ENHANCING WAREHOUSE PERFORMANCE BY EFFICIENT ORDER PICKING

This thesis studies order picking in warehouses. Order picking, the process of retrieving products from their storage locations to fill customer orders, is regarded as the most critical operation in a warehouse. Using stochastic modelling, we develop a model for zoned pick-and-pass systems to estimate order picking performance of various design alternatives and operating policies. The model is fast, flexible, and sufficiently accurate for practical purposes. The thesis also introduces a Dynamic Storage concept. In a Dynamic Storage System (DSS), orders are picked in batches and only those products needed for the current pick batch are retrieved from a reserve area and stored in the pick area, just in time. Through analytical and simulation models, we demonstrate a DSS can substantially improve order throughput and reduce labour cost simultaneously over conventional order picking systems, where all the products required during a pick shift are stored in the pick area. The thesis also studies an internal distribution process at a flower auction company. We introduce a zoned distribution system, analogous to pick-and-pass. Based on simulation and optimization models, we propose ways to reduce congestion and improve order lead time.

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Enhancing Warehouse Performance by Efficient Order Picking
Enhancing Warehouse Performance by Efficient Order Picking

Verbetering van magazijnprestaties door efficiënte orderverzamelprocessen

Proefschrift

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Rotterdam, October 2008
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1. INTRODUCTION

Logistics management is one of the most important activities for many companies. According to the Material Handling Institute of America (MHIA 2007), the mission of logistics is to achieve efficient flows of materials and information over the entire supply chain (logistics network), which consists of the physical and communication paths connecting multiple, inter-related businesses from their points of origin to the final end consumer. In a typical supply chain, raw materials are procured, items are produced at one or more factories, shipped to warehouses for intermediate storage, and then shipped to retailers or customers (Simchi-Levi et al. 2000). Warehouses play an important role in a supply chain, as products need to be put somewhere along the supply chain for temporary storage before reaching the end user. As a vital component in a supply chain, a warehouse mainly has five roles (according to Bartholdi and Hackman 2007). First, to consolidate products to reduce transportation cost by combining shipment in full capacity. Second, to realize economies of scale in manufacturing or purchasing. Vendors may give a price break to bulk purchases and the savings may offset the expense of storing the product. Similarly, the economics of manufacturing may dictate large batch sizes to amortize large setup costs, so that excess product must be stored. Third, to provide value-added processing. Increasingly, warehouses are being forced to incorporate value-added processing such as light assembly. This is a result of manufacturing firms adopting a policy of postponement of product differentiation, in which the final product is configured to the customer’s requirements as close to the delivery location as possible. Fourth, to reduce response time. A warehouse acts as a buffer between producers and customers to meet the changing market condition and to hedge against uncertainties (e.g., seasonality, demand fluctuations). Fifth, to act as a single source of supply to customers.

With the development of technology and the increasing globalization, logistics is becoming a competitive area for companies. Those who can deliver fast and accurately
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will gain advantages in the market. New trends appear in the supply chain, which subsequently impact the operations at warehouses. We distinguish the following supply chain trends with great impacts on warehousing: the increasing customer power, the change of customer order profiles, and the consolidation of warehouses.

One of the major trends in supply chains is the increasing power of customers. In early times, producers made their products and customers bought what was available in the market. Nowadays, customers have more choices. If one brand is not available at a shop, customers shift easily to other brands with similar functions. The existence of various substitutable products makes companies realize that a fast delivery ability is of great importance for them to maintain their market share and at the same time to use as a competitive weapon against their competitors. The requirement of fast delivery has large impact on warehousing operations. Products need to be stored and picked in a way that the customer order sojourn time in a warehouse can be shortened. New material handling equipments are introduced to assist the put-away and order picking process.

Another major trend in supply chains is the reduction in customer order sizes and the increase in order frequencies (i.e., higher order arrival rates). The major driver for this trend is that companies tend to keep their inventories low and hence release orders to their suppliers frequently and in small quantities. In order to serve such customers, companies tend to accept orders arriving at their warehouses late (see De Koster et al. 2007). To provide high customer service level and to achieve economics of scale in transportation, these small size, late-arrival orders need to meet the tight shipment time fence. Hence the time available for picking orders at warehouses becomes shorter, which imposes higher requirements on order processing time at warehouses.

In parallel to the changes in customer order profiles, the last two decades are also characterized by the centralization of facilities in the supply chain. Compared to a decentralized network, centralized facilities have the advantages of lower overhead cost, decreased inventory level, and a higher fill rate. The result of centralization is fewer warehouses with greater variety of products. The direct impact of the larger product variety on warehouses is the pressure on space utilization. To accommodate the products, the
warehouses become larger, which causes longer travel time in picking orders and subsequently impacts order response time.

Besides the new trends in supply chains, technology development also has a large impact on warehousing. Many operations which used to be done manually have now been mechanized or even automated. In an automated warehouse, robots might be used to stack incoming products on pallets. The content of each pallet is communicated to the central computer which assigns the pallet to an empty location in the storage area. Conveyors or Guided Vehicles (GV) are used to transport the incoming pallets to the storage buffer and automated Storage/Retrieval (S/R) machines are used to store the pallet at the right position in the storage area. S/R machines are also used to assist order picking processes, combining a large variety of picking methodologies like zone picking, and batch picking (see section 1.3). The picking process is also supported by systems such as pick-to-light, pick-to-voice, Radio Frequency Identification (RFID), etc. The picked orders can be transferred via conveyor systems to an automated sorting system which contains multiple chutes. The orders for a specific destination are sorted automatically to the same chute. Sorted orders are then packed and grouped for shipments. The development of information technology and warehouse management software systems integrates the above processes seamlessly. The automation of warehouses has the advantage of saving labor costs, reducing errors, and generating higher productivities. Many major warehousing solution providers in Europe, such as Witron, Swisslog, and Vanderlande provide such automated warehouse solutions.

The discussion on the impact of new supply chain trends on warehousing reveals new challenges on the warehouse order picking (or order selection) process: the process of retrieving individual articles from storage locations for the purpose of fulfilling an order for a customer. On the one hand, small but frequent customer orders arrive late at the warehouse requiring products to be selected from a large assortment. On the other hand, the order picking time has to be squeezed in order to provide fast response to customers. In order to handle these new challenges, warehousing researchers and practitioners are continuously endeavoring to develop and implement new picking systems and picking policies to warehouses. This thesis introduces new methods and concepts to model and
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analyze the performance of order picking systems. Specifically, in this thesis, we develop a fast and accurate tool to estimate the impact of various design parameters and operational policies on the order picking system performance. The tool can be used to estimate alternatives at the design phase of an order picking system. We also introduce a new storage method which can significantly save land space and labor hours used in order picking and, at the same time, achieve higher pick throughput comparing to the conventional storage methods used in the picking system. By studying a real-life internal distribution process, we provide solutions to reduce distribution congestion so as to ensure the order delivery time to meet the shipment time fence. From both academic and practical points of view, the concepts and the models discussed in this thesis provide deeper insights for warehouse researchers and practitioners to understand and improve the order picking system performance. We come back to the thesis’s contribution in section 1.4. In the next sections, we first give an introduction about order picking process and its classification in section 1.1. In section 1.2, we introduce the order picking system studied by this thesis. Section 1.3 reviews the literature on the topics that impact the order picking system performance. The contributions and the outline of this thesis are given in section 1.4.

1.1. Order picking

1.1.1. Order picking systems classification

As previously discussed, order picking is the process of picking products from their storage locations to fill customer orders. It involves the scheduling and releasing of customer orders, the picking of items from their storage locations and the disposal of the picked items. Order picking often consumes a large part of the total labor activities in the warehouse (Drury 1988, even claims up to 60%), and for a typical warehouse, order picking may account for 55% of all operating costs (Tompkins et al. 2003). The majority
of warehouses employ humans for order picking. According to the movement of human and products, order picking is classified into *picker-to-parts systems*, *parts-to-picker systems*, and *put systems* (refer to Figure 1.1). We will explain Figure 1.1 in this and the next sub-section.

![Figure 1.1: Classification of order picking systems.](image)

The most common order picking system is the picker-to-parts system (De Koster 2008), where order pickers walk or drive along the aisles to pick items. We can distinguish two types of picker-to-parts systems: low-level and high-level (man-on-board) picking. In low-level order picking systems, products are stored in bins on shelves, storage drawers in cabinets, or cartons on flow racks. The height of the storage system is limited by the reaching height of a human being. Order pickers pick the requested items from storage racks or bins (bin-shelving storage), while traveling along the storage aisles. Low-level order picking systems are widely used in warehouses because of their low initial cost, easy installation, easy reconfigurability, and low maintenance cost. High-level picking systems employ high storage racks. Shelves or storage cabinets can be stacked as high as floor loading, weigh capacity, throughput requirements, and/or ceiling heights will permit. Order pickers travel to the pick locations on board of a lifting order picking truck or crane. The machine (automatically) stops in front of the appropriate pick location and waits for the
order picker to perform the pick. Compared to a low-level system, a man-on-board system has higher installation and maintenance cost and lower reconfigurability. Ergonomic factors should also be taken into consideration when choosing the proper system since order picking can be heavy work. Repetitively lifting heavy articles, stretching and bending may easily bring physical and mental fatigue to pickers. Ergonomic issues are more important for man-on-board systems due to limitation in the freedom of movement and rapid weariness brought to pickers by lifting and sudden acceleration and deceleration.

The two most popular parts-to-picker systems are carousels and Automated Storage and Retrieval Systems (AS/RS). A carousel consists of a number of bins and shelves that rotate either horizontally or vertically. It is fit for small load storage and retrieval. Control of the carousal can be either manual by the order picker or automatic. AS/RS use aisle-bound cranes traveling vertically and horizontally simultaneously in a storage aisle, transporting storage containers to a pick station located at the end of the aisle. At the pick station, the order picker takes the required number of pieces, after which the storage container is transported by the crane to a storage location. The automated crane can work under different operating modes: single, dual and multiple command cycles. The single-command cycle means that either a load is moved from the depot to a rack location or from a rack location to the depot. In the dual-command mode, first a load is moved from the depot to the rack location, and next another load is retrieved from the rack. In multiple command cycles, the S/R machine has more than one shuttle and can pick up and drop off several loads in one cycle. A comprehensive literature review on AS/RS operating modes can be found from Sarker and Babu (1995), Van den Berg (1999), Rouwenhorst et al. (2000), Roodbergen (2001), and Gu et al. (2007).

In general, parts-to-picker systems are often easier for supervision, and offer higher productivity. On the other hand, they are often more expensive, are more difficult to reconfigure, and require more maintenance than picker-to-parts systems. It is important to mention that different order picking systems may exist simultaneously in a warehouse, for example, an AS/RS for slow movers and a manual-pick system for medium and fast movers.
Put systems often combine picker-to-parts and parts-to-picker systems. First, items have to be retrieved, which can be done in a parts-to-picker or picker-to-parts manner. Second, the carrier (usually a bin) with these pre-picked units is offered to an order picker who distributes them over customer orders (‘puts’ them in customer cartons). Put systems are particularly popular in case a large number of customer order lines have to be picked in a short time window (for example, at the Amazon Germany warehouse, or at flower auctions) and can result in about 500 picks on average per order picker hour (for small items) in well-managed systems (De Koster 2008). Newly developed systems indicate that up to 1000 put handlings per worker hour are feasible provided worker travel can be eliminated.

As mentioned before, low-level picker-to-parts order picking systems are the most common order picking system used in warehouses. We will detail our discussion on it in the next sub-section.

1.1.2. Low-level picker-to-parts order picking systems

Low-level picker-to-parts order picking systems have the advantage of low initial cost, easy installation, easy configurability, and low maintenance cost and hence they are used widely in practice. There are several organizational variants of low-level picker-to-parts systems. The basic variants include picking by article (batch picking) or picking by order. In the case of picking by article, multiple customer orders (the batch) are picked simultaneously by an order picker. Many in-between variants exist, such as picking multiple orders followed by immediate sorting (on the pick cart) by the order picker (sort-while-pick), or the sorting takes place after the pick process has finished (pick-and-sort).

Another important basic variant is zoning, which means dividing the whole pick area into a number of smaller areas (or zones) with one or more pickers assigned to each zone for picking the required items stored in the zone. The major advantages of zoning include familiarity of each picker with his/her zone, shortening travel distance (due to smaller traversed area), reducing congestion and the ease of administration and control (De Koster
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and Yu 2007; Jane and Laih 2005; and Petersen 2002). Depending on the picking strategy, zoning may be further classified into three types: **progressive zoning**, **synchronized zoning**, and **Bucket Brigades** (which can be regarded as a special form of progressive zoning).

Under the progressive zoning strategy, each order (or possibly a batch of orders) is processed only in one zone at a time; at any particular point in time each zone processes an order that is different from the others. Hence, the order is finished only after it sequentially visits all the zones containing its line items. This system is also referred to as a **pick-and-pass** system since orders are passed from one zone to another.

Under the synchronized zoning strategy, all zone pickers can work on the same order (or normally a batch of orders) at the same time and then consolidate the order in a designated location as it is completed. There may be some idle time of zone pickers waiting until all other zone pickers finish the current order. This synchronization of pickers intends to keep the orders from being mixed, and so to lessen the complexity of the following stages such as the accumulation and sortation.

A bucket brigade is a version of zoning where the workload, not a fixed break point, defines the zone. The bucket brigade concept initiates from the way of coordinating workers on a progressive assembling line. A balanced allocation of work will be automatically achieved if the workers are positioned from slowest to fastest along the line toward the direction of work flow (see Bartholdi and Eisenstein 1996; 2005; 2007; Bartholdi and Gue 2000; and Bartholdi et al. 2001). Bucket brigade can also be applied to order picking processes, assuming products are stored in a rack grouped in a line. Picking is done by pickers along the storage rack. When the last order picker (toward the flow of the picking line) completes an order, this picker pushes the order container away (e.g., onto a conveyor) and walks back to take over the order of the previous order picker, who in turn walks back and takes over the order of the previous picker. The process continues until the first order picker begins a new order. Bucket brigade can be seen as a version of progressive zoning where the zone sizes are variable. Bartholdi and Eisenstein (2007) implemented bucket brigades in the distribution center of Revco Drug Stores in North America and showed that bucket brigade increases the throughput rate and reduces management efforts comparing to conventional progressive zone-picking.
A pick-and-pass system is easy to implement and control. The orders are kept integral during the picking process which eliminates the sorting procedure at the end of the picking process. The synchronized zoning system usually gives a shorter response time at the expense of order integrity than the progressive zoning system (Jane and Laih 2005). However, the products need to be sorted per order after the picking process. This trade-off should be taken into consideration when choosing the appropriate zoning strategies. For both pick-and-pass and synchronized zoning systems, balancing workload between zones is an important issue which has large impact on the order picking performance. In a pick-and-pass order picking system, imbalance between zones may cause long waiting time and queues in front of the highly loaded zones and starving at zones with light load. In a synchronized zoning system, imbalance leads to longer idle time for those order pickers at zones with light load, which leads to lower utilization of the system and lower productivity of pickers. In a bucket brigade, a picker travels back to the previous picker or to the order release point to obtain the next order rather than waiting for another order to enter the zone. This has the effect of eliminating waiting time and backlogs. However, it takes training to assure that all items are picked during multiple handoffs. Another limitation of bucket brigade is it can only be used for a line-layout of the picking system.

Pick-and-pass systems are used widely in warehouses in Western Europe (e.g., at the European Distribution Center of YAMAHA Motor, and the warehouse of Nedac Sorbo, a Dutch non-food store merchandiser). The analyses of this thesis are mainly concerned with such order picking systems. In the next section, we discuss the pick-and-pass order picking system studied by this thesis.

1.2. Introduction of the pick-and-pass order picking system

As mentioned in section 1.1.2, pick-and-pass order picking systems have wide application in practice. However, literature on such order picking systems is not abundant. In this thesis, we will focus our analysis on such systems. The general conceptual model of the pick-and-pass order picking system discussed in this thesis is illustrated in Figure 1.2. Several variations of the system are discussed in the following chapters in this thesis. In a
common realization of a pick-and-pass order picking system, the pick area is divided into a number of smaller areas (pick stations or zones), each of which contains storage racks for products. The pick stations are connected by conveyors, through which customer orders can be transferred from one station to another. When a customer order (or a batch of orders) is released to the picking system, a bin is assigned to it together with a pick list. To fill the order, the order bin is set up on the conveyor passing various pick stations. If an article needs to be picked at a station, the conveyor will divert the order bin to the station, so that the main flow of order bins will not be blocked by bins waiting for picking. After entering the station, the order bin moves to the pick position. Order pickers are assigned to stations to pick the products in their stations to fill the bin. Having finished the pick list, the order picker pushes the bin back onto the main conveyor, which transports the bin to the next pick station. When all pick stations, where articles have to be picked, have been visited by the order bin, the content is checked and packed. Sorting is needed at the end when multiple orders are batched in one bin. Such pick-and-pass order picking systems are commonly used for small to medium sized items such as health and beauty products, household, office, or food products where the items can be stored in relatively small and accessible pick locations. They are typically applicable in case of a large daily number of multi-line orders. Summarized by De Koster (1996), the advantages of such pick-and-pass order picking systems include: flexibility in the number of order pickers that can be assigned to the pick stations and flexibility in the articles that can be handled; picking can be done directly in the final packing carton; high throughput can be achieved; cost-efficiency as compared with, for example, manual transportation; and ease of implementation.

Figure 1.2: Conceptual illustration of the pick-and-pass order picking system.
Many factors impact the performance of the pick-and-pass order picking system: the layout of the pick area; the number of zones and the size of each zone; the batching policy of orders for picking; the size of the storage face of the pick area and the storage policy of products at the pick area; the order picker routing policy at each pick station; the sorting and packing process at the end of the picking system; and the congestion in the pick area during the picking process. Measurements for the performance of an order picking system include: the throughput time of an order, the throughput of the system, makespan, the use of space, equipment or labor (see Le-Duc 2005). The analysis in this thesis mainly concentrates on the mean order throughput time in a pick-and-pass order picking system (chapter 2, 3, and 5). Chapter 4 focuses on the throughput and the labor cost of such a system. In the next section, we will elaborate the impact of the above factors on the performance of the pick-and-pass order picking system and briefly recapitulate the literatures on these topics.

### 1.3. Issues impacting the performance of a pick-and-pass system

As mentioned in the previous section, pick-and-pass order picking system has been used widely in warehouses. The following issues will impact the performance of a pick-and-pass order picking system:

- Layout of the pick area
- Zoning
- Batching
- Storage assignments
- Routing
- Order accumulation and sorting
- Congestion
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We will discuss the above topics in the following sub-sections with each topic in one sub-section. For each topic, we first give an introduction about the topic and then explain its impact on the performance of a pick-and-pass order picking system, followed by a brief discussion of the relevant literature on the topic. A comprehensive literature review on warehousing systems can be found in Ashayeri and Gelders (1985), Cormier and Gunn (1992), Gu et al. (2007), Van den Berg (1999), Van den Berg and Zijm (1999), and Rouwenhorst et al. (2000). Issues in design and control of order picking processes in particular, are mentioned in Goetschalckx and Ashayeri (1989), Le-Duc (2005), Roodbergen (2001), and De Koster et al. (2007).

1.3.1. Layout of the pick area

The layout problem for a pick area concerns the determination of the number of blocks, the number of aisles in a pick zone, the length and width of aisles in a block, and the position of the depot. See Figure 1.3 for the decision issues in pick area design. The common goal is to find a ‘best’ warehouse layout with respect to some certain objective functions among the layouts, which fit a given set of constraints and requirements. The most common objective function is the travel distance.
The number of aisles in a pick zone, i.e., the zone size, has large impact on the pick-and-pass order picking system performance. We will combine the discussion on this element with zoning in the next sub-section. The layout of the storage racks within a pick zone also impacts the performance of a pick-and-pass order picking system. We distinguish two types of layouts in this thesis: 1) a line layout where the storage face is located along the main conveyor line and 2) storage aisles are located perpendicular to the main conveyor line. We discuss these two layouts respectively in chapter 2 and chapter 3. The depot position at a zone influences the travel time of order pickers in the picking process. Jewkes et al. (2004) study the optimal pick position of an order bin at a pick station with a line
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layout in a pick-and-pass order picking system with objective to minimize order throughput time. The width of the picking aisles also influences picking performance. In a narrow aisle, blocking can occur when pickers can not pass each other. Even in a wide aisle, blocking may also happen at a pick location when two or more pickers need to pick at the same pick location. Picker blocking will negatively impact the throughput and the mean order throughput time in a pick-and-pass order picking system. The effect of aisle width on order picking system performance has been studied by Gue et al. (2006) and Parikh and Meller (2007).

Early publications on warehouse layout design can be found in Bassan et al. (1980), Rosenblatt and Roll (1984) and Rosenblatt and Roll (1988). Roodbergen (2001) proposes a non-linear objective function (i.e., average travel time in terms of number of picks per route and pick aisles) for determining the aisle configuration for random storage warehouses (including single and multiple blocks) that minimizes the average tour length. Also considering minimization of the average tour length as the major objective, Caron et al. (2000) consider 2-block warehouses (i.e., one middle cross aisle, see Figure 1.3) under the COI-based storage assignment (see section 1.3.4 for a discussion of storage assignment methods), while Le-Duc and De Koster (2005b) focus on the class-based storage assignment. For both random and volume-based storage assignment methods, Petersen (2002) shows, by using simulation, the effect of the aisle length and number of aisles on the total travel time. For a one-block warehouse, Roodbergen and Vis (2006) present analytical formulas to show the relationship between the order picking area and the average length of a picking route. They study several routing heuristics and find the best depot location is in the middle of the front cross-aisle when products are stored randomly in the picking area. Eisenstein (2008) analyzes the optimal depot positions and the corresponding optimal product assignments in a line-layout order picking system when single and dual depots are allowed along the pick line. The objective is to minimize the expected order picker travel distance. He also considers the optimal product assignment in the situation of no-depot but with conveyors installed along the pick line. In their recent paper, Gue and Meller (2006) propose two innovations in warehouse design, diagonal cross aisles and picking aisles having different orientations, to increase order picking throughput. The effect of the zone size on a pick-and-pass order picking system
performance is discussed in chapter 2 and 3 in this thesis. In chapter 4, we discuss the
determination of the pick area zone size in an order picking system using dynamic storage.
Much of the existing knowledge on warehouse layout is captured in the Erasmus-Logistica
website (http://www.fbk.eur.nl/OZ/LOGISTICA) that can be used to interactively
optimize warehouse layouts for various storage and routing strategies.

1.3.2. Zoning

As discussed in section 1.1.2, zoning is a method to divide the total order picking area into
smaller units. Order pickers only retrieve products located in their zone. Zone sizes and the
number of order pickers per zone have large impact on the performance of a pick-and-pass
system. With a fixed length of the whole order picking system (i.e., a fixed storage
capacity of the system) and a fixed number of order pickers, the larger the size of the pick
stations, the fewer number of stations we have in the system, and the more order pickers
are available at each pick station. Pick stations of larger size will increase the service time
due to longer travel time, and the fewer number of stations tends to increase the utilizations
of pick stations due to higher order arrival rates. Therefore they lead to an increase of the
mean order throughput time in the system. But on the other hand, fewer number of stations
leads to fewer station visits of an order (hence fewer queues and less setup time); more
order pickers per station implies decreasing utilizations at pick stations, which reduces the
mean order throughput time. Balancing the trade-off between these opposite effects by
deciding the appropriate zone size is important for a pick-and-pass system.

Little literature on zoning is available. Mellema and Smith (1988) use simulation to
examine the effects of the aisle configuration, stocking policy, batching and zoning rules
on order pickers’ productivity. They suggest that a combination of batching and zoning can
significantly increase the order pickers’ productivity (pieces per man-hour). Also, using
simulation, Petersen (2002) shows the zone shape, the number of items on the pick list, the
storage policy and the layout of the warehouse (with or without a back cross-aisle in the
pick zone) have a significant effect on the mean travel distance within the zone. Choe et al
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(1992) develop a queuing model to analyze the performance of a pick-and-sort order picking system with synchronized zoning. Brynzér and Johansson (1995) use case studies to analyze the impact of zoning and batching on picking efficiency. Malmborg (1995) studies the problem of assigning products to locations with zoning constraints. Using a mixed-integer programming method, Le-Duc and De Koster (2005a) consider the problem of determining the optimal number of zones (for a given pick area) in a pick-and-pack order picking system to minimize the mean order throughput time. Jewkes et al. (2004) determine the optimal stop position of an order bin in a pick station (zone), the optimal product location in pick stations, and the size of pick stations with the objective of minimizing the order throughput time. Using a general queuing network model, Yu and De Koster (2008b) analyze the impact of order batching and pick area zoning on a pick-and-pass order picking system performance (refer to chapter 3).

Balancing workload between zones is an important issue in determining zone size and assigning order pickers to zones. As discussed in the previous section, workload imbalance can cause serious deterioration of order throughput time and the order throughput in the system. Jane (2000) proposes several heuristic algorithms to balance the workload among order pickers and to adjust the zone size for order volume fluctuation in a pick-and-pass system. Jane and Laih (2005) consider the problem of heuristically assigning products to zones to balance the workload between zones in a synchronized order picking system. Their method is based on co-appearance of items in the same order (i.e., items often appearing in the same order are stored in the same zone). Meller and Parikh (2006) propose a mathematical model analogous to the classical dual bin-packing problem introduced by Coffman et al. (1978), to assign orders to pickers and picking waves in both batch picking and zone picking systems with an objective to minimize workload imbalance among order pickers and zones. Van Nieuwenhuyse et al. (2007) analyze the impact of workforce allocation between the picking and the sorting area in a pick-and-sort order picking system. Their research reveals that the minimum mean order throughput time is achieved when the workload is balanced between the picking and the sorting area. Yu and De Koster (2008a) show that balancing workload between pick stations in a pick-and-pass order picking system can improve order throughput time significantly (see chapter 2). De Koster and Yu (2007) design a heuristic to assign customers to zones to balance workload
between distribution zones for an internal distribution process at Aalsmeer Flower Auction company, the largest flower auction in the world (see chapter 5).

1.3.3. Batching

Order batching is the process of grouping customer orders together and jointly releasing them for picking. Batching is a popular strategy to improve productivity due to the reduction in order picking travel time. Instead of traveling through the warehouse to pick a single order, the picker completes several orders with a single trip. Hence, the travel time per pick can be reduced.

To batch orders, in a pick-and-pass system, these orders are assigned to a single bin and then released to the system for picking. Trade-offs exist in the order picking process: if batch sizes increase, the flow rates to pick stations will decrease (fewer bins to stations), leading to lower utilization of the stations and hence reducing the potential waiting time of bins in front of each station; on the other hand, a larger number of orders in a bin means longer service time at pick stations which tends to increase the mean order throughput time in the system. Also, a larger batch size implies longer queuing time for batch completion and longer processing time in the sorting process at the end of the pick-and-pass system. An interesting topic in pick-and-pass order picking systems is therefore to determine when to batch orders, how to batch orders, and to determine what the impact of batch size on the system performance is. These issues are studied in chapters 2 and 3.

According to the availability of order information, research on batching for general order picking systems is classified into two types: static batching and dynamic (online) batching. In static batching, the order information, i.e., the number of order lines (an order line is a certain number of pieces of one article) in each order, is known at the beginning of the planning horizon. The batching problem is then to decide the assignment of each order to a batch. The optimal solutions of order batching problems with the objective of minimizing the total travel time (distance) for picking a certain number of orders are difficult to obtain because the travel distance implication of assigning a specific order to a batch is dependent
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on the other orders that are assigned to the batch (see Rosenwein 1996). The reported results for optimization-based batching methods are limited to problems with small and medium sized sets of orders (see Armstrong et al. 1979; Chen and Wu 2005; Gademann et al. 2001; Gademann and Van de Velde 2005).

Due to the difficulties in solving the order batching problem to optimality, heuristics have been developed by researchers. Two types of heuristics are distinguished: *seed algorithms* and *savings algorithms*. In a seed algorithm, a seed order (initial order) is first selected based on *seed selection rules* for a batch and afterward unassigned orders are added into the batch according to *order addition rules* until the order picker is filled to capacity. Savings algorithms originate from Clarke and Wright (1964) and are based on the time saving that can be obtained by combining two orders in one picking tour as compared to the situation where both orders are picked individually. De Koster et al. (1999a) give a comprehensive review on these heuristics and perform a comparative study for the variants of both seed and saving algorithms. They conclude that even simple order batching methods lead to significant picking time savings compared to the first-come first-serve batching rule.

*Dynamic batching* takes the stochastic property of the order profile (i.e., the order arrival process and the number of order lines in a batch) into consideration. The batching problem is to determine the batch size or the batch time window (i.e., the time interval used to batch orders) such that the average throughput time of an arbitrary order is minimized. Literature on dynamic batching is not abundant, Chew and Tang (1999) and Tang and Chew (1997) model the order batching problem for a single-block warehouse as a queuing model and apply a series of approximations to calculate the lower bound, upper bound, and the mean value of the travel time of a picking tour. They consider the average throughput time of the first order in a batch as the estimation for the average throughput time of individual orders. Le-Duc and De Koster (2007) extend the work of Chew and Tang (1999) into a warehouse with 2-blocks. They perform a direct analysis on the average throughput time of an arbitrary order in the system. For a parallel-aisle warehouse with stochastic order arrivals, Gong and De Koster (2007) use polling models to analyze dynamic order picking, where order pickers travel around the pick area with the aid of RF equipment to pick all
outstanding order lines in their pick routes. They show dynamic order picking leads to shorter order throughput time compared with traditional batch picking with optimal batch size as described by Le-Duc and De Koster (2007). Gong and De Koster (2008) develop efficient heuristics to determine the optimal batch sizes with objectives of minimizing the total operational time for all batches, minimizing the average waiting time of a customer order in the system, and minimizing the total system cost per order which includes the operational cost and the waiting cost. Van Nieuwenhuyse et al. (2007) model the picking and the sorting processes as a tandem queue. They use a queuing network approach to analyze the factors influencing optimal batch size and the allocation of workers to the picking and the sorting processes. Recently, Schleyer (2007) proposes discrete time queuing models to analyze interdeparture and waiting time distributions under the different batch building modes, batch arrival processes, and batch service processes for material flow processes. In the work of Yu and De Koster (2008a), two consecutive orders with fewer than a certain number of order lines are batched and released to the pick-and-pass order picking system simultaneously. They analyze the impact of this batching rule on the mean order throughput time in the system (see chapter 2). Yu and De Koster (2008b) study the impact of batch size on mean order throughput time under the different zone sizes and the number of pickers per zone in a pick-and-pass order picking system by means of queuing network model (see chapter 3).

1.3.4. Storage assignment

Before the picking process can start, received products must be stored. A storage assignment policy is a set of rules used to assign products to storage locations. In a warehouse, the total available storage locations are normally divided into two parts: the forward area and the reserve area. The forward area, also referred as pick area, is a sub-region of the warehouse used for efficient order picking. The reserve area holds the bulk storage. It is used for replenishing the forward area and for picking those products not assigned to the forward area. Picking from the bulk area is usually less efficient than from the forward area.
The size of the forward area has a large impact on the performance of a pick-and-pass order picking system. A warehouse normally contains thousands of products. Reserving a pick slot for each product in the pick area results in a large area. Therefore, order pickers spend much time on travelling in the picking process, leading to low productivity and relatively low throughput of the order picking system. According to Tompkins et al. (2003), travel time accounts for around 50% of an order picker’s total time spent in a picking process. In chapter 4, we will introduce the concept of dynamic storage. It aims at making the forward pick area small in order to reduce travel time. It brings the products to the storage locations dynamically, just in time for the start of the picking process (by an automated crane). The number of locations available in the forward area is usually much smaller than the total number of Stock Keeping Units (SKUs). As these systems are capable of achieving very high picker productivity, they are becoming more and more popular in practice. Although research on dynamic storage is scarce, some literature exists on forward-reserve problems. Hackman and Platzman (1990) develop a model to decide which products should be assigned to the pick area and how much space must be allocated to each of the products given fixed capacity of the forward area, with an objective to minimize the total costs for order picking and replenishment. Frazelle et al. (1994) extend the problem and the solution method of Hackman and Platzman (1990) by treating the size of the forward area as a decision variable. Van den Berg et al. (1998) consider a warehouse with busy and idle periods and reserve-picking is allowed. Assuming unit-load replenishments, they develop a knapsack-based heuristic to find an allocation of products to the forward area that minimizes the expected total labor time related to order picking and replenishment during a busy period.

The storage assignment policy within a pick area also influences the performance of an order picking system since it determines the product locations and hence influences the travel time of order pickers in the system. The following three storage policies are mentioned frequently in literature: random storage, dedicated storage and class-based storage.
In a *random storage assignment*, items are allocated randomly over the available storage locations. Random storage has the advantage of high space utilization. Random storage policy is used in chapter 3 and chapter 4 in this thesis.

In a *dedicated storage assignment*, each item has a fixed storage location. Products are first sorted according to a certain rule. Mainly two types of rules exist in literature: Cube-per-Order Index (COI) and pick volume (frequency or turnover). The COI of an item is defined as the ratio of the item’s total required storage space to the number of trips required to satisfy its demand per period. The product with the lowest COI value is located closest to the depot. Next the product with the second lowest COI value is stored to the next closest location to the depot. The assignment continues until all the products are allocated. Similar principles apply to pick volume based assignment.

Literature on COI-based assignment includes Kallina and Lynn (1976), Malmborg and Bhaskaran (1987; 1989; 1990), Malmborg (1995; 1996) and Caron et al. (1998). Jarvis and McDowell (1991) prove the COI-based storage strategy is optimal in a single block warehouse to minimize the expected travel distance per order. Petersen (1997; 1999; 2000), Petersen and Schmenner (1999), and Petersen and Aase (2004) studied volume-based storage policies. The major disadvantage in implementing these storage methods is that items demand rates vary constantly and the product assortments change frequently, leading to a large amount of reshuffling of stock for each change. A solution might be to carry out reshuffling once per period. The loss of flexibility and consequently the loss of efficiency might be substantial when using these storage methods.

A storage method used frequently in practice is *class-based storage assignment*, which combines some of the methods mentioned so far. This method divides the products into a number of classes by some measures of demand frequency of the products, such as COI or pick volume. Each class is then assigned to a dedicated area in the warehouse. Two common types of assignment of product classes to a low level picker-to-parts warehouse, *within-aisle storage* and *across-aisle storage*, are illustrated by De Koster et al.(2007). Storage within a class is random. Some researchers study class-based storage in low-level picker-to-parts systems. Jarvis and McDowell (1991) suggest that each aisle should contain only one class of products, resulting in a within-aisle storage. Petersen and Schmenner
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(1999) and Petersen (1999) evaluate different class-based storage policies and conclude that the within-aisle storage strategy proposed by Jarvis and McDowell (1991) provides travel distance savings of 10-20% over the other class-based storage policies under different routing strategies. Petersen et al. (2004) recommend that the number of classes should be between 2 and 4. Le-Duc and De Koster (2005b) optimize the storage-class positioning based on a closed form travel-time estimate for the return routing policy. Le-Duc (2005) extends these results for other routing policies. Dekker et al. (2004) investigate routing and storage policies in a real picker-to-part warehouse. In this warehouse, different product groups have to be separated in the picking process. Further, a distinction must be made between breakable and unbreakable items. They develop a class-based storage assignment that leads to short travel time, while meeting the pick sequencing restrictions. Class-based storage policy is discussed in chapter 2 in this thesis.

1.3.5. Routing

A routing policy in an order picking process determines a visit sequence for order pickers to pick multiple products on the pick list. Routing policies impact the performance of a pick-and-pass order picking system by influencing the service time at pick stations. The routing problem is a special case of the well-known Travelling Sales Man problem, which is know to be NP-hard. Due to the specific structure of warehouses, the routing problem in a rectangular warehouse with one or two blocks is polynomial solvable. However, polynomial algorithms do not exist when the number of blocks increases or the warehouse structure is non-rectangular. Optimal routing solutions for warehouses with standard layouts (i.e., rectangular, single or two blocks) can be found in Cornuéjols et al. (1985), Ratliff and Rosenthal (1983), Roodbergen and De Koster (2001a; 2001b) and De Koster and Van der Poort (1998). In practice, heuristic routing methods are often used. They are studied by several researchers (see Hall 1993; Petersen 1997; Roodbergen and De Koster 2001a; 2001b). In general, five routing heuristics are distinguished: S-shape heuristics, return heuristics, midpoint method heuristics, largest gap heuristics, and combined...
heuristics. Detailed description of these heuristics can be found in Roodbergen (2001) and De Koster et al. (2007).

Taking the stochastic property of customer orders into consideration, some researchers analytically estimate the travel time in an order picker’s picking tour. The estimation of travel time provides insights on the design issues of an order picking system, such as determining the layout of the pick area. Chew and Tang (1999) and Tang and Chew (1997) analyze the mean, upper and lower bound of travel time in a single block warehouse with a general product-to-location assignment. Roodbergen (2001) estimates the mean travel time in a single block warehouse with S-shape routing policy and random storage. He later extends the analysis to a warehouse with multiple blocks. The estimation results are used to determine the optimal layout of the pick area with objective to minimize average picking travel distance. Caron et al. (1998) propose a travel time model for estimating the average travel distance of a picking tour when either S-shape or return routing heuristics are used with a COI-based storage method. Hwang et al. (2004) present analytical expressions for three routing policies (return, S-shape, and midpoint) under the COI-based storage rules. Le-Duc and De Koster (2007) develop a model to estimate the first and the second moment of travel time for a 2-block warehouse with S-shape routing policy and random storage method. The estimation enables them to determine the optimal batch size. Le-Duc and De Koster (2005b) estimate the average travel time for the return routing policy with class-based storage in a 2-block warehouse. Travel time estimation is an important part in this thesis when we derive the service time of an order at a pick station in a pick-and-pass order picking system. A detailed discussion on travel time estimation can be found in chapter 2, 3 and 4.

1.3.6. Sorting

Sorting is needed when multiple orders are picked together. It can be performed either during the picking process (sort-while-pick) or after the picking process (pick-and-sort). Sort-while-pick is straightforward but will lengthen the item extraction time as compared to single order picking. Its application can be limited by the product sizes, number of units
per order and the space available for multiple order bins on the pick cart. For a pick-and-sort process, a separate downstream sorting system is needed. In the implementation of a pick-and-pass order picking system, a sorting process is needed when orders are batched.

Sorting and the subsequent grouping per destination (order) can be done manually or automatically. An automated sorting system usually includes an accumulating conveyor, one or more inducts to the sorter, an identification scanner, a recirculation conveyor, and exit lanes. Items of a group of orders (a pick wave) arrive at the accumulating conveyor when they wait to be released into the sorting process. They are inducted onto the recirculation conveyor after the items of the previous pick wave finish their sorting process. The orders are assigned to sorting lanes according to order-to-lane assignment rules. Items circulate on the recirculation conveyor and enter the assigned sorting lane only if all items of the preceding order assigned to that lane have been removed. Eventually, sorted orders are taken from sorting lanes, checked, packed and shipped. The operational problem for sorting involves decisions such as wave-releasing and order-to-lane assignment so that the orders can be efficiently sorted in a given wave (see Bozer et al. 1988; Bozer and Sharp 1985; Johnson 1998; Le-Duc and De Koster 2005a; and Meller 1997).

Some researchers study the impact of sorting on the performance of an order picking system. Van Nieuwenhuyse et al. (2007) model a pick-and-sort order picking system into a general tandem queuing network and use the QNA model developed by Whitt (1983) to analyze it. They derive the expressions for the mean and the variance of sorting time as functions of the batch size and conclude that mean order throughput time is minimized when the workload is balanced between the picking and the sorting stations. Using a mixed integer programming model, Le-Duc and De Koster (2005a) determine the optimal number of zones in a synchronized zoning system such that the total order picking and the sorting time is minimized for a batch of orders. The impact of sorting on the mean order throughput time in a pick-and-pass order picking system is considered in chapter 3.
1.3.7. Congestion

Worker congestion in an aisle network is rarely studied in the specification of material handling systems. For a pick-and-pass order picking system, worker congestion in the pick aisles will lengthen the travel time of order pickers on their picking tours, leading to longer service time, and hence extend the order throughput time in the system. Taylor (1994) states “the measure of the congestion is jam times that can be translated by a supplementary time of transportation added to the minimum time to cross the element of the system”. Sources of system congestion include: (a) vehicles or pickers stopped at pick-up and drop-off locations may block the travel path, (b) intersections, (c) vehicle breakdowns and (d) two or more vehicles or pickers competing to use the same segment.

Studies on congestion in material handling systems mostly focus on Automated Guided Vehicles (AGVs) (see Cheng 1987; Choi et al. 1994; Lee et al. 1990; Maxwell and Muckstadt 1982; Vosniakos and Davies 1989; and Vosniakos and Mamalis 1990). Several researchers take the congestion issue into consideration when studying facility layout and location problems. Chiang et al. (2002) incorporate workflow congestion in layout design. In their work, a model formulated as a quadratic assignment problem, is developed to minimize workflow interference cost. Smith and Li (2001) formulate the facility layout problem as a stochastic quadratic assignment problem taking into account of random phenomena of congestions. Beamon (1999) develops an analytical method to measure congestion in a material handling system. He uses simulation and build up regression models to find the relationship between different material handling system variables and the system congestion.

Research on the impact of congestion on order picking is not abundant. Gue et al. (2006) use discrete time Markov chains to study the effect of pick density (the frequency that pickers stop to make picks) on narrow-aisle pick area congestion. Blocks occur in a narrow aisle when pickers are not able to pass each other due to the limited width of the aisle. In their paper, congestion is defined as the percentage of time pickers are blocked. They find that when the pick density is low, there is little congestion because pickers make few picking; for higher levels of pick density, congestion increases as pickers have to stop for
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picking, causing delays for other pickers. However, as the pick density continues to
increase, pickers tend not to block one another because they spend more time picking and
less time traveling. Using similar methods, Parikh and Meller (2007) analyze the effect of
pick density on congestion, which is defined as blocking at a pick position when two or
more pickers need to pick at the same position, in wide-aisle order picking systems. They
find when pickers pick only one SKU at a pick position, congestion is less in a wide-aisle
order picking system compared to that in a narrow-aisle system discussed by Gue et al.
(2006). However, when pickers pick more than one SKU at a pick-face, congestion
increases monotonically with an increase in the pick density. Chapter 5 of this thesis
introduces zoning and automation to an internal distribution process of a flower auction
company to reduce congestion. It shows that the customer response time improves
significantly when congestion is reduced.

1.4. Research contributions and outlines of the thesis

From the discussion in the previous sections, we see that pick-and-pass order picking
systems have many applications. However, there are several issues essential for the
understanding and the application of such systems that have not been studied adequately.
The following three issues are especially important and urgent.

1. In a pick-and-pass system, an order may visit multiple stations and multiple
orders may wait in front of stations for processing. Exact analysis of such systems
is difficult. It would be helpful to develop a fast approximation model to estimate
the performance of a pick-and-pass system (for example, mean order throughput
time, waiting time at stations, station utilizations, and so on) under different
warehousing operational strategies.

2. Increasing productivity and reducing labor cost in order picking are two major
concerns for warehouse managers. Conventional picker-to-parts order picking
methods lead to low productivity as order pickers spend much of their time in
traveling along the aisles. Designing an order picking system, which uses few
Introduction

worker hours while achieving high throughput, is an interesting study area for warehousing researchers.

3. Congestion often occurs in a real-life distribution process, but is often overlooked by researchers. Congestion in the pick area prolongs order lead time and makespan. Finding solutions to reduce congestion and applying them to practice bring both scientific and managerial contributions.

This thesis enriches the current research on order picking by providing analysis and solutions to the above mentioned issues. More specifically, the content of this thesis and its contributions are summarized as follows:

- Developing a fast, flexible, and accurate tool to estimate the mean throughput time of an arbitrary order in a pick-and-pass order picking system with stochastic order arrivals under different operational policies. According to our knowledge, in literature, this is the first attempt to model pick-and-pass systems with such policies and general service and arrival processes (see chapter 2). The discussion of this topic is based on Yu and De Koster (2008a).

- Zoning and batching are two important operations in a warehouse and are often implemented together. However, in literature, the analyses on zoning and batching are often separated. In this thesis, we use a general queuing network model to study the combined impact of zoning and batching on the pick-and-pass order picking system performance (see chapter 3). The discussion of this topic is based on Yu and De Koster (2008b).

- Although dynamic storage is used by more and more warehouses, literature on this concept is scarce. This thesis is the first to model and analyze a dynamic storage system with online order processing. Through analyzing the stability conditions of this system and applying the concept to a pick-and-pass order picking system, we demonstrate that dynamic storage leads to substantial improvements on order picking throughput and labor costs (see chapter 4). The discussion of this topic is based on Yu and De Koster (2008c).
Chapter 1

- The impact of congestion in travel aisles is often overlooked by researchers, but significantly influences the performance of a real material handling process. This thesis introduces the zoning concept with balanced workload between zones into an internal distribution process to reduce system congestion. We apply this to the case of VBA, a Dutch flower auction company. The resulting zoned distribution process resembles a reversed pick-and-pass order picking system. We show that zoning and workload balancing lead to improved order lead time and total makespan of the distribution process (see chapter 5). The discussion of this topic is based on De Koster and Yu (2007).

We draw conclusions and discuss suggestions for future research in chapter 6.
2. PERFORMANCE APPROXIMATION AND DESIGN OF PICK-AND-PASS SYSTEMS

2.1. Introduction

As discussed in the introduction chapter, pick-and-pass order picking systems have large application in practice. Many warehouses tend to accept late orders while providing rapid and timely delivery within tight time windows, which implies the time available for order picking becomes shorter (De Koster et al. 2007). Therefore minimizing order throughput time is an important objective. Exact analysis of a pick-and-pass system is difficult due to the large state space in modeling bin positions on the conveyor and difficulties in obtaining the exact distribution of service time at stations. At the design phase, a fast and flexible tool to estimate design alternatives on the mean order throughput time of a pick-and-pass order picking system is of importance for warehouse designers. In this chapter, we propose an approximation-based modeling and analysis method to evaluate the mean order throughput time in a pick-and-pass order picking system. The method provides a tool for fast evaluation of the impact of storage policies, sizes of pick stations, the number of order pickers per station, and the customer order profiles on order picking system performance. Additionally, the method also evaluates the effects of order batching and splitting on system performance. The modeling and the analysis of the system is based on the analysis of a $G/G/m$ queuing network by Whitt (1983). We show the approximation method leads to acceptable results by comparing it with both simulation and with the real order picking process at a parts DC of an international motor production company.
Chapter 2

In the rest of this chapter, we first describe the pick-and-pass system under consideration, and then review relevant literature briefly. After that, we elaborate the approximation model followed by validation of this model by means of simulation and comparison with a real order picking process. Next, we use this model to analyze the impact of different warehousing strategies on the order picking system performance and draw conclusions.

2.2. The pick-and-pass system under consideration

In this chapter, we consider a common type of pick-and-pass order picking system, which consists of a conveyor connecting all pick stations located along the conveyor line, as sketched in Figure 2.1. De Koster (1996) summarizes the advantages of such pick-and-pass order picking systems (refer to section 1.2). Storage shelves are used to store products at each pick station. A customer order contains several order lines (an order line is a number of units of one article). A bin is assigned to a customer order together with a pick list when it arrives at the order picking system. To fill an order, the order bin is transported on the conveyor passing various pick stations. If an order line has to be picked at a station, the transportation system automatically diverts the bin to the station, so that the main flow of bins cannot become blocked by bins waiting for picking. After entering the pick station, the order bin moves to the pick position. Order pickers are assigned to pick stations to fill customer orders. An order bin is processed by one order picker at a station and an order picker works on one order at a time. In this chapter, we assume the order picker picks one order line per picking trip. The picker starts his trip from the pick position, reads the next article on the bin’s pick list, walks to the storage shelves indicated, picks the required article, goes back to the pick position and deposits the picked article into the bin.

Although in some systems multiple lines may be picked in a picking trip, we model the simpler case where only one article is picked per trip. Systems which we have observed that adhere to this constraint include the parts Distribution Center (DC) of an international motor production company (we use this example in our model validation in section 2.5) where one article is picked per trip since articles are relatively heavy and need to be
Performance Approximation and Design of Pick-and-Pass Systems

barcode scanned. In another warehouse we studied, even light articles were not batched to reduce pick errors. In the next chapter, we relax this constraint and study the situation where multiple products are picked per trip. Having finished the pick list, the order picker pushes the bin back onto the main conveyor, which transports the bin to a next pick station. Such pick-and-pass systems are typically applicable in case of a large daily number of multi-line orders.

![Illustration of the pick-and-pass order picking system.](image)

Figure 2.1: Illustration of the pick-and-pass order picking system.

2.3. Related literature

The most relevant literature for the pick-and-pass system illustrated in Figure 2.1 is De Koster (1994). In that paper, he approximates pick-and-pass order picking systems by means of Jackson network modeling and analysis. His model assumes the service time at each pick station is exponentially distributed and customer orders arrive according to a Poisson process. The model described in this chapter generalizes the Jackson queuing network modeling of De Koster (1994) by allowing a general order arrival process and general service time distributions, which represents real-life warehouses more accurately and provide a deeper understanding of the pick-and-pass order picking system. Another relevant paper is Jewkes et al. (2004). They study the optimal pick position of an order bin at a pick station, the optimal product location at pick stations, and the size of pick stations in a pick-and-pass order picking system with the objective of minimizing the order throughput time. Since they consider a static setting, only stochastic travel time to pick
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orders is considered in their paper. In this chapter, we consider a dynamic setting, where the waiting times of order bins in front of pick stations are taken into account.

2.4. The approximation model

As illustrated in Figure 2.1, the pick-and-pass order picking system is represented by a sequence of pick stations connected by conveyor pieces. The service time for an order bin at a pick station consists of several components: setup time (time for starting and finishing the pick list, checking, weighing, labeling, etc.), travel time, and the picking time for order lines. Travel time depends on the number of order lines to be picked at the station, the location of these order lines in the pick station, and the travel speed of pickers. In this chapter, the route the pickers take is not an issue since storage aisles have a line-layout as illustrated in Figure 2.2 and pickers pick one line at a time. We will consider a different aisle layout in the next chapter, where routing needs to be considered. Picking time is proportional to the number of order lines to be picked in the station. We assume setup time and pickers’ travel speed are constants. We also assume the picking time per order line, which may consist of multiple units, is constant, and independent of the product type and the number of units picked. These assumptions will be reasonable when the variance of the number of units picked per order line and the pick time itself are relatively small. We assume a pick-frequency class-based storage policy (see section 1.3.4) in each station. Similar to other research (see e.g., Petersen et al. 2004), we assume demand is uniformly distributed over the products within a product class. The service time at a pick station is modeled as having a general distribution and is characterized only by its mean and Squared Coefficient of Variation (SCV). It is reasonable to use only two moments because in reality service time is hard to fit a theoretical distribution, whereas the information on mean and the variance of service time is relatively easy to obtain.

A conveyor piece $j$ can contain $k_j$ order bins and is assumed to have constant speed, $v_j$. We approximate it as $k_j$ servers in parallel, each of which has constant service rate of $v_j / k_j$. This means that the output rate of a conveyor piece $j$ equals exactly $v_j$ if and only if it is
completely full with bins. In the approximation, the output rate of a conveyor piece is proportional to the number of bins on it. At the end of a conveyor piece, a transition is made by the order bin to the subsequent conveyor piece, or it is pushed into a pick station. The transition probability of an order bin to enter a pick station depends on the bin’s pick list and the storage assignment of products in that station. We approximate this behavior by Markovian transition probabilities, which is justified in the case of a large number of independent bins processed per time unit (the typical application area of these systems). The transition probabilities at the end of a conveyor piece and at leaving a pick station are calculated in section 2.4.2. After finishing the picking at a station, the bin is pushed onto a conveyor piece downstream the pick station.

We assume each pick station has infinite storage capacity (buffer) for order bins. This assumption is reasonable because in reality order pickers at pick stations will ensure that the system will not be blocked when their stations become full. If a pick station tends to become full, the order pickers can temporarily put the bins on the floor. We also assume there is a buffer with infinite capacity in front of each conveyor piece, which means that the arrivals will not be lost and pick stations and conveyor pieces can not become blocked because of lack of output capacity. This assumption is also realistic because the conveyor pieces can normally contain a sufficiently large number of bins.

The whole pick-and-pass order picking system is modeled approximately as a $G/G/m$ queuing network consisting of $C+S$ nodes preceded by unlimited waiting space in front of them. Nodes 1, 2,...,$C$ represent conveyor pieces and nodes $C+1, C+2,..., C+S$ represent pick stations. The number of servers at each node equals the capacity of each conveyor piece or the number of order pickers working at the station.

The main notations used to analyze the queuing network are listed below. Other notations used in this chapter are defined in the context when they are needed.

\textit{Data}

$C$ : the number of conveyor pieces, with index $j$, from 1 to $C$.

$S$ : the number of pick stations, with index $j$, from 1 to $C+1$ to $C+S$. 

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$J$: the total number of nodes in the picking system, with index $j$, from 1 to $C+S+1$.

$h_j$: the number of order pickers at pick station $j$, $j = C+1, C+2, \ldots, C+S$.

$k_j$: the capacity of conveyor piece $j$, expressed in bins. $j = 1, 2, \ldots, C$.

$m_j$: the number of servers at node $j$, $j \in J$.

$I$: the number of product classes stored in the pick stations, with index $i$.

$f_i$: order frequency of the $i^{th}$ class products.

$l_{ij}$: space used to store the $i^{th}$ class products at station $j$, $j = C+1, C+2, \ldots, C+S$.

$n$: the number of order lines in an order, a random number.

$N$: the maximum number of order lines contained in a customer order, with index $n$.

$o_n$: probability that an order contains $n$ order lines, $n = 1, 2, \ldots, N$.

$pt$: picking time for one order line, expressed in seconds.

$st$: setup time per order bin at a pick station, expressed in seconds.

$v$: picker’s travel speed in zones (in meters per second).

$vl_j$: the velocity of conveyor piece $j$, expressed in bins per second. $j = 1, 2, \ldots, C$.

$\lambda_0$: external arrival rate of order bins to the system, entering node 1, expressed in bins/second.

Variables

$\lambda_j$: internal arrival rate of order bins to node $j$, $j \in J$.

$c_{e0}^2$: SCV of inter-arrival time of order bins to the system.

$c_{ej}^2$: SCV of inter-arrival time at node $j$, $j \in J$.

$c_{sj}^2$: SCV of service time at node $j$, $j \in J$.

$P_j$: probability that an order line is picked at station $j$, $j = C+1, C+2, \ldots, C+S$. 

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In the next two subsections, we will derive expressions for the mean and the SCV of the service time at each node and then calculate the mean throughput time of an order bin in the system.

2.4.1. Service time at pick stations and conveyor pieces

The mean service time at station $j$ if the order bin enters station $j$, has three components, setup time $st$, travel time $tr_j$, and the picking time $pk_j$. The mean service time is calculated by

$$E[se_j] = st + E[tr_j] + E[pk_j], \quad \forall j > C$$

We assume $st$ is constant. Next, we derive the expressions for the last two components in the equation above.

The probability that an order line of class $i$ is stored in station $j$ depends on the order frequency, $f_j$, of the $i$th class products and the space used to store the $i$th class products in station $j$, $l_i$. It is given by
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\[ p_{ij} = f_j \sum_{j=1}^{l_{ij}} l_{ij}, \quad \forall i, \forall j > C \quad (2.2) \]

Therefore, the probability that an order line is picked in station \( j \) is the summation of \( p_{ij} \) over \( i \).

\[ P_j = \sum_{i=1}^{j} p_{ij}, \quad \forall j > C \quad (2.3) \]

So the conditional probability of an order bin to enter station \( j \) given that there are \( n \) order lines in the order equals the probability that there is at least one order line to be picked at station \( j \):

\[ V_{mj} = 1 - (1 - P_j)^n, \quad \forall j > C, \forall n \quad (2.4) \]

Where \((1 - P_j)^n\) is the probability that none of the order lines in this order bin is to be picked in station \( j \). The probability of an order bin to enter station \( j \) now becomes:

\[ V_j = \sum_{n=1}^{N} V_{mj} \ast o_n, \quad \forall j > C \quad (2.5) \]

The number of order lines to be picked in station \( j \) given that the order bin contains \( n \) order lines is a random variable with binomial distribution, i.e.,

\[ P(X_j = x_j | n \text{ order lines in an order}) = \binom{n}{x_j} p_j^{x_j} (1 - P_j)^{n-x_j}, \quad \forall j > C \quad (2.6) \]

Canceling out the condition, we have

\[ P(X_j = x_j) = \sum_{n=1}^{N} P(X_j = x_j | n \text{ order lines in an order}) \ast o_n \]

\[ = \sum_{n=1}^{N} \binom{n}{x_j} p_j^{x_j} (1 - P_j)^{n-x_j} o_n, \quad \forall j > C \quad (2.7) \]

The expected number of lines to be picked at station \( j \) given the bin enters station \( j \) is:
To obtain the expected travel time, $E[tr_j]$, for an order bin, we need the information of the products’ locations in a pick station. Under the pick-frequency class-based storage policy, the optimal locations of products and the picker’s home base (stop position) of order bins in a pick station is illustrated in Figure 2.2 (see Jewkes et al. 2004), where class A refers to the class of those products with the highest order frequency, class B the second highest class, and so on.

Figure 2.2: Product locations in the storage rack at station $j$.

The expected travel time per order at station $j$ given that the order bin will enter station $j$ is:

$$E[tr_j] = \frac{1}{v} E[2 \sum_i d_{ij} X_{ij} \mid X_j > 0], \quad \forall j > C$$

where $v$ is the travel speed of order pickers expressed in meter/second, $X_{ij}$ is the number of lines of the $i$th class to be picked at station $j$, and $d_{ij}$ is the travel distance from the picker’s home base to the location of the $i$th class of products. $X_{ij}$ equals $X_j \frac{P_i}{P_j}$ in distribution. As mentioned we suppose that within each class, products are stored randomly and the demands are uniformly distributed over products. Hence $d_{ij}$ are uniformly distributed random variables with probability density function of:
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\[ f_{d_{ij}}(x) = \begin{cases} \frac{2}{T_j}, & \text{for } \frac{1}{2} \sum_{k=0}^{i-1} l_{kj} \leq x \leq \frac{1}{2} \sum_{k=0}^{i} l_{ij}, \forall i, \forall j > C \\ 0, & \text{elsewhere} \end{cases} \] (2.10)

We define \( l_{ij} = 0 \) in the equation above. Because \( d_{ij} \) are independent from \( X_j \) and \( \frac{P_{uj}}{P_j} \) are not random variables, we obtain

\[ E[tr_j] = \frac{2}{v} E[X_j | X_j > 0] \sum_{i} \frac{P_{uj}}{P_j} E[d_{ij}], \forall j > C \] (2.11)

where \( E[d_{ij}] \) is the expected value of \( d_{ij} \) given by

\[ E[d_{ij}] = \int_{-\infty}^{-\infty} x f_{d_{ij}}(x) dx = \frac{1}{2} \sum_{i=0}^{l_{ij}} l_{ij} + \frac{1}{4} l_{ij}, \forall i, \forall j > C \] (2.12)

Using equation (2.8), we can calculate the expected picking time at station \( j \) given that the order bin will enter station \( j \):

\[ E[pk_j] = pt \cdot E[X_j | X_j > 0], \forall j > C \] (2.13)

where \( pt \) is the constant picking time per order line. We assume \( pt \) is constant.

From equation (2.1), (2.8), (2.11) and (2.13), we can obtain the expected service time at station \( j \) given that the order bin will enter station \( j \).

To obtain the SCV of service time of an order bin at station \( j \), we need to calculate the second moment of service time, which is given by

\[ E[se_j^2] = E[(tr_j + pk_j + st)^2], \forall j > C \]


The second moment of \( tr_j \) is calculated as follows:
Performance Approximation and Design of Pick-and-Pass Systems

\[ E[tr_j^2] = \frac{4}{v^2} E[X_j^2 | X_j > 0] * E[\sum_{i=1}^l D_i^2] \]

\[ = \frac{4}{v^2} E[X_j^2 | X_j > 0] * E[2^* \sum_{i=1}^l (D_{ij} * D_{ij}) + \sum_{i=1}^l D_i^2] \]  
\[ = \frac{4}{v^2} E[X_j^2 | X_j > 0] * [2^* \sum_{i=1}^l \sum_{k=1}^l E[D_{ij}] * E[D_{ij}] + E[\sum_{i=1}^l D_i^2]]. \quad \forall j > C \]  

(2.15)

where \( D_{ij} = \frac{p_{ij}}{p_j} * d_{ij} \), and \( E[D_{ij}] = \frac{p_{ij}}{p_j} * E[d_{ij}] \) for \( \forall i, \forall j > C \)

The last step of the equation above follows from the independence of \( D_{ij} \) and \( D_{kj} \) if \( i \neq k \).

The conditional second moment of \( X_j \) is given by

\[ E[X_j^2 | X_j > 0] = \sum_{x_j = 1}^N x_j^2 * \frac{P(X_j = x_j)}{P(X_j > 0)} \]

\[ = \frac{\sum_{x_j = 1}^N \sum_{n=1}^r (n/x_j)^{P_j}(1-P_j)^{r-n} O_{x_j}}{P(X_j > 0)}. \quad \forall j > C \]  
\[ \text{ (2.16)} \]

The second moment of \( D_{ij} \) is given below:

\[ E[D_{ij}^2] = (\frac{D_{ij}}{P_j})^2 * E[D_{ij}^2] \]

\[ = \frac{2}{l_{ij}} (\frac{p_{ij}}{P_j})^2 \sum_{x_j = 0}^\frac{1}{2} l_{ij} x^2 dx \]  
\[ = \frac{1}{3} \left[ \frac{2}{l_{ij}} (\frac{p_{ij}}{P_j})^2 \left( \sum_{x_j = 0}^\frac{1}{2} l_{ij} x^3 \right) - \left( \frac{1}{2} \sum_{x_j = 0}^\frac{1}{2} l_{ij} \right)^3 \right]. \quad \forall i, \forall j > C \]  

(2.17)

From equation (2.15) to (2.17), we obtain \( E[tr_j^2] \). The second moment of \( pk_j \) is obtained by

\[ E[pk_j^2] = pr^2 * E[X_j^2 | X_j > 0] \]  
\[ \text{ (2.18)} \]
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The component $E[tr_j * pk_j]$ is calculated as

$$E[tr_j * pk_j] = \frac{1}{v} E[2^* (\sum_{i=1}^l d_{ij} * \frac{p_{ij}}{P_j} * X_j) * pt * X_j | X_j > 0]$$

$$= \frac{2^* pt}{v} E[X_j^2 | X_j > 0]^* (\sum_{i=1}^l \frac{p_{ij}}{P_j} * E[d_{ij}]), \quad \forall j > C$$

(2.19)

From equation (2.11)-(2.19), we can obtain the second moment of service time at a pick station given that the order bin will enter that station. With the value of the first and the second moment of service time, we can calculate the SCV of service time at station $j$

$$c_j^2 = \frac{E[se_j^2]}{E[se_j]}^2, \quad \forall j > C$$

(2.20)

As mentioned at the beginning of this section, the service rate of each server of a conveyor piece is constant; therefore the values of SCVs for conveyor pieces are zero, i.e.,

$$c_j^2 = 0, \quad \forall j \leq C$$

(2.21)

The mean service time of servers on a conveyor piece is the reciprocal of its service rate

$$E[se_j] = \frac{k_j}{vl_j}, \quad \forall j \leq C$$

(2.22)

With the information of the mean and SCV of service time at each node, we will calculate the order throughput time in the system in the next subsection.

2.4.2. Throughput time of an order bin

We calculate the mean throughput time of an order bin in the pick-and-pass order picking system under consideration based on the $G/G/m$ queuing network approximation model of Whitt (1983) and (1993) (see appendix A). The mean order throughput time consists of transportation times on conveyor pieces, service times at pick stations, and the waiting times in front of conveyor pieces and pick stations. The approximation analysis uses two
parameters to characterize the arrival process and the service time at each node, one to
describe the rate and the other to describe the variability. The two parameters for service
time are $E[se]$, and $c^2_s$, as we derived in section 2.4.1. For the arrival process, the
parameter is $\lambda_j$, the arrival rate, which is the reciprocal of the mean inter-arrival time
between two order bins to each node, and $c^2_{a_j}$, the SCV of the inter-arrival time.

Order bins arrive at the system at conveyor piece 1 (see Figure 2.1) with rate $\lambda_{01}$, and the
SCV of the inter-arrival time is $c^2_{01}$. To calculate the internal arrival rate and the SCV of
inter-arrival time at each node, we need to know the transition probabilities after the
service at each node to another node. At the end of a conveyor piece, an order bin is either
transferred to a subsequent conveyor piece for transportation or pushed into a pick station.
The transition probabilities between these nodes are given by

$$q_{j\rightarrow C} = V_{j\rightarrow C}, \quad \forall \ j < C$$  \hspace{1cm} (2.23)

$$q_{j\rightarrow 1} = 1 - V_{j\rightarrow C}, \quad \forall \ j < C$$  \hspace{1cm} (2.24)

$$q_{j\rightarrow S} = 1, \quad \forall \ C < j \leq C + S$$  \hspace{1cm} (2.25)

where the value of $V_{j\rightarrow C}$ is obtained from equation (2.5). The transition probabilities
between other nodes are zero. Because the order bins leave the system from the last
conveyor piece $C$, we have $q_{C\rightarrow j} = 0$ for $\forall 1 \leq j \leq J$. The matrix of the transition
probabilities is indicated by $Q$. As an example, consider a network with 3 pick stations
and 4 conveyor pieces, i.e., $C = 4$, and $S = 3$. Assuming that at the end of each conveyor
piece (except for piece 4, the last one), a bin has a probability of 0.6 to be pushed into the
next pick station. Bins enter the system from node 1 and leave the system from node 4.
The transition matrix is then given by

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\[
Q = \begin{pmatrix}
0 & 0.4 & 0 & 0 & 0.6 & 0 & 0 \\
0 & 0 & 0.4 & 0 & 0 & 0.6 & 0 \\
0 & 0 & 0 & 0.4 & 0 & 0 & 0.6 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0
\end{pmatrix}
\]

With the probability transition matrix, we can obtain the internal traffic rates \( \lambda_j \) and the SCV of the inter-arrival time between two bins to each node (see Appendix A).

The utilization of a conveyor piece and a pick station is given by

\[
\rho_j = \begin{cases}
\frac{\lambda_j}{v_{l_j}}, & \forall j \leq C \\
\lambda_j E[se_j]/h_j, & \forall j > C
\end{cases}
\]

(2.26)

The expected sojourn time of a bin at node \( j \) is given by

\[
E[T_j] = E[v_{t_j}] + (E[W_j] + E[se_j]), \quad \forall 1 \leq j \leq J
\]

(2.27)

where \( E[W_j] \) is the expected waiting time in front of node \( j \) as calculated by (A.9), and \( E[v_{t_j}] \) is the expected number of visits to node \( j \) of an order bin. The probability mass function of \( v_{t_j} \) is given by

\[
v_{t_j} = \begin{cases}
0, & \text{with probability } 1-V_j, \quad \forall 1 \leq j \leq J \\
1, & \text{with probability } V_j
\end{cases}
\]

(2.28)

where \( V_j \) is obtained from equation (2.5) for \( j > C \) and \( V_j = 1 \) for \( j \leq C \). Hence

\[
E[v_{t_j}] = 0 \times (1-V_j) + V_j = V_j, \quad \forall 1 \leq j \leq J
\]

(2.29)

The total expected order throughput time is the summation of the expected sojourn time at each node.
2.5. Model validation

To validate the quality of the approximation method described in section 2.4, we compare the results with both simulation and a real order picking process.

We built a simulation model in Automod® 10.0. For each scenario in the example, we use at least 20,000 orders, preceded by 2000 orders of initialization for the system to become stable, to guarantee that the 95%-confidence interval width for the Mean Order Throughput Time (MOTT) is below 1% of the average. The parameters used in the example are listed in Table 2.1.

Table 2.1: Parameters used in the example

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order arrival process</td>
<td>Poisson process (we evaluate different arrival rates)</td>
</tr>
<tr>
<td>Number of stations</td>
<td>18</td>
</tr>
<tr>
<td>Number of order pickers</td>
<td>18</td>
</tr>
<tr>
<td>Product classes and order frequency per class</td>
<td>Class 1: $f_1=0.8$, Class 2: $f_2=0.15$, Class 3: $f_3=0.05$</td>
</tr>
<tr>
<td>Total fraction of storage space for product classes</td>
<td>Class 1: 0.2, Class 2: 0.3, Class 3: 0.5</td>
</tr>
<tr>
<td>Size of order bins</td>
<td>60<em>40</em>35 cm</td>
</tr>
<tr>
<td>Conveyor speed</td>
<td>0.7 bins per second (0.1m minimum space between two bins)</td>
</tr>
<tr>
<td>Conveyor length</td>
<td>First piece 40 bins, 20 bins for others</td>
</tr>
<tr>
<td>Length of each pick station</td>
<td>28 meters (40 bins)</td>
</tr>
<tr>
<td>Walk speed of order pickers</td>
<td>1 meter/second</td>
</tr>
<tr>
<td>Picking time per line</td>
<td>18 seconds</td>
</tr>
<tr>
<td>Setup time</td>
<td>45 seconds</td>
</tr>
<tr>
<td>Maximum number of lines in an order bin</td>
<td>30</td>
</tr>
<tr>
<td>The number of order lines in an order</td>
<td>Empirical distribution (based on the data from a specific Dutch warehouse) with mean of 15.6 and standard deviation of 6.3</td>
</tr>
</tbody>
</table>

Table 2.2 illustrates the storage assignments in stations and the probability that an order bin has to be handled at a station as calculated from equation (2.5). We observe from Table 2.2 that stations have the same total storage space but use different storage space per product class (i.e., a non-uniform storage policy).
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Table 2.2: Storage space and the bin visit probabilities to stations under the non-uniform storage policy scenario

<table>
<thead>
<tr>
<th>$l_o$ (meter)</th>
<th>Class 1</th>
<th>Class 2</th>
<th>Class 3</th>
<th>Bin Visit Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>St. 1</td>
<td>4.9</td>
<td>5.6</td>
<td>6.3</td>
<td>0.36</td>
</tr>
<tr>
<td>St. 2</td>
<td>4.9</td>
<td>5.6</td>
<td>6.3</td>
<td>0.36</td>
</tr>
<tr>
<td>St. 3</td>
<td>4.9</td>
<td>5.6</td>
<td>6.3</td>
<td>0.36</td>
</tr>
<tr>
<td>St. 4</td>
<td>4.9</td>
<td>5.6</td>
<td>6.3</td>
<td>0.36</td>
</tr>
<tr>
<td>St. 5</td>
<td>4.9</td>
<td>5.6</td>
<td>6.3</td>
<td>0.36</td>
</tr>
<tr>
<td>St. 6</td>
<td>4.9</td>
<td>5.6</td>
<td>6.3</td>
<td>0.36</td>
</tr>
<tr>
<td>St. 7</td>
<td>4.9</td>
<td>5.6</td>
<td>6.3</td>
<td>0.36</td>
</tr>
<tr>
<td>St. 8</td>
<td>4.9</td>
<td>5.6</td>
<td>6.3</td>
<td>0.36</td>
</tr>
<tr>
<td>St. 9</td>
<td>4.9</td>
<td>5.6</td>
<td>6.3</td>
<td>0.36</td>
</tr>
</tbody>
</table>

We vary the arrival rates to the system to compare the performance of the approximation method to simulation under different workloads. The results are listed in Table 2.3.

Table 2.3 also illustrates the accuracy of $G/G/m$ modeling over Jackson network modeling as used in De Koster (1994). Jackson network modeling is a special case of the $G/G/m$ modeling. The results of Jackson network modeling in Table 2.3 are obtained by assuming Poisson order arrivals to the system and exponentially distributed service times at each node. The $G/G/m$ approximation method provides the same results as Jackson decomposition method when the above constraints are met.

Table 2.3 shows that the relative error between the approximation model and the simulation results are all below 6 percent under different workloads. It also shows that the larger the utilizations at stations, the more accurate $G/G/m$ modeling is over Jackson network modeling.
Table 2.3: Validation results for the example and comparisons to Jackson modeling

<table>
<thead>
<tr>
<th>Input rate (bin/sec)</th>
<th>MOTT (sec) G/G/m</th>
<th>numerical simulation</th>
<th>Rel.error</th>
<th>Station utilization (max)</th>
<th>MOTT (sec) (Jackson)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.008</td>
<td>1615.5</td>
<td>1556.2±4.6</td>
<td>3.8%</td>
<td>0.409</td>
<td>1867.5</td>
</tr>
<tr>
<td>0.011</td>
<td>1725.0</td>
<td>1647.3±5.2</td>
<td>4.7%</td>
<td>0.517</td>
<td>2119.9</td>
</tr>
<tr>
<td>0.013</td>
<td>1889.8</td>
<td>1789.5±6.1</td>
<td>5.6%</td>
<td>0.630</td>
<td>2518.6</td>
</tr>
<tr>
<td>0.016</td>
<td>2290.8</td>
<td>2171.5±8.3</td>
<td>5.5%</td>
<td>0.780</td>
<td>3559.0</td>
</tr>
<tr>
<td>0.018</td>
<td>3116.0</td>
<td>3023.4±15.7</td>
<td>3.1%</td>
<td>0.893</td>
<td>5792.4</td>
</tr>
<tr>
<td>0.019</td>
<td>4312.8</td>
<td>4247.4±24.4</td>
<td>1.5%</td>
<td>0.944</td>
<td>9078.5</td>
</tr>
</tbody>
</table>

We also conducted other experiments with different parameters: the number of pick stations varied from 4 to 18, with a step size of 2, and the utilization of pick stations varied from 0.2 to 0.9 with step size of 0.1. In all experimental settings, the relative errors between the approximation model and the simulation results were below 7%.

To further validate our approximation method, we compare our results to the performance of a real order picking process in the bulky storage area at the parts distribution center of an international motor production company. The bulky storage area stores in total 240 products divided into 3 classes. One class contains 48 heavy products and the other two classes each containing 96 products are categorized according to their order frequencies. The whole area is divided into four pick stations connected by conveyor pieces. Through analyzing the log file from the Warehouse Management System (WMS) for one picking day, which was chosen to be a representative for the typical picking process, we obtained the data for the order arrival process to the system, the service times at pick stations, and the routing probabilities of order bins to enter a station, which are listed in Table 2.4. We also measured the capacities of conveyor pieces and their moving speeds. We input these data into our approximation model. The result of MOTT is compared with the mean order throughput time obtained from the warehouse management system.

From Table 2.4, we find that the relative error is around 6 percent. We conclude that the quality of the approximation method is acceptable for practical purposes and hence we can use it as a tool to estimate the pick-and-pass order picking system performance. In the next
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section, we use this approximation method to evaluate various order picking, storage, and zoning strategies.

Table 2.4: Data and comparison with results of the real order picking system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of stations</td>
<td>4</td>
</tr>
<tr>
<td>Number of order pickers per station</td>
<td>1</td>
</tr>
<tr>
<td>Number of order lines to pick per order</td>
<td>Empirical distribution (mean 2.5, stdv 1.9)</td>
</tr>
<tr>
<td>Order inter-arrival time to the system (sec)</td>
<td>Empirical distribution (mean 28.9, stdv 52.4)</td>
</tr>
<tr>
<td>Service time at station A (sec)</td>
<td>Empirical distribution (mean 40.1, stdv 41.6)</td>
</tr>
<tr>
<td>Service time at station B (sec)</td>
<td>Empirical distribution (mean 51.0, stdv 51.1)</td>
</tr>
<tr>
<td>Service time at station C (sec)</td>
<td>Empirical distribution (mean 54.1, stdv 48.0)</td>
</tr>
<tr>
<td>Service time at station D (sec)</td>
<td>Empirical distribution (mean 38.8, stdv 35.0)</td>
</tr>
<tr>
<td>Prob. To enter station A</td>
<td>0.385</td>
</tr>
<tr>
<td>Prob. To enter station B</td>
<td>0.254</td>
</tr>
<tr>
<td>Prob. To enter station C</td>
<td>0.271</td>
</tr>
<tr>
<td>Prob. To enter station D</td>
<td>0.435</td>
</tr>
<tr>
<td>MOTT from G/G/m approximation model (sec)</td>
<td>302.1</td>
</tr>
<tr>
<td>MOTT from WMS (sec)</td>
<td>321.7</td>
</tr>
<tr>
<td>Relative error</td>
<td>6.1%</td>
</tr>
</tbody>
</table>

2.6. Scenario analysis

In this section we use the approximation method to analyze the impact of different warehousing strategies on the order picking system performance. These strategies include the storage assignment in pick stations, the pick station size and the number of order pickers in stations, and order batching and splitting decisions in the order release process. The parameters used for scenario analysis are the same as the example in the previous section and are listed in Table 2.1.

2.6.1. The effects of storage policies on system performance

Storage policies affect the order throughput time in the order picking system as they impact the workload balance in stations. In this subsection, we will compare the impact of
uniform (stations use identical storage spaces to store a certain class of products) and non-uniform (stations use different storage spaces to store a certain class of products) storage policies on mean order throughput time. We expect that the uniform storage policy leads to shorter order throughput time as it leads to workload balance between stations.

The storage space for each class of products in stations and the calculated probability using equation (2.5) for a bin to enter a pick station under the uniform storage policy is shown in Table 2.5.

Table 2.5: Storage space and the bin visit probabilities to stations under the uniform storage policy scenario

<table>
<thead>
<tr>
<th>l (meter)</th>
<th>St. 1</th>
<th>St. 2</th>
<th>St. 3</th>
<th>St. 4</th>
<th>St. 5</th>
<th>St. 6</th>
<th>St. 7</th>
<th>St. 8</th>
<th>St. 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>class 1</td>
<td>5.6</td>
<td>5.6</td>
<td>5.6</td>
<td>5.6</td>
<td>5.6</td>
<td>5.6</td>
<td>5.6</td>
<td>5.6</td>
<td>5.6</td>
</tr>
<tr>
<td>class 2</td>
<td>8.4</td>
<td>8.4</td>
<td>8.4</td>
<td>8.4</td>
<td>8.4</td>
<td>8.4</td>
<td>8.4</td>
<td>8.4</td>
<td>8.4</td>
</tr>
<tr>
<td>class 3</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>Bin visit prob.</td>
<td>0.39</td>
<td>0.39</td>
<td>0.39</td>
<td>0.39</td>
<td>0.39</td>
<td>0.39</td>
<td>0.39</td>
<td>0.39</td>
<td>0.39</td>
</tr>
</tbody>
</table>

Table 2.6 illustrates the resulting mean order throughput time in comparison with the results of the non-uniform storage policy (refer to Table 2.2 and Table 2.3).

Table 2.6: Comparison of system performance between uniform and non-uniform storage policies in pick stations

<table>
<thead>
<tr>
<th>Input rate (bin/sec)</th>
<th>Uniform MOTT (sec)</th>
<th>Non-uniform MOTT (sec)</th>
<th>Improvement</th>
<th>Uniform Utilization</th>
<th>Non-uniform Utilization</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.008</td>
<td>1613.0</td>
<td>1615.5</td>
<td>0.15%</td>
<td>0.376</td>
<td>0.409</td>
</tr>
<tr>
<td>0.011</td>
<td>1720.3</td>
<td>1725.0</td>
<td>0.27%</td>
<td>0.475</td>
<td>0.517</td>
</tr>
<tr>
<td>0.013</td>
<td>1876.4</td>
<td>1889.8</td>
<td>0.71%</td>
<td>0.579</td>
<td>0.630</td>
</tr>
<tr>
<td>0.016</td>
<td>2236.6</td>
<td>2290.8</td>
<td>2.3%</td>
<td>0.716</td>
<td>0.780</td>
</tr>
<tr>
<td>0.018</td>
<td>2849.1</td>
<td>3116.0</td>
<td>8.57%</td>
<td>0.821</td>
<td>0.893</td>
</tr>
<tr>
<td>0.019</td>
<td>3436.9</td>
<td>4312.8</td>
<td>20.31%</td>
<td>0.868</td>
<td>0.944</td>
</tr>
</tbody>
</table>
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As the stations are now balanced on average, we find from Table 2.6 that the mean order throughput times are shorter under the uniform storage policy than under the non-uniform storage policy. The improvement is substantial when the workload of the system increases. Because of the advantage of the uniform storage policy, we will focus our analysis on this storage policy in the following discussions.

2.6.2. The effects of station sizes and the number of pickers on system performance

The size of the pick stations and the number of order pickers in stations impact the mean order throughput time. With a fixed length of the whole order picking system (i.e., a fixed storage capacity of the system) and a fixed number of order pickers, the larger the size of the pick stations, the fewer stations we have in the system, and the more order pickers are available at each pick station. Pick stations of larger size will increase the service time due to longer travel time, and the smaller number of stations tends to increase the utilizations of pick stations due to higher order bin arrival rates. Therefore they lead to an increase of the mean order throughput time. But on the other hand, fewer stations lead to fewer station visits of an order bin (hence fewer queues and less setup time); more order pickers per station implies decreasing utilizations at pick stations, which reduces the mean order throughput time. In pick-and-pass order picking system design, a main question therefore is to find the right trade-off between these opposite effects by selecting the right number of stations. Table 2.7 shows the system performance for various combinations of station sizes and order pickers per station. It shows that under the current settings, the scenario of 6 stations with 3 order pickers per station has the best performance for all arrival rates used.
### Table 2.7: System performances under various station sizes and the number of order pickers per station

<table>
<thead>
<tr>
<th># of stations (# of picker per station) (station size in meters)</th>
<th>18(1)(28)</th>
<th>9(2)(56)</th>
<th>6(3)(84)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input rate (bin/sec) MOTT (sec) Utilization MOTT (sec) Utilization MOTT (sec) Utilization</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.008</td>
<td>1613.0</td>
<td>0.376</td>
<td>1370.7</td>
</tr>
<tr>
<td>0.011</td>
<td>1720.3</td>
<td>0.475</td>
<td>1407.9</td>
</tr>
<tr>
<td>0.013</td>
<td>1876.4</td>
<td>0.579</td>
<td>1463.7</td>
</tr>
<tr>
<td>0.016</td>
<td>2236.6</td>
<td>0.716</td>
<td>1586.9</td>
</tr>
<tr>
<td>0.018</td>
<td>2849.1</td>
<td>0.821</td>
<td>1765.7</td>
</tr>
<tr>
<td>0.019</td>
<td>3436.9</td>
<td>0.868</td>
<td>1904.3</td>
</tr>
<tr>
<td>0.020</td>
<td>4226.5</td>
<td>0.903</td>
<td>2052.3</td>
</tr>
<tr>
<td>0.021</td>
<td>6110.2</td>
<td>0.940</td>
<td>2294.8</td>
</tr>
</tbody>
</table>

### 2.6.3. The effect of batching orders on system performance

As we have seen from the analysis above, the input rate of order bins to the system has great impact on system performance. A large arrival rate results in higher workload to the system and will subsequently increase the mean order throughput time. One way to reduce the input rate to the system is to batch orders. We consider the following batching rules:

We batch two successive order bins each containing at most $L$ lines into one bin and then send it to the system. The order bins with more than $L$ lines are sent directly to the system. The batching threshold, $L$, can take any value between 1 and $\left\lfloor \frac{N}{2} \right\rfloor$, where $\left\lfloor \cdot \right\rfloor$ represents the floor function.
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means rounding down to the nearest integer. Otherwise, the number of lines in a batched bin may exceed the bin’s capacity. We assume that \( N \) (the maximum number of lines in an order) is also the capacity of an order bin. By batching small orders, we can decrease the input rate to the system, leading to a decrease in the mean order throughput time. On the other hand, the service time at each station and the probability of entering a pick station will increase because of more order lines to be picked. These factors lead to increase the mean order throughput time. The impact of order batching on system performance depends on the trade-off between these factors. We can analyze this impact with a slight modification of the approximation method discussed earlier. When we batch two successive bins, the first bin has to wait for one or more inter-arrival time periods to be processed. However, since the mean order inter arrival time is normally small compared to the total mean order throughput time and only those bins containing fewer than \( L \) lines are batched, this effect is small and can be neglected.

Assuming the original input process to the system is Poisson distributed with rate \( \lambda_{01} \), an order bin has a probability \( o_n \) of containing \( n \) order lines. Then the flow of order bins with \( n \) order lines is also a Poisson process with rate \( \lambda_{01} * o_n \). After batching, the original process is split into two sub-processes. The first sub-process refers to the batched bins and the second sub-process is the un-batched bins. According to the properties of Poisson process, the inter-arrival time of the first sub-process is Gamma distributed with parameters \( 2, \lambda_{01} * 1 * \sum_{n=1}^{L} o_n \). The input rate of this type of order flow is \( \tilde{\lambda}_{01} = \frac{1}{2} \lambda_{01} * \sum_{n=1}^{L} o_n \) and the SCV of the order inter-arrival time is \( c_{01}^2 = 0.5 \). The second sub-process is Poisson distributed with rate \( \tilde{\lambda}_{012} = \lambda_{01} * \sum_{n=L+1}^{N} o_n \), where \( N \) is the maximum number of lines in a bin. The SCV of the order inter-arrival time is \( c_{012}^2 = 1 \).

The basic idea to calculate the mean order throughput time with two input flows is derived from Whitt (1983). The procedure is first to calculate the mean and the SCV of service time at each pick station, the transition probabilities between nodes, and the internal traffic flows to each node separately for each input flow. Then we convert these two types of
flows into one (See Appendix B). The method of Appendix A is then again used to obtain the mean order throughput time. Following the example at the beginning of this section, we assume that $L$ equals 15. Table 2.8 compares the system performances between batching and non-batching scenarios.

Table 2.8 shows that the input rates decrease and the service times at pick stations increase when orders are batched. Batching orders can slightly reduce the utilizations of pick stations. The impact of pick station utilizations on waiting times in front of them is marginal when the utilizations are small, but become substantial when the utilizations become higher. We observe that when the system is not heavily loaded, order batching increases the mean order throughput time. This is mainly due to the longer service time at pick stations and the increased probability of entering pick stations. However, when the system is heavily loaded, the mean order throughput time decreases when we batch orders. Under a heavy load, the waiting time is the major component of the order throughput time; reducing pick station utilizations by batching orders can significantly reduce waiting time in front of pick stations and therefore reduce the mean order throughput time.
### Table 2.8: Comparison of system performances between batching and non-batching scenarios

<table>
<thead>
<tr>
<th>L=15</th>
<th>Order arrival rate (bins/sec)</th>
<th>Rate after batching (bins/sec)</th>
<th>MOTPT (sec)</th>
<th>Utilization</th>
<th>Mean waiting time (sec)</th>
<th>Mean service time (sec)</th>
<th>Bin visiting prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0083</td>
<td>0.0105</td>
<td>0.0128</td>
<td>0.0159</td>
<td>0.0182</td>
<td>0.0185</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0063</td>
<td>0.0079</td>
<td>0.0096</td>
<td>0.0119</td>
<td>0.0136</td>
<td>0.0138</td>
<td></td>
</tr>
<tr>
<td>Batching</td>
<td>1864.2</td>
<td>1973.9</td>
<td>2123.4</td>
<td>2435.8</td>
<td>2891.8</td>
<td>2971.6</td>
<td>0.358</td>
</tr>
<tr>
<td>Non-batching</td>
<td>1613.0</td>
<td>1720.3</td>
<td>1876.4</td>
<td>2236.6</td>
<td>2849.1</td>
<td>2968.2</td>
<td>0.376</td>
</tr>
<tr>
<td>Batching</td>
<td>0.358</td>
<td>0.452</td>
<td>0.551</td>
<td>0.682</td>
<td>0.781</td>
<td>0.792</td>
<td></td>
</tr>
<tr>
<td>Non-batching</td>
<td>0.376</td>
<td>0.475</td>
<td>0.579</td>
<td>0.716</td>
<td>0.821</td>
<td>0.833</td>
<td></td>
</tr>
<tr>
<td>Batching</td>
<td>22.1</td>
<td>31.0</td>
<td>43.1</td>
<td>68.4</td>
<td>105.3</td>
<td>111.7</td>
<td></td>
</tr>
<tr>
<td>Non-batching</td>
<td>23.7</td>
<td>34.3</td>
<td>49.7</td>
<td>85.2</td>
<td>145.5</td>
<td>157.3</td>
<td></td>
</tr>
<tr>
<td>Batching</td>
<td>83.4</td>
<td>83.4</td>
<td>83.4</td>
<td>83.4</td>
<td>83.4</td>
<td>83.4</td>
<td></td>
</tr>
<tr>
<td>Non-batching</td>
<td>80.1</td>
<td>80.1</td>
<td>80.1</td>
<td>80.1</td>
<td>80.1</td>
<td>80.1</td>
<td></td>
</tr>
<tr>
<td>Batching</td>
<td>0.69</td>
<td>0.69</td>
<td>0.69</td>
<td>0.69</td>
<td>0.69</td>
<td>0.69</td>
<td></td>
</tr>
<tr>
<td>Non-batching</td>
<td>0.56</td>
<td>0.56</td>
<td>0.56</td>
<td>0.56</td>
<td>0.56</td>
<td>0.56</td>
<td></td>
</tr>
<tr>
<td>L=15</td>
<td>Order arrival rate (bins/sec)</td>
<td>Rate after batching (bins/sec)</td>
<td>MOTPT (sec)</td>
<td>Utilization</td>
<td>Mean waiting time (sec)</td>
<td>Mean service time (sec)</td>
<td>Bin visiting prob.</td>
</tr>
<tr>
<td></td>
<td>0.0186</td>
<td>0.0189</td>
<td>0.0192</td>
<td>0.0200</td>
<td>0.0204</td>
<td>0.0208</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0139</td>
<td>0.0142</td>
<td>0.0144</td>
<td>0.0150</td>
<td>0.0153</td>
<td>0.0156</td>
<td></td>
</tr>
<tr>
<td>Batching</td>
<td>3004.5</td>
<td>3114.5</td>
<td>3162.0</td>
<td>3679.9</td>
<td>3991.1</td>
<td>4424.2</td>
<td>0.797</td>
</tr>
<tr>
<td>Non-batching</td>
<td>3018.4</td>
<td>3191.7</td>
<td>3436.9</td>
<td>4226.5</td>
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#### 2.6.4. The effect of splitting orders on system performance

As an alternative to batching orders, splitting an order into two small orders will reduce the service times in pick stations and the probabilities of entering pick stations. On the other hand, splitting orders increases the arrival flow rate because of more order bins entering the system. To analyze the impact of order splitting on system performance, we split an
order bin containing $R$ or more than $R$ lines into two bins, one containing $\lceil R/2 \rceil$ lines and the other containing $R - \lceil R/2 \rceil$ lines. Again, assuming the original arrival process is Poisson distributed, the input process is divided into two Poisson processes: the input flow of non-split bins with rate $\lambda_{01} = \lambda_{00} \sum_{n=1}^{\infty} \rho_n$, and the input flow of bins to be split with rate $\tilde{\lambda}_{012} = \lambda_{00} \sum_{n=1}^{\infty} \rho_n$. Before arriving at the first conveyor piece, we suppose the input flow of bins to be split will first pass through an artificial node with very small constant service time. A new order bin is created following the completion of service at the artificial node. According to the approximation method given at section 2.2 and 4.6 of Whitt (1983), the departure process, i.e., the arrival process to the first conveyor piece of this flow of split bins has rate of $2 \cdot \tilde{\lambda}_{012}$, and approximated SCV of inter-arrival time of 2.

The total arrival process to the first conveyor piece is therefore the combination of a Poisson process, with rate $\tilde{\lambda}_{011} = \lambda_{00} \sum_{n=1}^{\infty} \rho_n$ and a process with rate of $2 \cdot \tilde{\lambda}_{012}$, and SCV of inter-arrival time of 2. Similar to the approaches used to analyze batching orders, we can obtain the system performance for the order splitting scenario. The results with comparison to the non-splitting scenario, as illustrated in Table 9, show that splitting orders increases the input rates to the system and reduces the service times at pick stations and the probabilities of entering pick stations. Splitting orders increases the utilizations of pick stations. The mean order throughput time shortens when the station utilizations are low. This is mainly due to the reduction in service times and the probabilities of entering pick stations. When station utilization becomes high ($\rho > 0.75$ approximately for $R$ equals 15), order splitting increases the mean order throughput time because the waiting time in front of a station becomes longer due to higher utilization. Especially, when $\rho$ approaches 1, the system becomes unstable, and the mean order throughput time increases infinitely.

We note that the approximation model underestimates the mean order throughput time when we consider each split as a separate order. However, in reality, orders are only split
Chapter 2

when the number of order lines is large, and the impact on mean throughput time will be slight. The approximation model will give a reasonable estimation for the mean order throughput time from a practical point of view.

Table 2.9: Comparison of system performances between splitting and non-splitting scenarios

<table>
<thead>
<tr>
<th>R=15</th>
<th>Order arrival rate (bins/sec)</th>
<th>Rate after splitting (bins/sec)</th>
<th>MOTPT (sec)</th>
<th>Utilization</th>
<th>Mean waiting time (sec)</th>
<th>Mean service time (sec)</th>
<th>Bin visiting prob.</th>
</tr>
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<td></td>
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<td>Splitting</td>
<td></td>
<td>Non-splitting</td>
<td>Non-splitting</td>
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<th>R=15</th>
<th>Order arrival rate (bins/sec)</th>
<th>Rate after splitting (bins/sec)</th>
<th>MOTPT (sec)</th>
<th>Utilization</th>
<th>Mean waiting time (sec)</th>
<th>Mean service time (sec)</th>
<th>Bin visiting prob.</th>
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</tbody>
</table>

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2.7. Conclusions

In this chapter, we proposed an approximation method based on $G/G/m$ queuing network modeling to analyze performance of pick-and-pass order picking systems. The method can be used as a fast tool to estimate design alternatives on the mean order throughput time of the order picking system. These alternatives include the storage policies, the size of pick stations, the number of order pickers in stations, and the arrival process of customer orders. In general, the preference of one alternative over others is subject to a detailed specification of the order picking system. The quality of the approximation method is acceptable for practical purposes. Therefore, it enables planners to evaluate various system alternatives, which is essential at the design phase of the order picking system.
3. THE IMPACT OF BATCHING AND ZONING ON ORDER PICKING PERFORMANCE

3.1. Introduction

Batching and zoning are closely related issues, and are often applied simultaneously in a warehouse. However, in literature they are usually studied separately. In this chapter, we study the impacts of order batching and pick area zoning on the average throughput time of a random order in a pick-and-pass order picking system. The analysis is based on the approximation model developed in chapter 2. Because batches need to be sorted again by order, to provide a more accurate estimation, we also take the sorting process, after order picking, into consideration. Compared to the analysis in chapter 2, in this chapter, we consider a more general layout of the picking system with different routing, storage, and batching policies.

This chapter is organized as follows. We discuss the order picking system under consideration in the next section. Next we review related literature. Then we describe the approximation model, followed by numerical experiments. We draw conclusions at the end of this chapter.
3.2. The order picking system

We consider the pick-and-pass order picking system illustrated in Figure 3.1. Customer orders, each consisting of a number of order lines, arrive at the warehouse according to a Poisson process and are batched before being sent to the order picking system. When the batch size reaches a certain number of orders, the batch is released immediately for picking to the order picking system with a pick bin assigned to it. Batches are not queued at the picking system. We assume the batch size can take any value equal or larger than 1 and the pick bin is sufficiently large to contain all the items of the order batch. The pick area is divided into a number of zones of the same size, each of which consists of a number of picking aisles. Pick zones are connected by conveyor pieces. The pick bin is transferred on the conveyor and will enter a zone if there are items to be picked in the zone. When the picking of a bin has finished completely, it is transferred to the sorting station at the end of the conveyor. Sorting time is assumed proportional to the number of order lines in the batch (no sorting is needed when the batch size is one; in this case, the sorting time is zero).

Figure 3.1: Layout of the pick-and-pass system.
Roodbergen and Vis (2006) showed the optimal depot position minimizing average travel distance in picking is in the middle of the zone. Still, in practice the depot is often located at the zone boundary. Some companies use zone systems with depots alternatively at the left and right boundary, so that two adjacent zones have the depots close to each other (see Figure 3.2). This allows workers to pick orders at two stations at times of low workload. In line with other warehouse research (Chew and Tang 1999; Gibson and Sharp 1992; Petersen et al. 2004; Rosenwein 1996), we assume the depot of each zone to be located at the left-most aisle of the zone. This allows a much easier and more transparent analysis.

Figure 3.2: Example of Depot positions at alternative boundaries of the zones.

Products are stored randomly in the racks along the aisles in each pick zone. One or more order pickers are allocated to a zone. Order pickers work in parallel in a zone but each picker works on one order at a time and a pick bin is processed by one order picker at a zone. We assume the aisles are wide enough to allow two-way travel of order pickers. To simplify the analysis, we ignore congestion on travel routes when multiple pickers are assigned to a zone. While picking, an order picker starts from the depot with a pick list indicating the storage locations of the products to be picked; the order picker follows an S-shape travel route as illustrated in Figure 3.1. In the picking tour, the order picker picks all the lines required by the order, returns to the depot, and drops the products into the pick bin.
3.3. Related literature

Several researchers study the batching problem and travel time estimation in warehouses with various layouts by taking the stochastic property of the order profile and the service time in a picking tour into account. Chew and Tang (1999) estimate the lower bound, upper bound, and the approximation of mean travel time of a picking tour in a single-block warehouse with random storage and S-shape routing policy. They also estimate the second moment of travel times and then develop a queuing model to analyze the mean order throughput time of the first order in a batch. Le-Duc and De Koster (2007) extend the work of Chew and Tang (1999) to a warehouse with 2 blocks and analyze the average throughput time of a random order. Also considering a 2-block warehouse, Van Nieuwenhuyse et al. (2007) incorporate the sorting process and model the picking and the sorting processes as a tandem queue. They use a queuing network approach to analyze the factors influencing optimal batch size and the allocation of workers to the picking and the sorting areas. None of the literature above takes the zoning effect into consideration. De Koster (1994) considers a pick-and-pass system with pick stations connected by conveyors. Instead of going into the detailed analysis of service times at stations, he uses a Jackson network approximation to analyze the mean order throughput time of a random order. The analysis in this chapter extends the work of Chew and Tang (1999) and Le-Duc and De Koster (2007) into a multiple-zones situation. It differs from the previous chapter (Yu and De Koster, 2008) by considering a different aisle layout, storage strategy, routing policy, batching policy, and the inclusion of a sorting process.

3.4. The approximation model

Similar to chapter 2, we assume each pick zone has an infinite storage capacity (buffer) for order bins. We also assume there is a buffer with infinite capacity in front of each conveyor piece so that the arrivals will not be lost and pick zones and conveyor pieces can not become blocked because of lack of output capacity. Figure 3.3 gives an illustration of
The Impact of Batching and Zoning on Order Picking Performance

The conveyor system connecting the pick zones and the sorting station. The whole picking system is modeled as a $G/G/m$ queuing network with pick zones, sorting station, and conveyor pieces as nodes, preceded by unlimited waiting space in front of them. The number of servers at a node equals either the capacity of the conveyor piece or the number of workers working at the node.

![Figure 3.3: Illustration of the conveyor connecting pick zones and the sorting station.](image)

The following notations are used in this chapter; others notations are defined in the context when they are needed.

**Data**

- $C$: the number of conveyor pieces, with index $j$, from 1 to $C$.
- $S$: the number of pick zones, with index $j$, from $C + 1$ to $C + S$.
- $J$: the total number of nodes (pick zones, conveyor pieces, and the sorting station), with index $j$, from 1 to $S + C + 1$.
- $m_j$: the number of servers at node $j$, $j = 1, 2, ..., C + S + 1$.
- $M_j$: number of aisles in zone $j$.
- $L$: the length of picking aisles (in meters).
- $w$: center-to-center distance between two adjacent aisles (in meters).
- $v$: picker’s travel speed in zones (in meters per second).
- $vl_j$: the velocity of conveyor piece $j$ (in bins per second), $j = 1, 2, ..., C$.
- $n$: the number of order lines in an order, a random number.
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$\lambda_0$: external arrival rate of order bins to the system, entering node 1, expressed in bins/second.

**Variables**

$b$: batch size (given).

$\lambda_j$: pick bin input rate to each node (bins/second).

$Z$: the number of lines in a batch of $b$ orders.

$B_j$: number of visited aisles in zone $j$, given that there are $z_j$ lines to be picked, $j = C+1, C+2, \ldots C+S$.

$H_j$: the farthest visiting aisle in zone $j$, given that there are $z_j$ lines to be picked, $j = C+1, C+2, \ldots C+S$.

$P_1(z_j, Z)$: probability that there are $z_j$ lines to be picked at zone $j$, given that there are $Z$ lines in a batch.

$P_2(b, Z)$: probability that a batch has $Z$ lines given that it contains $b$ orders.

$V_j$: probability that an order bin to enter zone $j$, $j = C+1, C+2, \ldots C+S$.

$vt_j$: number of visits to node $j$ of an order bin, $j = 1, 2, \ldots C+S+1$.

$CA_j$: cross aisle travel time in zone $j$, given that zone $j$ is visited, and there are $z_j$ lines to be picked in zone $j$, $j = C+1, C+2, \ldots C+S$.

$WA_j$: within aisle travel time in zone $j$, given that zone $j$ is visited, and there are $z_j$ lines to be picked in zone $j$, $j = C+1, C+2, \ldots C+S$.

$tr_j$: travel time in zone $j$, given that zone $j$ is visited, and there are $z_j$ lines to be picked in zone $j$, $j = C+1, C+2, \ldots C+S$.

$se_j$: service time at zone $j$, given that zone $j$ is visited, and there are $z_j$ lines to be picked, $j = C+1, C+2, \ldots C+S$.

$SE_j$: service time at node $j$, given that node $j$ is visited, and there are $Q$ lines in a batch, $j = 1, 2, \ldots C+S+1$. 

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The methods used to analyze the impact of batching and zoning on the average order throughput time in the pick-and-pass order picking system are similar to the approximation model described in chapter 2, but the two moments of service time at each pick station differ since different layout, storage and routing policies are used in this chapter. In the following subsections, we derive the mean and the SCV of the service time of an order bin at pick zones, conveyor pieces, and the sorting station. Then we analyze of the mean order throughput time of a random order in the system with different batch size and the number of zones based on Whitt (1983).

3.4.1. Mean and SCV of service time at pick zones and conveyor pieces

A pick zone can contain one or more aisles. However, when a zone consists of a single aisle, the storage rack will be located along the conveyor piece. This case is studied in the
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previous chapter. In this chapter, we assume a pick zone consists of at least two aisles. The travel time in zone \( j \), given that zone \( j \) is visited and there are \( z_j \) lines to be picked, consists of two components: (1) travel time within the aisles \( WA_j \), and (2) travel time in the cross aisles \( CA_j \).

Under the assumption that products are randomly stored in slots along the aisles, we can easily obtain the number of aisles containing at least one pick location, \( B_j \). It has an expected value of

\[
E[B_j] = M_j - M_j (1 - \frac{1}{M_j})^{z_j}, \quad \forall C < j \leq C + S
\]

which is also the expected number of aisles to be visited in zone \( j \) by an order picker for the pick of an order bin in zone \( j \).

The expected within aisle travel time in zone \( j \) can then be stated as

\[
E[WA_j] = \frac{L}{v} E[B_j] + COR, \quad \forall C < j \leq C + S
\]

The correction factor \( COR \) accounts for the extra travel time in the last aisle that is visited. This extra travel time occurs if the number of aisles that has to be visited is an odd number. In this case, the last aisle is both entered and left from the front.

To estimate the correction term, we refer to Roodbergen (2001).

\[
COR = \sum_{g \in G} \left( \frac{M_j}{g} \right)^{z_j} \left( z_j + g - \frac{L}{v} \right) X \left( 2^g \left( \frac{g}{g - i} \right) \left( \frac{g - i}{g} \right)^{z_j} \right)
\]

where

\[
G = \{ g \mid 1 \leq g \leq M_j, g \leq z_j, \text{ and } g \text{ is odd} \}, \quad \text{and} \quad X = 1 - \sum_{i=1}^{g-1} (-1)^{i+1} \left( \frac{g}{g - i} \right) \left( \frac{g - i}{g} \right)^{z_j}
\]

which is 1 minus the probability that all of the \( z_j \) lines fall into fewer than \( g \) aisles.
The Impact of Batching and Zoning on Order Picking Performance

The expected travel time in cross aisles is twice the expected travel time from the depot to the furthest visited aisle in the zone since order pickers start picking from the depot and return to the depot when picking is finished. Therefore we have,

\[ E[CA_j] = 2 \times \frac{W}{v} (E[H_j] - 1), \quad \forall C < j \leq C + S \]  
(3.4)

where

\[ E[H_j] = M_j - \sum_{i=1}^{M_j-1} \left( \frac{i}{M_j} \right)^{\frac{1}{v}}, \quad \forall C < j \leq C + S \]  
(3.5)

is the expected farthest aisle visited in zone \( j \), according to Chew and Tang (1999).

Therefore, we can obtain the mean travel time in zone \( j \), given that zone \( j \) is visited and \( z_j \) lines are to be picked, by substituting equation (3.1) to (3.5) in the following equation,

\[ E[tr_j] = E[WA_j] + E[CA_j], \quad \forall C < j \leq C + S \]  
(3.6)

The expected service time in zone \( j \), given that \( z_j \) lines are to be picked in the zone is the summation of three components: travel time in the zone, picking time for the order lines, and the setup time for the pick bin:

\[ E[se_j] = E[tr_j] + z_j \times pt + st, \quad \forall C < j \leq C + S \]  
(3.7)

where \( pt \) is the picking time per order line and \( st \) is the setup time per order bin at a pick zone, both of which are assumed to be constant.

Because travel time is assumed to be independent of the picking time, the expected service time in zone \( j \) given that there are \( Z \) lines in a batch is calculated as

\[ E[SE_j] = \sum_{i=1}^{Z} P(z_j, Z) \times E[se_j] \]  

\[ = \sum_{i=1}^{Z} P(z_j, Z) \times E[tr_j] + \sum_{i=1}^{Z} P(z_j, Z) \times z_j \times pt + st, \quad \forall C < j \leq C + S \]  
(3.8)

where
Chapter 3

\[
P_i(z_j, Z) = \binom{Z}{z_j} \left( \frac{1}{S} \right)^{z_j} \left( 1 - \frac{1}{S} \right)^{Z-z_j}, \quad \forall C < j \leq C + S
\]

(3.9)

is the probability that there are \( z_j \) (\( z_j > 0 \)) lines to be picked in zone \( j \), given that there are \( Z \) lines in a batch.

The expected service time in zone \( j \) given that there are \( b \) orders in a batch is calculated as

\[
E[SE_j] = \sum_{Z=b}^{\infty} P_i(b, Z) \cdot E[SE_j], \quad \forall C < j \leq C + S
\]

(3.10)

where \( P_i(b, Z) \) is the probability that a batch has \( Z \) lines given that it contains \( b \) orders.

A customer order contains at least one order line. \( P_i(b, Z) \) is a function of batch size \( b \) and \( Z \). \( Z \) is the convolution of \( b \) independent and identically-distributed (i.i.d) random variables \( n \), the number of order lines in an order. As an example, if the number of lines in an order follows a shifted Poisson distribution of \( 1 + \text{Poisson}(a) \), then \( Z = b + \text{Poisson}(a \cdot b) \) in distribution and

\[
P_i(b, Z) = \frac{(ab)^{Z-b}}{(Z-b)!} e^{-ab}.
\]

The second moment of service time in zone \( j \) given that there are \( z_j \) lines to be picked in that zone is approximated as
The Impact of Batching and Zoning on Order Picking Performance

\[ E[se^2] = E \left[ \left( \frac{L}{v} * B_j + \frac{2w}{v} * (H_j - 1) + z_j * pt + st \right)^2 \right], \quad \forall C < j \leq C + S \]

\[ = \left( \frac{L}{v} \right)^2 * E[B_j^2] + \left( \frac{2w}{v} \right)^2 * E[H_j^2] + \frac{4wL}{v^2} * E[B_j * H_j] + \frac{4w * pt * z_j + 4w * st}{v} \left( \frac{8w^2}{v^2} \right) * E[H_j^2] + \frac{2L * pt * z_j + 2L * st}{v} \left( \frac{4wL}{v^2} \right) * E[B_j^2] + \left( z_j * pt \right)^2 + \left( 2pt * st - \frac{4w * pt}{v} \right) * z_j + \left( \frac{2w}{v} \right)^2 - \frac{4w * st}{v} + (st)^2 \]

(3.11)

where

\[ E[B_j^2] = M_j \left( (M_j - 1) \left( 1 - \frac{2}{M_j} \right)^{\gamma_j} + (1 - 2M_j) \left( 1 - \frac{1}{M_j} \right)^{\gamma_j} + M_j \right), \quad \forall C < j \leq C + S \]  

(3.12)

\[ E[H_j^2] = M_j^2 - \sum_{i=1}^{M_j-1} (2i+1) \left( \frac{i}{M_j} \right)^{\gamma_j}, \quad \forall C < j \leq C + S \]  

(3.13)

and

\[ E[B_j H_j] = (M_j + 1) \left( M_j - \sum_{i=1}^{M_j} (1-u_i)^{\gamma_j} \right) \left( M_j - \sum_{i=1}^{M_j-1} \left( \sum_{i+1}^{M_j} u_i \right)^{\gamma_j} \right) \]

\[ - \sum_{i=1}^{M_j-1} \left( i \left( \sum_{i=1}^{M_j} u_i \right)^{\gamma_j} + \sum_{i=1}^{M_j-1} \left( \sum_{i=1}^{M_j} u_i, u_{i+1} \right)^{\gamma_j} \right), \quad \forall C < j \leq C + S \]  

(3.14)

where \( u_i = 1/M_j \), for \( i = 1, 2, ..., M_j \). Equations (3.12), (3.13), and (3.14) are taken from Chew and Tang (1999). \( E[B_j] \) and \( E[H_j] \) are obtained from equation (3.1) and (3.5).

The second moment of service time at zone \( j \), given that zone \( j \) is visited and there are \( Z \) lines in a batch is:
Chapter 3

\[ E[SE_j^2] = \sum_{z_j=1}^{Z} E[se_j^2] * P_j(z_j, Z) \]

\[ = \sum_{z_j=1}^{Z} E[se_j^2] * \left( \frac{Z}{z_j} \right)^{z_j} \left( 1 - \frac{1}{S} \right)^{Z-z_j} \quad \frac{1 - \left(\frac{Z}{z_j}\right)^{z_j}}{1 - \left(\frac{1}{S}\right)^{z_j}} \quad \forall C < j \leq C + S \] (3.15)

The second moment of service time in zone \( j \), given that there are \( b \) orders in a batch is

\[ E[SE_j^2] = \sum_{b=1}^{\infty} P_j(b, Z) * E[SE_j^2], \quad \forall C < j \leq C + S \] (3.16)

The SCV of service time at node \( j \), given that there are \( b \) orders in a batch is

\[ c_j^2 = \frac{E[SE_j^2] - E[SE_j]^2}{E[SE_j]^2}, \quad \forall j > C \] (3.17)

The analyses of the service time for conveyor pieces are the same as we described in chapter 2. A conveyor piece \( j \) can contain \( k_j \) orders bins and is assumed to have constant speed of \( v_{l_j} \). We approximate it as \( k_j \) servers in parallel, each of which has constant service time of \( \frac{k_j}{v_{l_j}} \). Therefore, we have,

\[ c_j^2 = 0, \quad \forall j \leq C \] (3.18)

and

\[ E[SE_j] = \frac{k_j}{v_{l_j}}, \quad \forall j \leq C \] (3.19)
3.4.2. Mean and SCV of service time at the sorting station

When orders are batched, they need to be sorted again by order upon completion of the pick process. We assume a manual sorting process, where the service time of a pick bin at the sorting station is modeled as a constant setup time plus sorting time. We assume sorting time is linearly proportional to the number of lines in the bin and sorting time per line is constant. The service time of a pick bin, given there are $Z$ lines in a batch is

$$SE_j = st + so \cdot Z, \quad j = S + C + 1$$  \hspace{1cm} (3.20)

where $st$ is the bin setup time at the sorting station (constant) and $so$ (constant) is the sorting time per order line.

The expected service time of an order bin at the sorting station given that there are $b$ orders in a batch is then calculated as

$$E[SE_j] = st + so \cdot E[Z]$$

$$= st + so \cdot \sum_{Z=b} \infty Z \cdot P_2(b, Z), \quad j = S + C + 1$$  \hspace{1cm} (3.21)

The second moment of service time of an order bin at the sorting station, given that there are $b$ orders in a batch is

$$E[SE_j^2] = st^2 + 2st \cdot so \cdot E[Z] + so^2 \cdot E[Z^2], \quad j = S + C + 1$$  \hspace{1cm} (3.22)

where

$$E[Z^2] = \sum_{Z=b} \infty Z^2 \cdot P_2(b, Z)$$  \hspace{1cm} (3.23)

From equation (3.21), (3.22), and (3.17), we can obtain the SCV of service time of an order bin at the sorting station.
3.4.3. Mean throughput time of an order bin with $b$ orders

The method presented in Appendix A is used to calculate the mean order throughput time in this pick-and-pass order picking system. The two parameters for service time are $E[SE]$, the mean, and $c_{sj}^2$, the SCV, as we described in the previous sub-section. For the arrival processes, the parameters are $\lambda_j$, the arrival rate, and $c_{aj}^2$, the SCV of the inter-arrival times of bins at a node. Customer orders arrive at the system according to Poisson process with rate of $\lambda_{0i}$. After batching $b$ orders, the batched orders are assigned to an order bin, which is sent to the order picking system at conveyor piece 1 for picking. Therefore, the input of the order picking system has *Erlang* distribution with parameters $(b, \lambda_{0i})$. The input rate is

$$\lambda_i = \frac{\lambda_{0i}}{b}$$

(3.24)

The SCV of the order bin inter-arrival time is

$$c_{ai}^2 = \frac{1}{b}$$

(3.25)

Similar to the analysis in chapter 2, we next calculate the transition probabilities between nodes.

Because of the random storage policy in the pick area, the probability that an order line is stored in zone $j$ is,

$$P_j = \frac{1}{S}, \quad \forall C < j \leq C + S$$

(3.26)

The probability that an order bin enters zone $j$, given that there are $Z$ lines in the batch of orders, equals the probability that there is at least one order line to be picked at zone $j$

$$P_j = 1 - (1 - P_j)^Z, \quad \forall C < j \leq C + S$$

(3.27)
The probability that an order bin has to enter zone $j$, given that it contains $b$ orders is calculated as

$$V_j = \sum_{Z=0}^{\infty} P_z(b, Z) * P'_j, \quad \forall C < j \leq C + S$$

(3.28)

At the end of a conveyor piece, a bin is either transferred on a subsequent conveyor piece for transportation or pushed into a zone for picking. The transition probabilities between these nodes depend on the layout of the system. For the layout sketched in Figure 3.3, they are given by:

$$q_{j,C} = V_{j,C}, \quad \forall j < C$$

(3.29)

$$q_{j,1} = 1 - V_{j,C}, \quad \forall j < C$$

(3.30)

$$q_{j,C} = 1, \quad \forall C < j \leq C + S$$

(3.31)

$$q_{j,C+1} = 1, \quad j = C$$

(3.32)

From the transition probabilities between nodes, we can obtain the internal traffic rates $\lambda_j$ and the SCV of the inter-arrival time between two bins at each node (refer to Appendix A).

The utilization of each node is given by

$$\rho_j = \begin{cases} \lambda_j / \nu_j, & \forall j \leq C \\ \lambda_j E[SE_j] / \nu_j, & \forall j > C \end{cases}$$

(3.33)

The expected sojourn time of a bin at node $j$ is given by

$$E[T_j] = E[vt_j] * (E[W_j] + E[SE_j]), \quad \forall 1 \leq j \leq C + S + 1$$

(3.34)

Where $E[W_j]$ is the expected waiting time of a bin in front of node $j$ as calculated by (A.9), and $E[vt_j]$ is the expected number of visits to node $j$ of a bin. The probability mass function of $vt_j$ is given by
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\[ vt_j = \begin{cases} 
0, & \text{with probability } 1 - V_j, \\
1, & \text{with probability } V_j, 
\end{cases} \quad \forall 1 \leq j \leq C + S + 1 \quad (3.35) \]

Hence

\[ E[vt_j] = 0 \times (1 - V_j) + V_j = V_j, \quad \forall 1 \leq j \leq C + S + 1 \quad (3.36) \]

The expected throughput time of an order bin is the summation of the expected sojourn time at each node.

\[ E[tpt] = \sum_{j=0}^{C+S+1} E[T_j] \quad (3.37) \]

3.4.4. Mean throughput time of an arbitrary order

The mean throughput time of an arbitrary order in the system has two components: the mean waiting time to form a batch, \( E[W_b] \), and the mean throughput time of the order bin in the system, \( E[tpt] \), as we derived in the previous sub-sections. Therefore, the mean throughput time of an arbitrary order in the order picking system is,

\[ E[TPT] = E[W_b] + E[tpt] \quad (3.38) \]

where \( E[W_b] \) is approximated as

\[ E[W_b] = \sum_{i=0}^{b-1} \frac{1}{i^* \lambda_{0i}} \quad (3.39) \]

and \( \frac{1}{\lambda_{0i}} \) is the mean customer order inter-arrival time.
3.5. Numerical experiments

In this section, we use the method introduced in the previous sections to analyze the impact of batch and zone sizes on the mean throughput time of an arbitrary order. The parameters used in the example are listed in Table 3.1. We assume the sorting process is fast and there are enough workers at the sorting station so that the waiting time in front of the sorting station is short. We choose the number of pickers and the number of storage aisles to be integral multiples of the number of zones, since otherwise the system will become imbalanced, leading to increased throughput times. Imbalanced systems can always be improved by balancing (see chapter 2).

Table 3.1: Parameters used in the experiment

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number of aisles</td>
<td>36</td>
</tr>
<tr>
<td>Number of order pickers</td>
<td>12</td>
</tr>
<tr>
<td>Number of sorters</td>
<td>15</td>
</tr>
<tr>
<td>Length of an aisle</td>
<td>25 meters</td>
</tr>
<tr>
<td>Center-to-center distance between 2 aisles</td>
<td>3.5 meters</td>
</tr>
<tr>
<td>Size of order bins (L<em>W</em>H)</td>
<td>60<em>40</em>35 cm</td>
</tr>
<tr>
<td>Conveyor speed</td>
<td>0.7 bins per second (0.1m minimum space between two bins)</td>
</tr>
<tr>
<td>Conveyor length in total</td>
<td>220 bins</td>
</tr>
<tr>
<td>Picking time per line</td>
<td>12 seconds</td>
</tr>
<tr>
<td>Sorting time per line</td>
<td>12 seconds</td>
</tr>
<tr>
<td>Picking setup time in a zone</td>
<td>20 seconds</td>
</tr>
<tr>
<td>Sorting setup time at sorting station</td>
<td>20 seconds</td>
</tr>
<tr>
<td>Picker’s travel speed</td>
<td>1m/second</td>
</tr>
<tr>
<td>Distribution of the number of lines in a customer order</td>
<td>1+Poisson (4).</td>
</tr>
</tbody>
</table>

To study the impact of batch size on mean order throughput time with different zone settings, we vary the number of zones from 1 to 12 in the experiments, as illustrated in Figure 3.4. In each zone setting, we divide the storage aisles and the order pickers equally over the zones. This allows comparison of various batch and zone sizes. For every experiment of a specific zone setting, we choose the order arrival rate such that the utilizations at the pick zones are larger than 0.9 when the batch size is 1. We then vary the batch size from 1 to 7 with the same order arrival rate. We take the same orders across
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experiments. The results obtained from the approximation model are compared with simulation results. The simulation model is built in AUTOMOD 10.0. For each particular setting, the simulation results were obtained from runs of more than 10000 pick batches so that the 95% confidence intervals are within 1% of the mean order throughput time in the system.

Figure 3.4 shows the impact of batch size on mean order throughput time with different zone settings in comparison to simulation results. It appears that the average mean order throughput time is a convex function of batch size under different zone settings. This result is consistent with the findings of Chew and Tang (1999) who consider a single-block warehouse and of Le Duc and De Koster (2007) who consider a two-block warehouse with a single pick zone. The reason is as follows. The mean order throughput time has three components, the waiting time to form a batch, waiting time in front of nodes for service, and service time at nodes. When the batch size is small, the batch-forming time and the service time at nodes are small, but the arrival rates of flows to nodes are high, leading to high utilization of servers and long waiting time for service because of the limited number of servers at nodes. On the other hand, a large batch size results in long batch-forming time and service time, but low arrival rates of flows to nodes which lead to low utilization of servers and short waiting time for service. These trade-offs indicate that an optimal batch size exists.
We observe from Figure 3.4 and from other experiments that the quality of the approximation method is good with a maximum relative error less than 10 percent compared with simulation when the batch size is larger than one, in which case the utilizations at nodes are smaller than 0.9. The relative error increases up to the maximum of 30% when the batch size is 1 and the utilizations at nodes are larger than 0.9. The order picking system tends to become unstable when the utilizations are larger than 0.9. In order to further investigate the impact of utilization and the number of zones on the quality of the approximation, experiments are carried out based on the system of Table 3.1, with batch size 1. The worker utilizations are varied between 0.55 and 0.94 by varying the order arrival rates. Results are shown in Table 3.2. We find in general, the approximation
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method has acceptable quality for practical purpose. The relative errors increase with the number of zones and the utilization at zones.

Table 3.2: Relative approximation errors compared with simulation for varying the number of zones and worker utilizations

<table>
<thead>
<tr>
<th>Utilization</th>
<th>0.55</th>
<th>0.65</th>
<th>0.75</th>
<th>0.85</th>
<th>0.94</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 zone</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.0%</td>
<td>0.7%</td>
<td>2.3%</td>
</tr>
<tr>
<td>2 zones</td>
<td>0.7%</td>
<td>0.8%</td>
<td>1.2%</td>
<td>2.0%</td>
<td>13.8%</td>
</tr>
<tr>
<td>3 zones</td>
<td>1.1%</td>
<td>1.4%</td>
<td>2.1%</td>
<td>4.4%</td>
<td>25.2%</td>
</tr>
<tr>
<td>4 zones</td>
<td>1.2%</td>
<td>1.7%</td>
<td>2.7%</td>
<td>6.7%</td>
<td>30.2%</td>
</tr>
<tr>
<td>6 zones</td>
<td>1.3%</td>
<td>2.0%</td>
<td>3.8%</td>
<td>8.4%</td>
<td>31.2%</td>
</tr>
<tr>
<td>12 zones</td>
<td>1.2%</td>
<td>2.5%</td>
<td>4.9%</td>
<td>10.6%</td>
<td>29.4%</td>
</tr>
</tbody>
</table>

Figure 3.4 also shows that for a given number of order pickers and given storage space size, the mean order throughput time increases significantly with an increasing batch size beyond the minimum point when there are few zones with multiple servers in a zone, but the increase is less obvious when there are many zones with a single server per zone. The reason for this phenomenon can be explained as follows. In the first situation, the utilization at zones and hence the waiting time in front of zones decreases sharply with an increasing of batch size and soon reaches a small value. Beyond the minimum point, the increase of service time and batch-forming time is much larger than the decrease of waiting time at zones, leads to a significant increase of mean order throughput time. In the second situation, the utilization at zones and consequently the waiting time of order bins in front of zones decreases gradually with an increasing batch size. Beyond the minimum points, the increase of service time and batch-forming time is only slightly larger than the decrease of waiting time. Therefore, the increase of mean order throughput time over batch sizes is less obvious in this case.

Figure 3.4 shows zoning impacts the optimal batch size $b^*$ (it varies from $b^*=4$ for 12 zones to $b^*=2$ for other zone configurations) with different order arrival rates. Keeping the mean order arrival rates unchanged, we have also investigated three different two-stage hyperexponential order arrival distributions with squared coefficient of variation 1.22, 1.50, and 1.72. The results show the mean order throughput times change only marginally
The Impact of Batching and Zoning on Order Picking Performance

for these different arrival processes. The optimal batch size remains the same as for Poisson order arrivals.

Figure 3.5 shows the effect of zoning with different batch sizes on system performance. We find that when the batch size is small (b=1, in the example), zoning tends to reduce the mean order throughput time. When the batch size is larger, zoning has minor negative effect on mean order throughput time. The mean order throughput time starts to increase when the number of zones keeps on increasing. We can explain this phenomenon as follows. For a fixed storage area size, increasing the number of pick zones will decrease the arrival rate of flows to each zone and the service time at each zone, leading to a decrease in utilization at each zone, therefore decrease the waiting time in front of each zone. On the other hand, more zones imply more order bin visits to zones and hence more setup time, leading to increasing mean order throughput time. When the batch size is small, the flow arrival rates to zones and the server utilizations are high, waiting time accounts for a large part of the mean order throughput time, therefore reducing waiting time by zoning leads to significant improvement of mean order throughput time. When the batch size is large, waiting time in front of each zone is negligible due to a low arrival rate of flows. The reduction on service time per zone is compensated by the increased number of setups and hence the impact of zoning on mean order throughput time is marginal when the batch size is large. As the number of zones increases, the increased number of setups due to the increased number of visits to zones gradually compensates and exceeds the reduction on waiting time and service time at zones. Therefore the mean order throughput time will increase when the number of zones is large.

With the same mean order arrival rate, the model was also tested for hyperexponential order arrivals with different parameters. The mean order throughput time changes only slightly and the trends shown in Figure 3.5 remain. We carried out a sensitivity analysis on the setup time in pick zones varying from 5 to 30 seconds with a step size of 5 seconds. This has an impact on optimal batch sizes, but the trends shown in Figure 3.5 still hold. Approximation errors are comparable to those in Figure 3.5.
Figure 3.5: The impact of zoning on mean order throughput time under the different batch sizes $b$ with order arrival rate of 1.75 orders/min.

3.6. Conclusions

Batching and zoning are two warehousing operational policies used frequently in practice. In this chapter, we use the approximation model developed in chapter 2 to analyze the impact of these two policies on the order picking system performance. The approximation method shows to have acceptable quality for practical purpose. Errors are in general small,
but when the utilization or the number of zones becomes large, errors increase. Through the experiments carried out in the previous section with different input parameters, such as setup time at pick zones, different order arrival rates to the systems, and the different order arrival distributions, we find that an optimal batch size exists to minimize mean order throughput time. Batch sizes have a large impact on the mean order throughput time as shown in Figure 3.4. From Figure 3.5, we find the mean order throughput time in the system is quite robust for a varying number of zones around the optimum number of zones. We also find, for given order arrival rates, the precise shape of the order arrival distribution has only a slight impact on the mean order throughput time. This is especially true when the utilizations at zones are small. In general, many factors influence the system performance. This phenomenon reflects the complexity of the pick-and-pass system. The preference of one operational strategy over the other depends on the settings of the system. The approximation model developed in this paper can therefore be used as a fast tool to analyze these alternatives.
4. DYNAMIC STORAGE SYSTEMS

4.1. Introduction

In the previous chapters, we developed models to estimate the performance of conventional order picking systems, where all the SKUs ordered by customers during the entire day or a picking shift (which normally takes several hours) are located in the pick area. Due to the large storage space of the pick area, order pickers often spend much of their time on picking tours, leading to low throughput and low worker productivity in the picking process.

In this chapter, we introduce the concept of Dynamic Storage (DS). In a Dynamic Storage System (DSS), customer orders are batched in groups of \( b \) orders before they are released to the picking system. Only those products needed for the current picking batch are retrieved from a reserve area and stored in the pick area, just in time. In a DSS the size of the storage space of the pick area depends on the type and the number of products contained in the batch and hence it is a function of the batch size. Products ordered in two consecutive batches may be different. Automated Storage and Retrieval (S/R) machines reshuffle the products in the pick area before the picking process for a batch of orders starts. Depending on the products ordered by the current and the previous batches, products in the pick area need to be swapped, appended, or condensed (see section 4.3.3 for a detailed discussion).

The major advantage of such a DSS over a conventional order picking system is the higher throughput that can be achieved since only a small fraction of the SKUs are stored in the
pick area, which reduces the order pickers’ travel time. Also picker productivity increases compared to conventional order picking systems due to the smaller pick area. Between two pick batches the storage and reshuffle machines need some time to reshuffle the SKUs in the pick area. Depending on the time and frequency of this process, order pickers may then be assigned to other warehousing activities. However, this is only possible if the reshuffle time is sufficiently long for order pickers to carry out such other warehousing activities. This can be realized by choosing an appropriate batch size, which is discussed in section 4.3.6. Additionally, since S/R machines reshuffle the products, a DSS can eliminate potential worker congestion in the picking aisles caused by manual replenishment. DS has become very popular in Europe with high labor costs. All major warehousing solution providers now sell such systems. Figure 4.1 shows a DSS with some pick stations at the warehouse of Nedac Sorbo, a large non-food store merchandiser in the Netherlands. The bulk storage area is situated behind the pick area. S/R machines are used to automatically replenish and reshuffle the products in the pick area.

Although the number of DSS implementations increases rapidly, we have not found literature to model and analyze it. In this chapter, we show through mathematical and simulation modeling that a DSS can improve order picking throughput and increase picker productivity by comparing its performance with a conventional order picking system, where all the products are stored in the pick area. Of course the performance improvement is at the expense of an automated reshuffle system. Our model can therefore be used to evaluate the justification of such an investment. The chapter is organized as follows. In section 4.2, we review related literature. In section 4.3, we develop an approximation model to analyze the performance of a DSS order picking system with a single station. We compare the results with simulation and show the model is accurate. In section 4.4, we discuss two applications of DSS in a pick-and-pass order picking system. We draw conclusions in section 4.5.
4.2. Related literature

De Koster et al. (2007) point out that research on DS is a virgin area and mention several research topics related to it. The most related topic is the forward-reserve allocation problem, which basically discusses the separation of the bulk stock (reserve area) from the pick stock (forward area). Hackman and Rosenblatt (1990) develop a model to decide which products should be assigned to the pick area and how much space must be allocated to each of the products given a fixed capacity of the forward area with an objective to minimize the total costs for order picking and replenishment. Frazelle et al. (1994) extend the problem and the solution method of Hackman and Rosenblatt (1990) by treating the
size of the forward area as a decision variable. Van den Berg et al. (1998) consider a
warehouse with busy and idle periods where reserve-picking is allowed. Assuming unit-
load replenishments, they develop a knapsack-based heuristic to find an allocation of
products to the forward area that minimizes the expected total labor time related to order
picking and replenishment during a busy period.

The analysis in this chapter differs from the above literature in both the model and the
objective. One of the advantages of DS is that all the pickings are carried out from the pick
area (forward area). Therefore, our model does not consider direct picking from the bulk
area. We assume items will not be depleted during the pick cycle. Our objectives are to
compare the maximum throughput a DSS can achieve and the associated labor time needed
to a conventional order picking system.

4.3. Performance of a DSS with a single pick station

In this section, we illustrate the throughput improvement and labor reduction brought by a
DSS for an order picking system with a single pick station. The layout of the station is
illustrated in Figure 4.2. The products are stored in identical bins and are located along the
picking rack. We assume each SKU is equally likely to be ordered. Hence products are
stored randomly (or uniformly) in the pick area. The bulk storage area is located behind the
picking face, where one or more S/R machines are used to replenish the products to the
pick area (see Figure 4.1, for example). We also assume random storage in the bulk area
and the machine Pickup/Deposit (P/D) station is located at the lower left-hand corner of
the rack. Similar to the previous chapter, we assume the picker’s home base is located at
the boundary of the pick area (see Figure 4.2). The picker travels along the picking rack to
the product locations, picks the required quantity and then returns to his home base. We
assume all the required products in an order are picked in one tour and an order picker
picks one order per picking tour. It is possible to consider the case that the picker’s home
base is located in the middle of the pick area. In such a case, the picker first travels one
side of the home base to pick the products, then turns round, travels to the other side to
pick the products, and then returns to the home base. The calculation of travel time will be different in this case, the further analysis remains the same.

![Diagram of pick station]

Figure 4.2: Layout of a pick station.

We also assume orders arrive online, which means the order information is only available when it arrives at the order picking process. We first discuss the stability condition for the DSS, and then develop a mathematical model to find the maximum throughput that the DSS can achieve, and on top of that, the optimal batch size $b$ to minimize the labor time of order pickers required to pick a certain number of orders. The throughput improvement and the saving on labor time are compared to a benchmark system, where all the products in the warehouse are located in the pick area.

### 4.3.1. Stability condition of a DSS

We suppose customer orders arrive at a warehouse according to a Poisson process at a rate $\lambda$. They are batched in groups of size $b$, and then released simultaneously to the picking system. For batches of orders, Figure 4.3 illustrates the relation between batch-forming times, order service times (travel time + picking time), reshuffle times, and the potential waiting times to start reshuffling (since we can only start to reshuffle products when the whole batch of orders is available to the DSS). This explains waiting time $W_i$ in Figure 4.3. Reshuffling and order picking occur in alternating sequence at the station. To obtain a
stable system, we require the mean cycle time (reshuffle time plus service time of a batch of orders) for processing a batch of orders to be smaller than the mean batch-forming time. Our main objective is to determine the maximum throughput the DSS can achieve; on top of that, we are going to find the optimal batch size to minimize the total worker hours needed to pick a certain number of orders.

![Diagram of stability requirements for a DSS with a single station.](image)

**Figure 4.3: Illustration of stability requirements for a DSS with a single station.**

### 4.3.2. Mathematical formulation

The parameters and notations used in the formulation are as follows:

**Data**

- \( M \): the total number of products in the warehouse.
- \( N \): the maximum number of order lines in an order.
- \( O_n \): probability that an order contains \( n \) order lines, \( n \leq N \).
- \( \lambda \): customer order arrival rate to the DSS.
- \( d \): number of servers at the pick station.
- \( q \): number of reshuffle machines in use.

**Variables**

- \( b \): batch size.
- \( Z \): number of products needing to be reshuffled between 2 batches each consisting of \( b \) orders.
- \( R \): reshuffle time between 2 batches each consists of \( b \) orders.
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$Y$: the number of lines in an order.
$Y_b$: the number of lines in a batch of $b$ orders.
$Y_i$: the number of lines in the $i^{th}$ batch.

$$X_m = \begin{cases} 1, & \text{if product } m \text{ should be stored in the picking area.} \\ 0, & \text{otherwise.} \end{cases}$$

$$X_{i,m} = \begin{cases} 1, & \text{if product } m \text{ is ordered by the } i^{th} \text{ batch.} \\ 0, & \text{otherwise.} \end{cases}$$

$se$: the service time of an order in the DSS.
$SE$: service time of a batch of orders in the DSS.

For a stable system, the maximum throughput that can be achieved by a DSS during a certain time period is proportional to the customer order arrival rate $\lambda$. Equivalently, we have to minimize the inter-order arrival time $1/\lambda$. We therefore need to solve the following model,

$$\min \quad 1/\lambda$$

$$\text{st: } E[R] + E[SE] < b \cdot \frac{1}{\lambda}$$

where equation (4.2) is the stability condition. In order to solve the model, we need to obtain the expression of the mean reshuffle time between two batches, $E[R]$, and the mean service time of a batch of $b$ orders, $E[SE]$, both of which are functions of the batch size $b$.

We derive $E[R]$ in section 4.3.3 and $E[SE]$ in section 4.3.4.

4.3.3. Mean reshuffle time between two batches

To obtain the expression for $E[R]$, we first derive the expression of the expected number of products stored in the pick area for a batch of $b$ orders at the start of the reshuffle process, $E[X]$, then we analyze the expected number of products needing to be reshuffled between two consecutive batches, $E[Z]$. 

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We can calculate the expected number of order lines in an order $E[Y]$ by

$$E[Y] = \sum_{n=1}^{N} n \times O_n$$

(4.3)

The expected number of lines in a batch of $b$ orders can be obtained by

$$E[Y] = b \times E[Y]$$

(4.4)

since orders are independent of each other.

The expected value of $X_m$, i.e., the probability that product $m$ should be stored in the pick area for an order batch of $b$ orders, is calculated as

$$E[X_m] = E[E[X_m | Y]] = E[(1 - \frac{1}{M})^Y]$$

(4.5)

where $1 - (\frac{1}{M})^Y$ is the probability that product $m$ is not ordered in a batch of $Y$ order lines.

To calculate $E[X_m]$, we need to calculate the probability distribution of $Y$, which is a $b$-fold convolution of $Y$. It is often more convenient to use the moment generating function of $Y$.

$$\phi(t) = E[e^{tY}]$$

(4.6)

Where $\phi(t)$ is a function of batch size and the order profile $O_n$. As an example, if the number of lines in an order follows a shifted Poisson distribution of $1 + \text{Poisson}(a)$, then $Y = b + \text{Poisson}(a \times b)$ in distribution and $\phi(t) = e^{bt} \times e^{ab(e^t - 1)}$.

We let

$$w = \log(1 - \frac{1}{M})$$

(4.7)

and put it into equation (4.5), we have
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\[ E[X_m] = E[1 - e^{-\lambda T}] = 1 - \phi(w) \]  
(4.8)

The expected number of products stored in the pick area is

\[ E[X] = \sum_{m=1}^{M} E[X_m] = M \cdot [1 - \phi(w)] \]  
(4.9)

We suppose the picking rack is \( h \) layers high, and each storage bin is \( l \) meters long. The length of the pick area can be approximated as

\[ L = l \cdot E[X] / h \]  
(4.10)

Next we calculate the expected number of products needing to be reshuffled between two batches of orders, \( E[Z] \).

We distinguish 3 cases based on the relationship between the number of products stored in the \( i^{th} \) batch, \( \sum_{m=1}^{M} X_{i,m} \), and the \( (i+1)^{th} \) batch, \( \sum_{m=1}^{M} X_{i+1,m} \).

Case 1: \( \sum_{m=1}^{M} X_{i,m} = \sum_{m=1}^{M} X_{i+1,m} \)

The two batches have the same storage space in this case. Figure 4.4 shows the locations taken by the products in the \( i^{th} \) batch, the products ordered by the \( (i+1)^{th} \) batch, the products needing to be reshuffled and the final products locations in the \( (i+1)^{th} \) batch after reshuffling. Products ordered by both batches (4 and 6 in Figure 4.4, illustrated in bold) remain at their positions. Products ordered by the \( i^{th} \) batch but not by the \( (i+1)^{th} \) batch (products 1, 2, 3, and 5 in Figure 4.4) are swapped with those products ordered by the \( (i+1)^{th} \) batch but not by the \( i^{th} \) batch (products 7, 8, 10, and 9 in Figure 4.4). In this case,

\[ \sum_{m=1}^{M} X_{i,m} (1 - X_{i+1,m}) \], the number of products ordered by the \( i^{th} \) batch but not by the \( (i+1)^{th} \) batch, equals \( \sum_{m=1}^{M} X_{i+1,m} (1 - X_{i,m}) \), the number of products ordered by the \( (i+1)^{th} \) batch but
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not by the $i^{th}$ batch, therefore, the number of products need to be reshuffled between 2 batches is $\sum_{m=1}^{M} X_{i+1,m}(1 - X_{i,m})$.

Figure 4.4: Reshuffle products between two batches in case 1.

Case 2: $\sum_{m=1}^{M} X_{i,m} < \sum_{m=1}^{M} X_{i+1,m}$

The $i^{th}$ batch has smaller storage space than the $(i+1)^{th}$ batch, and $\sum_{m=1}^{M} X_{i,m}(1 - X_{i,m})$ is larger than $\sum_{m=1}^{M} X_{i,m}(1 - X_{i+1,m})$ in this case. Except for the $\sum_{m=1}^{M} X_{i,m}(1 - X_{i+1,m})$ products needing swapping ($1 \leftrightarrow 9$, and $5 \leftrightarrow 10$ in Figure 4.5), $\sum_{m=1}^{M} X_{i+1,m}(1 - X_{i,m}) - \sum_{m=1}^{M} X_{i,m}(1 - X_{i+1,m})$ products (products 12 and 7 in Figure 4.5) need to be appended to the pick area. Therefore, the number of products needing to be reshuffled in this case is also $\sum_{m=1}^{M} X_{i+1,m}(1 - X_{i,m})$. 

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Case 3: $\sum_{m=1}^{M} X_{i,m} > \sum_{m=1}^{M} X_{i+1,m}$

As illustrated in Figure 4.6, in this case, the storage space in the $i^{th}$ batch is larger than the storage space in the $(i+1)^{th}$ batch. $\sum_{m=1}^{M} X_{i,m} (1-X_{i+1,m})$ is larger than $\sum_{m=1}^{M} X_{i+1,m} (1-X_{i,m})$.

$\sum_{m=1}^{M} X_{i+1,m} (1-X_{i,m})$ products need to be swapped between the two batches. To reduce the travel distance for order pickers in picking the $(i+1)^{th}$ batch, we need to move those products ordered by both batches but located between the $(\sum_{m=1}^{M} X_{i+1,m} + 1)^{th}$ closest location from the depot and the $\sum_{m=1}^{M} X_{i,m}$ location to the first $\sum_{m=1}^{M} X_{i+1,m}$ slots. We select those slots occupied by products ordered in the $i^{th}$ batch but not ordered in the $(i+1)^{th}$ batch. As illustrated in Figure 4.6, we need to switch product 10 with product 7. This process is called condensing. The products ordered in the $i^{th}$ batch but not in the $(i+1)^{th}$ batch and located outside the closest $\sum_{m=1}^{M} X_{i+1,m}$ locations from the depot (product 8 in Figure 4.6) and the products being swapped out of the pick area for the $(i+1)^{th}$ batch (product 7 in Figure
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4.6) will be moved to the reserve area. Moving times of these products are not included in the reshuffle time as they can be carried out by the S/R machines parallel to the order picking process. Therefore, in this case, the number of products needing to be reshuffled before the start of the picking process is \( \sum_{m=1}^{M} X_{i+1,m} (1 - X_{i,m}) + CD \), where \( CD \) is the number of products needing to be condensed between two batches. In Appendix C, we compare the expected value of \( CD \) and the expected value of \( \sum_{m=1}^{M} X_{i+1,m} (1 - X_{i,m}) \) to show that the impact of \( CD \) is negligible for practical parameter settings.

![Diagram of product locations in the \( i^{th} \) batch, products ordered in the \( (i+1)^{th} \) batch, and product locations in the \( (i+1)^{th} \) batch.]

In conclusion, in all these cases, the number of products needing to be reshuffled between the \( i^{th} \) batch and the \( (i+1)^{th} \) batch can be approximated by the number of products ordered in the \( (i+1)^{th} \) batch, but not in the \( i^{th} \) batch, i.e.,

\[
Z = \sum_{m=1}^{M} [X_{i+1,m} * (1 - X_{i,m})]
\]

(4.11)
Using conditional probability and combining equation (4.6) and equation (4.7), the expected number of reshuffle products between two batches is calculated as

\[ E[Z] = M \cdot E[E[X_{i,m} | Y_{i+1}]] \cdot E[(1 - X_{i,m}) | Y_i] \]

\[ = M \cdot E[1 - \left(1 - \frac{1}{M}\right)^{Y_{i+1}}] \cdot E[1 - \frac{1}{M}] \]

\[ = M \cdot E[1 - E^r_{i+1}] \cdot E[1 - e^{-\phi(w)}] \]

\[ = M \cdot [1 - \phi(w)] \cdot \phi(w) \]  

(4.12)

where \( Y_{i+1} \) and \( Y_i \) are the number of order lines in batches \( i+1 \) and \( i \) respectively.

The mean reshuffle time between two batches is approximated as:

\[ E[R] = rs \times \left[ \frac{E[Z]}{q} \right] \]  

(4.13)

where \( [\cdot] \) means rounding up to the nearest integer value. \( q \) is the number of reshuffle machines, and \( rs \) the reshuffle time for one product.

Before analyzing \( rs \), we first need to understand the movement of the S/R machines in a reshuffle process. As we discussed above, a reshuffle process may be a swap process, an appending process, or a condensing process. Since Appendix C shows the impact of condensing can be neglected, we focus on the swap and the appending process. In a swap process, the S/R machine first travels to the location of the product needing to be removed from the pick area, picks the product bin, travels to an empty location in the reserve area, stores the product bin, travels to the location in the reserve area of the product needing to be stored in the pick area, retrieves the product bin, travels to the previously emptied storage slot in the pick area, and stores it there. In an appending process, the S/R travels to the location in the reserve area of the product needing to be stored in the pick area, picks the products bin, travels to the end of the pick area, and puts the bin into the first empty slot next to the end of the pick area.

For swapping bins, the S/R machine carries out double cycles. For appending bins, it carries out single cycles. In both cases, we approximate the cycle time of the S/R machine by formula 10.74 for dual command cycles of Tompkins et al. (2003), which is a worst case approximation in the case of appending bins. The formula is listed below:
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\[ T_{dc} = E(DC) + 4T_{PD} \]  \hspace{1cm} (4.14)

where, \( T_{dc} \) is the expected dual command cycle time; \( E(DC) \) is the expected travel time from the storage location to the retrieval location during a dual command cycle; and \( T_{PD} \) is the time required to either pickup or deposit the load. \( E(DC) \) is calculated by,

\[ E(DC) = \frac{T}{30} \left[ 40 + 15Q^2 - Q \right] \]  \hspace{1cm} (4.15)

where \( T \) (the scale factor) designates the longer (in time) side of the rack, i.e., the maximum of the travel time required to travel horizontally from the machine’s P/D station to the furthest location in the bulk storage area and the travel time required to travel vertically from the P/D station to the furthest location in the bulk storage area. \( Q \) (the shape factor) designates the ratio of the shorter (in time) side to the longer (in time) side of the rack.

We will show in the following sections that even under this assumption, a DSS can lead to significant improvements on order throughput and worker time needed.

4.3.4. Mean service time for a batch of \( b \) orders

To analyze the mean service time to pick a batch of \( b \) orders, \( E[SE] \), we first derive the expression for \( E[se] \), the mean service time to pick an order. \( E[se] \) is the summation of the mean travel time and the mean picking time, which is proportional to the number of lines to be picked.

The mean travel time to pick an order, \( E[tr] \), is calculated as

\[ E[tr] = \frac{(2 \sum_{n=1}^{N} L^* \left( \frac{n}{1+n} \right) * O_n)}{v} \]  \hspace{1cm} (4.16)
where, \( L \) is obtained from equation (4.10), and \( v \) is the picker’s travel speed. \( L = \frac{n}{1+n} \) is the expected travel distance given that \( n \) lines are picked and all the products are uniformly located in the interval \([0, L]\).

The mean service time of an order in the pick station is

\[
E[se] = E[tr] + E[Y] \cdot pk
\tag{4.17}
\]

where \( E[Y] \) is obtained from equation (4.3) and \( pk \) is the picking time for an order line (constant).

Therefore, the mean service time of a batch of \( b \) orders is approximated as

\[
E[SE] = \left( \frac{b}{d} \right) \cdot E[se], \quad B > d
\tag{4.18}
\]

where \( d \) is the number of servers in the pick station.

When the batch size is not larger than \( d \), the mean service time of a batch of \( b \) orders is modified to

\[
E[SE] = E[\max(se \mid b)], \quad b \leq d
\tag{4.19}
\]

which is the expected value of the largest service time of an order among \( b \) orders, because every picker picks at most one order and the batch is finished when the last picker finishes his order. We calculate it as

\[
E[SE] = E[\max(se \mid b)] = 2^b \left( \frac{E[Y_{\max}]}{1 + E[Y_{\max}]} \cdot L \right) + E[Y_{\max}] \cdot pk, \quad b \leq d
\tag{4.20}
\]

where \( Y_{\max} = \max\{\tilde{Y}_1, \tilde{Y}_2, \ldots, \tilde{Y}_{b-1}, \tilde{Y}_b\} \) is the maximum number of lines to be picked in an order among a batch of \( b \) orders. The first term in the equation above is an approximation of the mean travel time and the second term the expected value of picking time. The cumulative distribution function (CDF) of \( Y_{\max} \) is calculated by taking the \( b^{th} \) power of the CDF of \( \tilde{Y}_i \).
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\[ F_{\text{max}} (y) = P \left( (Y_1 \leq y) \cap (Y_2 \leq y) \cap \ldots \cap (Y_{b-1} \leq y) \cap (Y_b \leq y) \right) \]

\[ = \left[ F_{Y_i} (y) \right]^b \]

(4.21)

where \( Y_i \) are the number of lines in the \( i^{th} \) order in the batch. The last equality follows from the fact that \( Y_i \) are independent and identically-distributed (i.i.d) random variables.

The mean value of \( Y_{\text{max}} \), \( E[Y_{\text{max}}] \), can be obtained from its probability mass function

\[ p_{\text{max}} (y) = \left[ F_{Y_i} (y) \right]^b - \left[ F_{Y_i} (y-1) \right]^b. \]

Putting the results into equation (4.20), we obtain the value of \( E[SE] \) for \( b \leq d \).

In the next subsection, we discuss the procedures to obtain the maximum throughput modeled in equation (4.1) and (4.2) and the optimal batch size to minimize the total worker hours required for picking a batch of \( b \) orders while achieving the maximum throughput.

4.3.5. Solutions to obtain the maximum throughput

The value of \( 1/\lambda \) is the inter-arrival time between two orders with units of seconds. To solve the model expressed in equation (4.1) and (4.2). We start from \( 1/\lambda \) equal to 1 second, and increase this value with a step size of 1. For each value of \( 1/\lambda \), we determine the feasible range of \( b \) satisfying the stability condition in equation (4.2). The optimal value of \( \lambda_{\text{opt}} \), achieving the maximum throughput, is the largest value of \( \lambda \), for which a feasible value of \( b \) exists. Picking a batch of \( b \) orders requires \( b * E[SE] \) work hours. The expression of \( E[SE] \) is given by equation (4.17). So the optimal batch size \( b_{\text{opt}} \) to minimize the total worker hours is the minimum feasible value that \( b \) can take.

The total number of orders finished within a certain time period \( T \) in a DSS (suppose \( T \) starts with a new cycle), \( F_i \), is approximated as
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\[
Fi = \left\lfloor \frac{T}{b_{\text{opt}} \times \frac{1}{\lambda_{\text{opt}}}} \right\rfloor \max \left( \Phi, \frac{0}{E[se]} \right) \times d \tag{4.22}
\]

where \(\left\lfloor \cdot \right\rfloor\) means rounding down to the nearest integer, and

\[
\Phi = T - \frac{T}{b_{\text{opt}} \times \frac{1}{\lambda_{\text{opt}}}} \times b_{\text{opt}} - E[R] \tag{4.23}
\]

The first term in equation (4.22) is the number of orders finished by completed batches in a time interval of length \(T\), and the second term is an approximation of the number of orders completed in the current processing batch at time \(T\).

4.3.6. Numerical validation and comparison with a benchmark system

We use an example to validate the analytical model. The data used in the example is listed in Table 4.1. In order to obtain the reshuffle times per trip as listed in Table 4.1, we assume the layout of the bulk storage rack is square-in-time (SIT), which means the machine’s horizontal travel time from the P/D station to the furthest location in the storage aisle equals the vertical travel time from the P/D station to the highest location in the storage aisle. The size of the storage slot, the moving speeds of S/R machines, and the storage and retrieval times of a product bin are listed in Table 4.2. With the value given in Table 4.1 for the total number of products in the warehouse, we can obtain the layout of the bulk storage area as listed in Table 4.2. As we discussed in section 4.3.3, the reshuffle time for a product is approximated by the average cycle time of a swap process using equation (4.14) and (4.15). The resulting reshuffle time per trip with one machine is listed in Table 4.1. When multiple machines are used, mutual blocking may occur. Based on the DSS at Nedac
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Sorbo (see Figure 4.1), we estimate the cycle time prolongation to be 10\% of the average for each machine that is added. The resulting reshuffle times per trip for different numbers of machines are listed in Table 4.1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of order lines per order</td>
<td>1+Poisson(1)</td>
</tr>
<tr>
<td>Total number of products in the warehouse</td>
<td>600</td>
</tr>
<tr>
<td>Number of storage rack layers</td>
<td>4</td>
</tr>
<tr>
<td>Reshuffle time per trip with 1 machine (constant)</td>
<td>30.4 s</td>
</tr>
<tr>
<td>Reshuffle time per trip with 2 machines (constant)</td>
<td>33.4 s</td>
</tr>
<tr>
<td>Reshuffle time per trip with 3 machines (constant)</td>
<td>36.5 s</td>
</tr>
<tr>
<td>Number of pickers</td>
<td>2</td>
</tr>
<tr>
<td>Picking time per line (constant)</td>
<td>3 s</td>
</tr>
<tr>
<td>Picker’s travel speed (constant)</td>
<td>1 m/s</td>
</tr>
<tr>
<td>Experiment length $T$</td>
<td>20 days</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of a storage slot</td>
<td>0.6 m</td>
</tr>
<tr>
<td>Height of a storage slot</td>
<td>0.4 m</td>
</tr>
<tr>
<td>Length of the bulk storage area (in slots)</td>
<td>30</td>
</tr>
<tr>
<td>Height of the bulk storage area (in slots)</td>
<td>20</td>
</tr>
<tr>
<td>Horizontal moving speed of a reshuffle machine (constant)</td>
<td>2.25 m/s</td>
</tr>
<tr>
<td>Vertical moving speed of a reshuffle machine (constant)</td>
<td>1 m/s</td>
</tr>
<tr>
<td>Time required to either pickup or deposit of the load (constant)</td>
<td>4 s</td>
</tr>
</tbody>
</table>

We first derive the values of $\lambda_{opt}$ for different numbers of S/R machines from the model described above. From $\lambda_{opt}$, we find the optimal batch size $b_{opt}$ to minimize the total worker hours. For the optimal batch size $b_{opt}$, we calculate the average number of products stored in the pick area, the average number of products needing to be reshuffled between two batches, the average length of the pick area, the mean service time $E[SE]$, the mean
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reshuffle time $E[R]$, for a batch of $b_{opt}$ orders, the mean service time for an order $E[se]$, and the number of orders completed in the DSS in a time period $T$, $Fi$. These results are compared with simulations.

We built a simulation model in Automod® 10.0. The model takes the parameters listed in Table 4.1 and the values of $b_{opt}$ and $\lambda_{opt}$ obtained above from our analytical model as inputs. To obtain the products contained in a batch of orders, we first generate the number of order lines for each order in the batch according to the distribution in Table 4.1. For each order line, a product is chosen randomly from all the products in the warehouse. The products in a batch are randomly stored in the pick area. Products are swapped, appended, or condensed based on the products ordered by two consecutive batches. We take the cycle time for a swapping, an appending, and a condensing process from the calculation results of equation (4.14) and (4.15). To obtain the number of orders finished in a time interval $T$ of 20 days, we use 10 runs of 20 days with 2 days initialization time for each run. These runs ensure that the 95% confidence interval of the number of orders finished in 20 days is less than 1% of the average. All the other simulation results are obtained by using 1 run of 50 days with 2 days of initialization period. The simulation time span ensures the 95% confidence interval of all the measurements below 1% of their averages. The results are listed in Table 4.3. We conclude that the proposed method is accurate enough for practical purposes.
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Table 4.3: Model validation results

<table>
<thead>
<tr>
<th># of reshuffle machines</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum arrival rate (orders/hour)</td>
<td>66.7</td>
<td>94.7</td>
<td>124.1</td>
</tr>
<tr>
<td>Optimal batch size</td>
<td>172</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>Comparison</td>
<td>anal.</td>
<td>sim.</td>
<td>rel.error</td>
</tr>
<tr>
<td>Ave. length of pick area (m)</td>
<td>39.28</td>
<td>39.25</td>
<td>0.08%</td>
</tr>
<tr>
<td>Ave. # of products stored in the pick area</td>
<td>261.89</td>
<td>261.64</td>
<td>0.10%</td>
</tr>
<tr>
<td>Ave. # of products to be reshuffled between 2 batches</td>
<td>147.58</td>
<td>147.76</td>
<td>-0.12%</td>
</tr>
<tr>
<td># of orders finished in 20 days</td>
<td>31992</td>
<td>31384</td>
<td>1.93%</td>
</tr>
<tr>
<td>Ave. service time per order (sec)</td>
<td>55.66</td>
<td>55.87</td>
<td>-0.38%</td>
</tr>
<tr>
<td>Ave. service time per batch (sec)</td>
<td>4787.2</td>
<td>4820.1</td>
<td>-0.68%</td>
</tr>
<tr>
<td>Ave. reshuffle time per batch (sec)</td>
<td>4499.2</td>
<td>4638.6</td>
<td>-3.00%</td>
</tr>
</tbody>
</table>

We next use this model to show the improvements a DSS can bring on throughput, and on the number of worker hours to finish $P$ orders compared with a benchmark system. The pick area of the benchmark system has similar layout as the DSS, but all products in the warehouse are stored randomly at the pick area. Further layout parameters, the picker parameters, and the order profiles used in the benchmark system are identical to the DSS and are listed in Table 4.1. When an order arrives at the benchmark picking system, it is processed immediately by an order picker if available. It is queued otherwise. We use Automod® 10.0 to build a simulation model to obtain the performance of the benchmark system with the same orders, the same number of runs and the same initialization period for each run as used in the DSS. To obtain the maximum order arrival rate of the benchmark system, we set the order inter-arrival time (i.e., $1/\lambda$) at 1 second, and then increase it with a step size of 1 second, until the system becomes stable (i.e., the
utilizations of order pickers are less than, but close to 1). The largest value of $\lambda$ making the system stable is the maximum order arrival rate that can be handled by the benchmark system. The results are compared with the performance of the DSS using the formulations in the previous sub-sections. The total labor time needed to pick $P$ orders is the product of $P$ and $E[se]$, the mean service time for an order picker to finish an order. Since both the DSS and the benchmark system will pick the same number of orders $P$, it suffices to compare the values of $E[se]$ in the comparison. For the DSS, we vary the number of reshuffle machines from 1 to 3. The comparison results can be found in Table 4.4. We find the maximum throughput of the DSS to be substantially larger than the benchmark system and the throughput increases with the number of S/R machines. Also, the labor time needed for picking reduces significantly in the DSS. The improvement on throughput and the saving on worker hours are due to the shortened travel distance (time) in picking tours since we only store the products needed for the current batch of orders in the forward pick area.

We find from Table 4.3 that with the optimal batch size, $b_{opt}$, the average reshuffle time between two batches with two reshuffle machines is around 66 seconds, during which time period it might not be possible to assign order pickers to do other warehousing activities apart from simple cleaning work. In practice, we can choose a larger batch size which still meets the stability condition to address this issue. As an example of this case, the system is still stable when the batch size is 40. The reshuffle time between two batches is then 1102 seconds (around 18 minutes) which may be long enough for other warehousing activities. The saving on worker hours in picking is then still 83.2%.
Table 4.4: Performance comparison between the DSS and the benchmark system

<table>
<thead>
<tr>
<th># of reshuffle machines</th>
<th>DSS</th>
<th>Benchmark system (fixed storage)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Maximum order arrival</td>
<td>66.7</td>
<td>94.7</td>
</tr>
<tr>
<td>rate (orders/hour)</td>
<td>124.1</td>
<td>59.0</td>
</tr>
<tr>
<td>Number of orders finished in 20 days</td>
<td>31992</td>
<td>45472</td>
</tr>
<tr>
<td>Improvement of DSS on throughput in 20 days</td>
<td>12.6%</td>
<td>60.0%</td>
</tr>
<tr>
<td>Optimal batch size to minimize worker hours</td>
<td>172</td>
<td>2</td>
</tr>
<tr>
<td>Ave. service time per order (sec)</td>
<td>55.7</td>
<td>6.8</td>
</tr>
<tr>
<td>savings on worker hours</td>
<td>53.8%</td>
<td>94.4%</td>
</tr>
<tr>
<td>Ave. length of pick area (m)</td>
<td>39.28</td>
<td>0.60</td>
</tr>
<tr>
<td>Ave. number of products stored in the pick area</td>
<td>261.89</td>
<td>3.99</td>
</tr>
</tbody>
</table>

4.4. Application of DS to a pick-and-pass order picking system

We analyzed a DSS and its performance for an order picking system consisting of a single station. The concept of DS can also be applied to pick-and-pass order picking system as illustrated in Figure 4.7. In the pick-and-pass system, all the products in the warehouse are distributed evenly over the racks at the stations. Within a station, storage is random. A customer order is assigned to an order bin when it arrives at the warehouse. Order bins are released to the conveyor system and will be diverted to a pick station if items need to be picked there and if the buffer at that station is not full. Each station has one or more order pickers. Picked order bins will be pushed back to the conveyor system and travel downstream. Order bins that are not able to enter a station due to a full buffer will cycle in the system until room is available at the station’s buffer.
Dynamic storage has been implemented by several warehouses. In most applications we studied, products are reshuffled only once per shift with fairly large batch sizes. In such systems, the advantages of DS have not been explored fully since the pick area is still large. In the following sub-sections, we propose two alternatives for DS in a pick-and-pass system of multiple stations and compare their performances with the conventional pick-and-pass system described above. The analytical model developed in section 4.3 can be used to analyze the performance in the first alternative. For the second alternative, we resort to simulation.

4.4.1. Alternative 1

In this alternative, the order batch size is a multiple of the number of pick stations. Orders are assigned to stations sequentially before they are released to the order picking system and each station has the same number of orders to pick for a batch of orders. As an example, we suppose that there are 2 stations and the batch size is 6. We assign orders 1, 3, and 5 to station 1 and orders 2, 4, and 6 to stations 2. Each station has its own S/R machines. We assume each S/R can retrieve every product in the entire bulk storage area and a product can be assigned to multiple stations. At each station, we just in time store those products needed for the assigned orders in the pick area and use S/R machines to reshuffle products between batches. This implies each order visits only one pick station, and since the numbers of products in orders are i.i.d random variables, the operations at stations are identical. We can therefore analyze each station independently using the
methods described in the previous section. The throughput of the whole system equals approximately the throughput of a single station multiplied by the number of pick stations. Figure 4.8 shows the layout of this order picking system and the assignment of orders to stations with an order picking system of 5 stations.

Figure 4.8: System layout and the order assignments in application 1.

We use an example to compare the performance of the DSS to a benchmark conventional pick-and-pass system as we discussed at the beginning of this section. The data used by the DSS and the benchmark system is listed in Table 4.5 and Figure 4.7. We assume the bulk storage area has random storage. The dimension of a storage slot in the bulk area and the travel speeds of reshuffle machines are listed in Table 4.2. Again, we assume the bulk area is SIT, therefore we can calculate the dimension of the bulk storage area as listed in Table 4.5. The reshuffle machines work the same way as we described in previous sections in the entire bulk storage area. We use similar methods as in section 4.3.6 to obtain the reshuffle time per trip including congestions. The results are shown in Table 4.5. The performance of the benchmark system is obtained from simulation models built in Automod® 10.0. We first use 1 run of 50 days with 2 days initialization time to obtain the maximum order arrival rate of the benchmark system using the method described in section 4.3.6. We then input this value into the simulation model to obtain the number of orders finished in time $T$ and the number of worker hours used to finish a certain number of orders. We use 10 runs of 20 days for the simulation to ensure that the 95% confidence interval of these two...
measurements are less than 1% of the average values. For each experiment, we use 2 days initialization time.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>DSS Benchmark system</td>
<td>Fixed storage (fixed storage)</td>
</tr>
<tr>
<td>Number of order lines per order</td>
<td>1+Poisson(3)</td>
</tr>
<tr>
<td>Number of stations</td>
<td>5</td>
</tr>
<tr>
<td>Total number of products in the warehouse</td>
<td>3000</td>
</tr>
<tr>
<td># of storage rack layer</td>
<td>4</td>
</tr>
<tr>
<td>Length of the bulk storage area (in slots)</td>
<td>67</td>
</tr>
<tr>
<td>Height of the bulk storage area (in slots)</td>
<td>45</td>
</tr>
<tr>
<td>Conveyor moving speed</td>
<td>1 m/s</td>
</tr>
<tr>
<td># of pickers per station</td>
<td>1</td>
</tr>
<tr>
<td>Picker travel speed</td>
<td>1 m/s</td>
</tr>
<tr>
<td># of products stored in a station</td>
<td>600</td>
</tr>
<tr>
<td>Buffer capacity between stations</td>
<td>8 order bins</td>
</tr>
<tr>
<td>Reshuffle time per trip with 1 machine</td>
<td>48.4 s</td>
</tr>
<tr>
<td>Reshuffle time per trip with 2 machines</td>
<td>53.2 s</td>
</tr>
<tr>
<td>Reshuffle time per trip with 3 machines</td>
<td>58.1 s</td>
</tr>
</tbody>
</table>

The comparison between the two systems is shown in Table 4.6. We note all results for the DSS in Table 4.6 are obtained from the mathematical models developed in the previous sections. Compared to the performance of the benchmark system, the improvement on throughput and the saving on the number of worker hours in DSS is substantial. The saving on worker hours in picking in Table 4.6 is obtained from using the optimal batch sizes. In practice, to allow order pickers to have enough time to handle other warehousing activities during the reshuffle time period between two batches, we can choose larger batch sizes while not violating the stability conditions.
Table 4.6: Performance comparison between the DSS application 1 and the benchmark system

<table>
<thead>
<tr>
<th></th>
<th>DSS</th>
<th>Benchmark system (fixed storage)</th>
</tr>
</thead>
<tbody>
<tr>
<td># of reshuffle machines per station</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Maximum order arrival rate (orders/hour)</td>
<td>85.7</td>
<td>150.0</td>
</tr>
<tr>
<td>Optimal batch size to minimize worker hours</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td># of orders to pick at each station for a batch of orders with optimal batch size</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Number of orders finished in 20 days in the system</td>
<td>41140</td>
<td>72000</td>
</tr>
<tr>
<td>Improvement of DSS on throughput</td>
<td>64.0%</td>
<td>187.0%</td>
</tr>
<tr>
<td>Ave. service time per batch (sec)</td>
<td>12.96</td>
<td>12.96</td>
</tr>
<tr>
<td>Ave. reshuffle time per batch (sec)</td>
<td>193.6</td>
<td>106.4</td>
</tr>
<tr>
<td>Total worker hours used to finish 10000 orders</td>
<td>35.9</td>
<td>35.9</td>
</tr>
<tr>
<td>Savings on worker hours over the benchmark system to finish 10000 orders</td>
<td>95.8%</td>
<td>95.8%</td>
</tr>
</tbody>
</table>

4.4.2. Alternative 2

The application of DSS in this alternative has similar system layout as the conventional benchmark pick-and-pass system illustrated in Figure 4.7. To make it work properly, its operating rules are defined as follows.

1. The products in the bulk storage are divided equally over pick stations. Products stored at a station at a specific moment can only be chosen from the station’s bulk storage assortment.

2. A product can only be stored at one station, i.e., product splitting is not allowed.

3. The products stored at each station at the start of an order batch depend on the products ordered in the batch and the station’s assortment.
4. Each station has its own S/R machines to reshuffle products between batches.

5. The next order batch is released to the system for picking when picking for the previous batch has finished at all stations. No cross-batch picking is allowed in the system.

6. Reshuffling at stations is not synchronized. Each station starts reshuffling after a batch of orders has formed and the picking process for the previous batch has been finished at the station.

7. At each pick station, reshuffling and picking are carried out sequentially.

To have a stable system in this case, we require that the mean cycle time (the time period from the batch of orders entering the system until the picking for this batch has finished at all stations) of a batch of orders in the system is smaller than the mean batch forming time. Our objective is to find the maximum order arrival rate that the system can handle and the minimum batch size to achieve it.

Some difficulties exist in developing mathematical models to analyze the DSS performance in this case. First, the labor time used to pick a batch of orders contains not only the picking time and the travel time, but also the potential waiting time between two orders arriving at a station, which is difficult to derive. Second, because we do not synchronize reshuffles between stations, reshuffling and picking can be carried out simultaneously at different stations in the system; therefore, it is difficult to develop the cycle time for a batch of orders in a closed mathematical form.

We therefore use simulation to analyze the performance and compare it with the performance of the conventional benchmark pick-and-pass system, as we described at the beginning of this section with parameters listed in Table 4.5 and Figure 4.7. The DSS has the same parameters as the benchmark system and the stations’ assortments are identical to that of the benchmark system. Since products are evenly distributed over stations and the products stored at a station at a specific moment can only be chosen from the station’s assortment, a reshuffle machine assigned to a station will only travel in the specific bulk storage area located behind the pick station. Therefore, the average reshuffle times per trip will take the values listed in Table 4.1. The simulation model and the results for the
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benchmark system have already been discussed in section 4.4.1. For the simulation model of the DSS, we first use 1 run of 50 days with 2 days initialization time to obtain the maximum order arrival rate, $\lambda_{opt}$, that the DSS can achieve and the optimal batch size, $b_{opt}$, to minimize the worker hours while achieving the maximum order arrival rate using the similar methods as we described in section 4.3.5. The simulation time span ensures the 95% confidence interval of all the measurements below 1% of their averages. We then input $\lambda_{opt}$ and $b_{opt}$ into the simulation model, and use 10 runs of with 2 days initialization period to obtain the maximum throughput in a certain time period $T$ and the total number of worker hours used to finish a certain number of orders. The mean value of these two measurements can be found in Table 4.7. The 95% confidence intervals of these two measurements are both below 1% of their mean values.

Comparing the results between Table 4.6 and Table 4.7, alternative 2 yields a higher throughput than alternative 1 with the current settings. This might be due to the relatively smaller percentage of the reshuffle time in the total cycle time. In general, the preference of one alternative over the other depends on the settings. Alternative 1 may be cheaper in investment as it needs fewer conveyors. All the orders visit only one station. The operation at a station is independent of other stations. Although it is difficult to quantify differences analytically, the existence of alternatives provides warehouse managers a choice to select the appropriate implementation according to their working situation.
Table 4.7: Performance comparison between the DSS application 2 and the benchmark system

<table>
<thead>
<tr>
<th></th>
<th>DSS</th>
<th>Benchmark system (fixed storage)</th>
</tr>
</thead>
<tbody>
<tr>
<td># of reshuffle machines per station</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Maximum order arrival rate (order/hour)</td>
<td>112.50</td>
<td>163.64 189.47 58.06</td>
</tr>
<tr>
<td>Optimal batch size to minimize worker hours</td>
<td>62</td>
<td>44 37</td>
</tr>
<tr>
<td>Number of orders finished in 20 days</td>
<td>53744</td>
<td>78178 90628 25086</td>
</tr>
<tr>
<td>Improvement of DSS on throughput</td>
<td>93.1%</td>
<td>211.6% 261.3%</td>
</tr>
<tr>
<td>Total worker hours used to finish 10000 orders</td>
<td>223.35</td>
<td>235.45 247.13 859.75</td>
</tr>
<tr>
<td>Savings on worker hours over the benchmark system to finish 10000 orders</td>
<td>74.0%</td>
<td>72.6% 71.3%</td>
</tr>
<tr>
<td>Ave. # of products stored in the pick area at each station</td>
<td>48.0 34.2 28.9 600</td>
<td></td>
</tr>
<tr>
<td>Ave. storage length of pick area at each station</td>
<td>7.40</td>
<td>5.40 4.65 90</td>
</tr>
<tr>
<td>Mean batch forming time (sec)</td>
<td>1984</td>
<td>968 703</td>
</tr>
<tr>
<td>Mean cycle time per batch (sec)</td>
<td>1977.96</td>
<td>964.95 696.15</td>
</tr>
<tr>
<td>Mean reshuffle time per batch at stations in (sec)</td>
<td>1321.34</td>
<td>545.08 345.84</td>
</tr>
</tbody>
</table>

4.5. Conclusions

Improving order throughput and saving labor cost (increasing productivity) in order picking processes are the two major concerns for warehouse managers. This chapter discusses the concept of Dynamic Storage, which can improve order throughput and reduce labor cost simultaneously due to shorter travel distance in picking tours, with the aid of S/R machines. In a DSS, orders are released to the system in batches and only those products for the current picking batch are stored in the pick area. S/R machines reshuffle
products between batches. For a single-station order picking system, we derive a mathematical model to obtain the maximum throughput a DSS can achieve and, on top of that, find the optimal batch size to minimize order picking worker hours needed. The application of DS to multi-station order picking systems is discussed through two alternatives. For both order picking systems, the performance of a DSS is compared with conventional order picking where all the products are stored in the pick area. Through our mathematical and simulation models, we are able to demonstrate that a DSS can substantially improve throughput and reduce labor cost at the same time. Our results confirm and quantify the advantage of these systems over conventional picking systems and explain why so many companies are switching to dynamic storage to enhance performance.
5. REDUCING CONGESTION IN MATERIAL HANDLING SYSTEMS BY ZONING

5.1. Introduction

In previous chapters, we developed models to analyze the performance of pick-and-pass order picking systems. In this chapter, we use the case of Bloemenveiling Aalsmeer (VBA), a large flower auction company in the Netherlands, to illustrate the advantage of a pick-and-pass order picking system with zoning over an order picking system with a single zone. The analysis of this real case also considers congestion of pickers in aisles, which is not taken into consideration in previous chapters. Specific sources of congestion in a material handling system often include: 1) intersections, 2) vehicle breakdowns, 3) vehicle or worker stop at pick-up or drop-off locations, which may block the guide path and, 4) two or more vehicles or workers competing for the same aisle. In literature, some papers incorporate congestion issues into layout design problems (see for example, Chiang et al. 2002; and Smith and Li 2001). For an order picking system, Gue et al. (2006) and Parikh and Meller (2007) study the effect of pick density on narrow-aisle and wide-aisle pick area congestion.

The current internal distribution process at VBA resembles an order picking process with a single large zone and many order pickers in the zone, which leads to much congestion in the process. The problem is that it can not meet the customer order lead time requirements. By introducing a zoning concept and using AGVs for transportation between zones, the new distribution process is similar to a pick-and-pass order picking system. We show by simulation that congestion is reduced significantly in this zoned distribution process and
customer response times decrease. On top of that, we develop an algorithm to balance workload between zones to further improve the performance of the distribution process.

This chapter is organized as follows. In the next section, we describe the problems VBA is facing and explain our suggested solutions in brief. Then we elaborate the current distribution process at VBA. After that, we discuss the zone implementation at VBA followed by a description of customer-to-aisle assignments to balance workload between zones. Next, we describe our simulation model used to study the problems at VBA followed by explanation of simulation results and sensitivity analysis. We draw conclusions and discuss the implementation at VBA in the last section.

5.2. The problems at VBA and suggested solutions

VBA, the largest flower auction in the world (refer to http://en.wikipedia.org/wiki/Aalsmeer_Flower_Auction.), auctions about 19 million flowers per day between 6 am and 11 am, which equals 60,000 daily transactions on average (in 2005). Every night and early morning, flowers from the growers arrive at VBA in trolleys, to be auctioned the same morning. In the auction, customers can buy not only a whole trolley of flowers, but also parts of individual vases or boxes of flowers. After the auction process, a trolley can contain as many as 26 customer transactions with an average of 3.2. The auctioned flowers need to be distributed to customers’ Distribution Centers (DCs) situated elsewhere in the auction building within a few hours. After auctioning, trolleys are transported to one of the work buffers, where trolleys containing single and double transactions are separated from trolleys containing multiple transactions (3 or more). Trolleys with one or two transactions are towed directly to the customers’ DCs. Trolleys with multiple transactions need prior distribution of the transactions over so-called ‘customer trolleys’. As illustrated in Figure 5.1, the total distribution process consists of three parts. The first part refers to the distribution of trolleys with one or two transactions from a work buffer directly to customers’ DCs. The second part is the distribution of the trolleys with multiple transactions by tow-truck drivers from a work buffer to customer trolleys located in the distribution area. The third part of the distribution
Reducing Congestion in Material Handling Systems by Zoning

The process handles the transport of complete customer trolleys to the customers’ DCs, where flowers are bundled, labeled and packed before being shipped to retailers.

Due to the increasing daily number of transactions and the fixed space of the distribution area, the second part of the distribution process has become more and more congested, leading to exceeding the agreed customer lead time and makespan. When all the transactions ordered by a customer have arrived at the customer’s DC, the flowers are shipped to retailers around the globe. A delay at the second part of the distribution process will cause a customer to miss the fixed plane or truck departure time. VBA has realized that the second part of the distribution process is the bottleneck and seeks solutions to improve its performance. We focus on the second part of the distribution process in the rest of our discussion. VBA has strict service level agreements with customers on time fences: the 0.95-quantile transaction lead time (the time period, in which 95% of the total transactions have been delivered to customers’ trolleys since they are generated at the auction clocks) should be within 2 hours and the 0.98-quantile makespan (the time lapse from the start of the auction process until 98% of the total transactions have been distributed to customers’ trolleys) within about 7 hours. Unfortunately, with the increasing number of transactions, these targets can no longer be achieved with the current organization. So far, the growth has been accommodated by bringing in more workers. This solution is no longer sufficient. More workers lead to more congestion in the already crowded distribution area and, as a consequence, to more worker stress, and higher labor.

Figure 5.1: Illustration of the distribution process.
turnover rate, which leads to lower worker productivity reflected in increased transaction times in transferring flowers from the trolleys with auctioned flowers to customer trolleys. At the request of VBA, we investigated the possibility to improve the makespan and the transaction lead time by introducing zoning to the distribution process. Zoning is an accepted practice in order picking processes in warehouses. Well-known advantages of zoning include reducing congestion, shortening travel distance, and ease of administration and control (see section 1.1.2). In a zoned distribution process, the distribution area is divided into zones each containing a number of customer trolleys. Distributors are divided into groups and assigned to fixed zones to distribute the transactions to customer trolleys (i.e., travel to the customer trolley locations in the zone and move the flowers from the trolleys containing auctioned flowers to customer trolleys). Automated Guided Vehicles (AGVs) would then transport the trolleys containing auctioned flowers from the work buffer to zones and between zones. In section 1.3.2 and chapter 2, we have shown balancing workload between zones will shorten order throughput time in a warehouse. We therefore design new customer-to-aisle assignment methods in the distribution area to further improve the zoning performance. Within each zone, distributors can now work as teams responsible for organizing all the work in their zone. Teamwork is widely used in industrial environments. The advantages of teamwork include improving productivity through enhanced motivation and flexibility, improving quality, encouraging innovation, and increasing satisfaction by allowing individuals to contribute more effectively (Meyer 1994; Moses and Stahelski 1999; Schilder 1992).

In the next section, we describe the current distribution process at VBA. We will see this internal distribution process resembles an order picking process with multiple workers working in a single zone.

5.3. VBA’s current distribution process

This section describes the current distribution process at VBA. As illustrated in Figure 5.2, the distribution area consists of two parts: the customer area and the central transport aisle.
Reducing Congestion in Material Handling Systems by Zoning

The customer area consists of 37 customer aisles. VBA randomly assigns customers to these customer aisles. A customer aisle can contain as many as 18 customer trolleys. The central transport aisle, consisting of 2 one-directional lanes, connects these customer aisles. In the central transport aisle, distributors travel anti-clockwise. All trolleys with auctioned flowers in work buffers contain a document with transaction information, indicating at which position which transaction has to be distributed to customer trolleys. A distributor on an electric tow-truck (see Figure 5.3) picks up a trolley with auctioned flowers from a work buffer, travels to the customers’ trolleys and transfers the transactions to customers’ trolleys. Figure 5.4 shows a detailed layout of a customer aisle.

Figure 5.2: Layout of the distribution area in the current distribution process.
Chapter 5

Figure 5.3: Work buffer, customer aisles and flower distributors.

![Image of work buffer, customer aisles and flower distributors.]

Figure 5.4: Customer aisle layout.

The above distribution process is similar to a reversed order picking process. Instead of picking products to fill customer orders, in the distribution process, distributors obtain the distribution list and distribute transactions to the customer trolleys. The central transport aisle functions as a cross aisle in a warehouse; the customer aisles are similar to picking aisles in an order picking system; distributors work similarly as order pickers; the locations
Reducing Congestion in Material Handling Systems by Zoning

of customer trolleys are like product locations at a pick area; and the two work buffers resemble depot positions in a warehouse. The whole distribution process therefore can be regarded as an order distribution process with multiple order distributors working in a single order distribution zone.

Once a customer trolley is full, another worker exchanges it for an empty one, and drops the full trolley in a pick-up buffer at the rear end of the customer aisle (see Figure 5.2). From there, they are towed in a line to the customers’ DCs. When all the transactions on a trolley with auctioned flowers have been distributed, the distributor drops off the empty trolley near the last transaction position and picks up a new trolley with auctioned flowers from the nearest work buffer. Customer transactions occupying more than half of the trolley space are called restkoop. A restkoop customer is the last one to be visited and the distributor leaves the whole remaining trolley with auctioned flowers in the pick-up buffer at the rear end of this customer’s aisle instead of manually transferring the flowers to the customer’s trolley.

Distributors work individually in this current distribution process. They can travel through the entire distribution area. With more than 200 distributors, congestion occurs throughout the entire area, which leads VBA to exceed the required distribution time fences. Mainly two types of congestion can be distinguished for this distribution process. The first type of congestion refers to delays at intersection points between the central transport aisle and the customer aisles, which occurs when a tow-truck traveling in the central transport aisle is detained by a tow-truck intending to enter or leave a customer aisle to avoid collision and vice versa. The second type of congestion often happens in the customer aisles and refers to vehicles stopped at a drop-off location preventing other vehicles behind them to go through the aisle. In the next section, we will discuss the implementation of zoning at VBA to reduce congestion and therefore improve the system performance.
5.4. Zoning implementation at VBA

The implementation of zoning comprises a split of the distribution process (part 2 in Figure 5.1) into a transportation and a distribution sub-process. The first sub-process can be fully automated, by using AGVs to transport trolleys with auctioned flowers from the work buffers to the customer area, which is now divided in zones. The AGVs are also responsible for transporting trolleys with auctioned flowers between the customer zones. Each customer zone contains a fixed number of customer aisles. Distributors work in self-managed teams in a fixed zone to finish the rest of the distribution process.

Because of physical layout restrictions, a zone consists of 2 or 3 adjacent customer aisles, leading to 15 zones in total as illustrated in Figure 5.5: 7 zones of 3 aisles and 8 zones of 2 aisles. Each aisle can accommodate 18 customers. Therefore, each large zone with 3 aisles can hold 54 customers and each small zone with 2 aisles can hold 36 customers, with $18 \times 37 = 666$ customers in total. The number of distributors assigned to a zone depends on the number of transactions in the zone (i.e., on the allocation of customers to the zone). On top of this, some flexible workers can be added, depending on the total number of transactions to be handled. These flexible workers can travel around the whole distribution area and help distributors only when the traffic at their zone tends to be blocked. Distributors stay in their own zone; AGVs are responsible to pick up trolleys at the work buffers and drop them in the inbound buffers at the zones, or retrieve them from the zones’ outbound buffers to bring them to the next zone.
AGVs travel anti-clockwise in the inner high-speed lane (see Figure 5.5). To avoid congestion, when an AGV reaches its next destination (a zone), it moves to an outer lane and drops the trolley with auctioned flowers at the inbound buffer. There is some room for AGVs to queue here if necessary, without blocking the inner lane. After dropping off the trolley at the inbound buffer of a zone, the AGV first checks the zone’s outbound buffer for work. If no trolley is to be picked up, it moves to the inner lane and continues travel anti-clockwise until it encounters the closest trolley unallocated for transport either at a zone’s outbound buffer or at a work buffer. Within a zone, a distributor picks up the trolley with auctioned flowers from the inbound buffer according to a First-Come-First-Served rule (we investigate scenarios with or without a tow-truck). The distribution process inside an aisle is similar to the current distribution process. If multiple transactions on a trolley need to be handled in a zone, the distributor traverses the aisles in an S-shape curve. After distribution of the last transaction, the distributor drops a non-empty trolley in the zone’s outbound buffer and returns to the inbound buffer for the next trolley.

The above introduced zoned distribution process is similar to a reversed pick-and-pass order picking system, where customer zones are connected by an AGV system. Distributors are confined to their assigned zones to distribute transactions to customers. AGVs are used to transport trolleys from one zone to another.
Chapter 5

Through the implementation of zoning, vehicle congestion in the distribution process is reduced substantially as the AGVs move in an inner lane for transport and in an outer lane for moving to inbound or outbound buffers. All these buffer locations have bypass possibilities and some waiting positions for AGVs. Compared to the current distribution process, congestion also reduces in the customer aisles, as the maximum number of distributors that is allowed in an aisle is smaller than the number of distributors that could enter the aisle in the current situation. Additionally, as distributors are working in teams in a fixed zone during the whole day, they are more adapted to specific customer demands and therefore may reduce the transaction time. Hence, we expect zoning to improve VBA’s distribution performance.

5.5. Customer-to-aisle assignments

Currently, VBA randomly assigns customers to aisles, such that all aisles contain, as much as possible, the same number of customers. Random assignment (RAN) has a drawback that the number of AGV visits to zones varies considerably among zones, which leads to AGV congestion in the outer and possibly the inner lane, and therefore has negative impacts on distribution performance. Several ways exist to minimize imbalance between zones. We present two variants, denoted by BAL and BALMIN(x). BAL attempts to balance the number of transactions between zones, whereas BALMIN(x) attempts to minimize the total number of AGV visits to zones, while keeping the imbalance of transactions between zones below a threshold level x. Both models are formulated as 0-1 linear programming problems with a large number of binary decision variables. Appendix E shows that the problem is NP-hard. We therefore use heuristics to solve them. The models and their solution procedures are presented in Appendix D. For both models, we randomly assign customers allocated to a zone to an aisle in the zone and a position in the aisle.

In reality, we only know the exact number of transactions of each customer when the auction process has finished. However, all customers are regular, which means the number
Reducing Congestion in Material Handling Systems by Zoning

of transactions for each customer is relatively stable over a certain period. Based on historical data, we can therefore assign customers to zones (model BAL) with a roughly balanced workload. The detailed number of transactions on each trolley is impossible to obtain until after the auction process. Therefore, the BALMIN(x) model, which needs these detailed transaction data, primarily serves as a reference. If this model were to be used, we might assign customers based on trolleys that have already passed the auction process but have not yet arrived at the buffers and a combined forecast for remaining customers.

5.6. Simulation experiments

As vehicles and worker congestion is very important in simulating our model, we choose Automod® 10.0 to evaluate the effect of zoning on system performance. This software is particularly accurate in simulating real vehicle dimensions, movements, and congestion behaviors. We measure system performance by the 0.98-quantile makespan and the 0.95-quantile transaction lead time.

5.6.1. Simulation data

All data used for the simulation are based on measurements of the auction and the distribution process on a number of busy days in June 2000, which were selected by VBA to be representative of the future average daily throughput of 2005.

From these data we can obtain the probability that a trolley contains multiple transactions after the auction process, the probability of a trolley with multiple transactions to enter either of the two work buffers and the probability that a transaction consists of boxes. It is necessary to make a distinction between box and vase transactions, as the handling time for boxes is significantly longer. These three probabilities are measured over every quarter (15 minutes) during the auction process. We also obtain the distribution of the number of transactions on a trolley, the transaction order frequency, which is defined as the
probability that a transaction on a trolley belongs to a specific customer, and the probability that a trolley contains a restkoop. This probability depends on the number of transactions on the trolley and is hard to fit with any theoretical distribution. We therefore use the empirical distribution in the simulation model.

We have measured the transaction times of moving flowers to customer trolleys over different distributors. The resulting data have been analyzed and are hard to fit with any theoretical distribution. Therefore we use the empirical distribution for transaction time in the simulation model. Speeds of the tow-trucks have also been measured in a large number of measurements. Data for AGVs speeds are based on supplier information. We find they can travel at the same speeds as the current tow-trucks in the central aisles. Table 5.1 summarizes the parameters used in the model. We keep all the parameters identical for each scenario in the simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average auction rate</td>
<td>1 trolley per second</td>
</tr>
<tr>
<td>Total number of trolleys passing the auction process per day</td>
<td>20,700</td>
</tr>
<tr>
<td>Average daily number of trolleys with multiple transactions</td>
<td>7,500</td>
</tr>
<tr>
<td>Average percentage of trolleys containing a restkoop</td>
<td>32%, empirical distribution</td>
</tr>
<tr>
<td>Transaction time (seconds) for vases</td>
<td>$\mu=34.2$, $\sigma=8.0$, empirical distribution</td>
</tr>
<tr>
<td>Transaction time (seconds) for boxes</td>
<td>$\mu=55.5$, $\sigma=28.4$, empirical distribution</td>
</tr>
<tr>
<td>Total number of customers</td>
<td>666</td>
</tr>
<tr>
<td>Number of distributors</td>
<td>210</td>
</tr>
<tr>
<td>Number of customer aisles</td>
<td>37</td>
</tr>
<tr>
<td>Number of AGVs</td>
<td>110</td>
</tr>
<tr>
<td>Travel speed and acceleration/deceleration of electric tow-trucks and AGVs in the central transport aisles.</td>
<td>3.3 m/s, 1.7 m/s²</td>
</tr>
<tr>
<td>Travel speed and acceleration/deceleration of electric tow-trucks in customer aisles.</td>
<td>1.5 m/s, 1.5 m/s²</td>
</tr>
<tr>
<td>Travel speed and acceleration/deceleration without electric tow-trucks in customer aisles</td>
<td>1.0 m/s, 5 m/s²</td>
</tr>
<tr>
<td>Time for dropping off or hooking on a trolley by an AGV</td>
<td>5 seconds</td>
</tr>
<tr>
<td>Buffer sizes at the zones (for the zoned process)</td>
<td>5 trolleys</td>
</tr>
</tbody>
</table>
5.6.2. Model description

We distinguish two distribution processes: the current distribution process (CUR) and the zoned distribution process (ZON), and three customer-to-aisle assignment methods: RAN, BAL, and BALMIN(x) as discussed in the previous section. Balancing the number of transactions between aisles rather than zones can also be implemented in the current situation. Its formulation and solution approach are similar to the balancing problems for zones (Appendix D), albeit with a larger number of variables and constraints. The resulting imbalance of workload is below 5% of the average number of transactions in each aisle, which is sufficiently accurate for the purpose of the evaluation. Depending on the combinations of the distribution process and the assignment method, we evaluate 5 scenarios in the experiments: CUR-RAN, CUR-BAL, ZON-RAN, ZON-BAL and ZON-BALMIN. We first randomly generate all the trolleys and then determine the trolleys containing multiple transactions according to our measurements of the probability. For each trolley with multiple transactions we generate the number of transactions on it and the buyer for each transaction. With this information, we create the RAN, BAL and BALMIN customer-to-aisle assignments using MATLAB® 7.0 for general calculation and Lingo 8.0 for solving the 0-1 integer programming models. The results are then used as inputs for the simulation model. Figure 5.6 illustrates the relation between the simulation data, customer-to-aisle assignment methods, and the simulation models. We use 10 independent runs for each scenario while keeping the number of trolleys identical between scenarios.
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Figure 5.6: Illustration of the solution procedure.

The specifications for the simulation model are:

- Vehicles (electric tow-trucks and AGVs) operate continuously without any breakdown.
- Vehicles choose the shortest path to pick up and deliver loads.
- Trolleys with auctioned flowers at work buffers are processed on a first come first served basis.
- There is sufficient waiting space at work buffers for trolleys.
- The lane width of all the aisles only allows for one trolley to pass by at the same time.
- Distributors use electric tow-trucks in customer aisles. We analyze the scenario without tow-trucks in a sensitivity analysis.

The number of AGVs and the buffer size at the zones used in the model are chosen to ensure that the congestion (i.e., the percentage of time that vehicles are blocked or slowed
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down by a preceding vehicle) in the central transport aisle is below 10%. We evaluate the impact of the number of AGVs and buffer size on system performance in the next section.

5.6.3. Verification and validation

All the input data and the parameters used for the simulation model are based on the real distribution process at VBA and have been checked by the distribution process manager. As we simulate a large number of vehicles (AGVs and tow-trucks, 320 vehicles in total in the ZON models) care had to be taken to properly model the congestion behavior, taking into account the physical vehicle dimensions, speeds, acceleration/deceleration, yield behavior at intersections, moving behavior in curves, and collision prevention. The 3-dimensional animated simulation model has also been validated by VBA. To further validate our model, we compare the statistics of the 0.98-quantile makespan and the 0.95-quantile transaction lead time for scenario CUR-RAN to the real system performance in June 2000. The relative errors for both measurements are below 3%. Therefore we can use the simulation model to represent the real distribution process.

5.7. Results and sensitivity analysis

The simulation results in Table 5.2 confirm our expectation that zoning leads to less congestion, which in turn contributes to the large improvements on system performance. Comparing the two scenarios ZON-BAL and ZON-RAN, paired t-tests (with $p=0.0001$) show that the BAL assignment leads to better system performance in zoning scenarios. Paired-t tests show no significant difference ($p=0.1$) in system performance between the two current scenarios, CUR-RAN and CUR-BAL. Scenario ZON-BALMIN (with a threshold of 0.1) has only slightly better performance than scenario ZON-BAL ($p=0.001$ for makespan, $p=0.08$ for lead time). As mentioned earlier, ZON-BALMIN has been included as a reference. Therefore, we will further analyze the ZON-BAL scenario in the
Chapter 5

following parts of this section. The further analyses include the impact of transaction time on system performance, savings on the number of distributors in the zoning distribution process, the effect of distribution speed in customer aisles on system performance, the impact of buffer size and the impact of the number of AGVs on system performance. In every experiment, only one parameter is varied, the others remain unchanged.

Table 5.2: 95% Confidence interval for makespan, transaction lead time and congestion in each scenario

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>CUR-RAN</th>
<th>CUR-BAL</th>
<th>ZON-RAN</th>
<th>ZON-BAL</th>
<th>ZON-BALMIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.98-quantile makespan (hrs)</td>
<td>7.61±0.05</td>
<td>7.57±0.03</td>
<td>6.68±0.06</td>
<td>6.42±0.03</td>
<td>6.35±0.03</td>
</tr>
<tr>
<td>0.95-quantile transaction lead time (hrs)</td>
<td>2.86±0.05</td>
<td>2.83±0.03</td>
<td>2.13±0.13</td>
<td>1.73±0.06</td>
<td>1.69±0.05</td>
</tr>
<tr>
<td>Congestion in central aisle</td>
<td>9.1%±0.55%</td>
<td>7.3%±0.18%</td>
<td>7.4%±0.20%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Congestion in customer aisles</td>
<td>29.9%±0.51%</td>
<td>29.3%±0.45%</td>
<td>6.6%±0.23%</td>
<td>7.3%±0.20%</td>
<td>7.4%±0.17%</td>
</tr>
</tbody>
</table>

5.7.1. Impact of transaction time on system performance

As discussed before, the benefits of teamwork may be reflected in reduction in transaction times. We tested the impact of different percentages of transaction time reductions on system performance in scenario ZON-BAL. It shows that although reducing transaction time can improve system performance, the improvement is slight (around 2% for the 0.98-quantile makespan and 7% for the 0.95-quantile transaction lead time with a transaction time reduction of 10%). Further reduction on transaction time has only marginal benefits on system performance. This is due to the fact that transaction time (refer to Table 5.1) only accounts for a small part of the total transaction lead time. The majority of transaction lead time consists of waiting time in various buffers and the travel time in the aisles.
5.7.2. Savings on distributors

As distributors work in a specific zone in ZON scenarios, their travel distances become shorter, and congestion reduces. The ZON scenarios can therefore achieve a performance equal to the current situation, but with fewer distributors. We also investigate the trade-off between the number of distributors and system performance. The results indicate that scenario ZON-BAL with 150 distributors has approximately an equal performance with scenario CUR-RAN of 210 distributors. The saving on distributors is 28.5% in this case.

5.7.3. Impact of distributor speeds on system performance

As an alternative to using electric tow-trucks to drive trolleys in the customer aisles, we analyze the situation of pushing the trolleys manually in scenario ZON-BAL. This allows the sale of all current tow-trucks. The resulting travel speed will decrease to walk speed of about 1 m/s (the acceleration and deceleration is set to 5 m/s²). Simulation results show that compared with scenario CUR-RAN, we still have 13% improvement on the 0.98-quantile makespan and 33% improvement on the 0.95-quantile transaction lead time.

5.7.4. Impact of buffer sizes on system performance

The size of inbound buffers at the zones impacts the amount of AGV congestion, and the size of outbound buffers impacts the amount of tow-truck congestion in the customer aisles. Although larger buffers will reduce congestion, they occupy more space and therefore reduce the number of customers that can be accommodated. We therefore seek for the smallest possible buffer size yielding acceptable performances and vary the buffer sizes from 1 to 9 with step-size of 2 in scenario ZON-BAL. The results in Table 5.3 indicate that when the buffer size is larger than 7, improvements rapidly become negligible.
5.7.5. Impact of the number of AGVs on system performance

Fewer AGVs cause longer trolley waiting time in outbound buffers leading to longer lead times. Obviously, increasing the number of AGVs improves system performance. However, sensitivity analysis shows that the improvement becomes marginal when the number of AGVs is larger than 130. As the number of AGVs grows beyond 130, congestion increases, which has negative impact on lead times.

5.8. Conclusions and implementations at VBA

In this chapter, we analyze the distribution process at VBA and find solutions for it to reduce congestion and improve its distribution performance. At the request of VBA, we investigated the potential benefits of zoning on the distribution process performance. The resulting zoned distribution process resembles a reversed pick-and-pass order picking process as we discussed in the previous chapters. On top of that, we also investigate new customer-to-aisle assignment methods with the main objective to balance workload among zones. We use simulations to demonstrate that under the balanced workload scenario, the zoned distribution process can reduce makespan by 16 percent and transaction lead time by 40 percent compared with the current distribution process. Even with a random assignment of customers, the improvements by introducing zoning are 12 and 26 percent respectively. Sensitivity analysis shows that with a system performance equal to the current process, zoning with balanced workload among zones can save about 60 distributors (28.5% of the
Reducing Congestion in Material Handling Systems by Zoning

distributors used currently). We expect a reduction of transaction times by letting distributors work in self-organized teams within each zone. Although simulation results show that it has limited contribution in improving makespan and transaction lead time, teamwork adds to more social contacts between distributors. This will ease their stress, increase job satisfaction, and reduce labor turnover rate at VBA.

The results became clear by mid 2002. However, after investigating the automated transportation process further, it appeared that the investment on AGVs was larger than expected. The simulation model indicated that the long lead time was due to congestion and waiting effect in the distribution process. This inspired us to find another solution in which the top 10% trolleys that visit the largest number of customers are removed and handled in a separate and parallel system. We also evaluate this scenario in our simulation model. The results show that makespan and transaction lead time reduce by 11 percent and 27 percent respectively. Although this reduction is smaller than with the zoning concept and requires a few more workers, no large investments are required (space investments were not taken into account). For the short term this solution is sufficient. However when the daily number of transaction keeps on growing, zoning and teamwork combined with automation may become an attractive alternative.
6. CONCLUSIONS AND FUTURE RESEARCH

Order picking is one of the most important warehousing operations as it consumes a large amount of the total labor activities and accounts for a huge percentage of the total operational cost in a warehouse. In order to make the picking process efficient, the order picking system needs to be well designed and controlled. This thesis aims to provide models and analyses to support the design and the control of efficient order picking systems with focus on pick-and-pass systems, which have many advantages and are therefore widely used in warehouses (refer to section 1.2). The thesis includes the design of an approximation model for performance estimation of pick-and-pass order picking systems, an analysis on Dynamic Storage Systems, and a case study of Bloemenveiling Aalsmeer (VBA), a large flower auction company in the Netherlands, to illustrate the advantage of pick-and-pass systems over order picking systems with a single zone.

6.1. An approximation model for performance estimation and design of pick-and-pass order picking systems

Many factors influence the performance of a pick-and-pass order picking system. From a design point of view, the storage methods in each pick station, the size of a pick station, the number of stations, the number of order pickers per station, and the customer order profiles are of special importance. From an operational point of view, it is important to decide how to batch (or split) orders to minimize mean order throughput time in the order picking system. In this thesis, a tool is developed for fast modeling and analysis of possible alternatives for the design phase of a pick-and-pass system and to analyze the impact of order batching and order splitting on the system performance.
Chapter 6

In chapter 2, we develop an approximation-based method to analyze a pick-and-pass order picking system. The model relaxes the Jackson queuing network modeling of De Koster (1994) by allowing a general order arrival process and general service time distributions, which represent real-life warehouses more accurately and provide a deeper understanding of the pick-and-pass order picking system. The modeling and analysis of the system is based on the analysis of a $G/G/m$ queuing network by Whitt (1983). The pick-and-pass system is decomposed into conveyor pieces and pick stations, each of which is analyzed in isolation as a single $G/G/m$ queue. To determine the mean throughput time of an arbitrary order, the mean and the second moment of service time of an order bin at a station is developed. Although in chapter 2 we assume a line layout of the pick area, a class-based storage policy, and a one-order-line per picking trip routing policy, it is possible to extend the model into other layouts, storage and routing policies. We show the approximation method leads to acceptable results by comparing it with both simulation and with the real order picking process at a parts distribution center of an international motor production company.

Order batching and zoning of the pick area are two important factors that influence the order picking efficiency. Using the approximation method developed in chapter 2, chapter 3 studies the impact of batch size, zone size, and the number of pickers per zone on the pick-and-pass order picking system performance. In this chapter, we consider a different aisle layout in the pick stations and an S-shape routing policy, which are illustrated in Figure 3.1. We also take the impact of the sorting process at the end of the picking system into consideration. We find the average mean order throughput time is a convex function of batch size under different zone settings. This result is consistent with the findings of Chew and Tang (1999) who consider a single-block warehouse and of Le-Duc and De Koster (2007) who consider a two-block warehouse with a single pick zone. We find the mean order throughput time in the system is quite robust for a varying number of zones around the optimum number of zones. We also find, for given order arrival rates, the precise shape of the order arrival distribution has only a slight impact on mean order throughput time. This is especially true when the utilizations at zones are small. In general, the preference of one setting of batch sizes, zone sizes and number of pickers per zone
Conclusions and Future Research

over another is subject to a detailed specification of the pick-and-pass system. The approximation model provides a fast and accurate tool to evaluate these alternatives.

6.2. Dynamic Storage Systems

In a conventional order picking system, all the SKUs ordered by customers during the entire day or a picking shift (which normally takes several hours) are stored in the pick area. This requires much storage space. Order pickers spend much time on traveling to pick locations, leading to low throughput and low productivity. Chapter 4 introduces the concept of Dynamic Storage (DS). In a Dynamic Storage System (DSS), orders are released to the system in batches and only those products for the current picking batch are stored in the pick area just in time. S/R machines reshuffle products between batches. Because only a small fraction of the total SKUs is stored in the pick area, a DSS can reduce order pickers’ travel significantly in their picking tours, leading to increased throughput of the order picking system and reduced total number of worker hours needed for picking. Since the reshuffle process is completely automated, order pickers can be assigned to do other warehousing activities during the reshuffle time between two batches if the reshuffle time is sufficiently long, which can be achieved by choosing an appropriate batch size. Additionally, since the reshuffle work is done by S/R machines, a DSS can eliminate potential worker congestion in the picking aisles caused by manual replenishment.

In chapter 4, we derive a mathematical model to obtain the maximum throughput a DSS can achieve and, on top of it, to find the optimal batch size to minimize the number of order picking worker hours for a single-station order picking system. Then we apply the concept of DS to a pick-and-pass order picking system with multiple pick stations. Through our mathematical and simulation models, we are able to demonstrate that a DSS can improve throughput and reduce labor time significantly at the same time. Our results confirm and quantify the advantage of these systems over conventional picking systems.
and explain why so many companies are switching to dynamic storage to enhance performance.

6.3. Reducing material handling system congestion by zoning

Congestion in travel aisles prolongs the time on traveling, leading to longer order lead time, and hence has large impact on the performance of a material handling system. In chapter 5, we use a case study at a flower auction company in the Netherlands, to show zoning and balancing workload between zones as we discussed in the previous chapters can reduce congestion and improve order lead time.

As illustrated in Figure 5.2, the internal distribution process at flower auction VBA resembles a reversed order picking process (distribution instead of picking) with multiple distribution aisles, a cross aisle, and multiple distributors in a single distribution zone. Due to the presence of multiple distributors in the distribution area at the same time, congestion is a big problem in its distribution process leading to prolonged order transaction lead time and makespan. To solve this problem, we introduce zoning to divide the distribution area into separate work zones. A zone consists of several distribution aisles. Distributors are divided accordingly and are assigned to their own zones during the entire day. AGVs are introduced to transport distribution trolleys between zones. The resulted new distribution process is similar to a reversed pick-and-pass order picking system. Through simulation models, we conclude that the introduced new distribution process reduces congestion and therefore can improve order lead time and makespan significantly. In order to balance the workload between work zones, we develop an integer programming model and design a heuristics to solve it. Simulation shows balancing workload between work zones can further improve the performance of the distribution process.
6.4. Future research

The research carried out in this thesis has some limitations. These limitations shed lights on future research directions.

In chapter 2 and 3, we develop an approximation model to analyze the impact of various warehousing operational policies on the mean order throughput time. We also estimated the standard deviation of order throughput time using the method described in Whitt (1983). However, the method did not provide good estimation results. It would also be interesting to find a more accurate approach to estimate the standard deviation of order throughput time, which together with mean order throughput time provides a better description of the order picking system performance. Another interesting extension is to consider the situation that an order picker is responsible for picking at multiple pick stations. Furthermore, in reality, the buffer capacity in front of each pick station is finite, which influences performance in high-utilization situations. It might be possible to derive estimates for the mean throughput time using approximation methods for finite-buffer queuing networks.

In chapter 4, we performed a worst case analysis by using the average cycle time of a swap process as the reshuffle time for a product and treat it as a constant. It would make the approximation more accurate if we can find the distribution of the reshuffle time and input the mean value into the model for analysis. Also in chapter 4, we assume that reshuffling and picking are in sequence at a pick station. This assumption is strong since in reality, picking can start in parallel with the reshuffling process for those orders, whose required products have already been reshuffled into the pick area. The maximum throughput that can be achieved by a DSS in our analysis in chapter 4 is therefore underestimated. It would be interesting to know how much improvement can be achieved by relaxing this assumption.

We analyze the impact of congestion in a material handling system by means of simulation in chapter 5. It would be interesting to quantify this impact and incorporate the congestion effect into travel time models as we developed in chapter 2 and 3. The integration will lead to a better estimation of the service time at stations and the mean order throughput time in
the pick-and-pass system. Although our heuristics developed to solve the workload balancing issue in chapter 5 is accurate enough for the VBA case, it could be more convincing to find an efficient algorithm to solve the problem to optimality.

Several issues on order picking, which have not been treated in this thesis and in literature might be interesting for future research.

First, compared to synchronized zoning and none-zoned order picking systems, pick-and-pass order picking systems also have disadvantages, such as difficulties in separating big and small, urgent and normal orders, and relatively longer order throughput time when the system is heavily loaded due to longer waiting time and circulations on the conveyors. It would be instructive to analytically compare the performance of a pick-and-pass system, a synchronized zoning system, and a none-zoned system and determine when to use which system.

Second, a warehouse may have different storage and order picking systems, for example, storage areas for pallets and cases, and manual and automated picking areas. In such a hybrid warehouse, interesting research questions include: to determine the size for each storage and picking area; which products should be stored in which area with what quantity; the rules for allocating order pickers in different picking areas; The objectives can be maximizing order throughput or worker productivities, minimizing order throughput time or labor costs.

Third, much research on order picking considers only one specific objective, like minimizing order throughput. However, in practice, warehouse managers need to consider other objectives at the same time, such as, tardiness and total picking cost. The resulting solution for the design and operational policies of an order picking system should be a global “optimal” outcome. Literature on the design of order picking systems with multiple objectives is still scarce.

Finally, the order picking and sorting process are closely related to truck dispatching problems since orders for a specific destination need to be picked, sorted, packed, and loaded before the truck departure time. Integrating order picking and truck dispatching
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problems into one model and consider them as a whole to see the impact of order picking on customer response time would be a challenging research problem.
7. REFERENCES


Chapter 7


Chapter 7


Chapter 7


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Chapter 7


APPENDIX A  QUEUING NETWORK MODELS

According to Whitt (1983), to estimate the mean order throughput time in this $G/G/m$ queuing network system, we need to calculate the internal flow parameters. The internal flow rate to each node $\lambda_j$ is obtained by solving the following linear equations

$$\lambda_j = \lambda_0 + \sum_{i=1}^{J} \lambda_i q_{ij}, \quad \forall 1 \leq j \leq J$$

(A.1)

where $\lambda_0$ is the external arrival rate to node $j$, $J$ is the total number of nodes (conveyor pieces and pick stations) in the system, and $q_{ij}$ is the transition probability from node $i$ to node $j$.

The arrival rate to node $j$ from node $i$ is given by

$$\lambda_j = \lambda_i q_{ij}, \quad \forall 1 \leq i \leq J, \forall 1 \leq j \leq J$$

(A.2)

The proportion of arrivals to $j$ that come from $i$, is calculated by

$$pr_{ij} = \frac{\lambda_i}{\lambda_j}, \quad \forall 0 \leq i \leq J, \forall 1 \leq j \leq J$$

(A.3)

The variability parameters for the internal flows, i.e., the SCV for the arrival process to node $j$, are calculated by solving the following linear equations

$$c_{ij}^2 = a_j + \sum_{i=1}^{J} c_{ii}^2 b_{ij}, \quad \forall 1 \leq j \leq J$$

(A.4)

where

$$a_j = 1 + \omega_j \left\{ (pr_{ij} c_{ij}^2 - 1) + \sum_{i=1}^{J} pr_{ij} [ (1 - q_{ij}) + q_{ij} \rho_j^2 x_i ] \right\}$$

(A.5)

$$b_{ij} = \omega_j pr_{ij} q_{ij} (1 - \rho_j^2)$$

(A.6)
Appendix A

c_{_j}^2 is the SCV of the external inter-arrival time to node $j$, and $c_{_j}^2 = 0$ for $\forall j > 1$ since the order bins enter the system from the first conveyor piece.

$\rho_i$ is the utilization of node $i$ obtained from equation (2.26), and

$$x_i = 1 + m_i^{0.5} (\max\{c_{_i}^2, 0.2\} - 1) \tag{A.7}$$

with $m_i$ the number of servers at node $i$, and $c_{_i}^2$ the SCV of service time at node $i$ obtained from equation (2.20) and (2.21).

$$\omega_j = [1 + 4(1 - \rho_j)^2(\nu_j - 1)]^{-1} \tag{A.8}$$

with $\nu_j = \sum_{i=0}^{J} p_{ij}^2^{-1}$.

With the internal flow parameters, $\lambda_j$ and $c_{_j}^2$, and the service time parameters, $E[\tau_j]$, and $c_{_j}^2$, Whitt (1983) decomposes the network into separate service facilities that are analyzed in isolation. Each service facility is a $G/G/m$ queue. Whitt (1993) provides the following approximation for the expected waiting time in queues. Since we are focusing on a single node, we omit the subscript indexing the node in deriving the expected waiting time in front of a node.

For a multi-server node with $m$ servers, the expected waiting time is given by

$$E[W]_{G/G/m} = \phi(\rho, c_{_u}^2, c_{_j}^2, m) \left( \frac{c_{_u}^2 + c_{_j}^2}{2} \right) E[W]_{M/M/m} \tag{A.9}$$

where $c_{_u}^2$ and $c_{_j}^2$ are obtained from equation (A.4), equation (2.20) and (2.21) respectively, $\rho$ is given by equation (2.26), $E[W]_{M/M/m}$ is the waiting time in queue of a multi-server node with Poisson arrivals and exponential service distribution. The exact expression for $E[W]_{M/M/m}$ is given by

$$E[W]_{M/M/m} = \frac{P(N \geq m)}{\mu m(1 - \rho)} \tag{A.10}$$
where $\mu$ is the reciprocal of mean service time at each node.

$P(N \geq m)$ is the probability that all servers are busy and is given by

$$P(N \geq m) = \left(\frac{(m\rho)^m}{m!(1-\rho)}\right)^{\zeta} \tag{A.11}$$

with

$$\zeta = \left(\frac{(m\rho)^m}{m!(1-\rho)} + \sum_{k=0}^{m-1} \frac{(m\rho)^k}{k!}\right)^{-1}$$

The expression for $\phi$ in equation (A.9) is given by

$$\phi(\rho, c_1^2, c_2^2, m) = \begin{cases} 
\phi_1(m, \rho) + \frac{c_1^2}{4c_2^2 + 3c_1^2} \phi_3(m, \rho) + \frac{c_2^2}{4c_2^2 - 3c_1^2} \psi(c_1^2, m, \rho), & c_2^2 \geq c_1^2 \\
\phi_1(m, \rho) + \frac{c_2^2}{2(c_1^2 + c_2^2)} \phi_3(m, \rho) + \frac{c_1^2}{2(c_1^2 + c_2^2)} \psi(c_2^2, m, \rho), & c_2^2 \leq c_1^2 
\end{cases} \tag{A.12}$$

with

$$\phi_1(m, \rho) = 1 + \gamma(m, \rho) \tag{A.13}$$

$$\phi_3(m, \rho) = \frac{(1 - 4\gamma(m, \rho)) e^{-2(1-\rho)\rho}}{3\rho} \tag{A.14}$$

$$\gamma(m, \rho) = \min \{ 0, 24, \frac{(1-\rho)(m-1)((4+5m)^{0.5} - 2)}{16m\rho} \} \tag{A.15}$$

and

$$\psi(c^2, m, \rho) = \begin{cases} 
1, & c^2 > 1 \\
\phi_1(m, \rho)^{(1-c^2)}, & 0 \leq c^2 \leq 1
\end{cases} \tag{A.16}$$

with $c^2 = \frac{c_1^2 + c_2^2}{2}$, and $\phi_1(m, \rho) = \min \{ 1, \frac{\phi_3(m, \rho) + \phi_3(m, \rho)}{2} \}$. 
Based on the work of Whitt (1983), we convert the two input flows into one input flow. The external arrival rate to the system is given by

$$\lambda_0 = \lambda_{01} + \lambda_{02}$$  \hspace{1cm} (B.1)

where $\lambda_0$ is the combined external arrival rate to the system, $\lambda_{01}$ and $\lambda_{02}$ are the two separate external arrival rates to the system. The internal traffic rate to node $j$ is given by

$$\lambda_j = \lambda_{j1} + \lambda_{j2}, \quad \forall 1 \leq j \leq J$$  \hspace{1cm} (B.2)

where $\lambda_{j1}$ and $\lambda_{j2}$ are the internal traffic rates to node $j$ for each input flow solved from linear equations of (A.1).

The mean service time at pick station $j$ is the weighted combination of the service times for two separate input flows

$$E[se_j] = \frac{\lambda_{j1} E[se_{j1}] + \lambda_{j2} E[se_{j2}]}{\lambda_{j1} + \lambda_{j2}}, \quad \forall j > C$$  \hspace{1cm} (B.3)

where $E[se_{j1}]$ and $E[se_{j2}]$ are the mean service time for each separate input flow derived from equation (2.1).

The second moment of service time at pick station $j$ is derived by

$$E[se_j^2] = \frac{\lambda_{j1} E[se_{j1}^2] + \lambda_{j2} E[se_{j2}^2]}{\lambda_{j1} + \lambda_{j2}}, \quad \forall j > C$$  \hspace{1cm} (B.4)

where $E[se_{j1}^2]$ and $E[se_{j2}^2]$ are the second moments of service time at pick station $j$ for each input flow given by equation (2.14).
Appendix B

The SCV of service time at pick station \( j \), \( \sigma^2_{s_j} \), can then be calculated from equation (2.20), (B.3), and (B.4). Because the service time is constant at conveyor pieces, the SCV and the mean of service time are obtained from equation (2.21) and (2.22).

The SCV of inter-arrival time to each node, \( \sigma^2_{a_j} \), is again obtained from equation (A.4).

The required parameters are calculated as follows:

The transition probabilities from node \( i \) to node \( j \) are calculated as

\[
q_{ij} = \begin{cases} 
\frac{\lambda_j}{\lambda_i}, & \forall 1 \leq i \leq C, \ j = i+1, \ and \ j = i+C \\
1, & \forall C+1 \leq i \leq J, \ j = i-C 
\end{cases}
\]

(B.5)

\( \lambda_j \), the arrival rate from node \( i \) to node \( j \) is given by

\[
\lambda_j = \lambda_{ij1} + \lambda_{ij2}, \quad \forall 1 \leq i \leq J, \ \forall 1 \leq j \leq J
\]

(B.6)

where \( \lambda_{ij1} \) and \( \lambda_{ij2} \) are the arrival rates from node \( i \) to node \( j \) for each separate input flow derived from equation (A.2).

The utilizations \( \rho_j \) at each node \( j \), are calculated from equation (2.26).

\( pr_j \), the proportion of arrivals to \( j \) that come from \( i \), \( i \geq 0 \) is obtained by equation (A.3).

The SCV for the inter-arrival time of orders to the system is given by

\[
\sigma^2_{ai} = (1 - \omega_h) + \omega_h \left[ \sigma^2_{011} \left( \frac{\lambda_{011}}{\lambda_{011} + \lambda_{012}} \right) + \sigma^2_{012} \left( \frac{\lambda_{012}}{\lambda_{011} + \lambda_{012}} \right) \right]
\]

(B.7)

where \( \omega_h = [1 + 4(1 - \rho_j)^2 (v_{j1} - 1)]^{-1} \) with \( \rho_j = \lambda_{01j} E[x_{j1}] / m_j \) and

\[
v_{j1} = \left[ \left( \frac{\lambda_{011}}{\lambda_{011} + \lambda_{012}} \right)^2 + \left( \frac{\lambda_{012}}{\lambda_{011} + \lambda_{012}} \right)^2 \right]^{-1}
\]
Appendix B

c_{011}^2 \text{ and } c_{012}^2 \text{ are the SCV for the inter-arrival time of orders to the system of each separate input flow.}

At this point, we have converted the two input flows into one. We can apply the procedures in Appendix A to calculate the expected waiting time in front of each node and subsequently use equation (2.27) to obtain the expected sojourn time of a bin at a node.
APPENDIX C PROOF OF CHAPTER 4

In this appendix, we compare the expected value of \( CD \), the number of products needing to be condensed in case 3, and the expected value of \( \sum_{m=1}^{M} X_{i+1,m} (1 - X_{i,m}) \), the number of products needing to be swapped, to show that the impact of \( CD \) is negligible.

\( CD \) is a discrete random variable expressed in distribution as,

\[
CD = \left( \sum_{m=1}^{M} X_{i,m} * X_{i+1,m} \right) * \frac{\sum_{m=1}^{M} X_{i,m} - \sum_{m=1}^{M} X_{i+1,m}}{\sum_{m=1}^{M} X_{i,m}} \left| \sum_{m=1}^{M} X_{i,m} > \sum_{m=1}^{M} X_{i+1,m} \right.
\]

where \( \left( \sum_{m=1}^{M} X_{i,m} * X_{i+1,m} \right) \) is the number of products ordered by both batches and

\[
\frac{\sum_{m=1}^{M} X_{i,m} - \sum_{m=1}^{M} X_{i+1,m}}{\sum_{m=1}^{M} X_{i,m}}
\]

is the probability that a product ordered by both batches is located between the \( \sum_{m=1}^{M} X_{i+1,m} + 1^{th} \) closest location from the depot and the \( \sum_{m=1}^{M} X_{i,m} \) \( i^{th} \) location in the \( i^{th} \) batch. The expected value of \( CD \) is approximated as,

\[
E[CD] = E\left[ \sum_{m=1}^{M} X_{i,m} * X_{i+1,m} \right] * \frac{E\left[ \sum_{m=1}^{M} X_{i,m} \right] - E\left[ \sum_{m=1}^{M} X_{i+1,m} \right]}{E\left[ \sum_{m=1}^{M} X_{i,m} \right]} \left| \sum_{m=1}^{M} X_{i,m} > \sum_{m=1}^{M} X_{i+1,m} \right.
\]

where
Appendix C

\[ E[\sum_{m=1}^{M} X_{i,m} * X_{i+1,m}] = M * E[X_{i,m} * Y_i] * E[X_{i+1,m} * Y_{i+1}] \]

\[ = M * E[1 - (1 - \frac{1}{M})^{Y_i}] * E[1 - (1 - \frac{1}{M})^{Y_{i+1}}] \]

\[ = M * \left( \frac{E[Y_i]}{M} \right) * \left( \frac{E[Y_{i+1}]}{M} \right) \]

\[ \approx \frac{E[Y_i] * E[Y_{i+1}]}{M} \]  

(C.3)

\[ Y_i \text{ and } Y_{i+1} \text{ in the equation above are the number of order lines in the } i^{th} \text{ and the } (i+1)^{th} \]

batch and \( Y_i > Y_{i+1} \). The approximation in equation (C.3) is obtained by approximating

\[ (1 - \frac{1}{M})^{Y_i} \text{ to } 1 - \frac{Y_i}{M} \] using Taylor Polynomial.

\[ E[\sum_{m=1}^{M} X_{i,m}] = M * E[X_{i,m} | Y_i] = M * E[1 - (1 - \frac{1}{M})^{Y_i}] \]

\[ = E[Y_i] \]  

(C.4)

Putting equation (C.3) and (C.4) into equation (C.2), we get

\[ E[CD] = E[Y_{i+1}] \left( \frac{E[Y_i] - E[Y_{i+1}]}{M} \right) \]  

(C.5)

The expected value of \( \sum_{m=1}^{M} X_{i+1,m} (1 - X_{i,m}) \) is approximated as,

\[ E[\sum_{m=1}^{M} X_{i+1,m} (1 - X_{i,m})] = M * E[X_{i+1,m} | Y_{i+1}] * E[E(1 - X_{i,m}) | Y_i] \]

\[ = M * E[1 - (1 - \frac{1}{M})^{Y_{i+1}}] * E[1 - \frac{1}{M})^{Y_i}] \]

\[ \approx E[Y_{i+1}] * E[1 - \frac{Y_i}{M}] \]  

(C.6)

Comparing the expected value of \( E[CD] \) and \( E[\sum_{m=1}^{M} X_{i+1,m} (1 - X_{i,m})] \), we have,

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In general, the impact of $CD$ increases with the batch size (large batch sizes lead to large value of $E[Y]$, the expected number of order lines for a batch of orders) and decreases with $M$ (total number of products in the warehouse). According to the characteristic of a DSS as discussed in chapter 4, the batch size could not be very large and the value of $M$ is much larger than the value of $E[Y]$. The difference between $E[Y]$ and $E[Y_{i+1}]$ could not be too large since $Y_i$ and $Y_{i+1}$ have identical distribution, the quotient is much smaller than 1. The impact of $CD$ can therefore be neglected. With parameters listed in Table 4.1, simulation results show $E[CD]$ accounts for less than 3% of $E[\sum_{m=1}^{M} X_{i+1,m} (1 - X_{i,m})]$, when the batch size is less than 200. The percentage increases with the batch size and reaches around 5% when the batch size is 300 and around 10% when the batch size is 600. The simulation results confirmed our assumption.
APPENDIX D CUSTOMER-TO-AISLE ASSIGNMENTS

Problem BAL

Suppose we know the number of transactions of each customer. Then we formulate the problem as an integer programming model as follows:

Objective: \( \min Z \)

\[
\begin{align*}
\text{s.t.} & & \sum_{i \in I} x_{ij} = 54, & \forall j \in J \\
& & \sum_{i \in I} x_{ik} = 36, & \forall k \in K \\
& & \sum_{i \in I} x_{il} = 1, & \forall i \in I \\
& & \sum_{i \in I} x_{il} \cdot a_i - \text{AVE} \leq Z, & \forall l \in L \\
& & \sum_{i \in I} x_{il} \cdot a_i - \text{AVE} \geq -Z, & \forall l \in L
\end{align*}
\]

where

\( I = \) the set of customers, index \( i \).
\( J = \) the set of zones consisting of 3 aisles, index \( j \).
\( K = \) the set of zones consisting of 2 aisles, index \( k \).
\( L = J \cup K \) the entire set of zones, index \( l \).
Appendix D

\( a_i \) = the number of transactions for customer \( i \).

\[
AVE = \frac{\sum a_i}{|L|}, \quad l \in L, \text{ and } i \in I, \text{ the average number of transactions per zone.}
\]

The variables are:

\( x_{il} = 1 \) if customer \( i \) is assigned to zone \( l \), otherwise 0.

\( Z \) = the maximum workload imbalance among zones.

The objective of this model is to minimize the maximum workload imbalance among zones. Constraints (D.1) and (D.2) are the zone capacity constraints. Constraint (D.3) states that a customer can only be assigned to one zone. Constraints (D.4) and (D.5) define the maximum workload imbalance.

At VBA, \(|I| = 666, \quad |J| = 7, \quad |K| = 8, \quad \text{and} \quad |L| = 15\), therefore this problem has 666*15=9990 binary variables and 15+666+15+15=711 constraints. As commercial optimization software takes too much time to solve the problem to optimality, we use a heuristic approach. The heuristics divides the problem into two sub-problems. First, the 15 zones are divided into 2 parts. Part 1 includes those 7 zones each containing 3 aisles. Part 2 includes those 8 zones each containing 2 aisles. Therefore, part 1 accommodates 7*3*18=378 customers, and part 2 accommodates 8*2*18=288 customers. The first sub-problem is to assign customers to parts, such that \( T_1 \), the number of transactions in part 1 equals the total number of transactions \( *7/15 \), and \( T_2 \) the number of transactions in part 2 equals the total number of transactions \( *8/15 \). We formulate this sub-problem as follows.

Objective: \( \min Z \)

s.t. \[
\sum_{i \in I} x_{i1} = 378 \quad \text{(D.6)}
\]

\[
\sum_{i \in I} x_{i2} = 288 \quad \text{(D.7)}
\]

\[
x_{i1} + x_{i2} = 1, \quad \forall i \in I \quad \text{(D.8)}
\]
where,

\( I \) = the set of customers, index \( i \).

\( a_i \) = the number of transactions for customer \( i \).

\( T_1 \) = the total number of transactions in part 1.

\( T_2 \) = the total number of transactions in part 2.

The variables are:

\( x_{i1} = 1 \) if customer \( i \) is assigned to part 1, otherwise 0.

\( x_{i2} = 1 \) if customer \( i \) is assigned to part 2, otherwise 0.

\( Z \) = maximum workload deviation over all parts.

The objective of this model is to minimize the maximum workload imbalance over all parts. We use Lingo 8.0 to solve it to optimality in a few seconds.

Now, we know the customer assignment in each part. The next sub-problem is to assign customers to zones in part 1 and part 2 with the objective of balancing workload among zones. We do this separately for part 1 and part 2. The formulation of the sub-problem is similar to the original problem but with fewer variables and constraints. We use Lingo 8.0 to solve each problem and obtain acceptable results in a few seconds. The workload imbalance among zones is about 1 percent of the average number of transactions in each zone, which is sufficiently accurate for VBA.

**Problem BALMIN(c)**
Appendix D

Objective

\[
\min \sum_{w \in W} \sum_{l \in L} y_{wl}
\]

s.t.

\[
\sum_{i \in I} x_{ij} = 54, \quad \forall j \in J
\]

(D.13)

\[
\sum_{i \in I} x_{ik} = 36, \quad \forall k \in K
\]

(D.14)

\[
\sum_{i \in I} x_{il} = 1, \quad \forall i \in I
\]

(D.15)

\[
AVE = \frac{\sum_{w \in W} \sum_{l \in L} y_{wl}}{|L|}
\]

(D.16)

\[
\sum_{w \in W} y_{wl} - AVE \leq c \cdot AVE, \quad \forall l \in L
\]

(D.17)

\[
\sum_{w \in W} y_{wl} - AVE \geq -c \cdot AVE, \quad \forall l \in L
\]

(D.18)

\[
b_{wi} x_{il} \leq y_{wl}, \quad \forall w \in W, \forall i \in I, \forall l \in L
\]

(D.19)

\[
x_{ij}, x_{ik}, x_{il}, y_{wl} \text{ binary, } \quad \forall i \in I, \forall j \in J, \forall k \in K, \forall l \in L
\]

where

\[I = \text{the set of customers, index } i.\]

\[J = \text{the set of zones consisting of 3 aisles, index } j.\]

\[K = \text{the set of zones consisting of 2 aisles, index } k.\]

\[L = J \cup K \text{ the entire set of zones, index } l.\]

\[W = \text{the set of trolleys, index } w.\]

\[b_{wi} = 1 \text{ if customer } i \text{ has a transaction on trolley } w, \text{ otherwise } 0\]

\[c = \text{constant number. } 0 \leq c \leq 1\]

The variables are:
Appendix D

\[ x_{il} = 1 \text{ if customer } i \text{ is assigned to zone } l, \text{ otherwise } 0 \]

\[ y_{wl} = 1 \text{ if trolley } w \text{ visits zone } l, \text{ otherwise } 0. \]

The objective of this model is to minimize the total number of visits to zones. The last constraint implies that if customer \( i \) has an order on trolley \( w \), and customer \( i \) is located in zone \( l \), then the trolley will visit zone \( l \).

As shown in Appendix E, this problem is NP-complete. We therefore design a heuristics to solve it. The result of the previously discussed customer-to-aisle assignment method (BAL) provides us with a good starting solution. We then apply 2-opt improvement by switching the locations of every pair of customers in two different zones. The algorithm iterates until we can not further improve the objective.

We set the value of \( c \) in the above model to 0.1, which means we confine the imbalance of the number of visits among zones to 10% of the average number of visits.
APPENDIX E  PROOF OF THE NP-COMPLETENESS OF THE BALMIN(c) PROBLEM

We show the decision variant of problem BALMIN is NP-complete. For this purpose we consider a sub-problem of the original problem. The sub-problem excludes constraints (D.16), (D.17), and (D.18) and keeps the rest unchanged. We consider the special case of this sub-problem where every trolley contains exactly 2 customer transactions. Given a set $I$ of customers, $W$ of trolleys, $L$ of zones, zone capacity of $C_{\text{max}}$, and an integer $k$, the problem is to find if there is a clustering of customers to zones with fewer than or equal to $W+k$ total zone visits to finish all the transactions. Given a clustering of customers to zones, for every trolley with both customers in the same zone, add one to the objective function; for all other trolleys, add two to the objective function. Summing over all the trolleys, we are going to verify if the total is less than $W+k$. The problem is transformed into a graph partitioning problem as follows: Given a graph $G=(V,E)$, $M$ subsets, maximum subset size $n$, and an integer $k$, is there a partition of vertices into $M$ subsets of size at most $n$ with fewer than $k$ edges going between subsets? Let each vertex be a customer ($V\rightarrow I$), each edge $e=(u,v)$ represents a trolley containing customers $u$ and $v$ ($E\rightarrow W$), each subset be a zone ($M\rightarrow L$), and the maximum subset size be the zone capacity ($n\rightarrow C_{\text{max}}$). It is clear that there is an assignment of customers to zones with fewer than $W+k$ total zones visits for all trolleys if and only if there is a partition of vertices into subsets with at most $k$ edges between subsets. Hyafil and Rivest (1973) shows that this problem is NP-complete. Therefore, the special case where every trolley contains exactly two customers is NP-complete and the general case is as well. Consequently, the original problem is NP-complete.
Summary

Order picking is the most critical operation in a warehouse. It involves the scheduling and releasing of customer orders, the picking of items from their storage locations and the disposal of the picked items. Order picking often consumes a large part of the total labour activities and accounts for a substantial percentage of the total operating cost in a warehouse. Therefore, the performance of the order picking system has large impact on the performance of a warehouse. The thesis provides models and analyses to support the design and the control of efficient order picking systems with focus on pick-and-pass systems. In a pick-and-pass order picking system, the whole pick area is divided into pick stations (zones) connected by conveyors pieces with order pickers assigned to zones for picking. A customer order is assigned an order bin with a pick list to store articles when it is released to the picking system. The order bin gets on the conveyor, travels from one station to another and only visits a station when pick is needed. When all pick stations, where articles have to be picked, have been visited by the order bin, the content is checked and packed. Sorting is needed at the end when multiple orders are batched in one bin. Such order picking systems have many advantages (refer to De Koster 1996) and are widely used in practice.

In chapter 1, we discuss the classification of order picking systems with emphases on the explanation of pick-and-pass order picking systems studied in this thesis (refer to Figure 1.1). The factors, influencing the performance of a pick-and-pass system, include the layout of the pick area, the zoning of the pick area, storage assignments of products in the pick area, routing of order pickers in the pick area, order batching policies, order accumulation and sorting processes at the end of the picking, and congestions in the pick area. We recapitulate these topics and review the relevant literature. At the end of the chapter, we highlight the contribution and the outlines of the thesis.
Summary

As discussed in chapter 1, many factors influence the performance of a pick-and-pass order picking system. At the design phase, due to the absence of accurate operational data, an approximation tool is desired to estimate the system performance under the different design alternatives. Chapter 2 discusses such a tool to estimate the mean throughput time of an arbitrary order in a pick-and-pass order picking system. We first estimate the mean and the standard deviation of service time of an order bin at nodes (stations and conveyor pieces) and then develop the routing probabilities between nodes. These values are used to calculate the mean and the standard deviation of inter-arrival time of order bins to each node. The analysis is based on G/G/m queuing network modelling by Whitt (1983). The method provides a tool for fast evaluation of the impact of storage policies, sizes of pick stations, the number of order pickers per station, and the customer order profiles on order picking system performance. Additionally, the method also evaluates the effects of order batching and splitting on system performance. We validate the approximation method by extensive simulations and a real order picking process at an International Motor Production company in the Netherlands. The validation results demonstrate the quality of the approximation method is acceptable for practical purposes. Although we assume a line layout of the pick area, a class-based storage policy, and an order picker picks only one order line per trip in the analysis (refer to Figure 2.2), the model can be easily extended to other operating situations.

Chapter 3 uses the approximation model developed in the previous chapter to study the impacts of order batching and pick area zoning on the average throughput time of a random order in a pick-and-pass order picking system. Comparing to the analysis in chapter 2, in this chapter, we consider a parallel layout of the pick area with different routing, storage and batching policies (refer to Figure 3.1). Because batched orders need to be sorted again by order, we take the sorting process after the order picking into consideration. We carried out experiments with different input parameters, such as setup times at pick zones, different order arrival rates to the systems, and the different order arrival distributions. We find the batch size has large impact on the system performance, and an optimal batch size exists to minimize the mean order throughput time. We find the mean order throughput time in the system is quite robust for a varying number of zones around the optimum number of zones. We also find, for a given mean order arrival rate,
Summary

The precise shape of the order arrival distribution has only slight impact on the mean order throughput time. This is especially true when the utilizations at zones are small. In general, many factors influence the system performance. This phenomenon reflects the complexity of the pick-and-pass system. The preference of one operational strategy over the other depends on the settings of the system. The approximation model developed in chapter 2 and 3 can therefore be used as a fast tool to analyze these alternatives.

In a conventional order picking system, all the SKUs ordered by customers during the entire day or a picking shift (which normally takes several hours) are located in the pick area. Due to the large storage space of the pick area, order pickers often spend much of their time on picking tours, leading to low throughput and low worker productivity in the picking process.

In chapter 4, we introduce the concept of Dynamic Storage (DS). In a Dynamic Storage System (DSS), customer orders are batched in groups with fixed size before they are released to the picking system. Only those products needed for the current pick batch are retrieved from a reserve area and stored in the pick area, just in time. Automated Storage and Retrieval (S/R) machines resuffle the products in the pick area before the picking process for a batch of orders starts. In a DSS, since only a small fraction of the SKUs are stored in the pick area, order pickers’ travel time reduces significantly, leading to higher order throughput and higher picker productivities (lower labour cost) comparing to a conventional order picking system. In this chapter, we derive a mathematical model to obtain the maximum throughput a DSS can achieve and, on top of that, find the optimal batch size to minimize labour cost needed for order picking in a single-station order picking system. The application of DS to pick-and-pass order picking systems is discussed through two alternatives using the results developed for the single-station system and simulation respectively. For both order picking systems, the performance of a DSS is compared with conventional order picking systems. Through our mathematical and simulation models, we demonstrate that a DSS can substantially improve throughput and reduce labour cost at the same time.

In chapter 5, we use the case of Bloemenveiling Aalsmeer (VBA), a large flower auction company in the Netherlands, to illustrate the advantages of a pick-and-pass order picking
Summary

system with zoning over an order picking system with a single zone. At VBA, the auctioned flowers on trolleys need to be distributed to customers with their own trolleys located in a distribution area within VBA. The current distribution process at VBA is analogous to an order picking process with a single large zone (the whole distribution area). Many distributors travel in the zone with trolleys towed by small electric trucks for distributing flowers to customers (refer to Figure 5.3), leading to much congestion in the process. The problem is the distribution process can not meet the customer order lead time requirements. At the request of VBA, we investigate the potential benefits of zoning on the distribution process performance. In the zoned distribution process, the entire distribution area is divided into zones with distributors assigned to them. The distributors are only responsible for the distribution within their assigned zone. Automated Guided Vehicles (AGVs) are used to transport trolleys with flowers between zones. This resulted zoned distribution process resembles a pick-and-pass order picking process as we discussed in the previous chapters. Through extensive simulation, we evaluate the performance of VBA’s current distribution process and scenarios of the zoned process. By formulating integer programming models and designing heuristics for their solutions, we also investigate customer assignment methods to zones in order to balance workload among zones. The outputs of the integer programming models are used as inputs of the simulation models. We show by simulation that introducing zoning to the distribution process can significantly improve order lead time and makespan. Simulation shows balancing workload between zones can further improve the distribution performance.

In the concluding chapter 6, we summarize the findings and contributions of this thesis. We also discuss the limitation of each chapter and give suggestions for future research. Some interesting issues on order picking, which have not been treated in this thesis and in literature, are also discussed in this chapter.
拣货是仓库管理中最重要的一个环节。它包括计划和释放用户的订单，从仓库中提取货物，及对提取货物的配置。拣货过程通常占用了大量的劳力和仓库管理的成本。所以拣货系统的性能很大程度的影响仓库的运行性能。本篇论文集提供模型和分析方法来支持设计和控制高效的拣货系统，其重点是在于接力式拣货系统的分析。在一个接力式拣货系统中，整个拣货区被分为若干个拣货站(或拣货带)，站内有拣货员。各拣货区用传送带相连。每个客户订单会被分配到一个拣货单来指示要提取的货物和一个订单箱来存放将要提取的货物。订单箱通过传送带系统在拣货站之间传递，订单箱只进入需要提货的拣货站。当所有要提货的拣货箱被订单箱访问过后，订单箱内的货物被核实并被打包。如果一个订单箱内被分配了多个客户的订单进行批处理，则当拣货结束后，要增加一个订单分拣系统来分离客户的订单。这种接力式拣货系统有很多优点(参见 De Koster 1996)因此在实际中得到广泛的应用。

在第一章中，我们讨论拣货系统的分类并着重介绍本论文集中要分析的接力式拣货系统(见图 1.1)。影响接力式拣货系统性能的因素包括：拣货区的结构布置，拣货站的划分，拣货区货物的存放方式，拣货员的拣货路线，客户订单的分批处理策略，拣货处理结束后客户订单的存储和分拣，及拣货区的阻塞情况。我们简要的介绍这些概念并回顾相关的参考文献。在本章的最后，我们突出了本论文集的贡献和概要。

如第一章所述，许多因素影响接力式拣货系统的性能。在其设计阶段，由于缺少精确的运行数据，我们需要一个估算工具来评估不同设计方案下接力式拣货系统的性能。第二章讨论这样的工具来估算一个随机客户订单在该系统的平均停留时间。我们首先估算出每个订单箱在各节点(各拣货站和各段传送带)停留时间的均值和方差，然后推导出订单箱访问各节点的概率。由此我们推导出访问每个节点的订单箱的间隔时间的均值和方差。对此系统的分析建立在 Whitt(1983)关于 G/G/m 队列论的基础之上。我们的分析提供了个快速的工具来估算货物的存放策略，拣货站的大小，站内拣货员的人数，以及客户订单的特性对于拣货系统性能的影响。在此之上，我们的估算模型还可用于衡量批处理客户订单和分割处理客户订单对于拣货系统性能的影响。我们通过大量的仿真和对一个在荷兰的国际性发动机生产企业的拣货系统的分析来验证我们的估算模型。结果表明，估算的质量对于实际应用而言足够精确。尽管在分析中我们假设拣货区是线型排列，货物分级存放，及拣货员在每次拣货的往返中只提取订单中要求的一种物品(见图 2.2)，我们的分析模型可以很方便的拓展到其它的运行方式。
第三章应用上一章建立的模型来研究订单批处理和拣货区的划分对于一个随机的客户订单在接力式系统中的平均停留时间的影响。这一章我们考虑一个平行排列的拣货区和不同于第二章的拣货路径,货物存放及订单批处理方式(见图 3.1)。由于批处理的订单数被分拣成独立的订单，我们考虑了拣货过程后的分拣操作。我们用不同的参数进行了大量的实验,比如设置拣货站中不同的准备时间,订单的不同到达速度及概率分布。我们发现批处理的订单数对于系统的性能有很大影响,并存在一个使得订单在系统中停留时间的均值最小化的最优值。订单停留时间均值在最优个数的拣货站附近具有很强的鲁棒效应。我们同时发现,对于固定的平均订单到来速度,订单到达速度的精确概率分布对于系统性能仅有很小的影响,尤其是在拣货站的使用率很低时,这种影响更小。总而言之,很多因素影响系统的性能,这反映了接力式拣货系统的复杂性。对于某种运行策略的选择依赖于系统的参数。而我们在第二、第三章中建立的模型可以被用作一个快速的评估不同策略的工具。

在传统的拣货系统中，客户在一天或一个轮班(通常为几个小时)中订购的各种物品都被存放在拣货区。由于拣货区占用了大量的存储面积,拣货员浪费了大量时间在拣货途中,导致拣货过程中的低吞吐量和低效率。

在第四章,我们介绍动态存储(DS)的概念。在一个动态存储系统(DSS)中,客户订单在被释放到拣货系统之前先被划分成批次,每一批次包含固定数量的订单。只有当前要处理的批次中要提取的货物被从库存区取出,放入拣货区(JIT)。在每批订单被拣货处理之前,自动存取机械来重组放入拣货区中的物品。在动态存储系统中,由于仅有一小部分物品存储在拣货区,大大缩短了拣货员的拣货路途,较之于传统的拣货系统,提高了吞吐量和生产率(降低了劳力成本)。在本章中,对于只有一个拣货站的拣货系统,我们先建立数学模型来推导动态存储可实现的最大吞吐量。在此基础上,我们研究了实现最低劳力成本的批处理订单的数量;对于动态存储在接力式拣货系统中的应用,我们分析了两种情况,分别基于对于只有一个拣货站的拣货系统和应用仿真的手段。我们比较了动态存储和传统的拣货系统,通过数学分析和仿真,我们证明动态存储可以同时大幅度的提高拣货系统的订单吞吐量和劳力成本。

在第五章中,我们用一家大型的鲜花拍卖公司 Bloemenveiling Aalsmeer (VBA) 的实例来证实拥有多个拣货站的接力式拣货系统相对于只有一个拣货站的拣货系统的优点。在 VBA,客户在插车上的拍卖后的鲜花需要被从库存区取出,放入拣货区。VBA 以前的分发过程类似于只有一个拣货区(即整个分发区)的拣货过程。许多工人驾驶电单车来拖动拖车在整个分发区内把鲜花分发到客户。在文献 5.3 中,我们定义了分发系统的个数,使得当前的分发过程不能满足客户对鲜花传送时间的要求。在文献 5.3 的要求下,我们研究了分区对于分发过程的潜在的好处。在分类的分发过程中,整个分发区被分成若干个区,工人们也被分配到各个区并只负责其所在的区域的分发工作。自动搬运用(AGV)来负责分区之间拖车的传递。这个分了区的分发过程类似于我们前几章讨论的接力式拣货过程,通过大量的仿真,我们评估了当前的分发系统和分区分发系统的不同运行方案。通过建立整数规划模型和设计启发式算法,我们研究了客户拖车在分区中的划分方法,来平衡分区之间的工作量。
整数规划模型的结果被用作仿真程序的输入。通过仿真，我们证明分区的引入能明显的缩短鲜花的传送时间和整个分发过程的持续时间。仿真结果表明，平衡分区之间的工作量能进一步提高分发系统的性能。

在总结性的第六章，我们概述了本论文集的发现和贡献，讨论了各章的局限及未来的研究方向。一些未出现在本文集和文献中的关于拣货的主题也在本章中进行了讨论。
Mengfei Yu was born in 1973 in Shanghai. He graduated from Hohai University, China in 1994 with a Bachelor degree of Electrical Engineering and then worked as an Electrical Engineer in Wuxi Power Supply Company. In the autumn of 2001, he resumed his academic life at Katholieke Universiteit Leuven, Belgium, where he obtained his master degrees on Electrical Engineering and Industrial Engineering (specialized in Transportation and Logistics) in July 2003. In March 2004, Mengfei Yu started his Ph.D. research project at the Erasmus Research Institute of Management (ERIM) under the supervision of Professor René de Koster. His research areas focus on warehousing and material handling. He presented his research results in international conferences and workshops in Asia, Europe and North America. One of his papers co-authored with Professor de Koster won the second best student paper award in the International Conference on Industrial Engineering and Systems Management, 2007. His research results have been accepted for publications in the Journal of the Operational Research Society and IIE Transactions. Mengfei Yu is currently working as a consultant on supply chain management at Buck Consultants International, the Netherlands.


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ENHANCING WAREHOUSE PERFORMANCE BY EFFICIENT ORDER PICKING

This thesis studies order picking in warehouses. Order picking, the process of retrieving products from their storage locations to fill customer orders, is regarded as the most critical operation in a warehouse. Using stochastic modelling, we develop a model for zoned pick-and-pass systems to estimate order picking performance of various design alternatives and operating policies. The model is fast, flexible, and sufficiently accurate for practical purposes. The thesis also introduces a Dynamic Storage concept. In a Dynamic Storage System (DSS), orders are picked in batches and only those products needed for the current pick batch are retrieved from a reserve area and stored in the pick area, just in time. Through analytical and simulation models, we demonstrate a DSS can substantially improve order throughput and reduce labour cost simultaneously over conventional order picking systems, where all the products required during a pick shift are stored in the pick area. The thesis also studies an internal distribution process at a flower auction company. We introduce a zoned distribution system, analogous to pick-and-pass. Based on simulation and optimization models, we propose ways to reduce congestion and improve order lead time.

ERIM

The Erasmus Research Institute of Management (ERIM) is the Research School (Onderzoeksschool) in the field of management of the Erasmus University Rotterdam. The founding participants of ERIM are Rotterdam School of Management, Erasmus University, and the Erasmus School of Economics. ERIM was founded in 1999 and is officially accredited by the Royal Netherlands Academy of Arts and Sciences (KNAW). The research undertaken by ERIM is focused on the management of the firm in its environment, its intra- and interfirm relations, and its business processes in their interdependent connections.

The objective of ERIM is to carry out first-rate research in management, and to offer an advanced doctoral programme in Research in Management. Within ERIM, over three hundred senior researchers and PhD candidates are active in the different research programmes. From a variety of academic backgrounds and expertise, the ERIM community is united in striving for excellence and working at the forefront of creating new business knowledge.