A Data-driven Approach to Enhance Worker Productivity by Optimizing Facility Layout

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Abstract

The facility layout problem (FLP) is the problem of determining non-overlapping positions of departments on the shop floor to minimize material handling costs. Traditional methods for solving FLPs consider pairwise (from-to) flows to optimize layouts. This paper shows that these traditional methods underestimate the total travel distance of a layout, when departments have more than a single input/output point and some flows consist of visits to more than two departments. To accurately calculate the traveled distances, the actual routes of the workers and transporters (so-called connected movements) in the system need to be determined. The connected movements of the workers in a facility can now be captured using the Internet of Things network and stored in the cloud server for analysis. We propose a mixed-integer non-linear programming model for the FLP that minimizes the total travel distance using these connected movements as the input data. Because of the complexity of the problem, a biased random key genetic algorithm is used to find the layout. To ensure the validity of the method, a case study is carried out at a fertilizer production company that implemented an Internet of Things network to capture worker movement data to minimize worker productivity loss via an improved layout. By using these connected movements, the best layout for the case company is found. The results of the proposed data-driven optimization method indicate that leveraging connected movements can reduce the total travel distance by 10.6% compared to the best possible layout generated by the traditional pairwise method in the case study.

Keywords: Facility layout problem, Data-driven optimization, Connected movements, Sequence, Internet of things.

1 Introduction

Worker productivity in production facilities is largely governed by the arrangement of the workstations. Optimizing the relative locations of workstations at a worksite (also known as the facility layout problem, FLP) is a crucial managerial decision because it directly affects worker travel time and productivity. This decision is typically taken during the commissioning and implementation stages. A good facility layout decision can result in significant operational cost savings via increased manufacturing productivity and reduced costs (Tompkins et al. 2010). Other objectives of the FLP include minimizing material handling costs, reducing work in process, congestion and lead times, reducing operating costs, improving workplace safety, and accommodating the plant to future changes (Drira et al. (2007), Zhang et al. (2008), Singh and Sharma (2006), Heragu (2016) and Ahmadi and Akbari Jokar (2016)). A good layout can boost sustainability by providing a safe environment for employees and reducing energy-drainage activities such as unnecessary movements.

Anjos and Vieira (2017) define the FLP as finding an optimal arrangement of a predefined number of departments (so that these departments do not overlap within a given area) to minimize costs. Due to the combinatorial nature of the FLP, the optimal layout identification problem is data intensive and computationally hard. Traditional quantitative methods consider pairwise flows between departments while planning for facility layout (Tompkins et al. 2010). These conventional approaches of the FLP, based on from-to flow charts, are referred as the pairwise method in this study. The pairwise method for the FLP has two drawbacks: 1) Although it leverages the anticipated from-to movements of entities (such as workers, vehicles, products, etc.), an employee may need to visit more than two workstations in a sequence to complete the task. 2) The 'from-to' pairwise travel distance estimates between the source and the destination departments can be determined by choosing the closest input/output (IO) points between these two departments and thereby ignore the sequences. As departments may have multiple IO points, the pairwise method may thus underestimate the total travel distance. Example 1 illustrates the second drawback when using the pairwise method. Therefore, it is important to include real connected movements between departments to determine a good layout. Sequential visits between departments can occur under several conditions, e.g., when products have to be moved between departments with short processing times within the departments. An example is a kitting process where a worker must add components from various departments to the kit. Until recently, it was challenging and expensive to obtain insights into these connected worker movements. However, using new data sources (such as data obtained using Internet of Things (IoT) sensors) that provide real-time employee tracking can identify connected movements and use these to optimize facility layouts. With a 90% drop in the average price of sensors between 1992 and 2014, sensors have increasingly been adopted in real-time asset data collection and performance monitoring (Olsen and Tomlin 2019). In this research, the connected movements of a worker are named a *sequence*.

Example 1. Consider a two-dimensional layout with three departments where Departments 1 and 2 have two IO points each and Department 3 has one IO point (see Figure 1). We assume that each IO point can serve as both input and output for the corresponding zone. Table 1 shows the flow and distance matrices in this example where the flow denotes the number of movements between each pair of departments (from-to flows) and distances are measured by a rectilinear metric. Based on Table 1 and the layout in Figure 1, the optimal choice that minimizes the total travel distance is to travel from IO_1 to IO_3 and from IO_4 to IO_5. These movements lead to a total travel distance of four units (see the highlighted cells in Table 1). Now, assume that the flow matrix in Table 1 is extracted from a route of a worker traveling from Department 1 to Department 2 and then to Department 3 (i.e., sequence of $[1, 2, 3]$). To complete this sequence, a worker can go from IO_1 to IO_4 and then to IO_5, with a total travel distance of nine units or travel from IO_1 to IO_3, and then to IO_5 with a total travel distance of seven units which is the optimal choice in this case. The worker can also start at IO_2, resulting in a greater travel distance. It is clear that by considering pairwise movements instead of actual routes in the problem, the objective function underestimates the actual travel distance by 3 units. □

Table 1: Flow and distance matrices based on the pairwise method

Flow Matrix				Distance Matrix			
From \ To	Department 1	Department 2	Department 3	From \ To	Department 1	Department 2	Department 3
Department 1	0	1	0	Department 1	0	2 (from IO_1 to IO_3)	6 (from IO_2 to IO_5)
Department 2	0	0	1	Department 2	2 (from IO_3 to IO_1)	0	2 (from IO_4 to IO_5)
Department 3	0	0	0	Department 3	6 (from IO_5 to IO_2)	2 (from IO_5 to IO_4)	0

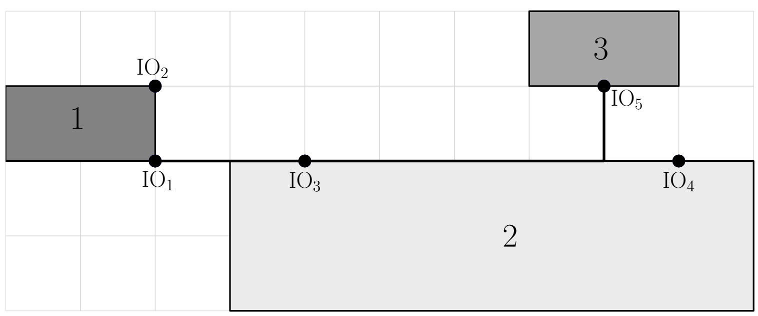


Figure 1: Underestimated travel distance example

This research is motivated by our collaboration with a fertilizer production company, Aries Agro (`ariesagro.com`) that manufactures nutritional products for plants and animals and has four plants in India. Its products have significant daily flow variations. A large portion of the work is carried out using warehouse trucks to transport pallets. In a preliminary analysis, the company identified unnecessary movements, high working temperatures, and strewed chemicals resulting from storing raw materials in the main building and the operation of the blender machines as the key factors affecting workforce productivity and safety. In addition, the company thinks that the current layout may need to be revised, as several processes and related movements have changed over time. For instance, the raw materials area and the blender, pulveriser and bagging area have high levels of physical activity but are not not located close to each other in the current layout, leading to unproductive excess travel. Therefore, the company has implemented sensors in one of its facilities to track, measure, and monitor worker activities (see Figure 2). Figure 3 shows the layout and data capturing process in building 1 of the Aries Agro main facility and Table 2 illustrates a slice of the data captured by the IoT based system implemented in the case company. The sensors may detect workers who are just passing by a department. For example, the data in row 6 captures a worker passing by the spray dryer area. We distinguish between visiting a department for production purposes or just passing by based on the length of each department along the aisles and the measured time a worker stays in that department. Hence, to find real connected movements, we exclude passing by movements when extracting data on workers' sequential visits to departments.

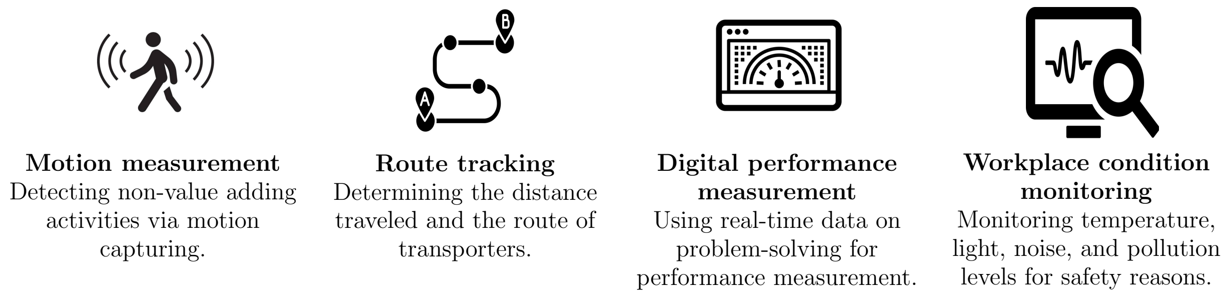


Figure 2: Applications of data obtained by IoT in workplace

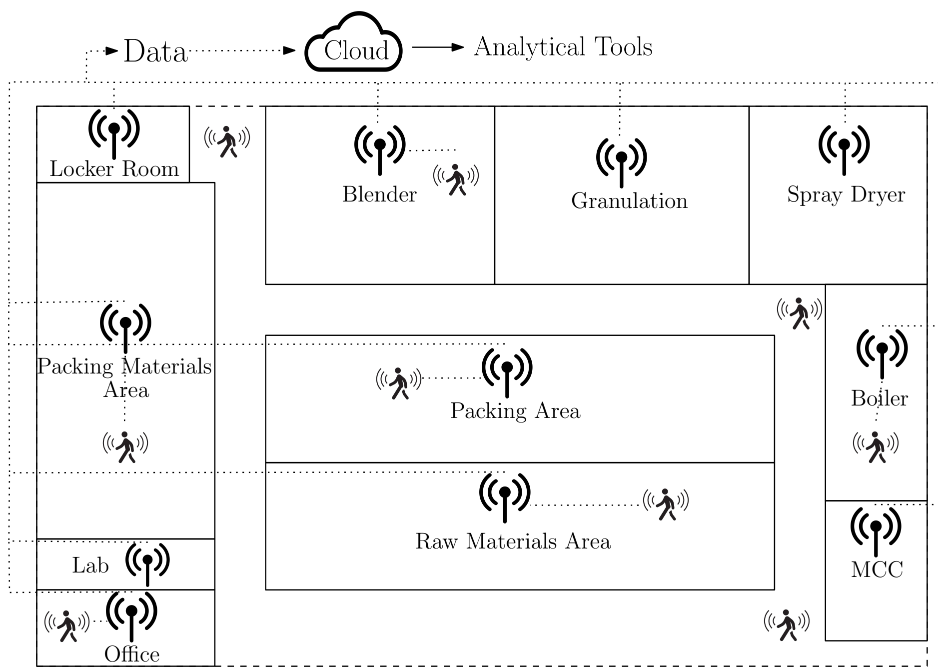


Figure 3: Layout of main facility (building 1) and data collection process using sensors

While analyzing the data of worker travel routes, we observed that some connected movements of workers are repeated during the work shifts. For example, a worker from the blending department starts traveling from the raw materials area to collect the materials and then proceeds to one of the four identical blending machines where the raw materials are blended. The worker waits for a small sample from the blender, delivers the sample to the quality control lab, returns to the blending department, collects the blended materials and takes them to the granulation department. This

Table 2: A slice of the route tracking data

No.	Date in time	Date out time	Worker name	Location name	Activity
1	2019-06-22 19:29:55.403	2019-06-22 19:32:38.780	Worker X	Raw material area	Retrieving raw materials for blending
2	2019-06-22 19:32:38.747	2019-06-22 19:38:27.270	Worker X	Blender, pulveriser and bagging area	Feeding the blending machine
3	2019-06-22 19:38:27.283	2019-06-22 19:45:55.687	Worker X	Raw material area	Retrieving raw materials for blending
4	2019-06-22 19:45:55.700	2019-06-22 19:46:25.203	Worker X	Blender, pulveriser and bagging area	Feeding the blending machine
5	2019-06-22 19:46:25.177	2019-06-22 19:50:13.697	Worker X	Granulation area	Delivering blended materials for granulation
6	2019-06-22 19:50:14.376	2019-06-22 19:50:20.726	Worker X	Spray dryer	Passing by

sequence is repeated multiple times. These observations motivate us to *revisit* the FLP by not just considering pairwise movements, but connected worker movements of varying lengths, due to subsequent department visits. A department may have multiple IO points (e.g., the blending machine department has four). Hence, the relative location of IO points within a department must be considered in the analysis. In addition, some movements (such as traveling from the blending area to the lab) may be unanticipated, but are still necessary. This can be caused by issues such as checking and collecting activities, missing data, insufficient quantities or space, that can easily be corrected by the worker. We include all such connected movements by collaborating with the IoT service provider (Alluvium IoT, `alluvium.in`) and use them to reoptimize the facility layout. Instead of using pairwise (from-to) flows, we use travel sequences of varying lengths with a data-driven optimization approach to identify better layouts that result in improved travel distances and hence worker productivity.

Traditional FLP approaches optimize static layouts using deterministic from-to flow input data. By considering connected worker movements obtained from IoT data, we can include 'unanticipated worker movements' (if defined essential) and real worker travel paths to obtain facility layouts. This research addresses the following research questions:(1) To what extent does including travel sequences in the FLP improve layout and travel distances? (2) What is the effect of using only pairwise travel distances on facility layout design? (3) To what extent can an improved plant layout reduce worker travel distances? The last question is of particular interest to the company.

To answer these questions, we formulate this problem as a mixed-integer non-linear programming (MINLP) optimization model and derive structural results that allow us to determine when the pairwise method underestimates the total travel distance, and to estimate the range of travel distance underestimation. The results indicate that optimizing the layout by using the travel sequences can reduce the real total travel distance in the case study by 10.6% compared to the pairwise method. The main contribution of this paper is to show that sequential travel paths of length greater than two can have an important impact on the best possible layout, and to identify the conditions under which such differences occur. We show that the difference is affected by the number and positions of departments and IO points.

The rest of this paper is organized as follows. Section 2 presents a literature review. Section 3 introduces a mathematical model for optimizing layouts based on the connected movements. In Section 4, we derive analytical results that illustrate the importance of using travel sequences instead of pairwise movements. Section 5 provides the solution methodology of this study. Section 6 illustrates the effect of leveraging connected workers movements in a case study. Section 7 provides sensitivity analyses. Policies and implications of this research to practice are discussed in Section 8. Section 9 provides insights, limitations, and extensions of this study, and finally, Section 10 concludes the paper.

2 Literature Review

This research contributes to the literature on the FLP. The objective of this paper is to minimize the total travel distance by locating the departments and their IO points in a single floor facility layout. Since no other academic literature studies connected movement paths of length longer than two, we focus on literature that studies similar layout problems with pairwise flow, and on positioning IO points within a layout. We discuss solution methods and constraints used in those papers.

This research studies a continuous FLP with departments of unequal sizes. FLPs with discrete layouts are usually modeled as a quadratic assignment problem (QAP), whereas FLPs with continual layout are often modeled as mixed integer programming (MIP) problems (Drira et al. 2007). Montreuil (1991) is one of the first who proposed a MIP model for FLP with a continual representation in which the dimensions of each department are determined by a given upper and lower bound for the width and length. The objective of this model is to minimize total travel distance that is calculated by multiplying from-to flows by the pairwise distances between departments. Meller et al. (1998) reformulate the binary variables regarding the non-overlapping constraints in the MIP model proposed by Montreuil (1991) to reduce model complexity. The model developed by Montreuil (1991) is optimally solvable for layouts up to five departments whereas the improved model proposed by Meller et al. (1998) can be solved for problems with seven departments (Gau and Meller 1999). Since the facility layout problem is NP-hard (Drira et al. 2007), using heuristic approaches for problems with more than 15 facilities is inevitable (Hosseini-Nasab et al. 2018). Genetic algorithms, tabu search, simulated annealing (SA), and ant colony optimization are the most frequently used metaheuristics for solving FLPs (Hosseini-Nasab et al. 2018). Kochhar et al. (1998) propose a genetic algorithm for FLPs with departments that have unequal sizes. They consider a discrete FLP and each department can be positioned into one or more square blocks. Their proposed algorithm improves the best-known solutions up to 5% but with a 30-50% higher computational time. Gonçalves and Resende (2015) develop the biased random-key genetic algorithm (BRKGA) to minimize total travel distance in facilities with rectangular departments that have unequal areas. The distance between departments is calculated from center to center, and the dimensions of departments are determined using maximally allowed aspect ratios. The results show that their approach obtains the best solution for 14 out of 16 benchmark datasets in case of low layout space utilization.

To find the efficient location of facilities in a given space, it is vital to determine the location of IO points of each department (Kim and Goetschalckx 2005). We focus here on literature that integrates locating IO points and departments to minimize the material handling costs or travel distance. Kim and Kim (1999) minimize the total travel distance between departments by finding the location of IO points in a given layout. The distance between departments is measured based on the shortest paths from the output to the input points. They consider intersections of aisles as possible locations of IO points and propose a branch and bound-based procedure to determine the optimal location of IO points. Kim and Goetschalckx (2005) propose an approach for integrated decisions on locating departments and IO points using contour distances and material flow paths. They determine the flow paths from the output to the input points by finding the shortest paths that follow the perimeter of the departments and use simulated annealing algorithm to find the final layouts. Yang et al. (2005) integrate the department location and flow-path design decisions to improve productivity in flexible manufacturing systems. They consider a green-field layout with a single-loop directed flow path and use a rectilinear distance measure between two adjacent cells from the drop-off to pick-up points. They first employ a simulated annealing algorithm to determine the block layout that minimizes the material handling costs with the directed flow path. The solution is then used as an input to the MIP model to find detailed layouts. Kelachankuttu et al. (2007) determine the location of a new department in a layout with given departments when it is not suitable to place the new facility on the optimal site due to practical constraints such as aisle locations. They use contour lines to find a suitable location for the new department. In their study, the new department has a single IO point whereas the existing departments can have more than one IO point. The distance between a pair of departments is measured by the shortest contour distance between the IO points of an existing department and the IO point of the new department. Sedehi and Farahani (2009) determine the location of departments, IO points, and automated guided vehicles (AGVs) flow paths concurrently in a single loop system. Materials are moved along the perimeter of departments and distances are calculated based on the shortest path that connects the output point of a department to the input point of the other department. Klausnitzer and Lasch (2019) propose two MIP models. The first integrates locating the departments, IO points and designing the paths, and the second considers aisles instead of flow paths. Their proposed approach tries to minimize the total travel distance from the output to the input points based on the from-to flows. Their MIP models are applicable to small-sized facilities such as medical companies with fewer than ten departments. Xiao et al. (2019) propose a hybrid robust optimization model for unequal-area dynamic facility layout problems, that takes the location of IO points into consideration. They develop a particle swarm optimization algorithm to solve it assuming from-to flows and a rectilinear distance measure between each pair of departments from the output to the input points. All mentioned studies consider the movements from the output (pick-up) to the input (delivery) points and ignore the real, possibly longer, travel distances. Table 3 summarizes the studies that consider the location of IO points in the FLP.

Table 3: Summary of key literature considering the IO point locations

Authors	Decisions	Type of data	Input data	Distance calculation	Modelling and approaches
Kim and Kim (1999)	Locating IO points in a given layout	Quantitative deterministic	From-to flows	Shortest path from output to input points	- Pairwise-based branch and bound (Exact)
Kim and Goetschalckx (2005)	Integrated IO points, department locations, and flow paths	Quantitative deterministic	From-to flows	Shortest path from output to input points	Pairwise-based MINLP simulated annealing
Yang et al. (2005)	Integrated department locations and flow paths	Quantitative deterministic	From-to flows	From output to input points of adjacent cells	Pairwise-based MIP simulated annealing
Kelachankuttu et al. (2007)	Locating the new facility in a layout with existing facilities	Quantitative deterministic	From-to flows	Shortest contour distance between the IO points of departments to the single IO point of the new department	-
Sedehi and Farahani (2009)	Integrated IO points, department locations, and flow paths	Quantitative deterministic	From-to flows	Shortest path from output to input points	- Pairwise-based heuristic algorithm
Klausnitzer and Lasch (2019)	Integrated IO points, department locations, flow paths, and aisle design	Quantitative deterministic	From-to flows	Shortest path from output to input points	Pairwise based MIP
Xiao et al. (2019)	Integrated IO points and department locations	Quantitative non-deterministic	Demands	From output to input points	Pairwise-based MINLP hybrid robust optimization solved by particle swarm optimization
This paper	Integrated IO points, department locations, and routing	Quantitative time-dependent flows	Travel sequences	Shortest route of travel sequences (by finding the optimal combination of IO points that workers should visit to complete travel sequences)	Sequence-based MINLP genetic algorithm (BRKGA)

Since facility layout is an important practical problem, several algorithms have been developed which are also used in commercial software, often as a decision-support tool. We summarize the most-well-known algorithms in Appendix A.

This study is the first that considers travel sequences in FLP. We propose an MINLP model for minimizing the total travel distance with connected movements as input data. We are not aware of studies that investigate the effect of a possible underestimation of travel distance, when the layout is based on only pairwise distances. We incorporate this by calculating the exact total travel distance when departments have multiple IO points and connected movements.

3 Layout Description, Assumptions, and Optimization Model

This section formulates a mathematical model and introduces key notations and main assumptions for the FLP with sequential movements. The model minimizes total travel distance. The model inputs are the number of departments, number of IO points per department, area of departments, maximum and minimum allowed aspect ratios of each department, and travel sequences (rather than pairwise flows). The decision variables are the positions of IO points on the periphery of the departments, the aspect ratios of departments, the sequence in which IO points should be visited for each connected movement, and the location of departments. The dimensions of each department in the layout are decision variables that depend on the maximum and minimum allowed aspect ratios and total needed area for that department (i.e., $e_n \leq \frac{W_n}{H_n} \leq u_n$).

To illustrate how the travel distance of each connected movement can be calculated, consider a travel sequence $[1, 3, 2, 3]$ in which Department 1 has IO_1 and IO_2, Department 2 has IO_3 and IO_4, and Department 3 has IO_5. To complete this sequence, a worker starts the route from IO_1 or IO_2 and travels to IO_5. Then the worker proceeds to IO_3 or IO_4. Finally, the worker completes the tour by traveling from the chosen IO point in the previous step (IO_3 or IO_4) to IO_5. The total travel distance corresponding to this sequence equals:
$\min : \Big\{(D_{1,5} + D_{5,3} + D_{3,5}), (D_{1,5} + D_{5,4} + D_{4,5}), (D_{2,5} + D_{5,3} + D_{3,5}), (D_{2,5} + D_{5,4} + D_{4,5})\Big\}$ where $D_{j,j'}$ is the rectilinear travel distance between IO points j and j'. Table 4 describes the notations. The main assumptions in this study are as follows.

Assumption 1. All departments are rectangular. Departments cannot overlap, but can vary in aspect ratios (the width and height of each department are decision variables that depend on the aspect ratios and required areas, i.e., $e_n \leq \frac{W_n}{H_n} \leq u_n$ and $W_n \times H_n = a_n$).

Assumption 2. IO points are located on the periphery of departments. The distance between IO points is calculated based on the rectilinear distance.

Assumption 3. Travel aisles are not considered in the model. However, we show how to compensate for this exclusion based on the focal company's layout in Section 6 (by adding a main U-shape aisle).

Each sequence, r, contains the departments that a worker visits in a connected movement to complete the tasks. In matrix π^r, the rows indicate the departments that appear in sequence r in chronological order. It means that the first row corresponds to the first department that should be visited in the sequence. The columns represent the IO points. Element $\pi^r_{o^r,j}$ of π^r equals 1 if IO point j is associated with the department visited in position o^r of the sequence r, otherwise, it is 0. Assume sequence $[1, 2, 1, 3]$ in the layout of Example 1. In this case, π^r equals:

$$\pi^r = \begin{array}{cc} & \begin{array}{ccccccc} \text{IO}_1 & \text{IO}_2 & \text{IO}_3 & \text{IO}_4 & \text{IO}_5 & \ldots & \text{IO}_J \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 1 \\ 3 \end{array} & \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & \cdot & 0 \\ 0 & 0 & 1 & 1 & 0 & \cdot & 0 \\ 1 & 1 & 0 & 0 & 0 & \cdot & 0 \\ 0 & 0 & 0 & 0 & 1 & \cdot & 0 \end{bmatrix} \end{array}$$

3.1 Integrated Location and Worker Routing Decisions Model

Model 1 minimizes the total travel distance by optimizing the facility layout based on sequences. The decision variables are the horizontal and vertical sizes of departments, the location of each department, the location of the IO points on the periphery of the departments, and the combination of IO points that should be visited in each sequence. Model 1 integrates these decisions to minimize

Table 4: Notations

Sets	Description
$\mathcal{N} = \{1, 2, ..., N\}$	Set of departments, indices n, n'.
$\mathcal{J} = \{1, 2, ..., J\}$	Set of IO points, indices j, j', j''
$\mathcal{R} = \{1, 2, ..., R\}$	Set of sequences, index r
$\mathcal{O}_r = \{1, 2, ..., l_r\}$	Set of steps in which IO points are visited in sequence r, index o^r.
Parameters	
M	A big number
N	Number of departments
R	Number of sequences
J	Total number of IO points in the problem (i.e. $J = \sum_n v_n$)
b^x	Length of the floor on x-axis
b^y	Length of the floor on y-axis
f_r	Frequency of sequence r
u_n	Maximum allowed aspect ratio of department n
e_n	Minimum allowed aspect ratio of department n
a_n	Area of department n
v_n	Number of IO points of department n
l_r	Length of sequence r
$\pi^r_{o^r,j}$	1, if IO point j can be visited in step o^r of sequence r to complete this sequence, 0 otherwise.
π^r	A matrix containing binary parameters $\pi^r_{o^r,j}$ associated with sequence r
$g_{j,n}$	Binary input parameter indicating if IO point j is associated with department n
Decision variables	
$X^r_{o^r,j,j'}$	1, if in the o^{th} step of the sequence r the worker leaves IO point j to go to IO point j', 0 otherwise.
$\theta_{j,n}$	1, if IO point j is located on the upper side of the rectangular department n, 0 otherwise.
$\theta'_{j,n}$	1, if IO point j is located on the lower side of the rectangular department n, 0 otherwise.
$\lambda_{j,n}$	1, if IO point j is located on the right side of the rectangular department n, 0 otherwise.
$\lambda'_{j,n}$	1, if IO point j is located on the left side of the rectangular department n, 0 otherwise.
$\delta_{n,n'}$	1, if departments n and n' are divided horizontally, 0 otherwise.
$\gamma_{n,n'}$	1, if departments n and n' are divided vertically, 0 otherwise.
W_n	Length of department n on x-axis
H_n	Length of department n on y-axis
C^x_n	Center location of department n on x-axis
C^y_n	Center location of department n on y-axis
I^x_j	Location of IO point j on x-axis
I^y_j	Location of IO point j on y-axis
$D_{j,j'}$	Rectilinear distance between IO point j and j'

the total travel distance based on the sequence method.

Model 1.

$$min\ Z_1 = \sum_{r\in\mathcal{R}} \sum_{o^r\in\mathcal{O}_r} \sum_{j\in\mathcal{J}} \sum_{j'\in\mathcal{J}} f_r \times D_{j,j'} \times X^r_{o^r,j,j'} \tag{1}$$

subject to

$$W_n \geq \sqrt{a_n \times e_n} \quad \forall n \in \mathcal{N} \tag{2}$$

$$W_n \leq \sqrt{a_n \times u_n} \quad \forall n \in \mathcal{N} \tag{3}$$

$$W_n \times H_n = a_n \quad \forall n \in \mathcal{N} \tag{4}$$

$$C^x_n + W_n \leq b^x \quad \forall n \in \mathcal{N} \tag{5}$$

$$C^y_n + H_n \leq b^y \quad \forall n \in \mathcal{N} \tag{6}$$

$$\mid C^x_n - C^x_{n'} \mid \geq \frac{W_n + W_{n'}}{2} - (1 - \delta_{n,n'}) \times M \quad \forall n, n' \in \mathcal{N}, n' \neq n \tag{7}$$

$$\mid C^y_n - C^y_{n'} \mid \geq \frac{H_n + H_{n'}}{2} - (1 - \gamma_{n,n'}) \times M \quad \forall n, n' \in \mathcal{N}, n' \neq n \tag{8}$$

$$\delta_{n,n'} + \gamma_{n,n'} \geq 1 \quad \forall n, n' \in \mathcal{N}, n' \neq n \tag{9}$$

$$I_j^x \leq C_n^x + \frac{W_n}{2} + (1 - \theta_{j,n} - \theta'_{j,n}) \times M + (1 - g_{j,n}) \times M \quad \forall j \in \mathcal{J}, n \in \mathcal{N} \tag{10}$$

$$I_j^x \geq C_n^x - \frac{W_n}{2} - (1 - \theta_{j,n} - \theta'_{j,n}) \times M - (1 - g_{j,n}) \times M \quad \forall j \in \mathcal{J}, n \in \mathcal{N} \tag{11}$$

$$I_j^y \leq C_n^y + \frac{H_n}{2} + (1 - \theta_{j,n}) \times M + (1 - g_{j,n}) \times M \quad \forall j \in \mathcal{J}, n \in \mathcal{N} \tag{12}$$

$$I_j^y \geq C_n^y + \frac{H_n}{2} - (1 - \theta_{j,n}) \times M - (1 - g_{j,n}) \times M \quad \forall j \in \mathcal{J}, n \in \mathcal{N} \tag{13}$$

$$I_j^y \leq C_n^y - \frac{H_n}{2} + (1 - \theta'_{j,n}) \times M + (1 - g_{j,n}) \times M \quad \forall j \in \mathcal{J}, n \in \mathcal{N} \tag{14}$$

$$I_j^y \geq C_n^y - \frac{H_n}{2} - (1 - \theta'_{j,n}) \times M - (1 - g_{j,n}) \times M \quad \forall j \in \mathcal{J}, n \in \mathcal{N} \tag{15}$$

$$I_j^x \leq C_n^x - \frac{W_n}{2} + (1 - \lambda_{j,n}) \times M + (1 - g_{j,n}) \times M \quad \forall j \in \mathcal{J}, n \in \mathcal{N} \tag{16}$$

$$I_j^x \geq C_n^x - \frac{W_n}{2} - (1 - \lambda_{j,n}) \times M - (1 - g_{j,n}) \times M \quad \forall j \in \mathcal{J}, n \in \mathcal{N} \tag{17}$$

$$I_j^x \leq C_n^x + \frac{W_n}{2} + (1 - \lambda'_{j,n}) \times M + (1 - g_{j,n}) \times M \quad \forall j \in \mathcal{J}, n \in \mathcal{N} \tag{18}$$

$$I_j^x \geq C_n^x + \frac{W_n}{2} - (1 - \lambda'_{j,n}) \times M - (1 - g_{j,n}) \times M \quad \forall j \in \mathcal{J}, n \in \mathcal{N} \tag{19}$$

$$I_j^y \leq C_n^y + \frac{H_n}{2} + (1 - \lambda_{j,n} - \lambda'_{j,n}) \times M + (1 - g_{j,n}) \times M \quad \forall j \in \mathcal{J}, n \in \mathcal{N} \tag{20}$$

$$I_j^y \geq C_n^y - \frac{H_n}{2} - (1 - \lambda_{j,n} - \lambda'_{j,n}) \times M - (1 - g_{j,n}) \times M \quad \forall j \in \mathcal{J}, n \in \mathcal{N} \tag{21}$$

$$\theta_{j,n} + \theta'_{j,n} \leq g_{j,n} \quad \forall j \in \mathcal{J}, n \in \mathcal{N} \tag{22}$$

$$\lambda_{j,n} + \lambda'_{j,n} \leq g_{j,n} \quad \forall j \in \mathcal{J}, n \in \mathcal{N} \tag{23}$$

$$\theta_{j,n} + \theta'_{j,n} + \lambda_{j,n} + \lambda'_{j,n} \geq g_{j,n} \quad \forall j \in \mathcal{J}, n \in \mathcal{N} \tag{24}$$

$$D_{j,j'} = \mid I_j^x - I_{j'}^x \mid + \mid I_j^y - I_{j'}^y \mid \quad \forall j, j' \in \mathcal{J} \tag{25}$$

$$X^r_{o^r,j,j'} \leq \pi^r_{o^r,j} \times \pi^r_{o^r+1,j'} \quad \forall \quad o^r \in \mathcal{O}_r - \{l_r\}, j \in \mathcal{J}, j' \in \mathcal{J}, r \in \mathcal{R} \tag{26}$$

$$\sum_{j=1}^{J} \sum_{j'=1}^{J} X^r_{o^r,j,j'} = 1 \quad \forall o^r \in \mathcal{O}_r - \{l_r\}, r \in \mathcal{R} \tag{27}$$

$$\sum_{j=1}^{J} X^r_{o^r,j,j'} - \sum_{j''=1}^{J} X^r_{o^r+1,j',j''} = 0 \quad \forall o^r \in \mathcal{O}_r - \{l_r - 1, l_r\}, j' \in \mathcal{J}, r \in \mathcal{R} \tag{28}$$

$$\sum_{i^r=1}^{l_r} \sum_{j=1}^{J} \sum_{j'=1}^{J} X^r_{o^r,j,j'} = l_r - 1 \quad \forall r \in \mathcal{R} \tag{29}$$

$$X^r_{o^r,j,j'} \in \{0,1\} \tag{30a}$$

$$\theta_{j,n}, \theta'_{j,n}, \lambda_{j,n}, \lambda'_{j,n}, \gamma_{n,n'}, \delta_{n,n'} \in \{0,1\} \tag{30b}$$

Objective function (1) minimizes the total travel distance based on the given sequences. Constraints (2), (3) and (4) determine the dimensions of each department based on the needed area, and the maximum and minimum allowed aspect ratios for each department. Constraints (5) and (6) guarantee that all departments are located within the boundary of the rectangular floor. Based on Constraint (7), the departments are divided horizontally and based on Constraint (8), they are divided vertically. Constraint (9) ensures that the departments do not overlap with each other by controlling the relaxation of Constraints (7) and (8). Departments can be divided vertically via relaxing Constraint (7), horizontally via relaxing Constraint (8), or diagonally without relaxing any of them. Constraints (10) to (21) determine the location of IO points in the problem. Parameter $g_{j,n}$ relaxes Constraints (10) to (21) when IO point j is not an IO point of department n. Constraints (10) and (11) determine the possible locations of an IO point on the x-axis when the IO point should be located on the upper or lower side of the department. Constraints (12) and (13) control the location of each IO point on the y-axis when the IO point is located on the upper side of the department. Constraints (14) and (15) determine the location of each IO point on the y-axis when the IO point is located on the lower side of the department. Constraints (16) and (17) determine the location of an IO point on the x-axis when the IO point is located on the right side of the department. Constraints (18) and (19) control the location of each IO point on the x-axis when the IO point is located on the left side of the department. Constraints (20) and (21) show the possible locations of an IO point on the y-axis when the IO point is located on the right or left side of the department. Based on Constraint (22), if IO point j is an IO point of department n, it can be located on the upper or lower side of that department. Based on Constraint (23), if IO point j is an IO point of department n, it can be located on the right or left side of that department. Based on Constraint (24), if IO point j is an IO point of department n, it should be located on at least one of the sides of that department. Note that an IO point can be located on two sides of a department when it is at the corner of a rectangular department. Constraint (25) calculates the distance between each pair of IO points.

Constraints (26) to (29) deal with finding the optimal combination of IO points that should be visited in the problem to complete the sequences. Constraint (26) controls the order of the departments that should be visited to complete sequences. It states that it is possible to go from IO point j of the current department to IO point j', only if the IO point j' is an IO point of the subsequent department in sequence r. Based on constraint (27), a worker must leave each department after entering it, excluding the last department. By arriving at the last department, the sequence is complete and there is no need to leave that department. Constraint (28) states that when a worker enters an IO point of a department, the worker should leave that department from exactly the same IO point, excluding the last department's IO points in the sequence. Based on constraint (29), the total number of movements needed for completing the connected movements of a sequence is controlled. Finally, Constraint (30) controls the values taken by the binary decision variables.

Model 1 is a non-linear version of the FLP MIP model with additional constraints on the routing of the workers. Solving the proposed model is computationally intensive even for small problem instances. In this study, we use a modified version of the state-of-the-art algorithm BRKGA, which is a constructive heuristic algorithm proposed by Gonçalves and Resende (2015), to determine the layouts based on pairwise and sequence methods.

4 Structural Results

As shown in Example 1, the pairwise method may underestimate the total worker travel distance in a layout. This section derives theoretical results associated with connected movements to highlight the benefits of using sequences instead of pairwise movements. We derive sufficient conditions under which the pairwise method fails to accurately calculate the total travel distance. Section 4.1 presents Models 2 and 3 that find the minimum travel distance in a given layout based on the sequence and the pairwise methods, respectively. Then, we determine when the pairwise method underestimates and when it accurately finds the total travel distance. We also provide the range of the possible underestimation in travel distance. Furthermore, Section 4.3 calculates the time complexity of finding the minimum travel distance based on a given sequence in a given layout.

4.1 Worker Routing Decision Models in a Given Layout

Model 1 can calculate the minimum travel distance based on sequences in a given layout, by selecting the optimal IO points that should be visited. When the layout is given, Constraints (2) to (25) that are related to the facility layout, can be dropped. In addition, the distance between each pair of IO points is no longer a decision variable but a fixed parameter based on the given layout. Therefore, we replace $D_{j,j'}$ by $t_{j,j'}$, the rectilinear distance between IO point j and j'. Model 2 minimizes the total travel distance based on sequences in a given layout by choosing the optimal combination of IO points that should be visited to complete each sequence.

Model 2.

$$min\ Z_2 = \sum_r \sum_{o^r} \sum_j \sum_{j'} f_r \times t_{j,j'} \times X^r_{o^r,j,j'} \tag{31}$$

S.t.

Constraints (26) - (30a)

Based on the pairwise method, a worker can arrive at one IO point of a department and leave that department from another IO point to reach the next department on the route. Entering a department by using an IO point and leaving it using another IO point is the source of underestimating the travel distances. In other words, to minimize the total travel distance based on the pairwise method, a given layout and a given sequence, we can use Model 2 but drop Constraint (28) that enforces the worker to leave each department from the same IO point that was used to enter it. Model 3 finds the minimum travel distance in a given layout based on pairwise flows and travel distances. This model chooses the closest pairs of IO points between departments to complete the route, which is exactly equal to the solution of a traditional pairwise method. Table 5 compares Models 1 to 3.

Model 3.

$$min\ Z_3 = \sum_{r} \sum_{o^r} \sum_{j} \sum_{j'} f_r \times t_{j,j'} \times X^r_{o^r,j,j'} \tag{32}$$

S.t.

$$\text{Constraints (26) - (27) , (29), (30a)}$$

Table 5: Model comparison

	Model 1	Model 2	Model 3
Objective function	Total travel distance based on sequences	Total travel distance based on sequences	Total travel distance based on pairwise movements
Layout	Optimizes the layout	Accepts a given layout as input	Accepts a given layout as input
IO points	Determines the optimal location of IO points	Accepts the given location of IO points as input	Accepts the given location of IO points as input
Decisions	- Optimal location of departments - Optimal width and height of departments - Optimal location of IO points - Optimal combination of IO points that should be visited to complete each connected movement	- Optimal combination of IO points that should be visited to complete each connected movement	- Choosing the closest pair of IO points between each pair of departments

Consider the layout of Figure 1. Assume the only input sequence is $r = [1, 2, 3]$ with frequency of one. Therefore, based on the pairwise method (from-to flows), in this problem the worker goes from Department 1 to Department 2 once, and from Department 2 to Department 3 once. The outputs of Models 2 and 3, and the traditional pairwise method are equal to:

$$\text{Model 2. } Z_2^* = \min \begin{cases} t_{1,3} + t_{3,5} \\ t_{1,4} + t_{4,5} \\ t_{2,3} + t_{3,5} \\ t_{2,4} + t_{4,5} \end{cases} \quad \text{Model 3. } Z_3^* = \min \begin{cases} t_{1,3} + t_{3,5} \\ t_{1,4} + t_{4,5} \\ t_{2,3} + t_{3,5} \\ t_{2,4} + t_{4,5} \\ t_{1,3} + t_{4,5} \\ t_{1,4} + t_{3,5} \\ t_{2,3} + t_{4,5} \\ t_{2,4} + t_{3,5} \end{cases} \quad \text{Pairwise. } Z'^* = \begin{cases} \min(t_{1,3},\ t_{1,4},\ t_{2,3},\ t_{2,4}) \\ \quad + \\ \min(t_{3,5},\ t_{4,5}) \end{cases}$$

As can be seen, the output of the Model 3 with the sequence r as the input equals the output of the pairwise method with from-to flows derived from the sequence as input. We now derive theoretical results, and use Model 3 to replicate the results of the pairwise method in this section. Lemma 1 states the conditions under which the traditional pairwise method of the FLP may underestimate the real travel distance in a given layout.

Lemma 1. *For a layout in which at least one of the departments has more than one I/O point, the optimal travel distance in the sequence method cannot be less than that in the pairwise method.*

Proof. We have to show that Z_3^*, under conditions (26)-(27), (29) is less or equal to Z_2^*, under the conditions (26)-(29), with $X^r_{o^r,j,j'}$ binary.
Let $X^r_{o^r,j,j'}$ satisfy conditions (26)-(29). Now $X^r_{i,j,j'}$ also satisfies conditions (26)-(27), (29). So $Z_2^* = \min\left\{Z_2\left(X\right)|X \text{ satisfies conditions (26)-(29)}\right\} \geq \min\left\{Z_3\left(X\right)|X \text{ satisfies conditions(26)} - \text{(27)}, \text{(29)}\right\} = Z_3^*$. □

Lemma 1 immediately leads to the following corollary.

Corollary 1. *When Constraint* (28) *is redundant, the optimal travel distance in both the sequence method and the pairwise method is the same.*

This follows from the fact that $\left\{X|X \text{ satisfies conditions(26)} - (29)\right\} = \left\{X|X \text{ satisfies conditions(26)} - (27), (29)\right\}$ when constraint (28) is redundant. The following sufficiency conditions make Constraint (28) redundant.

Sufficiency Condition 1: When all the departments have exactly one IO point. Note that constraint (28) in Model 2 is redundant if all the departments have one IO point.

Sufficiency Condition 2: When the length of all input sequences equals 2. When the length of all sequences equals 2, Constraint (28) in Model 2 is redundant, since $\mathcal{O}_r - \{l_r - 1, l_r\} = \emptyset \; \forall r$.

Sufficiency Condition 3: When all sequences consist of just two departments in each sequence.

Lemma 2. *For a given layout and travel sequences consisting of only two departments, the optimal travel distance in both the sequence method and the pairwise method is the same.*

Proof. We have to find optimal solution X_3^* of Model 3, with value Z_3^*, and show the solution satisfies conditions (28) as well. The found solution is then an optimal solution of Model 2 and then we have $Z_2^* = Z_3^*$.

Take an arbitrary travel sequence $r = [1, 2, 1, 2, ..., 1/2]$ with length of l_r ($l_r = 2k$ if l_r is even, or $l_r = 2k+1$ if l_r is odd, with $k = 1, ..., \lfloor l_r/2 \rfloor$), and label the first department in r as Department 1 and the other as Department 2.
Let d be the minimum distance between the IO points of Departments 1 and 2. Then, the minimum pairwise travel distance based on sequence r, $Z_3^* = (2k-1) \times d$, if l_r is even, or $Z_3^* = 2k \times d$ if l_r is odd. Note that Constraints (28) are met as well since the same IO point is chosen for entering and leaving an intermediate department. We already proved that $Z_2^* \geq Z_3^*$ and this solution is also

a solution for Model 2, therefore we have $Z_2^* = Z_3^*$. Since we took an arbitrary travel sequence, we can repeat the above arguments for all other travel sequences.

$\square$

Using the pairwise method may underestimate the total travel distance in a given layout. As a result, the optimal pairwise method solutions for the FLP may not be optimal in practice. It is important to know the degree of inaccuracy of travel distance in the pairwise method. The range of underestimation is determined in Lemma 3. Under the conditions of Corollary 1, when Constraint (28) is not an active condition, we have $Z_2^* = Z_3^*$. In general, based on Lemma 1, we have $Z_2^* \geq Z_3^*$. In particular, $Z_2^* - Z_3^* > 0$, when the solution of Model 3 violates Constraint (28) in Model 2. That is, when a transporter arrives at a department from an IO point, but leaves that department using another IO point based on a solution provided by the pairwise method. As a result, the underestimation occurs in intermediary departments where a transporter enters and then leaves, but does not occur in the first and last departments of a sequence.

Lemma 3. *For a given travel sequence r of length 3 in which the first and the last departments have one IO point and the second department has two IO points, the maximum gap between $Z_2^* - Z_3^*$ equals $W_{r_2} + H_{r_2}$ and the minimum gap between $Z_2^* - Z_3^*$ equals 0.*

Proof. As rectilinear distances are considered, we calculate the gap on each axis independently. Let $p_{i,k}^x$ be the location of IO point i of the k^{th} department in the sequence. Therefore, we have $p_{i,k}^x \in \left[C_k^x - \frac{W_k}{2}, C_k^x + \frac{W_k}{2}\right]$. Assume that, $p_{1,2}^x \leq p_{2,2}^x$ and $p_{1,1}^x \leq p_{1,3}^x$. In this case, $Z_3^{*x} = t_{p_{1,1}^x,p_{1,2}^x} + t_{p_{2,2}^x,p_{1,3}^x}$, while $Z_2^{*x} = \min\left(t_{p_{1,1}^x,p_{1,2}^x} + t_{p_{1,2}^x,p_{1,3}^x},\ t_{p_{1,1}^x,p_{2,2}^x} + t_{p_{2,2}^x,p_{1,3}^x}\right)$. Therefore, $Z_2^{*x} - Z_3^{*x} = \min\left(t_{p_{1,2}^x,p_{1,3}^x} - t_{p_{2,2}^x,p_{1,3}^x}, t_{p_{1,1}^x,p_{2,2}^x} - t_{p_{1,1}^x,p_{1,2}^x}\right)$. As $p_{i^k,k}^x \in \left[C_k^x - \frac{W_k}{2}, C_k^x + \frac{W_k}{2}\right]$, we have Maximize$\left(Z_2^{*x} - Z_3^{*x}\right) = W_2$ in both cases. In a similar fashion, it can be shown that Maximize$\left(Z_2^{*y} - Z_3^{*y}\right) = H_2$. Therefore, Maximize$\left(Z_2^* - Z_3^*\right) = W_2 + H_2 = W_{r_2} + H_{r_2}$.
In addition, $Z_2^{*x} - Z_3^{*x} = 0$ if $p_{1,1}^x = p_{1,2}^x = p_{2,2}^x = p_{1,3}^x$. In a similar fashion, it can be shown that Minimize$\left(Z_2^{*y} - Z_3^{*y}\right) = 0$. Therefore, Minimize$\left(Z_2^* - Z_3^*\right) = 0$. $\square$

Based on Lemma 3, maximum underestimation occurs when departments have diagonal positions with respect to each other, and the IO points of the intermediate departments have maximum distance (on the opposite corners of the rectangle). In addition, when departments in the connected movements have more IO points (> 1 for $k = 1, 3$ and > 2 for $k = 2$) we still have Maximize$\left(Z_2^{*x} - Z_3^{*x}\right) = W_{r_2}$ by choosing $p_1'^x = p_{1,1}^x$ and $p_{1,3}^x = p_3''^x$ or $p_1''^x = p_{1,1}^x$ and $p_{1,3}^x = p_3'^x$ where $p_k'^x$ is the $\max(p_{i^k,k}^x)$ $\forall i^k$ and $p_k''^x$ is the $\min(p_{i^k,k}^x)$ $\forall i^k$. Note that for sequences of length greater than three, we would have $Z_2^* - Z_3^* \leq \sum_{i=2}^{l_r-1} W_{r_i} + H_{r_i}$. The underestimation is maximum when departments are divided diagonally and the IO points of consecutive departments in the sequence are located in opposite corners of departments.

4.2 Travel Distance Bounds

The aim of this section is to provide upper and lower bounds for the total travel distance for pairwise flows and connected movements.

Upper bound

To obtain an upper bound, we first assume that each department has only one IO point located at the top left of the department. In this case, the total travel distance based on connected movements equals the distance between top left corners of consecutive departments that are visited in connected movements. Since we use the rectilinear travel distance, it is possible to calculate the horizontal and vertical travel distances separately, and find the following upper bound:

$$\mathrm{UB} = \sum_{r\in\mathcal{R}} \sum_{o^r=1}^{l_r-1} \left| \left(C^x_{o^r} - \frac{W_{o^r}}{2} \right) - \left(C^x_{o^r+1} - \frac{W_{o^r+1}}{2} \right) \right| + \sum_{r\in\mathcal{R}} \sum_{o^r=1}^{l_r-1} \left| \left(C^y_{o^r} - \frac{H_{o^r}}{2} \right) - \left(C^y_{o^r+1} - \frac{H_{o^r+1}}{2} \right) \right|$$

When all departments have exactly one IO point, Corollary 1 states that total travel distance based on the pairwise and sequence methods are equal.

Lower bound

To obtain a lower bound, we assume that each department has an infinite number of IO points, such that, each location on the periphery of a department can serve as an IO point. Then, define $\beta^w_{n,n'}$ and $\beta^h_{n,n'}$, as the horizontal and vertical distances between the closest points of departments n and n', respectively:

$$\beta^w_{n,n'} = \max \left\{ 0, \left| C^x_n - C^x_{n'} \right| - \left(\frac{W_n + W_{n'}}{2} \right) \right\}$$

$$\beta^h_{n,n'} = \max \left\{ 0, \left| C^y_n - C^y_{n'} \right| - \left(\frac{H_n + H_{n'}}{2} \right) \right\}$$

A lower bound for the total travel distance in the pairwise method, LB_p, equals the sum of distances between the closest points of consecutive departments in all connected movements, when each department has an infinite number of IO points, or:

$$\mathrm{LB}_p = \sum_{r\in\mathcal{R}} \sum_{o^r=1}^{l_r-1} \left(\beta^h_{o^r,o^r+1} + \beta^w_{o^r,o^r+1} \right)$$

From LB_p, we can also obtain a lower bound for sequences, LB_s. When the previous and next departments are located at two different sides of a middle department, it is necessary to traverse the width (or height) of the middle department to complete the connected movements. Define the following indicator functions:

$\omega^l_{n,n'} = \mathbb{1} \left\{ n \text{ is located on the left of department } n' \right\}$

$\omega^r_{n,n'} = \mathbb{1}\left\{ n \text{ is located on the right of department } n' \right\}$
$\omega^t_{n,n'} = \mathbb{1}\left\{ n \text{ is located on the top of department } n' \right\}$
$\omega^b_{n,n'} = \mathbb{1}\left\{ n \text{ is located on the bottom of department } n' \right\}$

Based on these indicator functions, we find:

$$\begin{aligned}
\text{LB}_s = \sum_{r\in\mathcal{R}} \sum_{o^r=1}^{l_r-1} \left(\beta^h_{o^r,o^r+1} + \beta^w_{o^r,o^r+1}\right) \\
+ \sum_{r\in\mathcal{R}} \sum_{o^r=2}^{l_r-1} \left[(\omega^l_{o^r,o^r+1} \times \omega^r_{o^r-1,o^r}) + (\omega^r_{o^r,o^r+1} \times \omega^l_{o^r-1,o^r}) \right] \times W_{o^r} \\
+ \sum_{r\in\mathcal{R}} \sum_{o^r=2}^{l_r-1} \left[(\omega^t_{o^r,o^r+1} \times \omega^b_{o^r-1,o^r}) + (\omega^b_{o^r,o^r+1} \times \omega^t_{o^r-1,o^r}) \right] \times H_{o^r}
\end{aligned}$$

Note that when two consecutive departments in a travel sequence are not divided horizontally (or vertically) it may still be necessary to traverse an additional distance. Since the goal is to obtain a lower bound, this type of additional travel distance is neglected.

4.3 Time Complexity of Finding the Travel Distance of a Sequence

Calculating the minimum travel distance of a given route is equivalent to solving a shortest path problem, where one IO point must be chosen for each stage (department). We therefore first need to define the graph corresponding to the problem. Assume sequence r with length l_r, $r = [r_1, r_2, ..., r_{l_r}]$, and a layout in which all departments have the same number of IO points equal to v. Figure 4 depicts the graph for finding the shortest path of these connected movements. Two dummy nodes are defined for the start and the end of the route, with distances 0 to the first and last departments of the sequence, respectively. Each vertex in the graph represents an IO point. In addition, the weight of each edge equals the distance between the IO points which the edge connects. As a result, the graph has $(l_r \times v) + 2$ nodes and $(v^2 \times (l_r - 1)) + 2v$ edges. Table 6 shows the time complexity of three well-known algorithms for finding the shortest path of this sequence, based on Wang (2018).

Table 6: Time complexity

Approach	Dijkstra's algorithm	The Bellman-Ford algorithm	The Floyd-Warshall algorithm
Time complexity	$O(V^2)$	$O(VE)$	$O(V^3)$
$V = (l_r \times v) + 2 \qquad E = (v^2 \times (l_r - 1)) + 2v$			

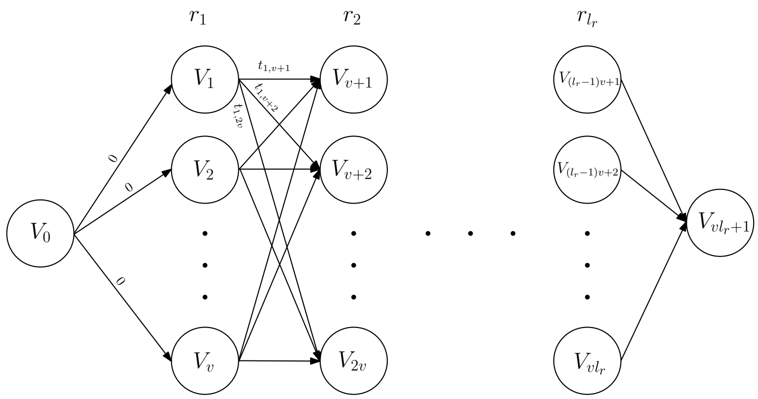


Figure 4: Shortest path problem graph

Finding the travel distance, i.e., the shortest path of all sequences can be time-consuming, even when using the best possible approach, since this algorithm should be used for every connected movement in the problem. To have acceptable calculation time, long routes may be broken into smaller travel subsequences without affecting the total travel distance in the whole route. As an example, assume a worker visits the departments in the facility plant in sequence $r = [\underline{4, 1, 2, 4}, 3, \underline{4, 1, 2, 4}]$ and Department 4 has just one IO point whereas the other departments have more than one IO point. In this case, it is possible to break this sequence into two different subsequences in which first subsequence, $[4, 1, 2, 4]$, has a frequency of two and the second one, $[4, 3, 4]$, has a frequency of one. These extracted travel subsequences then can be used as an input to the optimization model.

5 Methodology

Solving the FLP based on connected movements and multiple IO points per department is at least as hard as solving the traditional FLP, which is a known NP-hard problem. Therefore, we use the state-of-the art genetic algorithm, BRKGA, introduced by Gonçalves and Resende (2015). We make two main modifications since the primary algorithm proposed by Gonçalves and Resende (2015) is based on pairwise analysis and without considering IO points. First, we add a segment to each chromosome that determines the location of the IO points on the periphery of departments. The second modification is related to calculating the fitness of each chromosome based on the pairwise or the sequence method while considering the location of IO points. Instead of center-to-center distances of departments, we use the distance of the closest pair of the IO points between each pair of departments to form the distance matrix in the pairwise method. Instead of using from-to movements, we now use travel sequences as input for the sequence model and calculate the fitness of chromosomes by finding a shortest path that can be traversed to complete each travel sequence. Appendix B describes the steps taken to calculate the total travel distance in detail. Figure 10 in Appendix C shows that each chromosome in the algorithm consists of $2N + 2J + 2$ genes that form four segments. The first segment (gene 1 to gene N) determines the facility placing sequence.

These genes are randomly generated, and the department with a lower associated number is located first. The second segment (gene $N+1$ to gene $2N$) determines the width of each department with respect to required areas, and the maximum and minimum allowed aspect ratios. The third segment (gene $2N+1$ to gene $2N+2J$) is related to the location of IO points, and the fourth segment (gene $2N+2J+1$ and gene $2N+2J+2$) determines the position of the first department that should be located. Algorithm 1 schematically describes the main steps of the modified BRKGA (MBRKGA; the modified parts are marked with *). Appendix C presents details on BRKGA and on the placement strategy proposed by Gonçalves and Resende (2015).

Algorithm 1: MBRKGA: Finding the optimal layout and IO points positions

```
Set parameters;
Generate the initial population;
for g = 1 to Max Generation do
    for p = 1 to Max population do
        Decode the first segment of the chromosome p to determine the facility placement
          sequence;
        Decode the second segment of the chromosome p to determine the width and height
          of each facility;
        Decode the fourth segment of the chromosome p to locate the first department;
        Locate other facilities using the placement procedure;
        *Decode the third segment of the chromosome p to find the location of IO points;
        *Calculate the fitness of the chromosome;
        Update best fitness;
    Selection;
    Crossover;
    Mutation;
    Move best individuals to population g + 1;
Return the best solution.
```

To calculate the improvement achieved by considering sequences instead of pairwise movements, two layouts based on two different objective functions must be found. First, we use the objective function based on sequences (Z_1) to find the best sequenced-based layout. Then, we obtain the best pairwise-based layout by using MRBKGA with an objective function based on pairwise movements (Z_3). This procedure is illustrated in Algorithm 2.

Algorithm 2: Procedure of finding the value of sequences (VS)

Result: *Value of sequences*

Initialization (Determining sequences);

1.*Find the best sequenced-based layout, L_1^*, with the MBRKGA (with objective value Z_1^*)*;

2.*Find the best pairwise-based layout, L_2^*, with the MBRKGA (with objective value Z_3^*)*;

3.*Find the shortest real travel distance, Z_2^*, in pairwise layout L_2^* by calculating the length of each sequence*;

4.$VS = Z_2^* - Z_1^*$

To find the actual travel distance in the pairwise layout L_2^*, (step 3 in Algorithm 2), we need to calculate the shortest possible travel distance to complete each sequence in that layout. As the location of IO points and the sequence of visiting departments is known, the optimal combination of IO points that must be visited to complete each sequence should be found. Since the lengths of travel sequences are relatively short, this optimal combination is calculated by full enumeration of all possible combinations. After finding the optimal route of sequences, Z_2^* is calculated by summing the travel distances of all sequences.

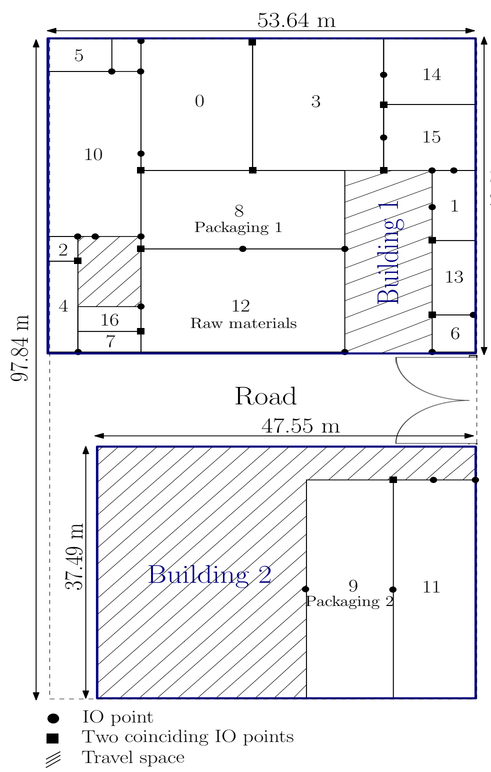
If we just consider pairwise movements, the plant's best layout would be layout L_2^* with a travel distance of Z_3^*. However, Z_3^* is not the actual travel distance, but an underestimate. As a result, by choosing layout L_2^*, the actual travel distance in the problem would equal Z_2^*. However, if we consider sequences, layout L_1^* with an actual objective function equal to Z_1^* would be the best. Therefore, by considering sequences, a layout can be obtained with a total travel distance Z_1^*. The difference, VS $= Z_2^* - Z_1^*$ is the value of considering sequences (VS), that is, the improvement in travel distance achieved by considering the sequences, where Z_2^* is the best actual travel distance in layout L_2^* that is found based on pairwise movements, and Z_1^* is the travel distance in layout L_1^* based on the sequences. Appendix B illustrates the procedure for calculating VS for a small numerical example.

6 Case Study

We apply our method to movement data captured by Aries Agro in its main plant that consists of 17 departments as shown in Figure 5b. The company expects sizeable business growth and is considering a production plant layout change to increase worker productivity and process efficiency. To measure current worker productivity and to map transport flows, Aries Agro has implemented an IoT-based solution provided by Alluvium IoT, `alluvium.in`) that allows accurate time-stamped tracking of all movements. Table 2 illustrates a slice of the collected data. We use worker movement data collected from June 2019 to February 2020 as input for improving the current layout.

6.1 Floor Plan and Current Layout

The area of the floor space is 53.64 m by 97.84 m and the factory consists of two buildings, that are separated by an internal road as depicted in Figure 5a. Figure 5a also shows the current layout of the case study. The door locations of Buildings 1 and 2 can be positioned flexibly, along the outside walls. Therefore, we do not consider the location of department doors in calculations. Currently, the facility's main inbound and outbound doors are located on the right side of the road. Based on the current layout and data acquired on the movements of the personnel, the total average travel distance based on pairwise flows equals 215,871 m over the planning horizon, but the actual total travel distance calculated using the travel sequences equals 249,904 m. According to the manager, the current layout was determined based on an estimate of pairwise travel distances. The raw material storage (Department 12) and packaging departments (Department 8) are located in the center due to the large physical workflow and high contact frequency with other departments.



(a) Current layout of the case study

No.	Department name	Area (m^2)	# IO points
0	Blender machine	279.45	4
1	Boiler area	58.53	4
2	Domino room	13.38	2
3	Granulation area	321.08	3
4	Lab	50.17	2
5	Locker room	37.63	1
6	Motor control room	30.10	2
7	Office	28.99	1
8	First packaging area	296.55	4
9	Secondary packaging area	347.92	4
10	Packing materials area	328.32	4
11	Bulk materials area	347.92	4
12	Raw materials area	398.00	4
13	Rotary drum dryer	56.86	3
14	Spray-dryer area	104.05	4
15	Secondary spray-dryer area	104.05	3
16	Attendance room	28.99	2

(b) Departments of the case study

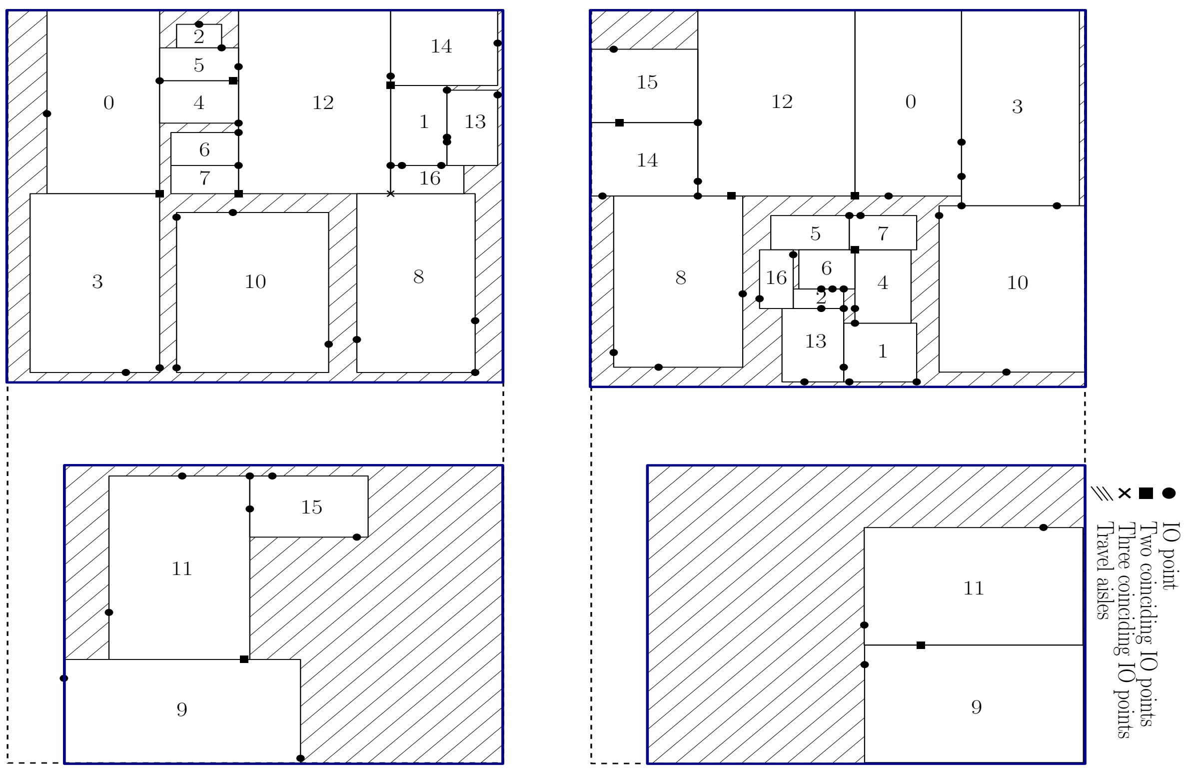
Figure 5: Current layout and departments of the Aries Agro

6.2 Results

We follow the steps suggested by Algorithm 2 to find VS. First we use Algorithm 1 to determine the layout for the case. Figure 5b shows the size of the departments and the number of IO points corresponding to each department. The maximum and minimum allowed aspect ratio for all departments equals 2 and 0.5, respectively. To ensure that the proposed layout of the case study is practical and avoids highly congested traffic, a U-shape main aisle must be included in the middle of the Building 1.

Best Found Layouts

Figures 6a and 6b show the best layout based on pairwise movements and sequences, respectively. Table 7 presents the underestimated total travel distance and actual total travel distance of these layouts over the planning horizon. It shows that a 10.62% improvement on total travel distance can be achieved by considering the sequences (VS $= \frac{Z_2^* - Z_1^*}{Z_1^*} * 100 = 10.62\%$). Compared to the current layout, the proposed layout (Figure 6b) reduces the total travel distance by 57.74%. In sum, leveraging travel sequences cuts the total travel distance by 11,220.11 m compared to the best pairwise layout and by 144,293 m compared to the current layout, leading to a significant productivity improvement.



(a) Pairwise layout

(b) Sequence layout

Figure 6: Layout of the case study problem

7 Sensitivity Analyses

The optimal layouts are affected by the number of IO points per department. In addition, if sequences in a given operation are long, the computation time grows rapidly. Therefore, in this section, we analyze how the number of IO points per department and breaking sequences into subsequences affect Z_1^*, Z_2^*, Z_3^* and the VS in the case study. Appendix D presents the final layouts

Table 7: Results of the case study problem

	Pairwise travel distance	Sequence travel distance	CPU time (min)
Pairwise layout	93,151.08 m (Z_3^*)	116,831.07 m (Z_2^*)	165
Sequence layout	101,164.38 m	105,610.96 m (Z_1^*)	206
Current layout	215,870.64 m	249,904.00 m	-

of each problem instance.

Number of IO points: When each department has one IO point, the outcomes of the pairwise and sequence methods are equal based on Lemma 1 and Sufficiency Condition 2. As Lemma 1 shows, the pairwise method underestimates total travel distance when at least one of the departments has more than one IO point. Table 8 shows Z_1^*, Z_2^*, Z_3^* and VS in a set of problems with an increasing number of IO points per department. In problem instance 1 to 8, we assume that all departments have the same number of IO points. Inputs from the case study problem such as department areas, aspect ratios, and travel sequences are used in these problem instances and only the number of IO points per department differs from the case study problem.

Table 8: Effect of the number of IO points on the VS

		Pairwise method			Sequence method		
Problem instance	Number of IO points (per department)	Z_3^* (m)	Z_2^* (m)	$Z_2^* - Z_3^*$ m (in % compared with Z_3^*)	Z_1^* (m)	$\lvert Z_2^* - Z_1^* \rvert$	VS (%)
1	1	120422.16	120422.16	0.00 (0.00%)	120422.16	0.00	0.00
2	2	79813.02	90337.74	10524.72 (13.19%)	87294.79	3042.95	3.49
3	3	72051.89	88976.13	16924.25 (23.49%)	79655.92	9320.22	11.7
4	4	66855.60	78695.48	11839.88 (17.71%)	77584.13	1111.35	1.43
5	5	68014.10	82726.08	14711.97 (21.63%)	75857.69	6868.38	9.05
6	6	66136.51	83004.32	16867.81 (25.50%)	74811.63	8192.69	10.95
7	7	64441.57	76523.72	12082.14 (18.75%)	73882.75	2640.97	3.57
8	8	62974.01	76497.39	13523.38 (21.47%)	73791.87	2705.52	3.67

Figure 7 shows that by increasing the number of IO points, Z_1^*, Z_2^*, and Z_3^* all decrease. This is obvious, since adding more IO points introduces more options for completing pairwise or sequence movements. However, the decrease in Z_1^* is smaller compared to that in Z_3^*, in particular for layouts with more than three IO points per department. Note that the decrease in Z_1^* is convex and the effect of adding more IO points decreases rapidly. The difference in travel distance in problems with five and eight IO points is less than 3%. The slight increase in Z_3^* from problem instances 4 to 5 is because of using a heuristic approach.

Breaking sequences into subsequences: We now investigate the effect of breaking sequences into subsequences on the $Z_2^* - Z_3^*$ and the impact on additional travel distances that can occur by deviating from the optimal layouts. In addition, we assess the effect of the length of sequences

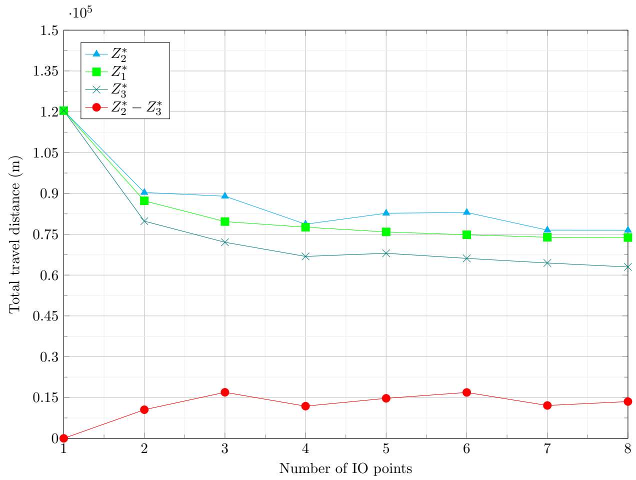


Figure 7: Effect of the number of IO points

on the computation time. To do so, we take five sequences of length 11 with varying frequencies. We distinguish three instances, 9, 10, and 11 in which we break these sequences of length 11 in subsequences of length 2, 3, and 6, respectively. Problem instance 12 contains the original travel sequences of length 11. The best layout based on the sequence method is found for problem instances 9 to 12. Table 16 in Appendix D presents the input sequences of each problem instance. The objective value based on the subsequences is indicated by Z_4^*. We also calculate the length of the original sequences, Z_2^*, in the layouts found for problem instances 9 to 11 to investigate how breaking the sequence into subsequences affects $Z_2^* - Z_4^*$. Note that breaking the sequences into subsequences of length two is identical to solving the problem based on the pairwise method. Table 9 shows the results of these problem instances.

Table 9: Effect of breaking sequences on the underestimation

Problem instance	Length of subsequences	Total travel distance, based on the subsequences, Z_4^* (m)	Total travel distance of the original sequences in the found layout, Z_2^* (m)	$Z_2^* - Z_4^*$ (m) (in % compared with fourth column)	Additional distance occurred (m) (difference between fourth column and highlighted cell)	CPU time (min)
9	2	8770.25	12843.62	4073.37 (31.72%)	991.84	77
10	3	10631.29	12611.37	1980.08 (15.70%)	759.59	75
11	6	11475.32	12268.17	792.85 (6.46%)	416.39	81
12	11	11851.78	11851.78	0 (0%)	0	3281

As Figure 8 shows, the value of $Z_2^* - Z_4^*$ increases when the original sequences are broken into smaller subsequences. Simultaneously, breaking the sequence into shorter sequences leads to a layout with higher actual total real travel distance (Z_2^*) compared to using the original long sequences. However, when sequences of length 6 are used (about half of the original length), the difference in the actual total travel distance between the exact optimal layout (gray cell) and the resulting layout based on the subsequences (Z_2^* in problem instance 11) is only 3.40%. This shows that we can indeed reduce CPU time considerably by breaking long sequences, without compromising too much on the quality of the solution. As Table 9 shows, the computation time increases rapidly with the length of the sequences. The computation effort can be divided into three parts. The first is handling chromosomes, including generation of the initial population, crossover,

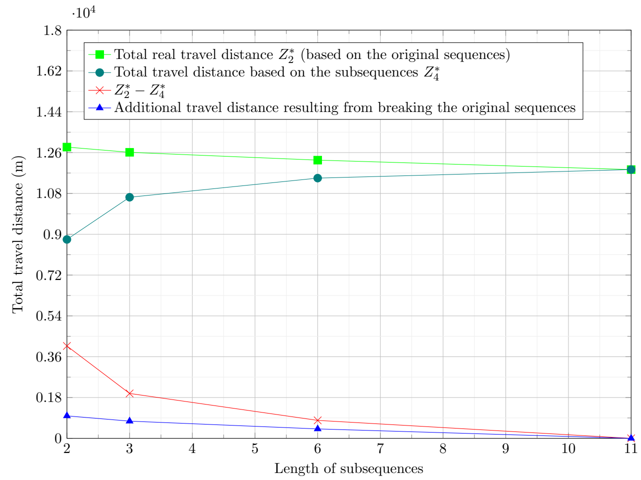


Figure 8: Effect of breaking sequences on the underestimation

and mutation. The second is decoding the chromosomes into the layouts, and the third is related to calculating the objective value of a feasible layout. The computation time of the third part grows exponentially when the length of input sequences increase. Note that the computation time of problem instance 10 is shorter than that of problem instance 9. This stems from the difference between the length of input subsequences (2 versus 3 that is not large). Problem instance 10 has 25 subsequences (of length 3), whereas problem instance 9 has 50 subsequences (of length 2).

8 Policies and Implications of Our Research to Practice

Many popular software tools exist for determining a facility layout. The algorithms behind the tools can be classified into three categories: 1) improvement, 2) construction, and 3) construction and improvement algorithms. The algorithms behind CRAFT, MCRAFT, MULTIPLE, and COFAD use improvement methods. ALDEP and CORELAP construct layouts, and BLOCPLAN and LOGIC construct or improve layouts. Appendix A presents an overview of of these algorithms.

All algorithms used in practice use pairwise relations between departments (e.g., from-to flows or a relationship matrix) as input (see Table 10 in Appendix A). Aries Agro also used pairwise flows to develop its layout by considering rectilinear travel distances. The flows were based on anticipated movements of raw materials and finished products. The raw material storage and packaging areas are located in the middle of the current layout since these departments had high anticipated from-to flows with the other departments. Heavy machines are located on the extreme right side of the current layout since these machines had a high anticipated pairwise flow interaction, and the right side is close to the inbound doors of the facility. However, flows have changed over time and more insights can now be obtained from the IoT. Therefore, optimizing the layout of the factory is required. We discussed the input data and model benefits with the Aries Agro management. The CEO remarked the following:

“The complexity of the operation lies not in the actual production lines, but in the

> scheduling and operations planning required to ensure timely availability of the product line, which faces seasonal pressures. Even with the standard operating procedures for the production lines, there was a gap that was felt in optimizing the layout of the factory and also monitoring the movement of labor for various processes. Data was required for building a more appropriate layout to maximize the productive time, lower sources of time wastage, keep a check on unauthorized movement in various zones and to set standard for monitoring productivity of worker groups. Considering the multiplicity of SKUs, common production lines for several products, interchangeability of workers across production processes, we wanted to create an IoT Platform that would 1) non-intrusively track the movement of workers when on duty within the facility, 2) build a database of per worker movement, downtime, and correlate with the corresponding output information, 3) use monitoring to set up performance standards for more quantitative performance appraisal systems for factory operations, and 4) rearrange machinery in a layout that optimizes the flow of material through the process and minimizes the unproductive movement of labor within the premises.
>
> With the pilot implementation of the IoT system at the Aries Agro's Ahmedabad facility, the data was initially collected for the workers on site over several months. Data on movement was traced back to those wearing ID badges and their individual data became available to the supervisor and factory manager. This IoT system that began as a monitoring tool for workers (beyond the CCTV and watchful eyes of the supervisor) has become a robust, data driven tool for creating more time and motion plans, better task scheduling and progress tracking, and developing an appraisal system based on information tracking. We plan to set up a reward and recognition program for workers with the best metrics for the day and week, with 'best worker of the week/month' recognitions and awards."

After collecting data on workers' sequential travel visits to departments, we identified a novel approach to optimize facility layout based on connected movements instead of traditional pairwise methods. We discussed our facility layout approach with the Aries Agro management team. The management team were interested in learning more about worker movement sequences to identify possible areas of improvement. In particular, understanding the worker travel sequences helped the management to understand the areas of improvement. For example, the raw materials area is located quite far from the blender location. This means that workers are subjected to several to-and-fro movements. Conveyor belts can reduce worker travel and increase their productivity. Although the company may not be able to completely overhaul the current layout, these sequential travel data can help them improve the current layout. Therefore, the management wants to modify the layout where possible to enhance the productivity of their employees. In addition, the management plans to use these insights when designing new sites. Using the appropriate number and location of the IO points could reduce the worker travel distance and improve productivity. In addition, modifying travel paths could have a positive effect on workers' travel times. Finally, the CEO of

Aries commented "if we can successfully implement these changes in one unit, Aries will consider implementing the IoT system in all its manufacturing facilities."

9 Insights and Discussion

Insights: The structural and numerical results of this study provide useful managerial insights which may have implications for practitioners. If the current layout is obtained based on from-to flow data with the pairwise method, and if some departments have more than a single IO point, the real distance that is traversed in the layout may be longer than anticipated. Such additional travel time can lead to longer job throughput times, higher internal transport utilization and, higher costs than anticipated. To assess whether a layout based on connected movements is beneficial, we first need to investigate the movement types. When employees and transporters travel in connected movements and visit more than two departments in a travel path and when departments have more than a single IO point, it may be worth investigating the impact of alternative layouts to see whether travel distances can be reduced.

Unfortunately, the computation time grows exponentially in the length of the sequences of connected movements (see Section 7). Although the path length of sequential departmental visits may be moderate in practice, it may be necessary to break down longer routes into smaller subsequences. When a department has a single IO point, the sequence can be broken at that point without affecting the solution quality (see Section 4.3). Then, it may still be necessary to break them into smaller subsequences. However, this can affect the quality of the solutions and increase costs. We show that there is a minimum impact of cost as long as the sequences are not too short. Increasing the number of IO points per department can also help to decrease the total travel distance, as long as the layout is determined using the sequence-based method. However, the effect of adding more IO points decreases rapidly. Our results indicate that when the number of IO points per department exceeds four (in our case example), the total travel distance based on connected movements hardly decreases further. As IO points require space and may be costly due to investments in facility infrastructure, a careful balance must be struck between the number of IO points per department and the aspired travel distance reduction.

Modifying or (re)designing a layout requires knowledge of the (anticipated) connected travel movements. Acquiring clean and reliable movement data comes with its own challenges. Workers or transport equipment may have sensors, and detection beacons may be positioned in the work zones. However, if the beacons are placed too close to each other, the exact location may be difficult to assess. When placed too far, movements may be missed or be misinterpreted. After placing detection beacons, it is necessary to check the accuracy of the collected data and revise the position of detection beacons if needed to address this challenge and acquire reliable data. Further, tracking employee movements may cause resistance or may be legally prohibited. Employee resistance may be reduced by raising their awareness of the benefits of the IoT system, such as safety and improved worker efficiency. Employees may interfere the tracking process by exchanging their tracking devices, thus botching up the entire data recording process. A camera-based authentication system could prevent this. In addition, as mentioned in Section 1, sensors could detect workers entering a department, while they are just passing by across the aisles around the department. This can be addressed by understanding which departments should be visited during the processes and by careful data cleaning (see the discussion in Section 1).

Extensions: This paper studies a static single floor FLP of a production facility. The method is equally applicable to other facility types, such as offices, warehouses, hospitals, educational, and service facilities. The proposed MBRKGA, which employs connected movements as an input can be extended to multi-floor FLPs. In this case, each floor can be represented by an empty maximal space, and it is possible to consider the locations of elevators on different floors by assuming that an elevator has the same coordinates in each empty maximal space. When the departments are located, the elevators could be added in connected movements between departments that are located on different floors. In this way, a transporter would use an elevator when it travels between the floors. This article studies a static FLP, while in practice, some facility layout decisions such as the arrangement of the workstations may not be static. New product introductions and variations in production volumes require facilities to be flexible and adaptable to changing work requirements. Although maximizing productivity and improving employee welfare is an important organizational goal, the effect of the location of workstations on workforce productivity is rarely assessed after the layout has been established. With new product introductions and fluctuations in product demand, product volumes, flows, and the resulting connected movements may change over time. Hence, the layout redesigning decision should be evaluated more frequently with new data sources such as connected sensors and an IoT network. If implemented appropriately, such a network can collect real-time data without involving human efforts, and is therefore a cost-effective solution for continuous data collection and performance monitoring. Real-time data can then be fed into computer programs to analyze the facility layout in real-time and to notify decision makers when it is beneficial to consider redesigning the layout. Redesigning the layout depends on two major aspects, 1) the possibility of reducing travel distances (or times) with the new layout, and 2) the possibility of reducing costs when redesigning the layout. Mobile robotic storage systems and workstations can be used in production and warehouse facilities to increase layout flexibility and facilitate layout redesigning in the *dynamic* facility layout problem. For example, managers of self-storage warehouses can redesign the modular layout of their warehouse within a relatively short time period, by repositioning the walls between storage compartments (see Gong et al. (2013)).

Limitations: The modified BRKGA used in this study has some limitations. It does not consider travel aisles in the layout and uses rectilinear travel distances for simplicity. Not considering travel aisles leads to an overly compact layout and may inflate the VS to a small extent, and using rectilinear travel distances may result in an inaccurate objective function value. Replacing rectilinear travel distances by contour distances, which is more detailed and accurate, can increase the calculated travel distances and may increase or decrease the VS. In addition, we assume that the shop floor and the departments are rectangular, whereas they can be differently shaped in practice.

These limitations may be overcome by adapting the algorithm, but this will cause increased calculation time and possible intractability. This paper modified the BRKGA for finding layouts based on travel sequences although the placement procedure of the BRKGA was developed based on the pairwise method without IO points. As a result, the quality of the sequence method solutions can be improved by developing a better, tailored, algorithm for the FLP with connected movements. Improving the quality of the results of sequence method will increase the VS as well, since Gonçalves and Resende (2015) have confirmed the quality of the layouts based on the pairwise method. In addition, full enumeration is used in the algorithm of this study to calculate the travel distance of each connected movement when searching the optimal combination of IO points that should be visited to complete each connected movement. Developing a more efficient procedure for finding the travel distance of each connected movement can reduce the solution identification time.

10 Conclusion

This paper studies the FLP with integrated decisions on determining the location of departments, IO points, and worker routes to minimize the total travel distance within a facility. When departments have more than a single IO point and more than two departments are visited in travel paths, the traditional pairwise method underestimates the total travel distance. This study is the first to consider connected worker movements as input to the FLP to calculate accurate travel distances and develop better layouts. Such connected worker movements can be collected in a facility via an IoT network. In our case company, we use nine months of captured data. Our data-driven optimization approach based on the real movements to shows that total travel distances can be reduced by up to 10.6% compared to traditional FLP methods. The structural and numerical results of our paper indicate that 1) the length of the movement paths impact the underestimation and the VS. Using the movement paths and a solution based on these movements paths makes sense only when the lengths of connected movement paths are longer than two. 2) The range of the underestimation depends on the sizes of departments in the connected movements. 3) The effect of adding more IO points per department on reducing travel distances is larger in pairwise-based method than in sequence-based. 4) Breaking sequences into subsequences may lead to layouts with additional travel distances, but it still pays off to reduce calculation times as long as the subsequences are not too short.

Based on the limitations of this study and capability of an IoT system for collecting data, the following directions can be considered for future research. Designing an efficient algorithm to solve FLP with connected movements is a research direction for future studies. A more efficient algorithm might allow us to consider contour distances to achieve more accurate results. Also, as this study shows, the solution time increases exponentially with the length of connected movements when using a full enumeration approach for finding the optimal path of each connected movement. As a result, it can be beneficial to develop an algorithm for finding the shortest route of each sequence, instead of the full enumeration used in this research. Finally, considering connected movements with time-tag data collected by a network of sensors can be used to solve the FLP with reduced congestion as an objective.

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Appendix A Popular FLP Algorithms in Practice

Improvement algorithms usually use the facility's current layout as the initial input layout and try to improve it. Computerized Relative Allocation of Facilities Technique (CRAFT) introduced by Buffa et al. (1964) is among the earliest algorithms for the FLP. This algorithm takes from-to flows and costs and an initial layout as the input and uses discrete representation without restricting departments to rectangular shapes. The objective of this algorithm is determined by multiplying rectilinear distances (from center to center), handling cost, and the flow between each pair of departments. CRAFT improves the initial layout by exchanging the adjacent departments. Automated Layout Design Program (ALDEP) and Computerized Relationship Layout Planning (CORELAP) are well-known construction algorithms. ALDEP, developed by Seehof and Evans (1967), constructs a layout by a discrete representation. This algorithm accepts a department relationship matrix as the input and uses the closeness score as an objective. CORELAP, proposed by Lee and Moore (1967), uses a relationship matrix as the input to construct a layout by a discrete representation whereas allowing irregular shaped departments. The objective of CORELAP is to maximize the closeness score. CORELAP attempts to find the best layout while ALDEP provides decision-makers with a set of layouts by constructing different layouts and reporting their associated scores. In addition, ALDEP solves multi-floor FLPS.

Computerized Facilities Design (COFAD), developed by Tompkins and Reed (1976), is similar to CRAFT except that it considers alternative material handling equipment. COFAD's input requirements include from-to flows, an initial layout, and alternative material handling equipment with associated costs for each movement between pairs of departments. This algorithm aims to minimize cost by jointly identifying the optimal layout and the choice of material handling equipment. MICRO-CRAFT (MCRAFT), proposed by Hosni et al. (1980), is similar to CRAFT but with a slight difference that allows exchanging non-adjacent departments in the improvement process. Another difference between CRAFT and MCRAFT is that the latter uses bands, leading to some restrictions in defining the initial layout. BLOCPLAN, presented by Donaghey and Pire (1990), can be used to construct or improve a layout using a continuous representation. BLOCPLAN first divides the layout into two or three bands and then sequentially assigns a rectangular department to a band. This algorithm improves a layout by considering all the possible two-way exchanges of departments. BLOCPLAN accepts from-to flows and relationship matrices and computes the objective based on distance or adjacency. Tam (1992) proposes Layout Optimization with Guillotine Induced Cuts (LOGIC) algorithm. This algorithm accepts from-to flows as the input to construct or improve a layout with rectangular departments by a continuous representation using a (center-to-center) distance-based objective function. LOGIC first divides the initial space into smaller sections using vertical and horizontal cuts named "guillotine cuts" and forms a slicing tree in the construction procedure. Then, it assigns departments to the sides of the cuts. LOGIC can also improve a layout by exchanging two departments while maintaining the slicing tree intact. Multifloor Plant Layout Evaluation (MULTIPLE), developed by Bozer et al. (1994), is similar to CRAFT. The main difference between MULTIPLE and CRAFT is the exchange procedure by which MULTIPLE can exchange any two departments. In addition, MULTIPLE extends the CRAFT algorithm to accommodate multi-floor FLPs, but it is still possible to use MULTIPLE for single-floor FLPs by setting the number of floors equal to 1. Table 10 presents an overview of these algorithms.

Table 10: Well-known algorithms in practice

Algorithm	Reference	Type	Input(s)	Objective
CRAFT	Buffa et al. (1964)	Improvement	- From-to flows - Initial layout - From-to costs	Minimizing material handling costs
ALDEP	Seehof and Evans (1967)	Construction	- Relationship matrix	Maximizing closeness score
CORELAP	Lee and Moore (1967)	Construction	- Relationship matrix	Maximizing closeness score
COFAD	Tompkins and Reed (1976)	Improvement	-From-to flows - Initial layout - Alternative material handling equipment with associated costs	Minimizing material handling costs (by choosing the layout and appropriate material handling equipment for each pairwise movement)
MCRAFT	Hosni et al. (1980)	Improvement	- From-to flows - Initial layout - From-to costs	Minimizing material handling costs
BLOCPLAN	Donaghey and Pire (1990)	Improvement and construction	- From-to flows (or relationship matrix)	Minimizing material handling costs (or maximizing closeness score)
LOGIC	Tam (1992)	Improvement and construction	- From-to flows	Minimizing material handling costs
MULTIPLE	Bozer et al. (1994)	Improvement	- From-to flows - Initial layout - From-to costs	Minimizing material handling costs

Appendix B Calculating VS in a Small Example

In this appendix, the VS is calculated for a small FLP with four departments using the procedure presented in Algorithm 2. Assume Departments 1, 2, and 3 each have one IO point in their center and Department 4 has two IO points that must be located on its periphery. Department 4 is twice as big as the other departments. Table 11 introduces the data for this problem instance. The frequency of each sequence in Table 11b represents the number of times that a sequence is repeated over the planning period. The pairwise flow matrix, based on the Table 11b is shown in Table 12.

Table 11: Input data

(a) Input data: dimensions of the departments

Department	Width	Height	Number of IO points
Department 1	2	2	1
Department 2	1	2	1
Department 3	2	2	1
Department 4	5	5	2

(b) Input data: sequences

No.	Sequence	Frequency
1	[1, 4]	57
2	[4, 3]	55
3	[2, 4, 1]	53
4	[4, 3, 1]	34
5	[1, 4, 2, 3]	58
6	[4, 3, 1, 3, 1]	31
7	[4, 2, 4, 2, 4]	51

Table 12: Input data: pairwise flow matrix

From \ To	Department 1	Department 2	Department 3	Department 4
Department 1	0	0	31	115
Department 2	0	0	58	155
Department 3	96	0	0	0
Department 4	53	160	120	0

Step 1: Optimal Layout Based on Sequences

We use full enumeration to obtain the optimal location of all departments and the optimal location of Department 4's IO points based on the sequences, and use a discrete representation and fixed aspect ratios to examine all possible layouts. All feasible layouts are generated and the layout with minimum objective function is determined. The objective function is the sum of travel distances of the travel sequences. Table 14 presents the travel distance of each sequence based on the layout Figure 9a. The total travel distance, Z_1^*, equals 1128.5.

Step 2: Optimal Pairwise Layout

We now change the objective function of this example to the total pairwise travel distance to obtain layout based on the pairwise movements. This is done again by generating all feasible layouts and choosing the layout with the minimum total pairwise travel distance. This objective function is calculated by multiplying the pairwise flow and the pairwise distance between each pair of departments. Figure 9b shows the results. Table 13 presents the pairwise distances resulting from Figure 9b. The optimal total travel distance based on the pairwise method Z_3^* equals 964.5 (see Table 14).

Table 14: Travel distance of each sequence in best layouts, all travel distances measured in metres

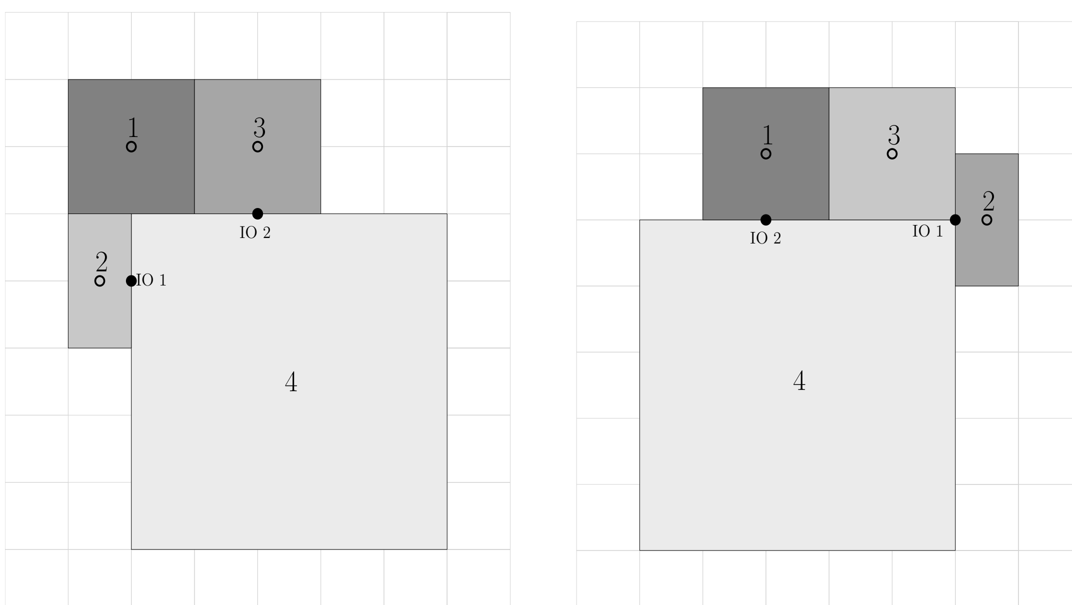
No.	Sequence	Frequency	Sequences TD in L_1^* (TD $\times$ frequency)	Sequences TD in L_2^* (TD $\times$ frequency)
1	[1, 4]	57	2 (114)	1 (57)
2	[4, 3]	55	1 (55)	2 (110)
3	[2, 4, 1]	53	2.5 (132.5)	4.5 (238.5)
4	[4, 3, 1]	34	3 (102)	4 (136)
5	[1, 4, 2, 3]	58	7 (406)	7 (406)
6	[4, 3, 1, 3, 1]	31	7 (217)	8 (248)
7	[4, 2, 4, 2, 4]	51	2 (102)	2 (102)
Total travel distances		$Z_3^* = 964.5$	$Z_1^* = 1128.5$	$Z_2^* = 1297.5$

Table 13: Pairwise distance matrix

From \ To	Department 1	Department 2	Department 3	Department 4
Department 1	0	4.5	2	1
Department 2	4.5	0	2.5	2.5
Department 3	2	2.5	0	2
Department 4	1	0.5	2	0

Step 3: Real Travel Distance in Optimal Pairwise Layout

The calculated objective function (Z_3^*) may underestimate the real travel distance. To calculate the actual travel distance of the layout L_2^* (Figure 9b), we must calculate each sequence's travel distance. Table 14 shows the travel distance of each sequence. This results in $Z_2^* = 1297.5$m.



(a) Sequence method, L_1^*

(b) Pairwise method, L_2^*

Figure 9: Optimal layouts

Step 4: Calculate the VS

In this example, the total travel distance by using the traditional pairwise method is $Z_2^* = 1297.5$. When using the sequential method, layout L_1^* is achieved with the total travel distance $Z_1^* = 1128.5$. Therefore, the proposed method improves the layout and the resulting total travel distance by 169 units or 14.97% (VS $= \frac{Z_2^* - Z_1^*}{Z_1^*} * 100$). By employing the MBRKGA algorithm instead of full enumeration, we find $Z_3^* = 997.17$, $Z_2^* = 1273.98$, $Z_1^* = 1206.38$, and VS $= 5.6\%$. To ensure that the results are comparable, the same fixed aspect ratios are used in the MBRKGA, but the MBRKGA uses a continuous IO point representation.

Appendix C BRKGA and Placement Strategy

Biased Random-Key Genetic Algorithm Bean (1994) proposed the random-key genetic algorithms (RKGA), where each chromosome of the population is constructed by randomly generated numbers. Thereafter, an algorithm is used to connect each chromosome to a solution of an combinatorial optimization problem. The main difference between RKGA and BRKGA is that both parents are selected randomly from the population in the former whereas in the latter, one parent should be selected from the elite members of the population and the other parent can be selected from the rest of the population (Gonçalves and Resende 2011). The BRKGA and the placement strategy proposed by Gonçalves and Resende (2015) outperform many of the existing heuristics for the FLP in several benchmark problems. Therefore, we employ the proposed placement strategy proposed by Gonçalves and Resende (2015) along with BRKGA for finding the location of each department in this study.

The steps of the BRKGA in this study are as follows:

1. First, p (size of the population) individual chromosomes of the first generation is randomly constructed. Each member of the population is constructed from $2N + 2J + 2$ random keys (genes) where N is the number of departments and J is the total number of IO points.

2. After decoding each chromosome, the associated layout and its objective value can be realized by using the placement strategy.

3. Evolve process: elite members of the current population (p_e members that have the best objective value) are chosen for the next generation. Also, p_m mutants are introduced to the population for the next generation. Finally, to complete the population of the next generation, $p - p_e - p_m$ offsprings are produced using crossover.

4. Repeat step 3 to reach g generations.

Chromosome representation and decoding: In this study, we assume that each department can have more than one IO point. These IO points can be located at each side of the rectangular departments. Each chromosome is obtained from $2N+2J+2$ random keys as represented by Figure 10.

$$Chromosome = (\underbrace{gene_1, ..., gene_N}_{\text{Facility placement sequence}}, \underbrace{gene_{N+1}, ..., gene_{2N}}_{\text{Length of each facility}}, \underbrace{gene_{2N+1}, ..., gene_{2N+2J}}_{\text{IO points locations}}, \underbrace{gene_{2N+2J+1}, gene_{2N+2J+2}}_{\text{Center coordinates of the first department}})$$

Figure 10: Chromosome representation

$gene_1$ to $gene_N$ are randomly generated numbers between 0 and 1000. The order of placing departments is based on their associated numbers from low to high. $gene_{N+1}$ to $gene_{2N}$ are the horizontal size (W_n) of each department. $gene_{N+n}$ is a random number between $\sqrt{a_n \times e_n}$ and $\sqrt{a_n \times u_n}$. Therefore, the vertical size (H_n) of department n equals to: $\frac{a_n}{gene_{N+n}}$.
$gene_{2N+1}$ to $gene_{2N+2J}$ describe the location of IO points corresponding to the departments. As mentioned earlier, IO points can be located at any point on the periphery of a department. Each IO point location is determined using two genes (location on the x-axis and y-axis) that describe the location of IO points with respect to the length and width of the department. Assume (C_n^x, C_n^y) and (W_n, H_n) are the coordinates, and length and width of the department n, respectively. Also, assume $gene_{2N+j}$ and $gene_{2N+j+1}$ are the related genes of the IO point location of that department. Therefore, these genes can be decoded as:
$(I_j^x, I_j^y) = (C_n^x + gene_{2N+j} * W_n, C_n^y + gene_{2N+j+1} * H_n)$

Crossover Process:

For evolution from generation g to $g+1$, $p - p_e - p_m$ individuals should be produced using crossover. First, one parent from the elite members of generation g, and one parent from the rest of the population in generation g are chosen. Then, each characteristic of the offspring chromosome is inherited by probability of ρ_e from the elite parent or probability of $1 - \rho_e$ from the other parent. In the BRKGA, ρ_e is greater than 0.5.

Input Parameters:

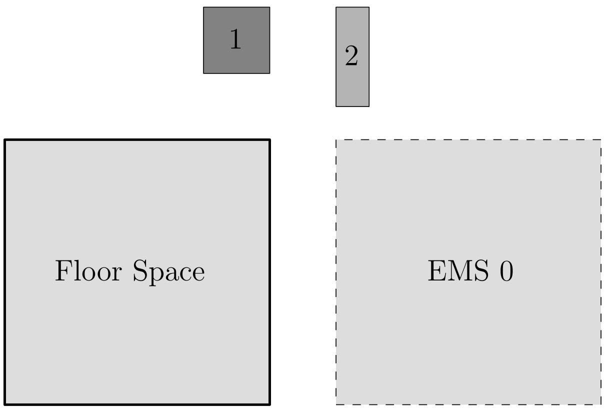
Table 15 presents the input parameters of the BRKGA for solving the problem instances of this study. The reported results in Sections 7 and 6 are based on the best found solutions of 20 runs of the algorithm on each generated problem.

Table 15: BRKGA parameters

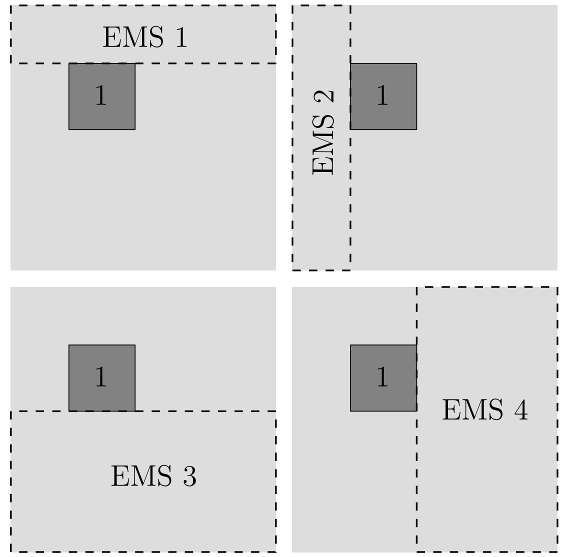
Parameter	Description	Quantity
p	Population size	$300 * N$
p	Number of generations	200
p_e	Number of elite members in each population	$min(0.25 * p, 50)$
p_m	Number of mutants	$0.25 * p$
ρ_e	Probability of inheriting a characteristic from the elite member	0.7

Placement Strategy:

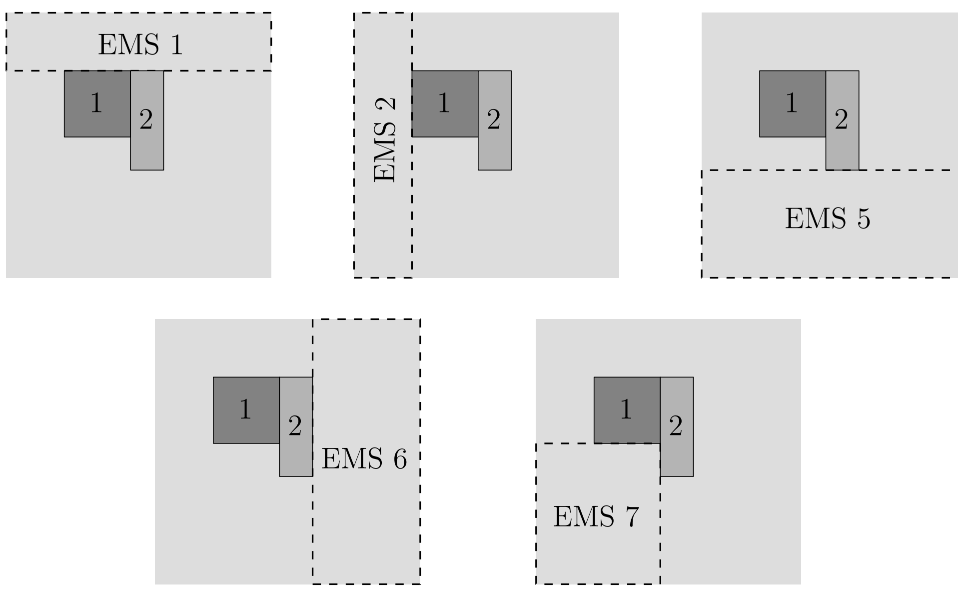
We use the placement strategy proposed by Gonçalves and Resende (2015) in this research in which, departments are placed in the layout based on empty maximal spaces (EMSs). An EMS is a largest possible rectangular free space in the layout, and the list of these rectangular spaces are stored and updated in a list during the placement of facilities. Figure 11 shows these EMSs in a problem when placing the first two facilities.



(a) Floor space, initial EMS, and departments



(b) EMSs after placing the first department



(c) EMSs after placing departments 1 and 2

Figure 11: An example of updating EMSs.

First, we determine the unconstrained optimal location for placing a department (for more details about unconstrained optimal location please refer to Heragu (1997)). As suggested by Gonçalves and Resende (2015), the possible locations of the departments are determined on each

EMS of the list as close as possible to this location. Then, based on the associated costs of these possible locations, the best location with the lowest cost is chosen for placing the current department. If the current department has no flow with already placed departments, the geometric center of all located departments can be considered as its unconstrained optimal location. After locating the current department, the EMS list should be updated. More details on finding and updating EMSs can be found in Lai and Chan (1997).

To ensure that the proposed layout of the case study is practical and avoids highly congested areas, a U-shape main aisle is added to building 1 of the company. The BRKGA starts with five EMSs instead of one by considering this aisle and two buildings. Figure 12, shows the initial EMSs and location of the aisle.

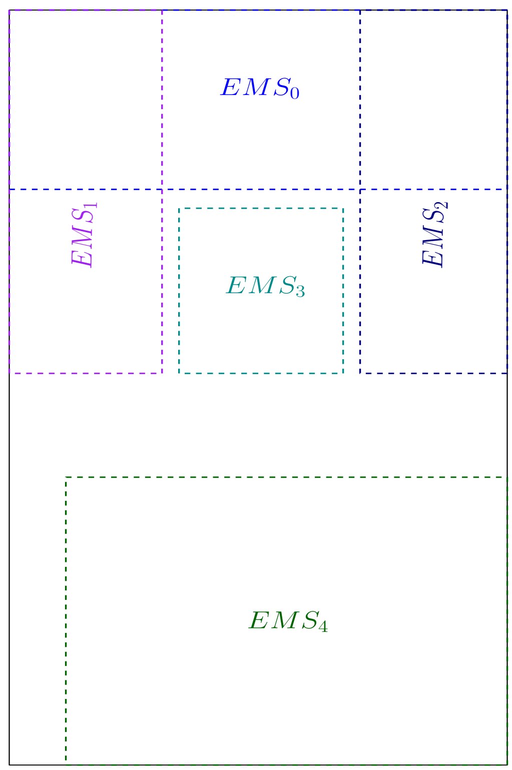
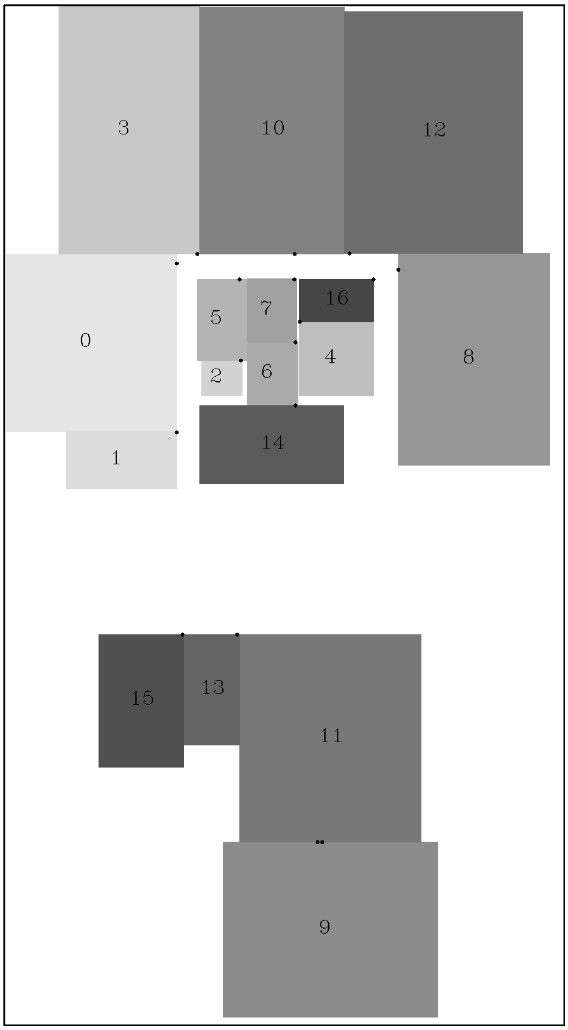


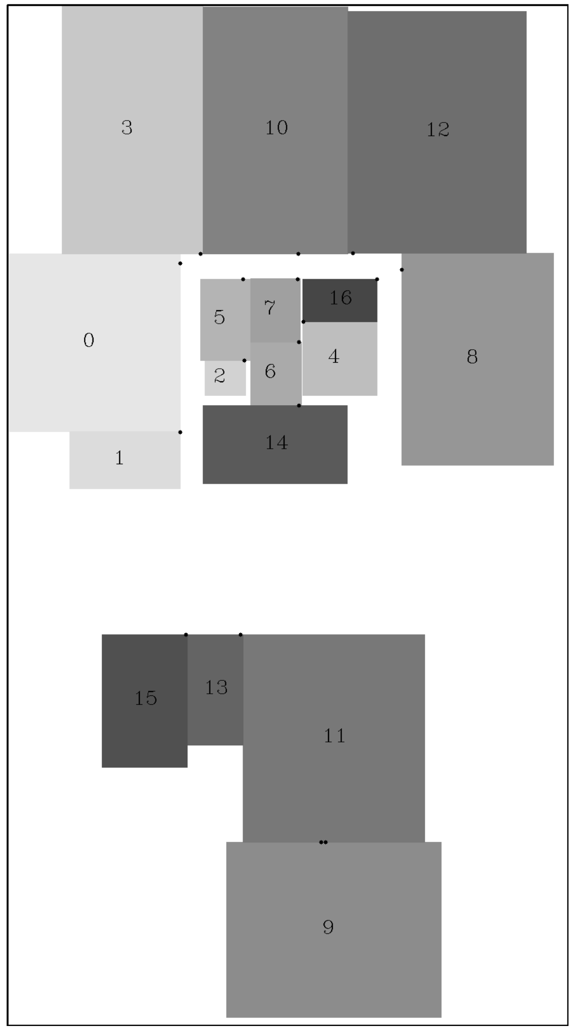
Figure 12: Floor design

Appendix D Input Parameters and Final Layouts

Problem Instances 1 to 8:

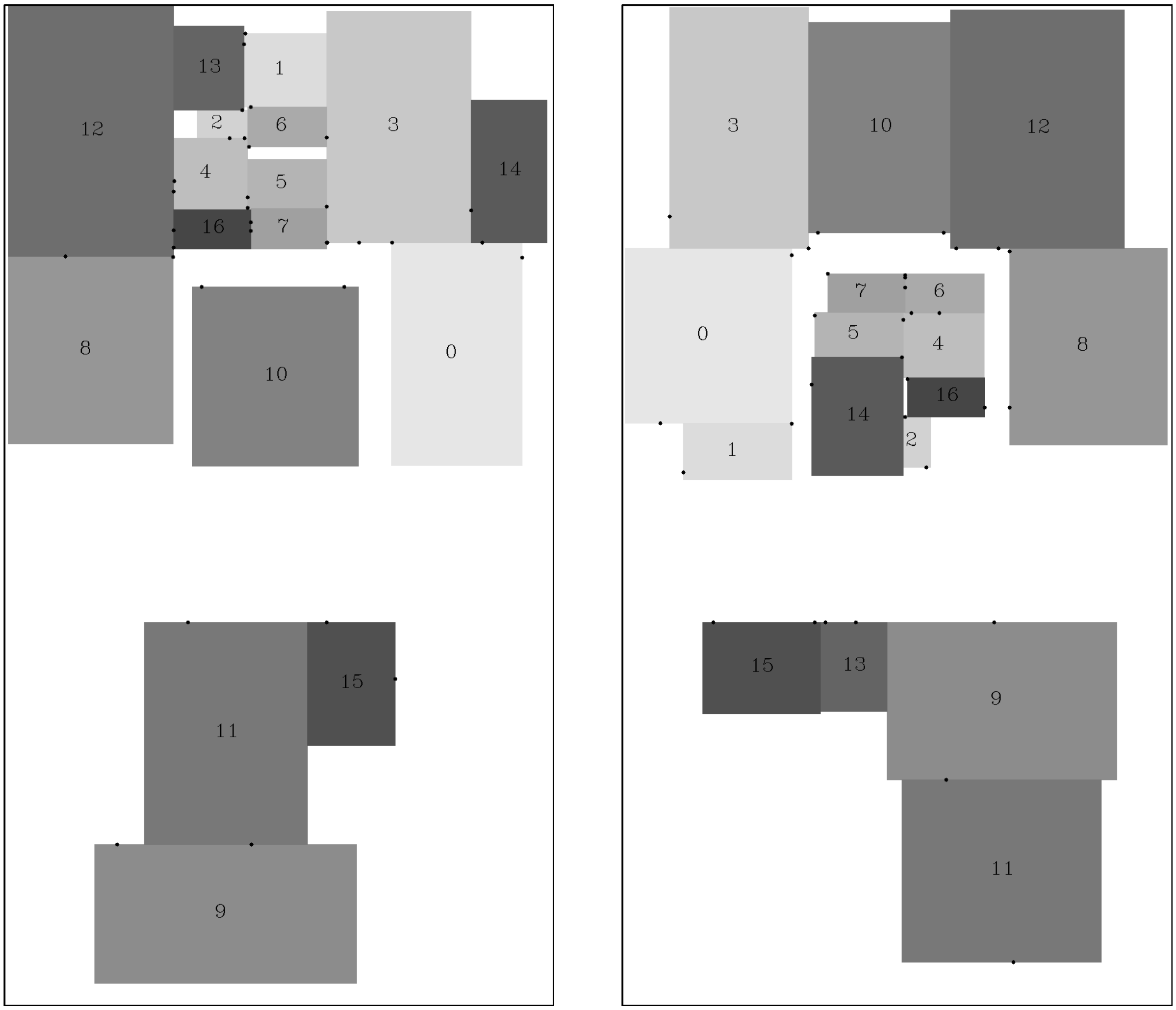


(a) Layout based on pairwise movements



(b) Layout based on sequences

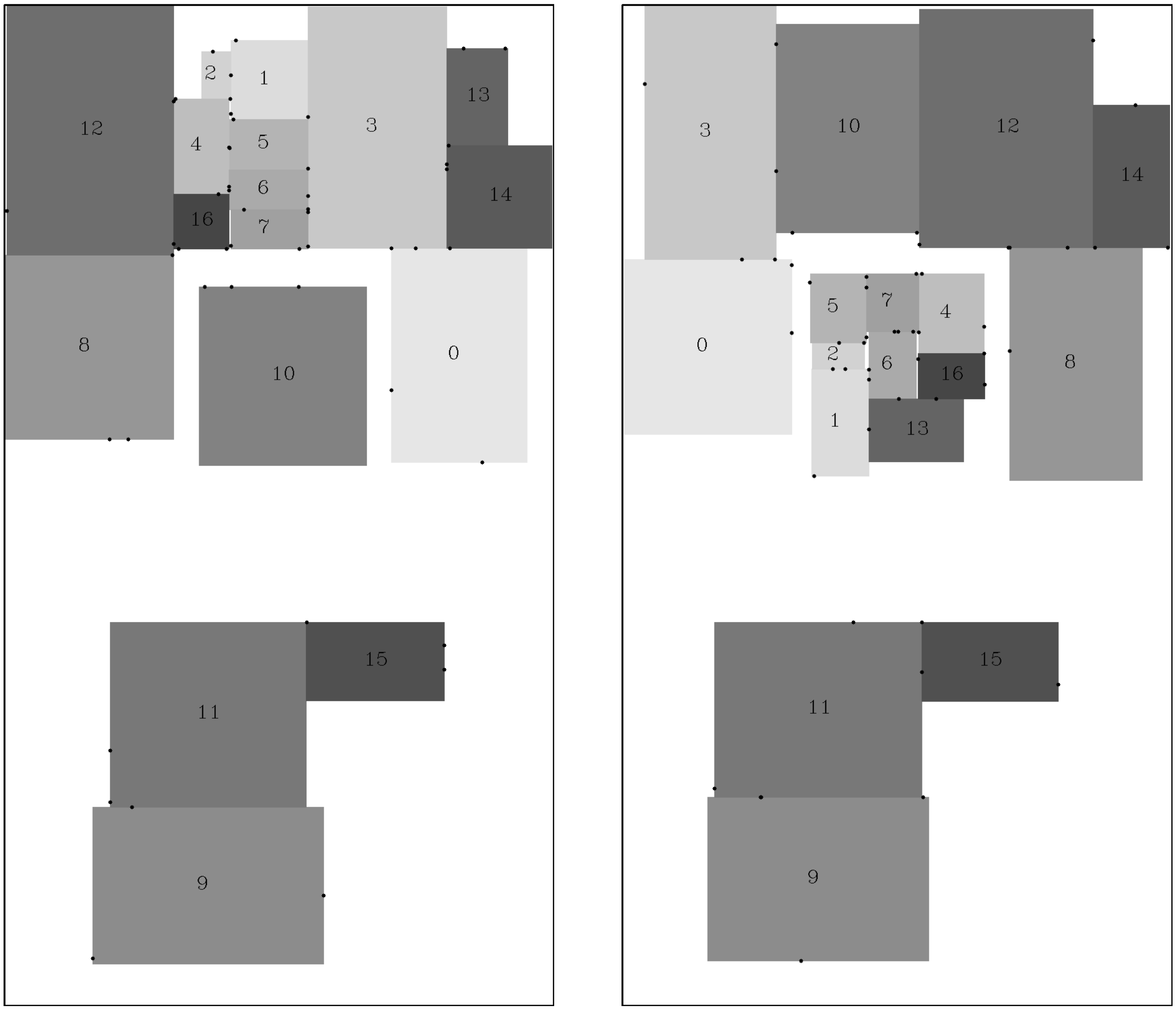
Figure 13: Layouts of problem instance 1



(a) Layout based on pairwise movements

(b) Layout based on sequences

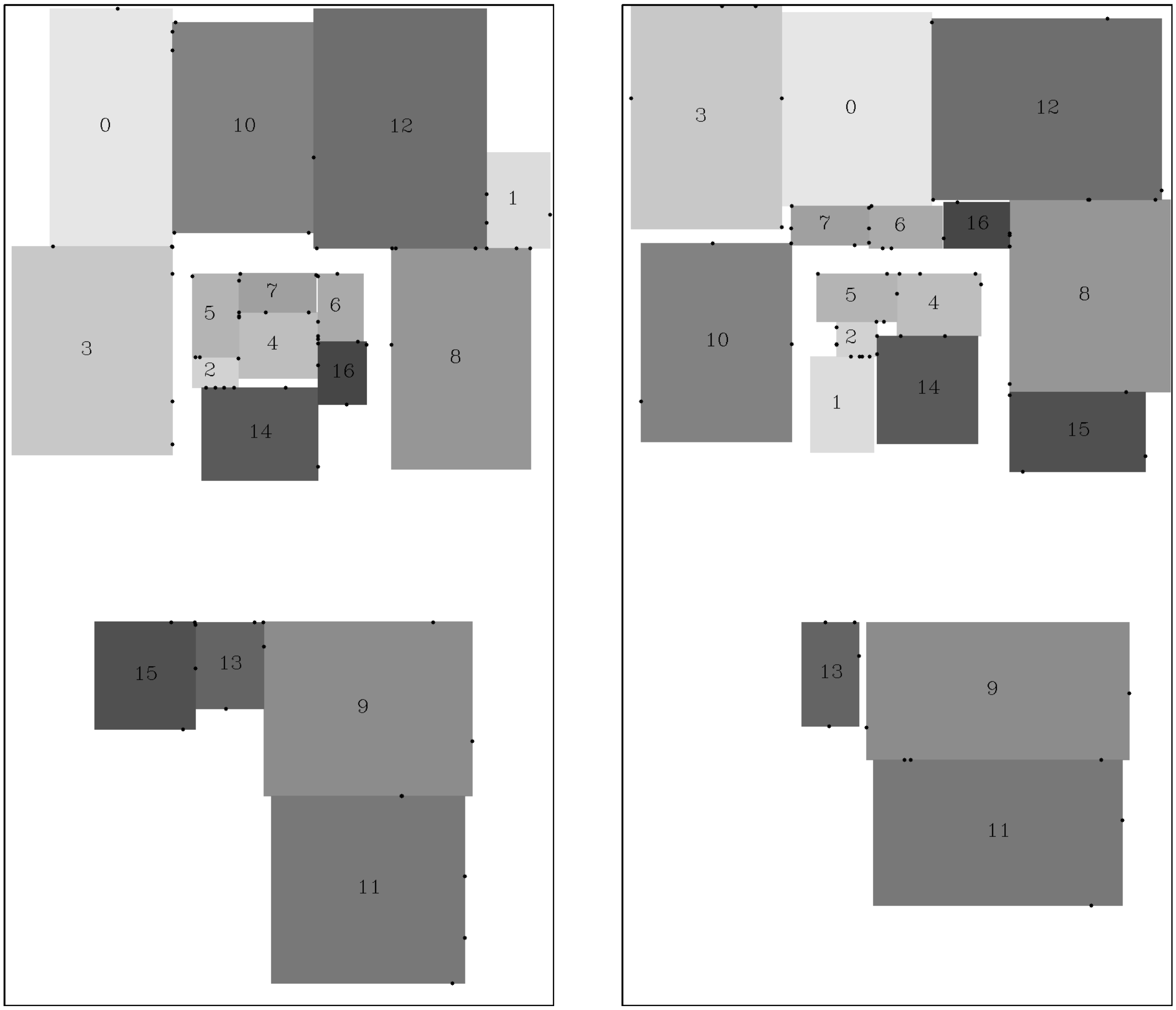
Figure 14: Layouts of problem instance 2



(a) Layout based on pairwise movements

(b) Layout based on sequences

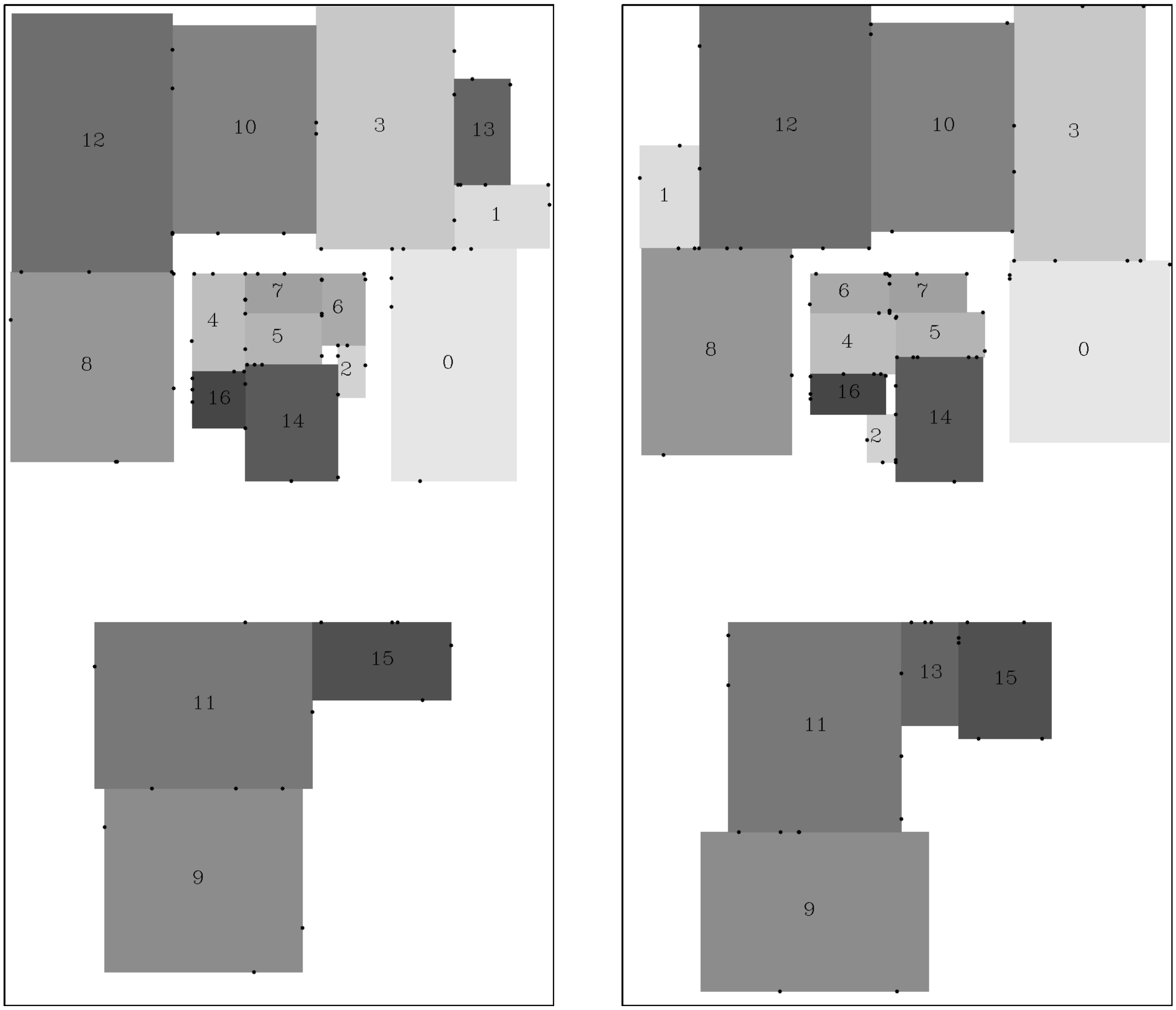
Figure 15: Layouts of problem instance 3



(a) Layout based on pairwise movements

(b) Layout based on sequences

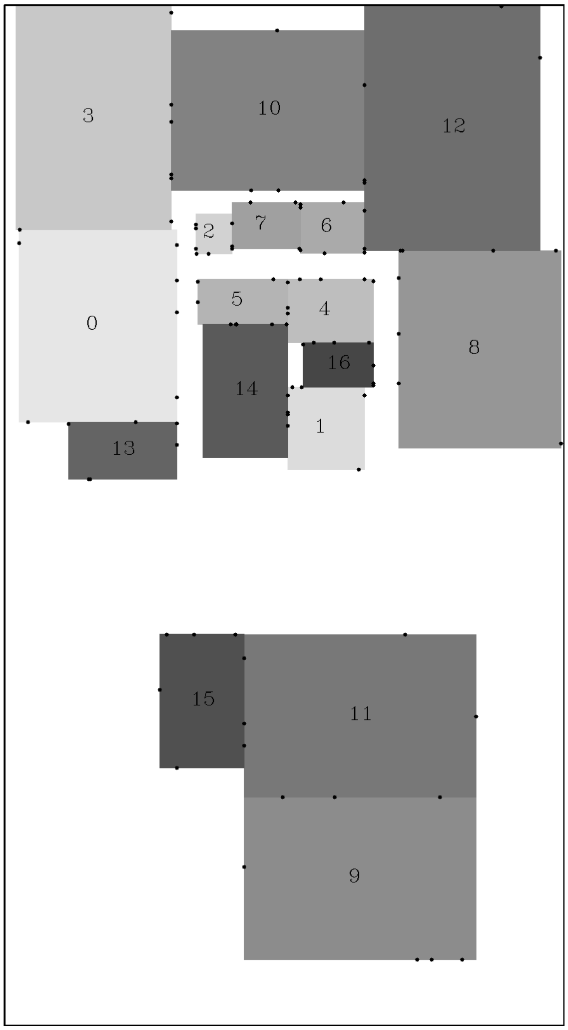
Figure 16: Layouts of problem instance 4



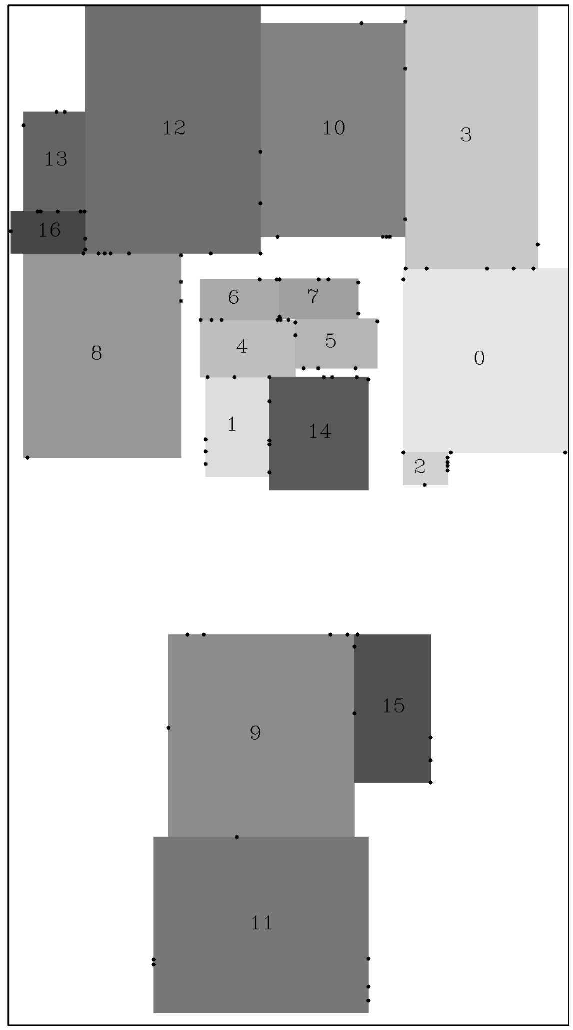
(a) Layout based on pairwise movements

(b) Layout based on sequences

Figure 17: Layouts of problem instance 5

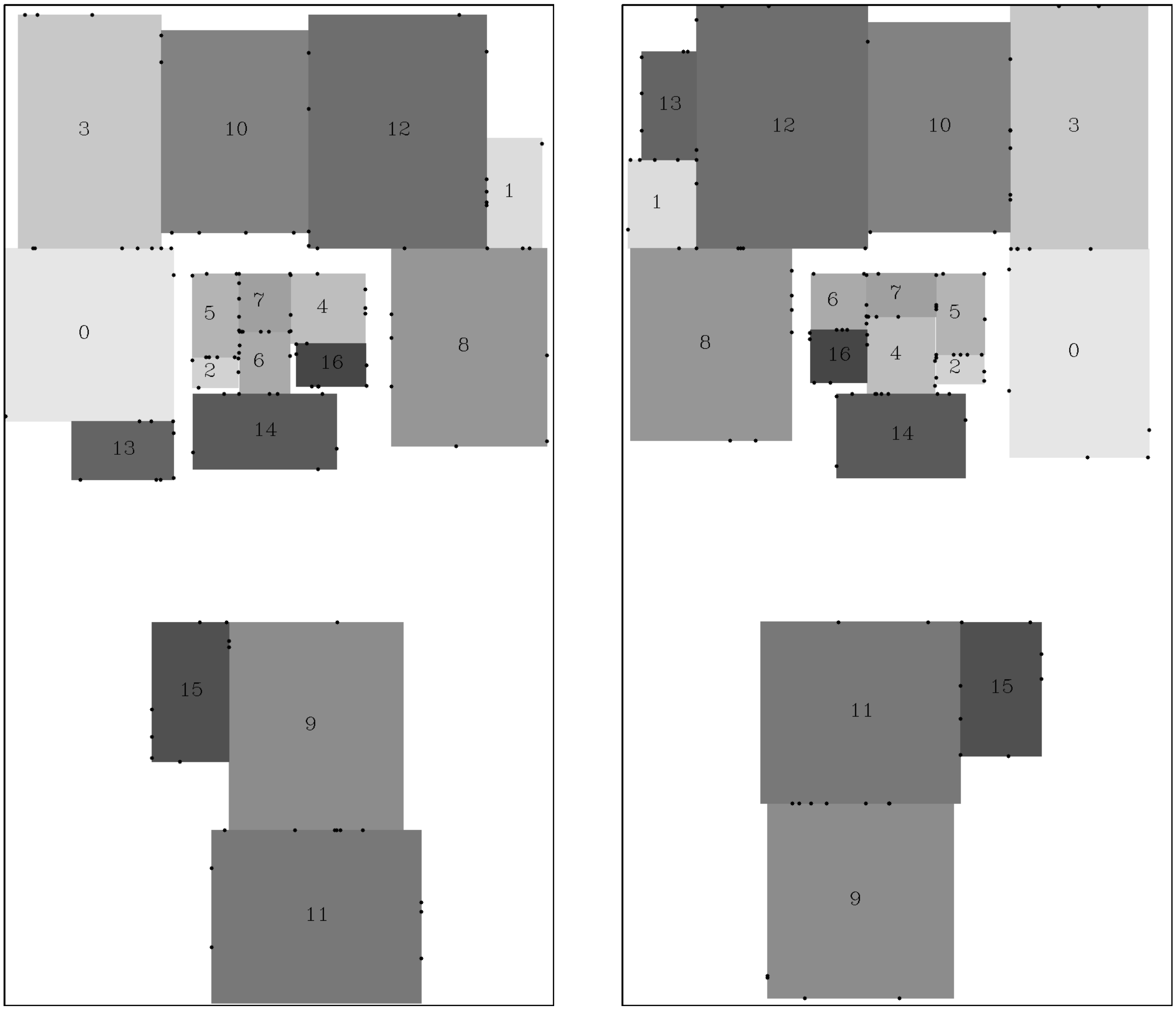


(a) Layout based on pairwise movements



(b) Layout based on sequences

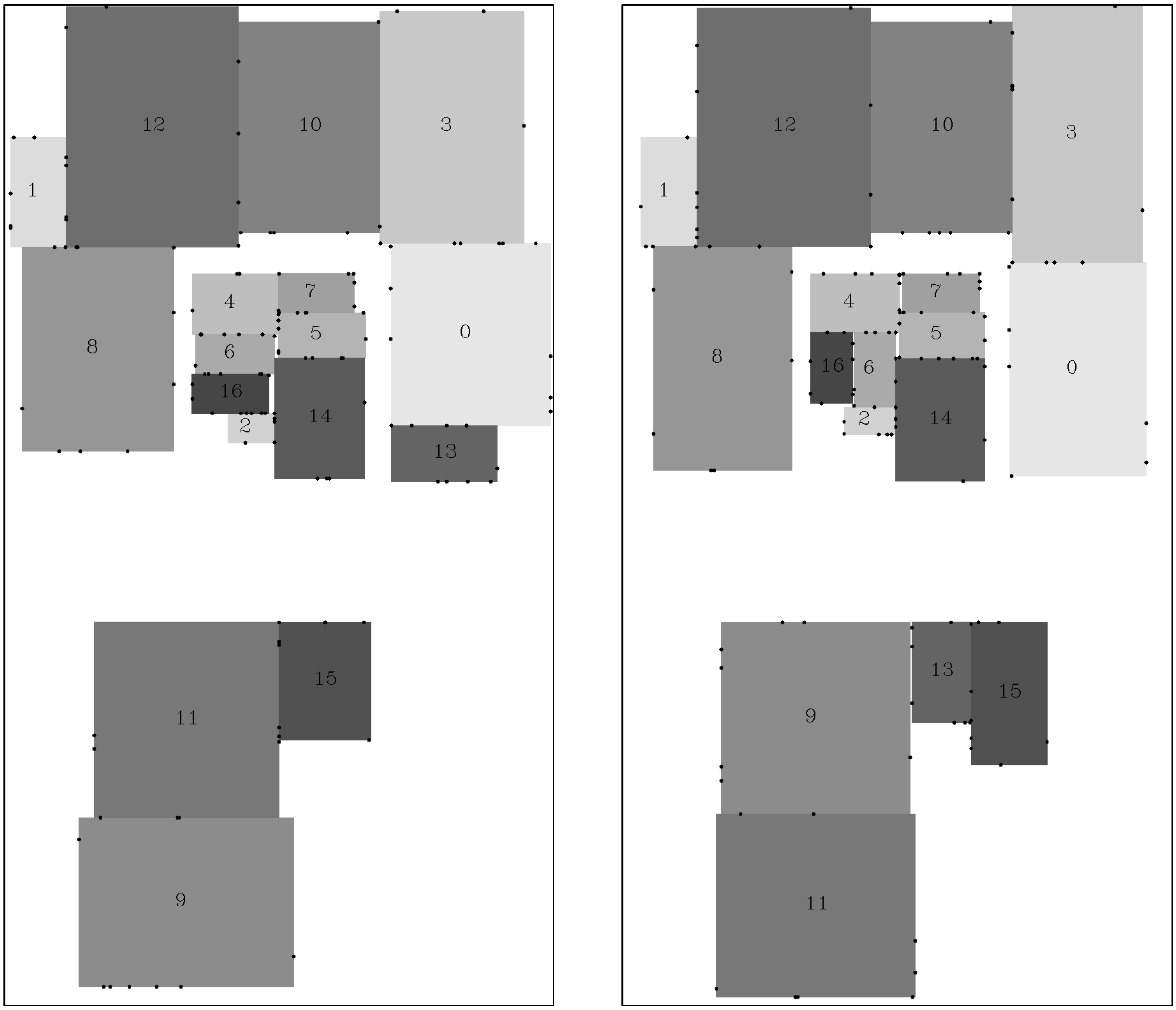
Figure 18: Layouts of problem instance 6



(a) Layout based on pairwise movements

(b) Layout based on sequences

Figure 19: Layouts of problem instance 7



(a) Layout based on pairwise movements

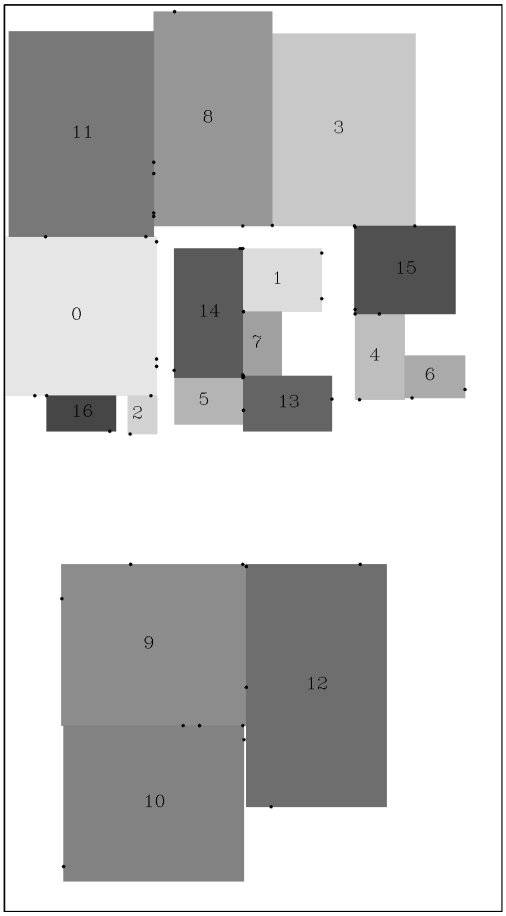
(b) Layout based on sequences

Figure 20: Layouts of problem instance 8

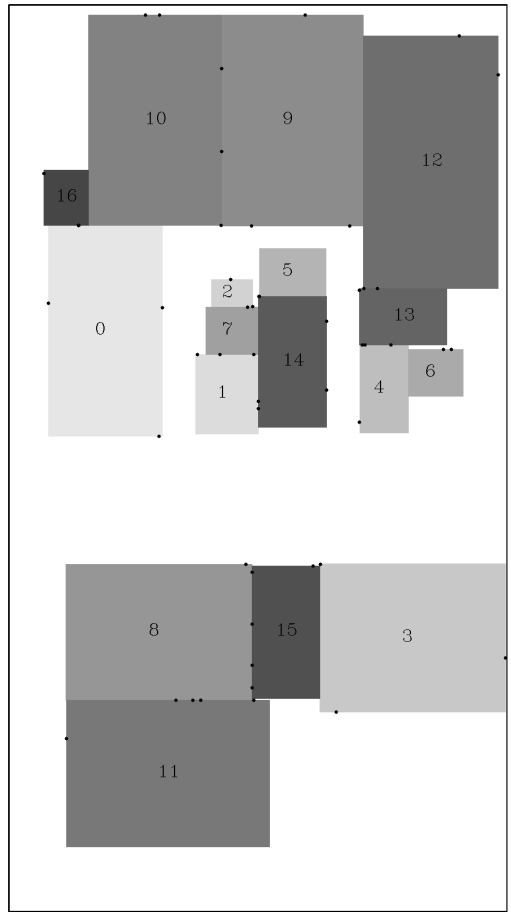
Problem Instances 9 to 12:

Table 16: Input sequences of problem instances 9 to 12

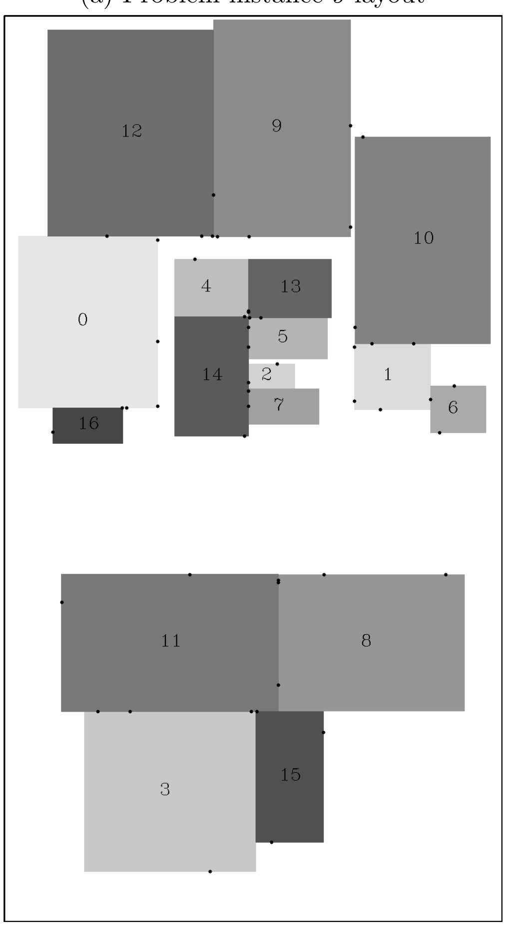
Problem instance	Input sequences										Frequency
	[10, 5]	[5, 14]	[14, 3]	[3, 15]	[15, 8]	[8, 0]	[0, 14]	[14, 7]	[7, 1]	[1, 8]	24
	[14, 9]	[9, 11]	[11, 0]	[0, 4]	[4, 15]	[15, 5]	[5, 9]	[9, 16]	[16, 0]	[0, 16]	14
9	[4, 13]	[13, 7]	[7, 2]	[2, 9]	[9, 5]	[5, 14]	[14, 13]	[13, 9]	[9, 10]	[10, 7]	18
	[4, 14]	[14, 2]	[2, 11]	[11, 8]	[8, 14]	[14, 1]	[1, 4]	[4, 13]	[13, 12]	[12, 9]	14
	[13, 5]	[5, 7]	[7, 0]	[0, 3]	[3, 11]	[11, 8]	[8, 5]	[5, 0]	[0, 12]	[12, 4]	19
	[10, 5, 14]		[14, 3, 15]		[15, 8, 0]		[0, 14, 7]		[7, 1, 8]		24
	[14, 9, 11]		[11, 0, 4]		[4, 15, 5]		[5, 9, 16]		[16, 0, 16]		14
10	[4, 13, 7]		[7, 2, 9]		[9, 5, 14]		[14, 13, 9]		[9, 10, 7]		18
	[4, 14, 2]		[2, 11, 8]		[8, 14, 1]		[1, 4, 13]		[13, 12, 9]		14
	[13, 5, 7]		[7, 0, 3]		[3, 11, 8]		[8, 5, 0]		[0, 12, 4]		19
	[10, 5, 14, 3, 15, 8]					[8, 0, 14, 7, 1, 8]					24
	[14, 9, 11, 0, 4, 15]					[15, 5, 9, 16, 0, 16]					14
11	[4, 13, 7, 2, 9, 5]					[5, 14, 13, 9, 10, 7]					18
	[4, 14, 2, 11, 8, 14]					[14, 1, 4, 13, 12, 9]					14
	[13, 5, 7, 0, 3, 11]					[11, 8, 5, 0, 12, 4]					19
	[10, 5, 14, 3, 15, 8, 0, 14, 7, 1, 8]										24
	[14, 9, 11, 0, 4, 15, 5, 9, 16, 0, 16]										14
12	[4, 13, 7, 2, 9, 5, 14, 13, 9, 10, 7]										18
	[4, 14, 2, 11, 8, 14, 1, 4, 13, 12, 9]										14
	[13, 5, 7, 0, 3, 11, 8, 5, 0, 12, 4]										19



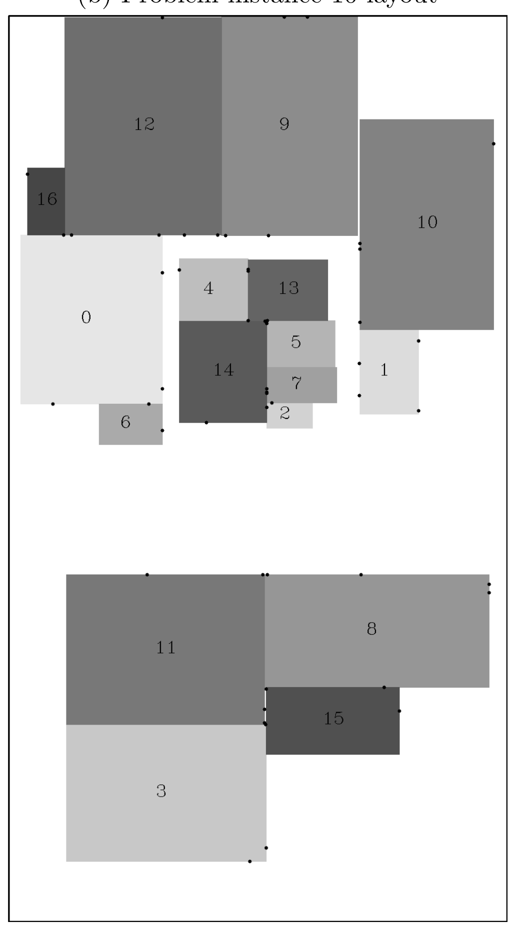
(a) Problem instance 9 layout



(b) Problem instance 10 layout



(c) Problem instance 11 layout



(d) Problem instance 12 layout