



Disentangling the enigma of multi-structured economic cycles - A new appearance of the golden ratio

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ABSTRACT

We study whether there is an interrelationship between the lengths of economic cycles. Such an interrelationship would be helpful to signal future economic downturns, thus to alleviate economic and societal distress. To detect the lengths of economic cycles, we introduce an improved method, where Fourier analysis is coupled with GARCH regression, mixed distribution estimation, and harmonic regression. We apply our methodology to detect cycles in percentage GDP growth in 25 OECD countries, and in Europe. The results indicate that in each economy, between two and five cycles are present. Cycles with a length between 5–6 years and between 9–10 years appear most frequently. A meta-analysis on the detected cycle lengths reveals that the ratio between the lengths of consecutive cycles often closely matches the golden ratio, ϕ . Interestingly, this finding opposes several existing theories about multi-cycle structures, which imply that the lengths of shorter cycles should be integer fractions of the lengths of longer cycles. Our paper thus provides a new direction for theory development regarding economic cycles and dynamic stability.

1. Introduction

Originating from the seminal work of [Kitchin \(1923\)](#), [Schumpeter \(1939\)](#) and [Burns and Mitchell \(1947\)](#), many researchers have demonstrated that economies systematically alternate between periods of expansion and contraction, that is, that economic growth exhibits cyclic behavior. See [Beveridge and Nelson \(1981\)](#), [Hillmer and Tiao \(1982\)](#), [Nelson and Plosser \(1982\)](#), and [Watson \(1986\)](#) among others. It is also widely accepted that macro-economic variables often display not one, but multiple subcycles with varying length. For example, several subcycles have been found in world consumption ([Kuczynski, 1978](#)), in world industrial production ([Haustein and Neuwirth, 1982](#)), in US, UK, French and German GDP ([Solomou, 1998](#)), in US and world inflation ([Berry et al., 2001](#); [Berry, 2000](#)), in US, UK and Dutch Gross Domestic Product (GDP), employment, interest rates and consumption ([De Groot, Franses, 2008](#)), in basic innovations ([De Groot, Franses, 2009](#)), in world GDP ([Korotayev and Tsirel, 2010](#)), and in commodity currencies ([Sani-das, 2014](#)).

Over the past 100 years, many fine scholars have attempted to disentangle the enigma of multi-structured economic cycles. Within this field, an interesting research angle concerns the question whether there

exists an interrelationship between the lengths of subcycles. Indeed, if such a relationship would be strong and stable over time, this would significantly improve cycle detection. Improved cycle detection would in turn facilitate signalling future turning points. This would alleviate economic and societal distress.

In the literature on multiple cycle structures, we find one body of work where researchers set out to detect a relationship between the lengths of well-documented subcycles, such as the Kitchin cycle, the Juglar cycle, the Kuznets cycle, the Kondratieff wave, and the war and hegemony cycle. A second body of work concerns theoretical explanations for the existence of subcycles and their potential interrelationship. We find that this literature is inconclusive about the particular subject of an interrelation between cyclical lengths. In this paper, we therefore pursue a different approach. We first measure the lengths of subcycles from macro-economic time series, using an improved econometric cycle detection method, without any prejudgement regarding the interrelationship between subcycles. We then aim to discern a pattern in the lengths of the detected subcycles. Any detected pattern between the cycle lengths provides a direction for future theory development.

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1.1. The quest for an interrelationship between subcycles

To substantiate our observation that the existing literature is inconclusive on the interrelationship between cyclical lengths, we briefly discuss the relevant scientific works here.

Fitting the waves together

Several researchers have attempted to fit well-documented subcycles together “like the pieces of a jigsaw puzzle” (Reijnders, 1990). In an illustrative example, Schumpeter (1939) already suggested that a Kondratieff wave of approximately 60 years may cycle in sync with six Juglar cycles of 7–11 years, and each Juglar cycle with three Kitchin cycles of 3–5 years. In a similar notion Van Duijn (1983) states that a Kondratieff wave consists of five Juglar cycles. The research in Berry and Kim (1994); Berry et al. (1993) combines leadership generations and long wave crises, respectively inspired by the 90-year generation waves presented in Strauss and Howe (1992) and the Kondratieff wave. The work of Tylecote (1994, 2013) argues that Kuznets waves with length between 15–25 years are embedded in the Kondratieff wave, and that Kondratieff waves are again embedded in lengthier hegemony waves. The relation between Kondratieff waves, hegemony and war is also described by Goldstein (1988); Thompson (1990); Wallerstein (1983). Their work conjectures that war and hegemony cycles of 150 years encompass three Kondratieff waves.

Theories inspired by physics

Next, we discuss several theoretical works about the interrelationship between cycles.

One direction in the literature postulates that the number of subcycles in an economy is likely to be small and that these subcycles tend to be closely linked. Forrester (1977) was perhaps the first to elaborate on this idea, suggesting that the underlying mechanism might be similar to that of entrainment in physics. This is the mechanism whereby a small amount of coupling in a system tends to draw together two or more periodic modes that are of about the same period of oscillation. Entrainment is also referred to as mode-locking. Building upon this idea, Mosekilde et al. (1992) developed a mathematical model for the interaction between oscillatory modes. Larsen and Haxholdt (1997) extended this work, demonstrating mode-locking in the context of the well-known Goodwin business cycle (Goodwin, 1951). Finally, Berry (2000) motivated the presence of mode-locking by the notion of a pacemaker: a rhythm-forcing agent that is itself cyclic and that functions to entrain weaker cycles to itself or to force other cycles to assume synchronization. In subsequent work, Berry et al. (2001) found empirical evidence for the presence of mode-locking cycles in world inflation and detected cycle lengths of 6, 9 and 18 years. According to the theory, these cycles are embedded in the 55-year Kondratieff wave (Berry and Dean, 2012).

Theories inspired by evolution

The work of Devezas and Modelski (2003); Devezas and Corredine (2001, 2002); Modelski (2006) hypothesizes that social change is a self-similar process, a pattern of regularity in global politics that chart change rather than exact repetition. Evolutionary systems in the world are nested multilevel processes that exhibit fractal and power law behavior. These topics are conscientiously discussed in Bak et al. (1987). Supposedly, this self-organizing system guides transition periods. The transition periods are identified to respectively lie 30, 60, 120, 250, 500, 1000, 2000, 4000 and 8000 years apart.

Theories inspired by dynamic stability

According to Schumpeter (1939), there exist many, or perhaps even an indefinite number of subcycles in an economy. In the long run, however, only a few subcycles remain stable and persist, whereas others become unstable and cease. De Groot, Franses, 2008 adopt this view, suggesting that business cycles are partly deterministic and coexist independently of one another. It is the sum of these cycles that creates

erratic behavior. It is also the sum of the cycles that absorbs exogenous shocks in the economy. The underlying arrangement of the cycles institutes stability in the economy. The empirical analysis in De Groot, Franses, 2008 reveals four cycle periods: 10, 26, 58 and 92.

1.2. Econometric cycle detection

Detecting cycles in macro-economic time series is not trivial. It requires the researcher to infer which cycle lengths are present in the signal, but also to determine the number of subcycles. Selecting the cycle lengths and the number of cycles using regression models is generally unfeasible. This requires iterating over all possible permutations of cycle lengths and the number of cycles. Fourier analysis, also referred to as spectral analysis, is more suitable for the task of cycle detection. There exists a vast amount of literature on this topic, see for example Berry et al. (2001); Bodger et al. (1986); Martin (2001); Stoica and Moses (2005). A drawback of spectral analysis is that the results are unreliable when trends and irregular components vary over time, that is, when the time series of interest displays signs of non stationarity (Gabor, 1946).

In this paper, we account for the effects of non stationarity by estimating a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model (Bollerslev, 1986) on the data. The weakly stationary standardized residuals of the GARCH model are then used as the input signal in a Fourier analysis. Secondly, to identify the number of prominent cycles in the data, corresponding with the peaks in the Fourier periodogram, a mixed distribution is estimated using the Expectation-Maximization (EM) algorithm (Dempster et al., 1977). Finally, given the obtained cycles and their estimated lengths, we estimate a classical trend-cycle model, where the cycles are captured using harmonic terms. This allows us to obtain the amplitudes and phases of our cycles.

We apply our cycle detecting method to a panel of GDP growth rates for 25 OECD countries, and for Europe. A meta-analysis on the detected cycle lengths allows us to investigate whether an interrelationship between the lengths of subcycles can be discerned.

The paper is organized as follows. Section 2 provides a discussion on Fourier analysis, which is at the heart of our cycle detection method. In Section 3, we present our cycle detection method for macro-economic time series. Section 4 presents the results of our empirical application. In Section 5 we discuss the implications of our findings for research and society. Finally, Section 6 concludes.

2. Fourier analysis

In this section we discuss Fourier analysis, also referred to as spectral analysis, which is at the heart of our cycle detection method. The basic concept of Fourier analysis is that every time series, or signal, can be perfectly decomposed into cosine waves. The technique was originally developed in the nineteenth century by Joseph Fourier to model heat dissipation (Fourier, 1822). It has since been widely applied in numerous fields where oscillations occur, for example in electrical engineering (Abraham, 1898), quantum mechanics (Bloch, 1928), and acoustics (Helmholtz, 1878). More recently, spectral analysis has been applied in macroeconomic analysis (de Jong and Dave, 2011) and in finance (Racicot, 2011).

2.1. The Fourier transform

Fourier analysis enables us to represent a function or a time series y_t , that is defined on the *time* domain, in the *frequency* domain. The latter is called the Fourier Transform (FT) of y_t , and will be denoted by F_y . The FT is again a data set or function. The domain of y_t is a finite set of N values at evenly spaced moments in a time interval $[0, T]$. The domain of F_y consists of N evenly spaced frequencies ranging from $[0, (N - 1)]$. The FT maps this domain to a set of N complex numbers of the form $z = a + bi =$

$$\sqrt{a^2 + b^2} e^{i \arctan(b/a)};$$

$$F_y(k) := \sum_{t=0}^{N-1} y_t e^{-2\pi i k t / N} = \sum_{t=0}^{N-1} y_t \left(\cos(2\pi k t / N) - i \sin(2\pi k t / N) \right) \quad (1)$$

The Fourier transform has a well-defined inverse, denoted by F^{-1} :

$$F_z^{-1}(t) := \frac{1}{N} \sum_{k=0}^{N-1} z_k e^{2\pi i k t / N} = \frac{1}{N} \sum_{k=0}^{N-1} z_k (\cos(2\pi k t / N) + i \sin(2\pi k t / N)). \quad (2)$$

Both of these equations can be derived using Euler's formula and De Moivre's formula (Markushevich, 2013). Without explaining the intermediate steps we present a short derivation to show that $F_{F_y}^{-1}(t) = y_t$:

$$\begin{aligned} F_{F_y}^{-1}(t) &= \frac{1}{N} \sum_{k=0}^{N-1} \left(\sum_{s=0}^{N-1} y_s e^{-2\pi i k s / N} \right) e^{2\pi i k t / N} \\ &= \frac{1}{N} \sum_{k=0}^{N-1} \sum_{s=0}^{N-1} y_s e^{2\pi i k (s-t) / N} = \frac{1}{N} \sum_{s=0}^{N-1} \sum_{k=0}^{N-1} y_s e^{2\pi i k (s-t) / N} \\ &= \frac{1}{N} \sum_{s=0}^{N-1} y_s \sum_{k=0}^{N-1} e^{2\pi i k (s-t) / N} \\ &= \sum_{s=0}^{N-1} y_s \delta_{st} = y_t. \end{aligned} \quad (3)$$

Hence, no information is lost by taking the FT of y_t .

2.2. Harmonic representation

The FT and the inverse FT can be combined to derive an alternative representation of y_t as a sum of harmonic terms. Assuming that N is odd, we derive:

$$\begin{aligned} y_t = F^{-1} F_y(t) &= \frac{1}{N} \sum_{k=0}^{N-1} F_y(k) e^{2\pi i k t / N} = \frac{1}{N} F_y(0) + \frac{1}{N} \sum_{k=1}^{N-1} F_y(k) e^{2\pi i k t / N} \\ &= \frac{F_y(0)}{N} + \frac{1}{N} \left(\sum_{k=1}^{(N-1)/2} F_y(k) e^{2\pi i k t / N} + \sum_{k=(N+1)/2}^{N-1} F_y(k) e^{2\pi i k t / N} \right) \\ &= \frac{F_y(0)}{N} + \frac{1}{N} \left(\sum_{k=1}^{(N-1)/2} F_y(k) e^{2\pi i k t / N} + \sum_{l=1}^{(N-1)/2} F_y(N-l) e^{2\pi i (N-l) t / N} \right) \\ &= \frac{F_y(0)}{N} + \frac{1}{N} \left(\sum_{k=1}^{(N-1)/2} F_y(k) e^{2\pi i k t / N} + \sum_{l=1}^{(N-1)/2} F_y(N-l) e^{2\pi i (N-l) t / N} \right) \\ &= \frac{F_y(0)}{N} + \frac{1}{N} \sum_{k=1}^{(N-1)/2} \left(F_y(k) e^{2\pi i k t / N} + F_y(N-k) e^{-2\pi i k t / N} \right) \\ &= \frac{F_y(0)}{N} + \frac{1}{N} \sum_{k=1}^{(N-1)/2} \left(F_y(k) e^{2\pi i k t / N} + F_y^*(k) e^{-2\pi i k t / N} \right) \\ &= \frac{F_y(0)}{N} + \frac{1}{N} \sum_{k=1}^{(N-1)/2} \left(|r_k| e^{i\phi_k} e^{2\pi i k t / N} + |r_k| e^{-i\phi_k} e^{-2\pi i k t / N} \right) \\ &= \frac{F_y(0)}{N} + \frac{2}{N} \sum_{k=1}^{(N-1)/2} \text{Re} \left(|r_k| e^{i(2\pi k t / N + \phi_k)} \right) \\ &= \frac{F_y(0)}{N} + \frac{2}{N} \sum_{k=1}^{(N-1)/2} |r_k| \cos(2\pi k t / N + \phi_k). \end{aligned} \quad (4)$$

In this derivation, we have applied the Euler notation for complex numbers: $z = a + bi = r e^{i\phi}$. Here, $r = a^2 + b^2$ and $\phi = \arctan(b/a)$ which allows to write $F_x(k) = r_k e^{i\phi_k}$. A similar derivation can be done for the case when N is even. For every k we can compute $F_y(k)$, hence the amplitude of the cosine wave is given by $r_k = |F_y(k)|$, and the phase of the cosine wave is equal to $\phi_k = \arctan(\text{Im } F_y(k) / \text{Re } F_y(k))$. This result demonstrates that we are able to exactly represent the original time series y_t as the sum of cosine waves. A graph plotting the amplitude and phase of the FT against the Fourier frequencies is called the *spectrogram*.

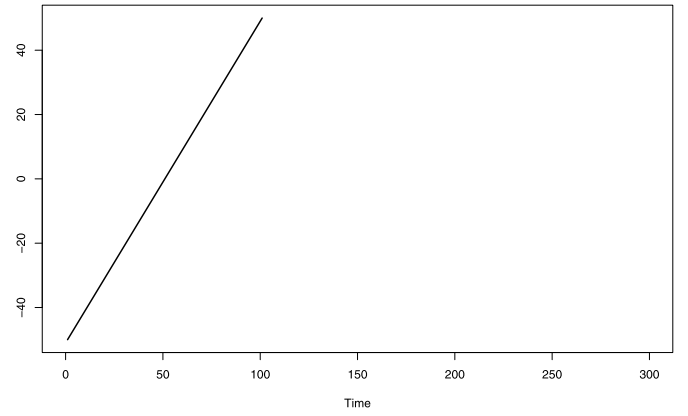


Fig. 1. Linear trend y_t .

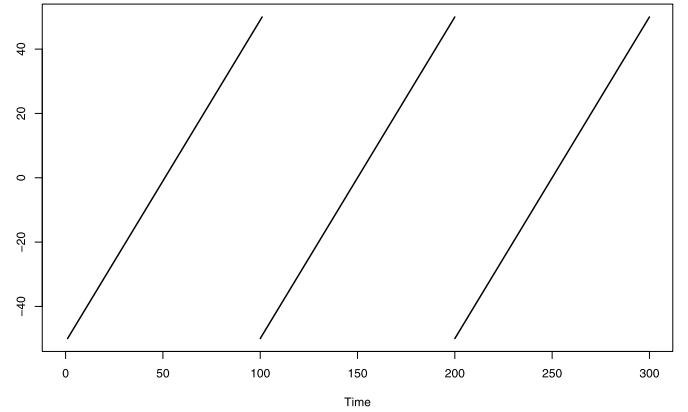


Fig. 2. Periodic continuation of y_t .

This is commonly used to graphically depict the properties of the harmonic representation.

2.3. Properties of the Fourier transform

The FT is an excellent tool to extract sinusoidal frequency components of any time series, as long as the time series is sufficiently stationary, see Gabor (1946), among others. In order for the Fourier transform and the harmonic representation to be accurate, weak stationarity of the time series y_t has to be guaranteed. This means that the time series should have a constant mean and constant variance. In this section, we illustrate why this is important.

2.3.1. A time-varying mean

The FT reviews a time series y_t as if it was sampled from a periodic function that completes one or more complete cycles in the observed domain $[0, 1, \dots, N-1]$. The harmonic representation of y_t in Equation (4) then allows for an extension of the domain from $[0, 1, \dots, N-1]$ to \mathbb{Z} , which is the periodic continuation of y_t . Fig. 1 and Fig. 2 give an example for the case where y_t is only a linear trend: $y_t = t - \frac{N}{2}$ on $t \in [0, 1, \dots, N-1]$.

Although there is no harmonic behavior present in the time series, the FT aims to model the trend, and the corresponding discontinuous periodic extension, as if it consisted of continuous sinusoids. This process will typically have a strong perturbing effect on the FT values. Therefore, it is necessary to remove any trend in the time series. Further discontinuities (in the periodic extension) arise when the boundaries are unequal: $y_0 \neq y_{N-1}$. This can be solved by subtracting the linear function $g(t) = y_0 + t \frac{y_{N-1} - y_0}{N}$ from the time series before taking the FT.

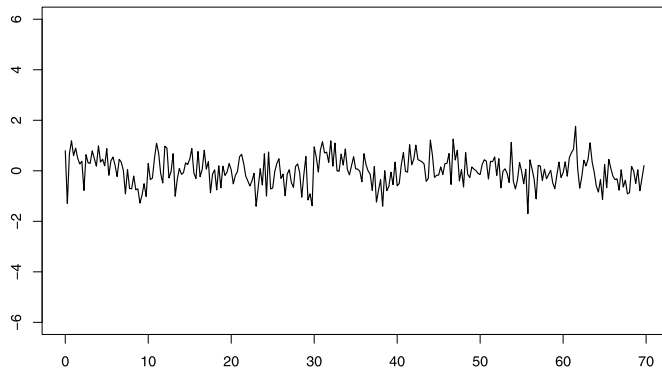


Fig. 3. Time series with constant variance.

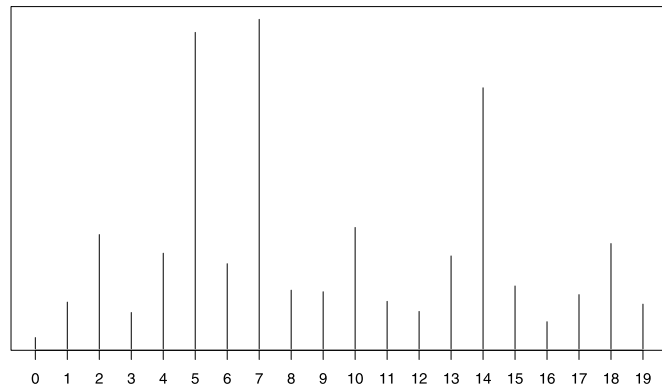


Fig. 4. Spectrogram of the time series with constant variance.

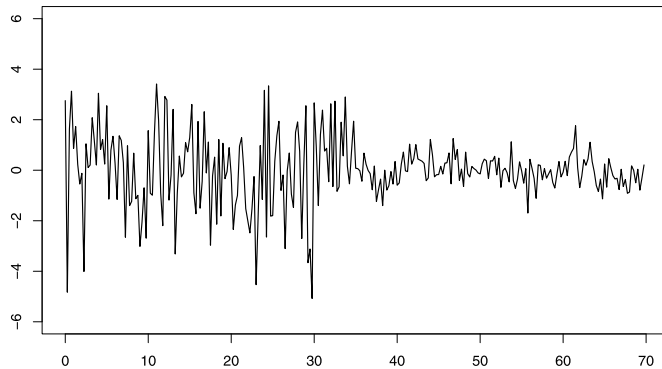


Fig. 5. Time series with non-constant variance.

2.3.2. Time-varying volatility

To illustrate the effect of non-constant variance we look at an example. We start with a time series composite of three sinus waves and a noise term:

$$y_t = 0.25\sin\left(\frac{5 \times 2\pi t}{N}\right) + 0.25\sin\left(\frac{7 \times 2\pi t}{N}\right) + 0.25\sin\left(\frac{14 \times 2\pi t}{N}\right) + \varepsilon_t \quad (5)$$

where $\varepsilon_t \sim N(0, 0.25)$, and $N = 280$. The corresponding series is shown in Fig. 3. In Fig. 4 we display its spectrogram, showing clear peaks at the frequencies 5, 7 and 14.

Next, the same time series is generated, but the error terms are now distributed differently over time. For $t \leq 140$ the error terms are multiplied by $\sqrt{12}$, and thus exhibit higher volatility: $\varepsilon_t \sim N(0, 3)$. The frequency of the composite sinusoid is unchanged, but the variance of

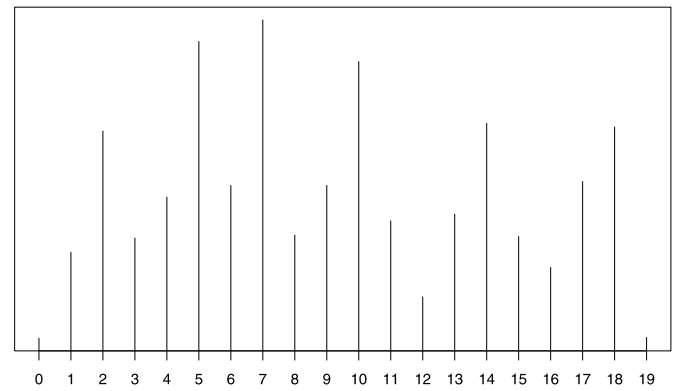


Fig. 6. Spectrogram of the time series with non-constant variance.

the error term differs over time. The corresponding series is shown in Fig. 5 and Fig. 6 displays the spectrogram. In the spectrogram, which is evidently disturbed, other frequencies than 4, 7 and 14 are detected. Perhaps most notably, it contains an incorrect peak at frequency 10.

The illustrated problem arises whenever y_t has different local properties. The FT correlates with sinusoids that are non-local, so all local information is spread over the entire domain. The example demonstrates how strong spurious results can grow under the effect of non-constant variance, that is, when the time series of interest displays heteroscedastic behavior over time.

3. Methodology

The aim of our research is to detect and measure the cycles that are present in economic time series. Commonly, economic variables that exhibit cyclical behavior, are decomposed as follows (Hillmer and Tiao, 1982; Nelson and Plosser, 1982; Watson, 1986):

$$y_t = T_t + C_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \quad (6)$$

where y_t is the time series of interest, T_t is the trend component, C_t is the cyclical component, and ε_t is an error term with variance σ_ε^2 . This paper focuses on the estimation of the cyclical component with harmonic regression, discussed in Section 3.1. Fourier analysis is a useful tool to determine the appropriate cycle lengths that are used in the harmonic regression. However, non stationarity in the data of interest and trivial values in the Fourier frequencies form genuine obstacles that have to be overcome in the application of Fourier analysis. Therefore, Section 3.2 introduces a method to account for non stationary behavior in the signal. Finally, Section 3.3 specifies how the peaks in the periodogram, that represent the most prominent cycle lengths, are detected.

3.1. Harmonic regression

To model the cyclical component in Equation (6), we employ a harmonic regression model (Artis et al., 2007). A harmonic regression model is a special case of Equation (6) where the cyclical component is formulated as:

$$C_t = d + \sum_{j=1}^k \alpha_j \cos\left(\frac{2\pi t}{\omega_j} + c_j\right) \quad (7)$$

where d is a constant shift parameter, α_j is the amplitude, defined as half the bandwidth of the j^{th} cycle, ω_j is the length of the j^{th} cycle, c_j is its shift relative to the starting point $t = 0$, and k is the number of cycles to be included. The cosine terms in the harmonic regression are also referred to as Fourier terms, due to the strong resemblance between harmonic regression and Fourier analysis. Note that the cycles in Equation (7) are deterministic, which is tractable for out-of-sample estimation. Stochastic

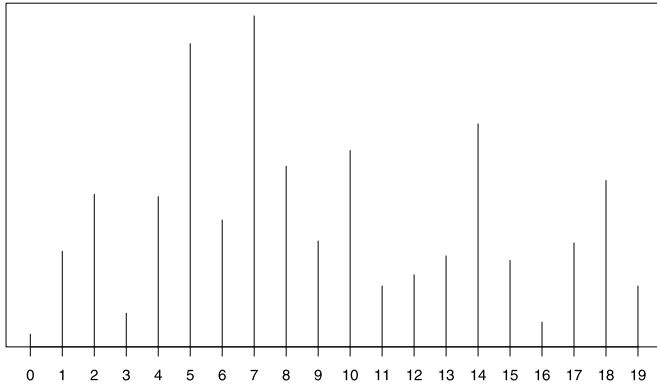


Fig. 7. Spectrogram of the standardized residuals.

cycles have been introduced and popularized by Harvey and Trimbur (2003), among others.

3.2. Accounting for potential non stationarity

To account for potential non stationary in the data before applying Fourier analysis, we estimate a model for the mean and variance of the time series of interest. We then calculate the standardized residuals of the model, defined as the data minus the trend term scaled with the estimated standard deviations. In this paper, we estimate the variances with a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model (Bollerslev, 1986; Engle, 1982). The GARCH model is a well-studied representation of a time series with conditional heteroskedasticity. The mean equation of the model is specified as follows:

$$y_t - \mu_t = \varepsilon_t$$

$$\mu_t = \mu + \sum_{i=1}^{m-n} \delta_i x_{i,t} + \sum_{i=m-n+1}^m \delta_i x_{i,t} \sqrt{h_t} \quad (8)$$

where

$$V[y_t | \mathcal{I}_{t-1}] = E[\varepsilon_t^2 | \mathcal{I}_{t-1}] = h_t. \quad (9)$$

Here δ_i is the parameter corresponding to the exogenous variable x_i , and \mathcal{I}_{t-1} is the set of information up until t . In the specification for the mean function μ_t , optionally m variables are scaled with the conditional standard deviation $\sqrt{h_t}$. In Equation (9) the error term ε_t follows a GARCH process. The GARCH(r,s) process is formulated as an ARMA(r,s) model for the conditional variance h_t :

$$\varepsilon_t = \sqrt{h_t} z_t, \quad z_t \sim i.i.d.N(0, 1)$$

$$h_t = \zeta_0 + \sum_{i=1}^r \psi_i \varepsilon_{t-i}^2 + \sum_{i=1}^s \nu_i h_{t-i} \quad (10)$$

The GARCH model is equipped to describe the variability in the long-run mean and variance of many time series. Provided that this is true for a specific time series at hand, the standardized residuals are weakly stationary but still contain the same cyclic behavior as the original signal. As a consequence, we can apply spectral analysis on these standardized residuals to detect multi-structured cycles.

To illustrate this, we model the heterogeneity in the disturbed time series shown in Fig. 5 with a simple variance equation which includes a dummy variable that equals one for first half of the observations. Fig. 7 presents the resulting spectrogram. The peaks in this spectrogram correspond to the actual frequencies of the sinusoid, 5, 7, and 14, and the disturbance over the other frequencies is drastically reduced.

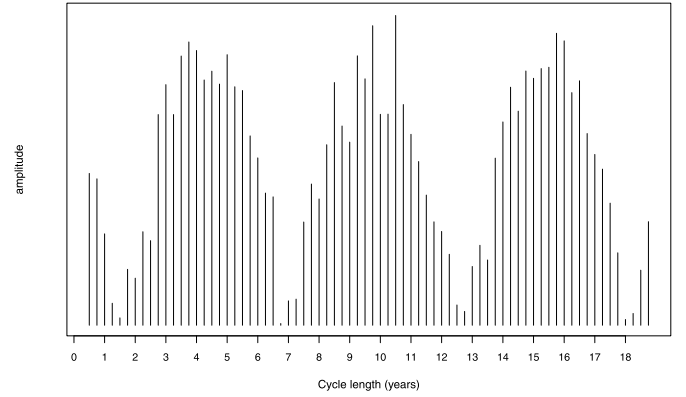


Fig. 8. Example periodogram.

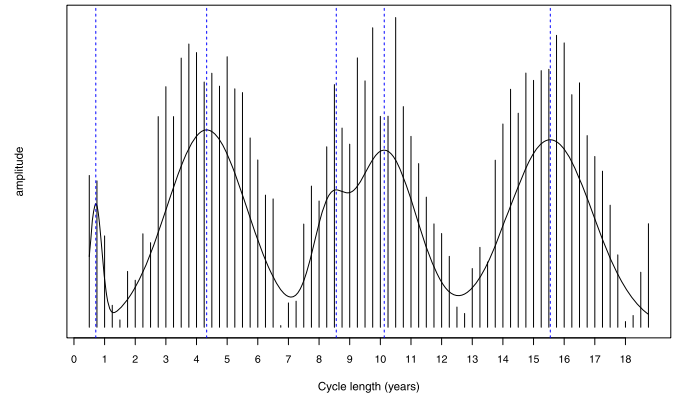


Fig. 9. Example periodogram with selected cycles.

3.3. Determining the number of cycles and their corresponding cycle lengths

The FT results in a set of combinations of frequencies and amplitudes. We review the periodogram, where the cycle lengths, the reciprocal of the frequencies divided by N , are presented on the x-axis. An example is given in Fig. 8. The significant cycles in the economy are indicated by the cycle lengths with the largest amplitude. However, because of the effects of *leakage* it is not evidently clear which exact cycle lengths have the largest amplitude. For example in Fig. 8, the local maxima of the periodogram can be anywhere between 3–6, 8–11 and 14–17. Leakage is a problem that occurs when the length of the time series does not match the actual frequencies of the data. The effect of leakage is similar to that of a non-constant variance, and as a result, the FT will assign part of the signal to neighbouring frequencies, giving a disturbed view of the actual data generating process. Furthermore, the periodogram provides little information about the number of cycles that construct the signal. For example, there could be either one or two modes between cycle lengths 8–11 in Fig. 8.

We therefore adopt the following data-driven approach to select the number of cycles and their lengths. First, we estimate a mixture of Gaussian distributions with different means and standard deviations on the periodogram, using the Expectation-Maximization (EM) algorithm of Dempster et al., 1977. The local maxima of the mixed distribution, which are the modes of the components of the mixed distribution, represent the lengths of the cycles. Secondly, to determine the number of Gaussian distributions that construct the mixture, that is, to determine the number of components m , we estimate the mixed distribution for different choices of m . The number m that strikes the best balance between the fit of the mixture distribution and the corresponding number of parameters is then determined using the Bayesian Information

Table 1
Summary of the OECD panel data.

Country	Available years	Country	Available years
Australia	1961 Q1 - 2018 Q4	Japan	1961 Q1 - 2018 Q4
Austria	1961 Q1 - 2019 Q1	Korea	1961 Q1 - 2019 Q1
Belgium	1961 Q1 - 2019 Q1	Luxembourg	1961 Q1 - 2018 Q4
Canada	1962 Q1 - 2018 Q4	Mexico	1961 Q1 - 2019 Q1
Czech Republic	1961 Q1 - 2018 Q4	Norway	1961 Q1 - 2019 Q1
Denmark	1961 Q1 - 2019 Q1	OECD Europe	1962 Q1 - 2018 Q4
Finland	1961 Q1 - 2019 Q1	Portugal	1961 Q1 - 2019 Q1
France	1961 Q1 - 2019 Q1	South Africa	1961 Q1 - 2018 Q4
Germany	1961 Q1 - 2019 Q1	Spain	1961 Q1 - 2019 Q1
Greece	1961 Q1 - 2018 Q4	Sweden	1961 Q1 - 2018 Q4
Iceland	1961 Q1 - 2018 Q4	The Netherlands	1961 Q1 - 2019 Q1
Ireland	1961 Q1 - 2018 Q4	UK	1956 Q1 - 2019 Q1
Italy	1961 Q1 - 2019 Q1	USA	1948 Q1 - 2019 Q1

Criterion (BIC):

$$\text{BIC} = \ln(N)k - 2\ln(\hat{L}) \quad (11)$$

where N is the number of observations, k the number of parameters, and \hat{L} the estimated Log Likelihood of the estimated mixture distribution. Fig. 9 shows the estimated mixed distribution on the periodogram of Fig. 8. In this example, the method selected five cycles, where the cycle length are indicated by the dotted vertical lines.

Table 2
Selected cycle lengths for each of the 25 countries and Europe.

Country	Cycle length in years											
	3–4	4–5	5–6	6–7	7–8	8–9	9–10	10–11	11–12	12–13	13–14	14–15
Australia	-	-	5	-	-	-	9.3	-	-	-	-	14.7
Austria	-	-	-	6	-	-	9.4	-	-	-	-	14.9
Belgium	-	4	-	-	-	-	9.3	-	-	-	-	-
Canada	3.7	-	-	6.2	-	-	-	-	-	12	-	-
Czech Republic	-	4.4	-	-	7.1	-	9.3	-	-	-	-	-
Denmark	3.9	-	-	-	7.1	-	-	-	11.5	-	-	-
Finland	-	-	5.3	-	-	-	9	-	-	12.9	-	-
France	-	-	5.6	-	-	-	9.6	-	-	-	-	-
Germany	-	-	5.1	-	-	-	9.6	-	-	-	13.1	-
Greece	3.2	-	-	6.4	-	-	9	-	-	-	-	-
Iceland	-	-	-	-	-	8.4	-	10.6	-	-	-	-
Ireland	3.5	-	5.4	-	7.1	-	9.5	-	-	-	-	14.7
Italy	-	-	5.4	-	-	-	9.9	-	-	-	-	-
Japan	-	-	5.5	-	-	-	9.3	-	-	-	-	-
Korea	-	4.7	-	-	-	-	9.2	-	-	-	-	-
Luxembourg	-	4	-	-	7.2	-	9.1	-	-	-	-	14.3
Mexico	-	-	5.1	-	-	8.7	-	-	11.9	-	-	-
Norway	-	-	-	6.2	-	-	-	10.4	-	-	-	-
OECD Europe	-	-	5.5	-	-	-	9.5	-	-	-	-	-
Portugal	3.2	-	-	-	-	-	9.6	-	-	-	-	-
South Africa	-	-	5.4	-	-	8.2	-	-	-	12.8	-	-
Spain	-	4.9	-	-	-	-	9.2	-	-	-	-	-
Sweden	-	-	5.1	-	-	-	9.9	-	-	-	-	-
The Netherlands	-	-	5.4	-	-	-	9.5	-	-	-	13.6	-
UK	-	-	5	-	-	8.7	-	-	-	-	-	14
USA	3.9	-	5.4	6.9	-	-	-	-	11.2	-	-	-
Count	6	5	14	5	4	4	18	2	3	3	2	5

4. Results

We apply our methodology to measuring economic cycles in percentage GDP growth, since GDP is the most widely-used indicator for economic welfare (Stiglitz et al., 2017). Quarterly GDP growth is conveniently administered by the Organisation for Economic Co-operation and Development (OECD, 2019) for all its members. We focus on 25 countries, since for these countries quarterly GDP growth rates are available from at least 1962. Additionally, we analyse GDP growth in Europe as a whole. In Table 1 we provide a summary of the panel data set.

4.1. Cycle detection

Firstly, we estimate the following model with time-varying mean and GARCH(1,1) distributed error terms on the panel of GDP growth rates, y_t :

$$\begin{aligned} y_t - \mu_t &= \varepsilon_t, \\ \mu_t &= \mu + \log(t), \quad t = 0, 1, 2, \dots, n \\ \varepsilon_t &= \sqrt{h_t} z_t, \quad z_t \sim i.i.d. N(0, 1) \\ h_t &= \zeta_0 + \psi \varepsilon_{t-1}^2 + \nu h_{t-1} \end{aligned} \quad (12)$$

The logarithmic trend, $\log(t)$, is motivated by the decreasing pattern in GDP growth across many countries, and is often used in the literature (see Ayres, 2006; Modis, 2007; Modis, 2013, among others).

Secondly, for each of the 25 countries in the data set and for Europe, the FT is calculated on the standardized residuals of the GARCH model in Equation (12). Using the resulting amplitudes and cycle length combinations, we can then construct the periodogram. As discussed in the methodology, the most prominent cycles and their lengths are derived from a mixed distribution that is estimated on the periodogram. We evaluate mixtures that contain up to six components. Note that we only select cycles with lengths between 3 and 15 years. This is because cycles with lengths between 0.5 and 3 years are likely to describe noise instead of actual business or economic cycles. Moreover, the limited time span of the data does not enable us to accurately detect cycles with

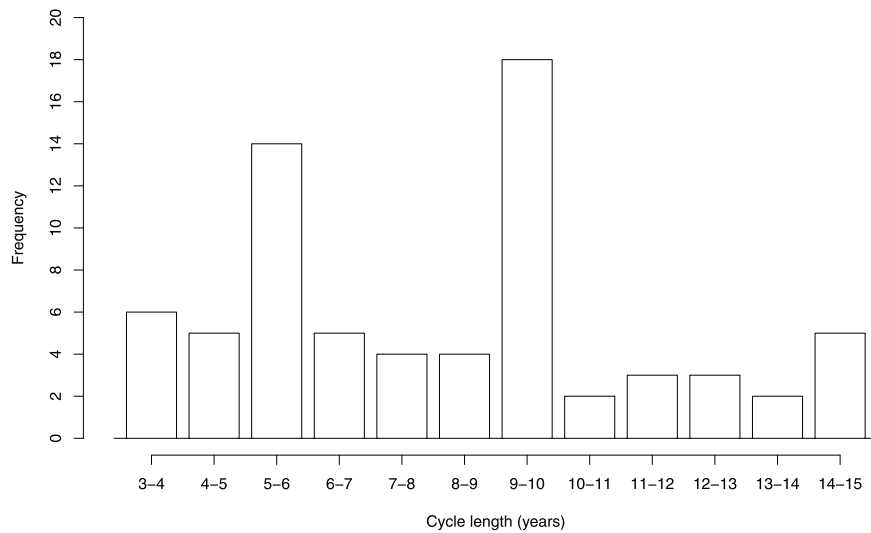


Fig. 10. Frequency distribution of the cycle lengths.

lengths larger than 15 years. Ideally the time span of the series should be at least four times the cycle length of interest. This implies that our data does not allow us to detect Kuznets swings (Kuznets, 1930) or Kondratieff waves (Kondratieff, 1926; 1928). For recent investigations of cyclic behavior over a longer time span, we refer to Metz (2011), Modis (2017), and Grinin et al., 2020. Appendix A presents the periodograms with their estimated mixed distributions, separately for all 25 countries and for Europe.

Table 2 presents an overview of the selected cycle lengths. Multiple cycle structures are found for all 25 countries. There are 11 countries with 2 cycles, 12 countries with 3 cycles, 2 countries with 4 cycles and 1 country with 5 cycles. Interestingly, some cycle lengths occur more frequently than others. Fig. 10 shows a frequency distribution of the cycle lengths that are found in the data. Clearly, cycles of 5–6 years, and cycles of 9–10 years occur most frequently in the panel, with 14 and 18 occurrences, respectively. In fact, in 9 countries and in Europe, we detected both a cycle of 5–6 years and a cycle of 9–10 years. Note that 5–6-year cycles have been found in GDP by Solomou (1998) as well, and by De Groot, Franses, 2009 in innovations. These are often referred to as business cycles. The 9–10-year cycles may be interpreted as Juglar cycles (Juglar, 1862). The argumentation behind the Juglar cycle lies in the relation between commerce and monetary expansions. Excess speculation and disproportionate growth of industry and trade lead to an unstable boom (Besomi, 2009; Legrand and Hagemann, 2007). Juglar cycles are reported to have a length between 7 and 11 years.

4.2. Harmonic regression analysis

We use the selected cycle lengths in Table 2 to model the cyclic behavior of the original GDP growth rates. After careful model selection, we opt to employ the trend-cycle model of Equation (6) with the following trend component T_t and cyclical components C_t here:

$$T_t = d + \gamma \log(t)$$

$$C_t = \sum_{j=1}^k \alpha_j \cos\left(\frac{2\pi t}{\omega_j} + c_j\right)$$

where d is a constant shift, and γ is a trend parameter. The model is estimated on the GDP growth rates of the 25 countries of interest and of Europe, where the harmonic parameters α_j and c_j are initialised with the values that result from the spectral analysis. Table 3 present the estimated parameters of the cyclical components for all 25 countries and Europe. Almost all of the estimated amplitudes and phases are

statistically significant at the 5% level, indicating that there is a strong correlation between the detected cycles and the time series. Specifically, for the G7 countries, only Cycle 1 for the USA (3.9 years) is not statistically significant, and Cycle 3 (13.1 years) for Germany is significant at the 10% level. The amplitudes of the detected cycles range from 0.15 to 2.47 percentage points. This means that a single cycle can describe swings in GDP growth rates of up to $2 \times 2.47 = 4.94$ percentage points. Appendix B shows the in-sample fit of our model on the GDP growth rates of the selected 25 countries and of Europe, and an out-of-sample fit for 12 years ahead. The out-of-sample estimates of the harmonic regression model can help to predict future turning points of an economy. For the year 2020, our results indicate a decline in economic activity across many nations.

4.3. Analyzing the interrelationship between cyclical lengths of subcycles

The general notion about the interrelationship between cyclical lengths is that the shorter waves compromise the longer waves. More precisely, the shorter wave lengths are expected to be integer fractions of the longer wave lengths. Table 5 presents the ratio's of the consecutive cycle lengths, as documented in Table 2. Here ratio i is given by the length of sub cycle i divided by the length of sub cycle $i + 1$, where the sub cycles are ordered by length, from smallest to largest. The average of the calculated ratio's is equal to 0.619. This number is remarkably close to the reciprocal of the golden ratio, Φ :

$$\Phi = 1 / \phi = 1 / \left(\frac{1 + \sqrt{5}}{2} \right) = 0.618033... \quad (13)$$

The golden ratio $\phi = 1.618033$ is the unique ratio $\frac{b}{a}$ which satisfies $\frac{b}{a} = \frac{a+b}{a}$, where $b > a$ (Dunlap, 1997). It is the number which is worst approximated by its fractal decomposition. This leads us to hypothesize the following:

H: The ratio's between consecutive cycle lengths come from a distribution that is centered around Φ .

To investigate this hypothesis, a two-tailed Student's t -test is conducted. The test-statistic t is given by: $t = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$, where \bar{x} denotes the sample mean (0.619), $\mu = \Phi = 0.618$ the hypothesized value, σ the sample standard deviation (0.097), and n the sample size (45). This statistic follows a Student's t -distribution with $n - 1$ degrees of freedom, and equals:

Table 3
Harmonic regression results.

Country	Parameter	Cycle 1	Cycle 2	Cycle 3	Cycle 4	Cycle 5
Australia	Periodicity	5	9.3	14.7	-	-
	Amplitude	0.77***	0.59***	-0.65***	-	-
	Phase	-2.52***	-2.86***	-12.63***	-	-
Austria	Periodicity	6	9.4	14.9	-	-
	Amplitude	0.78***	-0.66***	0.61***	-	-
	Phase	-1.02***	-14.85***	-4.34***	-	-
Belgium	Periodicity	4	9.3	-	-	-
	Amplitude	0.45***	0.70***	-	-	-
	Phase	-2.87***	-4.68***	-	-	-
Canada	Periodicity	3.7	6.2	12	-	-
	Amplitude	-0.49***	-0.44**	0.66***	-	-
	Phase	-0.98***	-8.53***	-0.14**	-	-
Czech Republic	Periodicity	4.4	7.1	9.3	-	-
	Amplitude	0.45**	0.74***	1.38***	-	-
	Phase	-0.98***	-0.67***	-1.58***	-	-
Denmark	Periodicity	3.9	7.1	11.5	-	-
	Amplitude	-0.86***	0.84***	-0.53***	-	-
	Phase	-0.70***	-0.54**	-5.45***	-	-
Finland	Periodicity	5.3	9	12.9	-	-
	Amplitude	-1.14***	-1.52***	1.01***	-	-
	Phase	-4.67***	0.70***	-7.55***	-	-
France	Periodicity	5.6	9.6	-	-	-
	Amplitude	-0.51***	0.59***	-	-	-
	Phase	-0.82***	-4.53***	-	-	-
Germany	Periodicity	5.1	9.6	13.1	-	-
	Amplitude	1.04***	-0.80***	-0.36*	-	-
	Phase	-1.17***	-2.21***	-7.27***	-	-
Greece	Periodicity	3.2	6.4	9	-	-
	Amplitude	-1.25***	-1.33***	1.40***	-	-
	Phase	-1.77***	-3.28***	-6.28***	-	-
Ireland	Periodicity	3.5	5.4	7.1	9.5	14.7
	Amplitude	0.36	1.59***	0.3	-1.93***	1.58***
	Phase	-8.67***	-12.95***	-0.56*	1.12***	-2.73***
Iceland	Periodicity	8.4	10.6	-	-	-
	Amplitude	-2.47***	-1.93***	-	-	-
	Phase	-5.96***	-9.13***	-	-	-
Italy	Periodicity	5.4	9.9	-	-	-
	Amplitude	1.19***	1.00***	-	-	-
	Phase	-3.58***	-8.98***	-	-	-
Japan	Periodicity	5.5	9.3	-	-	-
	Amplitude	0.88***	1.03***	-	-	-
	Phase	-3.51***	-2.87***	-	-	-
Korea	Periodicity	4.7	9.2	-	-	-
	Amplitude	1.65***	-1.21***	-	-	-
	Phase	-0.98***	-1.82***	-	-	-
Luxembourg	Periodicity	4	7.2	9.1	14.3	-
	Amplitude	-0.15	1.20***	-0.81**	-1.34***	-
	Phase	-0.88*	-3.71***	-0.95***	2.66***	-
Mexico	Periodicity	5.1	8.7	11.9	-	-
	Amplitude	0.68**	-1.44***	1.19***	-	-
	Phase	-5.84***	-9.09***	-5.48***	-	-
Norway	Periodicity	6.2	10.4	-	-	-
	Amplitude	-0.48***	0.84***	-	-	-
	Phase	-5.34***	-2.07***	-	-	-
OECD Europe	Periodicity	5.5	9.5	-	-	-
	Amplitude	0.61***	0.75***	-	-	-
	Phase	0.05	-2.69***	-	-	-
Portugal	Periodicity	3.2	9.6	-	-	-
	Amplitude	0.15	1.75***	-	-	-
	Phase	-1.07**	-1.35***	-	-	-
Spain	Periodicity	4.9	9.2	-	-	-
	Amplitude	-0.39*	1.09***	-	-	-
	Phase	-2.87***	12.50***	-	-	-
Sweden	Periodicity	5.1	9.9	-	-	-
	Amplitude	1.36***	-0.50**	-	-	-
	Phase	3.67***	-9.61***	-	-	-
South Africa	Periodicity	5.4	8.2	12.8	-	-
	Amplitude	-0.94***	-1.35***	0.58***	-	-
	Phase	0.16***	-1.37***	-7.13***	-	-
The Netherlands	Periodicity	5.4	9.5	13.6	-	-
	Amplitude	1.01***	0.90***	0.17	-	-
	Phase	-0.71***	-2.85***	-4.46***	-	-
UK	Periodicity	5	8.7	14	-	-
	Amplitude	0.64***	1.10***	-0.61***	-	-

(continued on next page)

Table 3 (continued)

Country	Parameter	Cycle 1	Cycle 2	Cycle 3	Cycle 4	Cycle 5
USA	Phase	-2.25***	-11.94***	-5.83***	-	-
	Periodicity	3.9	5.4	6.9	11.2	-
	Amplitude	0.27	1.12***	1.07***	0.79***	-
	Phase	-5.44***	-0.88***	-3.82***	0.92***	-

The 6th column of this table is omitted, as for none of the countries a 6th cycle is selected.

The cycles lengths and phase coefficients are presented in years. The cycle lengths were not re-estimated and are the same as in Table 2.

* $p < .1$, ** $p < .05$, *** $p < .01$.

Table 5

Ratio's between the lengths of consecutive cycles.

Country	Ratio 1:	Ratio 2:	Ratio 3:	Ratio 4:
	Length of 1 st cycle	Length of 2 nd cycle	Length of 3 rd cycle	Length of 4 th cycle
	Length of 2 nd cycle	Length of 3 rd cycle	Length of 4 th cycle	Length of 5 th cycle
Australia	0.54	0.63	-	-
Austria	0.64	0.63	-	-
Belgium	0.43	-	-	-
Canada	0.60	0.52	-	-
Czech Republic	0.62	0.76	-	-
Denmark	0.55	0.62	-	-
Finland	0.59	0.70	-	-
France	0.58	-	-	-
Germany	0.53	0.73	-	-
Greece	0.50	0.71	-	-
Iceland	0.79	-	-	-
Ireland	0.65	0.76	0.75	0.65
Italy	0.55	-	-	-
Japan	0.59	-	-	-
Korea	0.51	-	-	-
Luxembourg	0.56	0.79	0.64	-
Mexico	0.59	0.73	-	-
Norway	0.60	-	-	-
OECD Europe	0.58	-	-	-
Portugal	0.33	-	-	-
South Africa	0.66	0.64	-	-
Spain	0.53	-	-	-
Sweden	0.52	-	-	-
The Netherlands	0.57	0.70	-	-
UK	0.57	0.62	-	-
USA	0.72	0.78	0.62	-

$$t = \frac{0.619 - 0.618}{0.097/\sqrt{45}} = 0.0745 \quad (14)$$

The probability value for the test statistic is equal to $P(t_{44} = 0.0745) = 0.94$. Therefore hypothesis H cannot be rejected. Opposed to what was previously suggested, the relative distance between cycle lengths is supposedly far from an integer fraction. Instead, the ratio between the cyclical lengths is remarkably close to Φ .

5. Discussion

Cyclic behavior has always been a part of economic reality. Presumably [Clarke \(1847\)](#) was the first to make us aware of economic cycles, detecting the Kondratieff wave in the price of wheat, as administered since 1202. Many scholars and practitioners have aimed to better understand these economic cycles ever since. [Schumpeter \(1939\)](#) already suggested that in many economic processes, there exist not one, but multiple cycles. This phenomenon alone causes future economic turning points to be notoriously difficult to predict.

The objective of our paper was to analyze in a data-driven manner whether an interrelationship can be discerned between the lengths of subcycles. Our results suggest that the ratio's between these cyclical lengths are often close to the reciprocal of the golden ratio, Φ . This finding provides a direction for theory development regarding economic cycles and dynamic stability. It may also serve to develop new econometric models for economic cycles. Research in both directions would

Table 6

The cycles of ϕ .

Periodicity	Label of the cycle
3 years	Kitchin cycle
5 years	Business cycle
8 years	Juglar cycle
13 years	Bootstrap cycle
21 years	Kuznets swing
34 years	Societal wave
55 years	Kondratieff wave
90 years	Four-generation cohort cycle
145 years	War and hegemony cycle

serve to improve signalling future economic downturns and to alleviate economic and societal distress.

5.1. The cycles of ϕ

If we accept that cycles with lengths that are multiples of ϕ apart are likely to co-exist in an economy, an interesting pattern occurs. When we start with a 5- or 8-year year cycle and multiply by ϕ , we obtain the series 5, 8, 13, 21, 34, 55, 90 and 145. Remarkably, all these cycles have been documented in the literature. Additionally, if we divide 5 by ϕ , we obtain a cyclical length of 3 years, which is close to the well-known Kitchin cycle ([Kitchin, 1923](#)). The obtained series also matches the

results of De Groot and Franses (2012). Based on a meta-analysis of 88 detected cycles, documented in 51 studies, these authors found cyclical lengths to be clustered around 8, 21, 32 and 55 years.

We outline some of the main connections with the literature on long waves here. Table 6 provides an overview.

The 13-year cycle has been attributed to the bootstrap phenomenon, as described by Forrester (1977) and Haustein and Neuwirth (1982). The bootstrap phenomenon arises when a sector uses part of its own sales as a production factor. The signal to increase the supply of the production factor leads to operations which cause a reduction in the supply. The 13-year cycle was also detected in industrial production (Kuczynski, 1978).

Cycles with lengths around 21 years were documented by de Wolff (1924), Kuznets (1930) and Kuczynski (1978). These cycles are attributed to investments in construction. These cycles initially occurred because of the large immigration in the US between 1870 and 1920. The 21-year cycle has also been documented by Abramovitz (1959) and Solomou (1998) in GDP, and by Haustein and Neuwirth (1982) and Kuczynski (1978) in innovations.

The 34-year cycle has been attributed to regularities in human behavior (Devezas and Corredine, 2001; 2002). This is otherwise described as “the cyclicity of social phenomena is inhibited in the human biological structure” (De Groot and Franses, 2012, p. 62). The 34-year cycle is also reported in Lösch (1937) and Haustein and Neuwirth (1982) and Thio (1991) and Dent (1999).

The 55-year cycle is the well-known Kondratieff wave (Kondratieff, 1926; 1928) in economic variables such as prices, interest rates, wages, consumption, foreign trade and coal, pig iron, lead and wheat. The literature on the Kondratieff wave is abundant. We mention a couple of relevant studies: Ayres (1990a,b); Berry and Dean (2012); Berry et al. (1993); Berry (2000); Clark et al., 1981, 1984; Coccia (2010, 2018); Dator (1999a, 1999b, 2006); Devezas (2010); Devezas and Corredine (2001, 2002); Devezas et al. (2005); Diebolt and Doliger (2006); Focacci (2017); Freeman and Perez (1988); Freeman and Soete (1987); Grinin et al. (2014); Grinin (2019); Grinin et al. (2017, 2010); Hirooka (2005); Kleinknecht (1987); Korotayev et al. (2011); Korotayev and Tsirel (2010); Linstone (2004, 2006); Marchetti (1980, 1986); Mensch (1979); Metz (2011); Modis (2013, 2017); Reijnders, 1990; Schumpeter (1939); Silverberg and Verspagen, 2003; Van Duijn (1983); Volland (1987).

The so-called four-phase four-generation cohort cycle, with a length of approximately 90 years, has been extensively researched by Howe and Strauss (2000); Strauss and Howe (1992, 2009). This cycle arises as a result of generational recessive and dominant generations alternating each other. Strauss and Howe (1992) delineate 17 different generations over the course of American history between 1584 and 1981 with generation 18 starting in 1982.

Finally, the 145-year cycle is attributed to war and hegemony. The concept of hegemony is described by Goldstein (1988, 2003, 2006) as the ability of one country to center the world economy around itself. Within this context, Modelski (1978, 2006) describes the leadership cycle, Wallerstein (1983) describes the world system. Organski (1958) attributes the 145-year cycle to the transition of power. Modelski and Thompson (1988), Reijnders (2006); Reijnders, 1990, and Mandel (1995) also studied the hegemony cycle, and found it to be independent of the Kondratieff wave.

5.2. Future research

Our analysis was necessarily confined to our choice of data, the time span of the data, our cycle detection method, and our research objectives. We therefore recommend the following directions for future research.

Firstly, we chose to analyze GDP, since GDP is the most widely used indicator for economic welfare. It would be interesting to apply the methodology of this paper to other key economic indicators and to other economies. Additionally, while quarterly GDP has typically only been measured accurately since 1960, other socio-economic indicators have been measured over a longer time span. Applying our methodology to time series that span more time would allow to detect longer cycles and to further investigate the interrelationship between the lengths of these cycles. Another interesting angle would be to assess whether the lengths of subcycles are constant over time. A state space model might be helpful to investigate this. See, for example, Valle e Azevedo et al. (2006); Creal et al. (2010). Thirdly, for the purpose of our analysis, our models did not impose an interrelationship between the lengths of our subcycles. Imposing the golden ratio relationship between cyclical lengths might improve forecasting future turning points. Finally, our analysis raises new theoretical questions. We would be interested to see whether a mathematical model could be formulated to explain the appearance of the golden ratio ϕ in our empirical study. Such a model would be instrumental to formulate new multi-structured cycle theory.

6. Conclusion

The starting point of this paper has been the various scientific works on the origins of multi-structured economic cycles, and the implied interrelationships between cyclical lengths. Our aim was to provide a data-driven approach to chart the structure behind multi-structured cycles, and, more particularly, to discern a pattern in the lengths of subcycles.

We have detected multi-structured cycles in percentage GDP growth. Macro-economic time series, like GDP growth, have a tendency to grow non-linearly over time and are often heteroscedastic across time. These stylized facts cause most series to be non stationary. As a consequence, applying Fourier analysis to macro-economic time series often yields unreliable results. We have therefore coupled Fourier analysis with a GARCH model. The GARCH model enabled us to flexibly model potential dynamics, trend growth, and time-varying volatility. The standardized residuals of the model were then used as the input signal in a Fourier analysis. We selected the most prominent cycles by estimating a mixed distribution to locate the peaks in the resulting Fourier periodogram. We then estimated the accompanying amplitudes and shifts of the cycles using harmonic regression. Finally, we conducted a meta-analysis on the detected cycle lengths, focusing on the ratio's between the lengths of consecutive cycles.

In our empirical application to measuring cycles in percentage GDP growth for 25 OECD countries, and for Europe, we detected multi-structured cycles in all cases. Cycles with lengths between 5–6 and 9–10 years appeared most frequently in the panel data set, namely in 14 and 18 countries, respectively. Country-specific harmonic regression models confirmed that in general the cycles are strongly correlated with the original time series. Interestingly, we found that the ratio's between consecutive cycle lengths often closely match the golden ratio, ϕ . This finding contradicts the notion in the literature that integer multiples of shorter waves are contained within the longer waves.

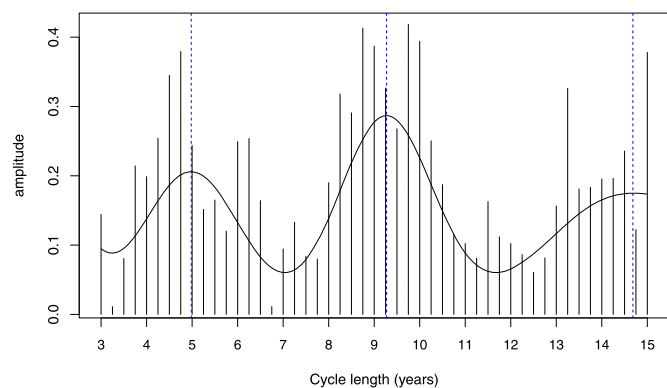


Fig. 11. Spectrogram and mixed distribution for Australia.

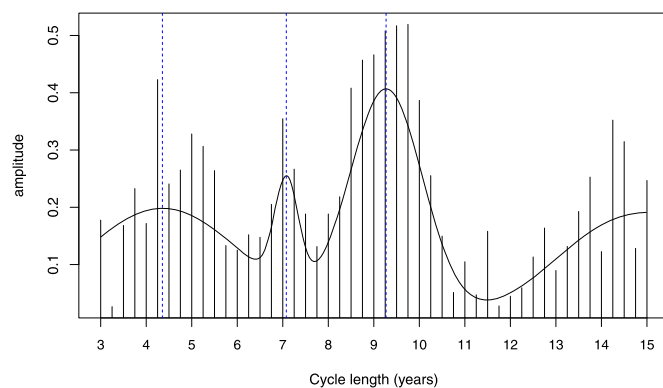


Fig. 15. Spectrogram and mixed distribution for Czech Republic.

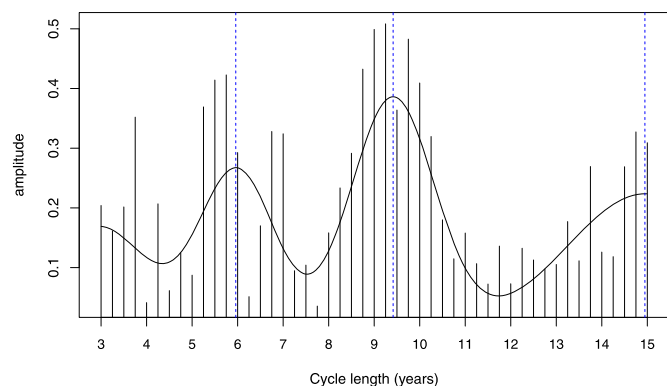


Fig. 12. Spectrogram and mixed distribution for Austria.

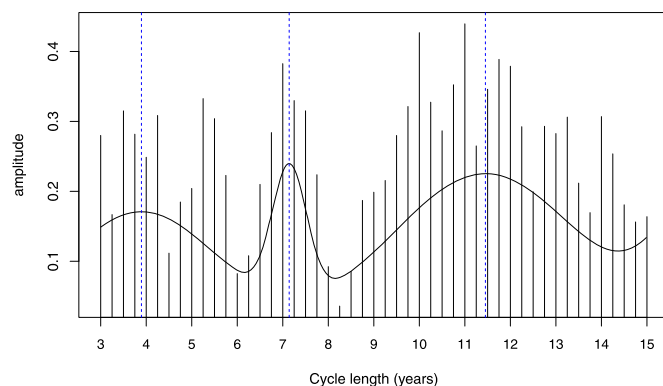


Fig. 16. Spectrogram and mixed distribution for Denmark.

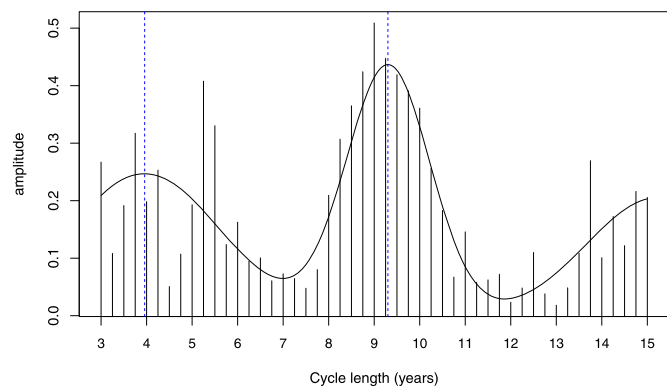


Fig. 13. Spectrogram and mixed distribution for Belgium.

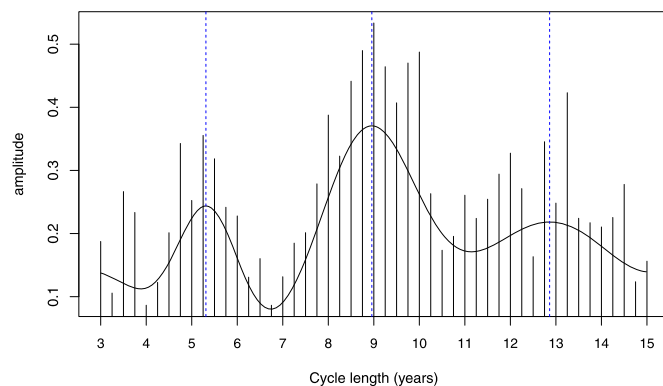


Fig. 17. Spectrogram and mixed distribution for Finland.

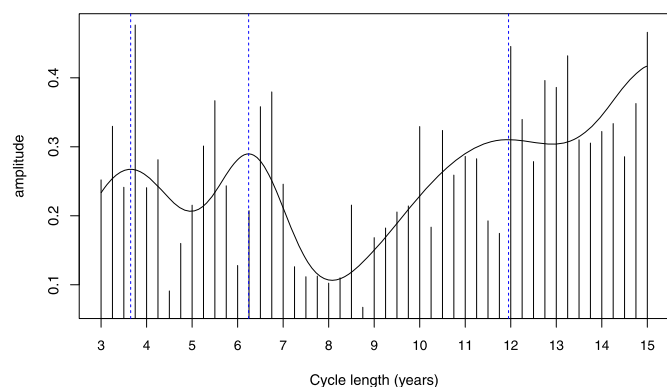


Fig. 14. Spectrogram and mixed distribution for Canada.

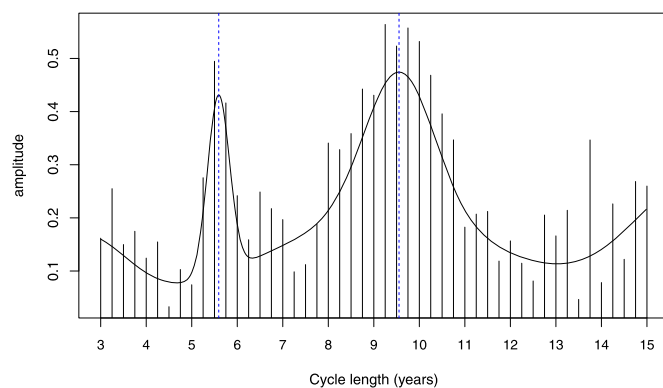


Fig. 18. Spectrogram and mixed distribution for France.

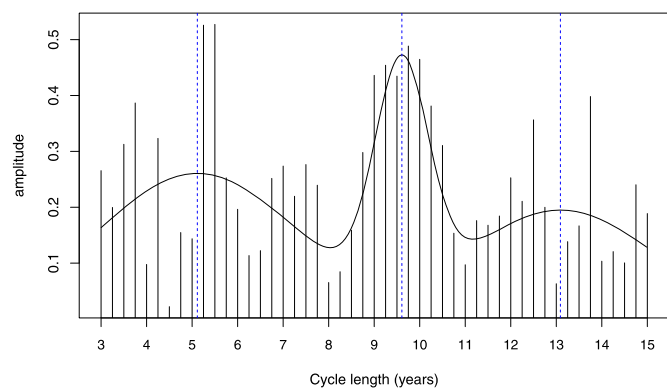


Fig. 19. Spectrogram and mixed distribution for Germany.

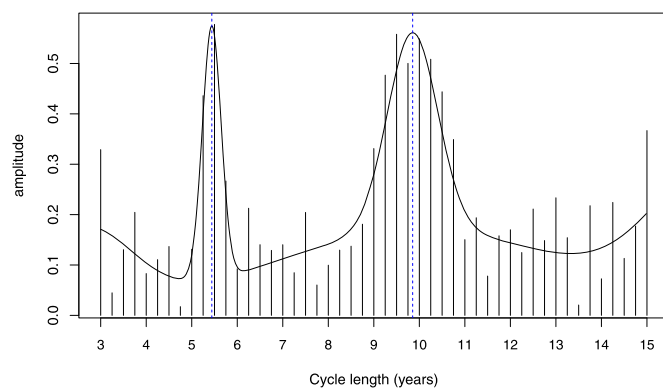


Fig. 23. Spectrogram and mixed distribution for Italy.

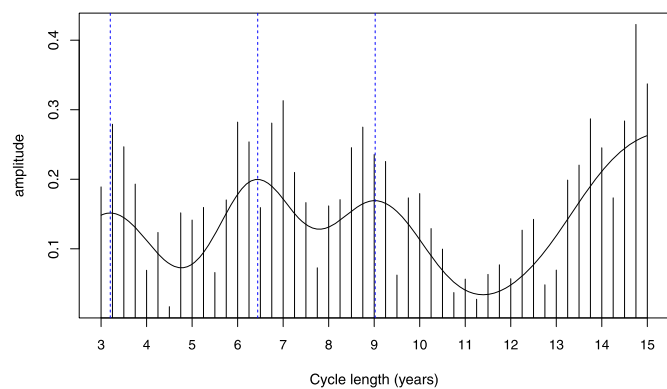


Fig. 20. Spectrogram and mixed distribution for Greece.

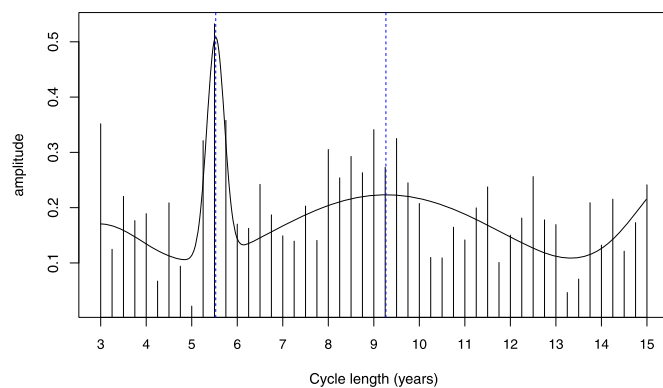


Fig. 24. Spectrogram and mixed distribution for Japan.

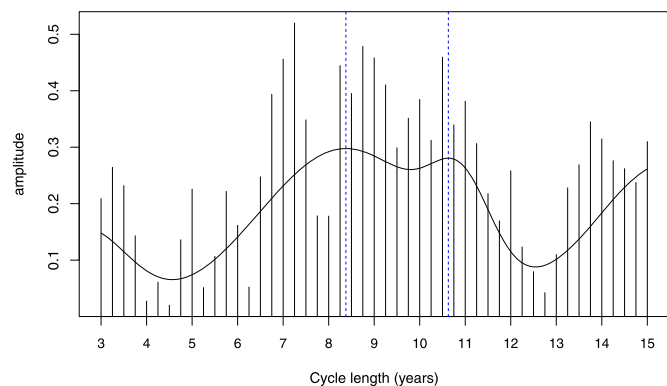


Fig. 21. Spectrogram and mixed distribution for Iceland.

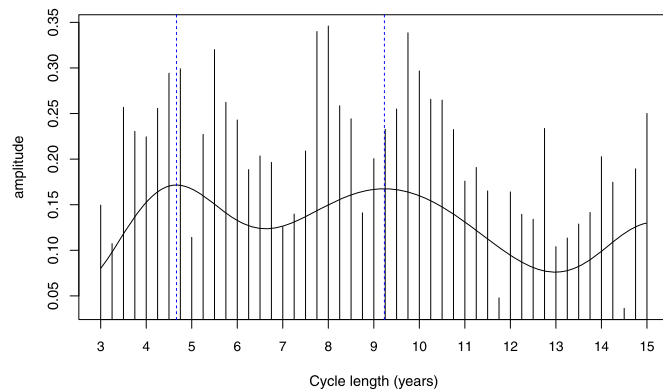


Fig. 25. Spectrogram and mixed distribution for Korea.

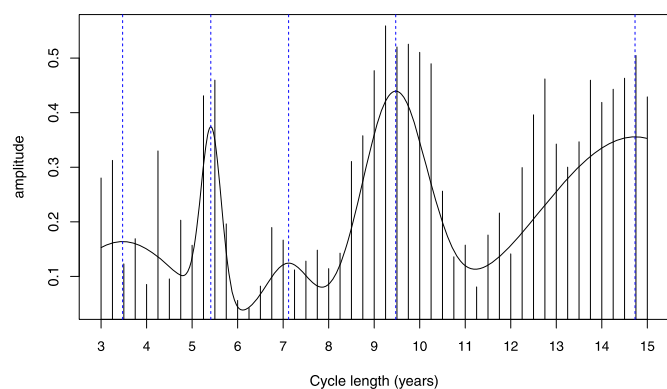


Fig. 22. Spectrogram and mixed distribution for Ireland.

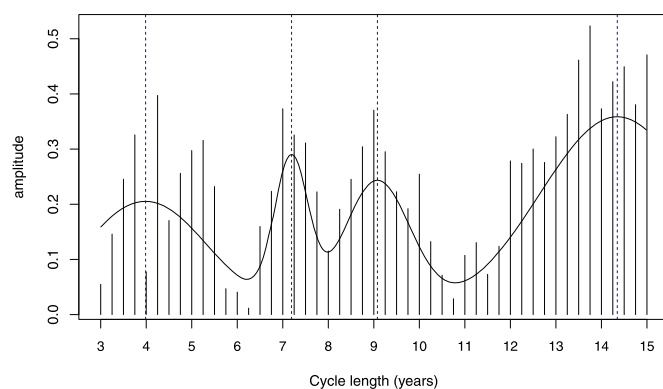


Fig. 26. Spectrogram and mixed distribution for Luxembourg.

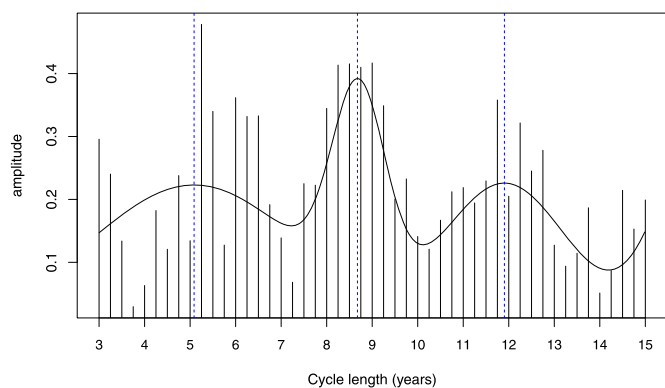


Fig. 27. Spectrogram and mixed distribution for Mexico.

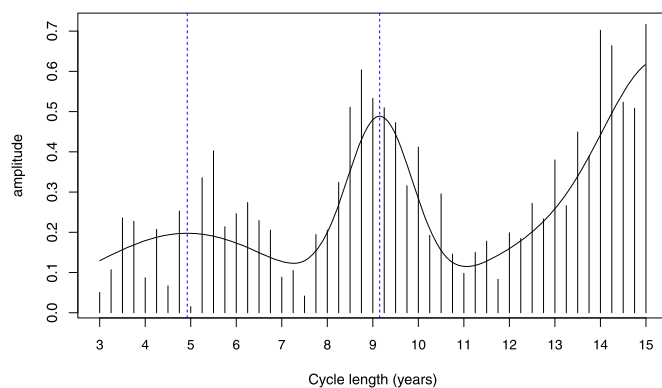


Fig. 31. Spectrogram and mixed distribution for Spain.

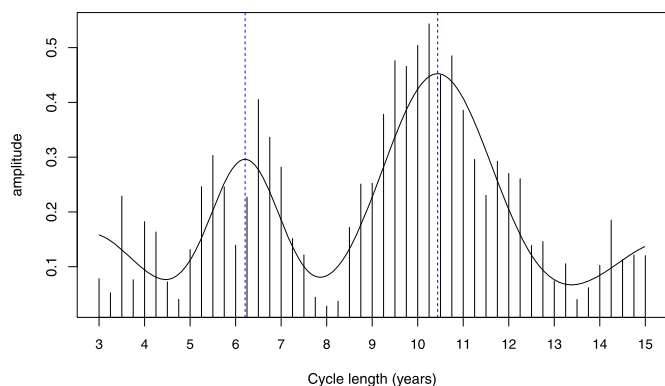


Fig. 28. Spectrogram and mixed distribution for Norway.

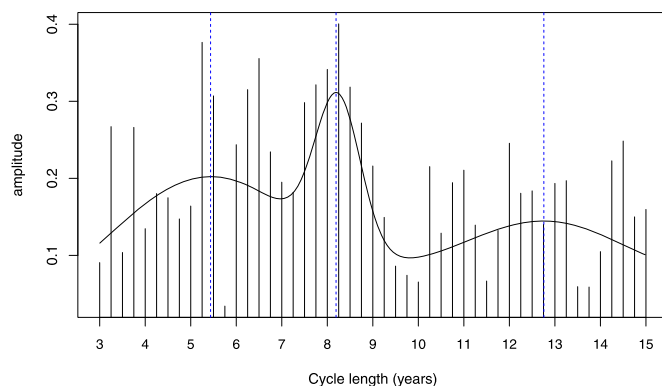


Fig. 32. Spectrogram and mixed distribution for South Africa.

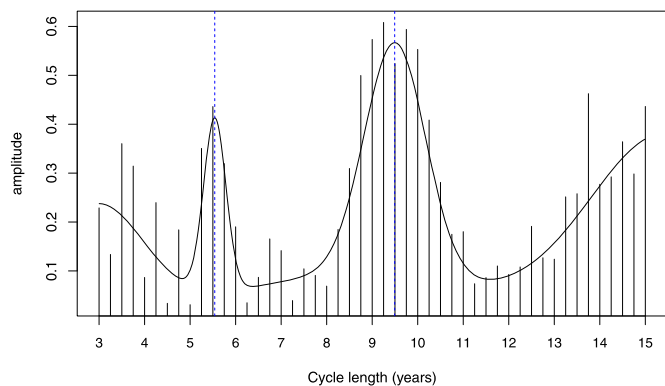


Fig. 29. Spectrogram and mixed distribution for the European OECD countries.

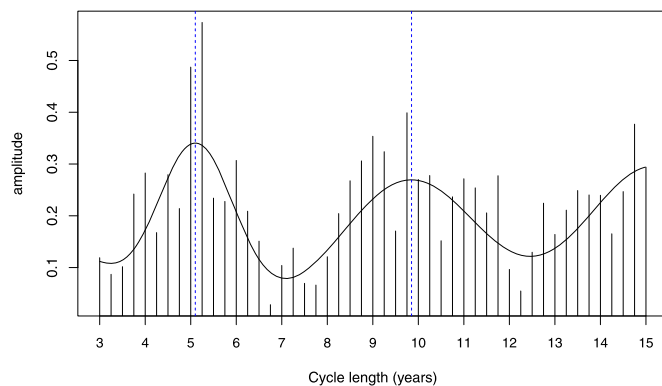


Fig. 33. Spectrogram and mixed distribution for Sweden.

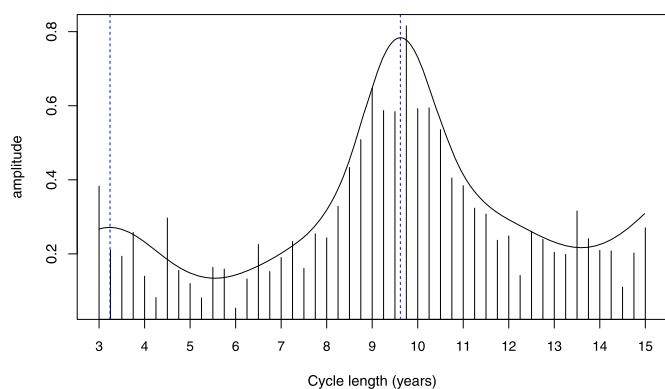


Fig. 30. Spectrogram and mixed distribution for Portugal.

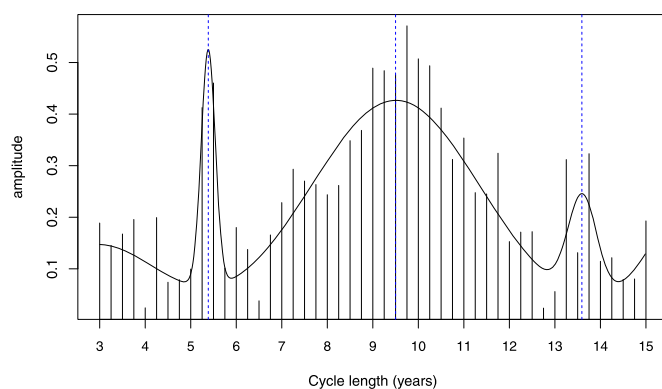


Fig. 34. Spectrogram and mixed distribution for The Netherlands.

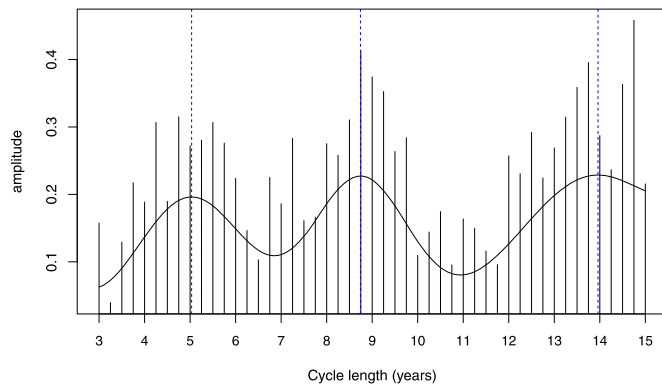


Fig. 35. Spectrogram and mixed distribution for UK.

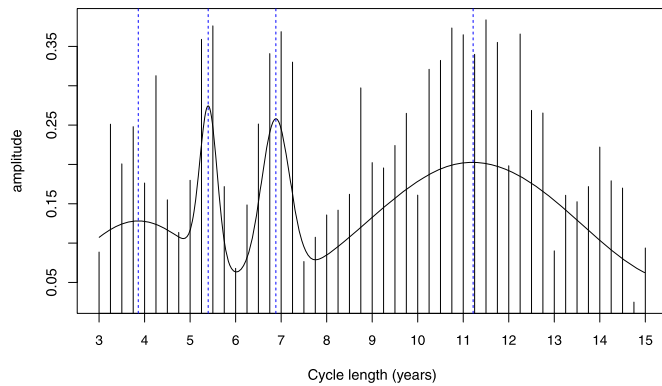


Fig. 36. Spectrogram and mixed distribution for USA.

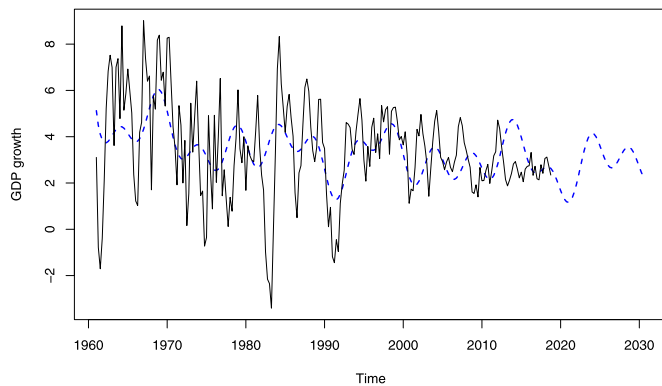


Fig. 37. Fitted model for Australia.

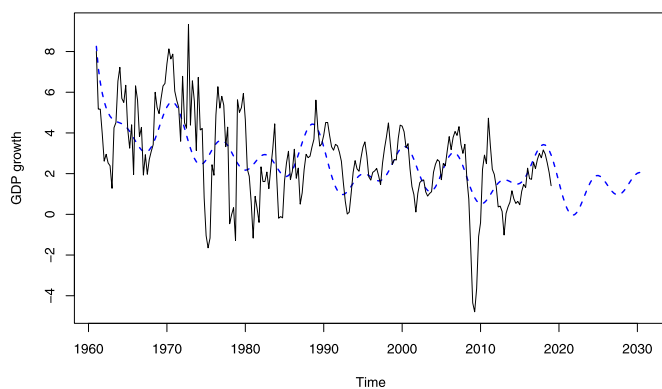


Fig. 38. Fitted model for Austria.

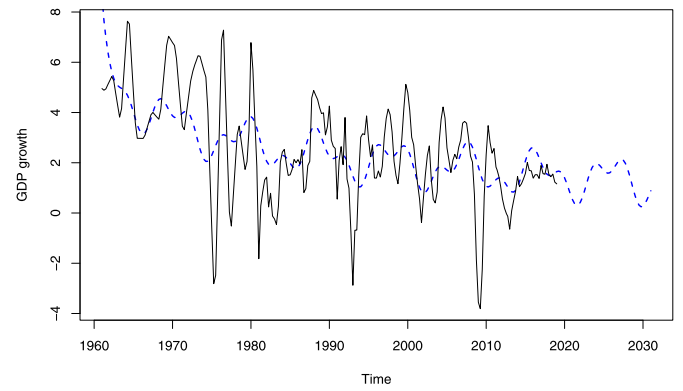


Fig. 39. Fitted model for Belgium.

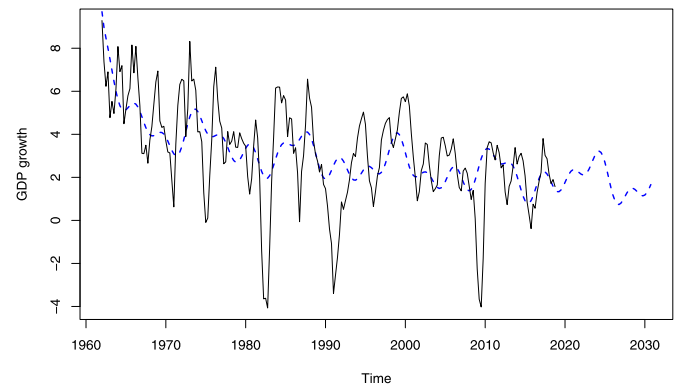


Fig. 40. Fitted model for Canada.

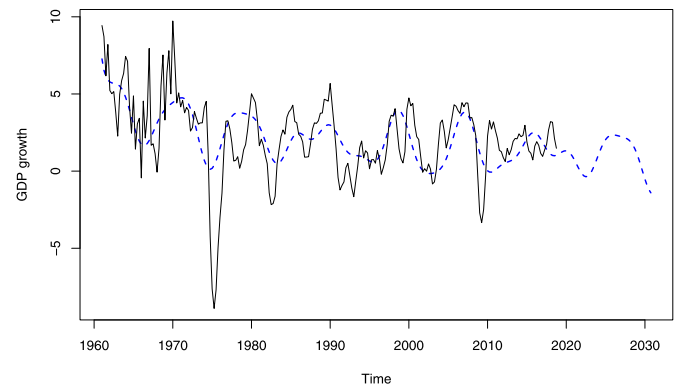


Fig. 41. Fitted model for Czech Republic.

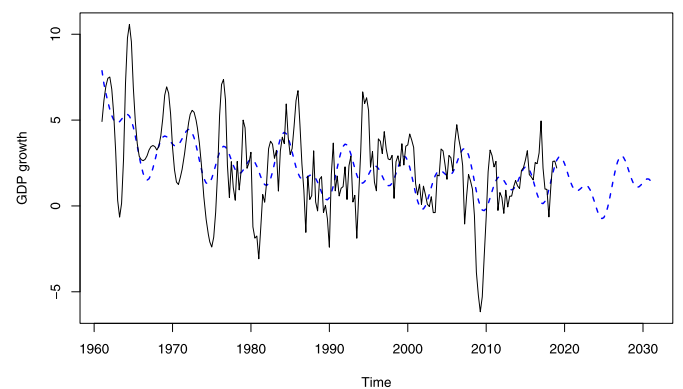


Fig. 42. Fitted model for Denmark.

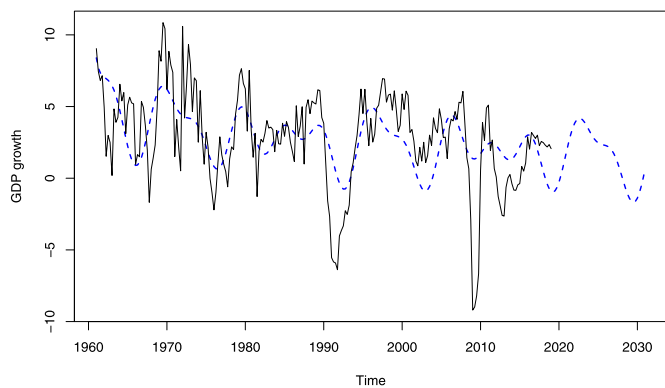


Fig. 43. Fitted model for Finland.

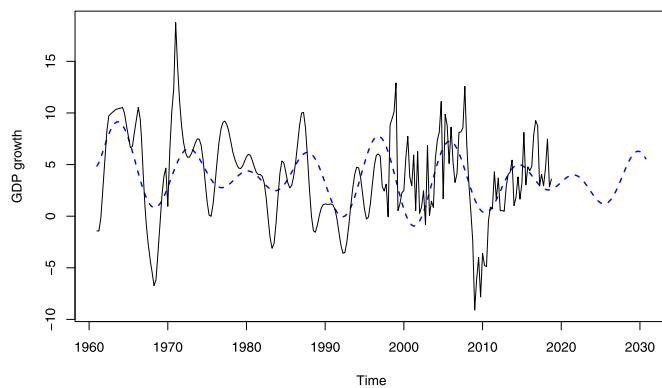


Fig. 47. Fitted model for Iceland.

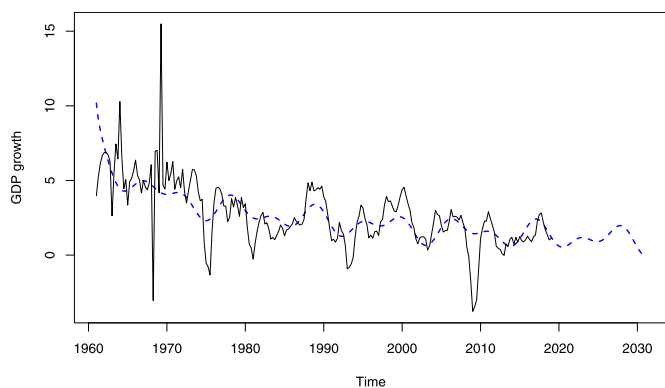


Fig. 44. Fitted model for France.

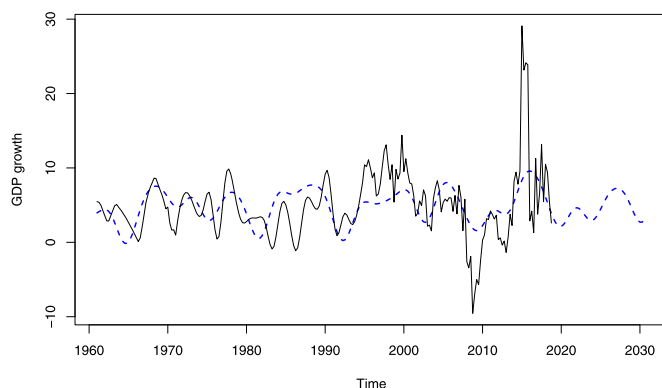


Fig. 48. Fitted model for Ireland.

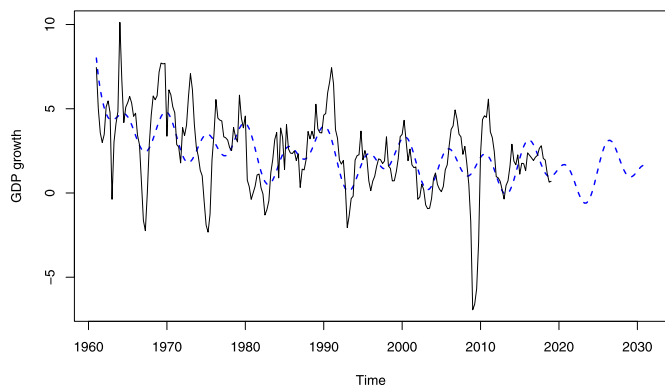


Fig. 45. Fitted model for Germany.

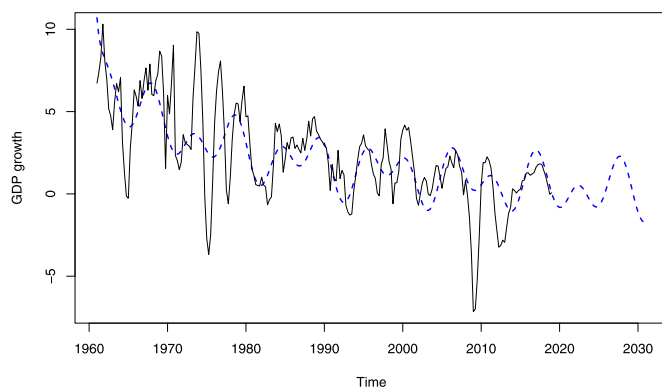


Fig. 49. Fitted model for Italy.

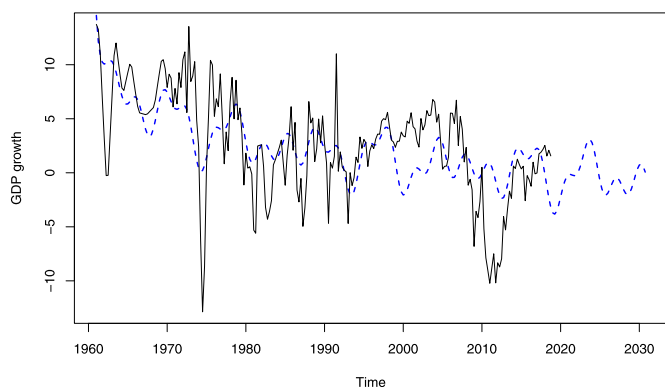


Fig. 46. Fitted model for Greece.

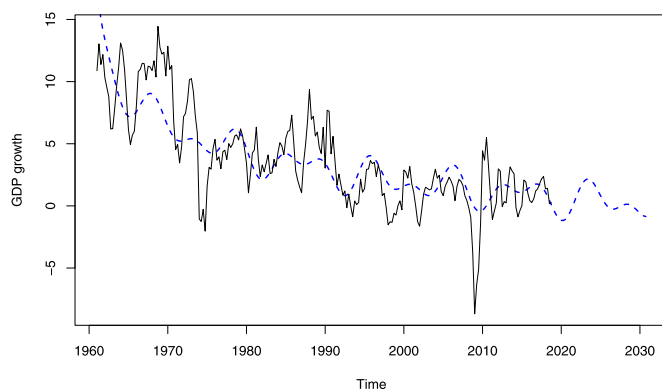


Fig. 50. Fitted model for Japan.

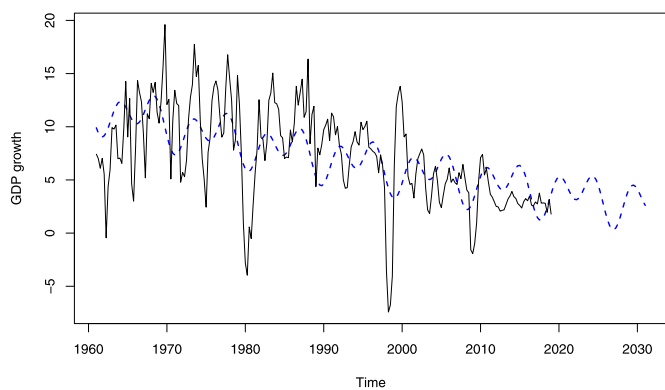


Fig. 51. Fitted model for Korea.

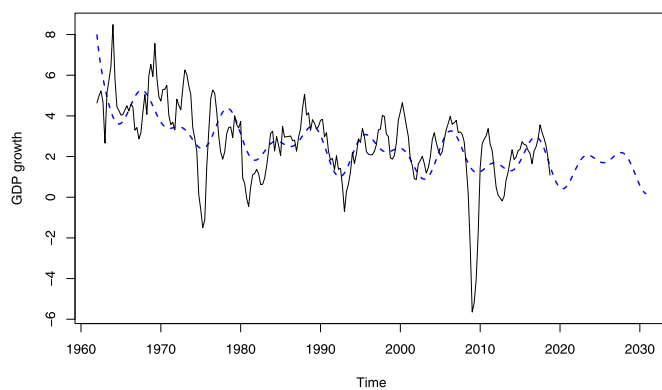


Fig. 55. Fitted model for the European OECD countries.

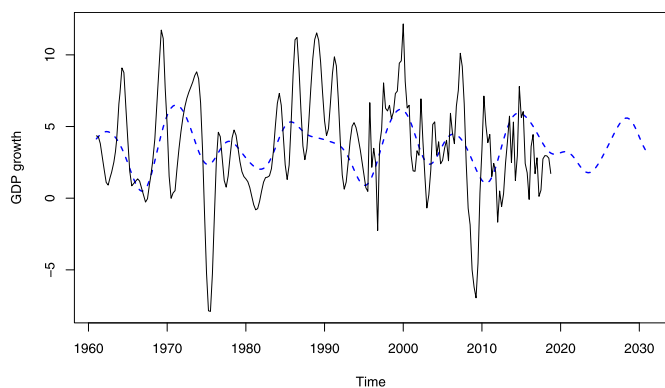


Fig. 52. Fitted model for Luxembourg.

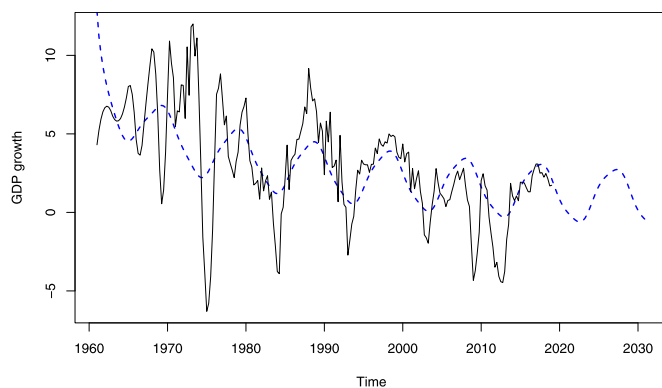


Fig. 56. Fitted model for Portugal.

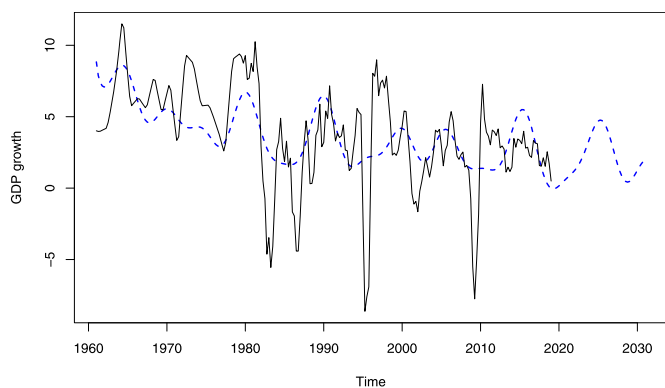


Fig. 53. Fitted model for Mexico.

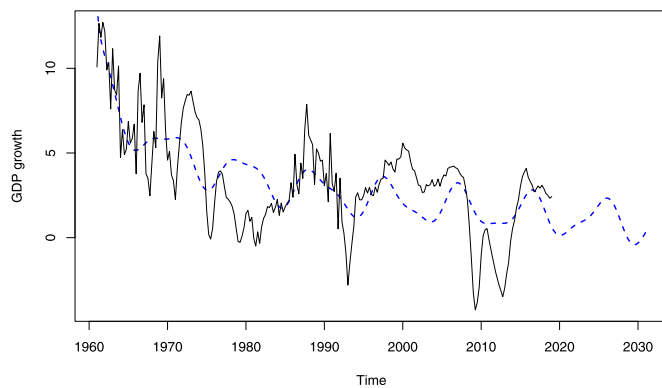


Fig. 57. Fitted model for Spain.

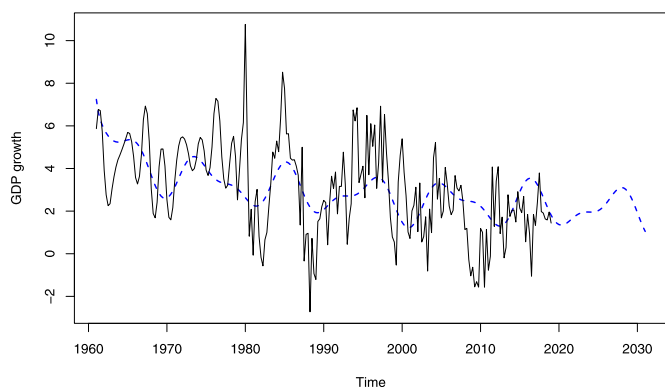


Fig. 54. Fitted model for Norway.

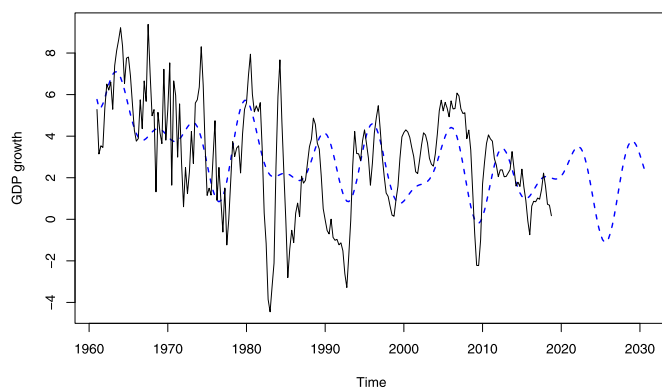


Fig. 58. Fitted model for South Africa.

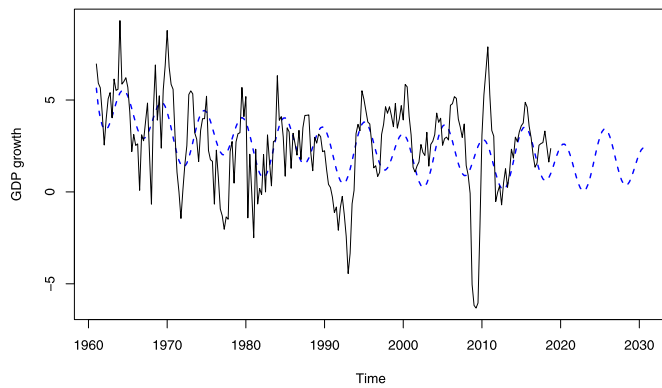


Fig. 59. Fitted model for Sweden.

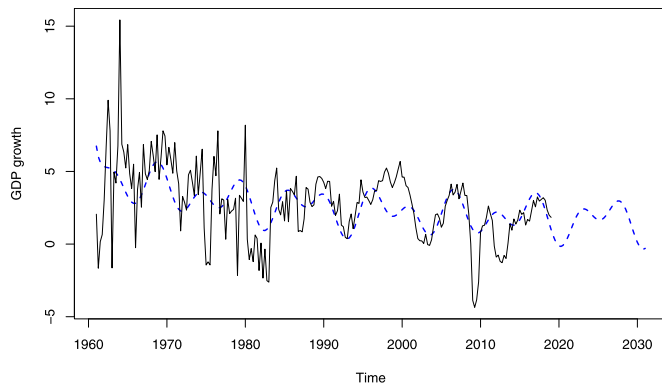


Fig. 60. Fitted model for The Netherlands.

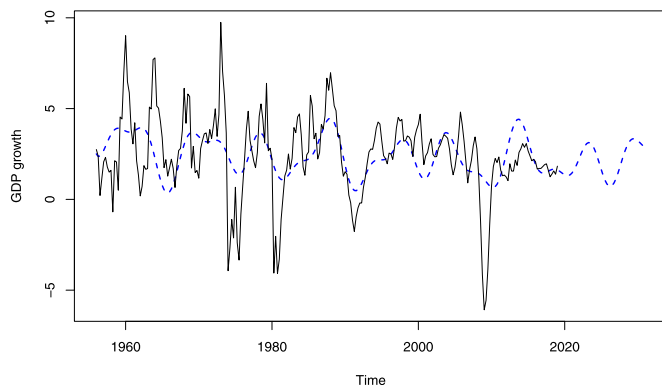


Fig. 61. Fitted model for UK.

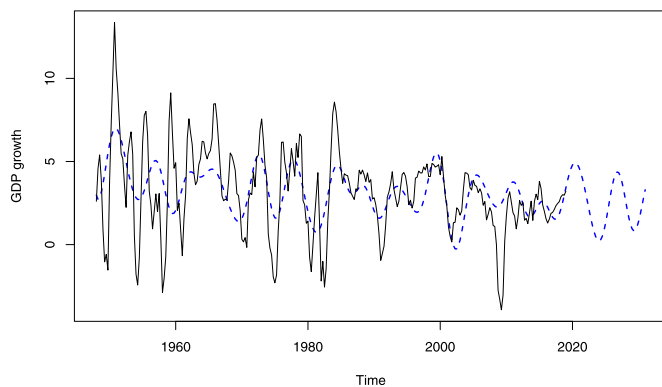


Fig. 62. Fitted model for USA.

Acknowledgments

We thank the Editors-in-Chief, two anonymous referees and Philip Hans Franses for helpful comments and suggestions on earlier versions of this paper.

Appendix A. Cycle detection

Figs 11–36

Appendix B. Fitted regressions

Figs 37–62

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