c_{ids} + \sum_{i=1}^{n_xr} \tau c_{ixr} + (\Delta \exp - \Delta \text{dom}) p$$

$$n_s - 1 + \exp - \Delta \text{dom}$$

$$= \frac{(\sum_{i=1}^{n_ds} c_{ids} + \sum_{i=1}^{n_xr} \tau c_{ixr}) (n_s - 1 + \exp - \Delta \text{dom})}{n_s - 1} (n_s - 1)$$

So, the effect through a change in the number of firms is zero. The direct effect remains which is positive. Using relative changes, one arrives at equation (4.26) in the main text.

**Equation 4.39: Welfare in free exit model**

Welfare is defined in equation (4.38) of the main text as:

$$U_s = \frac{I_s}{P_U s} = \frac{L + \Pi_s}{P_U s}$$

**B.3**

Labor income is fixed. All Cournot-sectors are equal. Therefore total profit $\Pi$ is equal to:

$$\Pi_s = Q \pi_s = Q \left( p_s q_{ds} + p_M q_{xs} - \sum c_{is} q_{ids} - \sum \tau c_{is} q_{ixs} \right)$$

**B.4**

$\pi_s$ is profit income in one Cournot-sector. To proceed one needs to assume that the two
countries are equal. This implies that (B.4) can be rewritten as:

\[
\frac{\Pi}{Q} = p(q_d + q_x) - \sum_{i=1}^{n_d} c_i q_{id} - \sum_{i=1}^{n_x} c_i \tau q_{ix}
\]

\[
\frac{\Pi}{Q} = pq - q \sum_{i=1}^{n_d} c_i \theta_{id} - q \sum_{i=1}^{n_x} c_i \tau \theta_{ix}
\]

\[
\frac{\Pi}{Q} = \frac{IP_\sigma^{-1}}{p^\sigma - 1} \left( \frac{p - \sum_{i=1}^{n_d} c_i \theta_{id} + \sum_{i=1}^{n_x} c_i \tau \theta_{ix}}{p} \right)
\]  \hspace{1cm} (B.5)

\[
\frac{\Pi}{Q} = (L + \Pi) \frac{p_\sigma^{-1}}{p^\sigma} (p - \bar{c})
\]  \hspace{1cm} (B.6)

The term \( \sum_{i=1}^{n_d} c_i \theta_{id} + \sum_{i=1}^{n_x} c_i \tau \theta_{ix} \) in (B.5) represents the market share weighted average of costs, \( \bar{c} \). Therefore, \( p - \sum_{i=1}^{n_d} c_i \theta_{id} - \sum_{i=1}^{n_x} c_i \tau \theta_{ix} \) is defined as the market share weighted average profit per unit of sales, \( \bar{\pi} \). Solving for \( L + \Pi \) from (B.6) yields:

\[
\frac{\Pi}{Q} - \Pi \frac{p_\sigma^{-1}}{p^\sigma} (p - \bar{c}) = L \frac{p_\sigma^{-1}}{p^\sigma} (p - \bar{c})
\]

\[
\Pi \left( 1 - \frac{QP_\sigma^{-1}}{p^\sigma} (p - \bar{c}) \right) = QL \frac{p_\sigma^{-1}}{p^\sigma} (p - \bar{c})
\]

\[
\Pi = \frac{QL \frac{p_\sigma^{-1}}{p^\sigma} (p - \bar{c})}{1 - \frac{QP_\sigma^{-1}}{p^\sigma} (p - \bar{c})}
\]

\[
L + \Pi = \frac{L}{1 - \frac{QP_\sigma^{-1}}{p^\sigma} (p - \bar{c})}
\]  \hspace{1cm} (B.7)

Substituting equation (B.7) into equation (B.3), one finds the following expression for welfare, equation (4.39) in the main text:

\[
W = \frac{L}{1 - \frac{QP_\sigma^{-1}}{p^\sigma} (p - \bar{c})} \frac{1}{P_U} = \frac{L}{1 - \frac{Q(p - \bar{c})}{P_U (1 + \sigma)}} \frac{1}{P_U} = \frac{L}{1 - \frac{Q(p - \bar{c})}{P_U (1 + \sigma)}} \frac{1}{P_U}
\]

\[
W = \frac{L (Qp + p^\sigma)}{p^\sigma + Q \bar{c}} \frac{1}{P_U}
\]  \hspace{1cm} (B.8)

Equation 4.40: Relative Welfare Change in free exit model
Log-differentiating equation (B.8) with respect to trade costs \( \tau \), treating the market price \( p \), the price index \( P_U \) and average costs \( \bar{c} \) as endogenous generates equation (4.40) in the
Starting from equation (B.9), one can elaborate on the term \( \hat{d} \hat{c} \). Using equations (4.9), (4.36), (4.37), \( \hat{d} \hat{c} \) can be rewritten as follows:

\[
d\hat{c} = \sum_{i=1}^{n_d} \frac{c_i}{p - c_i} \frac{p - c_i}{p} \varepsilon_{p, \tau} \hat{\tau} + \sum_{i=1}^{n_x} \frac{\tau c_i}{p - c_i} \frac{p - \tau c_i}{p} \left( \varepsilon_{p, \tau} - 1 \right) \hat{\tau} + \sum_{i=1}^{n_x} \frac{\tau c_i}{p} \frac{p - \tau c_i}{p} \hat{\tau} \hat{\hat{d}} \hat{c} (B.10)
\]

Using equation (4.27) one can substitute for the price elasticity in equation (B.10) to get:

\[
d\hat{c} = \sum_{i=1}^{n_d} c_i^2 \left( \sum_{i=1}^{n_x} \frac{\tau c_i}{c_i} \hat{\tau} + \sum_{i=1}^{n_x} \frac{\tau^2 c_i^2}{p} \hat{\tau} + \sum_{i=1}^{n_x} \frac{\tau^2 c_i^2}{p} \hat{\hat{d}} \hat{c} \right) + \sum_{i=1}^{n_x} \frac{c_i}{p} \left( \sum_{i=1}^{n_x} \frac{\tau c_i}{c_i} \hat{\tau} + \sum_{i=1}^{n_x} \frac{\tau^2 c_i^2}{p} \hat{\tau} + \sum_{i=1}^{n_x} \frac{\tau^2 c_i^2}{p} \hat{\hat{d}} \hat{c} \right) - \sigma \left( 2 \sum_{i=1}^{n_x} \frac{\tau^2 c_i^2}{p} - \sum_{i=1}^{n_x} \frac{p \tau c_i}{p} \hat{\tau} \right) \hat{\hat{d}} \hat{c} (B.11)
\]

Next, \( p = \frac{\sigma}{\sigma n - 1} \left( \sum_{i=1}^{n_d} c_i + \sum_{i=1}^{n_x} \tau c_i \right) \), \( \mu_c = \frac{1}{n} \left( \sum_{i=1}^{n_d} c_i + \sum_{i=1}^{n_x} \tau c_i \right) \), also implying that, \( p = \)
\[ \frac{\sigma n}{\sigma n - 1} \mu_c \text{ can be used to rewrite (B.11) as:} \]

\[ d\hat{c} = \frac{\sigma n}{p^2 (\sigma n - 1)} \left[ \left( \sum_{i=1}^{n_d} c_i^2 + \sum_{i=1}^{n_x} \tau^2 c_i^2 \right) \sum_{i=1}^{n_x} \tau c_i \hat{\tau} - \left( 2 \sum_{i=1}^{n_x} \tau^2 c_i^2 - p \sum_{i=1}^{n_x} \tau c_i \right) n \mu_c \hat{\tau} \right] \tag{B.12} \]

The following expression on the variance of costs is used:

\[ \text{Var}(c_i) = \frac{1}{n - 1} \left( \sum_{i=1}^{n_d} c_i^2 + \sum_{i=1}^{n_x} \tau^2 c_i^2 - \frac{1}{n} \left( \sum_{i=1}^{n_d} c_i + \sum_{i=1}^{n_x} \tau c_i \right)^2 \right) \tag{B.13} \]

\[ = \frac{1}{n - 1} \left( \sum_{i=1}^{n_d} c_i^2 + \sum_{i=1}^{n_x} \tau^2 c_i^2 - n \mu_c^2 \right) \tag{B.14} \]

Substituting equation (B.14) into equation (B.12) leads to the following expression:

\[ d\hat{c} = \frac{\sigma^2 n}{p^2 (\sigma n - 1)} \left[ \left( (n - 1) \text{Var}(c_i) + n \mu_c^2 \right) \sum_{i=1}^{n_x} \tau c_i \hat{\tau} - \left( 2 \sum_{i=1}^{n_x} \tau^2 c_i^2 - p \sum_{i=1}^{n_x} \tau c_i \right) n \mu_c \hat{\tau} \right] \tag{B.15} \]

Bringing the summation of \( \sum_{i=1}^{n_x} \tau c_i \) outside the brackets in equation (B.15) gives the final expression for \( d\hat{c} \) in equation (4.43) in the main text:

\[ d\hat{c} = \frac{\sigma n}{p^2 (\sigma n - 1)} \sum_{i=1}^{n_x} \tau c_i \left[ n \mu_c (\mu_c + p - 2 \tau c_i) + (n - 1) \text{Var}(c_i) \right] \hat{\tau} \]

Inequality (4.46) can be derived as follows. The \( d\hat{c} \) part of the welfare change in equation (B.9) can be written as:

\[ d\hat{c} = \sum_{i=1}^{n_d} c_i \hat{\theta}_{i_d} \hat{\theta}_{i_d} + \sum_{i=1}^{n_x} \tau c_i \hat{\theta}_{i_x} \hat{\theta}_{i_x} + \sum_{i=1}^{n_x} \tau c_i \hat{\theta}_{i_x} \hat{\tau} \]

\[ d\hat{c} = \sum_{i=1}^{n_d} \frac{\sigma c_i^2}{p} \hat{\theta} + \sum_{i=1}^{n_x} \tau^2 c_i^2 \left( \hat{\theta} - \hat{\tau} \right) + \sum_{i=1}^{n_x} \sigma \tau c_i \left( p - \tau c_i \right) \hat{\tau} \]

\[ d\hat{c} = \frac{1}{p} \sum_{i=1}^{n_d} \sigma c_i^2 \hat{\theta} - \sum_{i=1}^{n_x} \sigma \tau^2 c_i^2 \hat{\theta} - \sum_{i=1}^{n_d} c_i \hat{\tau} - \sum_{i=1}^{n_x} \tau c_i \hat{\tau} + \sum_{i=1}^{n_x} \sigma \tau c_i \left( p - \tau c_i \right) \hat{\tau} \]
\[ d \hat{c} = \frac{1}{p} \frac{\sigma}{\sum_{i=1}^{n_d} c_i + \sum_{i=1}^{n_x} \tau c_i} \]

\[
\left( \sum_{i=1}^{n_d} c_i^2 \sum_{i=1}^{n_x} \tau c_i \hat{\tau} - \sum_{i=1}^{n_d} \tau^2 c_i^2 \sum_{i=1}^{n_x} c_i \hat{\tau} + \sum_{i=1}^{n_x} \tau c_i \left( p - \tau c_i \right) \left( \sum_{i=1}^{n_d} c_i + \sum_{i=1}^{n_x} \tau c_i \right) \hat{\tau} \right) \]

\[
d \hat{c} = \frac{\sigma}{p^2 \left( n - 1 \right)} \]

\[
\left( \sum_{i=1}^{n_d} c_i^2 \sum_{i=1}^{n_x} \tau c_i \hat{\tau} - \sum_{i=1}^{n_d} \tau^2 c_i^2 \sum_{i=1}^{n_x} c_i \hat{\tau} + \left( p \sum_{i=1}^{n_x} \tau c_i - \sum_{i=1}^{n_x} \tau^2 c_i^2 \right) \left( \sum_{i=1}^{n_d} c_i + \sum_{i=1}^{n_x} \tau c_i \right) \hat{\tau} \right) \]

The third term between brackets in equation (B.16), the gain through lower trade costs, should be positive as \( p \geq \tau c_i \forall i \). This generates the following condition:

\[
\sum_{i=1}^{n_x} \tau^2 c_i^2 \sum_{i=1}^{n_x} c_i \leq p \sum_{i=1}^{n_x} \tau c_i \left( \sum_{i=1}^{n_d} c_i + \sum_{i=1}^{n_x} \tau c_i \right) - \sum_{i=1}^{n_x} \tau^2 c_i^2 \sum_{i=1}^{n_x} \tau c_i \quad (B.17)
\]

Substituting the condition in (B.17) into the first two terms of \( d \hat{c} \) in equation (B.16) one can proceed as follows:

\[
d \hat{c} \geq \left[ \sum_{i=1}^{n_d} c_i^2 \sum_{i=1}^{n_x} \tau c_i - p \sum_{i=1}^{n_x} \tau c_i \left( \sum_{i=1}^{n_d} c_i + \sum_{i=1}^{n_x} \tau c_i \right) + \sum_{i=1}^{n_x} \tau^2 c_i^2 \sum_{i=1}^{n_x} \tau c_i \right] \hat{\tau}
\]

\[
d \hat{c} \geq \sum_{i=1}^{n_x} \tau c_i \left[ \left( \sum_{i=1}^{n_d} c_i^2 + \sum_{i=1}^{n_x} \tau^2 c_i^2 \right) - p \left( \sum_{i=1}^{n_d} c_i + \sum_{i=1}^{n_x} \tau c_i \right) \right] \hat{\tau}
\]

\[
d \hat{c} \geq \sum_{i=1}^{n_x} \tau c_i \left[ \left( n - 1 \right) Var \left( c_i \right) + n \mu_c^2 - pn \mu_c \right] \hat{\tau}
\]

\[
d \hat{c} \geq \sum_{i=1}^{n_x} \tau c_i \left[ \left( n - 1 \right) Var \left( c_i \right) + n \mu_c \left( \mu_c - p \right) \right] \hat{\tau}
\]

\[
d \hat{c} \geq \sum_{i=1}^{n_x} \tau c_i \left[ \left( n - 1 \right) Var \left( c_i \right) + n \mu_c \left( \mu_c - \frac{\sigma n}{\sigma n - 1} \mu_c \right) \right] \hat{\tau}
\]

\[
d \hat{c} \geq \sum_{i=1}^{n_x} \tau c_i \left[ \left( n - 1 \right) Var \left( c_i \right) + \frac{\sigma n}{\sigma n - 1} \mu_c^2 \right] \hat{\tau}
\]

(B.18)

So, from the inequality in (B.18) \( d \hat{c} \) is positive whenever \( \frac{Var(c_i)}{\mu_c^2} \geq \frac{\sigma n}{(n-1)(\sigma n - 1)} \), condition (4.46) in the main text.
C Free Entry Model

Equation 4.52: Effect of sectoral trade liberalization on market price in free entry model

Assuming equal countries, the combined FE/ZCP condition, equation (4.50) is given by:

$$\left(\frac{\sigma^\sigma - 1}{p^\sigma} \int_0^p \left( p - 2c + \frac{c^2}{p} \right) f(c) \, dc + \frac{\sigma^\sigma - 1}{p^\sigma} \int_0^p \left( p - 2c + \frac{c^2}{p} \right) f(c) \, dc \right) = \delta_f$$

 Totally differentiating equation (C.1) towards $p$ and $\tau$ and considering the effect through the market share as negligibly small, one can calculate the effect of sectoral liberalization as:

$$\sigma L^\sigma \int_0^p \left( -\frac{(\sigma - 1)}{p^\sigma} + \frac{2\sigma c}{p^{\sigma+1}} - \frac{(\sigma + 1)c^2}{p^{\sigma+2}} \right) f(c) \, dc$$

$$+ \sigma L^\sigma \int_0^\frac{p}{\tau} \left( -\frac{(\sigma - 1)}{p^\sigma} - \frac{2\tau c(\sigma + 1)}{p^\sigma} \frac{\tau^2 c^2}{p^{\sigma+2}} \right) f(c) \, dc$$

$$- \left[ L^\sigma \int_0^\frac{p}{\tau} \left( \frac{2c}{p^\sigma} - \frac{2\tau c^2}{p^{\sigma+1}} \right) f(c) \, dc \right] d\tau = 0$$

Equation (C.2) can be rewritten as:

$$\int_0^p \sigma \left( 1 - \frac{c}{p} \right) \left( \frac{(\sigma + 1)c - (\sigma - 1)p}{p} \right) f(c) \, dc$$

$$+ \int_0^{\frac{p}{\tau}} \sigma \left( 1 - \frac{\tau c}{p} \right) \left( \frac{(\sigma + 1)\tau c - (\sigma - 1)p}{p} \right) f(c) \, dc$$

$$- \left[ \int_0^{\frac{p}{\tau}} \left( \frac{2c - 2\tau c^2}{p} \right) f(c) \, dc \right] d\tau = 0$$

Using the definition of market shares and multiplying by $\frac{p}{\tau}$ generates equation (4.52) in the main text.

Equation 4.53: Effect of economywide trade liberalization on market price in the free entry model

The effect of economywide liberalization, also takes into account the effect through the price index $P_U$. Totally differentiating the combined ZCP/FE in equation (C.1) towards
\[ p \text{ and } \tau \text{ considering } P_U \text{ as endogenous leads to:} \]
\[
\left[ \sigma L P_u^{\sigma-1} \int_0^p \left( -\frac{(\sigma - 1)}{p^\sigma} + \frac{2\sigma c}{p^{\sigma+1}} - \frac{(\sigma + 1)c^2}{p^{\sigma+2}} \right) f(c) \, dc \right] dp \\
+ \left[ \sigma L P_u^{\sigma-1} \int_0^\pi \left( -\frac{(\sigma - 1)}{p^\sigma} - \frac{2\tau c}{p^\sigma} - \frac{(\sigma + 1)\tau^2 c^2}{p^{\sigma+2}} \right) f(c) \, dc \right] d\rho \\
+ \frac{(\sigma - 1)LP_u^{\sigma-2}}{p^\sigma} \frac{\partial P_U}{\partial P} \frac{LP_u^{\sigma-1}}{p^\sigma} \times \\
\left( \int_0^p \left( p - 2c + \frac{c^2}{p} \right) f(c) \, dc + \int_0^\rho \left( p - 2c + \frac{c^2}{p} \right) f(c) \, dc \right) dp \\
- \left[ LP_u^{\sigma-1} \int_0^{\frac{\pi}{2}} \left( \frac{2c}{p^\sigma} - \frac{2\tau c^2}{p^{\sigma+1}} \right) f(c) \, dc \right] d\tau = 0 \tag{C.4} \]

One can calculate \( \frac{\partial P_U}{\partial P} = P_U \frac{Q_p^{\sigma-1}}{Q_p^{\sigma-1} + 1} \). Furthermore, the unconditional profits from domestic and exporting sales can be defined as \( \bar{\pi}_U = F(p) \tilde{\pi}_U \) and \( \bar{\pi}_x = \frac{F(\xi)}{F(p)} \tilde{\pi}_x \). Rewriting the first term as in the sectoral liberalization derivation, one arrives at the following expression with \( A, B \) and \( C \) defined as in the main text:

\[ \hat{p} = \frac{2 \int_0^{\frac{\pi}{2}} \tau c \left( 1 - \frac{zc}{p} \right) f(c) \, dc}{A + B + C} \tag{C.5} \]

**Equations 4.65 and 4.66:** Derivatives of average revenues with respect to trade costs

Average domestic and exporting revenues are defined in the main text in equations (4.63) and (4.64). Differentiating these expressions towards trade costs leads to:

\[
\frac{\partial \bar{\pi}_d}{\partial \tau} = \left[ P_U^{\sigma-1} L \sigma \int_0^p \left( -\frac{\sigma - 1}{p^\sigma} + \frac{\sigma c}{p^{\sigma+1}} \right) \mu(c) \, dc - \frac{P_U^{\sigma-1} L F(p)}{} \int_0^p \frac{p - c}{p} \mu(c) \, dc \right] \frac{\partial P}{\partial \tau} \\
+ \tau \frac{P_U^{\sigma-1} L}{p^{\sigma-1}} \left[ \sigma \int_0^p (1 - \theta_d(c)) \mu(c) \, dc - \frac{f(p) p}{F(p)} \int_0^p \frac{p - c}{p} \mu(c) \, dc \right] \varepsilon_{\rho, \tau} \tag{C.6} \]
\[
\frac{\partial \bar{r}_x}{\partial \tau} = P_{U_s}^{\sigma-1} L \int_0^{\frac{p_s}{\gamma}} \left( -\frac{(\sigma - 1)}{p^\sigma} + \frac{\tau p}{p^{\sigma+1}} \right) \mu(c) \, dc - \frac{f \left( \frac{p}{\gamma} \right)}{F \left( \frac{p}{\gamma} \right)} \int_0^{\frac{p}{\gamma}} \frac{p - \tau c}{p^\sigma} \mu(c) \, dc \]

\[
- P_{U_r}^{\sigma-1} L \int_0^{\frac{p_r}{\gamma}} \left( \frac{c}{p^\sigma F \left( \frac{p}{\gamma} \right)} \, dc + \frac{f \left( \frac{p}{\gamma} \right)}{F \left( \frac{p}{\gamma} \right)} \int_0^{\frac{p}{\gamma}} \frac{p - \tau c}{p^\sigma} \mu(c) \, dc \right)
\]

\[
= P_{U_s}^{\sigma-1} L \frac{1}{p^{\sigma-1}} \int_0^{\frac{p}{\gamma}} \left( 1 - \frac{\sigma}{\gamma} \frac{\tau c}{p} \right) \mu(c) \, \frac{\partial \tau}{\partial p} \, dc + \int_0^{\frac{p}{\gamma}} \frac{\tau c}{p} \mu(c) \, dc
\]

\[
- P_{U_r}^{\sigma-1} L \frac{f \left( \frac{p}{\gamma} \right)}{F \left( \frac{p}{\gamma} \right)} \int_0^{\frac{p}{\gamma}} \frac{p - \tau c}{p^\sigma} \mu(c) \, dc [\varepsilon_{p,\tau} - 1]
\]

\[
= - P_{U_s}^{\sigma-1} L \frac{1}{p^{\sigma-1}} \int_0^{\frac{p}{\gamma}} \left[ (1 - \theta_x(c)) \left( 1 - \varepsilon_{p,\tau} \right) + \frac{\sigma - 1}{\tau} \theta_x(c) \right] \mu(c) \, dc
\]

\[
- P_{U_r}^{\sigma-1} L \frac{f \left( \frac{p}{\gamma} \right)}{F \left( \frac{p}{\gamma} \right)} \int_0^{\frac{p}{\gamma}} \frac{p - \tau c}{p^\sigma} \mu(c) \, dc [\varepsilon_{p,\tau} - 1]
\]

(D.7)

D Unequal Countries Model

Equations 4.69 and 4.70: Effects of unilateral trade liberalization on market prices

The combined FE/ZCP’s in country s and r are given by:

\[
\frac{L_s P_{U_s}^{\sigma-1}}{p_s^{\sigma}} \int_{\varepsilon_s}^{p_s} \left( p_s - 2c + \frac{c^2}{p_s} \right) f_s(c) \, dc + \frac{L_r P_{U_r}^{\sigma-1}}{p_r^{\sigma}} \int_{\varepsilon_r}^{p_r} \left( p_r - 2\tau_{sr}c + \frac{\tau_{sr}^2 c^2}{p_r} \right) f_r(c) \, dc = \delta f_e
\]

(D.1)

\[
\frac{L_r P_{U_r}^{\sigma-1}}{p_r^{\sigma}} \int_{\varepsilon_r}^{p_r} \left( p_r - 2c + \frac{c^2}{p_r} \right) f_r(c) \, dc + \frac{L_s P_{U_s}^{\sigma-1}}{p_s^{\sigma}} \int_{\varepsilon_s}^{p_s} \left( p_s - 2\tau_{rs}c + \frac{\tau_{rs}^2 c^2}{p_s} \right) f_s(c) \, dc = \delta f_e
\]

(D.2)
Totally differentiating equation (D.1) towards $p_s$, $p_r$ and $\tau_{sr}$ gives:

$$\begin{align*}
\frac{\sigma L_s P_s^{\sigma-1}}{p_s^\sigma} & \int_0^{p_s} \left( 1 - \frac{c}{p_s} \right) \left( \frac{(\sigma + 1) c - (\sigma - 1) p_s}{p_s} \right) f(c) \, dc \, dp_s \\
+ \frac{\sigma L_r P_r^{\sigma-1}}{p_r^\sigma} & \int_0^{p_r} \left( 1 - \frac{\tau_{sr} c}{p_r} \right) \left( \frac{(\sigma + 1) \tau_{sr} c - (\sigma - 1) p_r}{p_r} \right) f(c) \, dc \, dp_r \\
- \frac{L_r P_r^{\sigma-1}}{p_r^\sigma} & 2 \int_0^{p_r} \left( c - \frac{2 \tau_{sr} c^2}{p_r} \right) f(c) \, dc \, d\tau_{sr} = 0 \quad (\text{D.3})
\end{align*}$$

Rewriting equation (D.3) in terms of relative changes and adding the equivalent expression for the other country, one arrives at:

$$\begin{align*}
q_s & \int_0^{p_s} \theta_{ds} (c) \left( (\sigma + 1) c - (\sigma - 1) p_s \right) f(c) \hat{p}_s + \\
q_r & \int_0^{p_r} \theta_{xs} (c) \left( (\sigma + 1) \tau_{sr} c - (\sigma - 1) p_r \right) f(c) \, dc \hat{p}_r - \frac{2q_r}{\sigma} \int_0^{p_r} \tau_{sr} \theta_{xs} (c) f(c) \, dc \hat{\tau}_{sr} = 0 \\
(D.4)
\end{align*}$$

$$\begin{align*}
q_r & \int_0^{p_r} \theta_{dr} (c) \left( (\sigma + 1) c - (\sigma - 1) p_r \right) f(c) \hat{p}_r + \\
q_s & \int_0^{p_s} \theta_{xr} (c) \left( (\sigma + 1) \tau_{rs} c - (\sigma - 1) p_s \right) f(c) \, dc \hat{p}_r - \frac{2q_s}{\sigma} \int_0^{p_s} \tau_{rs} \theta_{xr} (c) f(c) \, dc \hat{\tau}_{rs} = 0 \\
(D.5)
\end{align*}$$

Defining the marginal effects $dc_{ds}$, $dc_{dr}$, $dc_{xs}$, $dc_{xt}$, $dc_{xrs}$ and $dc_{xtr}$ as in the main text, equations (D.4) and (D.5) can be solved for $\hat{p}_s$, $\hat{p}_r$ as a function of $\hat{\tau}_{sr}$ and $\hat{\tau}_{rs}$, leading to equations (4.69) and (4.70) in the main text.
Chapter 5

Firm Heterogeneity and Endogenous Quality

5.1 Introduction

Empirical work by Schott (2004) shows that unit values of export goods within detailed product categories are related to exporting country characteristics. In particular, goods originating from richer countries display higher unit values. This empirical result poses a problem for the standard heterogeneous productivity monopolistic competition model, as introduced by Melitz (2003). With income likely to be strongly related to productivity, goods from higher productivity countries should have higher prices. In Melitz (2003) productivity is defined as the inverse of marginal costs. With a fixed markup, more productive firms charge lower instead of higher prices. The present chapter modifies the heterogeneous productivity monopolistic competition model in such a way that it can account for the empirical regularities found by Schott (2004) on income of the exporter country and unit values. Firms have a different productivity to produce quality. More productive firms produce higher quality goods charging higher prices.

The model is in the spirit of Melitz’s heterogeneous productivity model. Utility is CES with a CES quality parameter specific for each variety. So, the CES parameter measures the quality of a good. Firms can enter a market by drawing a productivity to produce quality parameter. This parameter can be seen as the natural appeal of a variety. The quality of a good can be increased by more investments in product development, product branding and marketing. The productivity to produce quality parameter determines the effectiveness of these investments. A larger quality also requires larger marginal costs. The model abstracts from between country differences: there are two identical countries trading with each other. Between country differences are introduced in the next chapter.

The model produces outcomes that can account for the empirical findings of Schott (2004). More productive firms have a larger quality and charge higher prices. Furthermore, only more productive firms and thus firms with a higher quality can export. Exporting firms are on average bigger. Trade creates a reallocation effect, more productive firms gain market share at the expense of less productive ones. As a result, average
quality of goods rises in the economy. Also trade liberalization reduces the average productivity and quality of exports, because also the less productive firms can export.

A possible critique to the presented model is that the definition of productivity is unnatural. There are two replies. First, empirically productivity is measured as value added divided by the value of inputs. So, also in the standard monopolistic competition model theoretical productivity is not well related to measured productivity. Second, there is a natural interpretation of the productivity to produce quality applied in this chapter. It is a measure for the inherent appeal of a product variety, so it can be seen as the productivity of a variety.

Another point of critique could be that the model features regular fixed costs besides fixed costs in product development, branding and marketing. These regular fixed costs are needed to ensure that only a part of the entering firms can produce profitably. The regular fixed costs can be motivated referring to standard fixed costs like those of overhead. Also in exporting regular fixed costs are required besides fixed costs in product development, branding and marketing to ensure that only some firms can export profitably. There is wide empirical support for the importance of sunk export costs (See for example Roberts and Tybout (1997) and Das et al. (2007)). This chapter only assumes that these sunk export costs can be split up in a part that does affect the quality of the good like product development costs and a part that does not like setting up distribution channels and costs to comply with local regulations.

The model in this chapter is related to the strand of literature on vertical product differentiation and trade. In particular there are three papers that follow an approach that is close to but different from the approach in this chapter. The first features in Hummels and Klenow (2005). In one of the models in their paper costs are also dependent on quality in a CES-framework. They also introduce a productivity to produce quality parameter and firms can choose the amount of resources invested in product quality. But in their model all firms in a country have the same quality productivity. So, it is a model of between country differences. Furthermore, quality is only related to marginal costs. Choosing the framework of Hummels and Klenow (2005) to model within country differences does not lead to sensible results. The price rises in their model only in quality, because firms in a country with larger quality productivity demand more labor leading to higher wages. So, the quality differences between countries are caused by differences in wage levels. Hence, one cannot generate within country differences in prices and quality with their modeling framework.¹

A second paper comes closest to the model in this chapter. Helble and Okubo (2006) also model vertical product differentiation in a Melitz type monopolistic competition model. More productive firms produce goods with higher prices, which reflects larger quality, confronting Schott’s critique on monopolistic competition models. They use

¹The model of Hummels and Klenow (2005) with quality only related to marginal costs can generate within country differences in quality and prices, when the marginal cost function would be made very complex. But then the model would not be solvable anymore and for example the price index can not be written anymore as an integral over the productivities. Besides the computational problems involved in making the quality productivity only dependent on marginal costs, the setup in the present model has more intuitive appeal. The quality of a good can be increased by investing more in product development and marketing which are costs that are fixed and sunk in nature.
5.2 CLOSED ECONOMY MODEL

This section proposes a model of monopolistic competition with heterogeneous productivity, where more productive firms produce goods of larger quality and with larger prices. The model is based on CES-preferences with firm specific CES-weights, their taste parameters. All firms have a productivity to produce quality. Firms can produce a higher level of quality reflected in their taste parameter when their productivity to produce quality is bigger. A larger taste parameter requires more investment in product development, branding and marketing. The price of a good is dependent on its quality, because the marginal cost of production varies with quality. The beneficial effect of international trade works in the same way as in Melitz. Due to iceberg trade costs and fixed trade costs, only more productive firms can export. International trade raises the expected profit of entry, induces more entry leading to higher real wages that squeeze
the least productive firms out of the market.

A representative consumer has CES utility with taste parameter $\alpha_v$ for variety $v$:

$$U = \left[ \int_{v \in V} \alpha_v x_v^{\frac{\sigma-1}{\sigma}} dv \right]^{\frac{\sigma}{\sigma-1}} \quad (5.1)$$

There are $L$ consumers in the economy. Normalizing the wage at 1, the market demand facing a firm is given by:

$$x_v = \alpha_v^\sigma p_v^{-\sigma} P^{\sigma-1} L \quad (5.2)$$

The price index $P$ is defined as:

$$P = \left[ \int_{v \in V} \alpha_v^\sigma p_v^{1-\sigma} dv \right]^{\frac{1}{1-\sigma}} \quad (5.3)$$

The cost function of a firm is given by:

$$C (x_v) = a_v (\alpha_v) x_v + f + f_{pbm,v} \quad (5.4)$$

Labor is the only production factor. Firms have a marginal cost $a_v$ that is dependent on the quality of the good $\alpha_v$ produced. In the remainder it is assumed that this relation is linear. The parameter restrictions to get a solution when a non-linear relation is chosen will be discussed below.

$$a_v = \alpha_v \quad (5.5)$$

There are two types of fixed cost, a regular fixed cost $f$ that consists of for example overhead costs and fixed costs of product development, branding and marketing $f_{pbm}$. The last type of fixed costs affects the quality of the good. There are several papers in industrial organization that make a similar assumption with fixed costs rising in quality, i.e. Mussa and Rosen (1978), Shaked and Sutton (1983), Gal-Or (1983) and Motta (1993)).

The taste parameter of a variety depends on the amount of product development, branding and marketing fixed costs spent $f_{pbm}$ and a ‘quality productivity’ parameter $\beta_v$. The relation is defined by the following expression:

$$\alpha_v = \beta_v \left( f_{pbm,v} \right)^\kappa, \quad 0 < \kappa < 1 \quad (5.6)$$

So, a firm can raise the attractiveness of its good by increasing the amount of product development and marketing expenditures. The effectiveness of these expenditures is larger for firms that have drawn a larger productivity to produce quality parameter, $\beta_v$. $\kappa < 1$ is a necessary assumption for equilibrium. This assumption is realistic as long as the equilibrium is in the range of product development, branding and marketing costs, where there are decreasing returns.

A firm can enter the market by incurring sunk entry costs $f_e$. After payment of the sunk entry costs a firm can draw the quality productivity parameter $\beta_v$. So, the sunk entry costs should be seen as a kind of market exploration costs, necessary for a firm to
get to know how popular its variety can become. After a firm knows \( \beta_v \) it either starts to produce or leaves the market immediately. When it starts to produce a firm has to incur product development costs and costs of marketing. There is a fixed death probability \( \delta \) of a certain variety.

The product development, branding and marketing costs \( f_{pbm} \) are partly sunk and partly fixed in nature. Because the amount of sunk investments in product development has to be decided on after the uncertainty of the quality productivity is released, the sunk part of \( f_{pbm} \) can be expressed as per period amortized costs using the probability of death parameter \( \delta \). So, \( f_{pbm} \) can be seen as a combination of costs that are more sunk like product development costs and marketing costs that are more fixed in nature.

In this model a firm has two choice variables, the price \( p_v \) and the amount of product development, branding and marketing fixed costs \( f_{pbm,v} \). The pricing equation is the familiar CES-markup equation:

\[
p_v = \frac{\sigma}{\sigma - 1} \alpha_v \tag{5.7}
\]

So, the price is rising in the endogenously determined quality level \( \alpha_v \). Substituting equations (5.6) and (5.7), the profit of the firm is given by:

\[
\pi_v = p_v x_v - \alpha_v x_v - f - f_{pbm,v} \tag{5.8}
\]

\[
\pi_v = \frac{\beta_v (f_{pbm,v})^\kappa}{\sigma} \left( \frac{\sigma - 1}{\sigma} \right)^{1-\sigma} P^{\sigma-1} L - f - f_{pbm,v} \tag{5.9}
\]

The first order condition of equation (5.9) with respect to \( f_{pbm,v} \) is given by:

\[
\beta_v (f_{pbm,v})^\kappa \left( \frac{\sigma - 1}{\sigma} \right)^{1-\sigma} P^{\sigma-1} L \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} = 1 \tag{5.10}
\]

The optimal \( f_{pbm,v} \) is:

\[
(f_{pbm,v})^\kappa = \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma - 1}{\kappa}} P^{\frac{\sigma - 1}{\kappa}} \left( \frac{L}{\sigma} \right)^{\frac{\sigma - 1}{\kappa}} (\kappa \beta_v)^{\frac{1}{\kappa}} \tag{5.11}
\]

Substituting equation (5.11) back into (5.6) and (5.9), one gets:

\[
\alpha_v = \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma - 1}{\kappa}} P^{\frac{\sigma - 1}{\kappa}} \left( \frac{L}{\sigma} \right)^{\frac{\sigma - 1}{\kappa}} (\kappa \beta_v)^{\frac{1}{\kappa}} \tag{5.12}
\]

\[
\pi_v = \left[ \kappa^{\frac{1}{\kappa}} - \kappa^{\frac{1}{\kappa}} \right] \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma - 1}{\kappa}} P^{\frac{\sigma - 1}{\kappa}} \left( \frac{L}{\sigma} \right)^{\frac{\sigma - 1}{\kappa}} (\beta_v)^{\frac{1}{\kappa}} - f \tag{5.13}
\]

The price is given by:

\[
p_v = \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma - 1}{\kappa}} P^{\frac{\sigma - 1}{\kappa}} \left( \frac{L}{\sigma} \right)^{\frac{\sigma - 1}{\kappa}} (\beta_v)^{\frac{1}{\kappa}} \tag{5.14}
\]
Equation (5.14) gives rise to the following observation:

**Observation 24** The market price \( p_v \) of an individual firm is rising in quality \( \alpha_v \) and in the productivity to produce quality \( \beta_v \).

Equation (5.7) shows that the market price is rising in quality and from equation (5.14) it is clear that the market price is rising in the productivity to produce quality. The empirical critique on the heterogeneous productivity monopolistic competition model discussed in the introduction was that more productive firms charge lower prices, whereas in reality more productive firms charge higher prices. Observation 24 shows that one can modify the monopolistic competition model to confront this critique. In the present model productivity is reformulated in terms of the ability to produce quality. In this way more productive firms charge higher prices, because they choose a higher level of quality.

Equilibrium in the economy can be found by combining a free entry condition and a zero cutoff profit condition. The free entry (FE) condition is given by:

\[
\tilde{\pi} = \frac{\delta f_e}{1 - G(\beta^*)} \tag{5.15}
\]

\( \tilde{\pi} \) is average profit and \( \beta^* \) is the cutoff quality productivity. The zero cutoff profit condition (ZCP) is given by:

\[
\pi(\beta^*) = \left[ \kappa^{1-\kappa} - \kappa^{1-\kappa} \right] \left[ \frac{\sigma - 1}{\sigma} \right]^{\frac{\sigma - 1}{\tau - \kappa}} P^{\frac{\sigma - 1}{\tau - \kappa}} \left( \frac{L}{\sigma} \right)^{\frac{1}{\tau - \kappa}} (\beta^*)^{\frac{1}{\tau - \kappa}} - f \tag{5.16}
\]

Equation (5.16) implies for \( \beta^* \):

\[
\beta^* = f^{1-\kappa} \left[ \kappa^{1-\kappa} - \kappa^{1-\kappa} \right]^{\kappa-1} \left[ \frac{\sigma - 1}{\sigma} \right]^{1-\sigma} P^{\sigma-1} \sigma \tag{5.17}
\]

Writing average profit as an integral over quality productivities one can express the ZCP as a function of average profit and the cutoff quality productivity. Average profit is given by:

\[
\tilde{\pi} = \left[ \kappa^{1-\kappa} - \kappa^{1-\kappa} \right] \left[ \frac{\sigma - 1}{\sigma} \right]^{\frac{\sigma - 1}{\tau - \kappa}} P^{\frac{\sigma - 1}{\tau - \kappa}} \left( \frac{L}{\sigma} \right)^{\frac{1}{\tau - \kappa}} \int_\beta^* \beta^{\frac{1}{\tau - \kappa}} \mu(\beta) d\beta - f \tag{5.18}
\]

\( \mu(\beta) = \frac{1}{1 - G(\beta^*)} g(\beta) \) is the distribution of productivities to produce quality of the firms that are in the market. Combining equations (5.17) and (5.18), the ZCP can now be rewritten as:

\[
\tilde{\pi} = f \left[ \left( \frac{1}{\beta^*} \right)^{\frac{1}{\tau - \kappa}} \frac{1}{1 - G(\beta^*)} \int_\beta^* \beta^{\frac{1}{\tau - \kappa}} g(\beta) d\beta - 1 \right] \tag{5.19}
\]

Or more concisely:

\[
\tilde{\pi} = f \left[ \left( \frac{\tilde{\beta}}{\beta^*} \right)^{\frac{1}{\tau - \kappa}} - 1 \right] \tag{5.20}
\]
5.2 CLOSED ECONOMY MODEL

With \( \widetilde{\beta} \) the average productivity to produce quality of the firms in the market, defined as:

\[
\widetilde{\beta} = \left[ \frac{1}{1 - G(\beta^*)} \int_{\beta^*}^{\beta_o} \beta \frac{1}{1-\kappa} g(\beta) d\beta \right]^{1-\kappa} \tag{5.21}
\]

Equations (5.15) and (5.20) together determine the cutoff quality productivity and average profit. It can be shown that (5.15) and (5.20) together yield a unique equilibrium in the same way as in appendix B of Melitz (2003). The proof is in appendix 5A. From the cutoff quality productivity and the average profit one can determine the average quality productivity, the average quality, the number of firms and all other endogenous variables. The number of firms can be derived from the condition for steady state of entry and exit. This leads to the following expression (derivations in appendix B):

\[
N = \frac{L}{\sigma (\bar{\pi} + f)} \tag{5.22}
\]

The only variable left to determine is the price index \( P \). Writing the price index as an integral over the quality productivity gives the following solution for the price index as a function of the cutoff productivity, the number of firms and parameters of the model:

\[
P = \frac{\sigma}{\sigma - 1} \left( \frac{1}{N} \right)^{\frac{1-\kappa}{\bar{\pi} \beta^*}} \left( \frac{\sigma}{L^\kappa} \right)^{\frac{\kappa}{\bar{\pi}}} \beta^{1-\sigma} \tag{5.23}
\]

The price index declines in the productivity to produce quality. At first sight this seems at odds with observation 24. But the price index is not only dependent on prices of individual firms, but also on their quality levels. Higher productivity leads to higher quality leading to a lower price index. This effect through quality dominates the effect through prices.

The model solves as follows: the FE and ZCP in equations (145.15) and (5.20) together determine the cutoff quality productivity and average profit. (5.22) can be used to determine the number of firms and (5.23) to calculate the price index. In equation (55.5) it was assumed that the relation between marginal costs and the taste parameter is linear. This can be generalized to a non-linear relation:

\[\alpha_v = (\alpha_v)\mu \tag{5.24}\]

Appendix 5C shows that one finds a positive solution for \( f_{pbm,v} \) that satisfies the second order condition of the firm if the following condition is satisfied:

\[0 < \sigma (1 - \mu) + \mu < \frac{1}{\kappa} \tag{5.25}\]

Appendix A shows that \( \kappa \) should be smaller than 1 to find an equilibrium for the cutoff quality productivity. So, when \( \mu = 1 \) as in the standard model, the SOC is satisfied. A larger \( \mu \) makes satisfaction of the SOC easier but can lead to a negative optimum. So, \( \mu \) cannot be too large. For \( \sigma = 2 \), \( \mu \) should be smaller than 2 for example to generate
a positive optimum. The condition on \( \mu \) is a restriction on the model. To generate large price differences between firms, the taste parameters also have to display large differences.

5.3 Open Economy Model

International trade is introduced in a standard way. There are two countries of equal size and with an equal quality productivity distribution, so as to guarantee equal wage levels. There are per unit iceberg trade costs \( \tau \) and a firm has to incur separate product development, branding and marketing costs for the exporting market, \( f_{pbm,x,v} \). There are also fixed costs of exporting \( f_x \) which can be seen as well as sunk entry exporting costs as there is no exporting uncertainty. It is assumed that the productivity to produce quality is equal to the quality productivity at home.\(^2\) The profit from exporting (substituting the CES pricing equation) is equal to:

\[
\pi_{v,x} = \frac{\tau^{1-\sigma} \beta_v (f_{pbm,x,v})^\kappa \left( \frac{\sigma}{\sigma - \tau} \right)^{1-\sigma} P^{\sigma-1} L}{\sigma} - f_x - f_{pbm,x,v} \tag{5.26}
\]

The equal countries assumption allows focusing on one country only. Country subscripts will be omitted therefore. The subscripts \( d, x \) indicate whether a good is for the domestic or exporting market. The first order condition generates the following solution for \( f_{pbm,x,v} \):

\[
f_{pbm,x,v} = \left( \frac{1}{\tau} \right)^{\frac{\sigma-1}{\sigma-\tau}} \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma-1}{\sigma-\tau}} P^{\frac{\sigma-1}{\tau}} \left( \frac{L}{\sigma} \right)^{\frac{1}{\tau}} (\kappa \beta_v)^{\frac{1}{\tau-\sigma}} \tag{5.27}
\]

Substituting this back into the profit equation leads to:

\[
\pi_{x,v} = \left( \frac{1}{\tau} \right)^{\frac{\sigma-1}{\sigma-\tau}} \left[ \kappa^{\frac{1}{\tau-\sigma}} - \kappa^{\frac{1}{\tau-\sigma}} \right] \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma-1}{\sigma-\tau}} P^{\frac{\sigma-1}{\tau}} \left( \frac{L}{\sigma} \right)^{\frac{1}{\tau}} (\beta_v)^{\frac{1}{\tau-\sigma}} - f_x \tag{5.28}
\]

This implies for the exporting ZCP:

\[
\beta_x^* = (f_x)^{1-\kappa} \tau^{\sigma-1} \left[ \kappa^{\frac{1}{\tau-\sigma}} - \kappa^{\frac{1}{\tau-\sigma}} \right] \left( \frac{\sigma - 1}{\sigma} \right)^{1-\sigma} P^{1-\sigma} \frac{\sigma}{L} \tag{5.29}
\]

Firms have to be more productive to export than for domestic production when:

\[
\beta_x^* > \beta^* \Leftrightarrow f_x \tau^{\sigma-1} > f \tag{5.30}
\]

The partition of firms between domestic producing and exporting firms works in the same way as in Melitz (2003). Equation (5.30) has the following implications:

\[^2\text{It can also be assumed that the quality productivity abroad is lower than in the home market. The motivation would be that it is more difficult to make a product appealing in a foreign market than in the domestic market. The literature on cultural proximity lends support for this approach. Making such an assumption creates problems for the proof of a unique equilibrium in the model where no specific distribution of productivities is assumed.}\]
5.3 OPEN ECONOMY MODEL

**Observation 25** Only more productive firms can export and firms producing a larger quality can export. Hence, the average quality of exported goods is larger than the average quality of all goods in the economy.

The partition of firms in equation (5.30) implies that only more productive firms can export. So, on average exporting firms are more productive. From observation 24 it is known that more productive firms have a larger quality. The size of a firm, i.e. its revenues, is equal to:

\[ r_v = p_v x_v = \alpha_v \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} P^{\sigma-1} L \]  
\[ (5.31) \]

The following observation follows from equation (5.31) and observation 25:

**Observation 26** Firms with higher quality are bigger. Hence, exporting firms are on average bigger than domestic producing firms.

An exporting ZCP similar to the domestic ZCP in equation (5.20) can be derived from equation (5.29). Adding the domestic and exporting ZCP leads to a relation between average profit and the two cutoff quality productivities:

\[ \bar{\pi} = f \left[ \left( \frac{\tilde{\beta}}{\beta^*} \right)^{\frac{1}{1-\kappa}} - 1 \right] + f_x \left[ \left( \frac{\tilde{\beta}_x}{\beta_x^*} \right)^{\frac{1}{1-\kappa}} - 1 \right] \]  
\[ (5.32) \]

With:

\[ \tilde{\beta}_x = \left[ \frac{1}{1 - G(\beta_x^*)} \int_{\beta_x^*}^{\infty} \beta^{\frac{1}{1-\kappa}} g(\beta) d\beta \right]^{1-\kappa} \]  
\[ (5.33) \]

The free entry condition remains as in (5.15). The open economy model can be solved by combining the FE in (5.15), the ZCP in (5.32) and the relation between the two cutoff quality productivities:

\[ \beta_x^* = \left( \frac{f_x}{f} \right)^{\frac{1}{1-\kappa}} P^{\sigma-1} \beta_x^* \]  
\[ (5.34) \]

Equations (5.15), (5.32) and (5.34) yield solutions for average profit \( \bar{\pi} \) and the domestic and exporting cutoff productivities, \( \beta_x^* \) and \( \beta_x^* \). Appendix A shows that one can find a unique equilibrium for the cutoff productivities from these equations. The number of firms can be derived as in the closed economy (see appendix B):

\[ N = \frac{L}{\sigma (\bar{\pi} + f + p_x f_x)} \]  
\[ (5.35) \]

\( p_x \) is the probability of exporting. \( p_x = \frac{1 - \kappa}{1 - G(\beta_x^*)} \). The expression for the price index becomes:

\[ P = \frac{\sigma}{\sigma - 1} \left( \frac{1}{N} \right)^{\frac{1}{\sigma - 1}} \left( \frac{\sigma}{\sigma - 1} \right)^{\frac{1}{\sigma - 1}} \bar{\pi}^{\frac{1}{\sigma - 1}} \left[ \beta_x^{\frac{1}{1-\kappa}} + \tau^{\frac{1}{1-\kappa}} \frac{1}{1 - G(\beta_x^*)} \beta_x^{\frac{1}{1-\kappa}} \right]^{\frac{1}{\sigma - 1}} \]  
\[ (5.36) \]
The following observations on the effect of trade liberalization hold in this model:

**Observation 27** Trade liberalization increases the average productivity to produce quality and the average quality in the economy.

**Observation 28** Trade liberalization decreases the average productivity to produce quality of exporting firms and decreases their average quality.

Appendix D proves that the domestic cutoff productivity $\beta^*$ rises and the exporting cutoff productivity $\beta_x$ declines with trade liberalization. Average productivity $\bar{\beta}$ and average exporting productivity $\bar{\beta}_x$, defined in equations (5.21) and equation (5.33) respectively, are rising in their respective cutoff productivities. This proofs that average productivity rises and average exporting productivity declines with trade liberalization. It can be shown that average quality and average exporting quality rise in average productivity and average productivity of exporting firms respectively, which proofs the last parts of observations 27 and 28.

The reallocation effects are familiar from Melitz (2003). The least productive firms are squeezed out of the market with trade liberalization increasing average productivity in the economy. Trade liberalization enables more firms to export, so also firms with a lower productivity and quality can start to export. Observations 27 and 28 are empirically testable. The predictions are that the economy wide average quality rises with trade liberalization, but the average quality of exporting declines with trade liberalization.

### 5.4 Concluding Remarks

The model proposed in this chapter provides a theoretical reply to recent empirical findings on exporting unit values, in particular in Schott (2004). The heterogeneous productivity Melitz model is modified in such a way that more productive firms produce higher quality goods and charge larger prices instead of lower prices. This is in line with empirical work showing that high income countries export goods with higher unit values. The CES-model with quality weights is used, a productivity to produce quality is introduced and a link between marketing expenditures and quality is created to produce this result. A larger quality of the good in turn implies a larger marginal cost and thus a larger price. This brings the heterogeneous productivity model more in line with the empirical finding that goods from high income countries display higher unit values. The model also generates some outcomes that are empirically testable. Exporting firms are more productive, produce a higher quality and are bigger. Trade liberalization increases the average quality in the economy, but it decreases the average quality of exports, because more firms can export.

### A Uniqueness of the Closed and Open Economy Equilibrium

To prove that the FE and ZCP, equations (5.15) and (5.20) yield a unique equilibrium for the cutoff quality productivity, one can proceed analogous to Melitz (2003) in his
appendix B. Equations (5.15) and (5.20) can be combined to get the following equation:

\[
(1 - G(\beta^*)) \left[ \left( \frac{\beta(\beta^*)}{\beta^*} \right)^{\frac{1}{1-\kappa}} - 1 \right] = \frac{\delta f_e}{f} \tag{A.1}
\]

It will be shown that the LHS of (A.1) is monotonically decreasing from \(\infty\) to 0 on \((0, \infty)\), which is sufficient to show that there is a unique equilibrium. Define \(k(\beta)\) as:

\[
k(\beta) = \left( \frac{\beta(\beta)}{\beta^*} \right)^{\frac{1}{1-\kappa}} - 1 \tag{A.2}
\]

Differentiating \(k(\beta)\) in equation (A.2) with respect to \(\beta\) gives:

\[
k'(\beta) = \frac{g(\beta)}{1 - G(\beta)} \left[ \left( \frac{\beta(\beta)}{\beta^*} \right)^{\frac{1}{1-\kappa}} - 1 \right] - \frac{1}{1 - \kappa} \left( \frac{\beta(\beta)}{\beta^*} \right)^{\frac{1}{1-\kappa}} \frac{1}{\beta} \tag{A.3}
\]

\(j(\beta)\) is defined as:

\[
j(\beta) = (1 - G(\beta)) k(\beta) \tag{A.4}
\]

\(j(\beta)\) is equal to the LHS of equation (A.1). \(j'(\beta)\) can be computed as:

\[
j'(\beta) = -\frac{1}{1 - \kappa} \left( \frac{\beta(\beta)}{\beta^*} \right)^{\frac{1}{1-\kappa}} \frac{1}{\beta} < 0 \tag{A.5}
\]

It is easy to see that \(\lim_{\beta \to 0} j(\beta) = \infty\) and \(\lim_{\beta \to \infty} j(\beta) = 0\). Hence, \(j(\beta)\) and thus the LHS of (A.1) is monotonically decreasing from \(\infty\) to 0 on \((0, \infty)\).

The equilibrium of the open economy model is constituted by equations (5.15), (5.32) and (5.34). Combining these three equations leads to the following equilibrium equation:

\[
f j(\beta^*) + f_x j(\beta^*_x (\beta^*)) = \delta f_e \tag{A.6}
\]

Equation (A.5) showed that \(j(\beta)\) is monotonically decreasing from \(\infty\) to 0 on \((0, \infty)\). Given that \(\beta^*_x\) is monotonically increasing in \(\beta^*\), the LHS of (A.6) is also decreasing from \(\infty\) to 0 on \((0, \infty)\) and hence there is a unique equilibrium.

B Number of Firms

First, the number of firms in the closed economy is derived and then in the open economy. In the closed economy the steady state of entry and exit dictates that the number of entering firms should be equal to the number of exiting firms:

\[
(1 - G(\beta^*)) N_e = \delta N \tag{B.1}
\]
$N_e$ is the number of all successful and unsuccessful entrants. Labor market equilibrium in the production sector and innovation sector are given by:

$$L_p = R - \Pi = N\bar{\tau} - N\bar{\pi} \quad (B.2)$$

$$L_e = N_e f_e = \frac{\delta N}{(1 - G(\beta^*))} f_e = N\bar{\pi} \quad (B.3)$$

$L_p$ and $L_e$ are the amount of labor used in the production and innovation sector, respectively. Adding the two labor market equilibria in (B.2) and (B.3) leads to:

$$L = L_p + L_e = N\bar{\tau} \quad (B.4)$$

Equation (B.4) can be rewritten to find the number of firms as a function of average profit:

$$N = \frac{L}{\sigma (\bar{\pi} + f)} \quad (B.5)$$

In the open economy the derivation is similar. Labor can be allocated in four different ways, in domestic production $L_p$, in domestic innovation $L_e$, in exporting production $L_{p,x}$, or in exporting innovation $L_{e,x}$:

$$L = L_p + L_e + L_{p,x} + L_{e,x} \quad (B.6)$$

The expression for domestic innovation $L_e$ is equal to the closed economy one, given in equation (B.3). The others become:

$$L_p = N\bar{\tau}_d - N\bar{\pi}_d \quad (B.7)$$

$$L_{p,x} = N_x\bar{\tau}_x - N_x\bar{\pi}_x - N_x f_x \quad (B.8)$$

$$L_{e,x} = f_{ex} N_{e,x} = f_{e,x} \delta N_x = f_x N_x \quad (B.9)$$

$N_x$ is the number of firms producing for the exporting market and $N_{e,x}$ is the number of firms entering the export market. So, adding the labor market allocations gives:

$$L = N (\bar{\tau}_d + p_x \bar{\tau}_x) \quad (B.10)$$

The expression for the number of firms becomes:

$$N = \frac{L}{\sigma (\bar{\pi} + f + p_x f_x)} \quad (B.11)$$

### C More General Marginal Cost Function

When the marginal cost function is given by (5.24) the profit of the firm becomes:

$$\pi_v = \left( \beta_v (f_{pbm,v})^{\kappa} \right)^{\sigma (1-\mu) + \mu} \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} p^{(1-\sigma)\frac{L}{\sigma}} - f_{pbm,v} - f \quad (C.1)$$
The first order condition with respect to \( f_{pbm,v} \) is:

\[
\kappa (\sigma (1 - \mu) + \mu) (\beta_v)^{\sigma(1-\mu)+\mu} (f_{pbm,v})^{\kappa(\sigma(1-\mu)+\mu)-1} \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} P^{\sigma - 1} L - 1 = 0 \tag{C.2}
\]

So, the solution for \( f_{pbm,v} \) is:

\[
f_{pbm,v} = \left[ \kappa (\sigma (1 - \mu) + \mu) P^{\sigma - 1} \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma - 1} \beta_v^{\sigma(1-\mu)+\mu} \right]^{1/\kappa(\sigma(1-\mu)+\mu)} \tag{C.3}
\]

So, a positive solution for \( f_{pbm,v} \) is found if:

\[
\sigma (1 - \mu) + \mu > 0 \quad \tag{C.4}
\]

The second order condition is satisfied when:

\[
\sigma (1 - \mu) + \mu < \frac{1}{\kappa} \quad \tag{C.5}
\]

Conditions (C.4) and (C.5) together imply condition (5.25) in the main text.

\section{The Effects of Trade Liberalization on Cutoff Productivities}

This appendix shows that the domestic cutoff productivity to produce quality \( \beta^* \) rises with trade liberalization and that the exporting cutoff productivity \( \beta_x^* \) declines with trade liberalization. Equation (A.6) is the equilibrium equation of the open economy model. Totally differentiating this equation towards the cutoff productivity and trade costs taking into account equation (5.34) for the relation between the domestic and exporting cutoff productivities \( \beta^* \) and \( \beta_x^* \), generates:

\[
\frac{d\beta_x^*}{d\tau} = -\frac{f_x j'(\beta_x^*) (\sigma - 1) \left( \frac{f_x}{F} \right)^{1-\kappa} \tau^{\sigma-2} \beta_x^*}{f j'(\beta^*) + f_x j'(\beta_x^*) \left( \frac{f_x}{F} \right)^{1-\kappa} \tau^{\sigma-1}} < 0 \tag{D.1}
\]

The RHS of (D.1) is negative as \( j'(\beta) \) is negative for all values of \( \beta \). Hence from (D.1) it follows that lower iceberg trade costs \( \tau \) lead to a higher cutoff productivity. Observation 28 can be proved by differentiating equation (5.34) towards trade costs \( \tau \):

\[
\frac{\partial \beta_x^*}{\partial \tau} = \left( \frac{f_x}{f} \right)^{1-\kappa} \tau^{\sigma-1} \left[ (\sigma - 1) \frac{\beta_x^*}{\tau} + \frac{d\beta_x^*}{d\tau} \right] \quad \tag{D.2}
\]
Substituting equation (D.1) into equation D.2 leads to:

\[
\frac{\partial \beta_x^*}{\partial \tau} = \left( \frac{f_x}{f} \right)^{\frac{1}{1-\kappa}} \tau^{\sigma-2} (\sigma - 1) \beta^* \left[ 1 - \frac{f_x j' (\beta_x^*) \left( \frac{f_x}{f} \right)^{1-\kappa} \tau^{\sigma-1}}{f j' (\beta^*) + f_x j' (\beta_x^*) \left( \frac{f_x}{f} \right)^{1-\kappa} \tau^{\sigma-1}} \right] \tag{D.3}
\]

The second term between brackets of the LHS of (D.3) is smaller than 1. Therefore, the exporting cutoff productivity \( \beta_x^* \) rises in trade costs \( \tau \), implying that also average exporting productivity rises in \( \tau \).
Chapter 6

Within Sector Specialization in a Monopolistic Competition Model of Trade

6.1 Introduction

Empirical work by Schott (2004) shows that unit values of export goods within detailed product categories are related to factor abundance of the exporting country. Hence, there is strong evidence for within-sector specialization: more skill-abundant and capital-abundant countries export goods with higher unit values. This chapter proposes a model that can account for the finding of Schott (2004). Production is non-homothetic in quality and higher quality goods require more skilled labor implying that skill abundant countries produce higher quality goods.

The model assumes homogeneous firms. There is one sector, two countries and two production factors, skilled and unskilled labor. Production is non-homothetic: a larger quality good requires relatively more labor. The model implies that more skill abundant countries produce higher quality goods. This result accounts for Schott’s (2004) empirical findings that within product categories skill abundant countries export higher quality goods. The model is a Hekscher-Ohlin factor abundance model, but factor abundance does not determine in which sectors a country has a comparative advantage but in which quality segment a country has a comparative advantage.

Two papers are related to the model in this chapter. Hummels and Klenow (2005) also model between country differences in the productivity to produce quality. They just assume that countries have different productivities. The present chapter relates the capability to produce quality to relative factor abundance and skill abundance in particular. Therefore, Hummels and Klenow’s model is in a Ricardian spirit, whereas the present model is of the Hekscher-Ohlin-type. Hummels and Klenow’s model can be linked to the empirical finding that goods from higher income countries and so from more productive countries display higher unit values, whereas the model in this chapter accounts for the empirical finding that goods from more skill abundant countries display higher unit values.
Verhoogen (2008) introduces a model with heterogeneous firms and quality differentiation where more productive firms pay higher wages to attract the most qualified workers. The Verhoogen (2008) model contains the same building blocks as the present model, but the setup is basically different. The demand system is different and moreover the model is partial equilibrium with given higher average wages for skilled workers.

The two models of this chapter and chapter 5 could be combined in an integrated model. Such a model could generate interesting results, in particular on the effect of trade liberalization on wage inequality. Solving the combined model requires numerics, which is left for future work. The remainder of the paper is as follows. The next section outlines the model in a closed economy. Section 3 goes into the open economy model introducing trade. Section 4 contains concluding remarks.

6.2 Closed Economy Model

Productivity differences between countries can be modeled in Ricardian fashion and a factor abundance fashion. In the former approach there are productivity differences between countries by assumption. In the latter approach productivity differences are related to factor abundance differences. A Dixit-Stiglitz monopolistic competition model with Ricardian differences in productivity is proposed by Hummels and Klenow (2005). Schott (2004) shows that quality differences between countries are related to factor abundance. Therefore, here a monopolistic competition model with productivity differences between countries based on factor abundance will be considered. The novelty is that production is non-homothetic in factors of production. Higher quality goods require relatively more high skilled labor. The model assumes equal firms and hence no firm heterogeneity to keep it analytically tractable.

Utility, the demand facing a firm and the price index are the same as in the previous chapter. A representative consumer has CES utility with taste parameter $\alpha_v$ for variety $v$:

$$U = \left[ \int_{v \in V} \alpha_v x_v^{\frac{1}{\sigma}} dv \right]^{\frac{\sigma}{\sigma-1}}$$

(6.1)

The market demand facing a firm is given by:

$$x_v = \alpha_v^\sigma p_v^{-\sigma} P^{\sigma-1} L$$

(6.2)

$I = w_s L_s + w_u L_u$ is total income in the economy and the sum of skilled and unskilled labor’s income. The price index $P$ is defined as:

$$P = \left[ \int_{v \in V} \alpha_v^\sigma p_v^{1-\sigma} dv \right]^{\frac{1}{1-\sigma}}$$

Production is increasing returns with a fixed cost of production. There are two factors of production, high-skilled labor and low-skilled labor. The cost function is a nonhomothetic CES function described for example in Shimomura (1999). The cost function is non-homothetic in quality. It is given by:
6.2 CLOSED ECONOMY MODEL

\[ C(\alpha, w_s, w_u) = (\alpha x + f) \left( \gamma(\alpha) w_s^{1-\theta} + (1 - \gamma(\alpha)) w_u^{1-\theta} \right)^{\frac{1}{1-\sigma}} \]  \hspace{1cm} (6.3)

\( w_s \) and \( w_u \) are the wage levels of skilled and unskilled labor, respectively. \( \theta \) is the substitution elasticity between skilled and unskilled labor and \( \gamma(\alpha) \) is the CES-weight in the unit cost function dependent on quality, constituting the non-homotheticity. So, in this specification costs depend on quality through the marginal costs and through the non-homotheticity in skilled and unskilled labor. The effect through fixed product development, branding and marketing costs present in the within country heterogeneity model of the previous chapter is omitted. The price of a good is given by the familiar markup equation:

\[ p = \frac{\sigma}{\sigma - 1} \alpha \left( \gamma(\alpha) w_s^{1-\theta} + (1 - \gamma(\alpha)) w_u^{1-\theta} \right)^{\frac{1}{1-\sigma}} \]  \hspace{1cm} (6.4)

Substituting the CES-pricing equation (6.4), profit of a firm becomes:

\[ \pi = \alpha \left[ \frac{\sigma}{\sigma - 1} \left( \gamma(\alpha) w_s^{1-\theta} + (1 - \gamma(\alpha)) w_u^{1-\theta} \right)^{\frac{1}{1-\sigma}} \right]^{1-\sigma} p^{\sigma-1} I - f \left( \gamma(\alpha) w_s^{1-\theta} + (1 - \gamma(\alpha)) w_u^{1-\theta} \right)^{\frac{1}{1-\sigma}} \]  \hspace{1cm} (6.5)

Taking the FOC of profit \( \pi \) in equation (6.5) with respect to \( \alpha \) gives:

\[ \frac{p x}{\sigma} \left[ \frac{1}{\alpha} - \frac{\sigma - 1}{\theta - 1} \gamma'(\alpha) \left( w_u^{1-\theta} - w_s^{1-\theta} \right) \right] - \frac{f}{\theta - 1} \left( \gamma(\alpha) w_s^{1-\theta} + (1 - \gamma(\alpha)) w_u^{1-\theta} \right)^{\frac{1}{1-\sigma}} \frac{\gamma'(\alpha) \left( w_u^{1-\theta} - w_s^{1-\theta} \right)}{\gamma(\alpha) w_s^{1-\theta} + (1 - \gamma(\alpha)) w_u^{1-\theta}} = 0 \]  \hspace{1cm} (6.6)

Assuming free entry and exit of firms a zero profit condition can be added to the model:

\[ \frac{p x}{\sigma} = f \left( \gamma(\alpha) w_s^{1-\theta} + (1 - \gamma(\alpha)) w_u^{1-\theta} \right)^{\frac{1}{1-\sigma}} \]  \hspace{1cm} (6.7)

Substituting the zero profit equation (6.7) into the FOC for \( \alpha \), equation (6.6), one finds:

\[ \frac{p x}{\sigma} \left[ \frac{1}{\alpha} - \frac{\sigma}{\theta - 1} \gamma'(\alpha) \left( w_u^{1-\theta} - w_s^{1-\theta} \right) \right] = 0 \]  \hspace{1cm} (6.8)

The first term between brackets in equation (6.8) measures the marginal benefit of larger quality for the firm, due to larger sales. The second term measures the loss due to a larger required use of skilled labor when quality is larger. To solve for \( \alpha \), an explicit function for \( \gamma(\alpha) \) should be chosen that is rising in \( \alpha \) and bounded between 0 and 1. The following function is chosen:

\[ \gamma(\alpha) = \alpha^\eta; \quad \eta > 1 \]  \hspace{1cm} (6.9)
The condition on $\eta$ is needed to satisfy the second order condition. This $\gamma(\alpha)$ yields the following solution for $\alpha$:

$$\alpha = \left[ \frac{\theta - 1}{\theta - 1 + \sigma \eta} \right]^\frac{1}{\eta}$$

Equation (6.10) implies that quality is rising in the relative wage of unskilled over skilled workers, $\frac{w_s}{w_u}$.\(^1\) Log differentiating equation (6.10) shows this:

$$\frac{\hat{w}_s}{w_u} = -\frac{1}{\theta - 1} \left( \left( \frac{w_s}{w_u} \right)^{\theta - 1} - 1 \right) \hat{\gamma}$$

Unit wage costs of production are equal to:

$$UC(\alpha, w_s, w_u) = (\gamma(\alpha) w_s^{1-\theta} + (1 - \gamma(\alpha)) w_u^{1-\theta})^{\frac{1}{1-\theta}}$$

Normalizing the unskilled wage $w_u$ at 1, this expression can be log differentiated as follows:

$$\frac{\hat{UC}}{\hat{w}_s} = \frac{1}{\theta - 1} \gamma \left[ 1 - w_s^{1-\theta} \right] \hat{\gamma} + \frac{\gamma w_s^{1-\theta}}{\gamma w_s^{1-\theta} + 1 - \gamma}$$

Substituting the log differentiation in (6.11) the relative change of unit wage costs in (6.13) becomes 0. A larger skilled wage $w_s$ leads to lower quality. The net effect is that unit costs do not change in response to a change in the relative wage because of the endogenous reaction in quality. This result is due to the specification chosen for $\gamma(\alpha)$. The following observation can be made:

**Observation 29** Unit wage costs do not change when relative wages change. An increase in the relative wage of skilled labor decreases the quality of goods implying that unit wage costs do not rise.

To solve for the endogenous variables in the model, two labor market equilibrium equations can be added. Applying Shepard’s lemma to the cost function in equation (6.3), one finds:

$$L_s = N (\alpha x + f) \left( \gamma(\alpha) w_s^{1-\theta} + (1 - \gamma(\alpha)) w_u^{1-\theta} \right)^\frac{\theta}{1-\theta} \gamma(\alpha) w_s^{-\theta}$$

$$L_u = N (\alpha x + f) \left( \gamma(\alpha) w_s^{1-\theta} + (1 - \gamma(\alpha)) w_u^{1-\theta} \right)^\frac{\theta}{1-\theta} (1 - \gamma(\alpha)) w_u^{-\theta}$$

Dividing the labor market equations (6.14) and (6.15) leads to:

$$\frac{L_s}{L_u} = \frac{\gamma(\alpha)}{1 - \gamma(\alpha)} \left( \frac{w_u}{w_s} \right)^\theta$$

\(^1\) is smaller than 1 as long as $\frac{w_u}{w_s} < \left( \frac{\sigma \eta}{\theta - 1 + \sigma \eta} \right)^\frac{1}{\theta - 1}$. So, it is assumed that $\theta$ is not too large relative to $\sigma$ and $\eta$. Unfortunately it is not possible to give a general condition for $\alpha < 1$ depending on parameters and exogenous variables only.
Combining equations (6.10) and (6.16) generates an implicit relation between $\alpha$ and the relative skill abundance $\frac{L_s}{L_u}$:

$$\frac{L_s}{L_u} = \frac{\gamma(\alpha)}{1 - \gamma(\alpha)} \left[ \frac{(\theta - 1) + \sigma \eta \gamma(\alpha) - (\theta - 1)}{(\theta - 1) + \sigma \eta \gamma(\alpha)} \right]^{\frac{\theta}{\sigma \eta}}$$

(6.17)

Log differentiating equation (6.17) with respect to $\frac{L_s}{L_u}$ and $\gamma$ shows that the RHS of (6.17) rises monotonically in $\gamma$:

$$\frac{\widehat{L_s}}{L_u} = \frac{1}{1 - \gamma} \gamma + \frac{\theta}{((\theta - 1) + \sigma \eta \gamma (\alpha) - (\theta - 1))} \gamma$$

(6.18)

Using (6.18) implies that the RHS of (6.17) rises monotonically. Also, it can be easily shown that the RHS of (6.17) rises from $-\frac{\theta - 1}{\theta - 1 + \sigma \eta}$ to $\infty$ for $\gamma \in [0, 1]$. So, there is a unique positive equilibrium for $\gamma$ and for $\alpha$. Equation (6.18) also implies that quality rises in relative factor abundance of high skilled labor. So, $\alpha$ and $\gamma$ rise when an economy becomes more skill abundant. Therefore, equation (6.18) has the following implication:

**Observation 30** The quality of goods produced rises in the skill abundance of the economy.

Equation (6.18) can be rewritten as:

$$\frac{\gamma}{1 - \gamma} \gamma - (1 - \gamma)^2 (\theta - 1) \frac{\widehat{L_s}}{L_u}$$

(6.19)

Equation (6.19) shows that the effect of skill abundance on quality declines when the elasticity of substitution between skilled and unskilled labor $\theta$ is larger. From equation (6.18) it can be easily seen that the effect of skill abundance on quality rises with the elasticity of substitution between consumed varieties $\sigma$ rises and with the parameter $\eta$ indicating the effect of quality on skill intensity of production (cf. equation (6.9)).

Log differentiating the market price equation (6.4) with respect to $p$ and skill intensity $\frac{L_s}{L_u}$, substituting equation 6.19 and using remark 29 that the unit cost does not change, one finds:

$$\frac{\widehat{p}}{\widehat{\alpha}} = \frac{1}{\gamma} \frac{(1 - \gamma) \gamma \sigma \eta - (1 - \gamma)^2 (\theta - 1) \frac{\widehat{L_s}}{L_u}}{1 - \gamma + \sigma \eta \gamma}$$

(6.20)

The implication of equation (6.20) is that:

**Observation 31** The market price rises in the skill abundance of the economy.

The model presented in this subsection contains a link between skill abundance, quality and market price. The relations are congruent with the findings by Schott (2004). More high skilled labor leads to higher quality and larger prices. The next subsection introduces trade in the model.
6.3 Open Economy Model

There are two countries, \( k, l = H, F \). There are per unit iceberg trade costs \( \tau \). There are also fixed costs of exporting \( f_x \). Venables (1994), Jean (2002) and Melitz (2003) include fixed export costs in their models as well, but their motivation is not grounded on firm empirical evidence. When there is no uncertainty, the fixed export costs can also be seen as per period equivalents of sunk export costs for which there is ample empirical evidence. The cost function of a firm in country \( k \) is given by:

\[
C_k = (\alpha_{kd} x_{kd} + f) UC(\alpha_{kd}, w_{sk}, w_{uk}) + (\alpha_{kx} x_{kx} + f_x) UC(\alpha_{kx}, w_{sk}, w_{uk})
\]

Country subscripts \( k, l = H, F \) indicate the country of origin and the subscripts \( d, x \) indicate whether a good is for the domestic or exporting market. Substituting the markup pricing equations, the profit of a firm in country \( k \) can be written as:

\[
\pi_k = \alpha_{kd} \left[ \frac{\sigma}{\sigma-1} UC(\alpha_{kd}, w_{sk}, w_{uk}) \right]^{1-\sigma} P^{\sigma-1} \frac{I_k}{\sigma} - f UC(\alpha_{kd}, w_{sk}, w_{uk})
\]

\[
+ \alpha_{kx} \left[ \frac{\sigma}{\sigma-1} \tau UC(\alpha_{kx}, w_{sk}, w_{uk}) \right]^{1-\sigma} P^{\sigma-1} \frac{I_l}{\sigma} - f_x UC(\alpha_{kx}, w_{sk}, w_{uk}) \quad (6.21)
\]

Taking the first order condition of profit in equation (6.21) with respect to quality in the domestic and exporting market one finds:

\[
\frac{p_{kd} x_{kd}}{\sigma} \left[ 1 - \frac{\sigma - 1}{\theta - 1} \gamma'(\alpha_{kd}) \left( w_{uk}^{1-\theta} - w_{sk}^{1-\theta} \right) \right] - \frac{f}{\theta - 1} \gamma'(\alpha_{kd}) \left( w_{uk}^{1-\theta} - w_{sk}^{1-\theta} \right) = 0 \quad (6.22)
\]

\[
\frac{p_{kx} x_{kx}}{\sigma} \left[ 1 - \frac{\sigma - 1}{\theta - 1} \gamma'(\alpha_{kx}) \left( w_{uk}^{1-\theta} - w_{sk}^{1-\theta} \right) \right] - \frac{f_x}{\theta - 1} \gamma'(\alpha_{kx}) \left( w_{uk}^{1-\theta} - w_{sk}^{1-\theta} \right) = 0 \quad (6.23)
\]

Zero profit equations can be added to the model. There are two zero profit equations, for domestic and exporting production. This can be motivated as follows. Suppose profits were negative in of the two markets. Then firms would leave that market. If profits were positive in one of the markets, firms would enter that market. The domestic and exporting market can be seen as separate markets, because there are fixed exporting costs besides regular fixed costs. One can also interpret both fixed costs as beachhead costs to enter the respective markets. Entering the domestic market and paying the domestic beachhead cost does not imply that one can also export as in the standard Krugman (1980) model without fixed export costs. Considering the domestic and exporting markets as separate markets leading to two zero profit conditions implies that the number of firms in the two markets can be different. The zero profit conditions are given by:

\[
\frac{p_{kd} x_{kd}}{\sigma} = f UC(\alpha_{kd}, w_{sk}, w_{uk}) \quad (6.24)
\]

\[
\frac{p_{kx} x_{kx}}{\sigma} = f_x UC(\alpha_{kx}, w_{sk}, w_{uk}) \quad (6.25)
\]
Combining the zero profit conditions with the first order conditions leads to:

\[
\frac{p_{kd}x_{kd}}{\sigma} \left[ \frac{1}{\alpha_{kd}} - \frac{\sigma}{\theta - 1} U' \left( \alpha_{kd}, w_{sk}, w_{uk} \right) \left( w_{uk}^{1-\theta} - w_{sk}^{1-\theta} \right) \right] = 0 \quad (6.26)
\]

\[
\frac{p_{kx}x_{kx}}{\sigma} \left[ \frac{1}{\alpha_{kx}} - \frac{\sigma}{\theta - 1} U' \left( \alpha_{kx}, w_{sk}, w_{uk} \right) \left( w_{uk}^{1-\theta} - w_{sk}^{1-\theta} \right) \right] = 0 \quad (6.27)
\]

Using the explicit expression for \( \gamma(\alpha) \) in equation (6.9), the solutions for the domestic and exporting quality level become:

\[
\alpha_{kd} = \left[ \frac{\frac{\theta - 1}{\theta - 1 + \sigma \eta}}{1 - \left( \frac{w_{uk}}{w_{sk}} \right)^{\theta - 1}} \right] \frac{1}{\eta} \quad (6.28)
\]

\[
\alpha_{kx} = \left[ \frac{\frac{\theta - 1}{\theta - 1 + \sigma \eta}}{1 - \left( \frac{w_{uk}}{w_{sk}} \right)^{\theta - 1}} \right] \frac{1}{\eta} \quad (6.29)
\]

Comparing equation (6.28) and (6.29) shows that quality for the domestic market and the exporting market are equal, which is convenient later on. This equality is due to the absence of firm heterogeneity that was imposed to keep the model analytically tractable.

As market size effects are not the focus of the present chapter, it is assumed that the two countries are of equal size, i.e. \( L_{sk} + L_{uk} = L_{sl} + L_{ul} \). This implies that market size effects are small. Abstracting from market size effects one can concentrate on differences in relative factor endowments instead of absolute factor endowments. The analysis continues by focusing on the effect of relative factor abundance differences between countries. The labor market equilibrium equations can be added to solve for the endogenous variables in the model:

\[
L_{sk} = N_{kd} \left( \alpha_{kd}x_{kd} + f \right) U' \left( \alpha_{kd}, w_{sk}, w_{uk} \right) \gamma \left( \alpha_{kd} \right) w_{sk}^{-\theta} \\
+ N_{kx} \left( \alpha_{kx}x_{kx} + f_{kx} \right) U' \left( \alpha_{kx}, w_{sk}, w_{uk} \right) \gamma \left( \alpha_{kx} \right) w_{sk}^{-\theta} \
(6.30)
\]

\[
L_{uk} = N_{kd} \left( \alpha_{kd}x_{kd} + f \right) U' \left( \alpha_{kd}, w_{sk}, w_{uk} \right) \gamma \left( \alpha_{kd} \right) w_{uk}^{-\theta} \\
+ N_{kx} \left( \alpha_{kx}x_{kx} + f_{kx} \right) U' \left( \alpha_{kx}, w_{sk}, w_{uk} \right) \gamma \left( \alpha_{kx} \right) w_{uk}^{-\theta} \
(6.31)
\]

From equations (6.28) and (6.29) follows that \( \alpha_{kd} \) and \( \alpha_{kx} \) are equal. Dividing the labor market equations, using as well the zero profit conditions, \( \alpha_{kd}x_{kd} + f = \sigma f \) and

---

2 Firm heterogeneity combined with an assumption on the size of the domestic versus the exporting beachhead costs would generate a larger average quality of exported goods as in the within differences model in the previous chapter.

3 A precise analytic condition for the absence of market size effects dependent only on the parameters of the model and factor endowments cannot be given.
\[ \alpha_{kx} x_{kx} + f_x = \sigma f_x, \] leads to:

\[ \frac{L_{sk}}{L_{uk}} = \frac{\gamma(\alpha_k)}{1 - \gamma(\alpha_k)} \left( \frac{w_{uk}}{w_{sk}} \right) \theta \]  \hspace{1cm} (6.32)

So, one arrives at the same expression as in the closed economy. Combining equations (6.28) and (6.32) and log differentiating towards relative factor abundance and quality generates:

\[ \hat{p}_{kd} = \hat{p}_{kx} = \hat{\alpha}_{kd} = \hat{\alpha}_{kx} = \frac{1}{\gamma} \left( \frac{(1 - \gamma) \gamma \sigma \eta - (1 - \gamma)^2 (\theta - 1) L_{sk}}{1 - \gamma + \sigma \eta \gamma} \right) \] \hspace{1cm} (6.33)

The market price rises in the quality of a good like in the closed economy model. So, observations 30 and 31 also hold in the open economy model:

**Observation 32** Export goods from relatively more skill abundant countries have a larger quality and a higher price.

### 6.4 Concluding Remarks

The model in this chapter extends the monopolistic competition Krugman model in such a way that productivity differences within sectors are related to factor abundance. Production is non-homothetic in quality in the between country differences model. The larger the quality of a good, the more skilled labor is needed. This model setup implies that more skill abundant countries produce higher quality goods. As marginal costs also rise in quality, higher quality goods also have a larger price. As such the model can account for the findings in Schott (2004) that more skill abundant countries export goods with higher unit values.

Combining the model of within country differences in chapter 5 and the between country differences model in this chapter would make it possible to study the effect of trade liberalization on the skill premium. The expected effect is that lower trade costs leads to an increasing market share of high quality producing exporting firms that are relatively skill abundant. As such the demand for skills would increase in both countries and the skill premium would increase in both countries. But combining the two models leads to an analytically intractable model that requires simulations and is left for future work. The present chapter’s main goal was to show that the heterogeneous productivity monopolistic competition model can be modified in a tractable way to bring this standard model in line with recent empirical findings on exporting unit values.
Chapter 7

Concluding Remarks

This thesis makes a contribution to the literature on firm heterogeneity in international trade, the so-called new new trade theory. New trade models include imperfect competition in trade models to be able to account for intra-industry trade. New new trade models generalize these models by accounting for differences between firms. By including firm heterogeneity the reallocation effect of trade can be modeled: more productive exporting firms gain market share at the expense less productive firms producing only for the domestic market. Before proposing new models chapter 2 first contained a survey of the most influential models on firm heterogeneity in international trade and compared them with each other. The other chapters in this thesis are mainly theoretical responses to different findings in the empirical literature. Chapter 3 proposes a model of heterogeneous popularity and exporting uncertainty to account for the large fraction of firms that exit the export market shortly after entry. Firms are heterogeneous with respect to the popularity of their good, the popularity across goods differs (but is correlated) and firms are uncertain about the popularity of their good before they enter a new market. The implication is that a fraction of the firms that start exporting based on the popularity of their good in the domestic market cannot sell profitably abroad and have to leave the export market.

Chapter 5 introduces endogenous quality in a firm heterogeneity model of trade to account for the findings by Schott (2004) that within detailed product categories richer countries export goods with higher unit values. Firms are heterogeneous with respect to the productivity to produce high quality goods. More productive firms will produce higher quality goods and charge higher prices.

Chapter 6 starts from another empirical finding by Schott (2004) that more skill-abundant countries export goods with higher unit values. A monopolistic competition model with a production function that is non-homothetic in quality is put forward to account for Schott’s finding. Higher quality goods require relatively more skills in production. The implication is that more skill abundant countries produce higher quality goods and given that marginal costs rise in quality also unit values rise with skill abundance.

Chapter 4, finally, proposes a different model of firm heterogeneity. The Brander and Krugman (1983) oligopoly model is extended with firm heterogeneity. There is Cournot competition between firms with different productivity. The model distinguishes
between the short-run and the long-run and derives several interesting results. There is an intuitive reallocation effect of trade: (freer) trade leads to more competition, lower prices and squeezes the least productive firms out of the market. The welfare effect of freer trade is unambiguously positive in the long-run but can be negative in the short-run due to the adverse effect of increased cross-hauling of goods. The chapter derives under what conditions on the distribution of productivities the welfare effect of lower trade costs is positive in the short-run. Unilateral liberalization can in the long-run lead to higher prices in the import liberalizing country due to a relocation effect of firms. The model makes clear predictions on the effect of importing country size and trade costs on the probability of zero trade flows and the unit values of trade. Finally, the model nests the Ricardian comparative advantage model and the Brander and Krugman (1983) model as special cases.

The work in this thesis provides opportunities for new research paths in different directions. On the one hand, the theory work can be extended in various interesting ways. First, the model on exporting uncertainty and heterogeneous popularity in its present set-up allows firms to enter the exporting market and get to know their popularity only by incurring sunk export costs. Recent empirical work by Eaton et al. (2007) shows that many firms enter the export market for the first time with very small sales. This suggests that firms have two possibilities to get to know their profitability on the exporting market: by doing market research and incurring sunk entry costs or by experimentation and learning. The model of chapter 3 could be extended to allow for the second possibility.

Second, the oligopoly model of chapter 4 could be extended to include the possibility of FDI. Firms do not only have to possibility to trade, but also to start producing abroad. But such an extension would require a significant change in the setup of the model, because fixed costs of production have to be included in the model to prevent that all firms will immediately start to produce abroad instead of exporting.

Third, a natural extension of the work in chapters 5 and 6 is to combine the two models from these chapters in a simulation. With such a combination of the two models, the effect of trade liberalization on the skill premium could be derived. The expected result is that trade liberalization squeezes the lowest quality firms out of the market leading to an increase of the average quality of goods. The demand for skilled labor will increase and therefore as well the skill premium. Such a finding would be similar to the skill-upgrading effect found in Verhoogen (2008).

Although most of the models in this thesis provide a theoretical account for existing empirical findings, the models generate various interesting additional predictions that can be tested. First, the predictions from the oligopoly chapter on zero trade flows and unit values depending on the size of the importer country and the size of trade costs can be tested. These predictions are different from the predictions derived in Baldwin and Harrigan (2007) in a Melitz-type model with quality differences. Baldwin and Harrigan (2007) find empirical support for their findings. But it could be that the predictions of the oligopoly model are still valid in sectors where the oligopoly model is a good approximation, i.e. in sectors with (nearly) homogeneous products without quality differences. The implication would be that the effect of importing country size
and trade costs on unit values and the probability of zero trade flows is sector dependent.

Second, the predictions derived in chapter 5 and 6 that richer countries and more capital and skill abundant countries export goods with higher unit values can be tested in a larger and wider dataset. Schott (2004) derived these empirical results with a dataset of American imports. It is by no means sure that these results hold through in a wider dataset.

A third and final empirical extension follows from the combination of the models of chapter 5 and 6 in a simulation. The expected prediction is that trade liberalization would lead to an increasing demand for skills and a rising skill premium. Do the data support this view? As studies of episodes of liberalization involve all kind of complications (Tybout (2001)) indirect evidence could be found by exploring the skill and capital intensity of exporters compared to importers. If their skill intensity is larger than of domestic producing firms, more trade would lead to an increased demand for skills and a rising skill premium. Such findings would lead to an interesting contribution to the heated debate about trade and wage inequality.
Samenvatting

Dit proefschrift levert een bijdrage aan de literatuur over de rol van heterogene bedrijven bij internationale handel, de zogenaamde nieuw nieuwe handelstheorie. De nieuwe handelstheorie neemt imperfecte concurrentie op in handelsmodellen om intra-industriële handel te kunnen verklaren. De nieuw nieuwe handelstheorie generaliseert deze modellen door ook rekening te houden met verschillen tussen bedrijven. Door verschillen tussen bedrijven expliciet op te nemen in de theorie, kan het reallocatie-effect van handel gemodelleerd worden: door meer internationale handel winnen meer productieve exporterende bedrijven marktaandeel ten koste van minder productieve bedrijven die alleen voor de binnenlandse markt produceren.


De imperfecte concurrentiemodellen in de nieuwe handelstheorie gaan er vanuit dat alle bedrijven gelijk zijn. Door deze aanname te verruimen, ontstaat een vijfde voordeel van internationale handel: het reallocatie-effect. Handel zorgt ervoor dat meer productieve exporterende bedrijven marktaandeel winnen ten koste van minder productieve
bedrijven die alleen voor de binnenlandse markt produceren. Het resultaat is dat de gemiddelde productiviteit omhoog gaat. Het reallocatie-effect kan ontstaan door meer concurrentie op de productmarkt of door meer concurrentie op de arbeidsmarkt. In het eerste geval leidt internationale handel tot meer concurrentie op de productmarkt, waardoor de minst productieve bedrijven uit de markt verdwijnen. In het tweede geval leidt internationale handel tot meer afzetmogelijkheden van de meest productieve bedrijven. Het gevolg is dat de vraag naar arbeid stijgt, waardoor de lonen omhoog gaan en de minst productieve bedrijven niet meer winstgevend kunnen produceren en verdwijnen.


Een eerste bijdrage van dit proefschrift in hoofdstuk 4 bestaat uit een model van heterogene bedrijven in een oligopoliesetting met Cournot concurrentie. Het model is een logische uitbreiding van het wederzijdse dumping model van Brander and Krugman (1983). Er is CES vraag tussen sectoren en Cournot concurrentie binnen sectoren tussen bedrijven met heterogene productiviteit. Productie wordt gekenmerkt door constante schaalvoordelen en bedrijven moeten verzonken toetredingskosten betalen om hun productiviteit te weten te komen. Er wordt onderscheid gemaakt tussen een lange en korte termijn analyse, afhankelijk van de aanwezigheid van een vrij toetredingsconditie. Alle resultaten worden afgeleid voor een algemene verdeling van de productiviteit van bedrijven. De belangrijkste resultaten die gevonden worden, zijn: in een model met weinig aannamen wordt een reallocatie-effect van handel afgeleid; zowel op de korte als op de lange termijn verdwijnen de minst productieve bedrijven van de markt en daalt de marktprijs; het model nest het wederzijdse dumping model en het Ricardiaanse model van comparatieve voordelen als speciale gevallen; Zoals in Brander and Krugman (1983) kan het welvaartseffect van lagere handelsbarrières negatief zijn op de korte termijn vanwege een toename in het 'zinloos transporteren' van goederen. Maar de analyse in het model in dit proefschrift kan preciezer aangeven wanneer het welvaartseffect negatief is, afhankelijk van de productiviteitsverdeling van bedrijven; het model bevat ook voorvertoning over de waarschijnlijkheid van nul handelsstromen tussen landen en de waarde van importen afhankelijk van de handelskosten en de economische omvang van het importerende land; een laatste interessant resultaat is dat unilateral liberalisatie leidt tot lagere prijzen op de korte termijn maar hogere prijzen op de lange termijn in het liberaliserende land. Dit wordt veroorzaakt door relocatie-effecten.

In hoofdstuk 3 wordt een model met exportonzekerheid en heterogene populariteit
voorgesteld. Het model start vanuit de empirische bevinding dat veel exportierende bedrijven de exportmarkt alweer verlaten kort nadat ze zijn toegetreden. Eaton et al. (2007) vinden bijvoorbeeld in een dataset van Colombiaanse bedrijven dat meer dan tweederde van de exportierende bedrijven de exportmarkt weer verlaat in het eerste jaar. Besedes and Prusa (2006) laten met Amerikaanse data zien dat een derde van de importen in een bepaalde productcategorie weer stopt binnen 1 jaar. Het model dat wordt voorgesteld om deze bevindingen te verklaren is een heterogene bedrijvenmodel waar bedrijven heterogen zijn in de populariteit van hun variëteit. De populariteit is verschillend in verschillende markten maar is wel gecorreleerd. Bedrijven kennen hun populariteit niet voordat ze beginnen met produceren en moeten verzonken toetredingskosten betalen om de populariteit te weten te komen. Dus als bedrijven de populariteit van hun goed op de binnenlandse markt weten, weten ze nog niet hoe succesvol ze zullen zijn op de exportmarkt. De implicatie is dat een gedeelte van de bedrijven probeert toe te treden tot de exportmarkt, maar de exportmarkt alweer moet verlaten kort na toetreding omdat ze niet winstgevend kunnen produceren.

Er zijn drie soorten handelskosten in het model, ijsberghandelskosten, verzonken exportkosten en vaste exportkosten. Een analyse van het model laat zien dat voor alle drie deze handelskosten lagere handelskosten leiden tot een hogere populariteit van het bedrijf dat net in de markt kan blijven en dus leiden tot een reallocatie-effect richting bedrijven met een hogere populariteit. Echter, de effecten van lagere vaste exportkosten en van lagere verzonken exportkosten op exportsucces (de kans om succesvol te exporteren gegeven dat een bedrijf is toegetreden tot de exportmarkt) hebben een verschillend teken. Lagere vaste exportkosten verhogen de kans op exportsucces, terwijl lagere verzonken exportkosten de kans op exportsucces verlagen.


Bibliography


Bekkers, Eddy and Joseph F. Francois (2008), ‘Heterogeneous Productivity and Trade under Oligopoly,’ Mimeo Erasmus University Rotterdam.


Menezes-Filho, Naercio Aquino and Muendler, Marc-Andreas, ‘Labor Reallocation in Response to Trade Reform,’ Mimeo University California San Diego.


